Statistik II - Exercise session 6
7.1.2015 & 14.1.2015

# Info

- Classroom: SPA1 203
- Time: Wednesdays, 16:15 - 17:45
- in English
- Assignments on webpage (lvb>staff>PB)

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Office: SPA1 R400 (upon agreement)

Schedule:

| Date | Week | Exercises |
|---|---|---|
| 15.10.14 | E1 | 4-11, 4-18, 4-19 |
| 22.10.14 | E1 | 4-11, 4-18, 4-19 |
| 29.10.14 | E2 | 5-6, 5-10, 5-11, 5-14 ,5-16 |
| 5.11.14 | E2 | 5-6, 5-10, 5-11, 5-14 ,5-16 |
| 12.11.14 | E3 | 5-16, 5-21, 5-23, 6-1 |
| 19.11.14 | E3 | 5-16, 5-21, 5-23, 6-1 |
| 26.11.14 | E4 | 6-3, 6-9, 6-13, 7-1 |
| 03.12.14 | E4 | 6-3, 6-9, 6-13, 7-1 |
| 10.12.14 | E5 | 7-3, 7-5, 7-26, 7-45 |
| 17.12.14 | E5 | 7-3, 7-5, 7-26, 7-45 |
| 07.01.15 | E6 | (7-45), 8-1, 8-4, 8-7 |
| 14.01.15 | E6 | (7-45), 8-1, 8-4, 8-7 |
| 21.01.15 | E7 | TBA |
| 28.01.15 | – | – |
| 04.02.15 | E8 | Review for exam |
| 11.02.15 | E8 | Review for exam |

# Review

- week 11 & week 12
- Slides: Testing of Hypotheses

**Statistical hypothesis** - A statement about the parameters describing a population (not a sample).

**Null hypothesis** ($H_0$) - A simple hypothesis associated with a contradiction to a theory one would like to prove.

**Statistic** - A sample function $V = V(X_1, \ldots, X_n)$, often to summarize the sample for comparison purposes.

**Region of rejection / Critical region** -The set of values of the test statistic for which the null hypothesis is rejected.

**Critical value** -The threshold value delimiting the regions of acceptance and rejection for the test statistic.

!NOTATION:
"$H_1$" - reject $H_0$
"$H_0$" - not reject $H_0$
"$H_1$"$|H_0$ - reject $H_0$ when it is true
"$H_0$"$|H_1$ - not reject $H_0$ when "$H_1$" is not true

**Power of a test** $(1\beta)$ - The test's probability of correctly rejecting the null hypothesis. The complement of the false negative rate, $\beta$=P( "$H_0$"$|H_1$ ).

**Size / Significance level of a test** ($\alpha$) For simple hypotheses, this is the test's probability of incorrectly rejecting the null hypothesis. P( "$H_1$"$|H_0$ )

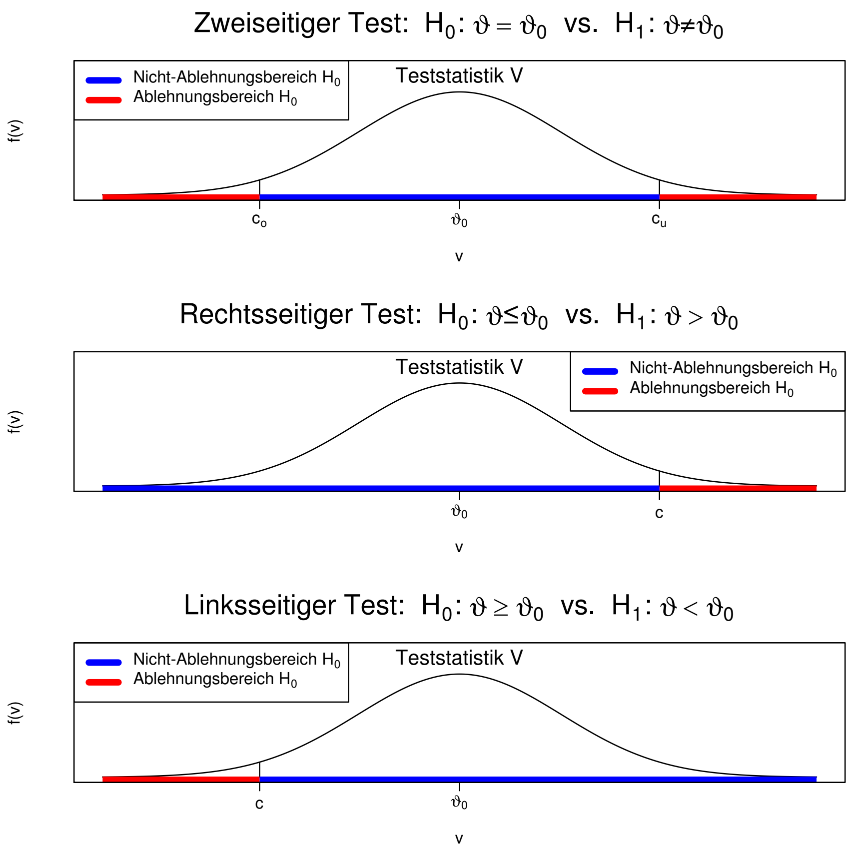
**Type I. Error** - reject $H_0$ when it is true
**Type II. Error** - not rejecting $H_0$ when it is wrong

| **Test** | **Null hypothesis** $H_0$ | **Alternative hypothesis** $H_1$ |
|---|---|---|
| Both-sided | $\vartheta = \vartheta_0$ | $\vartheta \neq \vartheta_0$ |
| One-sided (left-sided ) | $\vartheta \leq \vartheta_0$ | $\vartheta > \vartheta_0$ |

## Test for $\mu$

| Variance $\sigma^2$ | | known | unknown |
|---|---|---|---|
| Test statistic $V$ | | $\dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}}$ | $\dfrac{\overline{X} - \mu_0}{S/\sqrt{n}}$ |
| $X_i \sim N(\mu;\sigma)$ | $n \leq 30$ | $N(0,1)$ | $t_{n-1}$ |
| | $n > 30$ | $N(0,1)$ | |
| any distr. | $n > 30$ | $\approx N(0,1)$ | |

Note: t-distribution as a result of normal / chi-squared / sqrt(n)


Zweiseitiger Test: $H_0: \vartheta = \vartheta_0$ vs. $H_1: \vartheta \neq \vartheta_0$
Nicht-Ablehnungsbereich $H_0$
Ablehnungsbereich $H_0$
Teststatistik V
f(v)
$c_o$
$\vartheta_0$
$c_u$
v
Rechtsseitiger Test: $H_0: \vartheta \leq \vartheta_0$ vs. $H_1: \vartheta > \vartheta_0$
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Nicht-Ablehnungsbereich $H_0$
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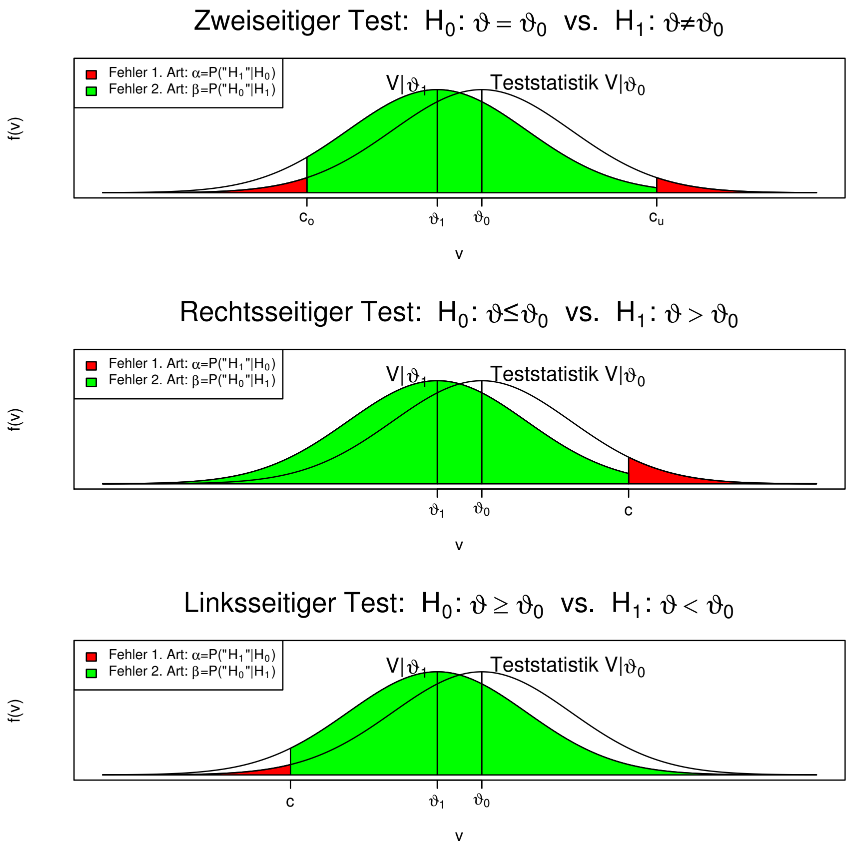
Zweiseitiger Test: $H_0: \vartheta = \vartheta_0$ vs. $H_1: \vartheta \neq \vartheta_0$
Fehler 1. Art: $\alpha$=P("$H_1$"|$H_0$)
Fehler 2. Art: $\beta$=P("$H_0$"|$H_1$)
$V|\vartheta_1$
Teststatistik $V|\vartheta_0$
f(v)
$c_o$
$\vartheta_1$
$\vartheta_0$
$c_u$
v
Rechtsseitiger Test: $H_0: \vartheta \leq \vartheta_0$ vs. $H_1: \vartheta > \vartheta_0$
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f(v)
c
$\vartheta_1$
$\vartheta_0$
v

## Power function

function of parameter, giving probability of rejection (based on the settings of hypothesis)

$$G(\vartheta) = P(\text{“}H_1\text{”}|\vartheta) \text{ with } \begin{cases} G(\vartheta) \leq \alpha & \text{for all } \vartheta \in \Theta_0 \\ G(\vartheta) = 1 - \beta(\vartheta) & \text{for all } \vartheta \in \Theta_1 \end{cases}$$

## Power function for test of $\mu$

| $G(\mu)$ for both-sided Test | |
|---|---|
| $1 - \left[ P\left( V \leq z_{1-\frac{\alpha}{2}} - \frac{\mu - \mu_0}{\sigma/\sqrt{n}} \right) - P\left( V < -z_{1-\frac{\alpha}{2}} - \frac{\mu - \mu_0}{\sigma/\sqrt{n}} \right) \right]$ | |
| $G(\mu)$ for left-sided Test | $G(\mu)$ for right-sided Test |
| $P\left( V < -z_{1-\alpha} - \frac{\mu - \mu_0}{\sigma/\sqrt{n}} \right)$ | $1 - P\left( V \leq z_{1-\alpha} - \frac{\mu - \mu_0}{\sigma/\sqrt{n}} \right)$ |

# Exercises

**Exercise 8-1 - Special refrigerators**

A company produces special refrigerators to conserve certain goods. The wished temperature for that type of refrigerators is -25 degrees. When goods are insufficiently cooled, they go bad very easily and since the client base of the company is not big, a defective products would cause the worst case-the ruin of the company. That is why the cooling performance of 100 randomly chosen produced refrigerators shall be tested on a significance level of 2,275% in order to decide whether the production can be carried on or if constructional changes on the refrigerators need to be made. Experience shows that the cooling temperature is normally distributed with standard deviation 2 degrees of Celsius.

a) What are the hypotheses for this test? Justify.

b) Formulate the underlying sample function formally and verbally and give its distribution under $H_0$.

c) What is the testing function and what is its distribution under $H_0$?

d) Determine the region of rejection.

e) Determine the value of power function if the true mean of cooling temperature is:
i) -24,8 ii) -25,8 iii) -29,0 degrees.

f) Sketch the power function.

g) Random sampling yielded a mean of cooling temperature of -26 degrees and standard deviation 1,5 degrees.

   i) What is the test decision?

   ii) Interpret the test result in an exact way statistically as well as from context point of view.

h) Random sampling yielded a mean of cooling temperature of -25,3 degrees.

   i) What is the test decision?

   ii) Interpret the test result in an exact way statistically as well as from context point of view.

   iii) Which mistake can be made by taking this test decision?

   iv) What is the probability that this mistake has really been made?

   v) How big is the probability to make this mistake when using this test procedure and the real $\mu$ is -29 degrees?

i) Why is it sufficient fo one-sided test to consider under the null hypothesis only the case $\mu = \mu_0$?

**Exercise 8-4** - **Average weight** A supermarket has chickens with average weight 1400g at a certain price. A dealer now makes an offer to deliver chickens of the same average weight for a lower unit price. Buyers/Customers C1 and C2 both know that the chicken weight is normally distributed, believing that the low price is due to a low average weight too. C1 then weights from 25 randomly selected chickens. It turns out that the arithmetic mean deviates from the target weight by -9 grams and standard deviation was found to be 50g. The significance level of test shall be 5 %.

a) Customer provides the following hypotheses:

   $H_0 : \mu \geq \mu_0 (= 1400)$ and $H_A : \mu < \mu_0 (= 1400)$.

   What is the risk to keep small in this hypothesis formulation?

b) State the sample function suitable for testing and verbalize.

c) Give the distribution and parameters under assumption that $H_0$ is true.

d) What is the test function and how is it distributed under $H_0$?

e) Determine the region of acceptance and rejection.

f) What is C1 decision?

g) What is the error C1 may have made?

C2 takes a second random sample of $n = 25$ which results in an average weight 1381g and the same standard deviation.

a) What is C2 decision?

b) What is the error C2 may have made?

**Exercise 8-7** - **Heavy-weight boxer** Two boxers Jim Knockout and Bill Uppercut are both the world's best boxer. A plaster company wants to offer the world's best boer an advertisement contract worth 1 Million Euro. The Chef of this company believes that Jim Knockout is the better boxer. To proof this hypothesis statistically, the chef organizes 11 fights between both boxers. In each fight there will be winner. (Ties are excluded.)

a) Formulate the hypothesis.

b) Define the test function.

c) How is test function distributed?

d) Determine accepting and rejecting regions for this test.

e) How do you decide this test, if Jim Knockout looses 3 fights?

f) Could you have made an error in your decision? If yes, which one?

g) The company plans to publish the test result in a short and comprehensive form. Formulate this press release.