Risk Patterns and Correlated Brain Activities

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Risk Perception

- Which part is activated during *risk related decisions* ?
- Can statistical analysis help to detect this area?
- Response curve (to stimuli)? classify “risky people”?

Risk Perception

- Survey conducted by Max Planck Institute

- 22 young, native German, right-handed and healthy volunteers

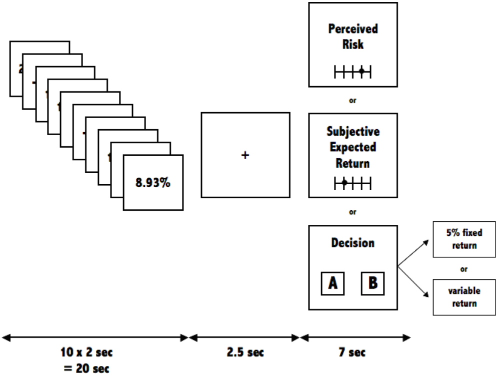
  3 subjects with extensive head movements ($> 5mm$)

  2 subjects with different stimulus frequency

  $n = 22 - (3 + 2) = 17$

- Experiment
  - Risk Perception and Investment Decision (RPID) task ($\times 81$)
  - fMRI images every 2.5 sec.
  - Analysis of the first part ($\times 45$)

Risk Perception



Returns Pause Decision

Risk Perception – Thermodynamics

Theoretical framework

- Risk-return model
  Mohr et al., 2010

- Mechanical Equivalent of Heat
  1st law of thermodynamics
  Mayer, 1841

Empirical evidence

- fMRI analysis

- Experiments "Joule apparatus"
  Joule, 1843

Risk Perception

- functional Magnetic Resonance Imaging

- Measuring Blood Oxygenation Level Dependent (BOLD) effect every 2-3 sec
  High-dimensional, high frequency & large data set

Risk Perception

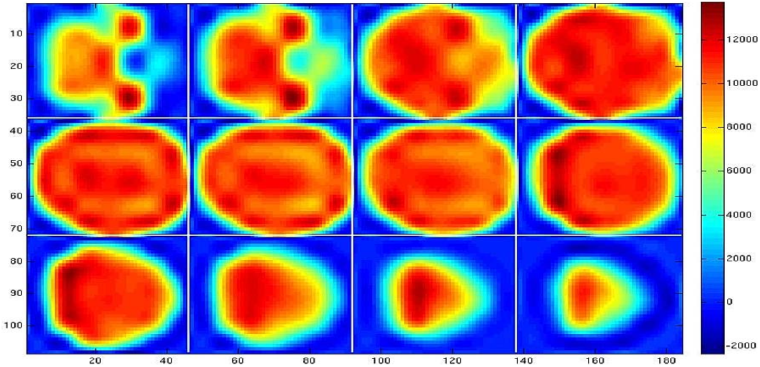
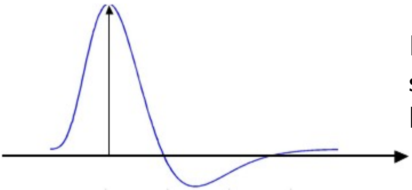


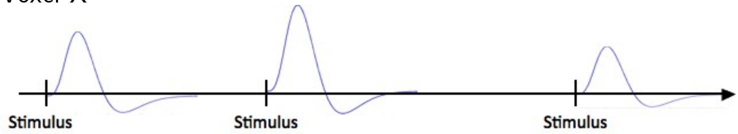
Figure 1: fMRI image observed every 2.5 sec, 12 horizontal slices of the brain's scan, $91 \times 92 \times 71 (x, y, z)$ data points of size 22 MB; scan resolution: $2 \times 2 \times 2mm^3$ fMRI

fMRI



Is there a significant reaction to specific stimuli in the hemodynamic response?

Voxel X



fMRI methods

- Voxel-wise GLM ▸ Voxel-wise GLM
  - linear model for each voxel separately
  - strong a priori hypothesis necessary

- Dynamic Semiparametric Factor Model (DSFM)
  - Use a "time & space" dynamic approach
  - Separate low dim time dynamics from space functions
  - Low dim time series exploratory analysis

Outline

Notation

$$\underbrace{(X_{1,1}, Y_{1,1}), \ldots, (X_{J,1}, Y_{J,1})}_{t=1}, \ldots, \underbrace{(X_{1,T}, Y_{1,T}), \ldots, (X_{J,T}, Y_{J,T})}_{t=T},$$

$X_{j,t} \in \mathbb{R}^d, \; Y_{j,t} \in \mathbb{R}$
T - the number of observed time periods
J - the number of the observations in a period t
$\mathsf{E}(Y_t | X_t) = F_t(X_t)$

Quantify $F_t(X_t)$. How does it move?

Dynamic Semiparametric Factor Model

$$\mathsf{E}(Y_t|X_t) = \sum_{l=0}^{L} Z_{t,l} m_l(X_t) = Z_t^\top m(X_t) = Z_t^\top A^* \Psi$$

$Z_t = (\mathbf{1}, Z_{t,1}, \ldots, Z_{t,L})^\top$ low dim (stationary) time series

$m = (m_0, m_1, \ldots, m_L)^\top$, tuple of functions

$\Psi = \{\psi_1(X_t), \ldots, \psi_K(X_t)\}^\top$, $\psi_k(x)$ space basis

A^* : $(L+1) \times K$ coefficient matrix

DSFM Estimation

$$Y_{t,j} = \sum_{l=0}^{L} Z_{t,l} m_l(X_{t,j}) + \varepsilon_{t,j} = Z_t^\top A^* \psi(X_{t,j}) + \varepsilon_{t,j}$$

- $\psi(x) = \{\psi_1(x), \ldots, \psi_K(x)\}^\top$ tensor B-spline basis

$$(\widehat{Z}_t, \widehat{A^*}) = \arg \min_{Z_t, A^*} \sum_{t=1}^{T} \sum_{j=1}^{J} \left\{ Y_{t,j} - Z_t^\top A^* \psi(X_{t,j}) \right\}^2 \qquad (1)$$

- Minimization by Newton-Raphson algorithm

B-Splines

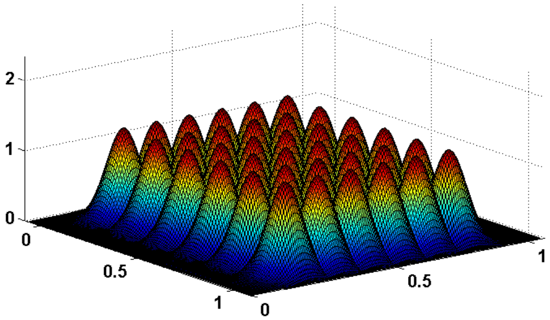


Figure 2: B-splines basis functions; order of B-splines: quadratic; number of knots: $6 \times 6 = 36$ ▸ B-Splines

DSFM Estimation

- Selection of L by explained variance

$$EV(L) = 1 - \frac{\sum_{t=1}^{T}\sum_{j=1}^{J}\left\{Y_{t,j} - \sum_{l=0}^{L} Z_{t,l}m_l(X_{t,j})\right\}^2}{\sum_{t=1}^{T}\sum_{j=1}^{J}\left\{Y_{t,j} - \bar{Y}\right\}^2}$$

number of B-splines (equally spaced) knots: $K = 12 \times 14 \times 14$

$L=2$	$L=4$	$L=5$	$L=10$	$L=20$
92.07	92.25	92.29	93.66	95.19

Table 1: EV in percent of the model with different numbers of factors L, averaged over all 17 analyzed subjects.

Panel DSFM

$$Y^i_{t,j} = \sum_{l=0}^{L} (Z^i_{t,l} + \alpha^i_{t,l}) m_l(X_{t,j}) + \varepsilon_{t,j},^i, \quad 1 \leq j \leq J,\ 1 \leq t \leq T,$$

- $n = 17$ weakly/strongly risk-averse subjects
- $Y_{t,j}$ - BOLD signal; X_j voxel's index
  $\alpha^i_{t,l}$ - fixed individual effect; ▸ Residual Analysis
- Identification condition: $\mathrm{E}\left\{ \sum_{i=1}^{n} \sum_{l=0}^{L} \alpha^i_{t,l} m_l(X_{t,j}) | X_{t,j} \right\} = 0$

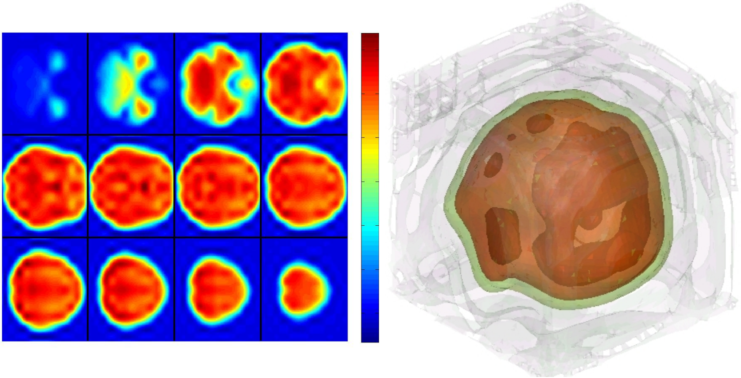
Panel DSFM Estimation

Feasible estimation algorithm:

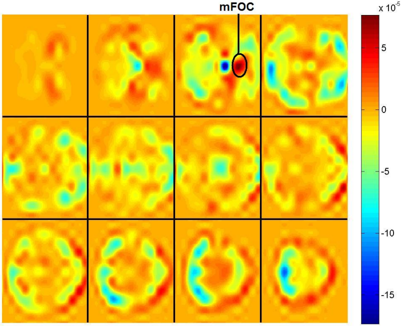
1. Average $Y^i_{t,j}$ over subjects i to obtain $\bar{Y}_{t,j}$
2. Estimate factors m_l for the "average brain" [via (1)]
3. Given $\widehat{m}_l$, for i, estimate $Z^i_{t,l}$

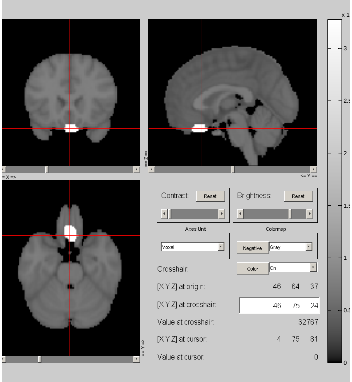
$$Y^i_{t,j} = \sum_{l=0}^{L} Z^i_{t,l} \widehat{m}_l(X_{t,j}) + \varepsilon^i_{t,j}$$

- 26h - computing time; CPU - 2 × 2.8GHz; data set of size 24.31 GB

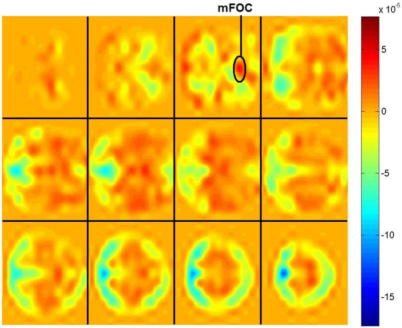


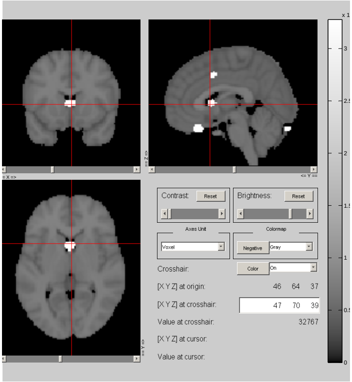
Estimated constant factor $\widehat{m}_0(X) = \sum_{k=1}^{K} \widehat{a}_{0,k}\psi_k(X)$ with $L = 20$



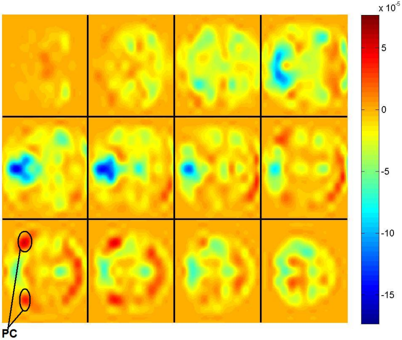


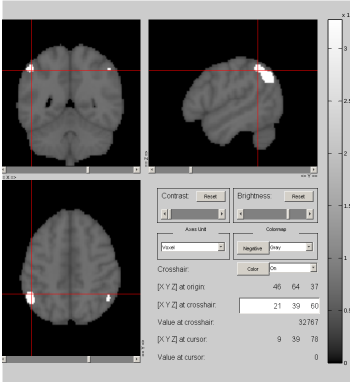
Estimated factor $\widehat{m}_5(X) = \sum_{k=1}^{K} \widehat{a}_{5,k} \psi_k(X)$ with $L = 20$
(MOFC = Medial orbitofrontal cortex)



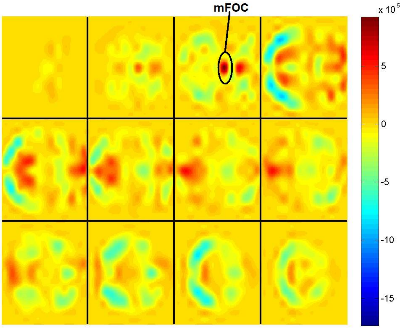


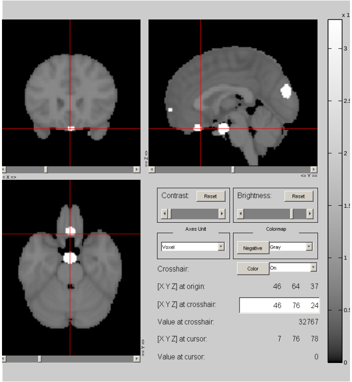
Estimated factor $\widehat{m}_9(X) = \sum_{k=1}^{K} \widehat{a}_{9,k}\psi_k(X)$ with $L = 20$



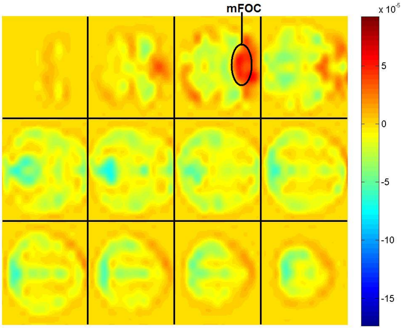


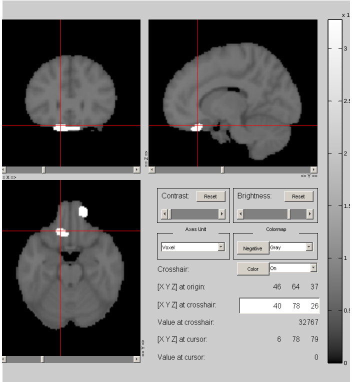
Estimated factor $\widehat{m}_{12}(X) = \sum_{k=1}^{K} \widehat{a}_{12,k}\psi_k(X)$ with $L = 20$
(PC = Paretial Cortex)



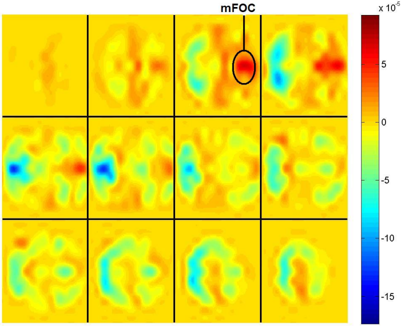


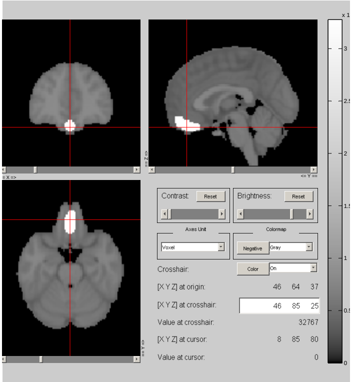
Estimated factor $\widehat{m}_{16}(X) = \sum_{k=1}^{K} \widehat{a}_{16,k}\psi_k(X)$ with $L = 20$





Estimated factor $\widehat{m}_{17}(X) = \sum_{k=1}^{K} \widehat{a}_{17,k}\psi_k(X)$ with $L = 20$





Estimated factor $\widehat{m}_{18}(X) = \sum_{k=1}^{K} \widehat{a}_{18,k}\psi_k(X)$ with $L = 20$

Estimated Factor Loading $\widehat{Z}_5$

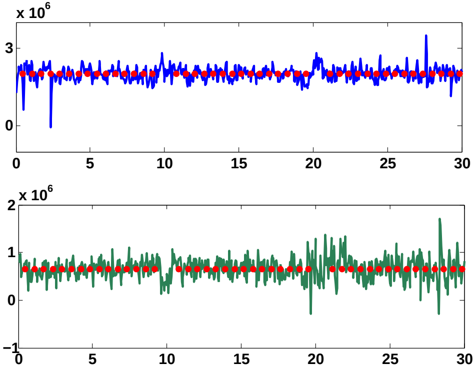


Figure 3: Estimated factor loading $\widehat{Z}_5$ for subjects within 30 minutes: 12 (upper panel) and 19 (lower panel) with $L = 20$; red dots denote stimulus

Estimated Factor Loading $\widehat{Z}_9$

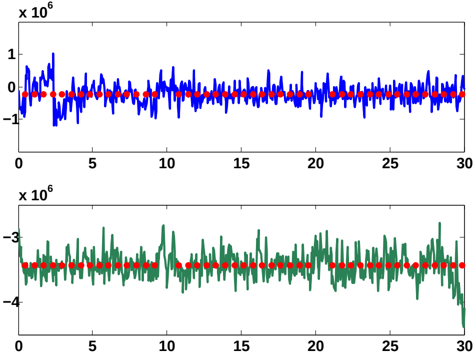


Figure 4: Estimated factor loading $\widehat{Z}_9$ for subjects within 30 minutes: 12 (upper panel) and 19 (lower panel) with $L = 20$; red dots denote stimulus

Estimated Factor Loading $\widehat{Z}_{12}$

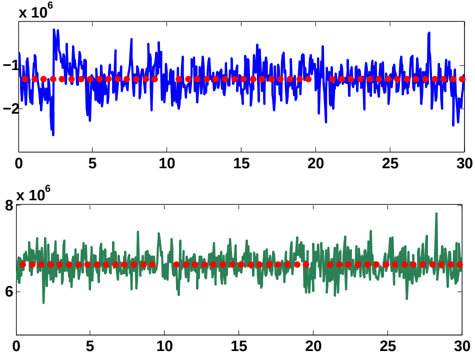


Figure 5: Estimated factor loading $\widehat{Z}_{12}$ for subjects within 30 minutes: 12 (upper panel) and 19 (lower panel) with $L = 20$; red dots denote stimulus

Estimated Factor Loading $\widehat{Z}_{16}$

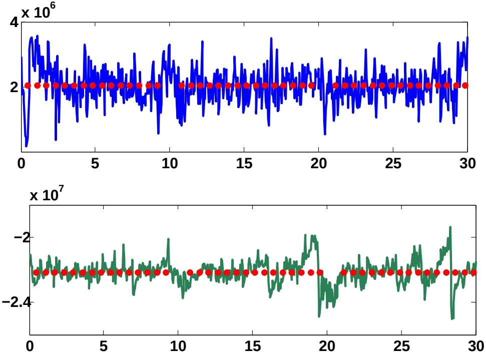


Figure 6: Estimated factor loading $\widehat{Z}_{16}$ for subjects within 30 minutes: 12 (upper panel) and 19 (lower panel) with $L = 20$; red dots denote stimulus

Estimated Factor Loading $\widehat{Z}_{17}$

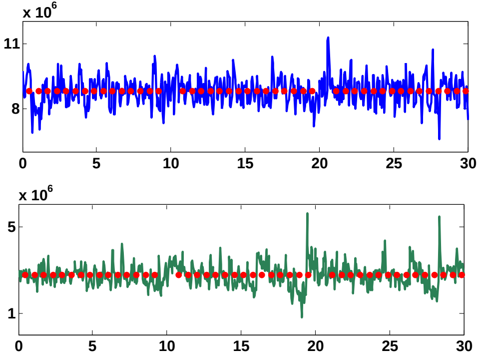


Figure 7: Estimated factor loading $\widehat{Z}_{17}$ for subjects within 30 minutes: 12 (upper panel) and 19 (lower panel) with $L = 20$; red dots denote stimulus

Estimated Factor Loading $\widehat{Z}_{18}$

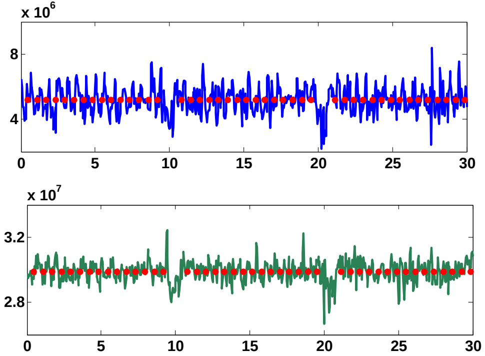


Figure 8: Estimated factor loading $\widehat{Z}_{18}$ for subjects within 30 minutes: 12 (upper panel) and 19 (lower panel) with $L = 20$; red dots denote stimulus

Reaction to the stimulus

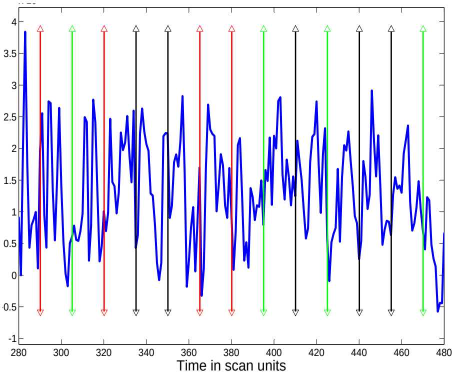


Figure 9: Detailed view of factor loading $\widehat{Z}_1$ for subject 12 with vertical lines in time points of stimuli of 3 different task: decision (red), subjective expected return (green) and perceived risk (black)

Reaction to the stimulus

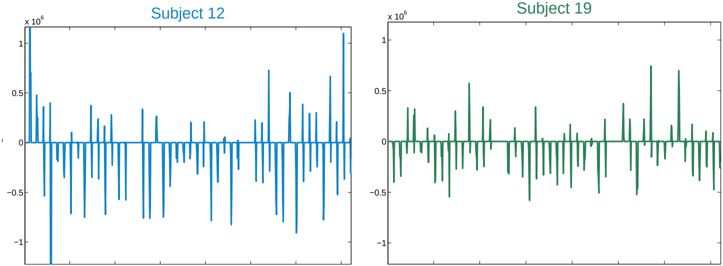


Figure 10: Reaction to stimulus $\overline{\Delta}\widehat{Z}^i_{s,l} = \frac{1}{3}\sum_{\tau=1}^{3}\Delta\widehat{Z}^i_{s+\tau,l}$, where $\Delta\widehat{Z}^i_{t,l} \stackrel{\text{def}}{=} \widehat{Z}^i_{s+t,l} - \widehat{Z}_{s,l}$, $t = 1, 2, 3$, s is the time of stimulus for factors loadings $\widehat{Z}^i_{t,12}$, for subjects 12 (left) and 19 (right) during the experiment (45 stimuli).

Risk attitude

- Subject's risk perception $\widetilde{R}_{i,s}$ - Risk Metrics
  - standard deviation
  - empirical frequency of loss (negative return)
  - difference between highest an lowest return (range)
  - coefficient of range (range/mean)
  - empirical frequency of ending below 5%
  - coefficient of variation (standard deviation/mean)

- Different subject - different risk perception

  fitted by correlation between risk metrics of return streams and $R_{i,j,s}$ - answers for "perceived risk" task $Q1$, $N = 27$

Risk attitude

- Subjective expected return $\widetilde{m}_{i,s}$ - ▸ Return Ratings
  - recency (higher weights on later returns)
  - primacy (higher weights on earlier returns)
  - below 0% (higher weights on returns below 0%)
  - below 5% (higher weights on returns below 5%)
  - mean

- Selecting return ratings for each subject individually

  best model selected by prediction power of one-leave-out cross validation procedure, $N = 27$

Risk attitude

- Each subject i has (R_i, m_i)
- Risk-return choice model

$$V_i(x_s) = m_i(x_s) - \beta_i R_i(x_s), \quad 1 \leq i \leq n, 1 \leq s \leq 27$$

x_s - return stream, m_i-subjective expected return, R_i - perceived risk , V_i - subjective value (unobserved), 5% - risk free return

- β Risk attitude parameter

Risk attitude

- Estimation of individual risk attitude by logistic regression

$$\begin{aligned} \mathrm{P}\left\{\text{risky choice}|(m,R)\right\} &= \frac{1}{1+\exp(m-\beta R-5)} \\ \mathrm{P}\left\{\text{sure choice}|(m,R)\right\} &= 1-\frac{1}{1+\exp(m-\beta R-5)} \end{aligned}$$

risky choice - unknown return, sure choice - fixed, 5% return

- $\widehat{\beta}$ derived by maximum likelihood method

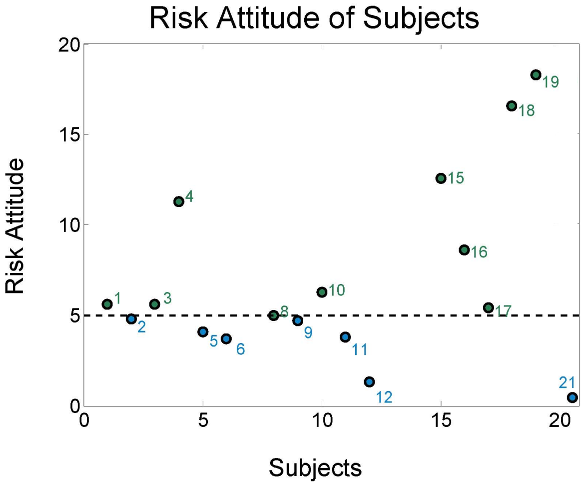


Figure 11: Risk attitude $\widehat{\beta}_i$ for 17 subjects; modeled by the softmax function from individuals' decisions, estimated by ML method ▸ Mohr et. al.

SVM Classification Analysis

- Support Vector Machines (SVM)
  17 subjects, 20 factor loading time series per subject

- Leave-one-out method to train and estimate classification rate
  SVM with Gaussian kernel; (R, C) chosen to maximize classification rate

- Weakly/strongly risk-averse subjects differ in reaction to stimulus $\Delta \widehat{Z}^i_{t,l}$ Reaction to Stimulus

SVM Classification Analysis

1. factors attributed to risk patterns: $l = 5, 9, 12, 16, 17, 18$
2. only "Decision under Risk" ($Q3$) stimulus
3. average reaction to s stimulus $\overline{\Delta}\widehat{Z}^i_{s,l} = \frac{1}{3}\sum_{\tau=1}^{3} \Delta\widehat{Z}^i_{s+\tau,l}$

SVM input data: volatility of $\overline{\Delta}\widehat{Z}^i_{s,l}$ over all $Q3$

Std		Estimated	
		Strongly	Weakly
Data	Strongly	1.00	0.00
	Weakly	0.14	0.86

Table 2: Classification rates of the SVM method, **without** knowing the subject's estimated risk attitude ▸ SVM Scores

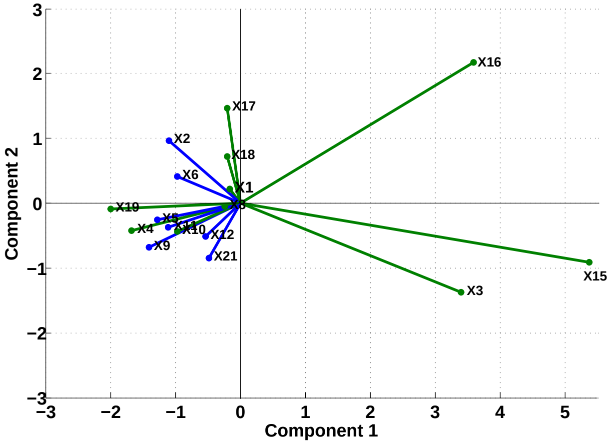


Figure 12: Normalized Principal Component Analysis on volatility of $\overline{\Delta}\widehat{Z}^{i}_{s,l}$ after stimulus for weakly/strongly risk-averse subjects; variance explained by the first and second components: 72%, 85%, respectively

Conclusion

- Factors $\widehat{m}$ identify activated areas, neurological reasonable
- Estimated factor loadings show differences for individuals with different risk attitudes (e.g. 12 vs. 19)
- SVM classification analysis of measurements in $\widehat{Z}_{t,l}$, $l = 5, 9, 12, 16, 17, 18$ after stimulus, can distinguish weakly/strongly risk-averse individuals with high classification rate, **without** knowing the subject's answers

Future Perspectives

- Comparison with the PCA/ICA (PARAFAC) approach
- Analysis of the second part of the experiment (under assumption of independency) to "generate" larger number of subjects
- Improvement of the classification criterion
- Penalized DSFM with seasonal effects

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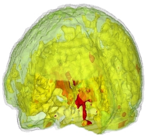

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Voxel-wise GLM

▸ fMRI methods

- FEAT - FMRI Expert Analysis Tool by Department of Clinical Neurology, University of Oxford

- GLM framework

$$Y = XB + \eta,$$

Y - single voxel BOLD time series, X - design matrix (regressors, i.e. visual, auditory)

- Significant, active areas (B) selected by *z-scores* $\equiv \frac{B_i - 0}{\sqrt{\mathrm{Var}(B_i)}}$ and grouping (20 neighbors) scheme

B-Splines

▸ B-Splines

Univariate **B-spline** basis $\Psi = \{\psi_1(X), \ldots, \psi_K(X)\}^\top$ is a series of $\psi_k(X)$ functions defined by $x_0 \leq x_2 \leq \ldots \leq x_{K-1}$, K knots and order p, i.e. for $p = 2$ (quadratic)

$$\psi_j(x) = \begin{cases} \frac{1}{2}(x - x_j)^2 & \text{if } x_j \leq x < x_{j+1} \\ \frac{1}{2} - (x - x_{j+1})^2 + (x - x_{j+1}) & \text{if } x_{j+1} \leq x < x_{j+2} \\ \frac{1}{2}\left\{1 - (x - x_{j+2})^2\right\} & \text{if } x_j \leq x < x_{j+1} \\ x & \text{otherwise} \end{cases}$$

B-Splines

▸ B-Splines

- Knots K and order p has to be specified in advance (EV criterion); K corresponds to bandwidth

- In higher dimensions, for $dim(X) = d > 1$

$$\Psi = \{\psi_1(X_1), \ldots, \psi_{K_1}(X_1)\} \times \ldots \times \{\psi_1(X_d), \ldots, \psi_{K_d}(X_d)\}$$

- Flexible and computationally efficient approach to capture various spatial structures

Residual Analysis

▸ PDSFM

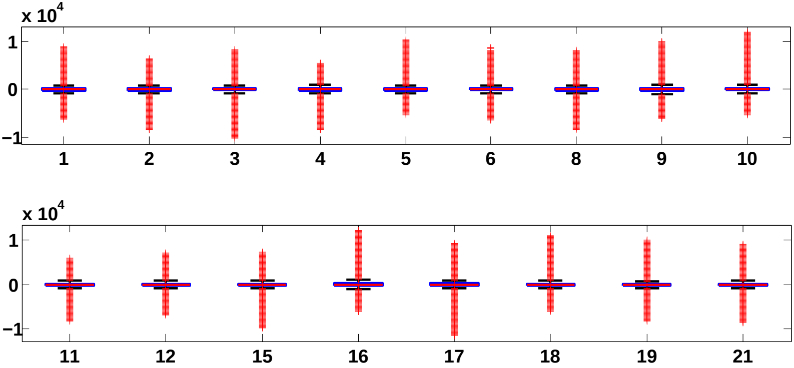

Figure 13: Boxplots of random subsets (size 3×10^7) from $\varepsilon^i_{t,j}$ (4.3×10^9 points) for all 17 analyzed subjects. Kurtosis exceeds 10

Residual Analysis

▸ PDSFM

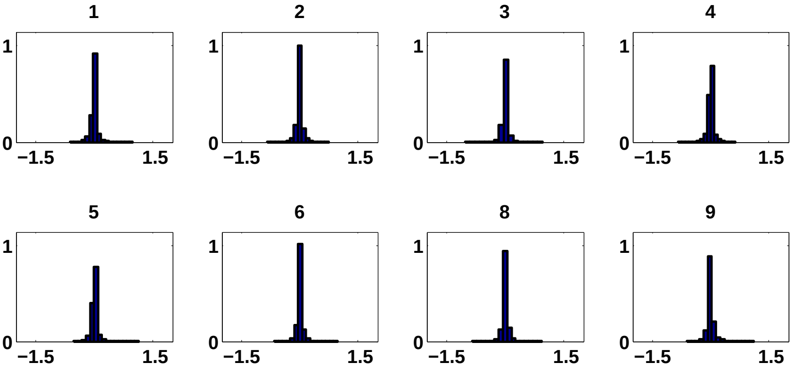

Figure 14: Histograms of random subsets (size 3×10^7) from $\varepsilon^i_{t,j}$ (4.3×10^9 points) for subjects $i = 1, 2, 3, 4, 5, 6, 8, 9$, respectively. Normality hypothesis (**KS test**) for standardized $\varepsilon^i_{t,j}$ rejected for all subjects, $\alpha = 5\%$

Residual Analysis

PDSFM

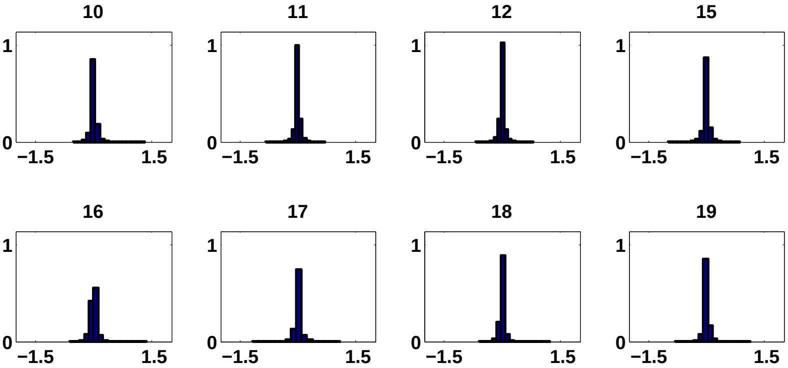

Figure 15: Histograms of random subsets (size 3×10^7) from $\varepsilon^i_{t,j}$ (4.3×10^9 points) for subjects $i = 10, 11, 12, 15, 16, 17, 18, 19$ respectively

Residual Analysis

PDSFM

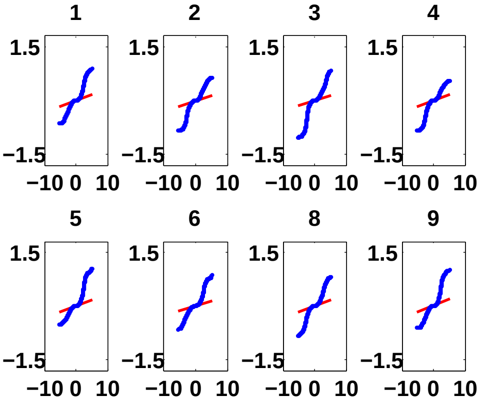

Figure 16: QQplots of random subsets (size 3×10^7) from $\varepsilon^i_{t,j}$ (4.3×10^9 points) for subjects $i = 1, 2, 3, 4, 5, 6, 8, 9$, respectively

Residual Analysis

▸ PDSFM

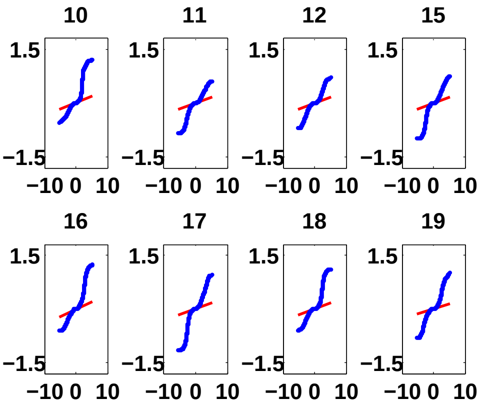

Figure 17: QQplots of random subsets (size 3×10^7) from $\varepsilon^i_{t,j}$ (4.3×10^9 points) for subjects $i = 10, 11, 12, 15, 16, 17, 18, 19$ respectively

Reaction to stimulus

SVM Analysis

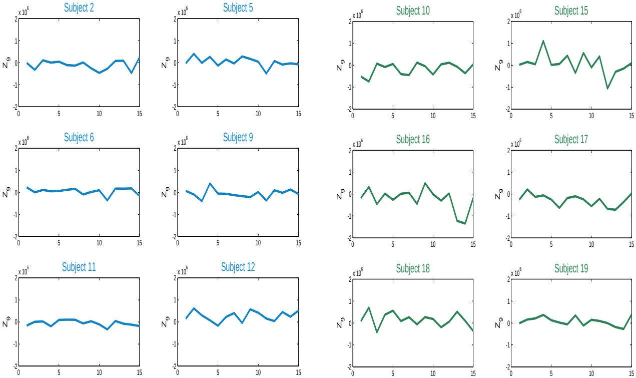

Figure 18: Averaged reaction $\overline{\Delta}\widehat{Z}^{i}_{s,9}$ to stimulus for all 15 $Q3$ questions for weakly/strongly risk-averse individuals

Reaction to stimulus

▸ SVM Analysis

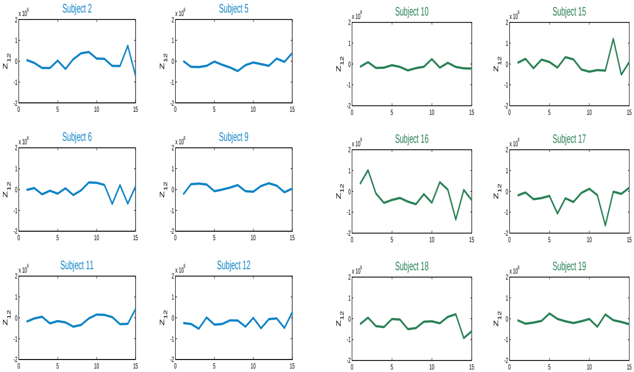

Figure 19: Averaged reaction $\overline{\Delta \widehat{Z}^{i}_{s,12}}$ to stimulus for all 15 $Q3$ questions for weakly/strongly risk-averse individuals

Return Ratings

▸ Risk Attitude

r_i, $i = 1, \ldots, 10$ denotes sequence of random returns in each trial
Subjective Expected Return (**SER**) models:

- Mean

$$SER = \frac{\sum_{i=10-m}^{10} r_i}{m}$$

  m-number of returns remembered, $2 \leq m \leq 10$

- Recency

$$SER = \frac{\sum_{i=10-m}^{10} r_i p}{\sum_{i=10-m}^{10} p}, \quad p = (i - 9 + m)^g$$

  g - weighting parameter of returns, $0 < g < 1$

Return Ratings

Risk Attitude

- Primacy

$$SER = \frac{\sum_{i=10-m}^{10} r_i p}{\sum_{i=10-m}^{10} p}, \quad p = (11-i)^g$$

m-number of returns remembered, $2 \le m \le 10$

g - weighting parameter of returns, $0 < g < 1$

- Overweight $< 0\%$

$$SER = \frac{\sum_{i=10-m}^{10} r_i p}{\sum_{i=10-m}^{10} p}, p = \begin{cases} 1, & \text{if } r_i \ge 0 \\ 1 + w, & \text{otherwise} \end{cases}$$

w - additional weight of returns , $0 < w < 1$; $1 \le m \le 9$

Return Ratings

Risk Attitude

- Overweight $< 5\%$

$$SER = \frac{\sum_{i=10-m}^{10} r_i p}{\sum_{i=10-m}^{10} p}, p = \begin{cases} 1, & \text{if } r_i \geq 5 \\ 1 + w, & \text{otherwise} \end{cases}$$

w - additional weight of returns , $0 < w < 1$; $1 \leq m \leq 9$

- Parameters fitted by Cross Validation over all 27 trials

Return Ratings

Risk Attitude

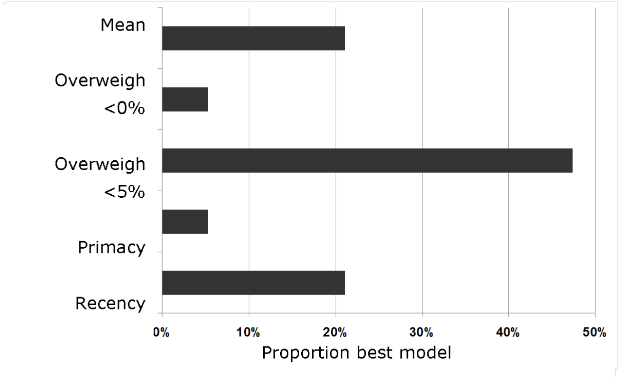

Figure 20: Distribution of return ratings over analyzed subjects

Risk Metrics

▸ Risk Attitude

Risk perception - risk metrics used by individuals

- Standard deviation of a return sequence
- Empirical frequency of loss (negative returns / all returns)
- Range - difference between highest an lowest return in a sequence
- Coefficient of range (range / mean)
- Empirical frequency of ending below 5% (returns $<$ 5% / all returns)
- Coefficient of variation (standard deviation / mean)

Risk Metrics

Risk Attitude

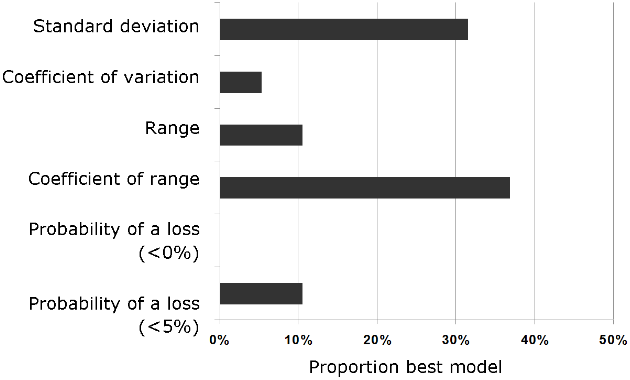

Figure 21: Distribution of risk metrices over analyzed subjects

SVM Scores

▸ SVM Classification

	Strongly									
i	1	3	4	8	10	15	16	17	18	19
β	5.6	5.6	11.3	5.0	6.3	12.6	8.6	5.4	16.6	18.3
Score	0.02	0.43	0.43	0.32	0.58	0.40	0.44	0.23	0.68	0.59
	Weakly									
i	2	5	6	9	11	12	21			
β	4.8	4.1	3.7	4.7	3.8	1.3	1.8			
Score	0.32	−1.03	−0.32	−0.44	−0.79	−0.04	−0.08			

Table 3: Estimated risk attitude and SVM scores (obtained **without** knowing the subject's answers)

SVM Scores

▸ SVM Classification

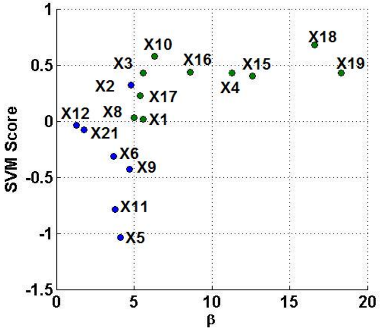

Figure 22: Scatter plot of $\widehat{\beta}_i$ vs SVM scores

Risk Metrics

▸ Risk Attitude

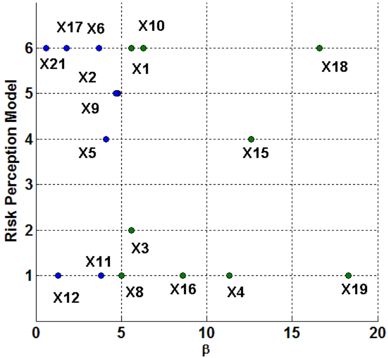

Figure 23: Scatter plot of $\widehat{\beta}_i$ vs risk perception models (vertical line). 1 - Standard deviation, 2 - Coefficient of variation, 3 - Empirical frequency of loss; 4 - Empirical frequency of ending below 5%, 5 - Coefficient of range, 6 - Coefficient of variation.