Quantile Regression in Risk Calibration

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Dependence Risk

Risk Calibration

- Quantification of risk: Value-at-Risk (VaR) and expected shortfall
- Drawbacks of usual VaR: Does not say much about dependence risk
- Need for other risk measure

Quantile Regression in VaR

- Parametric VaR: Chernozhukov and Umantsev (2001), Engle and Manganelli (2004)
- Nonparametric VaR: Cai and Wang (2008), Taylor (2008) and Schaumburg (2010)
- Parametric CoVaR: Adrian and Brunnermeier (2010)(AB)

Risk Calibration

- Marginal Expected Shortfall (MES): Acharya et al. (2010)
- Distressed Insurance Premium (DIP): Huang et al. (2010)

  Go to details

- AB: X_j and X_i are two asset returns,

$$\mathrm{P}\left\{X_j \leq \mathrm{CoVaR}_{j|i}(q) | C(X_i)\right\} = q.$$

  where $C(X_i) = \{X_i = \mathrm{VaR}_q(X_i)\}$.
- Advantages:
  1. Cloning property
  2. Conservative property
  3. Adaptiveness

Go to details

CoVaR Construction (AB)

$X_{j,t}$ and $X_{i,t}$ are two asset returns. Two linear quantile regressions:

$$X_{i,t} = \alpha_i + \gamma_i M_{t-1} + \varepsilon_{i,t}, \tag{1}$$

$$X_{j,t} = \alpha_{j|i} + \beta_{j|i} X_{i,t} + \gamma_{j|i} M_{t-1} + \varepsilon_{j,t}. \tag{2}$$

M_t: vector-valued state variables. $F^{-1}_{\varepsilon_{i,t}}(q|M_{t-1}) = 0$ and $F^{-1}_{\varepsilon_{j,t}}(q|M_{t-1}, X_{i,t}) = 0$.

$$\widehat{VaR}_{i,t} = \hat{\alpha}_i + \hat{\gamma}_i M_{t-1},$$

$$\widehat{CoVaR}_{j|i,t} = \hat{\alpha}_{j|i} + \hat{\beta}_{j|i} \widehat{VaR}_{i,t} + \hat{\gamma}_{j|i} M_{t-1}.$$

Nonlinear Dependence in Asset Returns

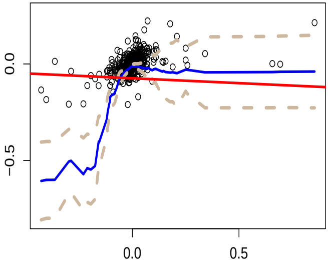

Figure 1: Goldman Sachs (GS) and Citigroup (C) weekly returns 0.05 (left) and 0.1(right) quantile functions. y-axis=GS returns; x-axis=C returns. LLQR lines. Linear parametric quantile regression line. 95% Confidence band. $N = 546$.

Nonlinear Dependence in Asset Returns

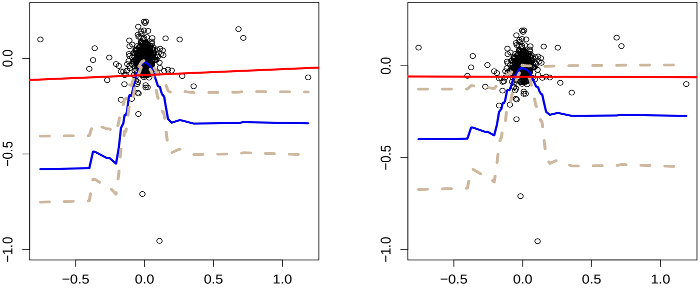

Figure 2: Lehman Brothers (LB) and AIG weekly returns 0.05 (left) and 0.1(right) quantile functions. y-axis=LB returns; x-axis=AIG returns. LLQR lines. Linear parametric quantile regression line. 95% Confidence band. $N = 546$.

Nonlinear Dependence in Asset Returns

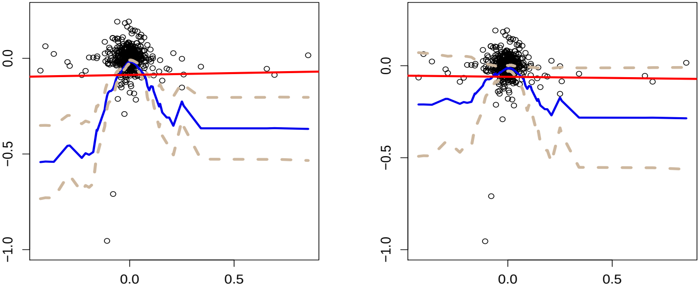

Figure 3: LB and C weekly returns 0.05 (left) and 0.1(right) quantile functions. y-axis=LB returns; x-axis=C returns. LLQR lines. Linear parametric quantile regression line. 95% Confidence band. $N = 546$.

Nonlinear Dependence in Asset Returns

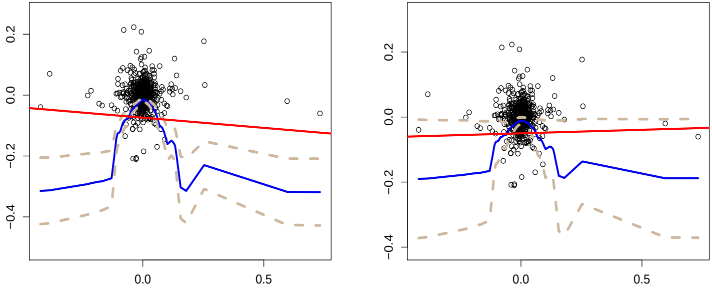

Figure 4: Bank of America (BOA) and GS weekly returns 0.05 (left) and 0.1(right) quantile functions. y-axis=BOA returns; x-axis=GS returns. LLQR lines. Linear parametric quantile regression line. 95% Confidence band. $N = 546$.

Nonlinear Dependence in Asset Returns

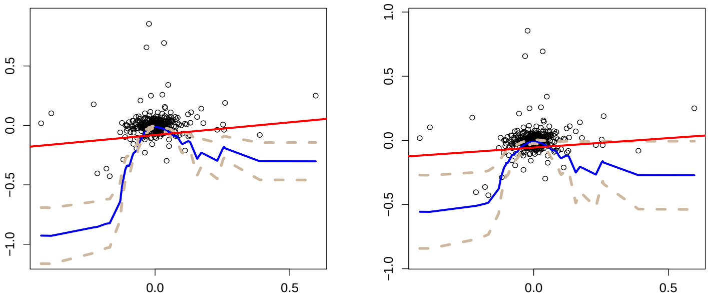

Figure 5: BOA and C weekly returns 0.05 (left) and 0.1(right) quantile functions. y-axis=BOA returns; x-axis=C returns. LLQR lines. Linear parametric quantile regression line. 95% Confidence band. $N = 546$.

General Specification

- More general, with functions f, g;

$$X_{i,t} = f(M_{t-1}) + \varepsilon_{i,t}; \tag{3}$$
$$X_{j,t} = g(X_{i,t}, M_{t-1}) + \varepsilon_{j,t}. \tag{4}$$

  M_t: vector-valued state variables. $F^{-1}_{\varepsilon_{i,t}}(q|M_{t-1}) = 0$ and $F^{-1}_{\varepsilon_{j,t}}(q|M_{t-1}, X_{i,t}) = 0$.

- Challenge:
  1. The curse of dimensionality for f, g
  2. Numerical Calibration of (3) and (4)

Goal

- Computing CoVaR (i.e. two step quantile regression) in a nonparametric (or semiparametric) fashion
- Testing the risk-measuring performance of the estimated CoVaR
- What can one learn from the semiparametric specification

Outline

1. Motivation ✓
2. Locally Linear Quantile Regression
3. A Semiparametric Model
4. Backtesting
5. Conclusions and Further Work

Locally Linear Quantile Estimation (LLQR)

- Locally Linear Quantile Regression (LLQR):

$$\underset{\{a_{0,0},a_{0,1}\}}{\operatorname{argmin}} \sum_{i=1}^{N} K\left(\frac{x_i - x_0}{h}\right) \rho_q \left\{ y_i - a_{0,0} - a_{0,1}(x_i - x_0) \right\}. \quad (5)$$

- Choice of Bandwidth: Yu and Jones (1998)
- Asymptotic Uniform Confidence Band: Härdle and Song (2010)

Macroeconomic Drives

Component of M_t:

1. VIX
2. Short term liquidity spread
3. The daily change in the three-month treasury bill rate
4. The change in the slope of the yield curve
5. The change in the credit spread between 10 years BAA-rated bonds and the treasury rate
6. The daily S&P500 index returns
7. The daily Dow Jones U.S. Real Estate index returns

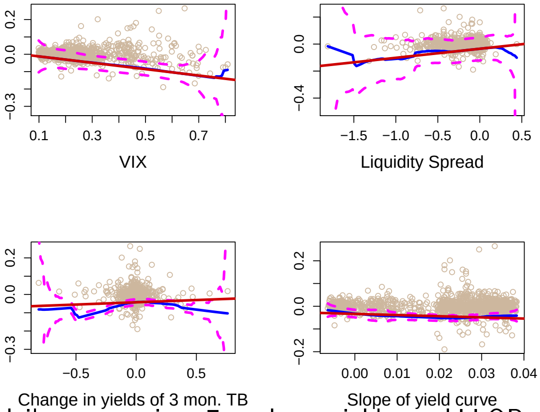

Figure 6: GS daily returns given 7 market variables and LLQR curves. Data 20060804-20110804. $N = 1260$. $\tau = 0.05$.

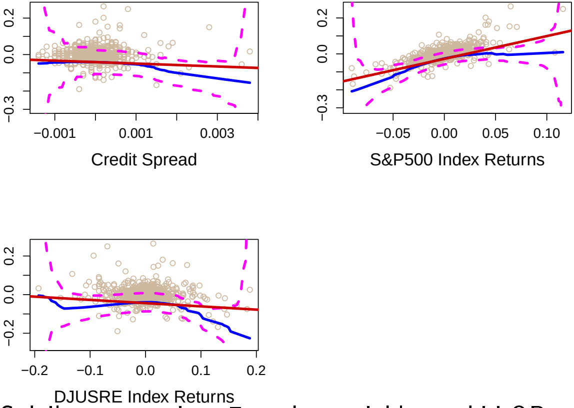

Figure 7: GS daily returns given 7 market variables and LLQR curves. Data 20060804-20110804. $N = 1260$. $\tau = 0.05$.

Partial Linear Model

- The aforementioned linearity tests imply

$$X_{i,t} = \alpha_i + \gamma_i M_{t-1} + \varepsilon_{i,t}; \qquad (6)$$

$$X_{j,t} = \beta_j M_{t-1} + l(X_{i,t}) + \varepsilon_{j,t}. \qquad (7)$$

l: a general function. M_t: state variables. $F^{-1}_{\varepsilon_{i,t}}(q|M_{t-1}) = 0$ and $F^{-1}_{\varepsilon_{j,t}}(q|M_{t-1}, X_{i,t}) = 0$.

- Advantage:
  1. Capturing nonlinear asset dependence
  2. Avoid curse of dimensionality

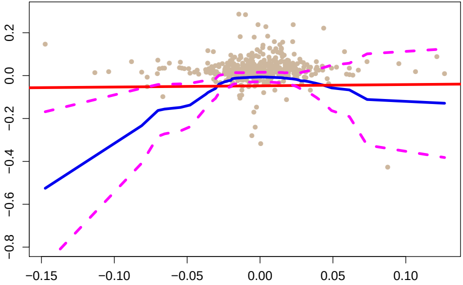

Figure 8: The nonparametric part of the PLM estimation. y-axis=GS daily returns. x-axis=C daily returns. The LLQR quantile curve. Linear parametric quantile line. 95% Confidence band. Data 20080625-20081223. N=126. h =0.2003. $q = 0.05$.

Estimation of Partial Linear Model

- Method: Liang, Härdle and Carroll (1999) and Härdle, Ritov and Song (2011)
- Estimation of l: LLQR
- j: GS daily returns,
  i: C daily returns
  Window Size: 126 days (half a year)
  Data 20060804-20110804

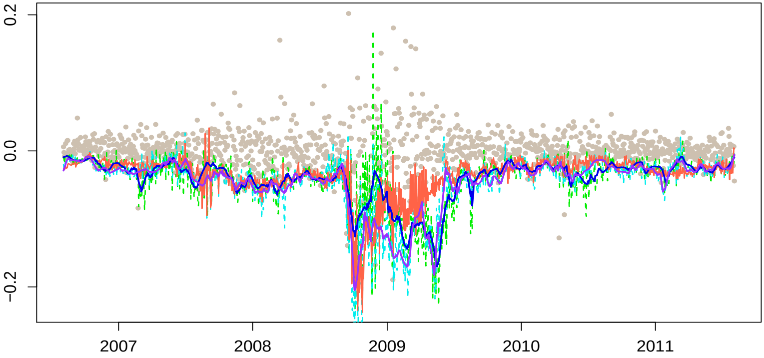

Figure 9: CoVaR of Goldman Sachs given the VaR of Citigroup. The x-axis is time. The y-axis is the GS daily returns. PLM CoVaR . AB (2010) CoVaR . The linear QR VaR of GS.

Backtesting Procedure

- Berkowitz, Christoffersen and Pelletier (2011): If the VaR algorithm is correct, violations should be unpredictable

$$I_t = \begin{cases} 1, & \text{if } R_t < \widehat{VaR}_{t-1}(q) \\ 0, & \text{otherwise.} \end{cases}$$

- Formally, violations I_t form a sequence of martingale difference

Box Tests

- $\hat{\rho}_k$ be the estimated autocorrelation of lag k of violation $\{I_t\}$ and N be the length of the time series.
- Ljung-Box test:

$$\mathrm{LB}(m) = N(N+2)\sum_{k=1}^{m} \frac{\hat{\rho}_k^2}{N-k} \tag{8}$$

- Lobato test:

$$\mathrm{L}(m) = N\sum_{k=1}^{m} \frac{\hat{\rho}_k^2}{\hat{v}_{kk}} \tag{9}$$

CaViaR Test

- Engle and Manganelli (2004)
- Berkowitz, Christoffersen and Pelletier (2011): CaViaR performs best overall
- Test procedure:

$$I_t = \alpha + \beta_1 I_{t-1} + \beta_2 VaR_{t-1} + u_t,$$

where VaR_{t-1} can be replaced by $CoVaR_{t-1}$ in the case of conditional VaR. The residual u_t follows a Logistic distribution.

- The null hypothesis is $\hat{\beta}_1 = \hat{\beta}_2 = 0$.

Summary of Backtesting Procedure

- LB(1): i.i.d. test
- LB(5): i.i.d. test
- L(1): Testing first one lag autocorrelation = 0
- L(5): Testing first five lags autocorrelation = 0
- CaViaR-overall: all data 20060804-20110804
- CaViaR-crisis: data 20080804-20090804

Table 1: Goldman Sachs VaR/CoVaR backtesting p-values.

Measure	LB(1)	LB(5)	L(1)	L(5)	CaViaR-overall	CaViaR-crisis
			Panel 1			
$\widehat{VaR}_{GS,t}$	0.3449	0.0253*	0.3931	0.1310	1.265×10^{-6}***	0.0024**
			Panel 2			
$\widehat{CoVaR}^{AB}_{GS\|SP,t}$	0.0869	0.2059	0.2684	0.6586	8.716×10^{-7}***	0.0424*
$\widehat{CoVaR}^{PLM}_{GS\|SP,t}$	0.0518	0.0006***	0.0999	0.0117*	2.2×10^{-16}***	0.0019**
			Panel 3			
$\widehat{CoVaR}^{AB}_{GS\|C,t}$	0.0489*	0.2143	0.1201	0.4335	3.378×10^{-9}***	0.0001***
$\widehat{CoVaR}^{PLM}_{GS\|C,t}$	0.8109	0.0251*	0.8162	0.2306	2.946×10^{-9}***	0.0535

*, ** and *** denote significance at the 5, 1 and 0.1 percent levels.

Conclusions and Further Work

- Semiparametric model may capture risk better than linear model during financial crisis
- Multivariate nonlinear part in PLM
- Other assets returns

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Macorprudential Risk Measures

- Marginal Expected Shortfall (MES): $R = \sum_i y_i R_i$, y_i: weights, R_i: asset return

$$\text{MES}^i_\alpha = \frac{\partial ES_\alpha(R)}{\partial y_i} = -\mathsf{E}\left[R_i | R \leq -VaR_\alpha\right]$$

- Distressed Insurance Premium (DIP): Huang et al. (2010) $L = \sum_{i=1}^N L_i$ total loss of a portfolio

$$DIP = \mathsf{E}^Q\left[L | L \geq L_{min}\right]$$

Return

Advantages of CoVaR

- Cloning Property: if dividing X_i into several clones, then the value of CoVaR conditioning on the individual large firm does not differ from the one conditioning on one of the clones
- Conservative Property: CoVaR conditioning on some bad event, the value would be more conservative than VaR
- Adaptive to the changing market conditions

Return

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