Statistics II: Exercise Session 8 (Review)

Exercises

5-26: *Dinner*

Student Fritzi goes shopping for a dinner with her friends. At a greengrocery she buys 1kg bag of apples and 1kg net of mandarins. Statistically skilled shop assistant Paul tells her that he always properly checks his 1kg packages. Over the years he has observed that the weight of the packages is normally distributed with the expected value of exactly 1kg. Interestingly, the apple bags have the variance of 400g^2 and the nets of mandarins have only the variance of 225g^2. What is the probability that by random selection of an apple bag and a mandarin net Fritzi comes home with more than 1950g of fruits?

7-9: *Accidents*

You are interested in the frequency of accidents on a transport hub. Both the police and the insurance companies have already studied this issue in the past, so you can rely on these reports:

Insurance company:	x	0	1	2	$x \neq 0,1,2$
	$P(X = x)$	0.7	0.1	0.2	0

Police:	x	0	1	2	$x \neq 0,1,2$
	$P(X = x)$	0.1	0.4	0.5	0

Unfortunately, both probability distributions differ significantly from each other. In order to be able to decide between the two of the distributions, you select 5 random days and count and note the number of accidents on the crossroads by yourself. Your sample has the following result: (0, 2, 0, 2, 1).

a) For which of the two distributions do you decide if you use the maximum likelihood method?

b) For which of the two distributions do you decide if you use the least squares method?

8-4: *Average weight*

A supermarket has been selling chickens with average weight of 1400g at a certain price. A dealer now makes an offer to deliver chickens of the same average weight for a lower unit price. Customers C_1 and C_2 of the supermarket both know that the chickens' weight is normally distributed and believe that the lower price is a result of a lowered average weight. C_1 then weights 25 randomly selected chickens. It turns out that the arithmetic mean deviates from the target weight by -9 grams and standard deviation is found to be 50g. The significance level of the test should be 5 %.

a) The customer constructs following hypotheses:

$$H_0 : \mu \geq \mu_0 (= 1400\text{g}) \quad \text{and} \quad H_1 : \mu < \mu_0 (= 1400\text{g})$$

What is the risk which is kept small in this hypothesis formulation?

b) State the sample function suitable for this test and verbalize your answer.

c) Give the distribution and parameters of the sample function under the assumption that H_0 is true.

d) What is the test function and how is it distributed under H_0?

e) Determine the region of acceptance and rejection of H_0.

f) What is the decision of the customer C_1?

g) Which error could have C_1 made?

The customer C_2 takes a second random sample of $n = 25$ chickens. It results in the average weight of 1381g and the same standard deviation as before.

h) What is the decision C_2 makes?

i) Which error could have C_2 made?