# Statistics in finance and computing

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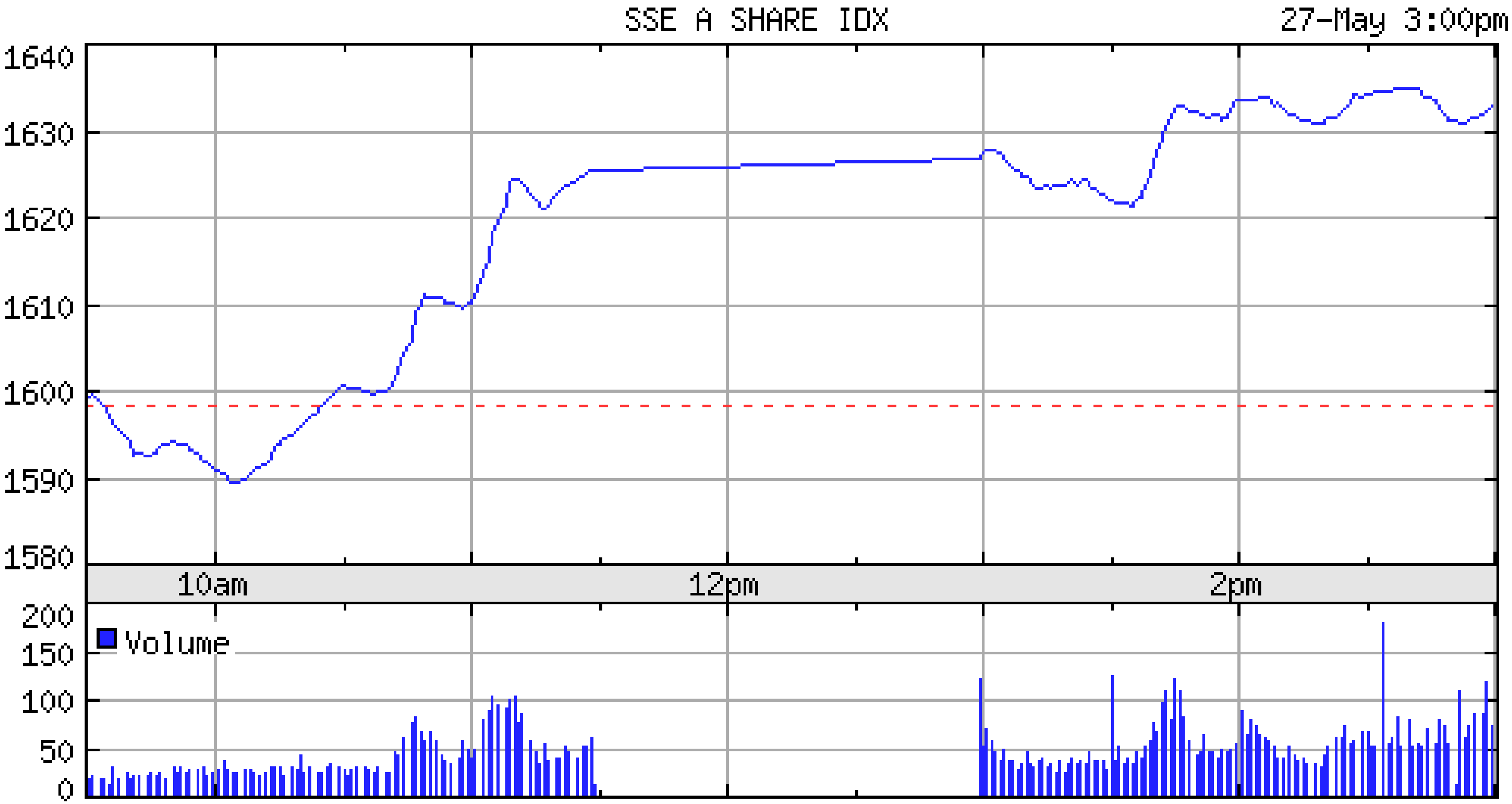
# 风险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# Outline

- Model and estimate implied volatility surface (IVS) for trading, hedging of derivatives positions and risk management, cooperation with Michal Benko and Matthias Fengler.
- Calibrate IVS and price option using fast Fourier transform (FFT), cooperation with Szymon Borak and Kai Detlefsen.
- Estimate value at risk (VaR) based on local homogeneous volatility and generalized hyperbolic (GH) distribution, cooperation with Ying Chen (陈颖) and Seok-Oh Jeong
- Skewness and kurtosis trading strategies, cooperation with Oliver Blaskowitz.

# Implied Volatility String Dynamics

Black and Scholes (1973) (BS) formula prices European options under the assumption that the asset price $S_t$ follows a geometric Brownian motion with constant drift and constant volatility coefficient $\sigma$:

$$C_t^{BS} = S_t \Phi(d_1) - K e^{-r\tau} \Phi(d_2) \ ,$$

where $d_{1,2} = \frac{\ln(S_t/K) + (r \pm \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}$. $\Phi(u)$ is the CDF of the standard normal distribution, $r$ a constant interest rate, $\tau = T - t$ time to maturity, $K$ the strike price.

# Implied volatilities

Volatility $\hat{\sigma}$ as *implied* by observed market prices $\tilde{C}_t$:

$$\hat{\sigma}: \quad \tilde{C}_t - C_t^{BS}(S_t, K, \tau, r, \hat{\sigma}) = 0 \; .$$

Unlike assumed in the BS model, $\hat{\sigma}_t(K,\tau)$ exhibits **distinct, time-dependent** functional patterns across $K$ (**smile or smirk**), and a **term-structure** $T-t$: Thus $\hat{\sigma}_t(K,\tau)$ is interpreted as a **random surface**: the implied volatility surfaces **(IVS)** .

# Why implied volatilities as a state variable?

1. easily observable,
2. shocks are highly correlated through $K, \tau$, the underlying asset, and across markets
3. practitioners quote options in 'terms of implied volatilities',
4. trading rules and strategies can be defined through implied volatilities,
5. tradable through volatility contracts, e.g. VDAX, VIX.
6. option markets behave increasingly self-governed, Bakshi et al. (2000); Cont and da Fonseca (2002).

# Purpose

A **modeling strategy** in terms of a **semiparametric factor model (SFM)** for the IVS $Y_{i,j}$ $(i = \text{day}, j = \text{intraday})$:

$$Y_{i,j} = m_0(X_{i,j}) + \sum_{l=1}^{L} \beta_{i,l} m_l(X_{i,j}) \quad . \tag{1}$$

Here $m_l(X_{i,j})$ are smooth factor functions and $\beta_{i,l}$ a multivariate loading time-series.

Key features:

- **modeling** and **estimation** in one approach
- suitable for IV data measured on a **degenerated design**.

# Degenerated Design of IV Data

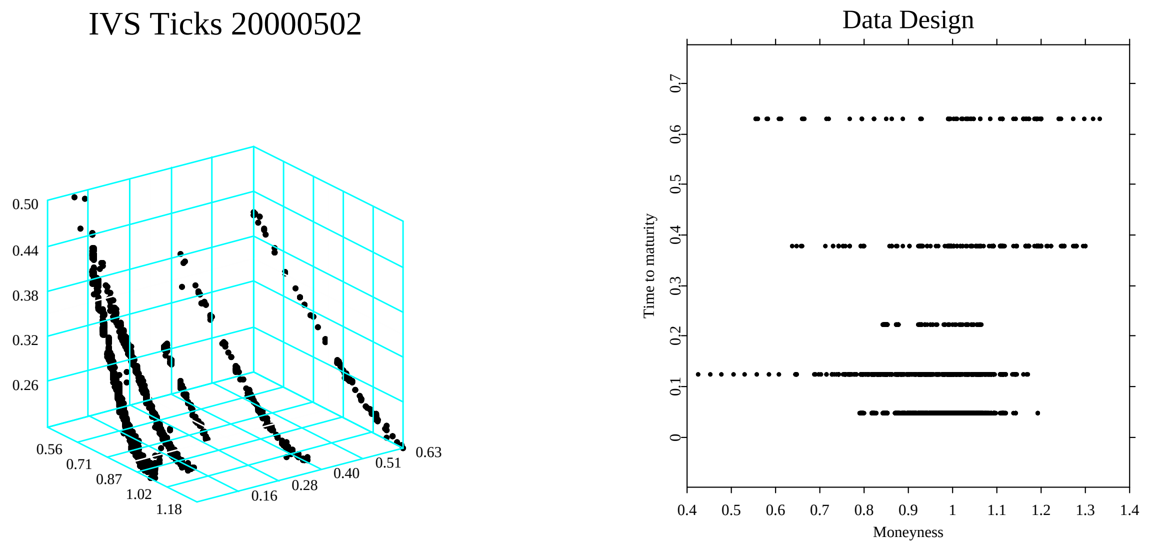



Figure 1: *Left panel: call and put implied volatilities observed on 20000502. Right panel: data design on 2000502; ODAX, difference-dividend correction according to Hafner and Wallmeier (2001) applied.*

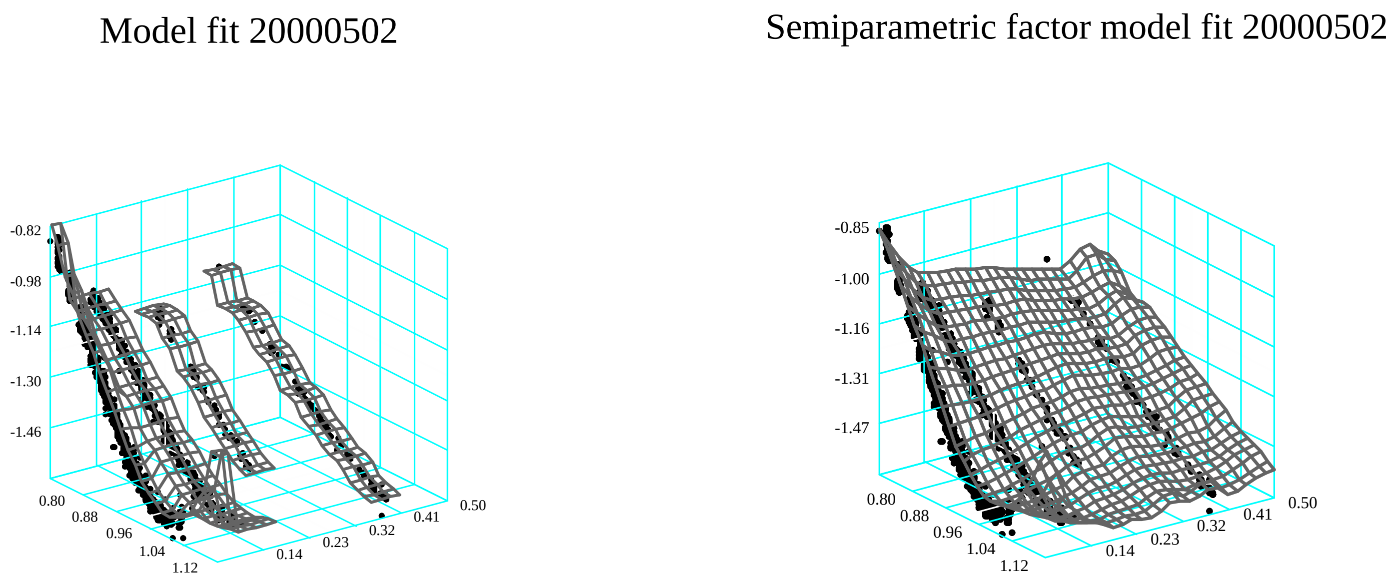



Figure 2: *Traditional model (Nadaraya-Watson estimator) and semi-parametric factor model fit for 20000502. Bandwidths for both estimates* $h_1 = 0.03$ *for the moneyness and* $h_2 = 0.08$ *for the time to maturity dimension.*

# The semiparametric factor model

Consider the generalized, additive model for the IVS:

$$Y_{i,j} = m_0(X_{i,j}) + \sum_{l=1}^{L} \beta_{i,l} m_l(X_{i,j}) \quad , \tag{2}$$

$Y_{i,j}$ is $\log -$implied volatility,
$i$ denotes the trading day $(i = 1, \dots, I)$,
$j = 1, \dots, J_i$ is an index of the traded options on day $i$.

$m_l(\cdot)$ for $l = 0, \dots, L$ are basis functions in covariables $X_{i,j}$,
and $\beta_i$ are time-dependent weights.

For $m_l(\cdot)$ for $l = 0, \ldots, L$ consider two different set-ups in $X_{i,j}$:

**(A)** $X_{i,j}$ is a two-dimensional vector containing time to maturity $\tau_{i,j}$ and forward moneyness, $\kappa_{i,j} = \frac{K}{F(t_{i,j})}$, i.e. strike $K$ divided by futures price $F(t_{i,j})$

**(B)** as in (A) but with one-dimensional $X_{i,j}$ that only contains $\kappa_{i,j}$.

Here, we focus on (A).

## Nadaraya-Watson smoothing

We define the estimates of $\widehat{m}_l$ and $\widehat{\beta}_{i,l}$ with $\widehat{\beta}_{i,0} \stackrel{\text{def}}{=} 1$, as minimizers of:

$$\sum_{i=1}^{I} \sum_{j=1}^{J_i} \int \left\{ Y_{i,j} - \sum_{l=0}^{L} \widehat{\beta}_{i,l} \widehat{m}_l(u) \right\}^2 K_h(u - X_{i,j}) \, du, \tag{3}$$

where $K_h$ denotes a two dimensional product kernel,
$K_h(u) = k_{h_1}(u_1) \times k_{h_2}(u_2), \quad h = (h_1, h_2)$ with a one-dimensional kernel $k_h(v) = h^{-1} k(h^{-1} v)$.

## Other representations

Replacing in (3) $\widehat{m}_l$ by $\widehat{m}_l + \delta g$ with arbitrary functions $g$ and $\widehat{\beta}_{i,l}$ by $\widehat{\beta}_{i,l} + \delta$ and taking derivatives with respect to $\delta$, the minimizer can be written as $1 \leq l' \leq L, 1 \leq i \leq I$:

$$\sum_{i=1}^{I} J_i \widehat{\beta}_{i,l'} \widehat{q}_i(u) = \sum_{i=1}^{I} J_i \sum_{l=0}^{L} \widehat{\beta}_{i,l'} \widehat{\beta}_{i,l} \widehat{p}_i(u) \widehat{m}_l(u), \tag{4}$$

$$\int \widehat{q}_i(u) \widehat{m}_{l'}(u)\, du = \sum_{l=0}^{L} \widehat{\beta}_{i,l} \int \widehat{p}_i(u) \widehat{m}_{l'}(u) \widehat{m}_l(u)\, du, \tag{5}$$

where

$$\widehat{p}_i(u) = \frac{1}{J_i} \sum_{j=1}^{J_i} K_h(u - X_{i,j}),$$

$$\widehat{q}_i(u) = \frac{1}{J_i} \sum_{j=1}^{J_i} K_h(u - X_{i,j}) Y_{i,j}.$$

# Algorithm

The algorithm exploits equations (4) and (5) iteratively:

1. for an appropriate initialization of
   $\beta_{l,i}^{(0)}, \quad i = 1, \ldots, I, \quad l = 1, \ldots, L$
   get an initial estimate of $\widehat{m}^{(0)} = (\widehat{m}_0, ..., \widehat{m}_L)^\top$
2. update $\beta_i^{(1)}, \quad i = 1, \ldots, I,$
3. estimate $\widehat{m}^{(1)}$.
4. go to step 2.

until minor changes occur during the cycle.

Optimization implemented in XploRe, , Härdle et al. (2000).

## Data Overview

| | Min. | Max. | Mean | Median | Stdd. | Skewn. | Kurt. |
|---|---|---|---|---|---|---|---|
| T. to mat. | 0.028 | 2.014 | 0.131 | 0.083 | 0.148 | 3.723 | 23.373 |
| Moneyness. | 0.325 | 1.856 | 0.985 | 0.993 | 0.098 | -0.256 | 5.884 |
| IV | 0.041 | 0.799 | 0.279 | 0.256 | 0.090 | 1.542 | 6.000 |

Table 1: *Summary statistics on the data base from 199801 to 200105. Source: EUREX, ODAX, stored in the CASE financial database MD*base.*

Total number of observations:
4.48 million contracts,
$J_i \approx 5\,200$ observations per day
total time series has $I \approx 860$ days.

## Estimation Results

We fit the model for $L = 4$, i.e. there is

- one invariant basis function $\widehat{m}_0$ and
- 4 'dynamic' basis functions $\widehat{m}_1, \ldots, \widehat{m}_4$
- 4 time series of $\{\beta_{l,i}\}_{i=1}^{I}$ with $l = 1, \ldots, 4$

Data: German DAX Index option implied volatility
for 1998 - 05/2001 (EUREX).
$Y_{i,j} \stackrel{\text{def}}{=} \ln\{\hat{\sigma}(\kappa, \tau)\}$
CASE financial database MD*base

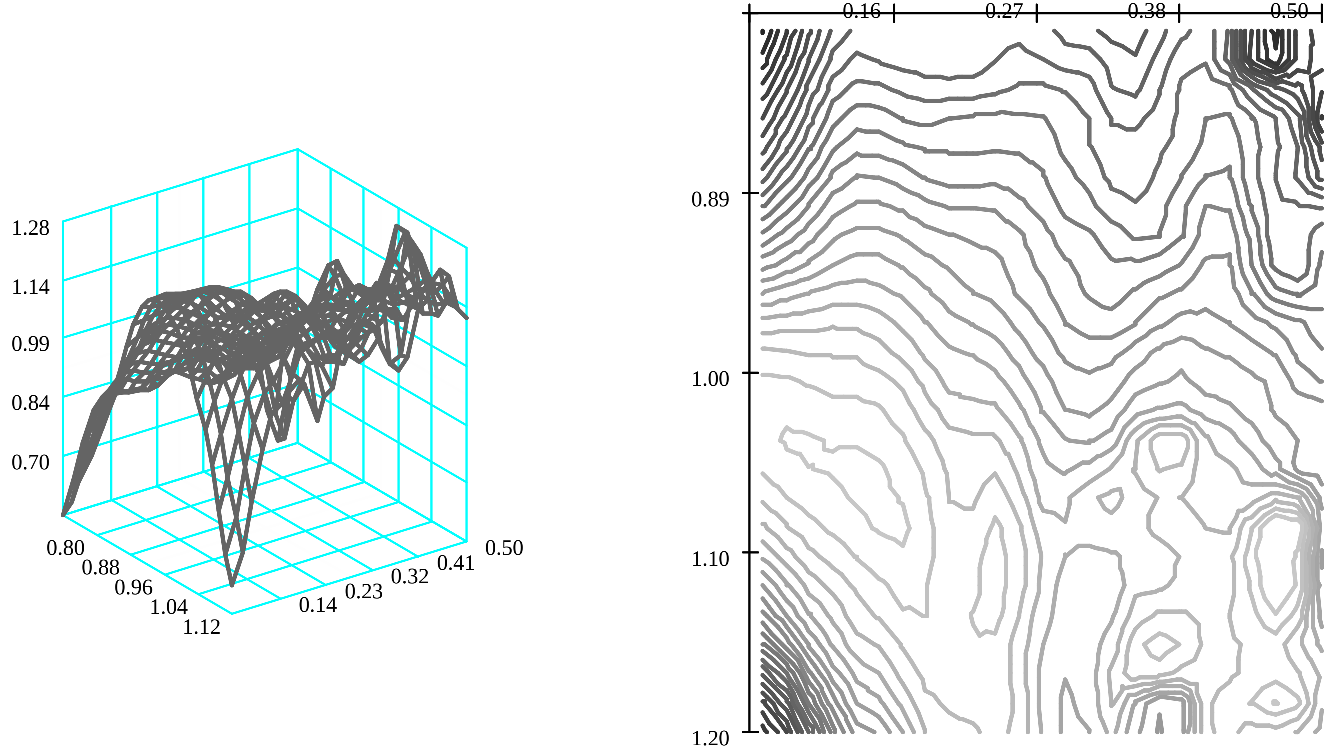



Figure 3: *Invariant basis function* $\widehat{m}_1$

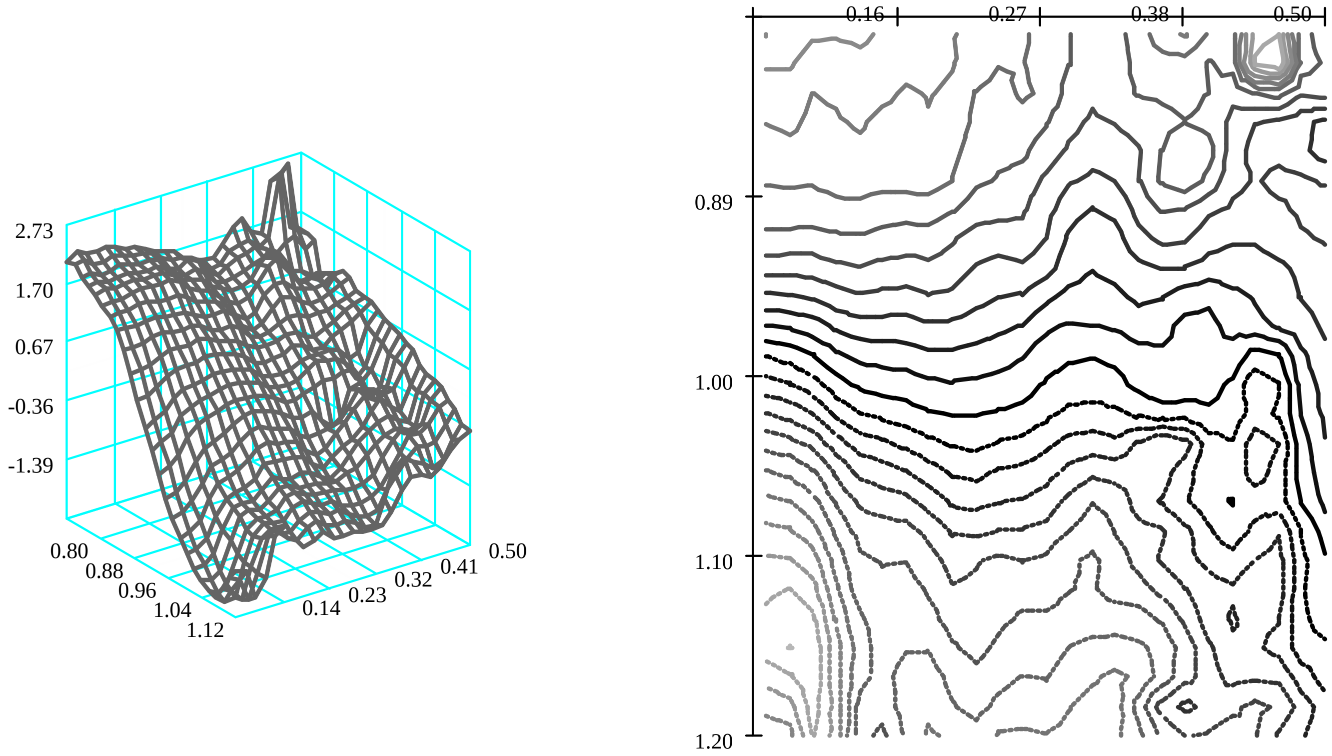



Figure 4: *Invariant basis function* $\widehat{m}_2$

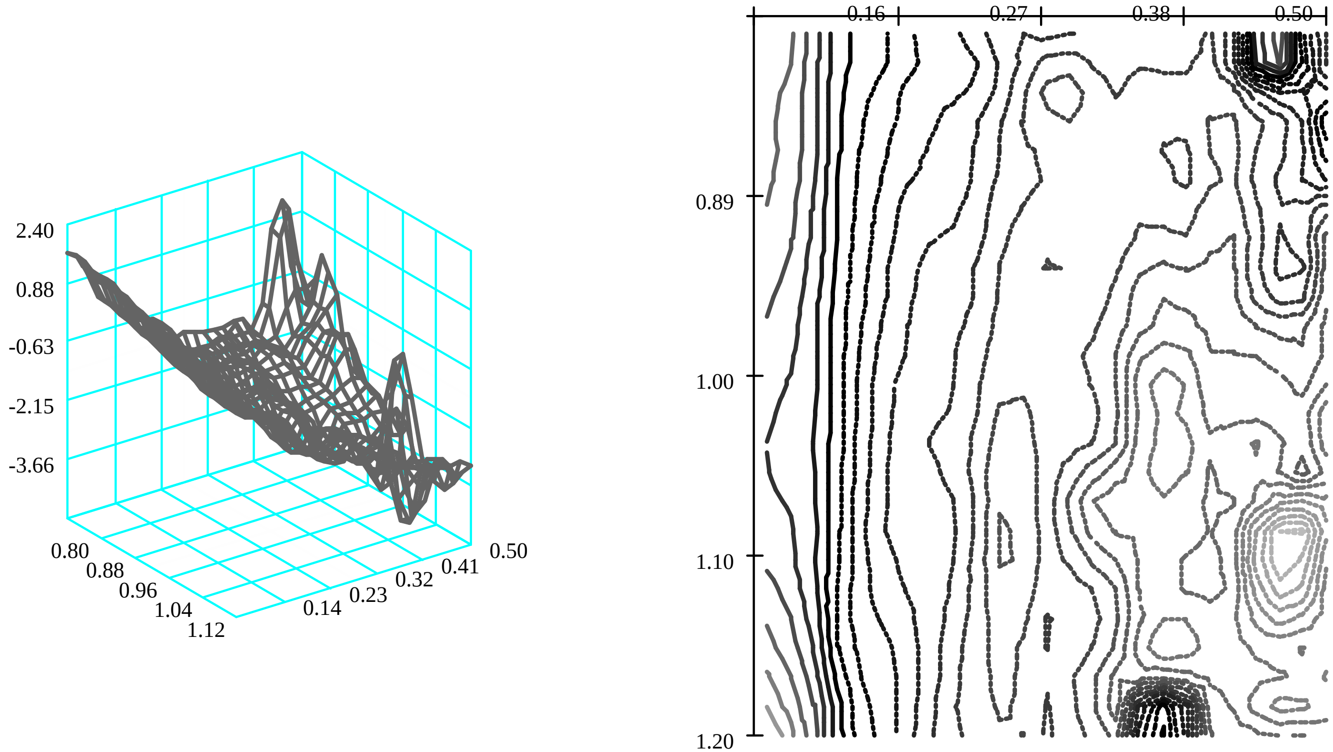
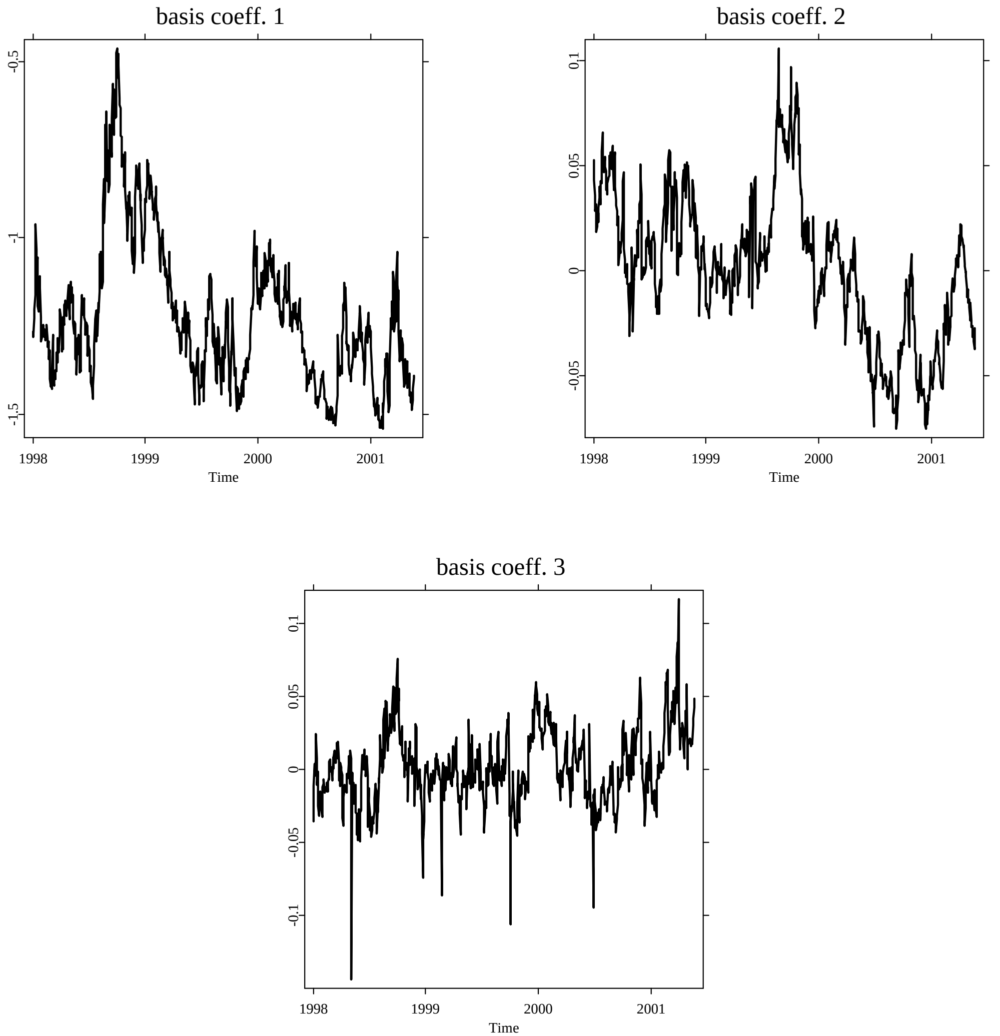



Figure 5: *Invariant basis function* $\widehat{m}_3$


basis coeff. 1
-0.5
-1
-1.5
1998
1999
2000
2001
Time
basis coeff. 2
0.1
0.05
0
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1998
1999
2000
2001
Time
basis coeff. 3
0.1
0.05
0
-0.05
-0.1
1998
1999
2000
2001
Time

# Unit Root Tests

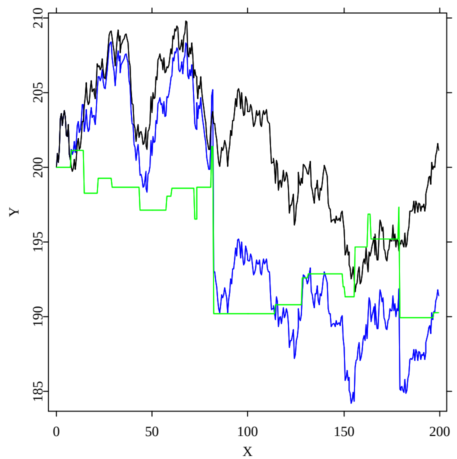
| Coefficient | Test Stat. | # of lags |
|---|---|---|
| $\widehat{\beta}_1$ | -2.67* | 3 |
| $\widehat{\beta}_2$ | -2.97 | 2 |
| $\widehat{\beta}_3$ | -6.30 | 2 |

Table 2: *ADF tests on* $\widehat{\beta}_1$ *to* $\widehat{\beta}_3$ *for the full IVS model, intercept included in each case. Third column gives the number of lags included in the ADF regression. MacKinnon critical values for rejection of the hypothesis of a unit root are -2.87 at 5% significance level, and -3.44 at 1% significance level.*

# Calibration of IVS and Option Pricing via Fast Fourier Transform (FFT)

Derivatives are financial products whose value depends on some underlying instrument, e.g. a stock.

Which models for the underlying instruments can replicate well derivative prices observed on real markets?



# Calibration

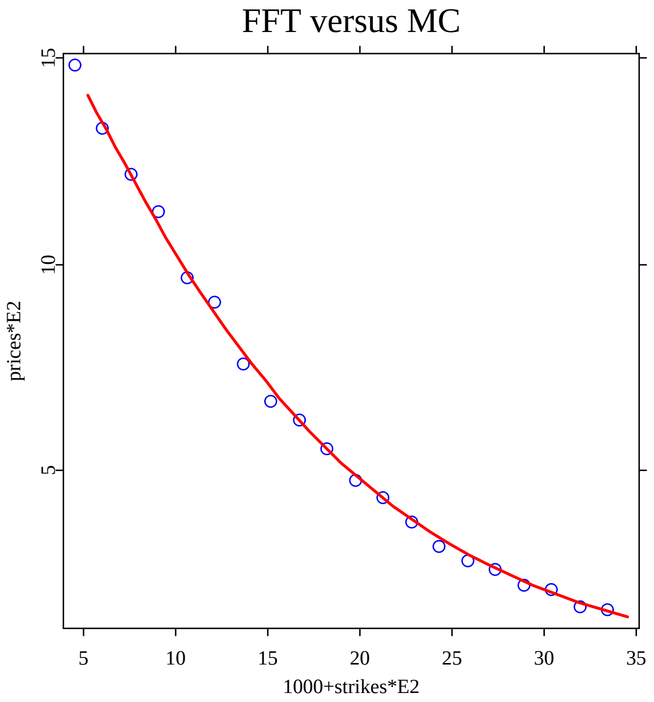
Given a model for the underlying price process $(S_t)$ we estimate parameters that minimize the distance between the IVS of the model and an IVS observed on the market.

- numerical minimization algorithms
- many evaluations of the function to be minimized
- fast algorithm for computing the option prices of the whole IVS

# FFT versus Monte Carlo (MC)

FFT time: 0.015 sec.

MC time: 36.531 sec. (5000 simulations, 500 time steps)



# Option pricing with FFT

Carr and Madan (1999) proposed a method to price option based on the fast Fourier transform (FFT).

Motivations for the use of FFT:

- considerable power of the FFT
- Fourier transform of the (log) price process is known for many models
- FFT allows to calculate prices for a whole range of strikes

The value $C_T(k)$ of a $T$-maturity call with strike $K = \exp(k)$ is given by

$$C_T(k) = \int_k^\infty e^{-rT}(e^s - e^k) q_T(s) ds$$

where $q_T$ is the risk-neutral density of the log price $S_T$.

However $C_T$ is not square-integrable we cannot apply the Fourier inversion directly. Thus we consider the modified function

$$c_T(k) = \exp(\alpha k) C_T(k)$$

which is square-integrable given a suitable $\alpha > 0$.

The Fourier transform of $c_T$ is defined by

$$\psi_T(v) = \int_{-\infty}^{\infty} e^{ivk} c_T(k) dk.$$

As $c_T$ is square-integrable we can get back the call price by applying the inverse Fourier transform

$$C_T(k) = \frac{\exp(-\alpha k)}{2\pi} \int_0^{\infty} e^{-ivk} \psi_T(v) dv.$$

The Fourier transform $\psi$ can be expressed as

$$\psi_T(v) = \frac{e^{-rT}\phi(v - (\alpha + 1)i)}{\alpha^2 + \alpha - v^2 + i(2\alpha + 1)v}$$

where $\phi$ is the Fourier transform of $q_T$.

# Pricing calls with different strikes

Consider now $N$ calls with maturity $T$ and strikes

$$k_u = -\frac{1}{2}N\lambda + \lambda u, \ \ u = 0, \ldots, N-1$$

where $\lambda > 0$ is the regular spacing size between each two log strikes.

The numerical approximation of the call price is

$$C_T(k_u) \approx \frac{\exp(-\alpha k_u)}{\pi} \sum_{j=0}^{N-1} e^{-i\lambda\eta j u} e^{i\frac{1}{2}N\lambda v_j} \psi_T(v_j)\eta, \ \ u = 0, \ldots, N-1.$$

This representation allows a direct application of the FFT which is an efficient algorithm for computing the sum

$$w_k = \sum_{j=0}^{N-1} e^{-i\frac{2\pi}{N}jk} x_j, \;\; k = 0, \dots, N-1.$$

The parameters $\lambda, \eta, N$ only need to satisfy the constraint

$$\lambda\eta = \frac{2\pi}{N}.$$

If we choose a small $\eta$ in order to obtain a fine grid for the numerical integration, then we get call prices at relatively large strike spacings, i.e. with few strikes lying in the desired region near the stock price.

Disadvantages of FFT:

- instable for fixed FFT parameter $\alpha, \eta, N$
- applicable only to European options

Modifications:

- Numerical integration with the Simpson rule instead of the trapezoid rule.
- Centralizing of the grid of $\log$ strikes around the spot price.
- Use of different modifications of the call price function $C_T$.

# Calibrate Bates model

## Bates model

$$
\begin{aligned}
\frac{dS_t}{S_t} &= \mu dt + \sqrt{V_t} dW_t^S + dZ_t \\
dV_t &= \xi(\eta - V_t)dt + \theta\sqrt{V_t} dW_t^V \\
\mathsf{E}(dW_t^V dW_t^S) &= \rho dt
\end{aligned}
$$

where $S_t$ underlying, $\mu$ drift of underlying, $V_t$ stochastic volatility, $\xi$ rate of mean reversion, $\eta$ average level of volatility, $\theta$ volatility of volatility, $\rho$ correlation of Wiener process $W_t^V$ for volatility and Wiener process $W_t^S$ for underlying. $Z_t$ is compound Poisson process with log-normal distribution of jumps:

$\ln(1+k) \sim \mathsf{N}(\ln(1+\overline{k}) - \frac{1}{2}\delta^2, \delta^2)$

The log-price $X_t = \ln S_t$ follows the dynamics:

$$dX_t = (r - \lambda\overline{k} - \frac{1}{2}V_t)dt + \sqrt{V_t}dW_t^S + d\tilde{Z}_t$$

where $\tilde{Z}_t$ is compound Poisson process with intensity $\lambda$ and normal distribution of jump size.

Characteristic function of this process is given and FFT method could be applied.

Parameters to estimate:

- $\lambda$ intensity of jumps
- $\overline{k}$ part of the mean of jumps
- $\delta$ deviation of jumps
- $\xi$ rate of mean reversion
- $\eta$ average level of volatility
- $\theta$ volatility of volatility
- $\rho$ correlation of Wiener processes
- $V_0$ initial volatility

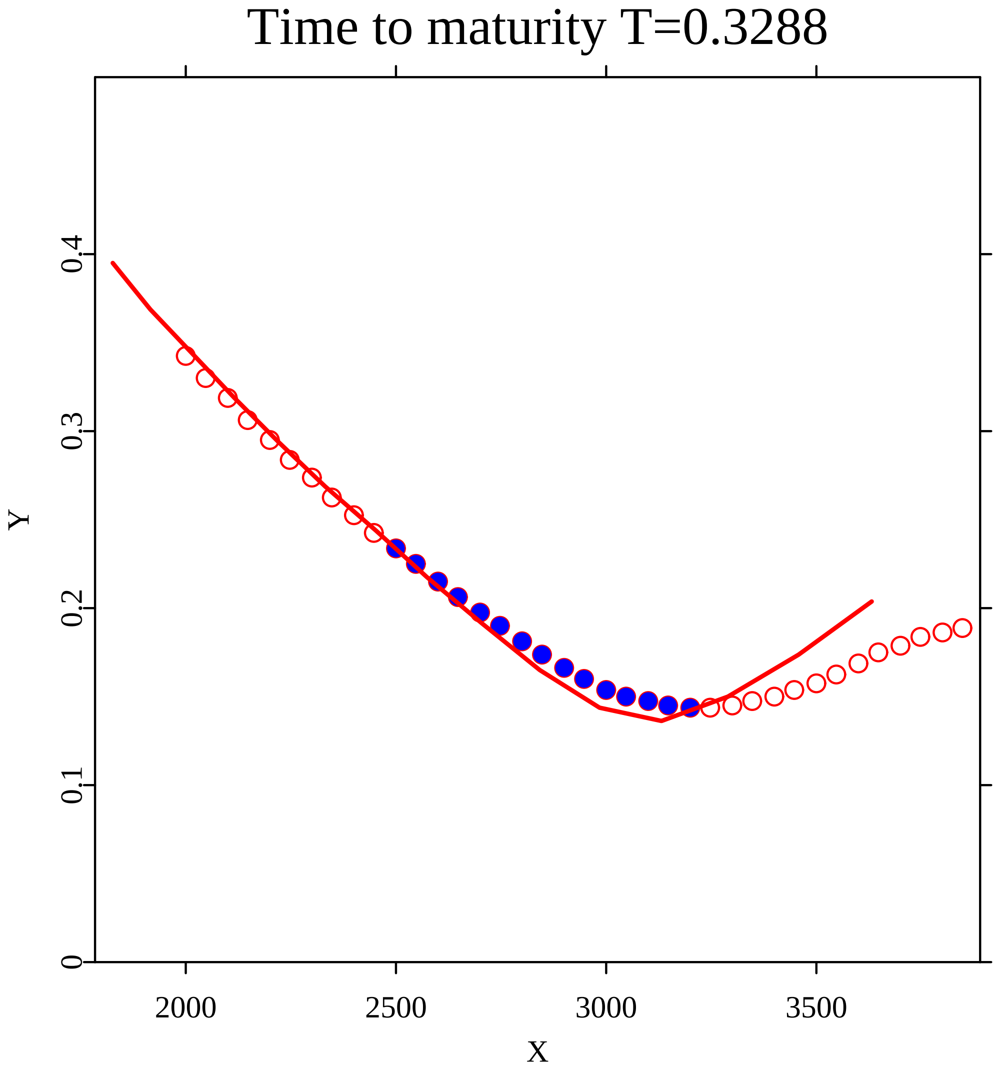
# Minimizing function

$$f(p) = \sum_i \frac{(C_i^M - C_i^B(p))^2}{C_i^M} \mathbf{I}_{S \leq K} + \sum_i \frac{(P_i^M - P_i^B(p))^2}{P_i^M} \mathbf{I}_{S > K}$$

where $p$ is the set of parameters, $C_i^M, P_i^M$ call/put option prices from the market, $C_i^B(p), P_i^B(p)$ option prices calculated with Bates model with parameters $p$ and $I$ is the indicator function.

Estimated parameters of the Bates model:

- $\hat{\lambda} = 0.008$
- $\hat{\bar{k}} = -0.08$
- $\hat{\delta} = 1.6012$
- $\hat{\xi} = 8.1262$
- $\hat{\eta} = 0.037$
- $\hat{\theta} = -0.579$
- $\hat{\rho} = 1.52$
- $\hat{V}_0 = 0.016$


Time to maturity T=0.3288
Y
X
0
0.1
0.2
0.3
0.4
2000
2500
3000
3500

# VaR based on GH distribution and adaptive volatility

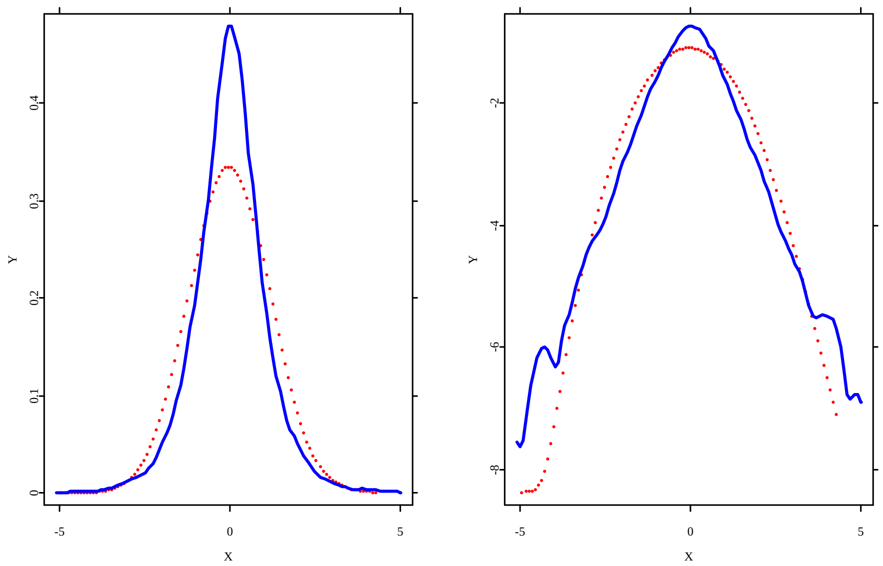



Figure 6: Graphical comparison of densities (left) and log-densities (right) of daily DEM/USD FX rate from 1979/12/01 to 1994/04/01 (3720 observations). The kernel density estimate of the standardized residuals (line) and the normal density (dots) with $h = 0.57$ (rule of thumb).

# Risk Management Models

## Heteroscedastic model

$$R_t = \sigma_t \varepsilon_t, \; t = 1, 2, \cdots$$

$R_t$ (log) return, $\sigma_t$ volatility, $\varepsilon_t$ identically and independently distributed (i.i.d.) stochastic term.

## Typical assumptions

1 The stochastic term is normally distributed, $\varepsilon_t \sim \mathrm{N}(0, 1)$.

2 A time homogeneous structure of volatility:

- ARCH model, Engle(1995)
- GARCH model, Bollerslev(1995)
- Stochastic volatility model, Harvey, Ruiz and Shephard(1995)

**Improvements**

The generalized hyperbolic (GH) distribution family can well replicate the empirical distribution observed in financial markets:

- Hyperbolic (HYP) distribution in finance, Eberlein and Keller(1995),
- GH distribution + (parametric) stochastic volatility model, Eberlein, Kallsen and Kristen(2003).

A time inhomogeneous model gives an appropriate volatility estimation.

- Adaptive volatility model + normal distribution, Mercurio and Spokoiny(2003).

**Combine A & B!**

# Generalized Hyperbolic (GH) Distribution

$X \sim GH$ with density:

$$f_{GH}(x;\lambda,\alpha,\beta,\delta,\mu) = \frac{(\gamma/\delta)^{\lambda}}{\sqrt{2\pi}K_{\lambda}(\delta\gamma)} \frac{K_{\lambda-1/2}\left\{\alpha\sqrt{\delta^2+(x-\mu)^2}\right\}}{\left\{\sqrt{\delta^2+(x-\mu)^2}/\alpha\right\}^{1/2-\lambda}} \cdot e^{\beta(x-\mu)}$$

Where $\gamma^2 = \alpha^2 - \beta^2$, $K_{\lambda}(\cdot)$ is the modified Bessel function of the third kind with index $\lambda$:

$$K_{\lambda}(x) = \frac{1}{2}\int_0^{\infty} y^{\lambda-1} exp\{-\frac{x}{2}(y+y^{-1})\}\, dy$$

Furthermore, the following conditions must be fulfilled:

- $\delta \geq 0$, $|\beta| < \alpha$ if $\lambda > 0$
- $\delta > 0$, $|\beta| < \alpha$ if $\lambda = 0$
- $\delta > 0$, $|\beta| \leq \alpha$ if $\lambda < 0$

# Subclass of GH distribution

*Motivation: the four parameters $\mu, \delta, \beta, \alpha$ can be interpreted as trend, riskiness, asymmetry and the likeliness of extreme events.*

**Hyperbolic (HYP) distributions:** $\lambda = 1$,

$$f_{HYP}(x; \alpha, \beta, \delta, \mu) = \frac{\gamma}{2\alpha\delta K_1(\delta\gamma)} e^{\{-\alpha\sqrt{\delta^2+(x-\mu)^2}+\beta(x-\mu)\}}, \qquad (6)$$

where $x, \mu \in \mathbb{R}$, $0 \leq \delta$ and $|\beta| < \alpha$.

**Normal-inverse Gaussian (NIG) distributions:** $\lambda = -1/2$,

$$f_{NIG}(x; \alpha, \beta, \delta, \mu) = \frac{\alpha\delta}{\pi} \frac{K_1\left\{\alpha\sqrt{\delta^2 + (x-\mu)^2}\right\}}{\sqrt{\delta^2 + (x-\mu)^2}} e^{\{\delta\gamma + \beta(x-\mu)\}}. \qquad (7)$$

**Example:** ML estimators of HYP distribution
$\hat{\alpha} \approx 1.744$, $\hat{\beta} \approx -0.017$, $\hat{\delta} \approx 0.782$ and $\hat{\mu} \approx 0.012$.

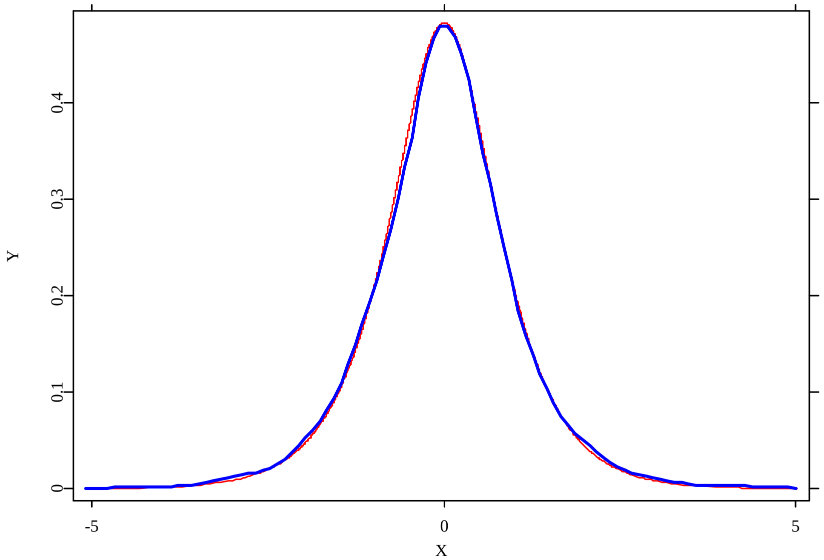
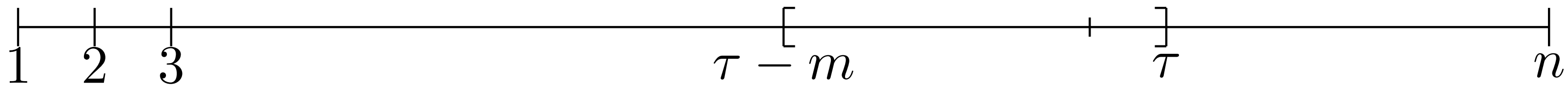



Figure 7: The estimated density of the standardize return of FX rates (blue) with nonparametric kernel ($h = 0.57$) and a simulated HYP density (red) with the maximum likelihood estimators $\hat{\alpha} \approx 1.744$, $\hat{\beta} \approx -0.017$, $\hat{\delta} \approx 0.782$ and $\hat{\mu} \approx 0,012$.

# Adaptive Volatility Estimation

**Assumption:**

For a fixed point $\tau$, volatility is locally time homogeneous in a short time interval $[\tau - m, \tau)$, thus we can estimate $\widetilde{\sigma}_\tau = \widetilde{\sigma}_I = \frac{1}{|I|}\sum_{i \in I} \sigma_i$, where $|I|$ is the number of observations in $I = [\tau - m, \tau)$.

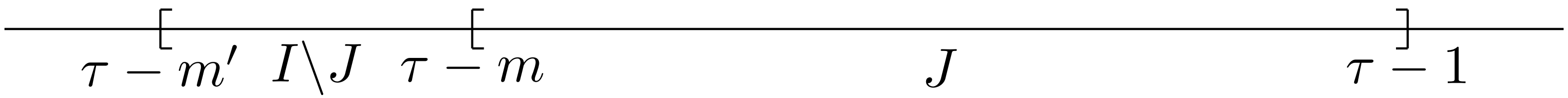


## Volatility estimation $\rightarrow$ Specify the interval $I$:

Split interval $I$ into $J \subset I$ and $I \backslash J \subset I$.


$$|\widetilde{\sigma}_{I \backslash J} - \widetilde{\sigma}_J| \leq T_I. \tag{8}$$

Once (5) holds, we increase the interval by increasing $m$ until time homogeneity is violated. $I^* = \max \{I : I \text{ fulfills (5)}\}$




**Specify the interval $I$ $\rightarrow$ Estimate $\sigma_{I \backslash J}$ and $\sigma_J$ by $\widetilde{\sigma}_{I \backslash J}$ and $\widetilde{\sigma}_J$**

**Specify the test statistic $T_I$**

# Volatility estimation

$$R_t = \sigma_t \varepsilon_t \quad \longrightarrow \quad \mathsf{E}(R_t^2 | \mathcal{F}_{t-1}) = \sigma_t^2$$

- Power transformation: for every $\gamma > 0$, we have

$$\begin{aligned}
\mathsf{E}(|R_t|^\gamma | \mathcal{F}_{t-1}) &= \sigma_t^\gamma \,\mathsf{E}(|\varepsilon|^\gamma | \mathcal{F}_{t-1}) = C_\gamma \sigma_t^\gamma && (9)\\
\mathsf{E}[(|R_t|^\gamma - C_\gamma \sigma_t^\gamma)^2 | \mathcal{F}_{t-1}] &= \sigma_t^{2\gamma} \,\mathsf{E}[(|\varepsilon|^\gamma - C_\gamma)^2 | \mathcal{F}_{t-1}] && (10)\\
&= \sigma_t^{2\gamma} D_\gamma^2 && (11)
\end{aligned}$$

$$|R_t|^\gamma = C_\gamma \sigma_t^\gamma + D_\gamma \sigma_t^\gamma \zeta_t, \qquad (12)$$

where $C_\gamma$ is the conditional mean and $D_\gamma^2$ the conditional variance of $|\varepsilon|^\gamma$ and $\zeta_t = (|\varepsilon|^\gamma - C_\gamma)/D_\gamma$ is i.i.d. with mean 0.

We choose $\gamma = 1$.

Set $\theta_t = C_\gamma \sigma_t^\gamma$ which is the **conditional mean** of $|R_t|^\gamma$, then in a time homogeneous interval $I$, the constant $\theta_I = C_\gamma \sigma_I^\gamma$ can be estimated by $\widetilde{\theta}_I$:

$$\widetilde{\theta}_I = \frac{1}{|I|} \sum_{t \in I} |R_t|^\gamma.$$

Replace $|R_t|^\gamma$, we get:

$$\begin{aligned} \widetilde{\theta}_I &= \frac{1}{|I|} \sum_{t \in I} \theta_t + \frac{s_\gamma}{|I|} \sum_{t \in I} \theta_t \zeta_t. \\ \mathrm{Var}[\widetilde{\theta}_I] &= \frac{s_\gamma^2}{|I|^2} \mathsf{E} \sum_{t \in I} \theta_t^2. \end{aligned}$$

where $s_\gamma = D_\gamma / C_\gamma$.

Here we denote $v_I^2 = \frac{s_\gamma^2}{|I|^2} \sum_{t \in I} \theta_t^2$ as the conditional variance of $\widetilde{\theta}_I$.

The estimator $\widetilde{v}_I = \frac{s_\gamma}{|I|^{1/2}} \widetilde{\theta}_I$.

The GH distribution has a moment generating function $\varphi_f(z)$:

$$\varphi_f(z) = e^{\mu z} \cdot \frac{\gamma^\lambda}{\gamma_z^\lambda} \cdot \frac{K_\lambda(\delta\gamma_z)}{K_\lambda(\delta\gamma)}, \quad |\beta + z| < \alpha, \qquad (13)$$

where $\gamma_z^2 = \alpha^2 - (\beta + z)^2$.

- $\varphi_f$ is infinitely many times differentiable near $0$, hence its every moment exists.
- there exists $a_\gamma > 0$ to fulfill

$$log \, \mathsf{E} \, e^{u\zeta_\gamma} \le \frac{a_\gamma u^2}{2} \quad \text{for every } \gamma \le 1$$

since GH distribution exponentially decay fast enough.

**Theorem (Mercurio and Spokoiny 2003):** Let $R_1, ..., R_\tau$ obey the heteroscedastic model and the volatility coefficient $\sigma_t$ satisfies the condition $b \leq \sigma_t^2 \leq bB$ with some positive constant $b$, $B$. Then it holds for the estimate $\widetilde{\theta}_I$ of $\theta_\tau$ :

$$\mathrm{P}(|\widetilde{\theta}_I - \theta_\tau| > \Delta_I(1 + \lambda s_\gamma |I|^{-1/2}) + \lambda \widetilde{v}_I)$$
$$\leq 4\sqrt{e}\lambda(1 + logB) \exp(-\frac{\lambda^2}{2a_\gamma(1 + \lambda s_\gamma |I|^{-1/2})^2}).$$

– Test statistics: Under homogeneity $|\widetilde{\theta}_I - \theta_\tau|$ is bounded by $\lambda \widetilde{v}_I$ provided that $\lambda$ is sufficiently large.

$$|\widetilde{\theta}_I - \theta_\tau| < \lambda \widetilde{v}_I$$

From the triangle inequality, we get: $\widetilde{\theta}_{I\setminus J} - \widetilde{\theta}_J$ is bounded by $\lambda(\widetilde{v}_{I\setminus J} + \widetilde{v}_J)$ for $J \subset I$, i.e.

$$|\widetilde{\theta}_{I\setminus J} - \widetilde{\theta}_J| \leq \lambda(\widetilde{v}_{I\setminus J} + \widetilde{v}_J) = \lambda'(\sqrt{\widetilde{\theta}^2_J |J|^{-1}} + \sqrt{\widetilde{\theta}^2_{I\setminus J} |I\setminus J|^{-1}}),$$

where $\lambda' = \lambda s_\gamma$.

– Cross-validation (CV) method:

$$\lambda'^* = argmin\{\sum_{t=t_0}^{n} \left(|R_t|^\gamma - \widetilde{\theta}_{(t,\lambda')}\right)^2\},$$

# Iteration

For any time point $t_0 \leq t \leq n$, where $t_0 > m$ is the starting point, use the following iteration:

1. Given a starting value of $m = m_0$, which indicates a local homogeneous interval $[t - m, t]$,
2. Given different critical values $\lambda'$, we can estimate the corresponding $\widetilde{\theta}_{\tau,\lambda'}$ and get simultaneously the forecast error

$$\sum_{t=t_0}^{n} \left( |R_t|^{\gamma} - \widetilde{\theta}_{(t,\lambda')} \right)^2 ,$$

3. Pick up the optimal lambda, which returns the minimal forecast error, as the critical value.

4. Multiple test the time homogeneity in (5). If (5) is not rejected, enlarge $m$ to $k * m_0$, $k \in \mathbb{N}$, otherwise the loop stops.

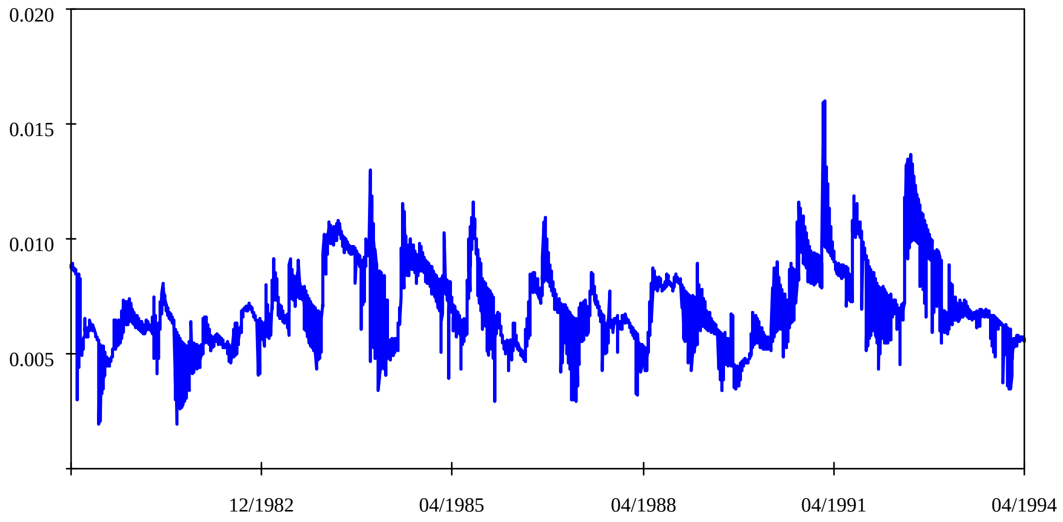



Figure 8: The adaptive volatility estimates of DEM/USD exchange rates.

# Value at Risk (VaR)

$q_p$ is the $p$-th quantile of the distribution of $\varepsilon_t$, i.e. $P(\varepsilon_t < q_p) = p$.

$$P(R_t < \sigma_t q_p \,|\, \mathcal{F}_{t-1}) = p$$

$$\text{VaR}_{p,t} = \sigma_t q_p$$

$\widetilde{\sigma}_t$ are estimated by adaptive model

$q_p$ is given be the quantile of HYP or NIG distribution

# ML estimator of GH distributions

Parameters estimation is based on the previous 500 observations (standardized returns), which varies little.

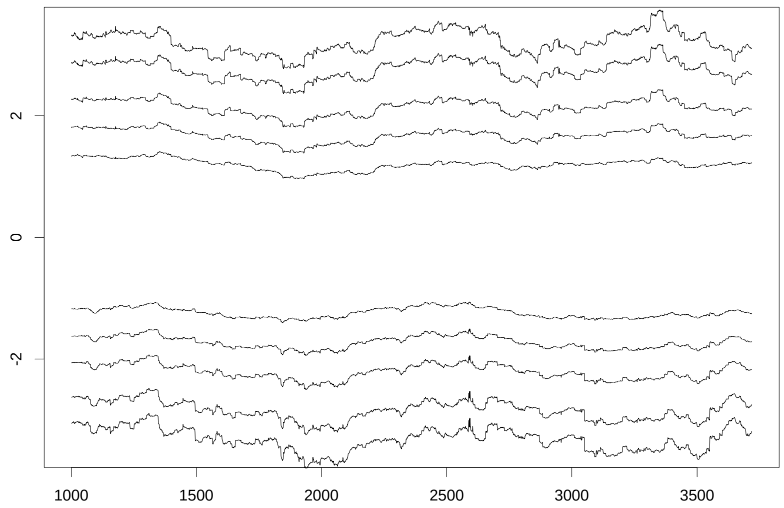



Figure 9: Quantiles varying over time. From the top the evolving HYP quantiles for $p = 0.995$, $p = 0.99$, $p = 0.975$, $p = 0.95$, $p = 0.90$, $p = 0.10$, $p = 0.05$, $p = 0.025$, $p = 0.01$, $p = 0.005$.

# GHADA model and normal model

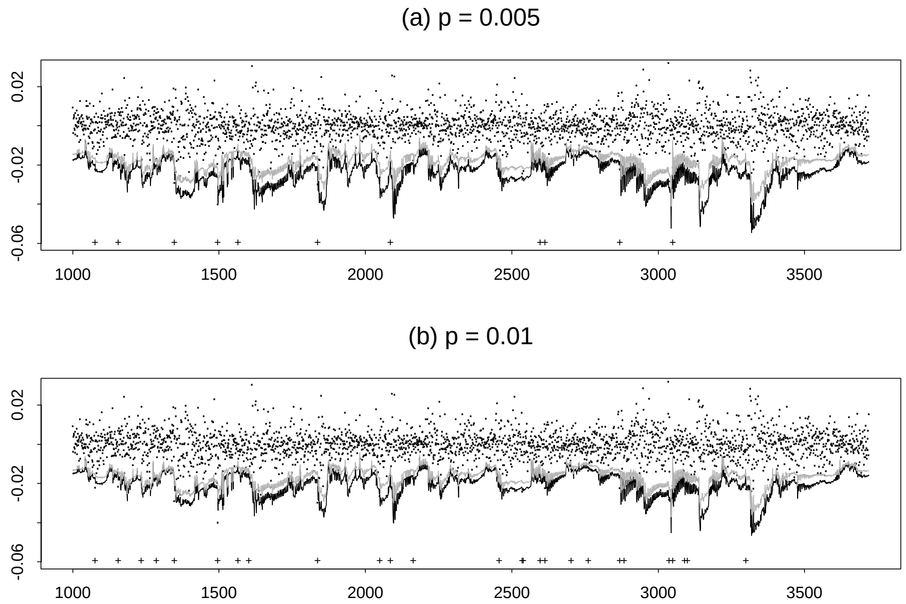



Figure 10: Value at Risk forecast plots for DEM/USD data. The dots are the returns, the solid line is the VaR forecast based on HYP underlying distribution, the gray line the VaR forecast based on normal distribution, and the crosses indicate the VaR excesses of HYP model. (a) $p = 0.005$. (b) $p = 0.01$.

# Backtesting VaR

**Testing VaR levels:**

$$H_0 : \mathsf{E}\, I = p \quad \text{vs.} \quad H_1 : \text{not } H_0 \tag{14}$$

where $I$ is the proportion of exceptions. Likelihood ratio statistic:

$$\mathrm{LR1} = -2 \log \left\{ (1-p)^{T-N} p^N \right\} + 2 \log \left\{ (1 - N/T)^{T-N} (N/T)^N \right\},$$

where $N$ is the number of exceptions, $T$ is the number of observations and $\mathrm{LR1}$ is asymptotically $\chi^2(1)$ distributed

## Testing Independence:

$$H_0 : \pi_{00} = \pi_{10} = \pi, \pi_{01} = \pi_{11} = 1 - \pi \quad \text{vs.} \quad H_1 : \text{ not } H_0$$

Likelihood ratio statistic:

$$\mathrm{LR2} = -2 \log \left\{ \hat{\pi}^{n_0} (1 - \hat{\pi})^{n_1} \right\} + 2 \log \left\{ \hat{\pi}_{00}^{n_{00}} \hat{\pi}_{01}^{n_{01}} \hat{\pi}_{10}^{n_{10}} \hat{\pi}_{11}^{n_{11}} \right\},$$

where $\hat{\pi}_{ij} = n_{ij}/(n_{ij} + n_{i,1-j})$, $n_j = n_{0j} + n_{1j}$, and $\hat{\pi} = n_0/(n_0 + n_1)$. Under $H_0$, $\mathrm{LR2}$ is asymptotically $\chi^2(1)$ distributed as well.

| Model | $p$ | $N/T$ | LR1 | p-value | LR2 | p-value |
|---|---|---|---|---|---|---|
| Normal | 0.005 | 0.01471 | 33.78275 | 0.0000* | 1.19500 | 0.2743 |
| | 0.01 | 0.02243 | 31.38546 | 0.0000* | 0.11446 | 0.7352 |
| | 0.025 | 0.03356 | 7.24351 | 0.0079* | 0.29236 | 0.5887 |
| | 0.05 | 0.05590 | 1.92431 | 0.1654 | 0.03239 | 0.8572 |
| HYP | 0.005 | 0.00405 | 0.53274 | 0.4655 | 0.08940 | 0.7649 |
| | 0.01 | 0.00956 | 0.05338 | 0.8173 | 0.50224 | 0.4785 |
| | 0.025 | 0.02464 | 0.01441 | 0.9044 | 0.07259 | 0.7876 |
| | 0.05 | 0.05112 | 0.07152 | 0.7891 | 0.11980 | 0.7293 |

Table 3: Backtesting results for DEM/USD example. * indicates the rejection of the model which is used.

# Skewness and Kurtosis Trading

Recall the option pricing theory

$$\text{European put: } P = e^{-r\tau}\int_0^\infty \max(K_1 - S_T, 0)q(S_T)dS_T$$

$$\text{European call: } C = e^{-r\tau}\int_0^\infty \max(S_T - K_2, 0)q(S_T)dS_T,$$

with time to maturity $\tau = T - t$, strike price $K$ and risk–free interest rate $r$.

$q(S_T)$? – A state price density (SPD) of the underlying!

In Black-Scholes world: $q(S_T)$ is lognormal and unique.

## General Comment

In an arbitrage free and complete market model exists exactly **one** risk-neutral density. If markets are not complete there are in general **many** risk-neutral measures.

Comparing **two** SPD's, as we do, amounts rather to compare two different models, and trades are initiated depending on the model in which one believes more.

Suppose there are two SPDs $f^*$, $g^*$ with $f^*$ more negatively skewed than $g^*$ and a European OTM put respectively call.

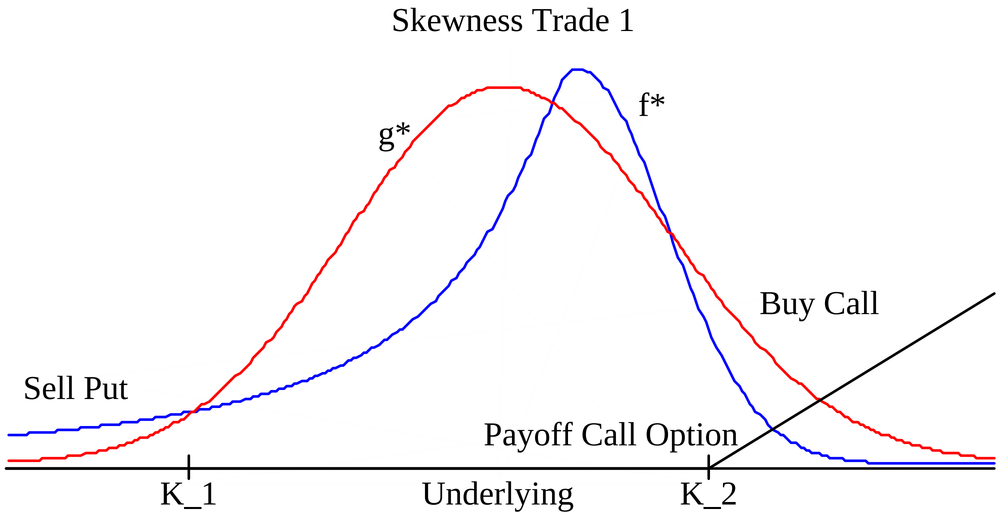



Figure 11: Skewness Trade 1.

This tail situation implies for a European call option with strike $K_2$

$$C(f^*) \quad < \quad C(g^*)$$

price computed with $f^*$ $<$ price computed with $g^*$

If the call is priced using $f^*$ but one regards $g^*$ as a better approximation of the underylings' SPD, one would **buy** the call.

Analogously: Short a European OTM put with strike $K_1$.

This motivates a **skewness trade 1**: Portfolio of a short OTM put and a long OTM call, which is also called a Risk Reversal, Will02.

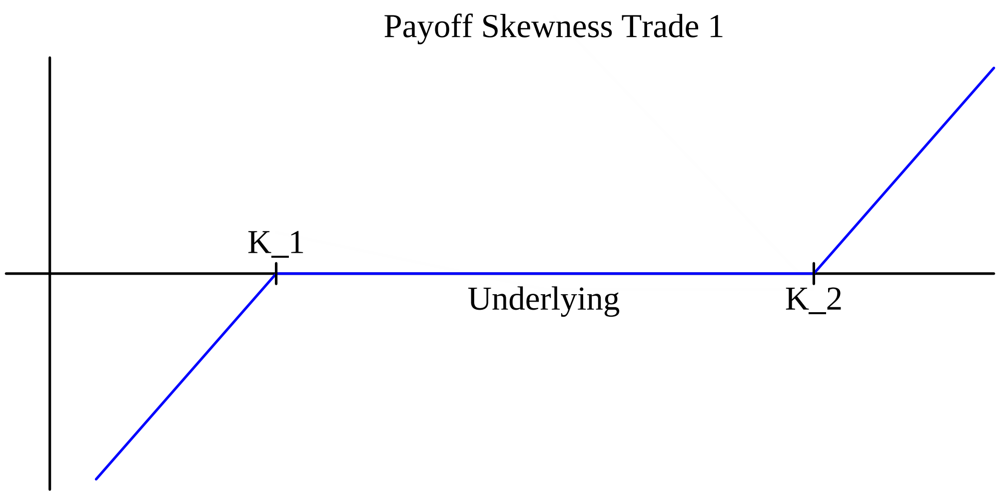



Figure 12: Payoff Skewness Trade 1

Comparing $f^*$ and $g^*$ leads also to **kurtosis trades**: Buy and sell calls and puts of different strikes.

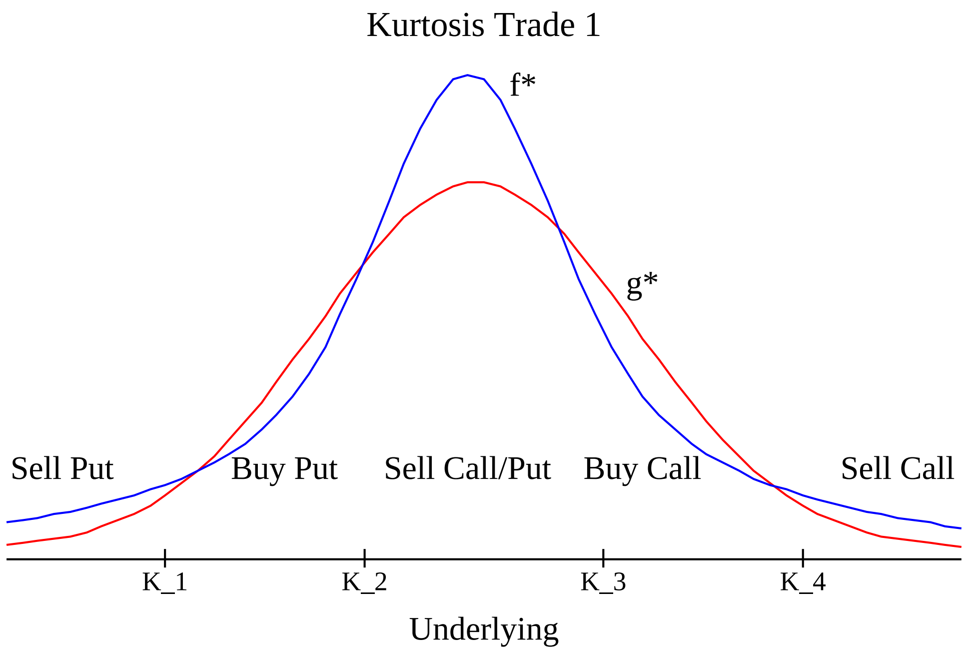



Figure 13: Kurtosis Trade 1.

The payoff profile at maturity is given in Figure 14, which is basically a modified butterfly.

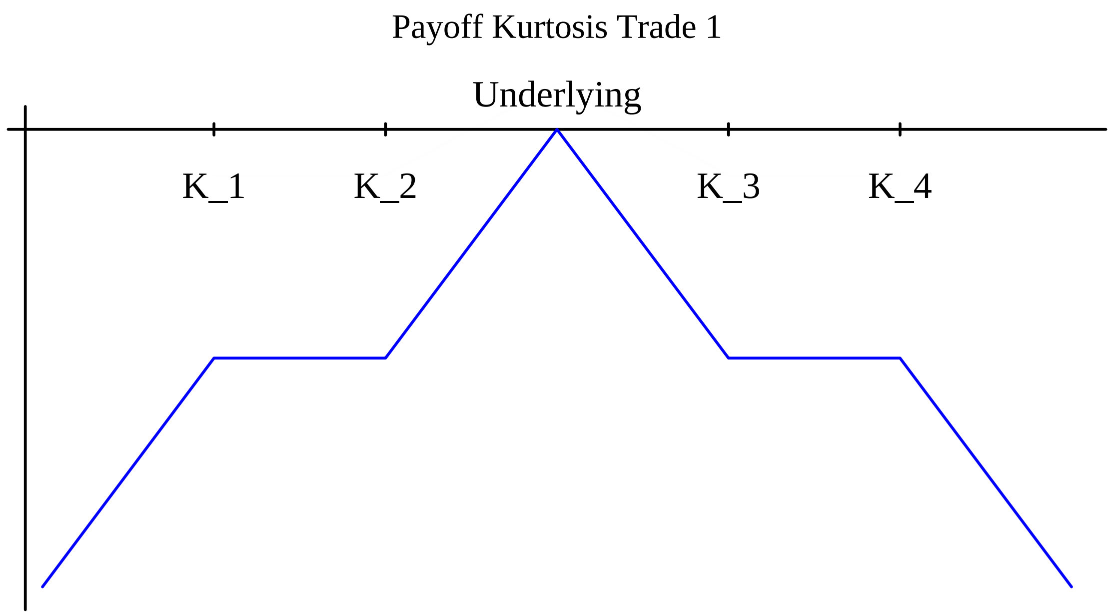



Figure 14: Payoff Kurtosis Trade 1

In this work, $f^*$ is an option implied SPD and $g^*$ is a (historical) time series SPD.

To compare implied to (historical) time series SPD's, we use Barle and Cakici's Implied Binomial Tree algorithm to estimate $f^*$ whereas $g^*$ is inferred from a combination of a non-parametric estimation from a historical time series of the DAX and a forward Monte Carlo simulation.

Later on we will specify in terms of moneyness $K/S_t e^{r\tau}$ where to buy or sell options.

Within such a framework, is it profitable to trade skewness and kurtosis? Does a SPD comparison contain information about the stock market bubble that burst in March 2000?

# Estimation of the Option-Implied SPD

## Implied Binomial Tree (IBT)

Numerical method to compute SPD adapted to volatility smile

Several approaches: Rubinstein $(1994)$, Dupire $(1994)$, Derman and Kani $(1994)$ and Barle and Cakici $(1998)$

XploRe compute Derman and Kani's `IBTdk` and Barle and Cakici's `IBTbc` IBT

Barle and Cakici's version proved to be more robust

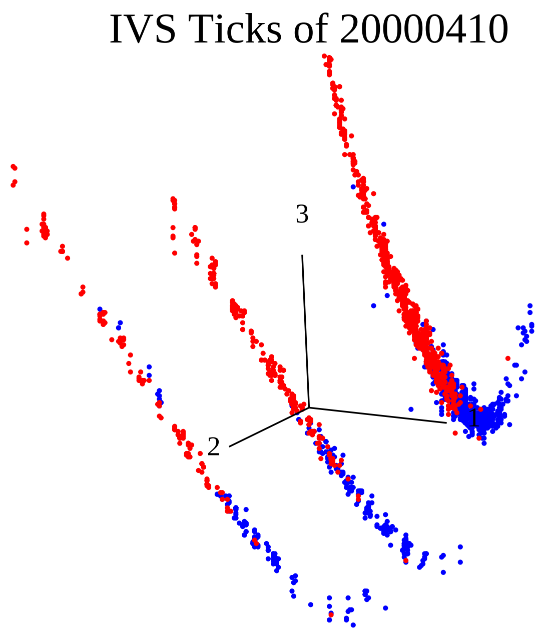



Figure 15: Implied volatility smile on 04/10/2000. Dimension 1: Time to Maturity, 2: Moneyness, 3: Implied Volatility.

# (Implied) Binomial Tree

Each (implied) binomial tree consists of $3$ trees (level $n$, node $i$):

- Tree of underlyings' values $s_{n,i}$
- Tree of transition probabilities $p_{n,i}$
- Tree of Arrow-Debreu (AD) prices $\lambda_{n,i}$
  Arrow-Debreu security: A financial instrument that pays off 1 at node $i$ at level $n$, and otherwise 0.

## Example

$T = 1$ year, $\triangle t = 1/4$ year

smile structure: $\sigma_{imp}(K, t) = 0.15 - 0.0005K$

| | | | | | |
|---|---|---|---|---|---|
| | | | | | 119.91 |
| | | | | 115.07 | |
| | | | 110.05 | | 110.06 |
| | | 105.13 | | 105.14 | |
| stock prices $s_{n,i}$ | 100.00 | | 100.00 | | 100.00 |
| | | 95.12 | | 95.11 | |
| | | | 89.93 | | 89.93 |
| | | | | 85.21 | |
| | | | | | 80.02 |

## Example (2)

$T = 1$ year, $\triangle t = 1/4$ year
smile structure: $\sigma_{imp}(K,t) = 0.15 - 0.0005K$

| | | | | |
|---|---|---|---|---|
| | | | | 0.596 |
| | | | 0.578 | |
| | | 0.589 | | 0.590 |
| transition probabilities $p_{n,i}$ | 0.563 | | 0.563 | |
| | | 0.587 | | 0.586 |
| | | | 0.545 | |
| | | | | 0.589 |

## Example (3)

$T = 1$ year, $\triangle t = 1/4$ year

smile structure: $\sigma_{imp}(K,t) = 0.15 - 0.0005K$

| | | | | | |
|---|---|---|---|---|---|
| | | | | | 0.111 |
| | | | | 0.187 | |
| | | | 0.327 | | 0.312 |
| | | 0.559 | | 0.405 | |
| AD prices $\lambda_{n,i}$ | 1.000 | | 0.480 | | 0.342 |
| | | 0.434 | | 0.305 | |
| | | | 0.178 | | 0.172 |
| | | | | 0.080 | |
| | | | | | 0.033 |

# Binomial Tree vs IBT

- **BT**: Discrete version of a diffusion process with constant volatility parameter:

$$\frac{dS_t}{S_t} = \mu_t dt + \sigma dZ_t$$

Constant transition probabilities: $p_{n,i} = e^{\sigma\sqrt{\Delta t}}$ (with $\Delta t$ fixed)

- **IBT**: Discrete version of diffusion process with a generalized volatility parameter:

$$\frac{dS_t}{S_t} = \mu_t dt + \sigma(S_t, t) dZ_t$$

Non constant transition probabilities $p_{n,i}$ (with $\Delta t$ fixed)

# IBT

Recombining tree divided into $N$ equally spaced time steps of length $\Delta t = \tau/N$

IBT constructed on basis of observed option prices, i.e. takes the smile as an input

IBT-implied SPD: at final nodes assign

$$f^*(s_{N+1,i}) = e^{r\tau}\lambda_{N+1,i} \qquad i = 1, \ldots, N+1$$

where $\lambda_{N+1,i}$ denote the Arrow-Debreu prices

# Application to EUREX DAX-Options

$20$ non overlapping periods from June $1997$ to June $2002$ ($\tau \approx 65/250$ fixed)

period from Monday following 3rd Friday to 3rd Friday 3 months later

Example: on Monday, $23/06/97$, we estimate $f^*$ of Friday, $19/09/97$

- `volsurf` estimates implied volatility surface using:
  - Option data of preceeding $2$ weeks (Monday, $09/06/97$, to Friday, $20/06/97$)
- `IBTbc` computes IBT with input parameters:
  - DAX on Monday June $23$, $1997$, $S_0 = 3748.79$
  - time to maturity $\tau = 65/250$ and interest rate $r = 3.12$

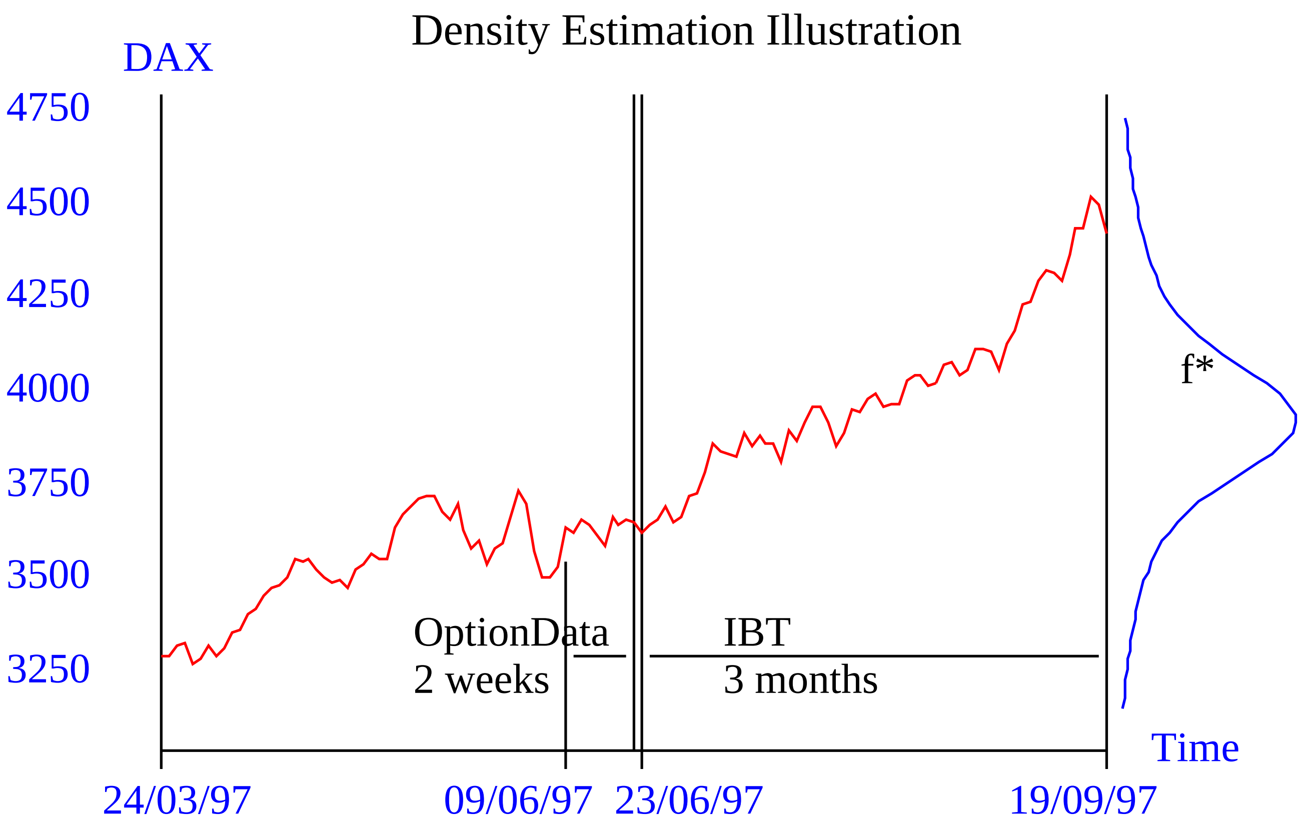



Figure 16: Procedure to estimate implied SPD of Friday, 19/09/97, estimated on Monday, 23/06/97, by means of 2 weeks of option data.

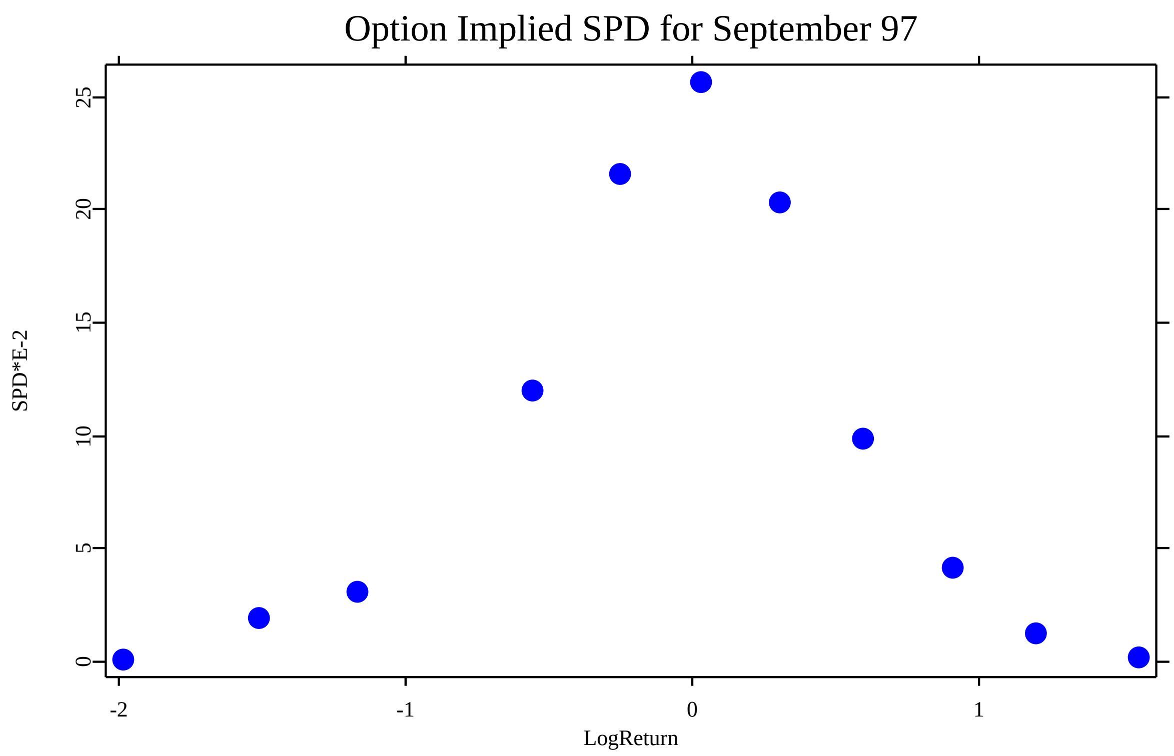



Figure 17: Implied SPD of Friday, 19/09/97, estimated on Monday, 23/06/97, by an IBT with $N = 10$ time steps, $S_0 = 3748.79$, $r = 3.12$ and $\tau = 65/250$.

# Estimation of the Time Series SPD

The estimation of the (historical) time series SPD is based on Ait01.

$S$ follows a diffusion process

$$dS_t = \mu(S_t)dt + \sigma(S_t)dW_t.$$

Further assume a flat yield curve and the existence of a risk-free asset $B$ which evolves according to

$$B_t = B_0 e^{rt}.$$

## Estimation of the Time Series SPD (2)

Then the risk-neutral process follows from Itô's formula and Girsanov's theorem (giving a SPD $g^*$ which will later be compared to the SPD $f^*$):

$$dS_t^* = rS_t^* dt + \sigma(S_t^*) dW_t^*$$

Drift adjusted but diffusion function is identical in both cases !

# Estimation of the Diffusion Function

Florens-Zmirou (1993), WHTsy1997 estimator for $\sigma$

$$\hat{\sigma}^2(S) \quad = \quad \frac{\sum_{i=1}^{N^*-1} K_\sigma(\frac{S_{i/N^*}-S}{h_\sigma}) N^* \{S_{(i+1)/N^*} - S_{i/N^*}\}^2}{\sum_{i=1}^{N^*} K_\sigma(\frac{S_{i/N^*}-S}{h_\sigma})}$$

$K_\sigma$ kernel (in our simulation: Gaussian), $h_\sigma$ bandwidth, $N^*$ number of observed index values $(N^* \approx 65)$ in the time interval $[0, 1]$

$\hat{\sigma}$ consistent estimator of $\sigma$ as $N^* \to \infty$

$\hat{\sigma}$ estimated using a $3$ month time series of DAX prices

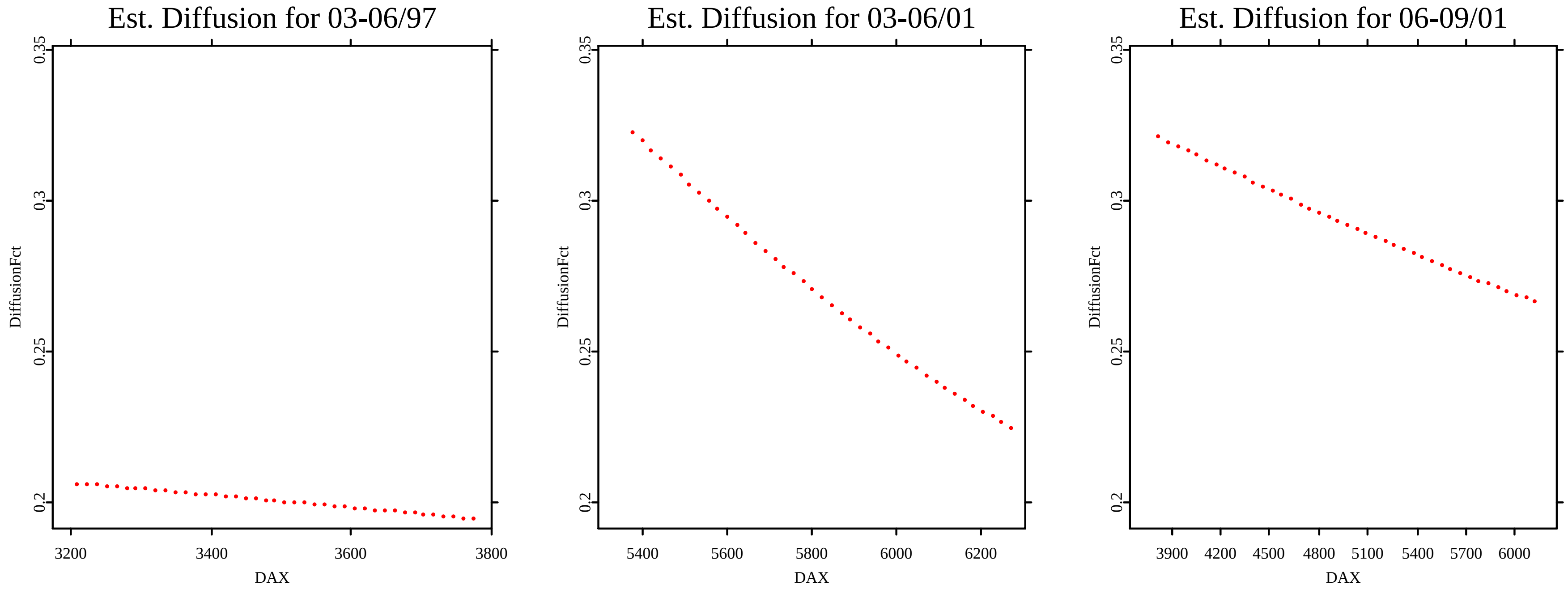



Figure 18: Estimated diffusion functions ("standardized" by $\hat{\sigma}/S_0$) for the periods 24/03/97-20/06/97 ($h_\sigma = 469.00$), 19/03/01-15/06/01 ($460.00$), 18/06/01-21/09/01 ($1597.00$) respectively.

## Simulation of the Time Series SPD

- Use Milstein scheme given by

$$
\begin{aligned}
S^*_{i/N^*} &= S^*_{(i-1)/N^*} + rS^*_{(i-1)/N^*}\Delta t + \hat{\sigma}(S^*_{(i-1)/N^*})\Delta W^*_{i/N^*} + \\
&\frac{1}{2}\hat{\sigma}(S^*_{(i-1)/N^*})\frac{\partial\hat{\sigma}}{\partial S^*}(S^*_{(i-1)/N^*})\left\{(\Delta W^*_{(i-1)/N^*})^2 - \Delta t\right\},
\end{aligned}
$$

  where $\Delta W^*_{i/N^*} \sim N(0, \Delta t)$ with $\Delta t = \frac{1}{N^*}$, drift set equal to $r$, $\frac{\partial\sigma}{\partial S^*}$ approximated by $\frac{\Delta\sigma}{\Delta S^*}$ , $i = 1, \ldots, N^*$

- Simulate $M = 10000$ paths for time to maturity $\tau = \frac{N^*}{250}$
- Compute annualized log-returns for simulated paths:

$$
u^*_{m,T=1} = \left\{\log(S^*_{m,T=1}) - \log(S^*_{t=0})\right\}\tau^{-1}, m = 1, \ldots, M
$$

# Simulation of the Time Series SPD (2)

- SPD $g^*$ obtained by means of nonparametric kernel density estimation $(t = 0)$:

$$
\begin{aligned}
g^*(S^*) &= \frac{\hat{p}_t^*\{\log(S^*/S_t^*)\}}{S^*}, \\
\hat{p}_t^*(u^*) &= \frac{1}{M h_{p^*}} \sum_{m=1}^{M} K_{p^*}\Big(\frac{u_{m,t}^* - u^*}{h_{p^*}}\Big),
\end{aligned}
$$

where $K_{p^*}$ is a kernel (here: Gaussian) and $h_{p^*}$ is a bandwidth.

- Note: $S_T^* \sim g^*$, then with $u^* = \ln(S_T^*/S_t^*)$ $\hat{p}_t^*$ is related to $g^*$ by

$$
P_{p^*}(S_T^* \leq S^*) = P_{p^*}(u^* \leq \log(S^*/S_t^*)) = \int_{-\infty}^{\log(S^*/S_t^*)} p_t^*(u^*) du^* .
$$

# Application to DAX

20 periods from June 1997 to June 2002 ($\tau \approx 65/250$ fixed)

period from Monday following 3rd Friday to 3rd Friday 3 months later

Example: on Monday, 23/06/97, we estimate $g^*$ of Friday, 19/09/97

- Friday, September 19, 1997, is the 3rd Friday
- $\hat{\sigma}$ estimated using DAX prices from Monday, March 23, 1997, to Friday, June 20, 1997
- Monte-Carlo simulation with parameters
  - DAX on Monday June 23, 1997, $S_0 = 3748.79$
  - time to maturity $\tau = 65/250$ and interest rate $r = 3.12$

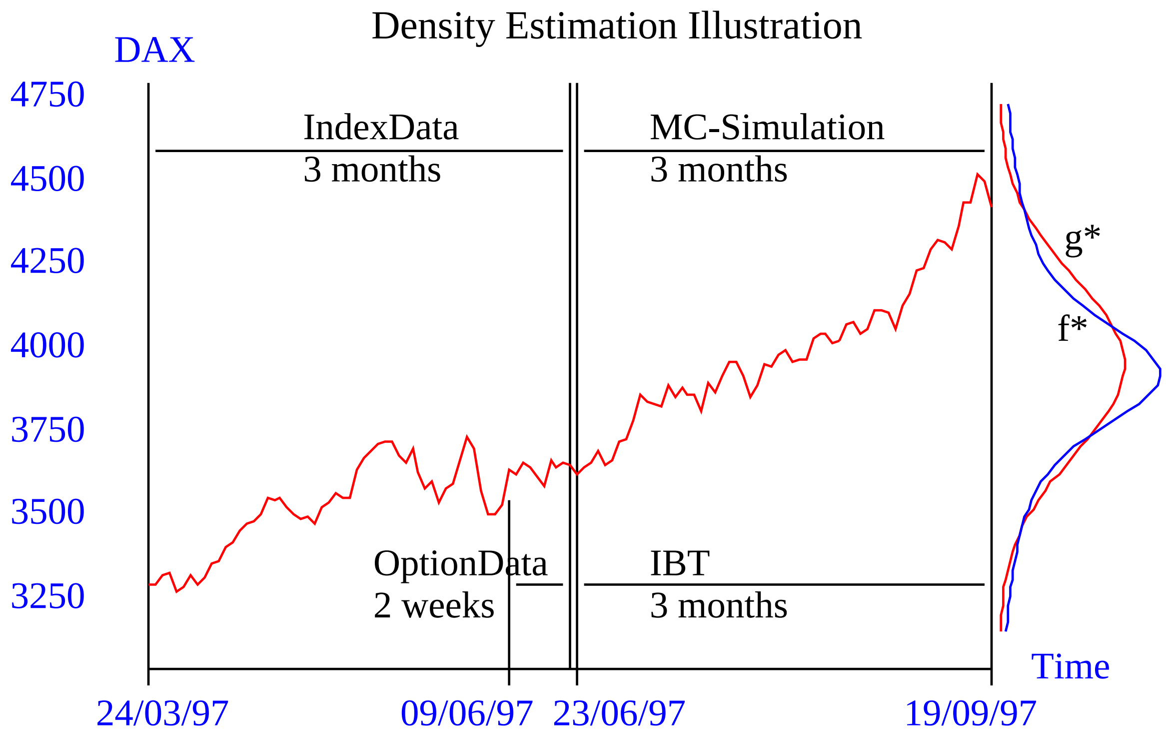



Figure 19: Comparison of procedures to estimate time series and implied SPD of Friday, $19/09/97$. SPD's estimated on Monday, $23/06/97$, by means of 3 months of index data respectively 2 weeks of option data.

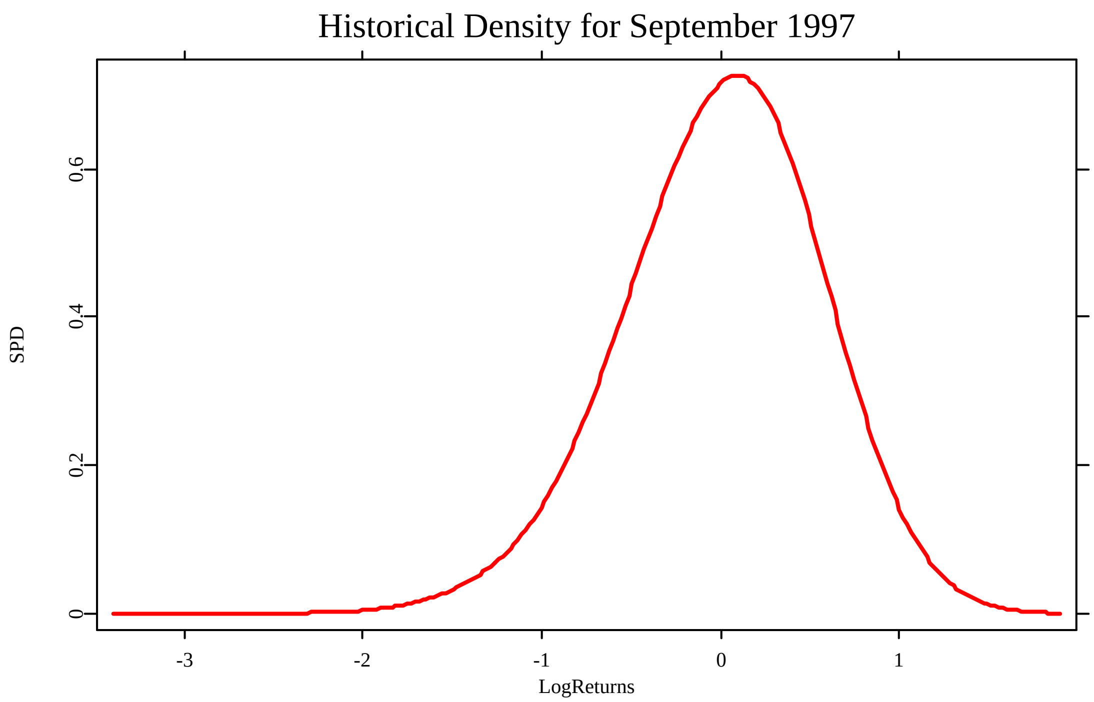



Figure 20: Estimated time series SPD of Friday, 19/09/97, estimated on Monday, 23/06/97. Simulated with $M = 10000$ paths, $S_0 = 3748.79$, $r = 3.12$ and $\tau = 65/250$.

# Comparison of Implied and Time Series SPD

Comparison of 20 non-overlapping 3 months periods from June 1997 to June 2002.

SPDs estimated only for most liquidly traded option contracts maturing in March, June, September and December.

SPDs compared by looking at standard deviation, skewness and kurtosis.

# Comparison of Standard Deviation

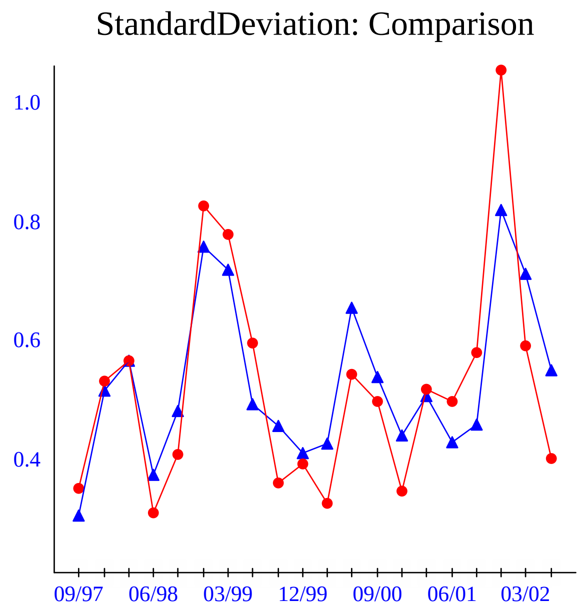



Figure 21: $f^*$ denoted by a triangle and $g^*$ denoted by a circle.

- standard deviation time series of $f^*$ and $g^*$ cross each other frequently
- it appears that standard deviations increase from 2000 on

# Comparison of Skewness

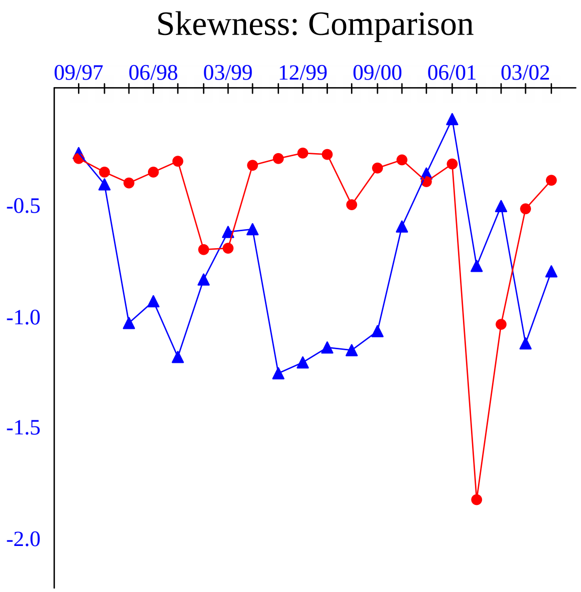
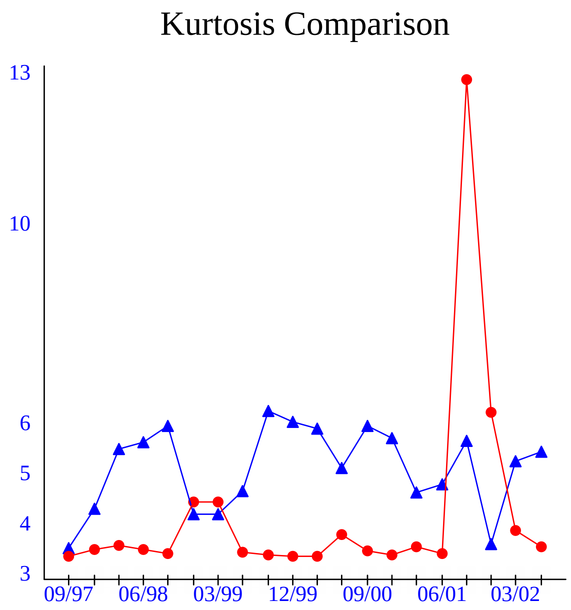



Figure 22: $f^*$ denoted by a triangle and $g^*$ denoted by a circle.

- $f^*$ and $g^*$ negatively skewed for all periods
- $f^*$ more negatively skewed than $g^*$

## Comparison of Kurtosis




- $f^*$ and $g^*$ leptokurtic
- $\text{Kurt}(f^*) < \text{Kurt}(g^*)$
- outlier of $g^*$ in 09/01

Figure 23: $f^*$ denoted by a triangle and $g^*$ denoted by a circle.

# Trading Strategies

General interest:

- Is it possible to exploit the SPD comparison by means of a skewness and/or kurtosis trade?
- Is the strategy's performance consistent with the SPD comparison?

Strategy features:

- Only European calls & puts with $\tau = \frac{65}{250}$ (3 months) considered
- All options are kept until expiration
- Buy/sell ONE option at each moneyness (strike) under consideration

What option to buy or to sell at the estimation day of $f^*$ and $g^*$ ?

| Skewness Trade 1 | | Kurtosis Trade 1 | |
|---|---|---|---|
| skew($f^*$) $<$ skew($g^*$) | | kurt($f^*$) $>$ kurt($g^*$) | |
| Position | Moneyness | Position | Moneyness |
| short puts | $< 0.95$ | short puts | $< 0.90$ |
| | | long puts | $0.90 - 0.95$ |
| | | short puts | $0.95 - 1.00$ |
| | | long calls | $1.00 - 1.05$ |
| | | short calls | $1.05 - 1.10$ |
| long calls | $> 1.05$ | long calls | $> 1.10$ |

Table 4: Definitions of moneyness $(K/S_t e^{r\tau})$ regions.

# Performance Measurement

Return for each the 20 three month periods measured by:

$$\text{portfolio return} = \frac{\text{net cash flow at } t = T}{\text{net cash flow at } t = 0} - 1.$$

Net EURO cash flow in $t = 0$ comprises:

- net cash flow from buying and selling puts and calls,
- for each short call sold buy one share of the underlying,
- for each put sold put the value of the puts' strike on a bank account.

Net EURO cash flow in $t = T$ results from:

- sum of options inner values,
- selling the underlying,
- receiving cash from the puts strikes.

One DAX index point = 1 EURO. (As in a DAX option contract consisting of 5 options one index point has a value of 5 EURO.)

No interest rate between $t = 0$ and $t = T$ considered.

Remark: Buy/sell all options available in the moneyness region in question.

Note: This approach amounts to a careful performance measurement. Applying EUREX margin deposit requirements would decrease the cash outflow for each short option.

# Performance S1 Trade

| | Skewness Trade 1 | | |
|---|---|---|---|
| Period | 06/97-03/00 | 06/00-03/02 | Overall |
| Number of Subperiods | 12 | 8 | 20 |
| Total Return | 4.85 | -8.53 | -2.05 |
| Return Volatility | 3.00 | 9.79 | 6.78 |
| Minimum Return | -3.66 | -25.78 | -25.78 |
| Maximum Return | 7.65 | 7.36 | 7.65 |
| Sharpe Ratio (Strategy) | 0.10 | -0.46 | -0.24 |
| Sharpe Ratio (DAX) | 0.38 | -0.35 | 0.02 |

Table 5: Skewness Trade 1 Performance. Only Total Return is annualized. Returns are given in percentages.

## Performance S1 Trade (2)

| Month | CashFlow in $t = 0$ | CashFlow in $t = T$ | NetCashFlow |
|---|---|---|---|
| Jun97 | 541.80 | 37.88 | 579.68 |
| Sep97 | 344.80 | 0.00 | 344.80 |
| Dec97 | 516.90 | 3781.62 | 4298.52 |
| Mar98 | 1042.00 | 942.68 | 1984.68 |
| Jun98 | 1690.80 | -6896.90 | -5206.10 |
| Sep98 | -1559.50 | 0.00 | -1559.50 |
| Dec98 | 714.30 | 13.27 | 727.57 |
| Mar99 | 923.90 | 286.41 | 1210.31 |
| Jun99 | 964.60 | 0.00 | 964.60 |
| Sep99 | 1019.40 | 3979.47 | 4998.87 |
| Dec99 | 2259.60 | 2206.92 | 4466.52 |
| Mar00 | 3537.40 | -864.48 | 2672.92 |

Table 6: Net option cash flows of S1 trade for 06/97-03/00 (no underlying, strikes considered). Cash flows are measured in EUROs.

## Performance S1 Trade (3)

| Month | CashFlow in $t = 0$ | CashFlow in $t = T$ | NetCashFlow |
|---|---|---|---|
| Jun00 | 197.80 | 0.00 | 197.80 |
| Sep00 | -283.20 | -307.17 | -590.37 |
| Dec00 | -321.90 | -645.92 | -967.82 |
| Mar01 | -420.10 | 0.00 | -420.10 |
| Jun01 | 293.60 | -21616.16 | -21322.56 |
| Sep01 | -5.80 | 4384.25 | 4378.45 |
| Dec01 | 1003.30 | 0.00 | 1003.30 |
| Mar02 | 813.50 | -9499.20 | -8685.70 |

Table 7: Net option cash flows of S1 trade for 06/00-03/02 (no underlying, strikes considered). Cash flows are measured in EUROs.

# DAX evolution from 01/97 to 01/03

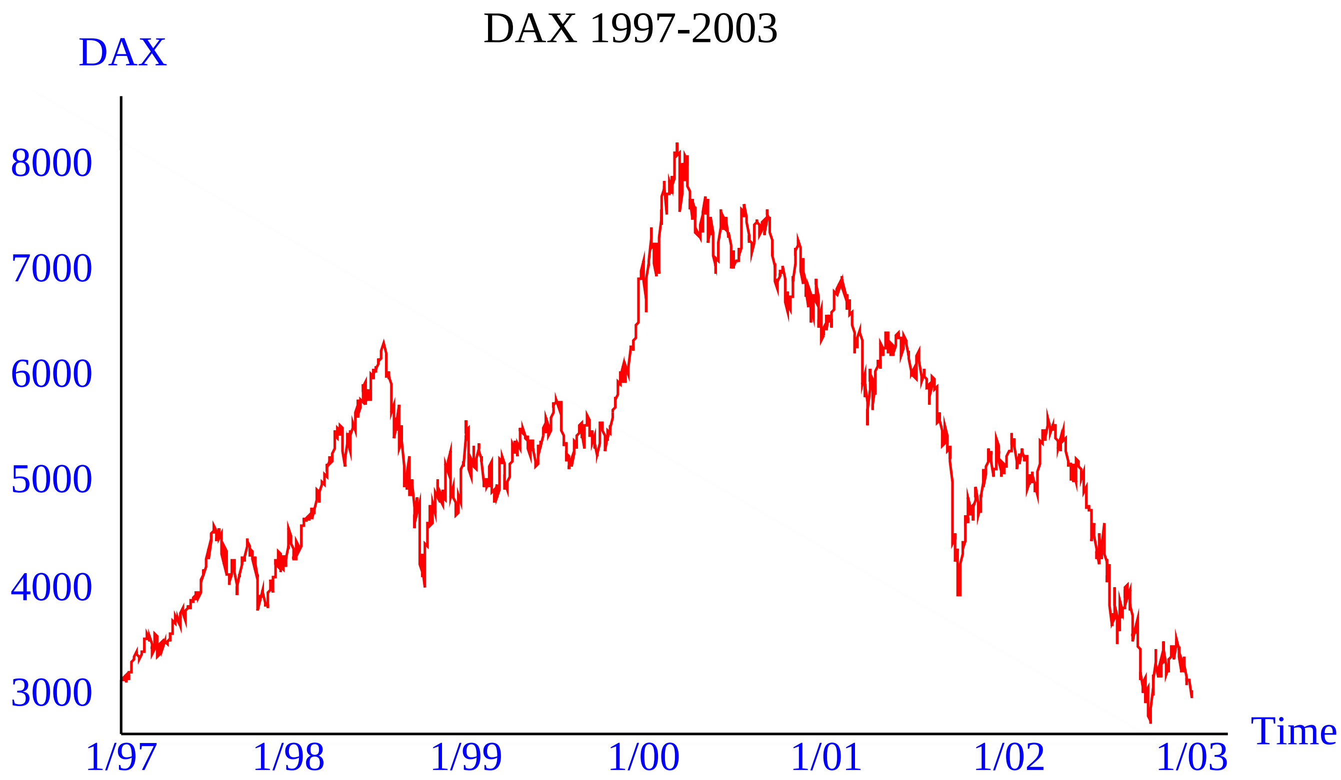



Figure 24: Evolution of DAX from 01/97 to 01/03

# Performance K1 Trade

| | Kurtosis Trade 1 | | |
|---|---|---|---|
| Period | 06/97-03/00 | 06/00-03/02 | Overall |
| Number of Subperiods | 12 | 8 | 20 |
| Total Return | 14.49 | -7.48 | 2.01 |
| Return Volatility | 3.87 | 13.63 | 9.33 |
| Minimum Return | -4.54 | -28.65 | -28.65 |
| Maximum Return | 8.79 | 18.14 | 18.14 |
| Sharpe Ratio (Strategy) | 0.55 | -0.32 | -0.05 |
| Sharpe Ratio (DAX) | 0.38 | -0.35 | 0.02 |

Table 8: Kurtosis Trade 1 Performance. Only Total Return is annualized. Returns are given in percentages.

## Performance K1 Trade (2)

| Month | CashFlow in $t = 0$ | CashFlow in $t = T$ | NetCashFlow |
|---|---|---|---|
| Jun97 | 1257.10 | -989.40 | 267.70 |
| Sep97 | 2047.20 | -58.22 | 1988.98 |
| Dec97 | 1345.70 | -2451.94 | -1106.24 |
| Mar98 | 1793.90 | -1744.51 | 49.39 |
| Jun98 | 2690.30 | -4612.70 | -1922.40 |
| Sep98 | 4758.10 | -541.60 | 4216.50 |
| Dec98 | 3913.40 | -803.08 | 3110.32 |
| Mar99 | 2233.50 | -1190.94 | 1042.56 |
| Jun99 | 1593.80 | -338.32 | 1255.48 |
| Sep99 | 1818.70 | -3194.15 | -1375.45 |
| Dec99 | 2745.50 | -3706.92 | -961.42 |
| Mar00 | 4940.50 | -2419.08 | 2521.42 |

Table 9: Net option cash flows of K1 trade for 06/97-03/00 (no underlying, strikes considered). Cash flows are measured in EUROs.

## Performance K1 Trade (3)

| Month | CashFlow in $t = 0$ | CashFlow in $t = T$ | NetCashFlow |
|---|---|---|---|
| Jun00 | 2178.20 | -555.30 | 1622.90 |
| Sep00 | 2039.90 | -2257.17 | -217.27 |
| Dec00 | 2477.90 | -2957.40 | -479.50 |
| Mar01 | 1853.70 | -502.50 | 1351.20 |
| Jun01 | 1674.10 | -12235.10 | -10561.00 |
| Sep01 | 2315.00 | -2935.25 | -620.25 |
| Dec01 | 2190.80 | -454.24 | 1736.56 |
| Mar02 | 1464.90 | -4976.00 | -3511.10 |

Table 10: Net option cash flows of K1 trade for 06/00-03/02 (no underlying, strikes considered). Cash flows are measured in EUROs.

# Conclusion & Outlook

Trading performance positive in subperiods 06/97 - 03/00 and negative 06/00 - 03/02 for S1 as well as K1 trade. However, a SPD comparison does not produce any signal ex ante.

SPD estimation methodology need to be fine tuned. For example, extent historical SPD estimation: In Monte Carlo simulation draw random numbers from the distribution of the residuals resulting from the estimation of $\sigma$ (WHYat2003).

Strategy design (hedging) and performance measurement to be improved.

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