Statistics II: Exercise Session 2

1 Probability Distributions

1.1 Discrete Probability Distributions

1.1.1 Bernoulli distribution

$$X \sim B(p) \qquad \mathrm{E}[X] = p \qquad \mathrm{Var}(X) = p \cdot (1-p)$$

$$f_B(x;p) = \begin{cases} 1-p & \text{for } x = 0; \\ p & \text{for } x = 1; \\ 0 & \text{otherwise} \end{cases}$$

$$F_B(x;p) = \begin{cases} 0 & \text{for } x < 0; \\ 1-p & \text{for } 0 \leq x < 1; \\ 1 & \text{for } x \geq 1 \end{cases}$$

Distribution of a a random variable which takes the value 1 with success probability of p and the value 0 with failure probability of $1-p$.

1.1.2 Binomial Distribution

$$X \sim B(n;p) \qquad \mathrm{E}[X] = n \cdot p \qquad \mathrm{Var}(X) = n \cdot p \cdot (1-p)$$

$$f_B(x;n,p) = \begin{cases} \binom{n}{x} \cdot p^x \cdot (1-p)^{n-x} & \text{for } x = 0, 1, \ldots, n; \\ 0 & \text{otherwise} \end{cases}$$

$$F_B(x;n,p) = \begin{cases} \sum_{k=0}^{x} \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k} & \text{for } x \geq 0; \\ 0 & \text{for } x < 0 \end{cases}$$

Models number of successes in n draws, <u>*with*</u> replacement, p is probability of a success in each draw.

1.1.3 Poisson Distribution

$$X \sim PO(\lambda) \qquad \mathrm{E}[X] = \lambda \qquad \mathrm{Var}(X) = \lambda$$

$$f_{PO}(x;\lambda) = \frac{\lambda^x}{x!} \cdot e^{-\lambda} \quad \text{for } x = 0, 1, 2, \ldots; \lambda > 0;$$

$$F_{PO}(x;\lambda) = \begin{cases} \sum_{k=0}^{x} \frac{\lambda^k}{k!} \cdot e^{-\lambda} & \text{for } x \geq 0; \lambda > 0; \\ 0 & \text{for } x < 0 \end{cases}$$

Models number of events occurring in a fixed interval of time and/or space.

1.2 Continuous Probability Distributions

1.2.1 Exponential Distribution

$X \sim EX(\lambda)$ $\mathrm{E}[X] = \frac{1}{\lambda}$ $\mathrm{Var}(X) = \frac{1}{\lambda^2}$

$$f_{EX}(x;\lambda) = \begin{cases} \lambda \cdot e^{-\lambda x} & \text{for } x \geq 0, \lambda > 0; \\ 0 & \text{for } x < 0 \end{cases}$$

$$F_{EX}(x;\lambda) = \begin{cases} 1 - e^{-\lambda x} & \text{for } x \geq 0, \lambda > 0; \\ 0 & \text{for } x < 0 \end{cases}$$

Models the time between events in a process in which events occur continuously and independently at a constant average rate (intensity).

Exercises

5-21: *Archer*

Archer A. Mor wants to impress his friends with his skills. He knows that on average he hits a target 3 times out of 5 shots. To show his skills, he shoots 3 times at a disc.

a) What distribution do the r.v. X: "Hit in the i-th shot" and the r.v. Y: "Sum of hits in 8 shots" have? Explain your answer.

b) Write the expected value of the sum of hits in general and specifically for this example.

c) What is the probability that A. Mor hits the disc exactly 3 times?

5-23: *Restaurant*

In a restaurant, which only contact-seeking singles can enter, a guest can choose from a rich menu.

A: On an ordinary Sunday, when the restaurant is open from 19.00 to 24.00, usually 25 guests come (independently from each other). One can assume that in every time period during this hours the same number of guests is expected to arrive.

   a) What is the probability that exactly one person comes during the first hour?

   b) How many minutes pass on average on a Sunday evening between arrivals of two guests?

B: A guest can make an order repeatedly as many times as he wants. The host assumes that every guest (independently from the others) makes an additional order at most once. He also knows that with a probability of 70 % a guest does not order anything additionally.

   a) What is the distribution of the r.v. X: "Number of additional orders on a Sunday evening" when one assumes the usual number of guests?

   b) If the usual number of guests arrives, the capacity of the kitchen will be exceeded by additional orders with a probability of 0.05 %. Determine the number of additional orders which reaches the limit of the kitchen.