CDO Surfaces Dynamics

Barbara Choroś-Tomczyk
Wolfgang Karl Härdle
Ostap Okhrin

Ladislaus von Bortkiewicz
Chair of Statistics
C.A.S.E. - Center for Applied Statistics and Economics
Humboldt-Universität zu Berlin

iTraxx over Time

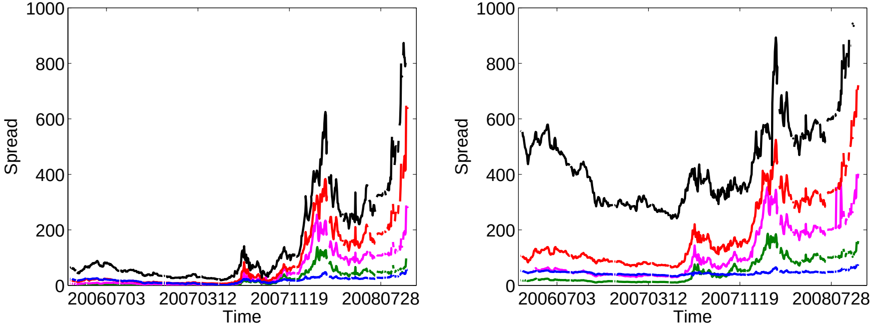


Figure 1: Spreads of iTraxx tranches, Series 5, maturity 5 (left) and 10 (right) years, data from 20060407-20081103. Tranches: 1, **2**, 3, 4, 5.

iTraxx Spread Surface

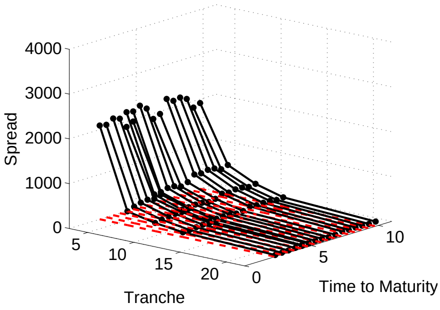


Figure 2: Spreads of tranches of all series observed on 20080909 (left) and 20090119 (right).

Research Goals

- Modelling the dynamics of CDO surfaces
    - spread surfaces
    - base correlation surfaces
- Applications in trading

Dynamic Semiparametric Factor Model

Applications:

1. Implied volatility surfaces in M. R. Fengler, W. Härdle and E. Mammen, *JFE* (2007) and B. Park, E. Mammen, W. Härdle, and S. Borak, *JASA* (2009)
2. Risk neutral densities in E. Giacomini, W. Härdle, and V. Krätschmer, *AStA* (2009)
3. Limit order book in W. Härdle, N., Hautsch, and A. Mihoci, *JEF* (2012)
4. Variance swaps in K. Detlefsen and W. Härdle, *QF* (2013)
5. fMRI images in A. Myšicková, S. Song, P. Majer, P. Mohr, H. Heekeren, W. Härdle, *Psychometrika* (2013)

Outline

1. Motivation ✓
2. CDOs
3. DSFM
4. Empirical Study
5. Applications
6. Conclusions

Risk Transfer

iTraxx Europe

- A static portfolio of 125 equally weighted CDS on European entities;
- Sectors: Consumer (30), Financial (25), TMT (20), Industrials (20), Energy (20), Auto (10);
- New series of iTraxx Europe issued every 6 months (March and September) and the underlying reference entities are reconstituted;
- Maturities: 3Y, 5Y, 7Y, 10Y.

Gaussian Copula Model

Default times are modelled from the Gaussian vector $(X_1, \dots, X_d)^\top$:

$$X_i = \sqrt{\rho}Y + \sqrt{1-\rho}Z_i,$$

where Y (systematic risk factor), $\{Z_i\}_{i=1}^d$ (idiosyncratic risk factors) are i.i.d. $N(0,1)$. Hence:

$$(X_1, \dots, X_d)^\top \sim N(0, \Sigma),$$

with

$$\Sigma = \begin{pmatrix} 1 & \rho & \cdots & \rho \\ \rho & 1 & \cdots & \rho \\ \vdots & \vdots & \ddots & \vdots \\ \rho & \rho & \cdots & 1 \end{pmatrix}.$$

Large Portfolio Framework

Assume that

- obligors have the same default probability and LGD,
- one dependence parameter ρ,
- d very large.

Computations are simplified significantly when the portfolio loss distribution is approximated:

$$\mathrm{P}(\tilde{L} \leq x) = \Phi\left\{\frac{\sqrt{1-\rho}\,\Phi^{-1}(x) - \Phi^{-1}(p)}{\sqrt{\rho}}\right\}.$$

Correlation's Types

Compound correlation $\rho(l_j, u_j)$, $j = 1, \ldots, J$.

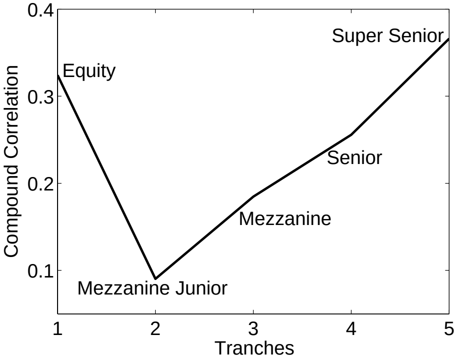


Figure 3: Implied correlation smile in the Gaussian one factor model, 20071022.

Correlation's Types

Base correlation (BC) $\rho(0, u_j)$, $j = 1, \dots, J$.

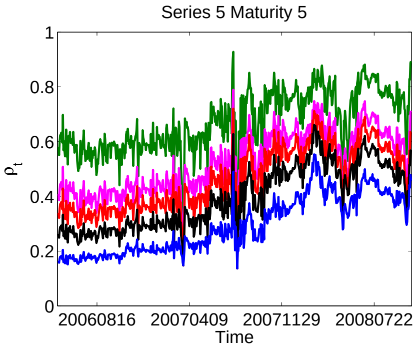
Represent the expected loss $\mathrm{E}\{L_{(l_j,u_j)}\}$ as a difference:

$$\mathrm{E}\{L_{(l_j,u_j)}\} = \mathrm{E}_{\rho(0,u_j)}\{L_{(0,u_j)}\} - \mathrm{E}_{\rho(0,l_j)}\{L_{(0,l_j)}\},\ j = 2, \dots, J.$$

of the expected losses of two fictive tranches $(0, u_j)$ and $(0, l_j)$.
Bootstrapping process: $\mathrm{E}\{L_{(0,3\%)}\}$ is traded on the market,

$$\begin{aligned}
\mathrm{E}\{L_{(3\%,6\%)}\} &= \mathrm{E}_{\rho(0,6\%)}\{L_{(0,6\%)}\} - \mathrm{E}_{\rho(0,3\%)}\{L_{(0,3\%)}\},\\
\mathrm{E}\{L_{(6\%,9\%)}\} &= \mathrm{E}_{\rho(0,9\%)}\{L_{(0,9\%)}\} - \mathrm{E}_{\rho(0,6\%)}\{L_{(0,6\%)}\}, \dots
\end{aligned}$$

Base Correlations over Time



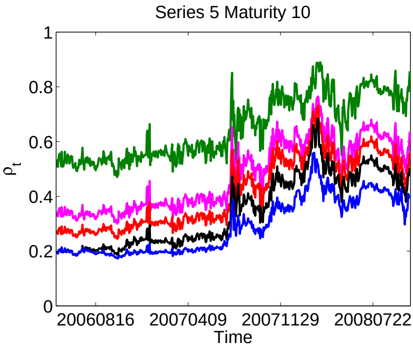


Figure 4: BC of iTraxx tranches, Series 5, maturity 5 (left) and 10 (right) years, data from 20060510-20081023. Tranches: 1, **2**, 3, 4, 5.

Base Correlation Surfaces

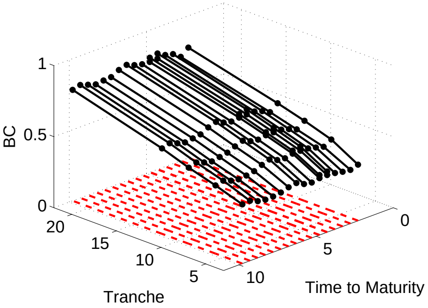


Figure 5: Implied base correlations on day 20080909 (left) and 20090119 (right).

Dynamic Semiparametric Factor Model

$$Y_{t,k} = m_0(X_{t,k}) + \sum_{l=1}^{L} Z_{t,l} m_l(X_{t,k}) + \varepsilon_{t,k} = Z_t^\top A \psi(X_{t,k}) + \varepsilon_{t,k}$$

$Y_{t,k}$	log-spreads and Z-transformed BC on day t, $t = 1, \ldots, T$
k	intra-day numbering of BCs on day t, $k = 1, \ldots, K_t$
$X_{t,k}$	two-dimensional vector of the tranche seniority and the time-to-maturity
m_l	factor functions, time invariant, nonparametric estimation
$Z_{t,l}$	time series, $l = 0, \ldots, L$, dynamic behavior
$\psi(X_{t,k})$	tensor B-spline basis
A	coefficient matrix

Estimation

Using an iterative algorithm:

$$(\widehat{Z}_t, \widehat{A}) = \arg\min_{Z_t, A} \sum_{t=1}^{T} \sum_{k=1}^{K_t} \left\{ Y_{t,k} - Z_t^\top A\psi(X_{t,k}) \right\}^2$$

Selection of L, the numbers of spline knots R_1, R_2 and the orders of splines k_1, k_2 by maximising the explained variance criterion:

$$\mathrm{EV}(L, R_1, r_1, R_2, r_2) = 1 - \frac{\sum_{t=1}^{T} \sum_{k=1}^{K_t} \left\{ Y_{t,k} - \sum_{l=1}^{L} Z_{t,l} m_l(X_{t,k}) \right\}^2}{\sum_{t=1}^{T} \sum_{k=1}^{K_t} \left\{ Y_{t,j} - \widetilde{m}_0(X_{t,k}) \right\}^2},$$

where $\widetilde{m}_0$ is an empirical mean surface.

DSFM without the Mean Factor

Reduce the number of factors estimated in the iterative algorithm by first subtracting the empirical mean $\widetilde{m}_0$ and then fitting the DSFM:

$$Y_{t,k} = \widetilde{m}_0(X_{t,k}) + \sum_{l=1}^{L} Z_{t,l} m_l(X_{t,k}) + \varepsilon_{t,k} = \widetilde{m}_0(X_{t,k}) + Z_t^\top A\psi(X_{t,k}) + \varepsilon_{t,k},$$

where m_l are new factor functions, $l = 1, \ldots, L$.

Data

- Series 2-10
- Maturities 5, 7, 10Y
- 1004 days between 20050330-20090202
- 49 502 data points

Year	3Y	5Y	7Y	10Y
2005	0	1478	715	1532
2006	181	3998	3739	4005
2007	75	5155	5170	5172
2008	232	5904	5916	5932
2009	0	260	263	263
All	488	16740	15803	16840

Table 1: Number of observed values of iTraxx tranches in the period 20050330-20090202.

Data Preparation

- Convert the upfront payment quotes of the equity tranche to standard spreads using the Gaussian copula model.
- Since the data are monotone in the tranche seniority direction and positive, use log-spreads and Z-transformed-BC.

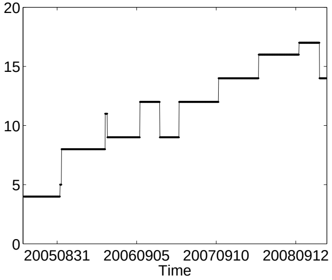
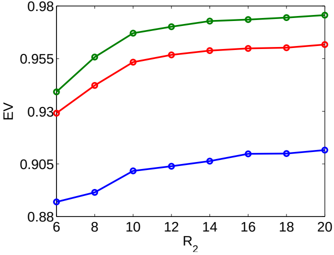
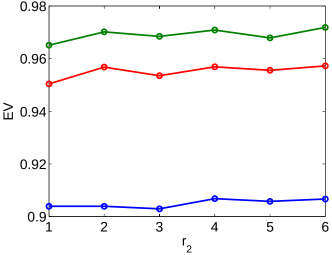


Figure 6: Daily number of curves for every surface during the period 20050330-20090202.

DSFM for Z-transformed-BC





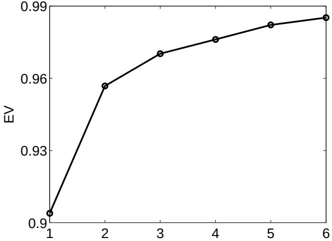


Figure 7: Proportion of the explained variance as a function of R_2 (up left) with $r_2 = 2$, as a function of r_2 (up right) with $R_2 = 10$, as a function of L (down) for $L = 1$, $L = 2$, $L = 3$, $r_1 = 2$ and $R_1 = 5$.

DSFM w/o Mean F. for Z-transformed-BC

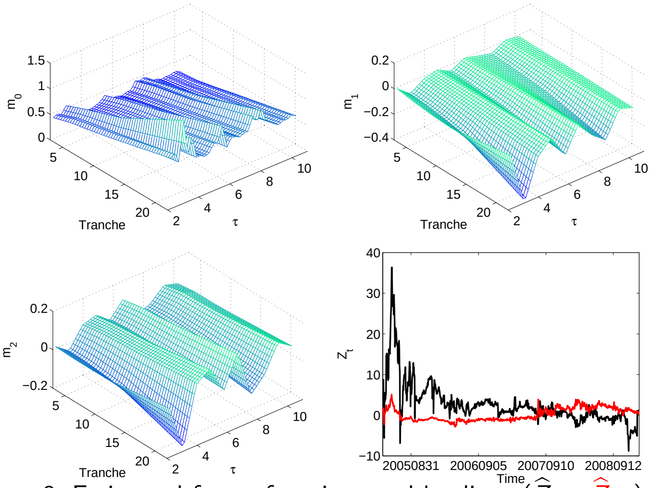


Figure 8: Estimated factor functions and loadings ($\widehat{Z}_{t,1}$, $\widehat{Z}_{t,2}$). App

DSFM Estimation Results

For DSFM for both data types

- $\widehat{Z}_{t,1}$ is a slope-curvature factor
- $\widehat{Z}_{t,2}$ is a shift factor

Model	Log-Spr	Z-BC
DSFM	0.016	0.004
DSFM w/o mean f.	0.045	0.006

Table 2: Mean squared error of the in-sample fit.

DSFM without the mean factor Fit

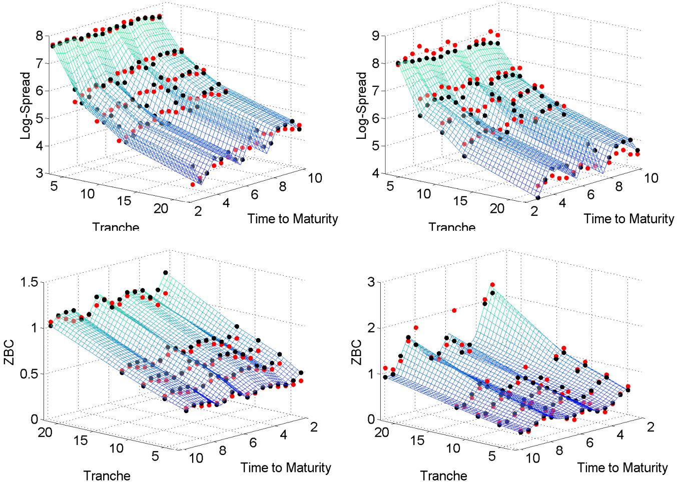


Figure 9: In-sample **fit** of the models to data on 20080909 and 20090119.

Curve Trades

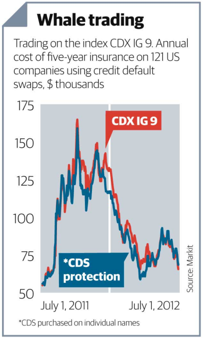
So, how can I make money with this?

Combine tranches of different time to maturity, see Felsenheimer et al. (2004) and Kakodkar et al. (2006):

- Flattener – sell a long-term tranche, buy a short-term tranche
  Example: sell 10Y 3-6% and buy 5Y 6-9%
  Outlook: bullish long-term, bearish short-term
- Steepener – opposite trade

JP Morgan Trading Loss, May 2012

J.P. Morgan's flattener – bought 5Y CDX IG 9 index, sold 10Y CDX IG 9 index in a 3:1 ratio. The final loss reached $6.2 billion.



Flattener

Sell protection at $s_1(t_0)$ for the period $[t_0, T_1]$ and buy protection at $s_2(t_0)$ for $[t_0, T_2]$, $T_1 > T_2$. At t_0 for $\ell = 1, 2$:

$$\mathrm{MTM}_\ell(t_0) = \sum_{t=t_1}^{T_\ell} \beta(t_0, t)\left[s_\ell(t_0)\Delta t \mathrm{E}\{F_\ell(t)\} - \mathrm{E}\{L_\ell(t) - L_\ell(t-\Delta t)\}\right] = 0.$$

At $\tilde{t} > t_0$, the market quotes $s_\ell(\tilde{t})$ and

$$\mathrm{MTM}_\ell(\tilde{t}) = \{s_\ell(t_0) - s_\ell(\tilde{t})\} \sum_{t=\tilde{t}_1}^{T_\ell} \beta(\tilde{t}, t)\Delta t \mathrm{E}\{F_\ell(t)\}.$$

Curve Trade

- A positive MTM means a positive value to the protection seller.
- If the protection seller closes the position at time $\tilde{t}$, then receives from the protection buyer $\mathrm{MTM}_{\ell}(\tilde{t})$.
- Flattener-trader aims to maximize the total MTM value

$$\mathrm{PL}(\tilde{t}) = \mathrm{MTM}_1(\tilde{t}) - \mathrm{MTM}_2(\tilde{t}).$$

Risk in Curve Trades

- If one buys 5Y 6-9% and sells 10Y 6-9%, then the trade is hedged for default until the maturity of the 5Y tranche. Defaults that emerge from 10Y 6-9% are covered by 5Y 6-9% till it expires.
- Series differ in the composition of the collateral.
- If one buys 5Y 6-9% and sells 10Y 3-6%, then these tranches provide protection of different portion of portfolio risk. If there is any default in 10Y 3-6%, then we must deliver a payment obligation and incur a loss.

Empirical Study

Idea

- Use DSFM to forecast spread and BC surfaces
- Calculate forecasted MTM surfaces
- Recover those tranches that maximise P&L

Remarks

- Because of many missing data and short data histories, the standard econometric methods cannot be used for the forecasting.
- Consider trades that generate no or a positive carry – the spread of the long tranche doesn't exceed the spread of the short tranche.
- Do not account for default payments (no data of historical defaults in iTraxx), do not account for the positive carry.

Forecasting with DSFM in Rolling Windows

Let Y_t be log-spreads or Z-transformed-BC.

- Consider a rolling window of $w = 250$.
- Estimate the DSFMs using $\{Y_\nu\}_{\nu=t-w+1}^{t}$ for $t = w, \ldots, T-h$.
- As a result, we get $T - w + 1$ times $\widehat{m} = (\widehat{m}_0, \ldots, \widehat{m}_L)^\top$ and $\widehat{Z}_t = (\widehat{Z}_{t,0}, \ldots, \widehat{Z}_{t,L})^\top$ of length w.
- Compute h-day forecast of the factor loadings using VAR.
- Due to the fixed issuing scheme, $X_{t+h,k}$ is not forecasted.
- Calculate the forecast $\widehat{Y}_{t+h}$ from the forecast $\widehat{Z}_{t+h}$.
- Transform $\widehat{Y}_{t+h}$ suitably to get $\hat{s}(t+h)$ or $\hat{\rho}(t+h)$.

Forecasting MTM Surfaces

For predicted $\{\hat{s}_k(t), \hat{\rho}_k(t)\}$, $t = w+h, \ldots, T$, $k = 1, \ldots, K_t$, compute $\widehat{\mathrm{MTM}}_k(t)$, where the initial spread $s_k(t_0)$ is observed on $t_0 = t - h$.

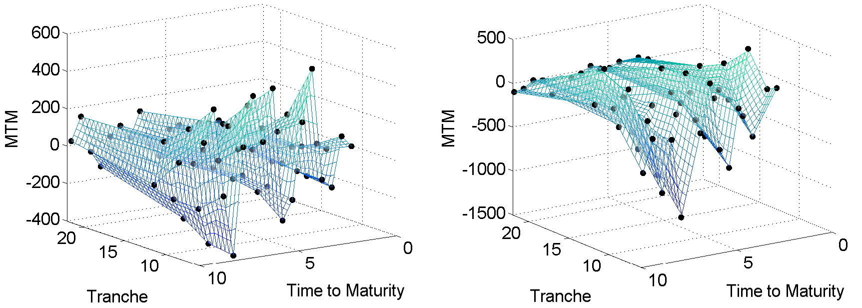


Figure 10: MTM surfaces on 20080909 (left) and 20090119 (right) calculated using one-day spread and BC predictions obtained with the DSFM.

Transaction Costs

Calculate the ask (bid) spread by increasing (reducing) the observed spread by the following percentage:

Maturity	1	2	3	4	5
5Y	1.88	1.78	2.52	3.77	6.28
7Y	1.49	1.65	2.31	2.97	4.87
10Y	1.41	1.66	1.83	2.52	4.09

Table 3: Average bid-ask spread excess over the mid spread as a percentage of the mid spread for Series 8 during the period 20070920-20090202.

Trading Strategies

Construct a curve trade

1. Fit and forecast the DSFM models to spreads and BC.
2. Calculate h-day forecasts of the MTM surfaces.
3. Recover which two tranches optimize a given strategy.

Strategies – restrict the choice to a flattener (or a steepener) with

1. a fixed tranche and fixed maturities,
2. a fixed tranche and all maturities,
3. all tranches and fixed maturities,
4. all tranches and all maturities (no restrictions),

or allow to combine flatteners and steepeners.

Backtesting

- Consider the time horizons $h = 1$, 5, 20 days.
- For the tranches that optimize a given strategy, check the corresponding historical market spreads, calculate the resulting MTM values, and the realised P&L.

Mean of Daily Gains in Percent

Strategy	DSFM						DSFM without the mean factor					
	1 day		1 week		1 month		1 day		1 week		1 month	
	LZ	Z	LZ	Z	LZ	Z	LZ	Z	LZ	Z	LZ	Z
FS-AllT-AllM	0.29	0.35	0.10	0.13	0.05	0.04	0.30	0.30	0.11	0.13	0.04	0.03
FS-T2-AllM	0.29	0.33	0.13	0.14	0.06	0.05	0.33	0.28	0.12	0.13	0.05	0.04
FS-T3-AllM	0.19	0.22	0.07	0.08	0.03	0.02	0.18	0.23	0.07	0.07	0.02	0.02
FS-T4-AllM	0.14	0.17	0.04	0.05	0.02	0.01	0.12	0.18	0.04	0.05	0.01	0.01
FS-T5-AllM	0.09	0.11	0.04	0.04	0.02	0.01	0.08	0.11	0.03	0.04	0.01	0.01
F-T2-AllM	0.30	0.34	0.12	0.12	0.06	0.04	0.28	0.32	0.12	0.11	0.05	0.04
F-T3-AllM	0.16	0.20	0.06	0.07	0.02	0.01	0.16	0.20	0.06	0.07	0.02	0.01
F-T4-AllM	0.10	0.15	0.03	0.04	0.01	0.01	0.10	0.15	0.03	0.04	0.01	0.01
F-T5-AllM	0.09	0.10	0.03	0.03	0.01	0.01	0.08	0.10	0.03	0.03	0.01	0.01
S-T2-AllM	0.39	0.43	0.15	0.17	0.07	0.06	0.45	0.46	0.13	0.16	0.05	0.06
S-T3-AllM	0.27	0.31	0.09	0.10	0.04	0.03	0.30	0.35	0.09	0.09	0.02	0.02
S-T4-AllM	0.20	0.25	0.06	0.07	0.03	0.02	0.20	0.24	0.05	0.06	0.01	0.01
S-T5-AllM	0.12	0.15	0.04	0.04	0.02	0.02	0.12	0.16	0.04	0.04	0.01	0.02
F-AllT-105	0.20	0.21	0.07	0.09	0.03	0.02	0.19	0.21	0.06	0.08	0.02	0.02
F-AllT-107	0.22	0.26	0.07	0.08	0.03	0.03	0.25	0.25	0.08	0.08	0.03	0.03
F-AllT-75	0.15	0.15	0.04	0.06	0.01	-0.00	0.14	0.15	0.04	0.05	0.01	0.00
S-AllT-510	0.16	0.17	0.05	0.08	0.02	0.01	0.16	0.18	0.05	0.08	0.01	0.00
S-AllT-710	0.17	0.23	0.05	0.10	0.02	0.03	0.21	0.25	0.07	0.09	0.02	0.02
S-AllT-57	0.11	0.13	0.03	0.03	0.01	-0.01	0.12	0.13	0.03	0.03	0.00	-0.01

Table 4: Calculations based on predictions of log-spreads and Z-transformed BCs marked as LZ; based only on Z-transformed BCs marked as Z.

Investor's Strategy

Follow a certain strategy over a year and constantly rebalance the portfolio. At t_0 enter an optimal (according to the DSFM) curve trade for h-day horizon. At $t_0 + h$ chose:

1. keep the current position for the next h-days,
2. close the current position and enter a new one.

Assume a margin of 10% of your notional. Every time the position is closed, add to the margin the realized P&L. If margin ≤ 0, quit the trade.

Investor's Strategy

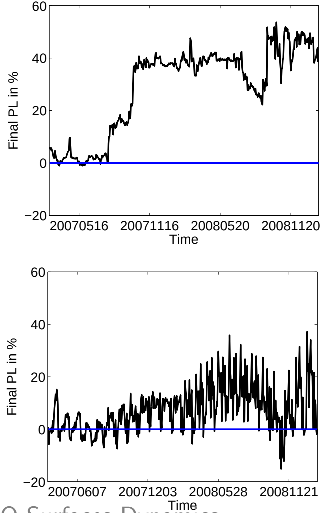
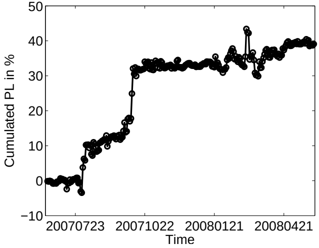
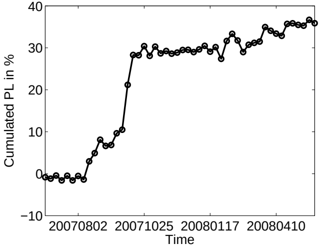


Figure 11: Combined flatteners and steepeners from all tranches and all maturities. Closing profits after one year. Rebalancing after: 1 day (upper left), 1 week (upper right), 1 month (lower). Calculations based on the DSFM predictions of log-spreads and Z-transformed BCs.

Investor's Strategy





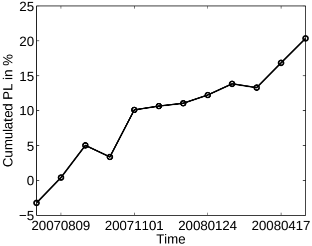


Figure 12: Daily cumulated P&L over one year 20070614–20080529. Rebalancing after: 1 day (upper left), 1 week (upper right), 1 month (lower). Calculations based on the DSFM predictions of log-spreads and Z-transformed BCs.

Conclusions

- Investigated evolution over time of tranche spread surfaces and base correlation surfaces using the DSFM.
- Empirical study is conducted using an extensive data set of 49,502 observations of iTraxx Europe tranches in 2005-2009.
- Proposed a modification to the classic DSFM.
- Both DSFMs successfully reproduce the dynamics in data.
- Used DSFM in constructing the curve trades.
- Analysed the performance of 43 strategies that combine different positions, tranches, and maturities.
- Backtesting showed high daily gains of the resulting curve trades.

References

C. Bluhm and L. Overbeck
Structured Credit Portfolio Analysis, Baskets and CDOs
Chapman & Hall/Crc Financial Mathematics Series, 2006

J. Felsenheimer, P. Gisdakis, and M. Zaiser
DJ iTraxx: Credit at its best!
Credit derivatives special HVB Corporates & Markets, 2004

M. R. Fengler, W. K. Härdle, and E. Mammen
A semiparametric factor model for implied volatility surface dynamics
Journal of Financial Econometrics, 2007

C. Gourieroux and J. Jasiak
Dynamic factor models
Econometric Reviews, 2001

A. Kakodkar, S. Galiani, J. G. Jónsson, and A. Gallo
Credit derivatives handbook, A guide to the exotics credit derivatives market
Technical report Merrill Lynch, 2006

B. Park, E. Mammen, W. K. Härdle, and S. Borak
Dynamic Semiparametric Factor Models
Journal of the American Statistical Association, 2009

CDO Surfaces Dynamics

Barbara Choroś-Tomczyk
Wolfgang Karl Härdle
Ostap Okhrin

Ladislaus von Bortkiewicz
Chair of Statistics
C.A.S.E. - Center for Applied Statistics and Economics
Humboldt-Universität zu Berlin

DSFM for Log-Spreads

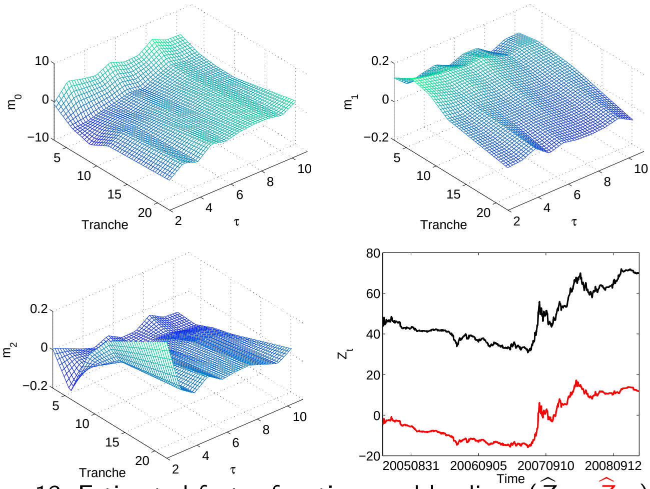


Figure 13: Estimated factor functions and loadings ($\widehat{Z}_{t,1}$, $\widehat{Z}_{t,2}$).

Talk

DSFM without the Mean Factor for Log-Spreads

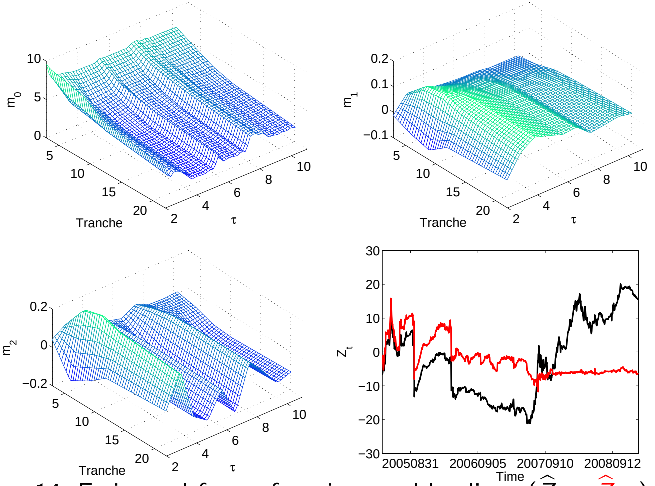


Figure 14: Estimated factor functions and loadings $(\widehat{Z}_{t,1}, \widehat{Z}_{t,2})$.

Talk

DSFM for Z-transformed-BC

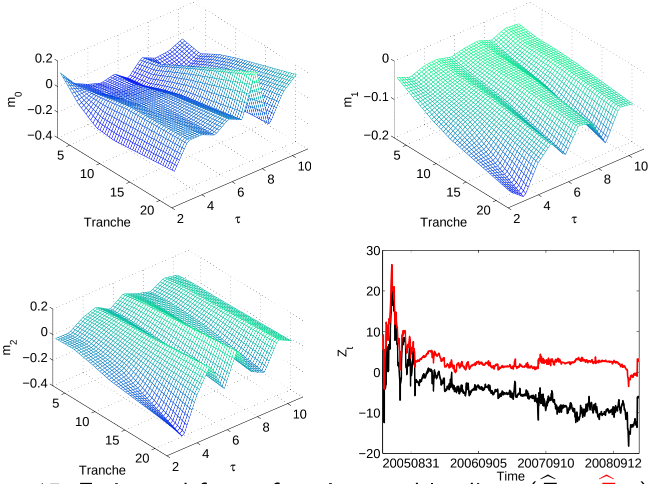


Figure 15: Estimated factor functions and loadings ($\widehat{Z}_{t,1}$, $\widehat{Z}_{t,2}$).

Talk