Penalized Adaptive Method in Forecasting with Large Information Set and Structure Change

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Excess bond premium

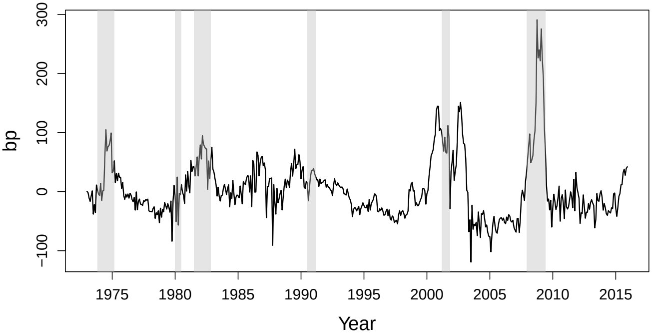
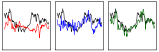


Figure 1: FEDS Notes: Excess bond premium, Jan 1973 - Mar 2016 (link), shaded areas are NBER designated recessions



Excess bond premium models

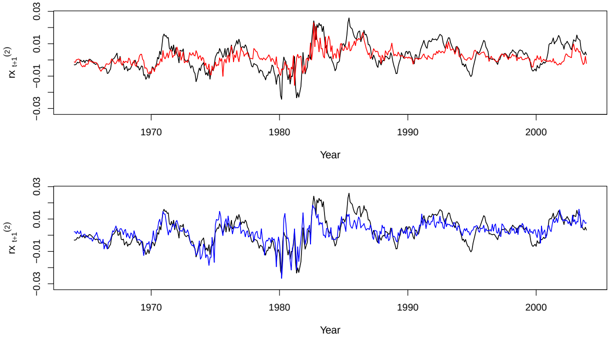


Figure 2: Real (black) vs. fitted excess bond premium for 2-year bonds by Cochrane and Piazzesi (2005) and Ludvigson and Ng (2009), Jan 1964 - Dec 2003

PAMinsam

Challenges

- High-dimensional data
  - Stock prices
  - Macroeconomic variables
- Dimension reduction
  - Systemic risk indicator
  - Excess bond premium modelling
- Non-stationarity

Dimension reduction

- Factor analysis
- Principal component analysis
- Penalized regression analysis
  - Selects important variables
  - Good interpretation of the fitted model
  - Tibshirani (1996): Lasso
  - Fan and Li (2001): SCAD penalty

Penalized likelihood

- Penalized likelihood

$$Q(\beta) = n^{-1} \sum_{i=1}^{n} l_i(\beta) - \sum_{j=1}^{p} p_\lambda(|\beta_j|)$$

  with $l_i(\cdot)$ non-penalized log-likelihood function and $p_\lambda(\cdot)$ a penalty function with parameter $\lambda > 0$

- Fan and Li (2001): Local quadratic approximation (LQA)
- Zou and Li (2008): Local linear approximation (LLA)

$$Q(\beta) \approx n^{-1} \sum_{i=1}^{n} l_i(\beta) - \sum_{j=1}^{p} p'_\lambda(|\beta_j|)|\beta_j|$$

Window selection

- Time varying coefficients
  - Heterogeneity throughout time
- Use of rolling windows with fixed window size
- Polzehl and Spokoiny (2004, 2006): Adaptive window choice
  - Data driven choice of the longest homogeneous interval
  - Propagation & separation
  - Change point analysis

Propagation-separation approach

- Series of nested intervals for a given time point t

$$I_t^{(1)} \subset I_t^{(2)} \subset I_t^{(3)} \subset \ldots \subset I_t^{(M)}$$

  with $n^{(m)}$ observations in $I_t^{(m)}$, $m = 1, \ldots, M$

- Propagation: Extension of local model in a (nearly) homogeneous situation
- Separation: Extension is restricted to the region of local homogeneity

Outline

1. Motivation ✓
2. Penalized adaptive method
3. Real data application
4. Concluding remarks

Penalized adaptive method

Combination of SCAD penalty with adaptive window choice

- Dimension reduction
- Longest homogeneous interval detection
- Prediction based on the estimated sparse coefficients

SCAD penalty

- Linear model $Y = X\beta + \varepsilon$
  with $Y_{(n\times 1)}$, $X_{(n\times p)}$, $\beta_{(p\times 1)}$, $\varepsilon_{(n\times 1)} \overset{iid}{\sim} (0, \sigma^2)$
- Fan and Li (2001): Quadratic spline function with knots at λ and $a\lambda$ with

$$\frac{\partial p_\lambda(\beta)}{\partial \beta} = \lambda \left\{ \mathbf{I}(\beta \leq \lambda) + \frac{(a\lambda - \beta)_+}{(a-1)\lambda} \mathbf{I}(\beta > \lambda) \right\}$$

  for $a > 2$ and $\beta > 0$
- Zou and Li (2008): LLA algorithm Details

Hypothesis

- Recall the series of nested intervals for a given time point t

$$I_t^{(1)} \subset I_t^{(2)} \subset I_t^{(3)} \subset \ldots \subset I_t^{(M)}$$

- $I_t^{(1)}$ homogeneous by assumption
- Hypothesis

$$H_0: \quad Y_t \sim \mathbb{P}_1, \quad \text{for } t \in I_t^{(m)}$$

$$H_1: \quad \begin{cases} Y_t \sim \mathbb{P}_1, & \text{for } t \in I_t^{(m-1)} \\ Y_t \sim \mathbb{P}_2, & \text{for } t \in I_t^{(m)} \setminus I_t^{(m-1)} \end{cases}$$

for $m = 2, \ldots, M$ with measures $\mathbb{P}_1, \mathbb{P}_2 \in \{\mathbb{P}(\theta), \theta \in \Theta \subseteq \mathbb{R}^p\}$

Generalized likelihood ratio

- Test statistic

$$
\begin{aligned}
T_t^{(m)} &= \frac{n_t^{(m-1)}}{n_t^{(m)}} \max_\beta Q(\beta, I_t^{(m-1)}) \\
&+ \frac{n_t^{(m)} - n_t^{(m-1)}}{n_t^{(m)}} \max_\beta Q(\beta, I_t^{(m)} \setminus I_t^{(m-1)}) \\
&- \max_\beta Q(\beta, I_t^{(m)})
\end{aligned}
$$

  for $m = 2, \ldots, M$
- SCAD penalty estimator $\tilde{\beta}_t^{(m)} = \operatorname{argmax}_\beta Q(\beta, I_t^{(m)})$
- Adaptive estimator $\widehat{\beta}_t^{(m)}$, for $m = 1, \ldots, M$

Penalized Adaptive Method

- Algorithm
  1. Assume $I_t^{(1)}$ homogeneous
  2. Initialization $\widehat{\beta}_t^{(1)} = \tilde{\beta}_t^{(1)}$
  3. $m = 2$
  4. While $T_t^{(m)} < \zeta_m$ and $m < M$
$$\widehat{\beta}_t^{(m)} = \tilde{\beta}_t^{(m)}$$
$$m = m + 1$$
  5. Final estimate $\widehat{\beta}_t = \widehat{\beta}_t^{(m)}$
- Critical values $\zeta_2, \ldots, \zeta_M$
- Q: How to find appropriate critical values?

Multiplier bootstrap

- Literature: Suvorikova et al. (2015), Chernozhukov et al. (2013), Spokoiny and Zhilova (2015)
- Bootstrapped penalized likelihood function

$$Q^{\circ}(\beta) = n^{-1} L^{\circ}(\beta) - \sum_{j=1}^{p} p_{\lambda}(|\beta_j|) = n^{-1} \sum_{i=1}^{n} l_i(\beta) u_i - \sum_{j=1}^{p} p_{\lambda}(|\beta_j|)$$

with $u_i \overset{iid}{\sim} (1, 1)$ for $i = 1, \ldots, n$

- Note $\arg\max_{\beta} \mathsf{E}[Q^{\circ}(\beta)|Y] = \arg\max_{\beta} Q(\beta) = \tilde{\beta}$

  and $\tilde{\beta}^{\circ} = \arg\max_{\beta} Q^{\circ}(\beta)$

Bootstrapped test statistic

Reproduction of homogeneous situation under H_0

$$
\begin{aligned}
T_t^{\circ(m)} \quad = \quad & \frac{n_t^{(m-1)}}{n_t^{(m)}} \max_{\beta} Q^{\circ}(\beta, I_t^{(m-1)}) \\
& + \frac{n_t^{(m)} - n_t^{(m-1)}}{n_t^{(m)}} \max_{\beta} Q^{\circ}(\beta, I_t^{(m)} \setminus I_t^{(m-1)}) \\
& - \max_{\beta} Q^{\circ}(\beta_{ts}, I_t^{(m)})
\end{aligned}
$$

with

$$
\beta_{ts} = \begin{cases} \beta & \text{for} \quad I_t^{(m-1)} \\ \beta + \tilde{\beta}_{t12} & \text{for} \quad I_t^{(m)} \setminus I_t^{(m-1)} \end{cases}
$$

where $\tilde{\beta}_{t12} = \operatorname{argmax}_{\beta} Q(\beta, I_t^{(m)} \setminus I_t^{(m-1)}) - \operatorname{argmax}_{\beta} Q(\beta, I_t^{(m-1)})$

Validity of multiplier bootstrap

Under H_0

$$\mathscr{L}\left(T_t^{(m)}\right) \approx \mathscr{L}\left(T_t^{\circ(m)} | Y\right)$$

Critical values ζ_m for $(1-\alpha)\%$ confidence level approximated by

$$\zeta_{t\alpha}^{\circ(m)} = \inf\left\{z \geq 0 : \mathrm{P}\left(T_t^{\circ(m)} > z | Y\right) \leq \alpha\right\}$$

for $m = 2, \ldots, M$

▸ Coverage probabilities ▸ Change points detection

Model definition

- $y_t^{(k)}$ log yield of k-year discount bond at time t, $k = 2, \dots, 5$
- $f_t^{(k)}$ log forward rate for loans between time $t + k - 1$ and $t + k$ specified at time t
- Excess log returns $rx_{t+1}^{(k)}$ of k-year bonds

$$rx_{t+1}^{(k)} = \beta_{0t}^{(k)} + \beta_{1t}^{(k)\top} f_t + \beta_{2t}^{(k)\top} M_t + \varepsilon_{t+1}^{(k)}$$

with $f_t = (y_t^{(1)}, f_t^{(2)}, \dots, f_t^{(5)})$ and M_t vector of macro variables Macro variables

- Cochrane and Piazzesi (2005): Single forward factor (CP1F)
- Ludvigson and Ng (2009): 5 macro factors with single forward factor (LN5F) or 6 macro factors (LN6F)

Model settings

- Constant increment $n_t^{(m)} - n_t^{(m-1)} = 48$, $n_t^{(1)} = 48$
- Dimension $p = 36$
- Multipliers $u_i \sim \text{Pois}(1)$ for $i = 1, \ldots, 1000$
- Confidence level $(1 - \alpha) = 99$ %
- Monthly data Jan 1961 - Dec 2011: CRSP, Global Insights Basic Economics Database, The Conference Board's Indicators Database

Out-of-sample fit

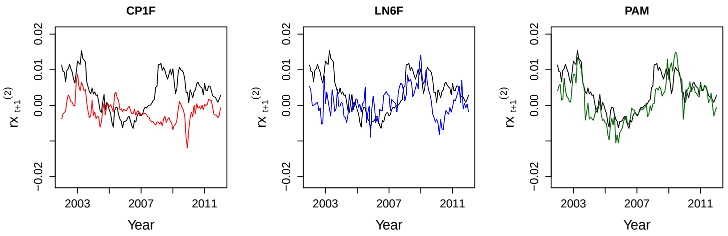


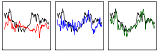
Figure 3: Real values (black) and 1-year ahead predictions (CP1F, LN6F, PAM) of 2-year bonds, Dec 2001 - Dec 2011 PAMoutsam

More results In-sample fit

Out-of-sample fit performance

		RMSPE	MAPE	$\frac{\text{RMSPE}_{\text{PAM}}}{\text{RMSPE}}$	$\frac{\text{MAPE}_{\text{PAM}}}{\text{MAPE}}$
$rx_{t+1}^{(2)}$	CP	0.008	0.007	0.50	0.43
	CP1F	0.008	0.006	0.50	0.50
	LN5F	0.008	0.006	0.50	0.50
	LN6F	0.006	0.005	0.67	0.60
	PAM	**0.004**	**0.003**	–	–
$rx_{t+1}^{(3)}$	CP	0.015	0.013	0.47	0.46
	CP1F	0.015	0.013	0.47	0.46
	LN5F	0.015	0.013	0.47	0.46
	LN6F	0.012	0.010	0.58	0.60
	PAM	**0.007**	**0.006**	–	–
$rx_{t+1}^{(4)}$	CP	0.021	0.017	0.57	0.59
	CP1F	0.021	0.018	0.57	0.56
	LN5F	0.021	0.018	0.57	0.56
	LN6F	0.017	0.013	0.71	0.77
	PAM	**0.012**	**0.010**	–	–
$rx_{t+1}^{(5)}$	CP	0.025	0.021	0.64	0.62
	CP1F	0.026	0.021	0.62	0.62
	LN5F	0.026	0.022	0.62	0.59
	LN6F	0.021	0.017	0.76	0.76
	PAM	**0.016**	**0.013**	–	–

Table 1: Forecasting performance of PAM, Cochrane and Piazzesi (2005) and Ludvigson and Ng (2009) models PAMoutsam



Conclusion

Penalized adaptive method

- Fully data-driven method
- Capturing non-stationarity and effective dimension reduction
- Improved performance in excess bond returns modelling

Outlook

- Inference for $p > n$ case
- Extension beyond linear models
- Optimal penalty parameter selection

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One-step sparse estimates in nonconcave penalized likelihood models
The Annals of Statistics **36**(4): 1509-1533, 2008

LLA algorithm

Back

- Zou and Li (2008): for $p < n$ set $\beta^{(0)}$ as unpenalized MLE
- Kim et al. (2008): for $p > n$ set $\beta^{(0)}$ as LASSO estimator
- Algorithm
  1. Set initial value $\beta^{(0)}$
  2. For $k = 0, 1, \ldots,$ repeatedly solve

$$\beta^{(k+1)} = \arg\max_{\beta} \left\{ \sum_{i=1}^{n} l_i(\beta) - n \sum_{j=1}^{p} p'_{\lambda}(|\beta_j^{(k)}|)|\beta_j| \right\}$$

until convergence

LLA estimator

Back

- Continuous
- Unbiased for large parameters
- Oracle properties
  - Consistency in variable selection
  - Asymptotic normality

under condition:

$$\sqrt{n}\lambda_n \to \infty \text{ and } \lambda_n \to 0$$

Macroeconomic variables I

Back

	Description	Transformation
1.	Personal Income	Δ log
2.	Real Consumption	Δ log
3.	Industrial Production Index (Total)	Δ log
4.	NAPM Production Index (Percent)	–
5.	Civilian Labor Force: Emplyed, Total	Δ log
6.	Unemployment Rate: All workers, 16 years & over (Percent)	Δ
7.	NAPM Employment Index (Percent)	–
8.	Money Stock M1	Δ^2 log
9.	Money Stock M2	Δ^2 log
10.	Money Stock M3	Δ^2 log
11.	S&P500 Common Stock Price Index: Composite	Δ log
12.	Interest Rate: Federal Funds (% p.a.)	Δ
13.	Commercial Paper Rate	Δ
14.	Interest Rate: US Treasury Bill, Sec Mkt, 3-m (% p.a.)	Δ
15.	Interest Rate: US Treasury Bill, Sec Mkt, 3-m (% p.a.)	Δ
16.	Interest Rate: US Treasury Const Maturities, 1-y (% p.a.)	Δ

Table 2: List of macroeconomic variables from Ludvigson and Ng (2009)

Macroeconomic variables II

Back

	Description	Transformation
17.	Interest Rate: US Treasury Const Maturities, 5-y (% p.a.)	Δ
18.	Interest Rate: US Treasury Const Maturities, 10-y (% p.a.)	Δ
19.	Bond Yield: Moody's Aaa Corporate (% p.a.)	Δ
20.	Bond Yield: Moody's Baa Corporate (% p.a.)	Δ
21.	cp90 - fyff Spread	–
22.	fygm3 - fyff Spread	–
23.	fygm6 - fyff Spread	–
24.	fygt1 - fyff Spread	–
25.	fygt5 - fyff Spread	–
26.	fygt10 - fyff Spread	–
27.	fyaaac - fyff Spread	–
28.	fybaac- fyff Spread	–
29.	Spot Market Price Index: all commodities	Δ^2 log
30.	NAPM Commodity Prices Index (Percent)	–
31.	CPI-U: All items	Δ^2 log

Table 3: List of macroeconomic variables from Ludvigson and Ng (2009)

Simulation settings

Back

- 1000 scenarios
- Design matrix $X_{(n\times p)}$

$$\{X_i\}_{i=1}^n \sim \mathrm{N}_p(0,\Sigma),$$

$n = 100, 200, 400,\ p = 10,\ q = \|\beta\|_0 = 3, 5$

- Covariance matrix $\Sigma_{(p\times p)}$

$$\sigma_{ij} = 0.5^{|i-j|}$$

$i, j = 1, \dots, p$

- $b = 1000$ bootstrap samples
- $u_i \sim$ Exp(1), Pois(1) or from a bounded distribution

Bounded distribution

Bootstrapped quantiles coverage probability

Back

				Confidence level			
n	p	q	$\mathscr{L}(u_i)$	90 %	95 %	97.5 %	99 %
			Bounded	81.2	87.1	91.0	94.9
100	10	3	Exp(1)	80.3	86.5	90.5	94.7
			Pois(1)	87.9	92.4	95.3	97.8
			Bounded	77.9	83.7	87.6	93.5
100	10	5	Exp(1)	76.7	83.0	88.3	93.4
			Pois(1)	85.7	90.5	94.8	97.5
			Bounded	90.7	94.8	97.3	98.6
200	10	3	Exp(1)	90.4	95.2	97.3	98.6
			Pois(1)	92.9	96.5	98.3	99.2
			Bounded	86.7	92.1	96.0	98.3
200	10	5	Exp(1)	85.9	91.9	96.0	98.1
			Pois(1)	90.4	94.8	97.4	99.2
			Bounded	97.1	98.6	99.6	99.8
400	10	3	Exp(1)	97.2	98.6	99:5	99.8
			Pois(1)	97.7	98.8	99.6	99.8
			Bounded	94.3	97.5	98.5	99.2
400	10	5	Exp(1)	94.4	97.6	98.5	99.3
			Pois(1)	95.2	98.1	98.5	99.4

Table 4: Empirical coverage probabilities PAMsimLR

Bounded distribution

Back

Random variable Z with values in the interval $[0, 4]$ with a probability density function defined as

$$f(z) = \begin{cases} \dfrac{3}{14} & \text{if} \quad 0 \leq z \leq 1 \\[2ex] \dfrac{1}{12} & \text{if} \quad 1 < z \leq 4 \end{cases} \tag{1}$$

Change points detection

Back

- 500 scenarios of $n = 500$ observations
- Design matrix $X_{(n \times p)}$ as before, $p = 10$
- Number of intervals $M = 10, 5$, $n^{(m+1)} - n^{(m)} = 100, 50$ for $m = 1, \ldots, M-1$, $n^{(1)} = 100, 50$
- Change point simulation

$$\beta_i^* = \begin{cases} (1,1,1,1,1,0,\ldots,0) & \text{if} \quad i < i_{cp} \\ (1,0.8,0.6,0.4,0.2,0,\ldots,0) & \text{if} \quad i \geq i_{cp} \end{cases}$$

where i_{cp} denotes observation with a change point

- $b = 1000$, $u_i \sim \text{Exp}(1)$, $\text{Pois}(1)$ or from a bounded distribution

Change points detection summary

Back

$M = 10$	i_{cp}	50	100	200	400
Bounded	Corr	99.6	100.0	99.8	100.0
	1stCorr	99.6	84.6	62.6	42.8
Exp(1)	Corr	99.6	100.0	99.8	100.0
	1stCorr	99.6	84.6	62.6	42.8
Pois(1)	Corr	99.4	100.0	100.0	100.0
	1stCorr	99.4	91.8	76.8	59.8

$M = 5$	i_{cp}	100	200	400
Bounded	Corr	100.0	99.8	100.0
	1stCorr	100.0	91.6	83.0
Exp(1)	Corr	100.0	99.8	100.0
	1stCorr	100.0	91.2	82.4
Pois(1)	Corr	100.0	99.8	100.0
	1stCorr	100.0	94.0	87.4

Table 5: Percentage of correctly identified change points

Change points detection

▸ Back

- 500 scenarios of $n = 500$ observations
- Design matrix $X_{(n\times p)}$ as before, $p = 10$
- Number of intervals $M = 10, 5$, $n^{(m+1)} - n^{(m)} = 100, 50$ for $m = 1, \dots, M-1$, $n^{(1)} = 100, 50$
- Change point simulation

$$\beta_i^* = \begin{cases} (1,1,1,1,1,0,\dots,0) & \text{if} \quad i < i_{cp} \\ (1,1,1,0,\dots,0) & \text{if} \quad i \geq i_{cp} \end{cases}$$

where i_{cp} denotes observation with a change point

- $b = 1000$, $u_i \sim \text{Exp}(1)$, $\text{Pois}(1)$ or from a bounded distribution

Change points detection summary

Back

$M = 10$	i_{cp}	50	100	200	400
Bounded	Corr	100.0	100.0	100.0	100.0
	1stCorr	100.0	85.4	64.0	41.8
Exp(1)	Corr	100.0	100.0	100.0	100.0
	1stCorr	100.0	84.4	63.0	41.0
Pois(1)	Corr	100.0	100.0	100.0	100.0
	1stCorr	100.0	91.8	77.8	58.8

$M = 5$	i_{cp}	100	200	400
Bounded	Corr	100.0	99.8	100.0
	1stCorr	100.0	91.0	82.4
Exp(1)	Corr	100.0	99.8	100.0
	1stCorr	100.0	91.4	82.0
Pois(1)	Corr	100.0	99.8	100.0
	1stCorr	100.0	94.0	87.8

Table 6: Percentage of correctly identified change points

Out-of-sample fit for 3-year bonds

Back

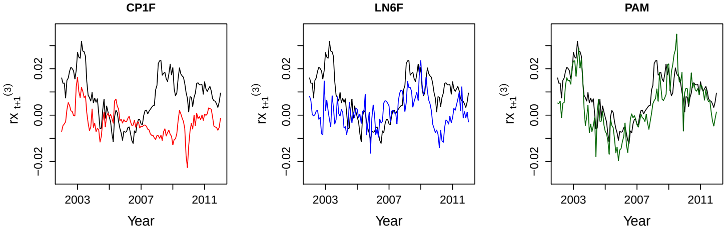
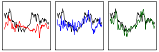


Figure 4: Real values (black) and 1-year ahead predictions (CP1F, LN6F, PAM) of 3-year bonds, Dec 2001 - Dec 2011

PAMoutsam



Out-of-sample fit for 4-year bonds

Back

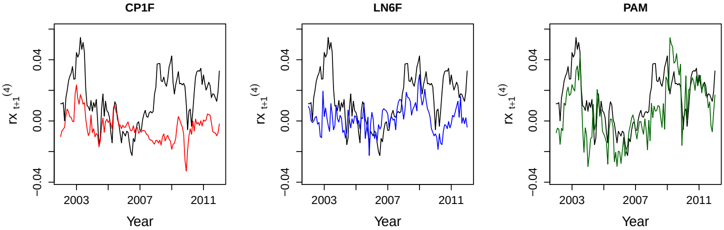


Figure 5: Real values (black) and 1-year ahead predictions (CP1F, LN6F, PAM) of 4-year bonds, Dec 2001 - Dec 2011

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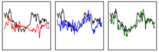

Out-of-sample fit for 5-year bonds

Back

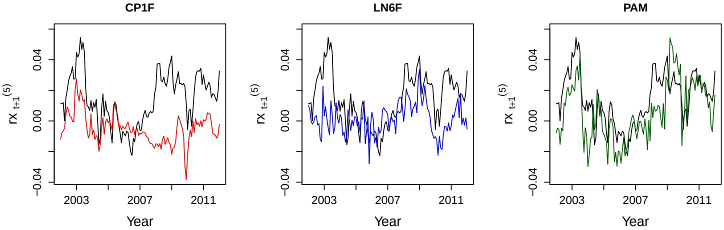

Figure 6: Real values (black) and 1-year ahead predictions (CP1F, LN6F, PAM) of 5-year bonds, Dec 2001 - Dec 2011

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In-sample fit for 2-year bonds

Back

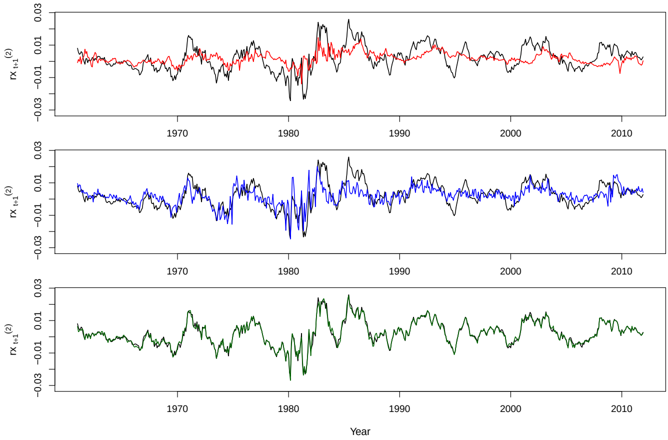

Figure 7: Real excess bond premium (black) and fitted CP1F, LN6F, PAM for 2-year bonds, Jan 1961 - Dec 2011 PAMinsam

In-sample fit for 3-year bonds

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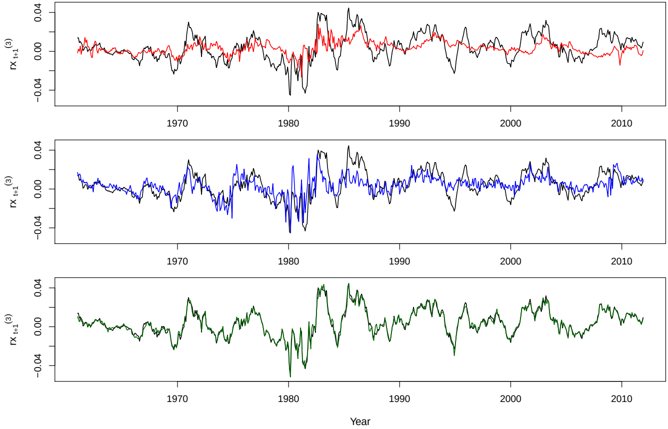
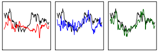


Figure 8: Real excess bond premium (black) and fitted CP1F, LN6F, PAM for 3-year bonds, Jan 1961 - Dec 2011 PAMinsam



In-sample fit for 4-year bonds

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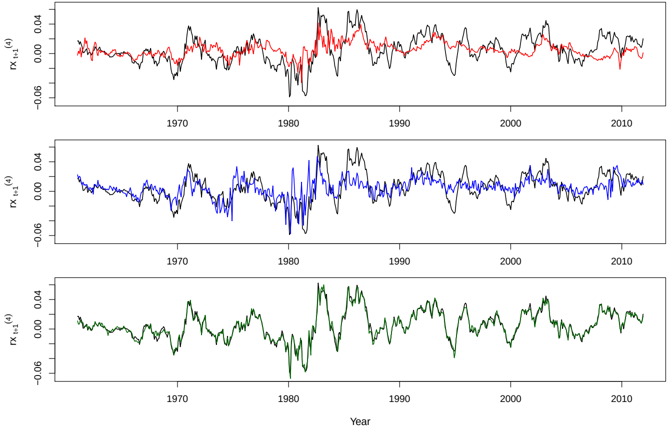

Figure 9: Real excess bond premium (black) and fitted CP1F, LN6F, PAM for 4-year bonds, Jan 1961 - Dec 2011 PAMinsam

In-sample fit for 5-year bonds

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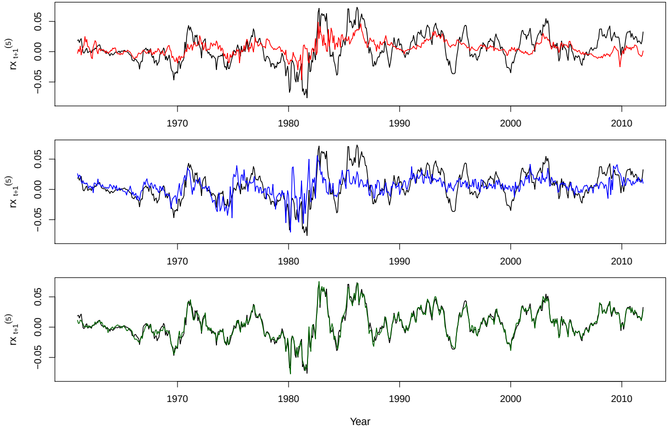
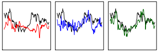


Figure 10: Real excess bond premium (black) and fitted CP1F, LN6F, PAM for 5-year bonds, Jan 1961 - Dec 2011 PAMinsam



In-sample fit performance

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		Jan 1964 - Dec 2003				Jan 1961 - Dec 2011			
		RMSE	MAE	R^2	R^2_{adj}	RMSE	MAE	R^2	R^2_{adj}
$rx^{(2)}_{t+1}$	CP	0.007	0.005	0.322	0.315	0.007	0.005	0.215	0.208
	CP1F	0.007	0.005	0.318	0.316	0.007	0.005	0.204	0.203
	LN5F	0.007	0.005	0.365	0.357	0.006	0.004	0.377	0.371
	LN6F	0.005	0.004	0.579	0.574	0.005	0.004	0.501	0.496
	PAM	**0.001**	**0.001**	**0.980**	**0.979**	**0.001**	**0.001**	**0.978**	**0.977**
$rx^{(3)}_{t+1}$	CP	0.012	0.010	0.340	0.333	0.012	0.010	0.224	0.217
	CP1F	0.012	0.010	0.338	0.336	0.012	0.010	0.220	0.219
	LN5F	0.012	0.009	0.385	0.377	0.011	0.008	0.383	0.377
	LN6F	0.010	0.008	0.532	0.526	0.010	0.008	0.463	0.458
	PAM	**0.003**	**0.002**	**0.970**	**0.970**	**0.002**	**0.002**	**0.970**	**0.970**
$rx^{(4)}_{t+1}$	CP	0.017	0.013	0.370	0.363	0.017	0.013	0.253	0.247
	CP1F	0.017	0.013	0.369	0.368	0.017	0.013	0.251	0.250
	LN5F	0.016	0.013	0.414	0.407	0.015	0.012	0.401	0.395
	LN6F	0.015	0.012	0.486	0.479	0.015	0.011	0.420	0.414
	PAM	**0.004**	**0.003**	**0.968**	**0.967**	**0.003**	**0.003**	**0.967**	**0.966**
$rx^{(5)}_{t+1}$	CP	0.021	0.016	0.344	0.337	0.021	0.016	0.231	0.225
	CP1F	0.021	0.016	0.344	0.343	0.021	0.016	0.229	0.228
	LN5F	0.020	0.016	0.386	0.378	0.019	0.015	0.368	0.362
	LN6F	0.019	0.015	0.461	0.454	0.018	0.014	0.398	0.392
	PAM	**0.005**	**0.003**	**0.965**	**0.964**	**0.005**	**0.003**	**0.962**	**0.961**

Table 7: Fitted PAM and models of Cochrane and Piazzesi (2005) and Ludvigson and Ng (2009) PAMinsam

In-sample fit summary

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- Different models for different times to maturity
- Average length of homogeneous intervals 5.8 years
- Average size of active sets 13.5
- Both forward rates and macro variables selected
- Evidence of time variation