

# Local Adaptive Multiplicative Error Models for High-Frequency Forecasts

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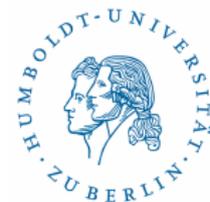
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# Statistical Challenges

- Understanding high-frequency dynamics
  - ▶ Time-varying parameters
    - ▶ Parameter Dynamics
  - ▶ Regime shifts
  
- Modelling using Procrustean assumptions
  - ▶ Time-invariant parameters
  - ▶ Transition form, number of regimes, transition variable type



## Objectives

### (i) Localising Multiplicative Error Models (MEM)

- ▶ Local parametric approach (LPA)
- ▶ Balance between modelling bias and parameter variability
- ▶ Estimation windows with potentially varying lengths

### (ii) Short-term forecasting

- ▶ Case study: trading volume
- ▶ Evaluation against standard approach - fixed estimation length on an ad hoc basis



## Example

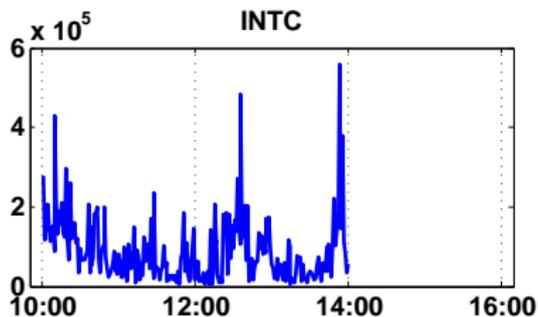


Figure 1: One-minute cumulated trading volume for Intel Corporation (INTC) on 20080902



## Example

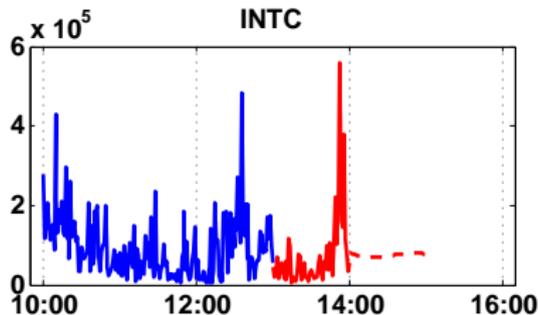


Figure 2: One-minute cumulated trading volume for Intel Corporation (INTC) on 20080902 with an **EACD(1,1)** estimation window length of **60** with volume forecasts to the next one hour (dashed)



## Example

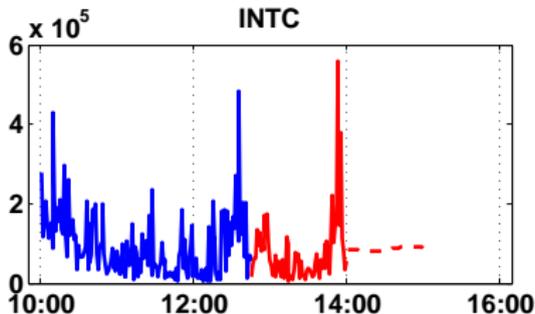


Figure 3: One-minute cumulated trading volume for Intel Corporation (INTC) on 20080902 with an **EACD(1,1)** estimation window length of **75** with volume forecasts to the next one hour (dashed)



## Example

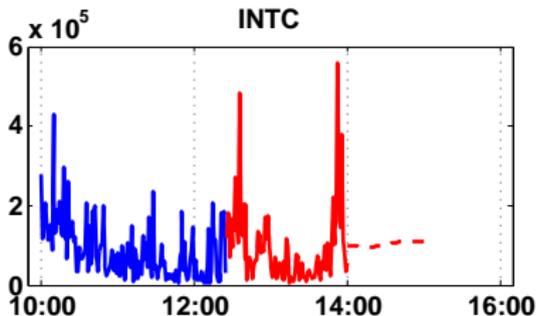


Figure 4: One-minute cumulated trading volume for Intel Corporation (INTC) on 20080902 with an **EACD(1,1)** estimation window length of **95** with volume forecasts up to the next one hour (dashed)



## Research Questions

▶ Example

- How strong is the variation of MEM parameters over time?
- What are typical interval lengths of parameter homogeneity?
- How good are LPA short-term forecasts?



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# Outline

1. Motivation ✓
2. Multiplicative Error Models (MEM)
3. Local Parametric Approach (LPA)
4. Forecasting Trading Volumes
5. Conclusions



## Multiplicative Error Models (MEM)

- Engle (2002),  $\text{MEM}(p, q)$ ,  $\mathcal{F}_i$  - information set up to  $i$

$$y_i = \mu_i \varepsilon_i, \quad E[\varepsilon_i | \mathcal{F}_{i-1}] = 1$$
$$\mu_i = \omega + \sum_{j=1}^p \alpha_j y_{i-j} + \sum_{j=1}^q \beta_j \mu_{i-j}, \quad \omega > 0, \alpha_j, \beta_j \geq 0$$

- Hautsch (2012) - comprehensive MEM literature overview
  - $y_i$  - squared (de-means) log return: GARCH( $p, q$ )
  - $y_i$  - volume, bid-ask spread, duration: ACD( $p, q$ )

Engle, Robert F. on BBI:



## Autoregressive Conditional Duration (ACD)

1. Exponential-ACD, Engle and Russel (1998)

▶ EACD

$$\varepsilon_i \sim \text{Exp}(1), \theta_E = (\omega, \alpha, \beta)^\top, \alpha = (\alpha_1, \dots, \alpha_p), \beta = (\beta_1, \dots, \beta_q)$$

2. Weibull-ACD, Engle and Russel (1998)

▶ WACD

$$\varepsilon_i \sim \mathcal{G}(s, 1), \theta_W = (\omega, \alpha, \beta, s)^\top$$

Weibull, E. H. Waloddi on BBI:



## Parameter Estimation

- Consistent parameter estimation
- Quasi maximum likelihood estimates (QMLEs) of  $\theta_E$  and  $\theta_W$

$$\tilde{\theta}_I = \arg \max_{\theta \in \Theta} L_I(y; \theta) \quad (1)$$

- $I = [i_0 - n, i_0]$  - interval of  $(n + 1)$  observations at  $i_0$
- $L_I(\cdot)$  - log likelihood, ▶ EACD and ▶ WACD



## Data

- NASDAQ Stock Market in 2008, 250 trading days
  - ▶ 5 stocks: AAPL, CSCO, INTC, MSFT and ORCL
  - ▶  $\check{y}_i$  - one-minute cumulated trading volume from 10:00-16:00
  - ▶  $y_i$  - seasonally adjusted trading volume
  
- Periodicity effect - FFS approximation, Gallant (1981)
  - ▶ Engle and Rangel (2008) - 30-days rolling window

$$y_i = \check{y}_i / [\delta \cdot \bar{v} + \sum_{m=1}^M \{ \delta_{c,m} \cos(\bar{v} \cdot 2\pi m) + \delta_{s,m} \sin(\bar{v} \cdot 2\pi m) \}]$$

$\bar{v} \in (0, 1]$  - number of minutes from opening until  $i/360$



## Intraday Periodicity

Figure 5: Estimated intraday periodicity components for AAPL, order  $M = 6$  selected by BIC



# Parameter Dynamics

▶ Statistical Challenges

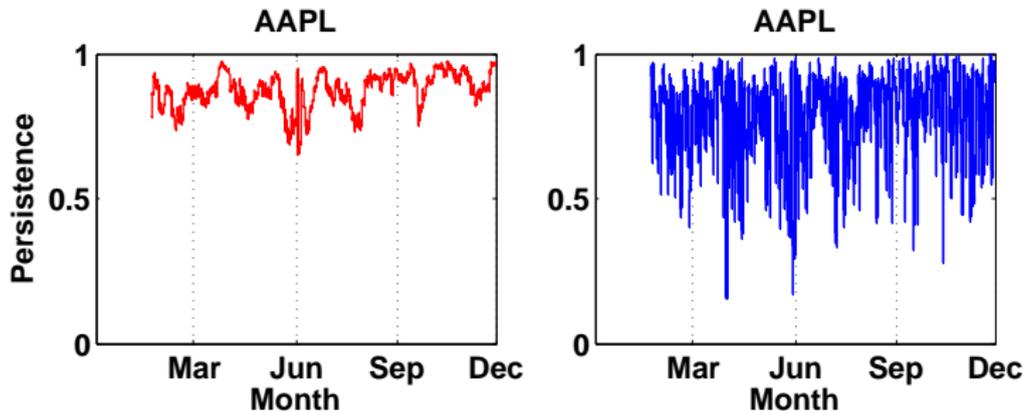


Figure 6: Estimated **weekly** ( $n = 1800$ ) and **daily** ( $n = 360$ ) persistence  $\tilde{\alpha}_i + \tilde{\beta}_i$  for seasonally adjusted trading volume using an EACD(1, 1) at each minute in 2008



## Estimation Quality

- Mercurio and Spokoiny (2004), Spokoiny (2009)
- Quality of estimating *true* parameter vector  $\theta^*$  by QMLE  $\tilde{\theta}_I$  in terms of Kullback-Leibler divergence;  $\mathcal{R}_r(\theta^*)$  - risk bound

$$E_{\theta^*} \left| L_I(\tilde{\theta}_I) - L_I(\theta^*) \right|^r \leq \mathcal{R}_r(\theta^*)$$

- ▶ 'Modest' risk,  $r = 0.5$  (shorter intervals of homogeneity)
- ▶ 'Conservative' risk,  $r = 1$  (longer intervals of homogeneity)

*Kullback, Solomon and Leibler, Richard A.* on BBI:



## Local Parametric Approach (LPA)

- LPA, Spokoiny (1998, 2009)
  - ▶ Time series parameters can be locally approximated
  - ▶ Finding the (longest) *interval of homogeneity*
  - ▶ Balance between modelling bias and parameter variability
  
- Time series literature
  - ▶ Volatility modelling - Mercurio and Spokoiny (2004)
  - ▶ GARCH(1,1) models - Čížek et al. (2009)
  - ▶ Realized volatility - Chen et al. (2010)



## Interval Selection

- $(K + 1)$  nested intervals with length  $n_k = |I_k|$

$$\begin{matrix} I_0 & \subset & I_1 & \subset & \cdots & \subset & I_k & \subset & \cdots & \subset & I_K \\ \tilde{\theta}_0 & & \tilde{\theta}_1 & & & & \tilde{\theta}_k & & & & \tilde{\theta}_K \end{matrix}$$

**Example:** Trading volumes aggregated over 1-min periods

$$\text{Fix } i_0, I_k = [i_0 - n_k, i_0], n_k = [n_0 c^k], c > 1$$

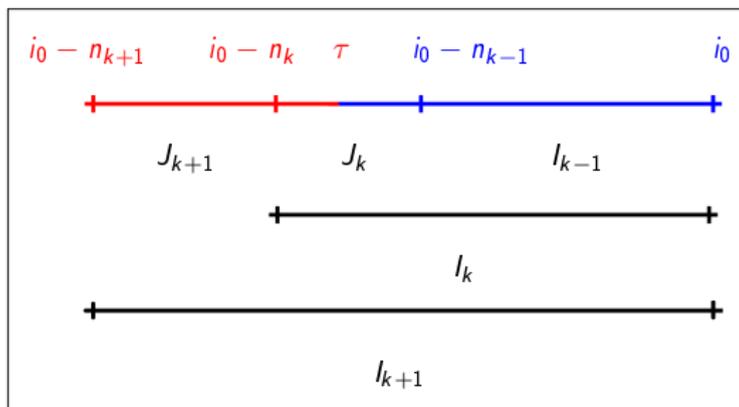
$$\{n_k\}_{k=0}^{13} = \{60 \text{ min.}, 75 \text{ min.}, \dots, 1 \text{ week}\}, c = 1.25$$



## Local Change Point Detection ▶ Example

□ Fix  $i_0$ , sequential test ( $k = 1, \dots, K$ )

$H_0$  : parameter homogeneity within  $I_k$  vs.  $H_1$  : change point within  $I_k$



$$T_k = \sup_{\tau \in J_k} \left\{ L_{A_{k,\tau}} \left( \tilde{\theta}_{A_{k,\tau}} \right) + L_{B_{k,\tau}} \left( \tilde{\theta}_{B_{k,\tau}} \right) - L_{I_{k+1}} \left( \tilde{\theta}_{I_{k+1}} \right) \right\},$$

with  $J_k = I_k \setminus I_{k-1}$ ,  $A_{k,\tau} = [i_0 - n_{k+1}, \tau]$  and  $B_{k,\tau} = (\tau, i_0]$



## Critical Values, $\mathfrak{z}_k$

▶ Critical Values

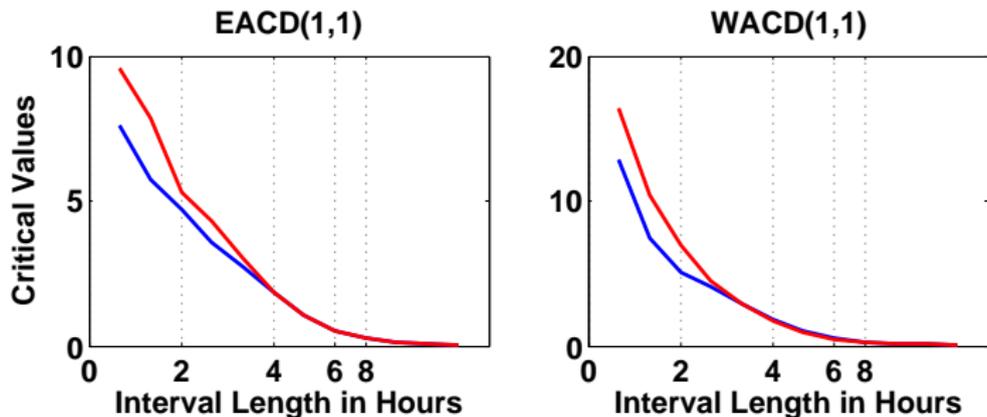


Figure 7: Critical values for low ( $\tilde{\alpha} + \tilde{\beta} = 0.84$ ) and high ( $\tilde{\alpha} + \tilde{\beta} = 0.93$ ) weekly persistence and 'modest' risk ( $r = 0.5$ ) with  $\rho = 0.25$



## Adaptive Estimation

- Compare  $T_k$  at every step  $k$  with  $\beta_k$
- Data window index of the *interval of homogeneity* -  $\hat{k}$
- Adaptive estimate

$$\hat{\theta} = \tilde{\theta}_{\hat{k}}, \quad \hat{k} = \max_{k \leq K} \{k : T_l \leq \beta_l, l \leq k\}$$

- Note: rejecting the null at  $k = 1$ ,  $\hat{\theta}$  equals QMLE at  $l_0$   
If the algorithm goes until  $K$ ,  $\hat{\theta}$  equals QMLE at  $l_K$



## Adaptive Estimation - Results

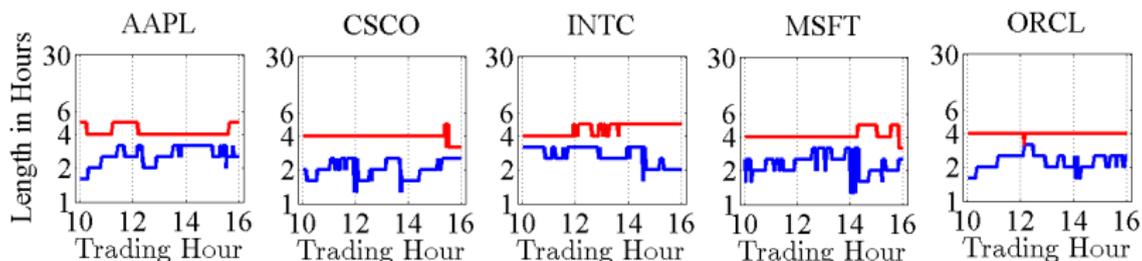


Figure 8: Estimated length  $n_{\hat{k}}$  of *intervals of homogeneity* given the **con-**  
**servative** ( $r = 1$ ) and **modest** ( $r = 0.5$ ) risk case on 20080222 using the  
EACD(1, 1) model with  $\rho = 0.25$



## Adaptive Estimation - Results

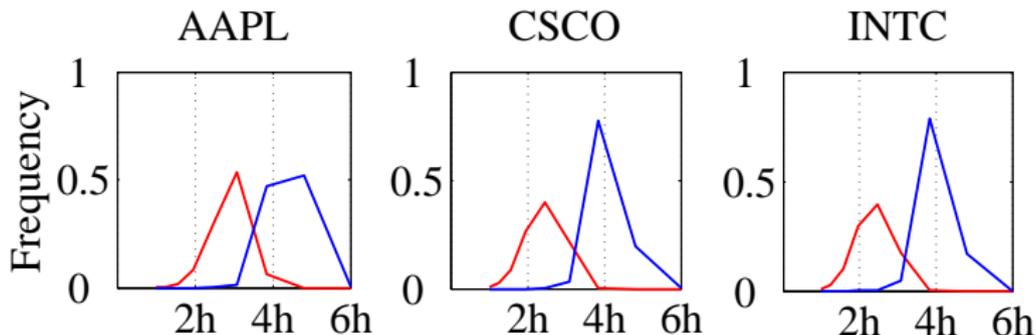


Figure 9: Relative frequency of the estimated interval length  $n_{\hat{k}}$  (discrete variable, in hours) given the **modest** ( $r = 0.5$ ) and **conservative** ( $r = 1$ ) risk case from 20080222 to 20081231 using the EACD(1,1) model with  $\rho = 0.25$



## Adaptive Estimation - Results

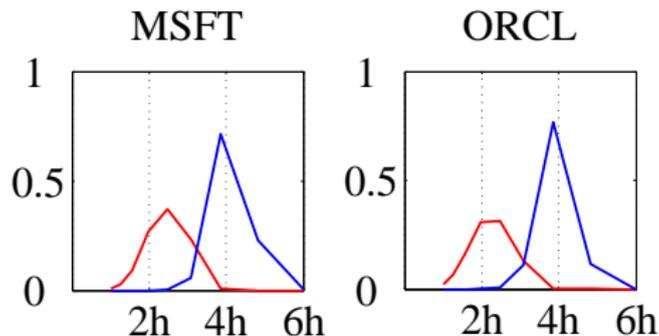


Figure 10: Relative frequency of the estimated interval length  $n_{\hat{k}}$  (discrete variable, in hours) given the **modest** ( $r = 0.5$ ) and **conservative** ( $r = 1$ ) risk case from 20080222 to 20081231 using the EACD(1,1) model with  $\rho = 0.25$



## Forecasting Trading Volumes

### Setup

- 5 stocks, forecasting period: 20080222 - 20081222 (210 days)
- Forecasts at each minute, horizon  $h = 1, \dots, 60$  min.
- EACD(1, 1) and WACD(1, 1),  $r \in \{0.5, 1\}$ ,  $\rho \in \{0.25, 0.5\}$

### Strategies

- LPA technique - prediction  $\hat{y}_{i+h}$ , error  $\hat{\varepsilon}_{i+h} = \check{y}_{i+h} - \hat{y}_{i+h}$
- 'Standard' method: 360 (1 day) or 1800 observations (1 week)  
- prediction  $\tilde{y}_{i+h}$ , error  $\tilde{\varepsilon}_{i+h} = \check{y}_{i+h} - \tilde{y}_{i+h}$



## Forecasting Superiority

- Diebold and Mariano (1995) tests, loss differential

$$d_h = \{d_{i+h}\}_{i=1}^n = \tilde{\varepsilon}_{i+h}^2 - \hat{\varepsilon}_{i+h}^2 \quad (2)$$

- Ratio of root mean squared errors

$$\sqrt{n^{-1} \sum_{i=1}^n \hat{\varepsilon}_{i+h}^2} / \sqrt{n^{-1} \sum_{i=1}^n \tilde{\varepsilon}_{i+h}^2} \quad (3)$$



## Forecasting Superiority

- Qualitative test ▶ EACD ▶ WACD

$$T_{ST,h} = \left\{ \sum_{i=1}^n \mathbf{1}(d_{i+h} > 0) - 0.5n \right\} / \sqrt{0.25n} \xrightarrow{\mathcal{L}} N(0, 1) \quad (4)$$

- Quantitative test,  $H_0 : E[d_h] = 0$

$$T_{DM,h} = \bar{d}_h / \sqrt{2\pi \hat{f}_{d_h}(0) / n} \xrightarrow{\mathcal{L}} N(0, 1) \quad (5)$$

$\bar{d}_h = n^{-1} \sum_{i=1}^n d_{i+h}$ ,  $\hat{f}_{d_h}(0)$  - spectral density estimate at frequency zero



## Forecasting Superiority

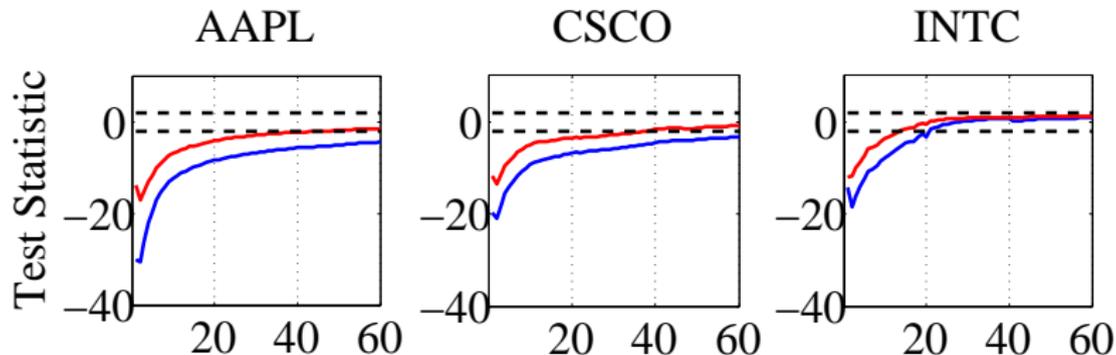


Figure 11: Test statistic  $T_{DM,h}$  across all 60 forecasting horizons from 20100222 to 20101222 (210 trading days): LPA against a fixed-window scheme using 360 and 1800 observations using the EACD(1,1) model,  $r = 0.5$  and  $\rho = 0.25$



## Forecasting Superiority

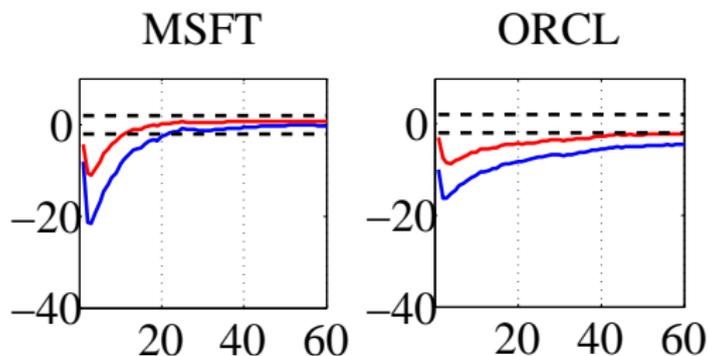


Figure 12: Test statistic  $T_{DM,h}$  across all 60 forecasting horizons from 20100222 to 20101222 (210 trading days): LPA against a fixed-window scheme using 360 and 1800 observations using the EACD(1,1) model,  $r = 0.5$  and  $\rho = 0.25$



## Forecasting Superiority

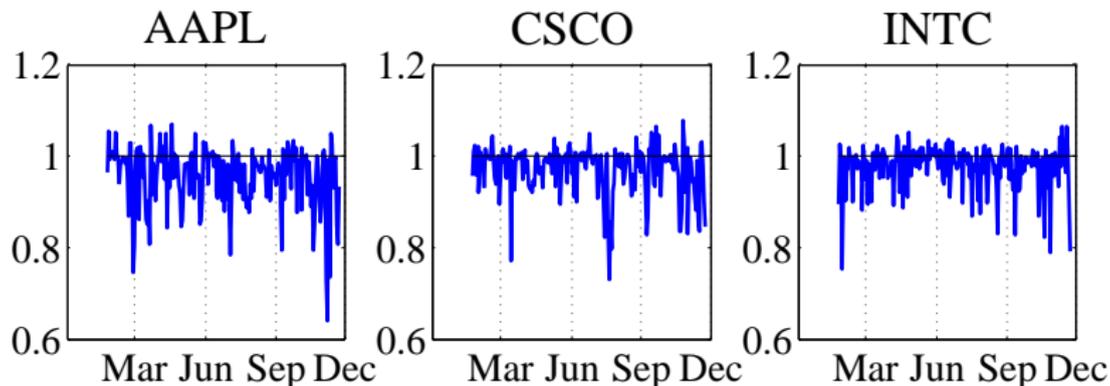


Figure 13: Ratio between the RMSPEs of the LPA and of a fixed-window approach (6 trading hours) over the sample from 20100222 to 20101222 (210 trading days) using an EACD(1,1) model,  $r = 0.5$  and  $\rho = 0.25$



## Forecasting Superiority

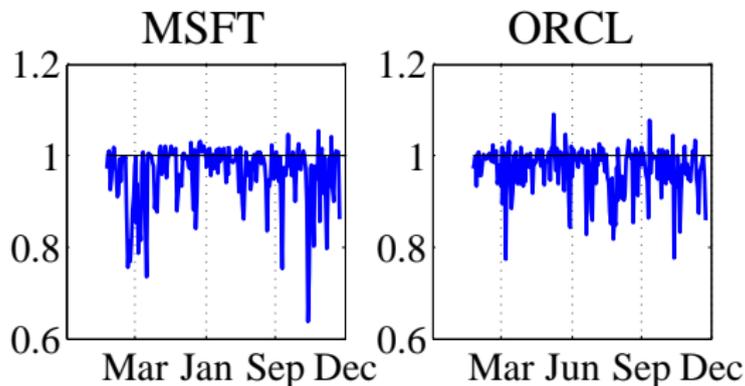


Figure 14: Ratio between the RMSPEs of the LPA and of a fixed-window approach (6 trading hours) over the sample from 20100222 to 20101222 (210 trading days) using an EACD(1,1) model,  $r = 0.5$  and  $\rho = 0.25$



## Forecasting Superiority

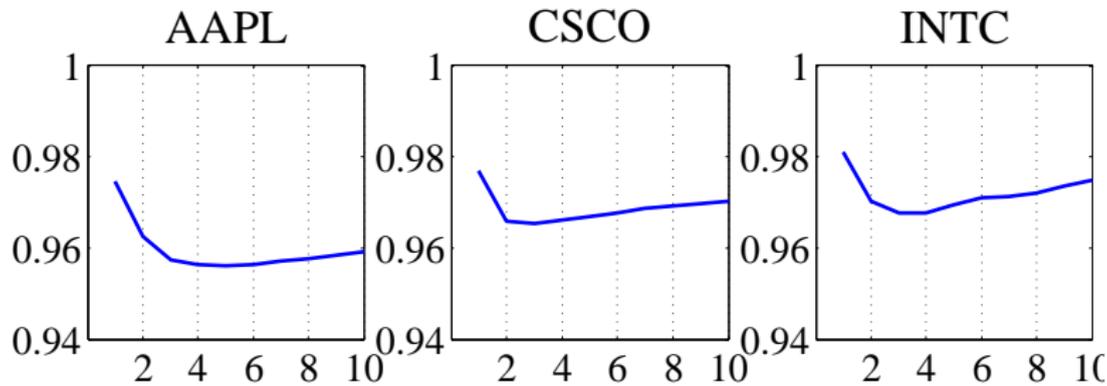


Figure 15: Ratio between the RMSPEs of the LPA and of a fixed-window approach (6 trading hours) across horizon from 20100222 to 20101222 (210 trading days) using an EACD(1,1) model,  $r = 0.5$  and  $\rho = 0.25$



## Forecasting Superiority

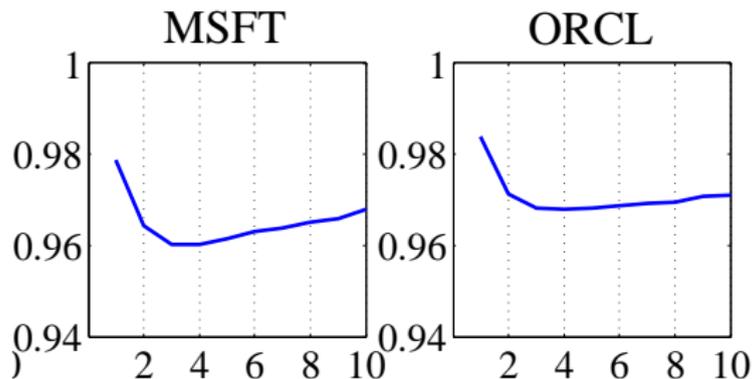


Figure 16: Ratio between the RMSPEs of the LPA and of a fixed-window approach (6 trading hours) across horizon from 20100222 to 20101222 (210 trading days) using an EACD(1,1) model,  $r = 0.5$  and  $\rho = 0.25$



## Conclusions

### (i) Localising MEM

- Time-varying parameters and estimation quality
- NASDAQ blue chips: AAPL, CSCO, INTC, MSFT and ORCL
- 'Conservative' adaptive estimation ( $r = 1$ ) requires 4-5 hours of data, modest risk approach ( $r = 0.5$ ) requires 2-3 hours

### (ii) Forecasting Trading Volumes

- LPA outperforms the 'standard' method
- Overall performance, horizon, trading day, tuning parameters



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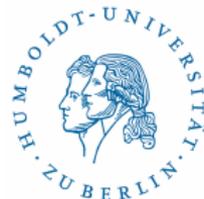
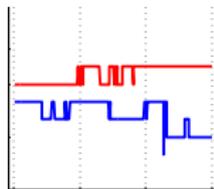
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## Example ▶ Research Questions

### Volume Weighted Average Price (VWAP) trading strategy

- Volume and market VWAP forecasting strategies
  - (i) 'Standard' method - fixed estimation window (one day/week)
  - (ii) LPA technique with selected interval of homogeneity
- VWAP trading strategy
  - ▶ Minimize liquidity impact costs (trading large orders at an inferior price compared to smaller orders) - order splitting
  - ▶ Benchmark the order price against (best predicted) market VWAP - trade at better than the market VWAP



## Exponential-ACD (EACD)

▸ ACD

▸ Parameter Estimation

□ Engle and Russel (1998),  $\varepsilon_i \sim \text{Exp}(1)$

$$L_I(y; \theta_E) = \sum_{i=\max(p,q)+1}^n \left( -\log \mu_i - \frac{y_i}{\mu_i} \right) \mathbf{1}\{i \in I\} \quad (6)$$

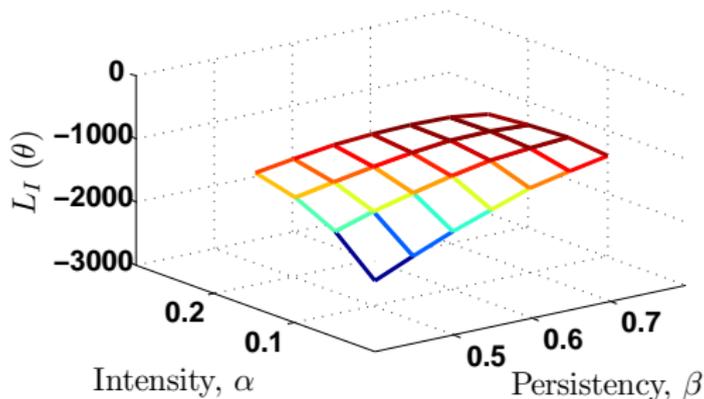


Figure 17: Log likelihood - EACD(1,1),  $\theta_E^* = (0.10, 0.20, 0.65)$



## Weibull-ACD (WACD)

▶ ACD

▶ Parameter Estimation

□ Engle and Russel (1998),  $\varepsilon_i \sim \mathcal{G}(s, 1)$

$$L_I(y; \theta_w) = \sum_{i \in I} \left[ \log \frac{s}{y_i} + s \log \frac{\Gamma(1+1/s) y_i}{\mu_i} - \left\{ \frac{\Gamma(1+1/s) y_i}{\mu_i} \right\}^s \right] \mathbb{I}\{i \in I\} \quad (7)$$

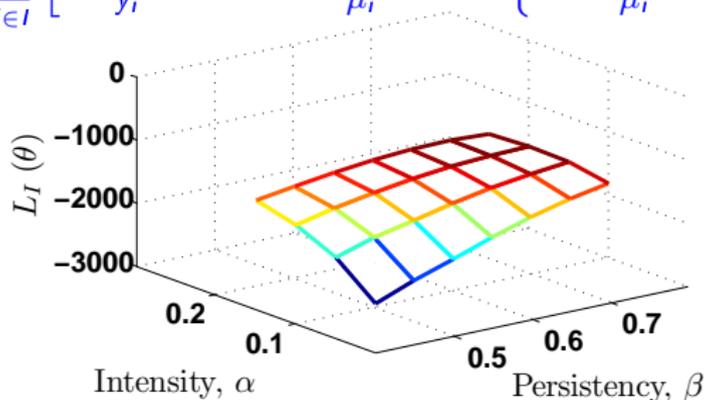
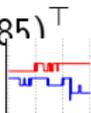


Figure 18: Log likelihood - WACD(1,1),  $\theta_w^* = (0.10, 0.20, 0.65, 0.85)^T$   
Local Adaptive MEM



## Parameter Dynamics

Estimation window	EACD(1, 1)			WACD(1, 1)		
	Q25	Q50	Q75	Q25	Q50	Q75
1 week	0.85	0.89	0.93	0.82	0.88	0.92
2 days	0.77	0.86	0.92	0.74	0.84	0.91
1 day	0.68	0.82	0.90	0.63	0.79	0.89
3 hours	0.54	0.75	0.88	0.50	0.72	0.87
2 hours	0.45	0.70	0.86	0.42	0.67	0.85
1 hour	0.33	0.58	0.80	0.31	0.57	0.80

Table 1: Quartiles of estimated persistence levels  $(\tilde{\alpha} + \tilde{\beta})$  for all five stocks at each minute from 22 February to 31 December 2008 (215 trading days). Calibration period: 2 January 2008 - 21 February 2008



## Parameter Dynamics ▶ Critical Values

Model	Low Persistence			Moderate Persistence			High Persistence		
	Q25	Q50	Q75	Q25	Q50	Q75	Q25	Q50	Q75
EACD, $\tilde{\alpha}$	0.28	0.22	0.18	0.30	0.23	0.19	0.31	0.24	0.20
EACD, $\tilde{\beta}$	0.56	0.62	0.67	0.59	0.66	0.71	0.62	0.68	0.73
WACD, $\tilde{\alpha}$	0.28	0.21	0.17	0.30	0.23	0.18	0.32	0.24	0.19
WACD, $\tilde{\beta}$	0.54	0.60	0.65	0.58	0.65	0.70	0.60	0.68	0.74

Table 2: Quartiles of 774,000 estimated ratios  $\tilde{\beta} / (\tilde{\alpha} + \tilde{\beta})$  (estimation windows covering 1800 observations) for all five stocks at each minute from 22 February to 31 December 2008 (215 trading days) conditional on the persistence level (low, moderate or high). Calibration period: 2 January 2008 - 21 February 2008



## Parameter Dynamics - Summary

- MEM parameters, their variability and distribution properties change over time
- Longer local estimation windows increase estimation precision and the misspecification risk
- Tradeoff between estimation (in)efficiency and local flexibility



## Critical Values, $\mathfrak{z}_k$ ▶ Critical Values

- Simulate  $\mathfrak{z}_k$  - homogeneity of the interval sequence  $l_0, \dots, l_k$
- 'Propagation' condition (under  $H_0$ )

$$E_{\theta^*} \left| L_{l_k}(\tilde{\theta}_{l_k}) - L_{l_k}(\hat{\theta}_{l_k}) \right|^r \leq \rho_k \mathcal{R}_r(\theta^*), \quad k = 1, \dots, K \quad (8)$$

$\rho_k = \rho k / K$  for given significance level  $\rho$

- Check  $\mathfrak{z}_k$  for (nine) different  $\theta^*$  ▶ Parameter Dynamics - Quartiles
  - ▶ EACD and WACD,  $K \in \{8, 13\}$ ,  $r \in \{0.5, 1\}$ ,  $\rho \in \{0.25, 0.50\}$
  - ▶ Findings:  $\mathfrak{z}_k$  are virtually invariable w.r.t.  $\theta^*$  given a scenario  
Largest differences at first two or three steps



## Local Change Point Detection ▶ LCP

**Example:** Trading volumes aggregated over 1-min periods

- Scheme with  $(K + 1) = 14$  intervals and fix  $i_0$
- Assume  $I_0 = 60\text{min.}$  is homogeneous
- $H_0$  : parameter homogeneity within  $I_1 = 75\text{min.}$ 
  - ▶ Define  $J_1 = I_1 \setminus I_0$  - observations from  $y_{i_0-75}$  up to  $y_{i_0-60}$
  - ▶ For each  $\tau \in J_1$  fit log likelihoods over  $A_{1,\tau}$ ,  $B_{1,\tau}$  and  $I_2$
  - ▶ Find the largest likelihood ratio -  $T_{I_1, J_1}$



## Forecasting Superiority

▶ Forecasting Superiority

	EACD(1, 1)				
	AAPL	CSCO	INTC	MSFT	ORCL
1 week					
$r = 0.5, \rho = 0.25$	-38.9	-28.6	-24.1	-33.8	-31.4
$r = 0.5, \rho = 0.50$	-38.7	-28.7	-24.2	-33.8	-31.4
$r = 1.0, \rho = 0.25$	-40.5	-31.4	-23.3	-39.1	-32.8
$r = 1.0, \rho = 0.50$	-40.4	-31.3	-23.3	-39.0	-32.9
1 day					
$r = 0.5, \rho = 0.25$	-10.8	-6.0	-13.1	-5.7	-15.1
$r = 0.5, \rho = 0.50$	-10.6	-6.0	-12.8	-5.5	-15.0
$r = 1.0, \rho = 0.25$	-6.9	-8.6	-8.7	-4.4	-12.9
$r = 1.0, \rho = 0.50$	-7.1	-8.6	-8.8	-4.4	-13.0

Table 3: Largest (in absolute terms) test statistic  $T_{ST,h}$  across all 60 forecasting horizons from 20080222-20081222 (210 trading days). LPA against a fixed-window scheme using 360 and 1800 observations



## Forecasting Superiority ▶ Forecasting Superiority

	WACD(1, 1)				
	AAPL	CSCO	INTC	MSFT	ORCL
1 week					
$r = 0.5, \rho = 0.25$	-22.6	-25.7	-20.2	-26.7	-26.6
$r = 0.5, \rho = 0.50$	-22.7	-25.5	-20.3	-26.7	-26.6
$r = 1.0, \rho = 0.25$	-27.9	-30.8	-21.5	-31.3	-29.8
$r = 1.0, \rho = 0.50$	-28.1	-30.8	-21.5	-31.5	-29.7
1 day					
$r = 0.5, \rho = 0.25$	-6.4	-3.5	-6.1	-4.9	-12.6
$r = 0.5, \rho = 0.50$	-6.3	-3.2	-6.2	-4.8	-12.7
$r = 1.0, \rho = 0.25$	-4.1	-5.1	-6.5	-4.2	-11.5
$r = 1.0, \rho = 0.50$	-3.9	-5.2	-6.5	-4.1	-11.4

Table 4: Largest (in absolute terms) test statistic  $T_{ST,h}$  across all 60 forecasting horizons from 20080222-20081222 (210 trading days). LPA against a fixed-window scheme using 360 and 1800 observations

