TENET: Tail-Event-driven NETwork Risk

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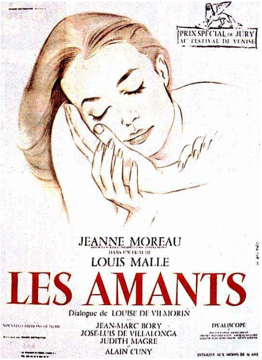
What is Systemic Risk?

"I know it when I see it".

Justice Potter Stewart, 1964.



What is Systemic Risk?

Systemic risk is a "risk of financial instability so widespread that it impairs the functioning of a financial system to the point where economic growth and welfare suffer materially".

ECB, Financial Network and Financial Stability, 2010.

"Financial institutions are **systemically important** if the failure of the firm to meet its obligations to creditors and customers would have significant adverse consequences for the financial system and the broader economy".

Daniel Tarullo, Regulatory Restructuring, 2009.

What is Systemic Risk?

Figure 1: Systemic Risk?

CoVaR as a Systemic Risk Measure

Step 1. Estimate linear quantile regressions

$$X_{i,t} = \alpha_i + \gamma_i M_{t-1} + \varepsilon_{i,t},$$
$$X_{j,t} = \alpha_{j|i} + \gamma_{j|i} M_{t-1} + \beta_{j|i} X_{i,t} + \varepsilon_{j|i,t},$$

where

- $X_{i,t}$ is the log return of a financial institution i,
- M_{t-1} are lagged macro state variables.

Adrian and Brunnermeier (2011)

Macro state variables

CoVaR as a Systemic Risk Measure

Step 2. Generate predicted values under assumption $F^{-1}_{\varepsilon_{i,t}}(\tau|M_{t-1}) = 0$ and $F^{-1}_{\varepsilon_{j|i,t}}(\tau|M_{t-1}, X_{i,t}) = 0$, $\tau = (0,1)$,

$$\widehat{\text{VaR}}^{\tau}_{i,t} = \hat{\alpha}_i + \hat{\gamma}_i M_{t-1},$$
$$\widehat{\text{CoVaR}}^{\tau}_{j|i,t} = \hat{\alpha}_{j|i} + \hat{\gamma}_{j|i} M_{t-1} + \hat{\beta}_{j|i} \widehat{\text{VaR}}^{\tau}_{i,t}.$$

Adrian and Brunnermeier (2011)

Elements of Systemic Risk

- Network Effects
- Single Institution's Contribution to Systemic Risk
- Single Institution's Exposure to Systemic Risk

Challenges

- Linear tail behavior
    - Adrian and Brunnermeier (2011)
    - Acharya et al. (2012)
    - Brownlees and Engle (2012)
- Linear tail behavior in **high dimensions**
    - Hautsch, Schaumburg, and Schienle (2014)
- **Non-linear** tail behavior in **ultra-high dimensions**
    - Method by Fan, Härdle, Wang, and Zhu (2014)

Non-Linearity

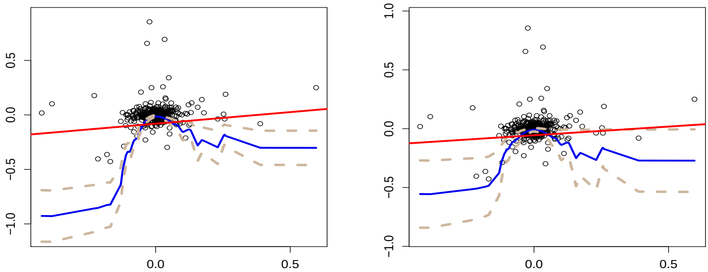


Figure 2: Bank of America (BOA) and Citi (C) weekly returns 0.05 (left) and 0.1 (right) quantile functions, y-axis = BOA returns, x-axis = C returns. Local linear quantile regression and Linear quantile regression. 95% confidence band, $T = 546$, weekly returns, 2005.01.31-2010.01.31. Chao, Härdle and Wang (2014).

Outline

1. Motivation ✓
2. Statistical Methodology
3. Systemic Risk Modelling
4. Empirical Analysis
5. Conclusion
6. References

Model Components

- **Tail Behavior:** Generalized Quantile Regression
- **Non-Linearity:** Single-Index Model
- **Ultra-High Dimensions:** Variable Selection

Generalized Quantile Regression

Let $\{X_i, Y_i\}_{i=1}^n$ be independent r. v., $X \in \mathbb{R}^p$, $\tau \in (0,1)$.

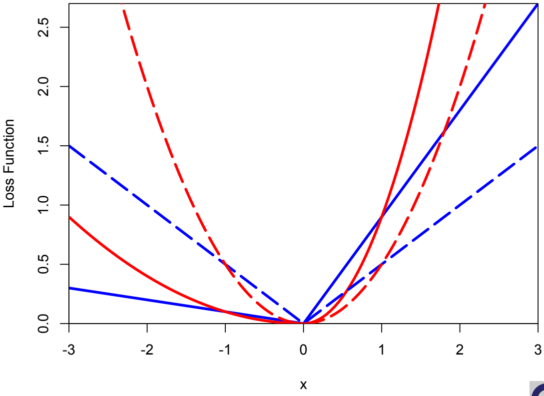
$$\begin{aligned} Y_i &= X_i^\top \theta + \varepsilon_i, \\ \hat{\theta} &= \arg\min_{\theta \in \mathbb{R}^p} \sum_{i=1}^n \rho_\tau(Y_i - X_i^\top \theta), \end{aligned}$$

where $\rho_\tau(\cdot)$ is an asymmetric loss function

$$\rho_\tau(u) = |u|^\alpha |\mathbf{1}(u \leq 0) - \tau|,$$

with $\alpha = 1$ corresponding to a quantile and $\alpha = 2$ corresponding to an expectile regression.

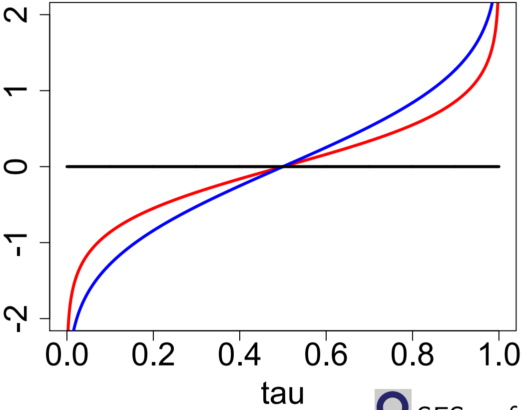
Asymmetric Loss Functions



LQRcheck

Figure 3: Asymmetric Loss Functions for Quantile and Expectile, $\tau = 0.9$: a solid line, $\tau = 0.5$: a dashed line.

Linear Quantile and Expectile



SFSconfexpectile0.95

Figure 4: Quantile and Expectile for $N(0, 1)$.

▸ Expectile-Quantile Correspondence

Single-Index Model

Let $\{X_i, Y_i\}_{i=1}^n$ be independent r. v., $X \in \mathbb{R}^p$.

$$Y_i = g(\beta^\top X_i) + \varepsilon_i,$$

where

- $g(\cdot)$ is the link function,
- $\beta \in \mathbb{R}^p$ is the vector of index parameters,
- $p = \mathcal{O}\{\exp(n^\alpha)\}$ for some $\alpha \in (0, 1)$.

Estimation

Recall (1):

$$Y_i = g(\beta^\top X_i) + \varepsilon_i$$

A quasi-likelihood approach under assumption $F_{\varepsilon_i}^{-1}(\tau|X) = 0$

$$\min_{\beta \in \mathbb{R}^p} \mathsf{E}\, \rho\{Y - g(\beta^\top X)\} \tag{1}$$

Further assumptions:
$\|\beta\|_2 = 1$ and first component of β is positive.

Estimation

Taylor approximation:

$$g(\beta^\top X_t) \approx g(\beta^\top x) + g'(\beta^\top x)\beta^\top (X_t - x) \tag{2}$$

Theoretically:

$$\begin{aligned} L_x(\beta) \stackrel{\text{def}}{=} & \; \mathsf{E}\, \rho\{Y - g(\beta^\top x) - g'(\beta^\top x)\beta^\top (X - x)\} \\ & K_h\{\beta^\top (X - x)\} \end{aligned} \tag{3}$$

Empirically:

$$\begin{aligned} L_{n,x}(\beta) \stackrel{\text{def}}{=} & \; n^{-1} \sum_{t=1}^{n} \rho\{Y_t - g(\beta^\top x) - g'(\beta^\top x)\beta^\top (X_t - x)\} \\ & K_h\{\beta^\top (X_t - x)\} \end{aligned} \tag{4}$$

where $K_h(.) = K(./h)/h$ with $K(.)$ a kernel and h a bandwidth.

Minimum Average Contrast Estimation

$$
\begin{aligned}
L_n(\beta) &\stackrel{\mathrm{def}}{=} n^{-1}\sum_{j=1}^{n} L_{n,X_j}(\beta) \\
&= n^{-2}\sum_{j=1}^{n}\sum_{t=1}^{n}\rho\left\{Y_t - g(\beta^\top X_j) - g'(\beta^\top X_j)\beta^\top(X_t - X_j)\right\} \\
&\quad K_h\{\beta^\top(X_t - X_j)\}
\end{aligned}
\tag{5}
$$

$$
\widehat{\beta} \approx \arg\min_{\beta} L_n(\beta) \tag{6}
$$

Variable Selection

$$\widehat{\beta} = \arg\min_{g,g',\beta} n^{-1} \sum_{j=1}^{n} \sum_{t=1}^{n} \rho\Big\{Y_t - g(\beta^\top X_j) - g'(\beta^\top X_j) X_{tj}^\top \beta\Big\} \omega_{tj}(\beta)$$
$$+ \sum_{l=1}^{p} \gamma_\lambda(|\beta_l|^\theta),$$

where

- $X_{tj} = X_t - X_j$,
- $\omega_{tj}(\beta) \stackrel{\text{def}}{=} \dfrac{K_h(X_{tj}^\top \beta)}{\sum_{t=1}^{n} K_h(X_{tj}^\top \beta)}$,
- $\theta \geq 0$,
- $\gamma_\lambda(t)$ is some nondecreasing function concave for $t \in [0, +\infty)$ with a continuous derivative on $(0, +\infty)$.

Numerical Procedure

Theory

Denote $\widehat{\beta}$ as the final estimate of β^*.

Theorem

Under A 1-5, the estimators $\widehat{\beta}^0$ *and* $\widehat{\beta}$ *exist and* $\mathrm{P}(\widehat{\beta}^0 = \widehat{\beta}) \to 1$. *Moreover,*

$$\mathrm{P}(\widehat{\beta}^0 = \widehat{\beta}) \geq 1 - (p - q)\exp(-C' n^{\alpha}). \tag{7}$$

Assumptions

Theory

Theorem

Under A 1-5, $\widehat{\beta}_{(1)} \stackrel{\text{def}}{=} \left(\widehat{\beta}_l\right)_{l\in\mathcal{M}_*}$, $b \in \mathbb{R}^q$, $\|b\| = 1$*:*

$$\|\widehat{\beta}_{(1)} - \beta^*_{(1)}\| = \mathcal{O}_p\{(\lambda D_n + n^{-1/2})\sqrt{q}\} \tag{8}$$

$$\boldsymbol{b}^\top C_{0(1)}^{-1}\sqrt{n}(\widehat{\beta}_{(1)} - \beta^*_{(1)}) \xrightarrow{\mathcal{L}} \mathrm{N}(0,\ \sigma^2) \tag{9}$$

where $\sigma^2 = \mathrm{E}[\psi(\varepsilon_i)]^2/[\partial^2\,\mathrm{E}\,\rho(\varepsilon_i)]^2$

$$\partial^2\,\mathrm{E}\,\rho(\cdot) = \frac{\partial^2\,\mathrm{E}\,\rho(\varepsilon_i - v)^2}{\partial v^2}\Big|_{v=0} \tag{10}$$

Go to details

Theory

Theorem

Under A 1-5, $\mathcal{B}_n \overset{\mathrm{def}}{=} \{\widehat{\beta} = \beta^*\} : \mathsf{P}(\mathcal{B}_n) \to 1$. *Let* $\mu_j \overset{\mathrm{def}}{=} \int u^j K(u) du$, $\nu_j \overset{\mathrm{def}}{=} \int u^j K^2(u) du$, $j = 0, 1, \ldots$. *If* $nh^3 \to \infty$ *and* $h \to 0$, *then*

$$\sqrt{nh}\sqrt{f_{Z_{(1)}}(z)/(\nu_0 \sigma^2)} \left\{ \widehat{g}(x^\top \widehat{\beta}) - g(x^\top \beta^*) - \frac{1}{2} h^2 g''(x^\top \beta^*) \mu_2 \partial \, \mathsf{E}\, \psi(\varepsilon) \right\} \xrightarrow{\mathcal{L}} \mathsf{N}(0, 1),$$

and

$$\sqrt{nh^3}\sqrt{\{f_{Z_{(1)}}(z)\mu_2^2\}/(\nu_2 \sigma^2)} \left\{ \widehat{g}'(x^\top \widehat{\beta}) - g'(x^\top \beta^*) \right\} \xrightarrow{\mathcal{L}} \mathsf{N}(0, 1).$$

▸ Go to details

Adaptive LASSO

$$\cdots \sum_{l=1}^{p} \gamma_\lambda(|\beta_l|^\theta) = \lambda \sum_{l=1}^{p} w_l |\beta_l|,$$

where

- λ is a penalty term,
- $\theta = 1$,
- $w_l = 1/|\widehat{\beta_l^0}|^\delta$ are weights, $l = 1, \ldots, p, \delta > 0$,
- $\widehat{\beta}^0$ is an initial estimator of β.

Zou (2006), Wu and Liu (2009)

Lambda

- Empirical choice of λ: $\lambda_n = 0.25\sqrt{||\beta_0||}\log n \vee p(\log n)^{0.5}$
- λ for ultra-high dimensions (Wang and Leng (2007))
- Schwarz Information Criteron (SIC)
  (Schwarz (1978), Koenker, Ng, and Portnoy (1994))

$$\mathrm{SIC}(\lambda) = \log[n^{-1}\sum_{i=1}^{n}\rho_\tau\{Y_i - f(X_i)\}] + \frac{\log n}{2n}\,\mathrm{df}$$

where df is a measure of the effective dimensionality of the fitted model.

Effective dimension

Bandwidth

Symmetrized nearest neighbor estimation implies

$$\widehat{m}_h(X_0) = (nh)^{-1} \sum_{i=1}^{n} Y_i K_h\{F_n(X_i) - F_n(x_0)\}$$

where

- $\widehat{m}(x)$ denotes an estimator of the regression function,
- h is some bandwidth tending to zero.

Härdle and Carroll (1989)

Methodology of AB

- *VaR*: $\widehat{\text{VaR}}_{i,t,\tau} = \hat{\alpha}_i + \hat{\gamma}_i M_{t-1}$,
- $\widehat{\text{CoVaR}}^{\text{AB}}$: $\widehat{\text{CoVaR}}_{s|i,t,\tau} = \hat{\alpha}_{s|i} + \hat{\gamma}_{s|i} M_{t-1} + \hat{\beta}_{s|i} \widehat{\text{VaR}}_{i,t,\tau}$,
  - AB's information set: firm i's VaR and macro state variables.
  - Systemic risk contribution: $\hat{\beta}_{s|i}$
- Limitations:
  - Linear assumption between a single firm and system.
  - Mechanical correlation between a single firm and the value-weighted system.

Methodology of TENET

- *VaR*: $\widehat{\text{VaR}}_{i,t,\tau} = \hat{\alpha}_i + \hat{\gamma}_i M_{t-1}$,
- $\widehat{\text{CoVaR}}^{\text{TENET}}$: $\widehat{\text{CoVaR}}_{j|\widetilde{R}_j,t,\tau} = \widehat{g}(\widehat{\beta}^\top_{j|\widetilde{R}_j}\widetilde{R}_{j,t})$,
  - TENET's information set: internal factors, many other firms' VaRs and macro state variables.
  - Spillover effects: $\widehat{g}\,'(\widehat{\beta}^\top_{j|\widetilde{R}_j}\widetilde{R}_{j,t})\widehat{\beta}_{j|\widetilde{R}_j}$.
- $\widehat{\text{CoVaR}}^{\textit{SYSTEM}}$: $\widehat{\text{CoVaR}}_{s|\widetilde{F}_j,t,\tau} = \widehat{g}(\widehat{\beta}^\top_{s|\widetilde{F}_j}\widetilde{F}_{j,t})$,
  - SYSTEM's information set: firm j's VaR, selected internal factors, selected other firms' VaRs and selected macro state variables.
  - Systemic risk contribution: $\widehat{g}\,'(\widehat{\beta}^\top_{s|\widetilde{F}_j}\widetilde{F}_{j,t})\widehat{\beta}_{s|\widetilde{F}_j}$

Advantages of TENET

- Nonlinear structure.
- High dimensional setting with variable selection.
- Network dynamics.

Step 1: VaR

Estimate linear QR

$$
\begin{aligned}
X_{i,t} &= \alpha_i + \gamma_i M_{t-1} + \varepsilon_{i,t}, && (11)\\
\widehat{\mathrm{VaR}}_{i,t,\tau} &= \hat{\alpha}_i + \hat{\gamma}_i M_{t-1}, && (12)
\end{aligned}
$$

- $X_{i,t}$ is the log-return of company i,
- M_{t-1} are macro state variables as in Adrian and Brunnermeier (2011).

Step 2: Spillover Effects based Network

Estimate SIM-based QRs with variable selection

$$X_{j,t} = g(\beta_{j|R_j}^{\top} R_{j,t}) + \varepsilon_{j,t}, \tag{13}$$

$$\widehat{\text{CoVaR}}^{TENET} \stackrel{\text{def}}{=} \widehat{\text{CoVaR}}^{SIM}_{j|\widetilde{R}_j,t,\tau} = \widehat{g}(\widehat{\beta}_{j|\widetilde{R}_j}^{\top} \widetilde{R}_{j,t}), \tag{14}$$

$$\widehat{D}_{j|\widetilde{R}_j} \stackrel{\text{def}}{=} \frac{\partial \widehat{g}(\widehat{\beta}_{j|R_j}^{\top} R_{j,t})}{\partial R_{j,t}} \Big|_{R_{j,t}=\widetilde{R}_{j,t}} = \widehat{g}\,'(\widehat{\beta}_{j|\widetilde{R}_j}^{\top} \widetilde{R}_{j,t}) \widehat{\beta}_{j|\widetilde{R}_j} \tag{15}$$

- $R_{j,t} = \{X_{-j,t}, M_{t-1}, B_{j,t-1}\}$ the p dimensional information set.
- $X_{-j,t} = \{X_{1,t}, X_{2,t}, \cdots, X_{k,t}\}$ log returns of all financial institutions except for a firm j, k: the number of financial institutions.
- $B_{j,t-1}$: the firm specific characteristics.

Step 2: Spillover Effects based Network

- $\beta_{j|R_j} \stackrel{\mathrm{def}}{=} \{\beta_{j|-j}, \beta_{j|M}, \beta_{j|B_j}\}^\top$.
- $\widetilde{R}_{j,t} \stackrel{\mathrm{def}}{=} \{\widehat{VaR}_{-j,t,\tau}, M_{t-1}, B_{j,t-1}\}$.
- $\widehat{\mathrm{VaR}}_{-j,t,\tau}$ are the estimated VaRs from (12) for financial institutions except for j in step 1.
- $\widehat{\beta}_{j|\widetilde{R}_j} \stackrel{\mathrm{def}}{=} \{\widehat{\beta}_{j|-j}, \widehat{\beta}_{j|M}, \widehat{\beta}_{j|B_j}\}^\top$.
- $\widehat{D}_{j|\widetilde{R}_j}$ is the gradient measuring the marginal effect of covariates evaluated at $R_{j,t} = \widetilde{R}_{j,t}$, and the componentwise expression is $\widehat{D}_{j|\widetilde{R}_j} = \{\widehat{D}_{j|-j}, \widehat{D}_{j|M}, \widehat{D}_{j|B_j}\}^\top$.
- $\widehat{D}_{j|-j}$ allows to measure spillover effects across the financial institutions and to characterize their evolution as a system represented by a network.

Step 2: Total Connectedness Matrix

$$A_t = \begin{array}{cc} & \begin{array}{ccccc} I_{1,t} & I_{2,t} & I_{3,t} & \cdots & I_{k,t} \end{array} \\ \begin{array}{c} I_{1,t} \\ I_{2,t} \\ I_{3,t} \\ \vdots \\ I_{k,t} \end{array} & \begin{pmatrix} 0 & |\widehat{D}_{1|2}| & |\widehat{D}_{1|3}| & \cdots & |\widehat{D}_{1|k}| \\ |\widehat{D}_{2|1}| & 0 & |\widehat{D}_{2|3}| & \cdots & |\widehat{D}_{2|k}| \\ |\widehat{D}_{3|1}| & |\widehat{D}_{3|2}| & 0 & \cdots & |\widehat{D}_{3|k}| \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ |\widehat{D}_{k|1}| & |\widehat{D}_{k|2}| & |\widehat{D}_{k|3}| & \cdots & 0 \end{pmatrix} \end{array}$$

Table 1: A $k \times k$ adjacency matrix for financial institutions at time t.

Step 2: Network Measures

- The firm level:
  - $DC_{j|i,t} \stackrel{\text{def}}{=} |\widehat{D}_{j|i}|$
  - $FC^{IN}_{j,t} \stackrel{\text{def}}{=} \sum_{i=1}^{k} |\widehat{D}_{j|i}|$
  - $FC^{OUT}_{j,t} \stackrel{\text{def}}{=} \sum_{j=1}^{k} |\widehat{D}_{j|i}|$
- The group level:

$$GC^{IN}_{g,t} \stackrel{\text{def}}{=} \sum_{i=1}^{k} \sum_{j \in g} |\widehat{D}_{j|i}|, \quad GC^{OUT}_{g,t} \stackrel{\text{def}}{=} \sum_{i \in g} \sum_{j=1}^{k} |\widehat{D}_{j|i}|$$

- The overall level:

$$TC_t = TC^{IN}_t = TC^{OUT}_t \stackrel{\text{def}}{=} \sum_{i=1}^{k} \sum_{j=1}^{k} |\widehat{D}_{j|i}|$$

Step 3: Systemic risk Contribution

Estimate SIM-based QRs without variable selection:

$$X_{s,t} = g(\beta_{s|F_j}^{\top} F_{j,t}) + \varepsilon_{s,t}, \tag{16}$$

$$\widehat{\text{CoVaR}}^{SYSTEM} \stackrel{\text{def}}{=} \widehat{\text{CoVaR}}^{SIM}_{s|\widetilde{F}_j,t,\tau} = \widehat{g}(\widehat{\beta}_{s|\widetilde{F}_j}^{\top} \widetilde{F}_{j,t}), \tag{17}$$

$$\widehat{D}_{s|\widetilde{F}_j} \stackrel{\text{def}}{=} \frac{\partial \widehat{g}(\widehat{\beta}_{s|F_j}^{\top} F_{j,t})}{\partial F_{j,t}} \Big|_{F_{j,t}=\widetilde{F}_{j,t}} = \widehat{g}\,'(\widehat{\beta}_{s|\widetilde{F}_j}^{\top} \widetilde{F}_{j,t}) \widehat{\beta}_{s|\widetilde{F}_j} \tag{18}$$

- $X_{s,t}$ refer to log returns of this financial system.

$$X_{s,t} = \frac{\sum_{i=1}^{k} X_{i,t} \cdot Asset_{i,t-1}}{\sum_{i=1}^{k} Asset_{i,t-1}}$$

Step 3: Systemic risk Contribution

- $F_{j,t} = \{X_{j,t}, C_{j,t}\}$, $C_{j,t} = \{X^*_{-j,t}, M^*_{t-1}, B^*_{j,t-1}\}$, $C_{j,t}$ includes control variables selected from step 2,
- $\beta_{s|F_j} = \{\beta_{s|j}, \beta_{s|C_j}\}^\top$.
- $\widetilde{F}_{j,t} = \{\widehat{VaR}_{j,t,\tau}, \widetilde{C}_{j,t}\}$, $\widetilde{C}_{j,t} = \{\widehat{VaR}^*_{-j,t,\tau}, M^*_{t-1}, B^*_{j,t-1}\}$,
- $\widehat{\beta}_{s|\widetilde{F}_j} = \{\widehat{\beta}_{s|j}, \widehat{\beta}_{s|\widetilde{C}_j}\}^\top$.
- $\widehat{D}_{s|\widetilde{F}_j} = \{\widehat{D}_{s|j}, \widehat{D}_{s|\widetilde{C}_j}\}^\top$ is the partial derivative of system CoVaR with respect to the variables in $F_{j,t}$ evaluated at level $F_{j,t} = \widetilde{F}_{j,t}$,
- $\widehat{D}_{s|j}$ is the the partial derivative of system CoVaR with respect to institution j. In terms of identification of the system risk contributions we focus here on $\widehat{D}_{s|j}$.

Dataset

- Asset log returns of 100 U.S. publicly traded financial firms.
- Firms classified by SIC codes: Depositories (25), Insurance (25), Broker-Dealers (25) and Others (25).
- 4 firm specific characteristics: LEV, MM, MTB, SIZE.
- 7 macro state variables: VIX, 3MTB, LIQUIDITY, YIELD, CREDIT, D_J, S&P.
- Time period: January 5, 2007 - January 4, 2013, $T = 266$, $n = 48$.
- Frequency: weekly.

Firms

Macro state variables

Network Dynamics

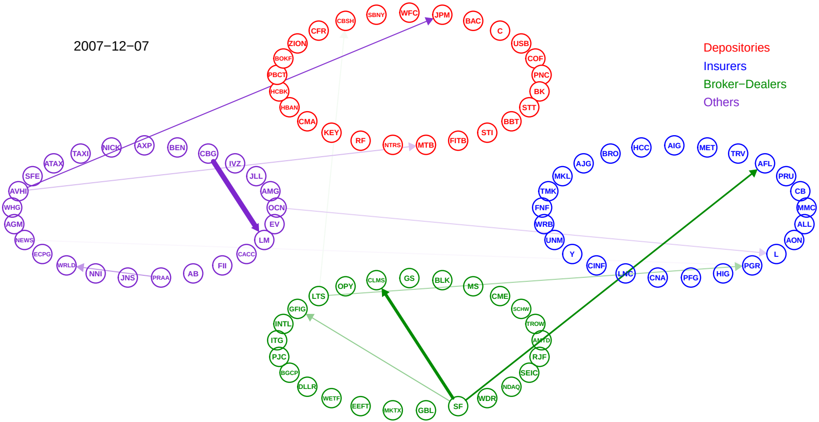
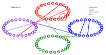


Figure 5: Financial risk network dynamics Depositories, Insurance, Broker-Dealers, Others ; $T = 266$, $\tau = 0.05$, $n = 48$.



Network Analysis–Overall Level

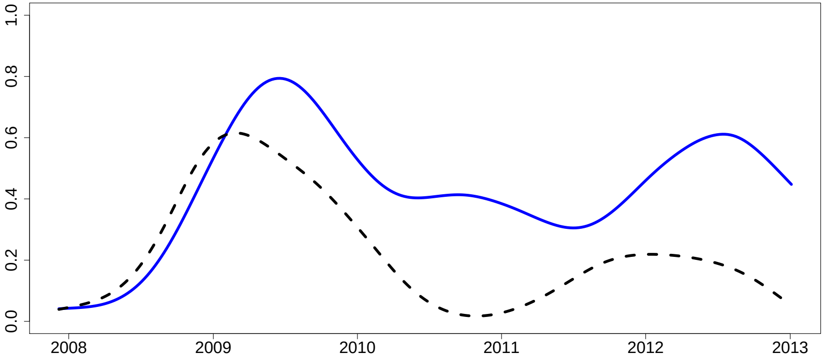


Figure 6: Total connectedness (solid line) and averaged λ of 100 financial institutions (dashes line): 20071207–20130105, both are standardized on $[0, 1]$ scale. Financial Risk Meter:http://sfb649.wiwi.hu-berlin.de/frm/index.html

Network Analysis–Group Level

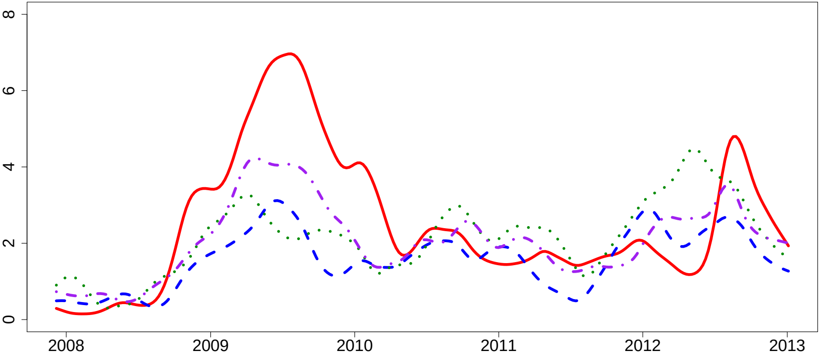


Figure 7: Incoming links for four industry groups. Depositories, Insurance, Broker-Dealers, Others ; . $\tau = 0.05$, window size $n = 48$, $T = 266$.

Network Analysis–Group Level

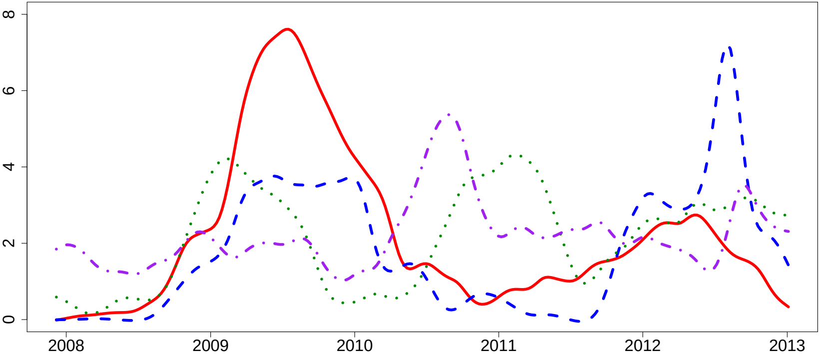


Figure 8: Outgoing links for four industry groups. Depositories, Insurance, Broker-Dealers, Others ; . $\tau = 0.05$, window size $n = 48$, $T = 266$.

Network analysis–Firm Level

- Most connected institution wrt Incoming links:
  Oppenheimer Holding, Inc. (OPY). IN-link
- Most connected institution wrt Outgoing links:
  Lincoln National Corporation (LNC). OUT-link
- Directional most connected institutions:
  from NewStar Financial, Inc. (NEWS) to Oppenheimer Holdings, Inc.(OPY). DIRECT-link

Summary of Network analysis

- The connections between institutions tend to increase before the financial crisis.
- The connections between institutions get weaker as the financial system stabilized.
- Whereas banks dominate both incoming and outgoing links, the insurers are less affected by the financial crisis and exhibit less contribution in terms of risk transmission.
- Several institutions with moderate or small sizes and also some non bank institutions received or transmitted more risk, as there are "too connected" firms.

Systemic Risk Contribution

Ranking of SRC	Ticker	Averaged Sum	Ranking of MC
1	JPM	0.27	2
2	BAC	0.26	3
3	WFC	0.23	1
4	C	0.19	4
5	PRU	0.17	13
6	L	0.16	28
7	GS	0.13	7
8	MET	0.12	9
9	MTB	0.11	33
10	AXP	0.10	5

Table 2: Top 10 financial institutions ranked according to the systemic risk contribution (SRC) calculated by the averaged sum of partial derivatives, and the Ranking of market capitalization (MC) in this 100 financial institutions' list is also shown in this table.

Link Function Dynamics

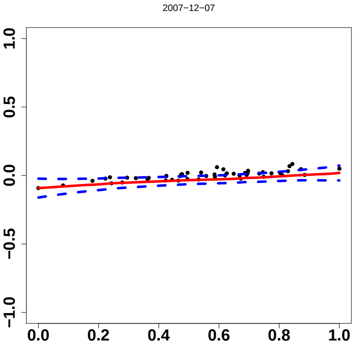


Figure 9: Link function dynamics for JPM, 5th Janary 2007 - 30th December 2011, $\tau = 0.05$, window size $n = 48$.

Conclusion

- Network can identify the interconnectedness among financial institutions.
- Partial Derivative of Systemic CoVaR can find the systemic risk contributors.
- Both major systemic risk contributors and major interconnected companies are systemically important.

TENET: Tail Event driven NETwork risk

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http://www.case.hu-berlin.de

Expectile-Quantile Correspondence

Let $v(x)$ represents expectile regression, $l(x)$ represents quantile regression.
Fixed x, define $w(\tau)$ such that $v_{w(\tau)}(x) = l(x)$ then $w(\tau)$ is related to $l(x)$ via

$$w(\tau) = \frac{\tau l(x) - \int_{-\infty}^{l(x)} y dF(y|x)}{2\,\mathrm{E}(Y|x) - 2\int_{-\infty}^{l(x)} y dF(y|x) - (1-2\tau)l(x)}$$

For example, $Y \sim U(-1,1)$, then $w(\tau) = \tau^2/(2\tau^2 - 2\tau + 1)$
Expectile corresponds to quantile with transformation w. Return

Numerical procedure

1. Given $\widehat{\beta}^{(t)}$, standardize $\widehat{\beta}^{(t)}$ so that $\|\widehat{\beta}^{(t)}\| = 1$, $\widehat{\beta}_1^{(t)} > 0$. Then compute

$$(\widehat{a}_j^{(t)}, \widehat{b}_j^{(t)}) \stackrel{\text{def}}{=} \underset{(a_j,b_j)'\text{s}}{\arg\min} \sum_{i=1}^{n} \rho\big(Y_i - a_j - b_j X_{ij}^\top \widehat{\beta}^{(t)}\big)\omega_{ij}(\widehat{\beta}^{(t)}),$$

where

- $\widehat{\beta}_0$ initial estimator of β^*,
- $X_{ij} = X_i - X_j$,
- $a_j = g(\beta^\top X_j)$,
- $b_j = g'(\beta^\top X_j)$,
- $\omega_{ij}(\widehat{\beta}_0^{(t)}) \stackrel{\text{def}}{=} \dfrac{K_h(X_{ij}^\top \beta_0^{(t)})}{\sum_{i=1}^{n} K_h(X_{ij}^\top \beta_0^{(t)})}$,
- $t = 1, 2, \ldots$ are iterations.

Numerical procedure

2. Given $(\widehat{a}_j^{(t)}, \widehat{b}_j^{(t)})$, solve

$$\widehat{\beta}^{(t+1)} = \arg\min_{\beta} n^{-1} \sum_{j=1}^{n} \sum_{i=1}^{n} \rho\big(Y_i - \widehat{a}_j^{(t)} - \widehat{b}_j^{(t)} X_{ij}^{\top} \beta\big) \omega_{ij}(\widehat{\beta}^{(t)}),$$

$$+ \textstyle\sum_{l=1}^{p} \widehat{d}_l^{(t)} |\beta_l|.$$

where

- $\widehat{d}_l^{(t)} = \gamma_\lambda(|\widehat{\beta}_l^{(t)}|)$,
- $\omega_{ij}(.)$ are from the step before.

Return

Effective dimension

Let $\{X_i, Y_i\}_{i=1}^{n}$ be independent r. v.
Given X, let $Y_i \sim (\mu(X), \sigma^2)$, where $\mu(X)$ is the true mean and σ^2 is the common variance.

$$\mathrm{df}(\hat{f}) = \sum_{i=1}^{n} \frac{\mathrm{Cov}\{\hat{f}(X_i), Y_i\}}{\sigma^2}.$$

Under certain mild conditions an unbiased estimator of df is

$$\mathrm{df}(\hat{f}) = \sum_{i=1}^{n} \frac{\partial \hat{f}(X_i)}{\partial Y_i}$$

Stein (1981)

Return

Assumptions

A1 K a cts symmetric pdf, $g(\cdot) \in C^2$.

A2 $\rho(x)$ convex. Suppose $\psi(x)$, subgradient of $\rho(x)$:
i) Lipschitz continuous; ii) $\mathsf{E}\,\psi(\varepsilon_i) = 0$ and
$\inf_{|v|\le c} \partial\,\mathsf{E}\,\psi(\varepsilon_i - v) = C_1$.

A3 ε_i is independent of X_i. Let $Z_i = X_i^\top \beta^*$ and $Z_{ij} = Z_i - Z_j$.
$C_{0(1)} \stackrel{\text{def}}{=} \mathsf{E}\{g'(Z_i)^2(\mathsf{E}(X_{i(1)}|Z_i) - X_{i(1)})(\mathsf{E}(X_{i(1)}|Z_i - X_{i(1)}))\}^\top\}$,
and the matrix $C_{0(1)}$ satisfies
$0 < L_1 \le \lambda_{\min}(C_{0(1)}) \le \lambda_{\max}(C_{0(1)}) \le L_2$ for positive constants L_1 and L_2. There exists a constant $c_0 > 0$ such that $\sum_{i=1}^n \{\|X_{i(1)}\|/\sqrt{n}\}^{2+c_0} \xrightarrow{p} 0$, with $0 < c_0 < 1$. Also
$\|\sum_i \sum_j X_{(0)ij}\omega_{ij}X_{(1)ij}^\top \partial\,\mathsf{E}\,\psi(v_{ij})\|_{2,\infty} = \mathcal{O}_p(n^{1-\alpha_1})$.

Assumptions

A4 The penalty parameter λ is chosen such that $\lambda D_n = \mathcal{O}\{n^{-1/2}\}$, with $D_n \stackrel{\text{def}}{=} \max\{d_l : l \in \mathcal{M}_*\} = \mathcal{O}(n^{\alpha_1 - \alpha_2/2})$, $d_l \stackrel{\text{def}}{=} \gamma_\lambda(|\beta_l^*|)$, $\mathcal{M}_* = \{l : \beta_l^* \neq 0\}$ be the true model. Furthermore assume $qh \to 0$ as $n \to \infty$, $q = \mathcal{O}(n^{\alpha_2})$, $p = \mathcal{O}(\exp\{n^\delta\})$, $nh^3 \to \infty$ and $h \to 0$. Also, $0 < \delta < \alpha < \alpha_2/2 < 1/2$, $\alpha_2/2 < \alpha_1 < 1$. For example, $\delta = 1/5$, $\alpha = 1/4$, $\alpha_2 = 3/5$, $\alpha_1 = 3/5$.

A5 The error term ε_i satisfies $\mathsf{E}\,\varepsilon_i = 0$ and $\mathrm{Var}(\varepsilon_i) < \infty$. Assume that $\mathsf{E}\left|\psi^m(\varepsilon_i)/m!\right| \leq s_0 c^m$ where s_0 and c are constants.

Return

Subgradient

If $f : U \to \mathbb{R}$ is a real-valued convex function defined on a convex open set in the Euclidean space $\mathbb{R}^n$, a vector v in that space is called a subgradient at a point x_0 in U if for any x in U one has

$$f(x) - f(x_0) \geq v \cdot (x - x_0)$$

where the dot denotes the dot product.

Return

Matrix norm

Assume A is a $m \times n$ matrix

$$\|A\|_{\alpha,\beta} = \max_{x \neq 0} \frac{\|Ax\|_\beta}{\|x\|_\alpha}$$

Return

Sparsistency

The result of (7) is stronger than the oracle property defined in Fan and Li (2001) once the properties of $\widehat{\beta}^0$ are established. It was formulated by Kim et al. (2008) for the SCAD estimator with polynomial dimensionality p. It implies not only the model selection consistency and but also sign consistency (Zhao and Yu, 2006; Bickel et al., 2008, 2009):

$$\mathrm{P}\{sgn(\widehat{\beta}) = sgn(\beta^*)\} = \mathrm{P}\{sgn(\widehat{\beta}^0) = sgn(\beta^*)\} \rightarrow 1$$

Return

The confidence interval

The $100(1-\alpha)\%$ confidence interval:

$$\Big[\widehat{g}(z) - \tfrac{1}{\sqrt{nh}} \cdot \frac{\sigma\sqrt{\nu_0}}{\sqrt{\widehat{f}_{Z_{(1)}}(z)}} \cdot \mathfrak{z}_\alpha + \tfrac{1}{2}h^2\widehat{g}''(z)\mu_2\partial\widehat{\mathsf{E}}\psi(\varepsilon);$$
$$\widehat{g}(z) + \tfrac{1}{\sqrt{nh}} \cdot \frac{\sigma\sqrt{\nu_0}}{\sqrt{\widehat{f}_{Z_{(1)}}(z)}} \cdot \mathfrak{z}_\alpha + \tfrac{1}{2}h^2\widehat{g}''(z)\mu_2\partial\widehat{\mathsf{E}}\psi(\varepsilon)\Big]$$

where $\mathfrak{z}_\alpha$ is the α-Quantile of the standard normal distribution, and

$\widehat{f}_{Z_{(1)}}(z) = n^{-1}\sum_{i=1}^{n} K_h(z - Z_{i(1)})$, where $Z_{i(1)} = X_{i(1)}^\top \widehat{\beta}_{(1)}$.

Return

Network Analysis: IN-link

Ranking of IN-link	Ticker	Total IN Sum	Ranking of MC
1	OPY	68.63	98
2	IVZ	67.54	35
3	SFE	65.38	93
4	FITB	64.64	30
5	KEY	64.01	40
6	JPM	54.81	2
7	WFC	50.31	1
8	ZION	48.95	63
9	COF	48.36	10
10	STI	47.41	29

Table 3: Top 10 financial institutions ranked according to Incoming links calculated by the sum of absolute value of the partial derivatives, and the Ranking of market capitalization (MC) in this 100 financial institutions' list is also shown in this table.

Return

Network Analysis: OUT-link

Ranking of OUT-link	Ticker	Total Out Sum	Ranking of MC
1	LNC	260.72	43
2	C	174.46	4
3	LTS	164.48	97
4	MS	163.91	12
5	CBG	121.48	32
6	AGM	114.38	89
7	FITB	97.21	30
8	RF	84.65	36
9	ZION	84.52	63
10	NNI	80.87	77

Table 4: Top 10 financial institutions ranked according to Outgoing links calculated by the sum of absolute value of the partial derivatives, and the Ranking of market capitalization (MC) in this 100 financial institutions' list is also shown in this table.

Return

Network Analysis: DIRECT-link

Ranking of Sum	From Ticker	To Ticker	Total Sum
1	NEWS	OPY	33.06
2	LNC	CBG	32.74
3	C	MS	28.26
4	RF	STI	23.72
5	C	BAC	22.99
6	LNC	SFE	17.61
7	MS	LM	16.82
8	C	OPY	16.36
9	CBG	JLL	15.54
10	LNC	CLMS	15.34

Table 5: Top 10 directional connectedness from one financial institution to another. The ranking is calculated by the sum of absolute value of the partial derivatives.

Return

Financial firms

	Depositories (25)		Insurances (25)
WFC	Wells Fargo & Company	AIG	American International Group, Inc.
JPM	J P Morgan Chase & Co	MET	MetLife, Inc.
BAC	Bank of America Corporation	TRV	The Travelers Companies, Inc.
C	Citigroup Inc.	AFL	Aflac Incorporated
USB	U.S. Bancorp	PRU	Prudential Financial, Inc.
COF	Capital One Financial Corporation	CB	Chubb Corporation (The)
PNC	PNC Financial Services Group, Inc. (The)	MMC	Marsh & McLennan Companies, Inc.
BK	Bank Of New York Mellon Corporation (The)	ALL	Allstate Corporation (The)
STT	State Street Corporation	AON	Aon plc
BBT	BB&T Corporation	L	Loews Corporation
STI	SunTrust Banks, Inc.	PGR	Progressive Corporation (The)
FITB	Fifth Third Bancorp	HIG	Hartford Financial Services Group, Inc. (The)
MTB	M&T Bank Corporation	PFG	Principal Financial Group Inc
NTRS	Northern Trust Corporation	CNA	CNA Financial Corporation
RF	Regions Financial Corporation	LNC	Lincoln National Corporation
KEY	KeyCorp	CINF	Cincinnati Financial Corporation
CMA	Comerica Incorporated	Y	Alleghany Corporation
HBAN	Huntington Bancshares Incorporated	UNM	Unum Group
HCBK	Hudson City Bancorp, Inc.	WRB	W.R. Berkley Corporation
PBCT	People's United Financial, Inc.	FNF	Fidelity National Financial, Inc.
BOKF	BOK Financial Corporation	TMK	Torchmark Corporation
ZION	Zions Bancorporation	MKL	Markel Corporation
CFR	Cullen/Frost Bankers, Inc.	AJG	Arthur J. Gallagher & Co.
CBSH	Commerce Bancshares, Inc.	BRO	Brown & Brown, Inc.
SBNY	Signature Bank	HCC	HCC Insurance Holdings, Inc.

Financial firms

Return

	Broker-Dealers (25)		others (25)
GS	Goldman Sachs Group, Inc. (The)	AXP	American Express Company
BLK	BlackRock, Inc.	BEN	Franklin Resources, Inc.
MS	Morgan Stanley	CBG	CBRE Group, Inc.
CME	CME Group Inc.	IVZ	Invesco Plc
SCHW	The Charles Schwab Corporation	JLL	Jones Lang LaSalle Incorporated
TROW	T. Rowe Price Group, Inc.	AMG	Affiliated Managers Group, Inc.
AMTD	TD Ameritrade Holding Corporation	OCN	Ocwen Financial Corporation
RJF	Raymond James Financial, Inc.	EV	Eaton Vance Corporation
SEIC	SEI Investments Company	LM	Legg Mason, Inc.
NDAQ	The NASDAQ OMX Group, Inc.	CACC	Credit Acceptance Corporation
WDR	Waddell & Reed Financial, Inc.	FII	Federated Investors, Inc.
SF	Stifel Financial Corporation	AB	Alliance Capital Management Holding L.P.
GBL	Gamco Investors, Inc.	PRAA	Portfolio Recovery Associates, Inc.
MKTX	MarketAxess Holdings, Inc.	JNS	Janus Capital Group, Inc
EEFT	Euronet Worldwide, Inc.	NNI	Nelnet, Inc.
WETF	WisdomTree Investments, Inc.	WRLD	World Acceptance Corporation
DLLR	DFC Global Corp	ECPG	Encore Capital Group Inc
BGCP	BGC Partners, Inc.	NEWS	NewStar Financial, Inc.
PJC	Piper Jaffray Companies	AGM	Federal Agricultural Mortgage Corporation
ITG	Investment Technology Group, Inc.	WHG	Westwood Holdings Group Inc
INTL	INTL FCStone Inc.	AVHI	AV Homes, Inc.
GFIG	GFI Group Inc.	SFE	Safeguard Scientifics, Inc.
LTS	Ladenburg Thalmann Financial Services Inc	ATAX	America First Tax Exempt Investors, L.P.
OPY	Oppenheimer Holdings, Inc.	TAXI	Medallion Financial Corp.
CLMS	Calamos Asset Management, Inc.	NICK	Nicholas Financial, Inc.

Macro state variables

1. VIX
2. Short term liquidity spread (liquidity)
3. Daily change in the 3-month Treasury maturities (3MT)
4. Change in the slope of the yield curve (yield)
5. Change in the credit spread (credit)
6. Daily Dow Jones U.S. Real Estate index returns (D_J)
7. S&P500 returns (S&P)

Source: Adrian and Brunnermeier (2011), Datastream.

Return to Introduction

Return to Empirical Analysis

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