Statistics I: Exercise Session 5

1.7.2015, 14-16, SPA1 21b

1 Probability Theory

1.1 Addition Theorems

General Addition Theorem

$$\begin{aligned}
\mathrm{P}(A \cup B) &= \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B) \\
\mathrm{P}(A \cup B \cup C) &= \mathrm{P}(A) + \mathrm{P}(B) + \mathrm{P}(C) - \mathrm{P}(A \cap B) - \mathrm{P}(A \cap C) - \mathrm{P}(B \cap C) \\
&\quad + \mathrm{P}(A \cap B \cap C)
\end{aligned}$$

Addition Theorem for Disjoint Events ($A_i \cap A_j = \emptyset$ for all $i \neq j$)

$$\begin{aligned}
\mathrm{P}(A_1 \cup A_2 \cup \ldots \cup A_n) &= \mathrm{P}(A_1) + \mathrm{P}(A_2) + \ldots + \mathrm{P}(A_n) \\
&= \sum_{i=1}^{n} \mathrm{P}(A_i)
\end{aligned}$$

1.2 Conditional Probability

$$\mathrm{P}(A|B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}, \quad \mathrm{P}(B) > 0 \text{ and } \mathrm{P}(B|A) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(A)}, \quad \mathrm{P}(A) > 0$$

1.3 Independent Events

$$\mathrm{P}(A|B) = \mathrm{P}(A|\overline{B}) = \mathrm{P}(A) \text{ and } \mathrm{P}(B|A) = \mathrm{P}(B|\overline{A}) = \mathrm{P}(B)$$

1.4 Multiplication Rules

General Multiplication Rule

$$\begin{aligned}
\mathrm{P}(A \cap B) &= \mathrm{P}(A) \cdot \mathrm{P}(B|A) = \mathrm{P}(B) \cdot \mathrm{P}(A|B) \\
\mathrm{P}(A \cap B \cap C) &= \mathrm{P}(A) \cdot \mathrm{P}(B|A) \cdot \mathrm{P}(C|A \cap B)
\end{aligned}$$

Multiplication Rule for Independent Events

$$\mathrm{P}(A \cap B) = \mathrm{P}(A) \cdot \mathrm{P}(B)$$

1.5 Total Probability

$$\begin{aligned}
\mathrm{P}(B) &= \mathrm{P}(A_1 \cap B) + \mathrm{P}(A_2 \cap B) + \ldots + \mathrm{P}(A_n \cap B) \\
&= \sum_{i=1}^{n} \mathrm{P}(A_i \cap B) \\
&= \mathrm{P}(B|A_1) \cdot \mathrm{P}(A_1) + \mathrm{P}(B|A_2) \cdot \mathrm{P}(A_2) + \ldots + \mathrm{P}(B|A_n) \cdot \mathrm{P}(A_n) \\
&= \sum_{i=1}^{n} \mathrm{P}(B|A_i) \cdot \mathrm{P}(A_i)
\end{aligned}$$

1.6 Bayes' Theorem

$$\mathrm{P}(A_j|B) = \frac{\mathrm{P}(B|A_j) \cdot \mathrm{P}(A_j)}{\sum\limits_{i=1}^{n} \mathrm{P}(B|A_i) \cdot \mathrm{P}(A_i)} \quad \forall j = 1, \ldots, n$$

Exercises

3-25: *Old building*

In an old building with an outdated electricity system it often happens that the power system fails or the water supply freezes. Although both situations occur independently of each other, they depend on the season. So, naturally, water freezes only in winter time with the probability of 80 %. The power system fails with 40% probability even if it is not winter. It is the same as the probability that the power system does not fail in winter. Assume that winter time lasts for 30 % of the whole year.

a) Formalize the events mentioned in the text. What are their probabilities?

What is the probability that

b) the water supply freezes?

c) the power system fails?

d) the water supply freezes and power system fails at the same time?

e) the water supply freezes when power system has already failed?

f) the power system fails when the water supply has already frozen?

g) at least one of the undesirable situations happens?

h) at most one of the undesirable situations happens?

3-37: *Cheating*

Prof. Antischumm dislikes cheating during exams. Therefore he invented the Cheating-diagnostics-machine which provides following information: 90 % of students that cheat are recognized as cheating and 90 % of students that do not cheat are recognized as being honest. From his experience Prof. Antischumm knows that 10 % of all students cheat.

a) Define the events and their probabilities according to the information given in the text.

b) What is the probability that the machine provides a suspicion of cheating?

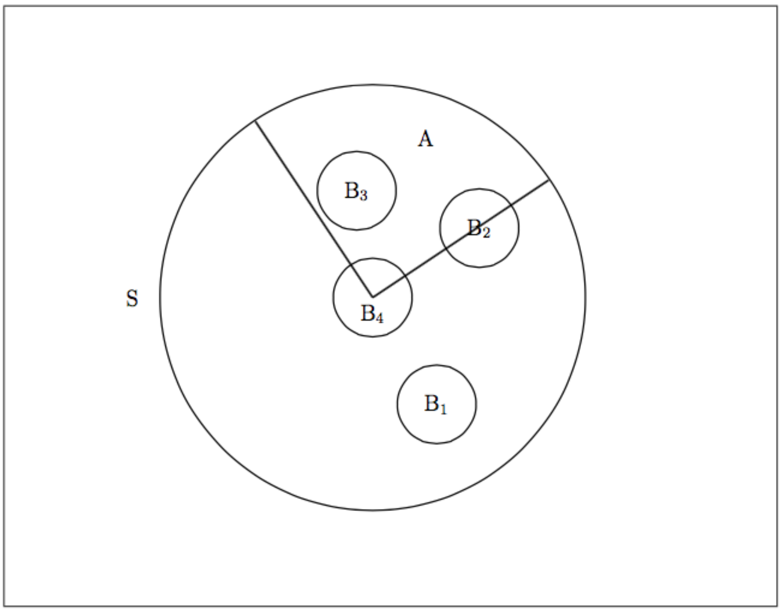
c) What is the probability that a student really cheated when the machine provided a corresponding suspicion?

3-55: *Weekend house*

The Sonne family has a weekend house on Rügen. They can come to the island of Rügen through Rügendamm or by a ferry. The family decides how to get there by flipping a (fair) coin: head is for Rügendamm, tail is for the ferry. If it is not rainy, the Sonnes always go to Rügen for a weekend. The probability of a traffic jam in Rügendamm is 25 %, for the ferry 10 %. When it rains, the family stays at home.
What is the probability that the Sonne family will not be stuck in a traffic jam, when the probability of rain is 0.20 for this time of the year?

3-61: *Independent events*

Probabilities of the events from the illustration below are proportional to their surface areas.



Which from the following pairs consist of independent events?

a) B_1, B_2	b) B_1, B_3	c) B_1, B_4	d) B_2, B_3	e) B_2, B_4
f) B_3, B_4	g) A, B_1	h) A, B_2	i) A, B_3	j) A, B_4