Tales of sentiment driven tails

Jozef Baruník Cathy Yi-Hsuan Chen Wolfgang Karl Härdle

Institute of Economic Studies Charles University in Prague Ladislaus von Bortkiewicz Chair of Statistics Humboldt-Universität zu Berlin



http://ies.fsv.cuni.cz http://lvb.wiwi.hu-berlin.de

"Forget the dot-com boom with its irrational exuberance and the real estate bubble that was supposed to be invincible: Current market sentiment eclipses all of that"

Jeff Cox, CNBC, March 1 2017

Sentiment moves market



John Maynard Keynes (1936): markets can fluctuate wildly under the influence of investors' "animal spirits," which move prices in a way unrelated to fundamentals.

Sentiment can cause mispricing

Fifty years later...

De Long, Shleifer, Summers, and Waldmann (1990) formalized the role of investor sentiment in financial markets.

- uninformed noise traders base their decisions on sentiment
 - ▶ greater mispricing (Stambaugh et al., 2012)
 - excess volatility (Dumas et al., 2009)

"Now, the question is no longer, as it was a few decades ago, whether investor sentiment affects stock prices, but rather how to measure investor sentiment and quantify its effects."

(Baker and Wurgler, 2007)

News moves markets

- □ Large literature Huang et al. (2014), Da et al. (2015), Shefrin
 (2007+)
- ☑ Zhang et al. (2016) textual sentiment provides incremental information about future stock reactions

Is average enough?

- Sentiment affects cross section of returns or volatility
- □ Grand average is OK for expected payoffs
- - bear vs. bull markets
 - extreme negative vs. positive returns

Is average man enough?

Contrarians vs. Trend followers





We already know that we can measure sentiment...

but how to quantify its effect on prices?

Contribution

- Provide decision-theoretic foundations of pricing in quantiles
- □ Nonlinear dynamic quantile asset pricing model

Outline

- 1. Motivation ✓
- 2. Theoretical Framework
- 3. Data Collection
- 4. Sentiment Projection
- 5. Calibration of weighting function
- 6. Quantile Panel Regressions
- 7. Outlook

Classical asset pricing

Investor maximizes utility subject to budget constraint. The FOC (Euler equation):

$$\mathsf{E}_{F}\left[M\times(1+R)\right]=1,\tag{1}$$

where M is a pricing kernel (PK), or stochastic discount factor (SDF), R is the total return on a risky asset with physical distribution F(R).

Probability weighting

Decisions under risk are more sensitive to changes in probability of events at extremes, Tversky and Kahneman (1992).

Polkovnichenko and Zhao (2013) use the rank-dependent expected utility (RDEU) $\mathcal{U}(R) = \mathsf{E}_F[u(R)g\{F(R)\}]$ with PK

$$M = u'(R)g\{F(R)\}, \tag{2}$$

where $g\{F(R)\} = G'\{F(R)\}$ is a probability weighting function.

Euler equation reads as:

$$\mathsf{E}_F[u'(R)g\{F(R)\}\ (1+R)] = 1. \tag{3}$$

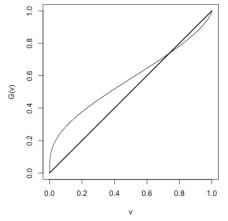


Figure: Probability weighting function $G(v) = \exp\{-(-\beta \log v)^{\alpha}\}$ with $\alpha = 0.7$ and $\beta = 0.6$

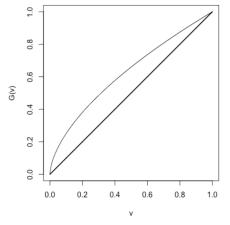


Figure: Probability weighting function $G(v) = \exp\{-(-\beta \log v)^{\alpha}\} = v^{0.6}$ $(\alpha = 1 \text{ and } \beta = 0.6)$

A route towards quantile preferences

$$X \succeq Y \text{ iff } \mathsf{E}_F[\mathcal{U}(X)] \ge \mathsf{E}_F[\mathcal{U}(Y)]$$
 (4)

 \square Manski (1988), Rostek (2010) look at τ -quantile preferences

$$X \succeq Y \text{ iff } Q_{\tau}[\mathcal{U}(X)] \ge Q_{\tau}[\mathcal{U}(Y)]$$
 (5)

 Maximising lower quantile is more risk-averse than higher quantile (example of portfolio), de Castro et al. (2017)

Example

Utility function $u(x) \stackrel{\text{def}}{=} x$

$$X = \begin{cases} 10^7 & \text{with } p = 10^{-6} \\ -1 & \text{with } q = 1 - p \end{cases} \quad Y = \begin{cases} 10 & \text{with } p = 9/10 \\ -1 & \text{with } q = 1 - p \end{cases}$$

$$X \succeq_{\mathsf{E}} Y \text{ since } \mathsf{E}[X] = 9 + 10^{-6} \text{ and } \mathsf{E}[Y] = 8 + 9/10$$

$$Q_{\tau}(X) \stackrel{\text{def}}{=} \inf\{\alpha \in \mathbb{R} : \mathbb{P}(X \leq \alpha) \geq \tau\}$$

$$X \begin{cases} \equiv_{\mathsf{Q}_{\tau}} Y \text{ for } \tau \le 1/10 \\ \preceq_{\mathsf{Q}_{\tau}} Y \text{ for } 1/10 < \tau \le 1 - 10^{-6} \\ \succeq_{\mathsf{Q}_{\tau}} Y \text{ for } \tau > 1 - 10^{-6} \end{cases}$$

A route towards a (dynamic) quantile model

Instead of classical preferences, look at an agent maximizing her stream of the future quantile utilities.

For a given $\tau \in (0,1)$, Euler equation reads:

$$Q_{\tau}\left[u'(R)g(v)\ (1+R)\right]=1,$$
 (6)

where v=F(R), $G(\cdot):[0,1] o [0,1]$ probability weighting fct and $g(\cdot)=G^{'}(\cdot).$

Can we relate $g(\cdot)$ to sentiment?

Probability weighting function and sentiment

Prelec (1998) weighting function:

$$G(v) = G(\alpha, \beta; v) = \exp\{-(-\beta \log v)^{\alpha}\}$$
 (7)

 α , β parameters govern the shape of $G(\cdot)$.

Link sentiment S_t to β_t :

$$\beta_t = \beta(S_t, \rho) = \exp\{-\rho(S_t^{-1} - 1)\} - 1$$
 (8)

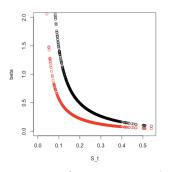


Figure: β versus S_t for $\rho=-0.1$ and $\rho=-0.05$

Fix $\alpha = 1$ to impose monotonicity and compute $v_t = (\operatorname{rank} R_t)/n$

$$G(v_t, S_t) = v_t^{\beta(S_t, \rho)} = v_t^{\exp\{-\rho(S_t^{-1} - 1)\} - 1}$$

$$G(v_t, S_t) = \exp\{(\exp\{-\rho(S_t^{-1} - 1)\} - 1) \log v_t\}$$
(9)

A dynamic quantile model with sentiments

Equation (6) is beneficial, since it can be log-linearized as for a general random variable W, $Q_{\tau}[\log(W)] = \log(Q_{\tau}[W])$. Hence

$$Q_{\tau} \left[u'(R_t) g(v_t, S_t) \ (1 + R_{t+1}) \right] = 1 \tag{10}$$

considering power utility function:

$$Q_{\tau} \left[-\gamma \log(R_t) + \log\{g(v_t, S_t)\} + \log(1 + R_{t+1}) \right] = 0.$$
 (11)

One can estimate the parameter driving $g(v_t, S_t)$ with nonlinear quantile regression.

How to estimate sentiment S_t ?

Data

- □ Panel of 100 most liquid constituents of S&P 500 stocks
- Sentiment variables: distilled from Nasdaq articles

Nasdaq Articles

- □ Terms of Service permit web scraping
- Currently > 580k articles between October 2009 and January 2017

There is a lot of news...



Dimensions of News

- Source of news
 - ▶ Official channel: government, federal reserve bank/central bank, financial institutions
 - Internet: blog, social media, message board
- □ Content of news: signal vs. noise
- - Scheduled vs. non-scheduled
 - Expected vs. unexpected
 - Specific-event vs. continuous news flows

The Power of Words: Textual Analytics

- Sentiment analysis
 - ▶ Lexica projection : positive, neutral and negative
 - Machine learning : text classification

Unsupervised Projection

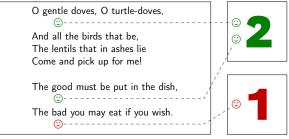


Figure: Example of Text Numerisization

- Many texts are numerisized via lexical projection
- Goal: Accurate values for positive and negative sentiment

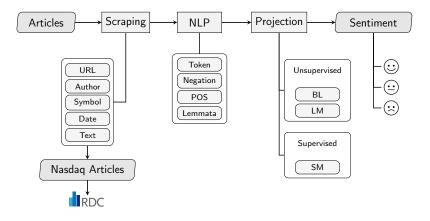
Examples

Supervised Projection

We use supervised projection (Zhang et al., JBES, 2016)

- - Sentence-level annotation of financial news
 - ► Manual annotation of 5,000 sentences by 16 annotators: to incorporate human knowledge
 - Example: "profit" with different semantic orientations
 - Neutral in "profit was 1 million"
 - Positive in "profit increased from last year"

How to gather Sentiment Variables?



Lexicon-based Sentiment

Consider document i, positive sentiment Pos_i , positive lexicon entries W_j (j = 1, ..., J) and count frequency of those entries w_j :

$$Pos_i = n_i^{-1} \sum_{j=1}^{J} I(W_j \in L) w_j$$
 (12)

with n_i : number of words in document i (e.g. sentence)

Equivalent calculation of negative sentiment Negi

Sentence-level Polarity

$$Pol_{i} = \begin{cases} 1, & \text{if } Pos_{i} > Neg_{i} \\ 0, & \text{if } Pos_{i} = Neg_{i} \\ -1, & \text{if } Pos_{i} < Neg_{i} \end{cases}$$

$$(13)$$

for sentence i

Measure sentiment on sentence level

Regularized Linear Models (RLM)

- □ Linear scoring function $s(X) = β^T X$ with $β ∈ ℝ^p$

Example

Regularized training error:

$$n^{-1} \sum_{i=1}^{n} \underbrace{L\{y_{i}, s(X)\}}_{\text{Loss Function}} + \lambda \underbrace{R(\beta)}_{\text{Regularization Term}}$$
(14)

with hyperparameter $\lambda \geq 0$

RLM Estimation

- Optimize via Stochastic Gradient Descent More
- Oversampling More
- \Box Choice of: $L(\cdot), R(\cdot), \lambda, X$ (*n*-gram range, features) . . .

Bullishness

$$B = \log \left\{ \frac{1 + n^{-1} \sum_{j=1}^{n} \mathbf{I} \left(Pol_{j} = 1 \right)}{1 + n^{-1} \sum_{j=1}^{n} \mathbf{I} \left(Pol_{j} = -1 \right)} \right\}$$
 (15)

by Antweiler and Frank (JF, 2004) with j = 1, ..., n sentences in document.

- \Box $B_{i,t}$ accounts for bullishness of company i on day t
- $oxed{\Box}$ Consider $BN_{i,t} = I(B_{i,t} < 0)B_{i,t}$

Calibration of probability weighting functions

Estimate ρ_{τ} using nonlinear quantile regressions.

Employ power utility $u(R) = R^{1-\gamma}/(1-\gamma)$.

$$Q_{\tau}[-\gamma \log(R_t) + \log\{g(v_t, S_t)\} + \log(1 + R_{t+1})] = 0, \quad (16)$$

recall

$$g(v, S) = G'(1, \beta; v) = \beta v^{\beta - 1},$$

 $\beta = \beta(S, \rho) = \exp\{-\rho(S^{-1} - 1)\} - 1.$

where

$$g(v_t, S_t) = (\exp\{-\rho(1/S_t - 1)\} - 1)v_t^{\exp\{-\rho(1/S_t - 1)\} - 2}.$$

Calibration of probability weighting functions

Expect ρ_{τ} to differ across τ since sentiment distorts beliefs of a τ -quantile preference maker.

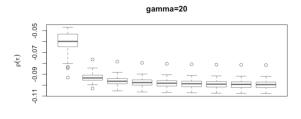
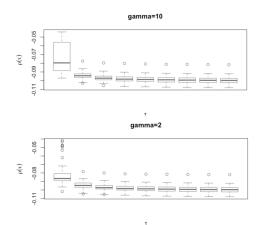


Figure: Variation over firms for $\gamma = 20$

Calibration of probability weighting functions



Message: ρ bigger for smaller au

Calibration of probability weighting functions

Higher values of ρ_{τ} in the left tail indicate that large negative sentiment is connected to higher overweighting of the PK.

Empirical Results: Pricing tails with Sentiment

$$Q_{\tau}\left[\widetilde{M}_{t}\times(1+R_{t+1})+1\right]=0$$

 \odot with $\widetilde{M}_t = \exp(-\alpha_{\tau} - \beta_{S,\tau} S_t - FF_t^{\top} \beta_{FF,\tau} - X_t^{\top} \beta_{X_t,\tau})$,

FF=Fama French 5 factors

 X_t - control variables including idiosyncratic factors

□ Factors are proxy for aggregate consumption

Empirical Results: Pricing tails with Sentiment

After log-linearization, we arrive to a simple linear model

$$Q_{\tau} \left[\log(1 + R_{t+1}) - \alpha_{\tau} - \beta_{S,\tau} S_t - FF_t^{\top} \beta_{FF,\tau} - X_t^{\top} \beta_{X_t,\tau} \right] = 0 \quad (17)$$

implying

$$Q_{\tau} \left[\log(1 + R_{t+1}) \right] = \alpha_{\tau} + \beta_{S,\tau} S_t + F F_t^{\top} \beta_{FF,\tau} + X_t^{\top} \beta_{X_t,\tau}$$
 (18)

with FF Fama-French Factors

Empirical Results: Sentiment as factor

- Aggregate market sentiment as possible risk factor.
- Control also for firm-specific sentiment and volatility
- Negative sentiment captures "fear", related to VIX (Da et al., 2015)
- □ Following high investor sentiment, aggregate returns are low (Baker and Wurgler, 2007)
- Overly optimistic beliefs about future cash flows is not justified by fundamentals.

A dynamic quantile model with sentiment

Linear asset pricing model Fama-French Factors

$$Q_{\tau}(r_{i,t+1}) = \alpha_{i,\tau} + \beta_{1,\tau} B_{i,t} + \beta_{2,\tau} \sigma_{i,t} + \beta_{3,\tau} |BN_t| + FF_t^{\top} \beta_{FF,\tau}$$
(19)

with $\sigma_{i,t}$ Garman & Klass (1980) range-based volatility

 $|BN_t|$ proxy for S_t (hence β_S from (18) is here β_3) $B_{i,t}$ proxy for idiosyncratic sentiment $\sigma_{i,t}$ proxy for volatility

 $B_{i,t}$, $\sigma_{i,t}$ control variables, contained in the matrix X in (18).

Eq (20) tests if sentiment prices quantiles of the excess asset returns.

- oxdot Coefficients varying across au imply marginal effect
- \odot Coefficients constant over τ : EU works?

A dynamic quantile model with sentiment

Linear asset pricing model Fama-French Factors

$$Q_{\tau}(r_{i,t+1}) = \alpha_{i,\tau} + \beta_{1,\tau} B_{i,t} + \beta_{2,\tau} \sigma_{i,t} + \beta_{3,\tau} |BN_t| + FF_t^{\top} \beta_{FF,\tau}$$
 (20)

with $\sigma_{i,t}$ - Garman & Klass (1980) range-based volatility.

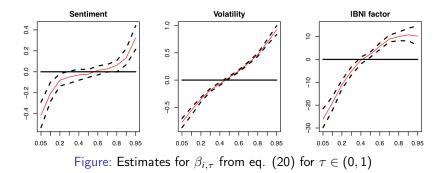
- (20) tests if sentiment prices quantiles of the excess asset returns.
 - Coefficients capture marginal effects of pricing factors
 - oxdot Coefficients varying across au imply marginal effect
 - \Box Coefficients constant over τ : EU works?

Results

Estimate (20) via QR

- □ Panel of 100 most liquid constituents of S&P 500 stocks
- oxdot Check sentiments across au

Results: Panel of 100 stocks



Full estimates of eq. (20) Further Graphics

Results: Panel of 100 stocks

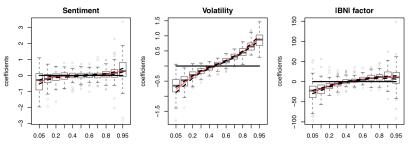
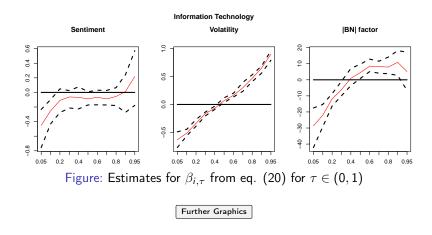
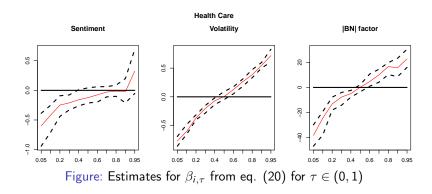


Figure: Estimates for $\beta_{i,\tau}$ together with box plots showing individual estimates with univariate individual I=1,...,100 QR estimates

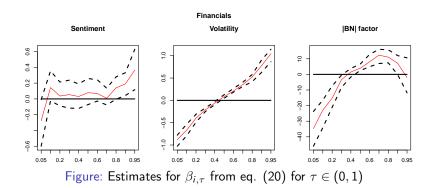
Empirical Results

- □ Tails are strongly influenced
- Sentiment and volatility effects similarly
- Asymmetric impact of market sentiment
- Increase in negative bullishness has positive effect on right tail, and negative effect on left tail
- Contrary to literature, factors explain daily data in quantiles

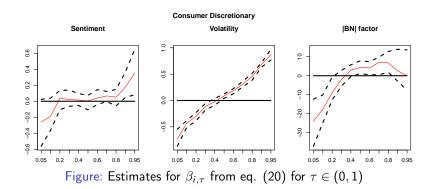




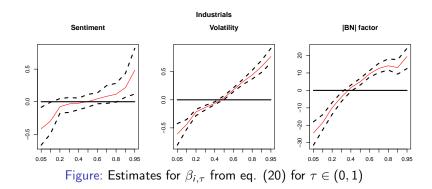
Full estimates of eq. (20) Further Graphics



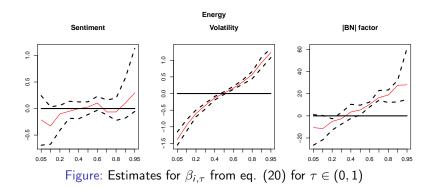
Full estimates of eq. (20) Further Graphics



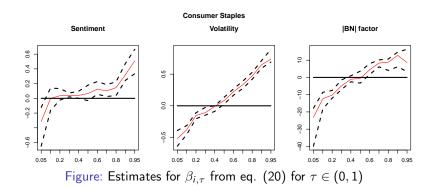
Full estimates of eq. (20) Further Graphics



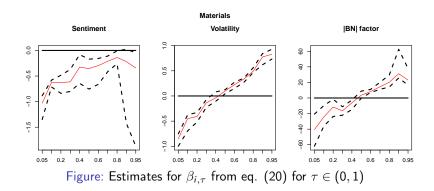
Full estimates of eq. (20) Further Graphics



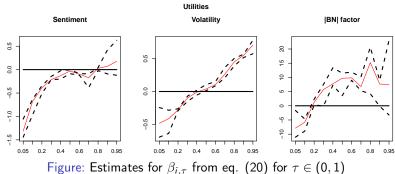
Full estimates of eq. (20) Further Graphics



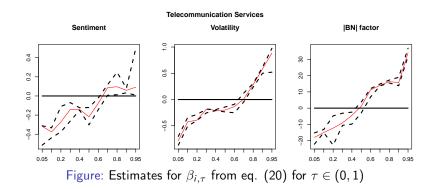
Full estimates of eq. (20) Further Graphics



Full estimates of eq. (20) Further Graphics



Full estimates of eq. (20) Further Graphics



Full estimates of eq. (20) Further Graphics

Outlook — 7-1

Summary

- Dynamic quantile model for asset pricing with sentiment
- Investor sentiment distilled from public news with cross-section of future return's quantiles.

Tales of sentiment driven tails

Jozef Baruník Cathy Yi-Hsuan Chen Wolfgang Karl Härdle

Institute of Economic Studies Charles University in Prague Ladislaus von Bortkiewicz Chair of Statistics Humboldt-Universität zu Berlin



http://ies.fsv.cuni.cz http://lvb.wiwi.hu-berlin.de

Bibliography



Baker, M., and J. Wurgler.

Investor sentiment and the cross-section of stock returns

Journal of Finance, 2006

de Castro, L. I. and A. F. Galvao

Dynamic quantile models of rational behavior
2017

Da, Z., Engelberg J. and Gao, P.

The Sum of All FEARS Investor Sentiment and Asset Prices
Review of Financial Studies, 2015



Dumas, B., Kurshev, A., Uppal, R.

Equilibrium Portfolio Strategies in the Presence of Sentiment Risk
and Excess Volatility
Journal of Finance. 2009

Fama, E. and K. French.

A Five-Factor Asset Pricing Model

J. Financial Econom., 2015

Huang, D., Jun Tu, J., Jiang, F., and Zhou, G.

Investor Sentiment Aligned: A Powerful Predictor of Stock Returns

Journal of Finance, 2014



The general theory of employment, interest and money. London: Macmillan, 1936

Koenker, R.

Quantile regression for longitudinal data. Journal of Multivariate Analysis, 2004

Manski, C.F.

Ordinal utility models of decision making under uncertainty.

Theory and Decision, 1988

Polkovnichenko, V. and Zhao, F.

Probability weighting functions implied in options prices

Journal of Financial Economics, 2013

Bibliography — 8-4



The probability weighting function Econometrica, 1998

Rostek, M.

Quantile maximization in decision theory. The Review of Economic Studies, 2010

Stambaugh, R.F., Yu, J.F., Yuan, Y.

The short of it: Investor sentiment and anomalies.

Journal of Financial Economics, 2012

Tversky, A. and Kahneman, D.

Advances in prospect theory: Cumulative representation of uncertainty

Journal of Risk and uncertainty, 1992



Zhang, J., Chen C. Y., Härdle, W. K. and Bommes, E. *Distillation of News into Analysis of Stock Reactions* J. Bus. Econom. Statist., 2016

Appendix

Appendix — 9-1

Tagging Example - BL

... McDonald's has an obesity **problem** that continues to get **worse**. And that's nothing to do with the food itself, but rather the huge menus that can now double as medieval fortification. For perspective, the chain's menu has grown 70% since 2007. And while more offerings might seem **like** a **good** thing, large menus result in **slower** service and more flare-ups between franchisees and the corporation.

Bloated menus raise inventory costs for smaller franchisees and **lead** to lower profit margins. The McDonald's corporate franchise fee is based upon sales instead of profits, making it a smaller **concern** for the company overall. ...

3 positive words and 5 negative words

TXTMcDbm Article source Appendix — 9-2

Tagging Example - LM

... McDonald's has an obesity **problem** that continues to get **worse**. And that's nothing to do with the food itself, but rather the huge menus that can now double as medieval fortification. For perspective, the chain's menu has grown 70% since 2007. And while more offerings might seem like a **good** thing, large menus result in **slower** service and more flare-ups between franchisees and the corporation.

Bloated menus raise inventory costs for smaller franchisees and lead to lower profit margins. The McDonald's corporate franchise fee is based upon sales instead of profits, making it a smaller **concern** for the company overall. ...

1 positive word and 4 negative words



Back

Appendix —————————————————9-3

Web Scraping

- Databases to buy?
- Automatically extract information from web pages
- Transform unstructured data (HTML) to structured data
- Use HTML tree structure to parse web page
- Legal issues
 - Websites protected by copyright law
 - Prohibition of web scraping possible
 - Comply to Terms of Service (TOS)

Back

Natural Language Processing (NLP)

- Text is unstructured data with implicit structure
 - ► Text, sentences, words, characters
 - ► Nouns, verbs, adjectives, ..
 - Grammar
- Transform implicit text structure into explicit structure
- Reduce text variation for further analysis
- ☑ TXTnlp

Back

Tokenization

String

"McDonald's has its work cut out for it. Not only are sales falling in the U.S., but the company is now experiencing problems abroad."

Sentences

"McDonald's has its work cut out for it.",
"Not only are sales falling in the U.S., but the company is
now experiencing problems abroad."

Words

```
'McDonald', '"s", 'has', 'its', 'work', 'cut', 'out' ...
```

Appendix ·

Negation Handling

- \Box "not good" \neq "good"
- ☐ Reverse polarity of word if negation word is nearby
- Negation words "n't", "not", "never", "no", "neither", "nor", "none"

Part of Speech Tagging (POS)

- Grammatical tagging of words
 - ▶ dogs noun, plural (NNS)
 - ▶ saw verb, past tense (VBD) or noun, singular (NN)
- □ Penn Treebank POS tags
- Stochastic model or rule-based

Appendix —————————————————————9-8

Lemmatization

- Determine canonical form of word
 - ▶ dogs dog
 - ▶ saw (verb) see and saw (noun) saw
- Reduces dimension of text
- - Porter stemmer: saw (verb and noun) saw

Loss Functions for Classification

Logistic: Logit

$$L\{y, s(X)\} = \log(2)^{-1} \log[1 + \exp\{-s(X)y\}]$$
 (21)

$$L\{y, s(X)\} = \max\{0, 1 - s(X)y\}$$
 (22)

Regularization Term

$$R(\beta) = 2^{-1} \sum_{i=1}^{p} \beta_i^2 \tag{23}$$

$$R(\beta) = \sum_{i=1}^{p} |\beta_i| \tag{24}$$

RLM Example

Sentence 1: "The profit of Apple increased."

Sentence 2: "The profit of the company decreased."

$$y = (1, -1) \quad (25)$$

$$X = Apple \begin{cases} 1 & 2 \\ 1 & 1 \\ 1 & 1 \\ 1 & 0 \\ increased \\ company \\ decreased \end{cases} \quad (26)$$

Back

k-fold Cross Validation (CV)

- □ Partition data into k complementary subsets
- No loss of information as in conventional validation
- Stratified CV: equally distributed response variable in each fold

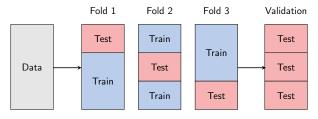


Figure: 3-fold Cross Validation

Back

Oversampling

- Balance size of neutral sentences and ones with polarity in sample
- Duplicate sentences within folds of stratified cross validation until the sample is balanced

Classification Error Rates

- □ Type I error rate = FP/(FP + TN)
- □ Type II error rate = FN/(FN + TP)

with TP as true positive, TN as true negative, FP as false positive and FN as false negative.

 Back

Stochastic Gradient Descent (SGD)

Approximately minimize loss function

$$L(\theta) = \sum_{i=1}^{n} L_i(\theta)$$
 (27)

$$\theta_i = \theta_{i-1} - \eta \, \frac{\partial L_i(\theta)}{\partial \theta} \tag{28}$$

SGD Algorithm

- 1. Choose learning rate η
- 2. Shuffle data
- 3. For i = 1, ..., n, do:

$$\theta_i = \theta_{i-1} - \eta \frac{\partial L_i(\theta)}{\partial \theta}$$

Repeat 2 and 3 until approximate minimum obtained.

SGD Example

 $X \sim N(\mu, \sigma)$ and $x_1, ..., x_n$ as randomly drawn sample

$$\min_{\theta} \ n^{-1} \sum_{i=1}^{n} (\theta - x_i)^2$$

Update step

$$\theta_i = \theta_{i-1} - 2\eta(\theta_{i-1} - x_i)$$

Optimal gain

Set $2\eta = 1/i$ and obtain $\theta_n = \bar{x}$ with \bar{x} as sample mean.

SGD Example ctd

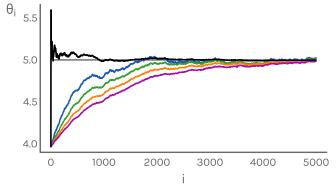


Figure: Estimate Mean via SGD, $x_t \sim N(5,1)$

$$\eta \in \{1/t, 1/1000, 1/1500, 1/2000, 1/2500\}$$
 Q TXTSGD

Back

Evaluation Supervised Learning

Pred True	-1	0	1	Total
-1	1,983	298	254	2,535
0	96	2,134	305	2,535
1	105	469	1,961	2,535
Total	2,184	2,901	2,520	7,605

Table: Confusion Matrix - Supervised Learning with Oversampling



Appendix —————————————————————9-20

Abbreviations

Sector	Abbreviation	
Consumer Discretionary	CD	
Consumer Staples	CS	
Energy	EN	
Financials	FI	
Health Care	HC	
Industrials	IN	
Information Technology	IT	
Materials	MA	
Telecommunication	TE	
Utilities	UT	

Table: Sector Abbreviations

back

Fama-French 5 factors

FF1 - the Mkt factor: excess return on the market index

FF2 - the SMB factor: (Small Minus Big) the average return on the nine small-stock portfolios minus that on the nine big-stock portfolios.

FF3 - the HML factor: (High Minus Low) the average return on the two value-stock portfolios minus that on the two growth-stock portfolios

Fama-French 5 factors cont.

FF4 - the RMW factor: (Robust Minus Weak) the average return on the two robust operating profitability portfolios minus that on the two weak operating profitability portfolios

FF5 - the CMA factor: (Conservative Minus Aggressive) the average return on the two conservative investment portfolios minus that on the two aggressive investment portfolios

Back

Garman & Klass range-based volatility

$$\sigma_{i,t} = 0.511(u - d)^{2} - 0.019\{c(u + d) - 2ud\} - 0.838c^{2}$$
with $u = \log(P_{i,t}^{H}) - \log(P_{i,t}^{L})$

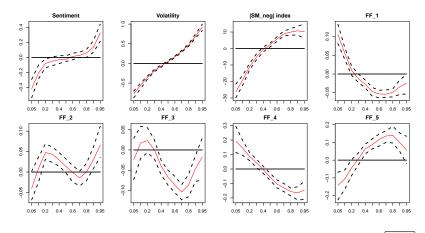
$$d = \log(P_{i,t}^{L}) - \log(P_{i,t}^{O})$$

$$c = \log(P_{i,t}^{C}) - \log(P_{i,t}^{O}),$$
(29)

where the $P_{i,t}^H, P_{i,t}^L, P_{i,t}^O, P_{i,t}^C$ are the daily highest, lowest, opening and closing stock prices.

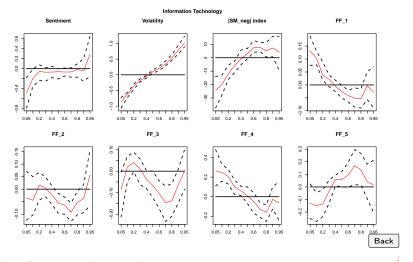
Back

Results: Panel of 100 stocks

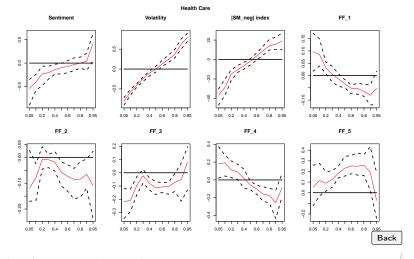


Back

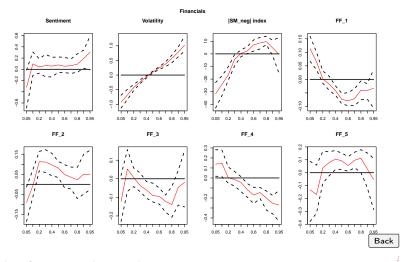
Results: Sectors



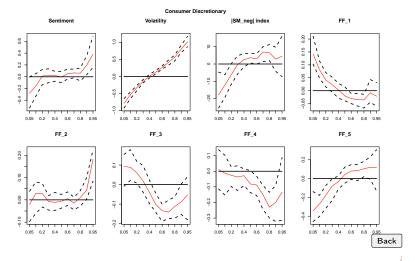
Results: Sectors



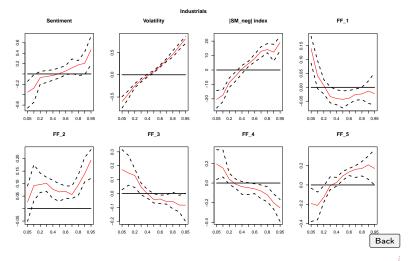
Results: Sectors



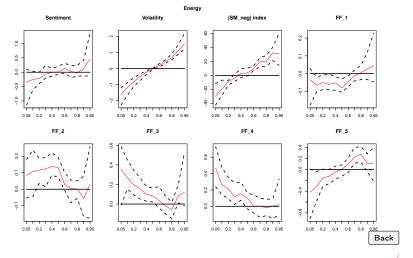
Results: Sectors



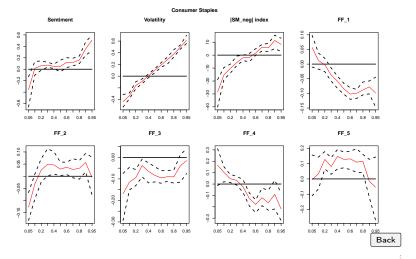
Results: Sectors



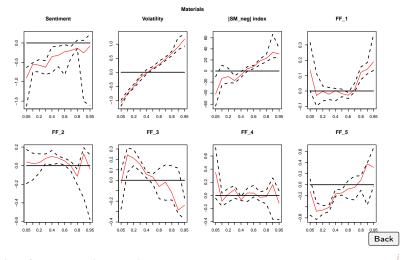
Results: Sectors



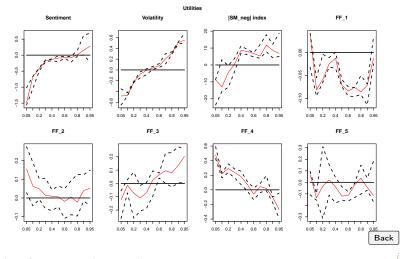
Results: Sectors



Results: Sectors



Results: Sectors



Results: Sectors

