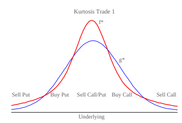
# Trading on Deviations of Implied and Historical Densities


Oliver Jim BLASKOWITZ[1]
Wolfgang HÄRDLE[1]
Peter SCHMIDT[2]

[1] Center for Applied Statistics and Economics (CASE)
[2] Bankgesellschaft Berlin, Quantitative Research



# Motivation

European Call $C(S_t, K, r, \tau)$:

$$C = e^{-r\tau} \int_0^\infty \max(S_T - K, 0) q(S_T) dS_T$$

with time to maturity $\tau = T - t$, strike price $K$ and risk–free interest rate $r$

What is $q(S_T)$? – A risk neutral density of the underlying!

Black–Scholes world: $q(S_T)$ lognormal.

Investors do not know $q(S_T)$. But from **option data** we can extract an **implied** State Price Density (SPD), $f^*(S_T)$.

Implied SPD may be different from **historical** SPD $g^*(S_T)$ estimated from **underlyings'** time series **data**

Suppose one knew $f^*$ and $g^*$. Are there profitable trading strategies to exploit differences in $f^*$ and $g^*$?

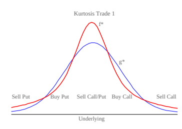
# Data

EUREX DAX–option settlement prices

MD*BASE (http://www.mdtech.de) database

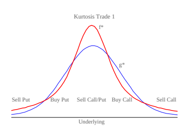
available data: $01/95$ – $12/01$

time period in this study: $01/97$ – $12/99$



# Outline of the talk

1. Motivation✓
2. Estimation of Implied SPD
3. Estimation of Historical SPD
4. Comparison of Implied and Historical SPD
5. Trading Strategies
6. A Word of Caution



# Estimation of Option–Implied SPD

## Implied Binomial Tree (IBT)

Numerical method to compute SPD adapted to volatility smile

Several approaches: Rubinstein $(1994)$, Dupire $(1994)$, Derman and Kani $(1994)$ and Barle and Cakici $(1998)$

XploRe Quantlets compute Derman and Kani's `IBTdk` and Barle and Cakici's `IBTbc` IBT

Barle and Cakici's version proved to be more robust

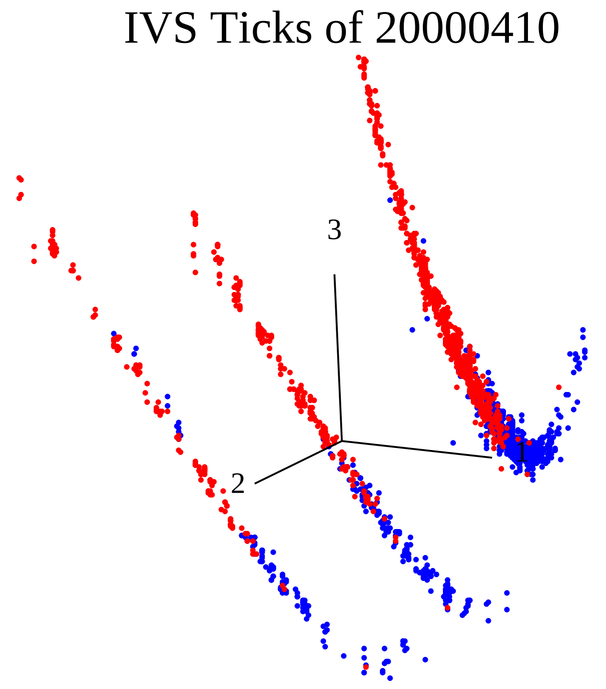



Figure 1: Implied volatility smile on 04/10/2000. Not contained in the data set.

# (Implied) Binomial Tree

Each (implied) binomial tree consists of $3$ trees (level $n$, node $i$):

- Tree of underlyings' values $s_{n,i}$
- Tree of transition probabilities $p_{n,i}$
- Tree of Arrow–Debreu (AD) prices $\lambda_{n,i}$
  Arrow–Debreu security: A financial instrument that pays off 1 $\mathrm{EUR}$ at node $i$ at level $n$, and otherwise 0.

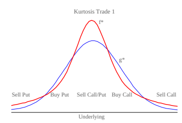
## Example

$T = 1$ year, $\triangle t = 1/4$ year

smile structure: $\sigma_{imp}(K, t) = 0.15 - 0.0005K$

| | | | | | |
|---|---|---|---|---|---|
| | | | | | 119.91 |
| | | | | 115.07 | |
| | | | 110.05 | | 110.06 |
| | | 105.13 | | 105.14 | |
| stock prices $s_{n,i}$ | 100.00 | | 100.00 | | 100.00 |
| | | 95.12 | | 95.11 | |
| | | | 89.93 | | 89.93 |
| | | | | 85.21 | |
| | | | | | 80.02 |





## Example (2)

$T = 1$ year, $\triangle t = 1/4$ year

smile structure: $\sigma_{imp}(K,t) = 0.15 - 0.0005K$

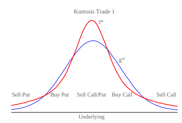
| | | | | |
|---|---|---|---|---|
| | | | | 0.596 |
| | | | 0.578 | |
| | | 0.589 | | 0.590 |
| transition probabilities $p_{n,i}$ | 0.563 | | 0.563 | |
| | | 0.587 | | 0.586 |
| | | | 0.545 | |
| | | | | 0.589 |

## Example (3)

$T = 1$ year, $\triangle t = 1/4$ year

smile structure: $\sigma_{imp}(K,t) = 0.15 - 0.0005K$

| | | | | | |
|---|---|---|---|---|---|
| | | | | | 0.111 |
| | | | | 0.187 | |
| | | | 0.327 | | 0.312 |
| | | 0.559 | | 0.405 | |
| AD prices $\lambda_{n,i}$ | 1.000 | | 0.480 | | 0.342 |
| | | 0.434 | | 0.305 | |
| | | | 0.178 | | 0.172 |
| | | | | 0.080 | |
| | | | | | 0.033 |



# Binomial Tree vs IBT

- **BT**: Discrete version of a diffusion process with constant volatility parameter:

$$\frac{dS_t}{S_t} = \mu_t dt + \sigma dZ_t$$

Constant transition probabilities: $p_{n,i} = p$ (with $\Delta t$ fixed)

- **IBT**: Discrete version of diffusion process with a generalized volatility parameter:

$$\frac{dS_t}{S_t} = \mu_t dt + \sigma(S_t, t) dZ_t$$

Non constant transition probabilities $p_{n,i}$ (with $\Delta t$ fixed)

# IBT

Recombining tree divided into $N$ equally spaced time steps of length $\Delta t = \tau/N$

IBT constructed on basis of observed option prices, i.e. takes the smile as an input

IBT–implied SPD: at final nodes assign

$$e^{r\tau}\lambda_{N+1,i} \text{ to } s_{N+1,i},\ i = 1, \ldots, N+1$$

where $\lambda_{N+1,i}$ denote the Arrow–Debreu prices

## Application to EUREX DAX–Options

$30$ periods from April $1997$ to September $1999$ ($\tau \approx 90/360$ fixed)

period from Monday following 3rd Friday to 3rd Friday 3 months later

Example: on Monday, $04/21/97$, we estimate $f^*$ of Friday, $07/18/97$

- `volsurf` estimates implied volatility surface using:
  - Option data of preceeding $2$ weeks (Monday, $04/07/97$, to Friday, $04/18/97$)
- `IBTbc` computes IBT with input parameters:
  - DAX on Monday April $21$, $1997$, $S_0 = 3328.41$
  - time to maturity $\tau = 88/360$ and interest rate $r = 3.23$

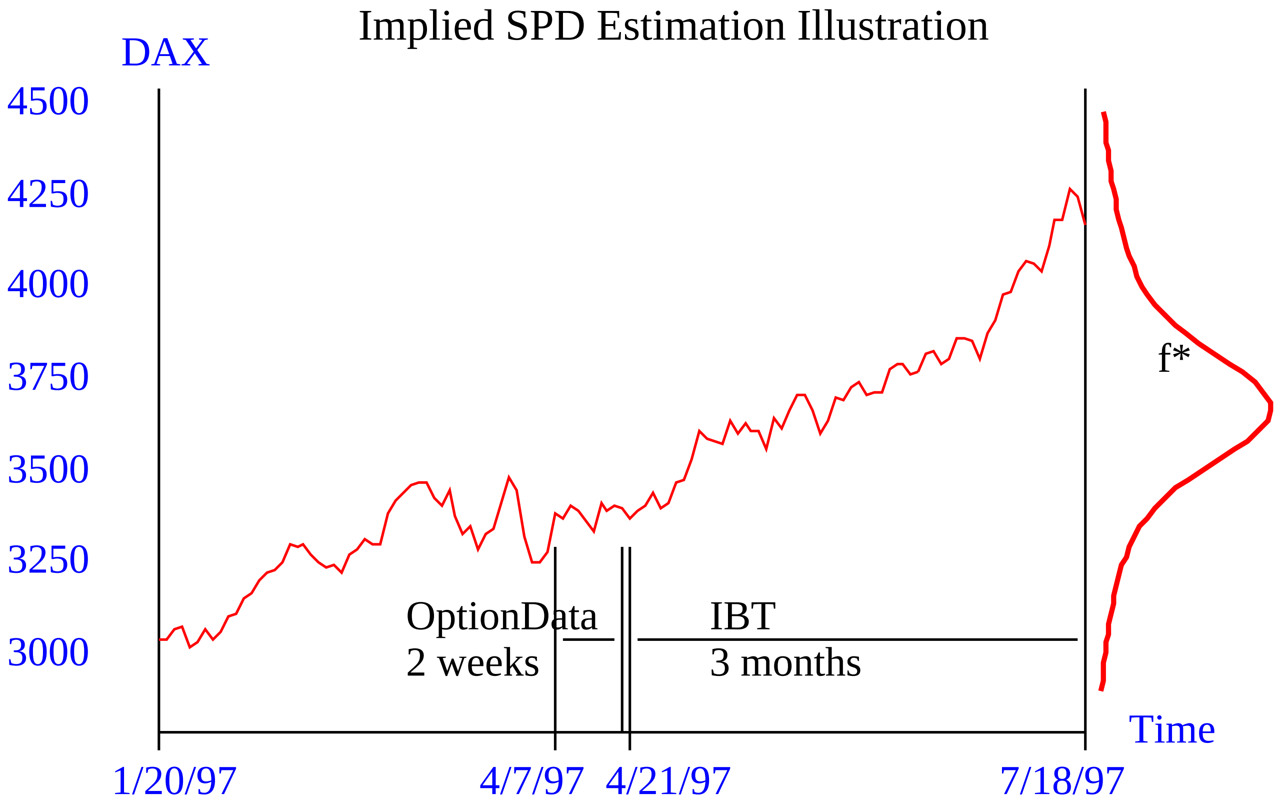



Figure 2: Procedure to estimate implied SPD of Friday, 07/18/97, estimated on Monday, 04/21/97, by means of 2 weeks of option data.

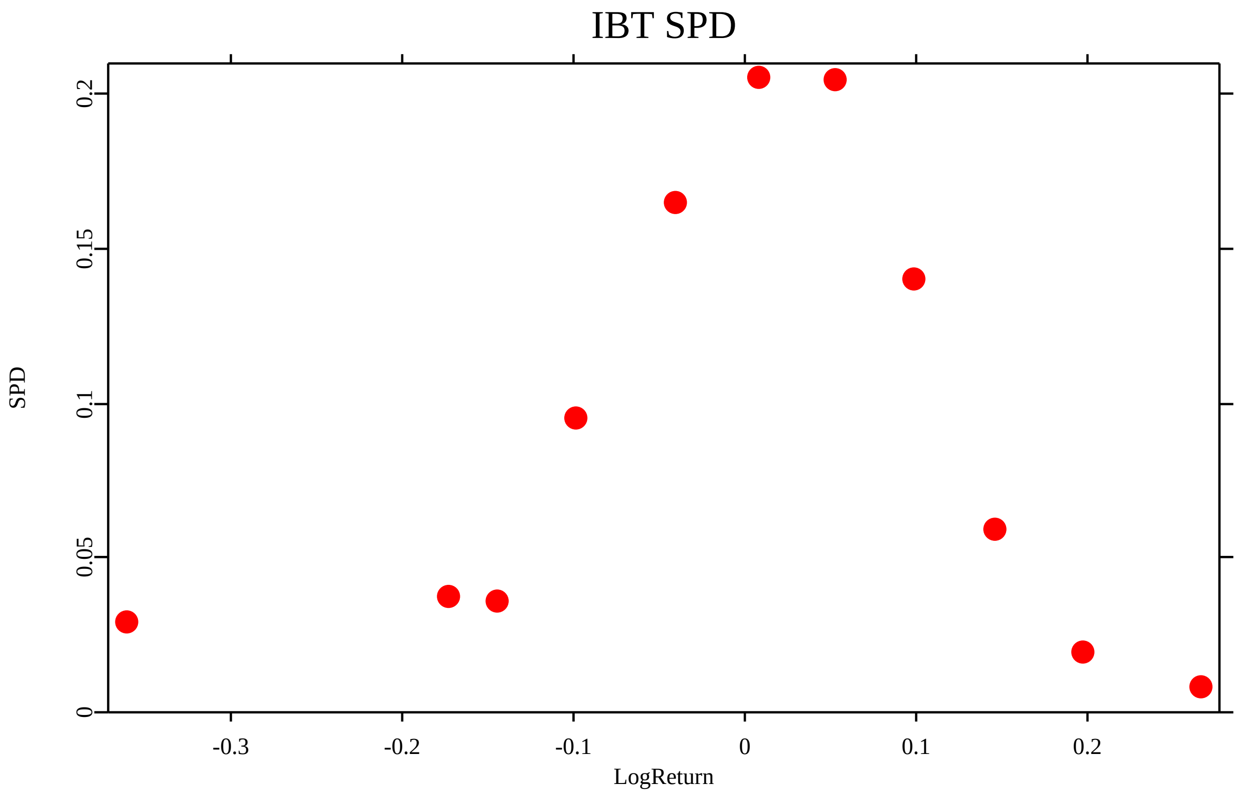



Figure 3: Implied SPD of Friday, 07/18/97, estimated on Monday, 04/21/97, by an IBT with $N = 10$ time steps, $S_0 = 3828.41$, $r = 3.23$ and $\tau = 88/360$.

# Estimation of Historical SPD

S follows a diffusion process

$$dS_t = \mu(S_t)dt + \sigma(S_t)dW_t$$

Change of measure (Girsanov) to obtain risk neutral process (giving a risk neutral SPD $g^*$ which will later be compared to the risk neutral SPD $f^*$)

$$dS_t^* = (r_{t,\tau} - \delta_{t,\tau})S_t^* dt + \sigma(S_t^*)dW_t^*$$

Drift adjusted but diffusion function is identical in both cases !

# Estimation of the Diffusion Function

Florens–Zmirou (1993), Härdle & Tsybakov (1997) estimator for $\sigma$

$$\hat{\sigma}(S) \quad = \quad \frac{\sum_{i=1}^{N^*-1} K_1(\frac{S_i - S}{h_1}) N^* \{S_{(i+1)/N^*} - S_{i/N^*}\}^2}{\sum_{i=1}^{N^*} K_1(\frac{S_i - S}{h_1})}$$

$K_1$ kernel, $h_1$ bandwidth, $N^*$ number of observed index values

$\hat{\sigma}$ unbiased estimator of $\sigma$ (without imposing restrictions on drift)

$\hat{\sigma}$ estimated using a $3$ month time series of DAX prices

# Simulation of Historical SPD

- Use Milstein scheme given by

$$\begin{aligned} S_i &= S_{i-1} + rS_{i-1}\Delta t + \sigma(S_{i-1})\Delta W_i + \\ &\quad \frac{1}{2}\sigma(S_{i-1})\frac{\partial \sigma}{\partial S}(S_{i-1})\Big((\Delta W_{i-1})^2 - \Delta t\Big), \end{aligned}$$

  where $\Delta W_i \sim N(0, \Delta t)$ with $\Delta t = \frac{1}{360}$, drift set equal to $r$, $\frac{\partial \sigma}{\partial S}$ approximated by $\frac{\Delta \sigma}{\Delta S}$ , $i = 1, \dots, TTM$ with $TTM \in \{87, \cdots, 91\}$

- Simulate $M = 10000$ paths for time to maturity $\tau = \frac{\mathsf{TTM}}{360}$

- Compute annualized log–returns for simulated paths:

$$u_{m,T} = \{\log(S_{m,T}) - \log(S_t)\}\,\tau^{-1}\,, m = 1, \dots, M$$

# Simulation of Historical SPD (2)

- SPD $g^*$ obtained by means of nonparametric kernel density estimation

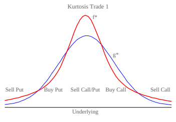
$$
\begin{aligned}
g^*(S) &= \frac{\hat{p}_t^*\{\log(S/S_t)\}}{S}, \\
\hat{p}_t^*(u) &= \frac{1}{Mh_1}\sum_{m=1}^{M} K_1\Big(\frac{u_{m,t}-u}{h_1}\Big)
\end{aligned}
$$

- Note: $S_T \sim g^*(S)$, then with $u = \ln(S_T/S_t)$ $\hat{p}_t^*$ is related to $g^*$ by

$$
P(S_T \leq S) = P(u \leq \log(S/S_t)) = \int_{-\infty}^{\log(S/S_t)} p_t^*(u)du\,.
$$

- $g^*$ is $\sqrt{N^*}$–consistent for $M \to \infty$
- $M$ : Number of simulated Monte Carlo paths.





# Application to DAX

30 periods from April 1997 to September 1999 ($\tau \approx 90/360$ fixed)

period from Monday following 3rd Friday to 3rd Friday 3 months later

Example: on Monday, 04/21/97, we estimate $g^*$ of Friday, 07/18/97

- Friday, April 18, 1997 is the 3rd Friday of April
- $\hat{\sigma}$ estimated using DAX prices from Monday, January 20, 1997, to Friday, April 18,1997
- Monte–Carlo simulation with parameters
  - DAX on Monday April 21, 1997, $S_0 = 3328.41$
  - time to maturity $\tau = 88/360$ and interest rate $r = 3.23$

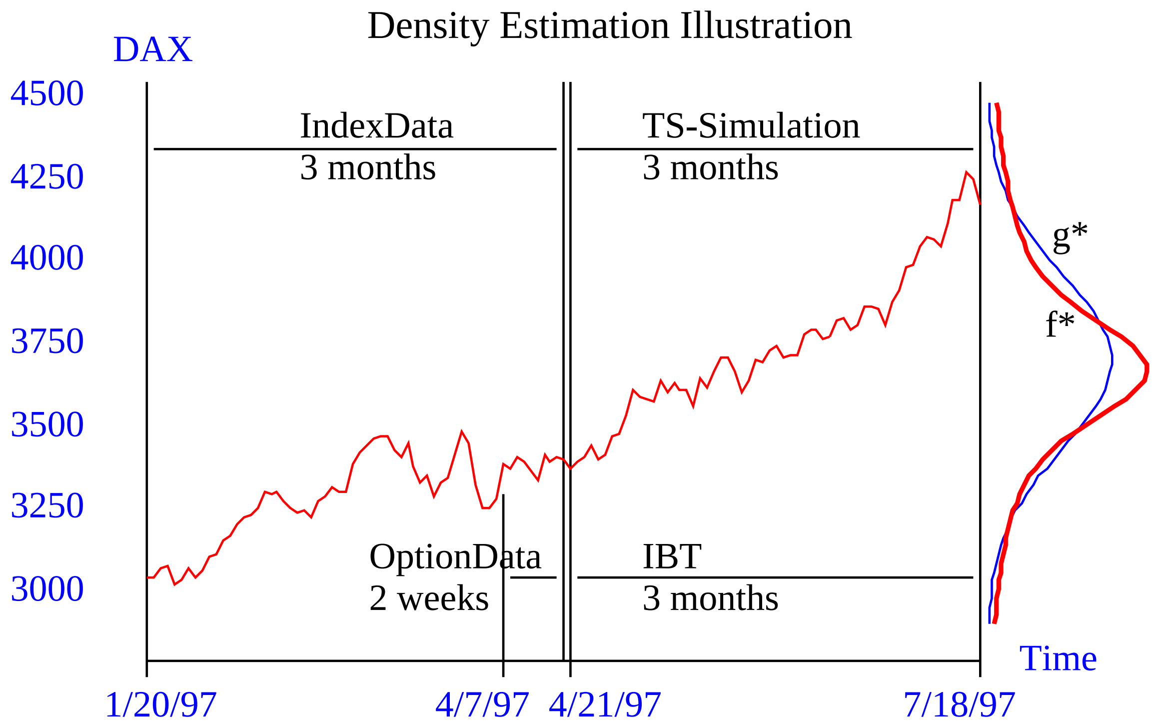
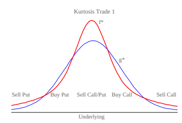



Figure 4: Comparison of procedures to estimate historical and implied SPD of Friday, 07/18/97. SPD's estimated on Monday, 04/21/97, by means of 3 months of index data respectively 2 weeks of option data.





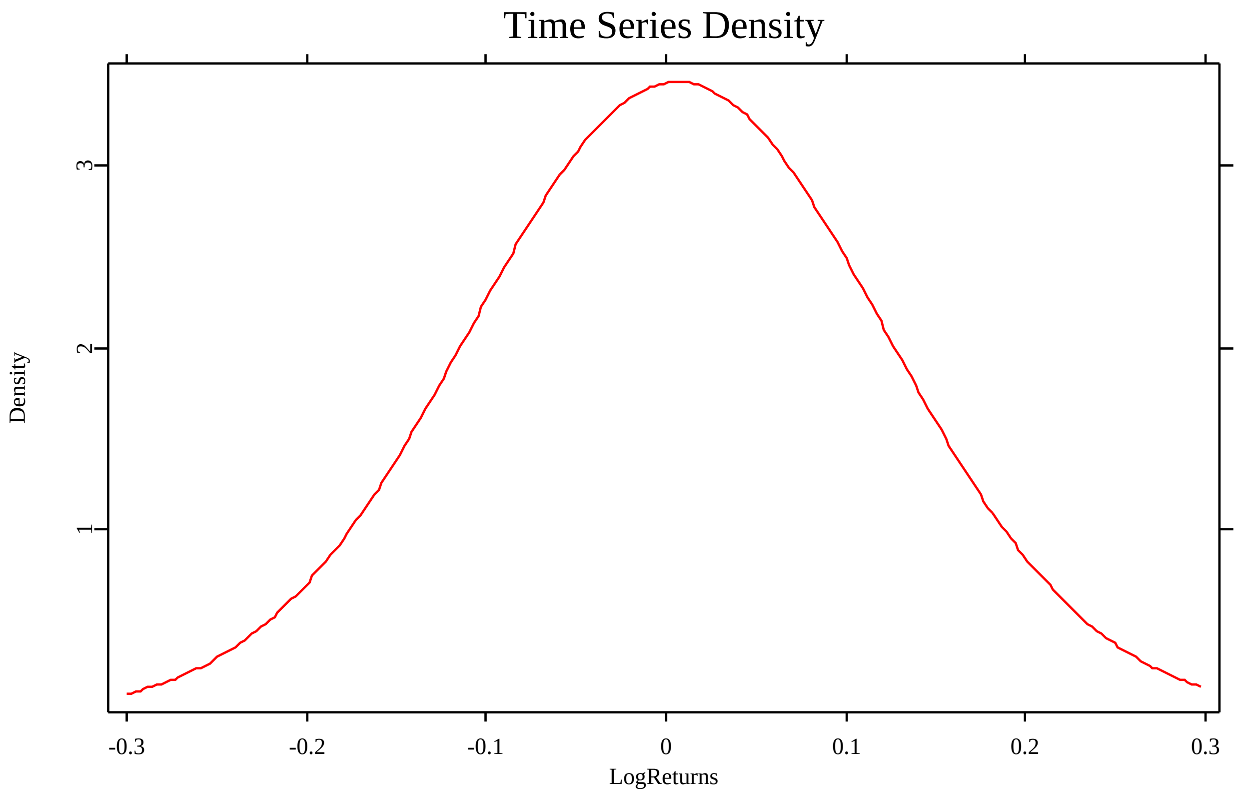


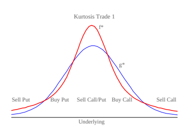

Figure 5: Estimated time series SPD of Friday, 07/18/97, estimated on Monday, 04/21/97. Simulated with $M = 10000$ paths, $S_0 = 3328.41$, $r = 3.23$ and $\tau = 88/360$.

# Comparison of Implied and Historical SPD

## Skewness

- Implied SPD $f^*$
  - Clearly negatively skewed for all periods but one
  - In September 1999 slightly positively skewed
- Historical SPD $g^*$
  - Systematically slightly negatively skewed
  - Skewness close to zero
- Comparison
  - Except for September 1999 $f^*$ systematically more negatively skewed than $g^*$.





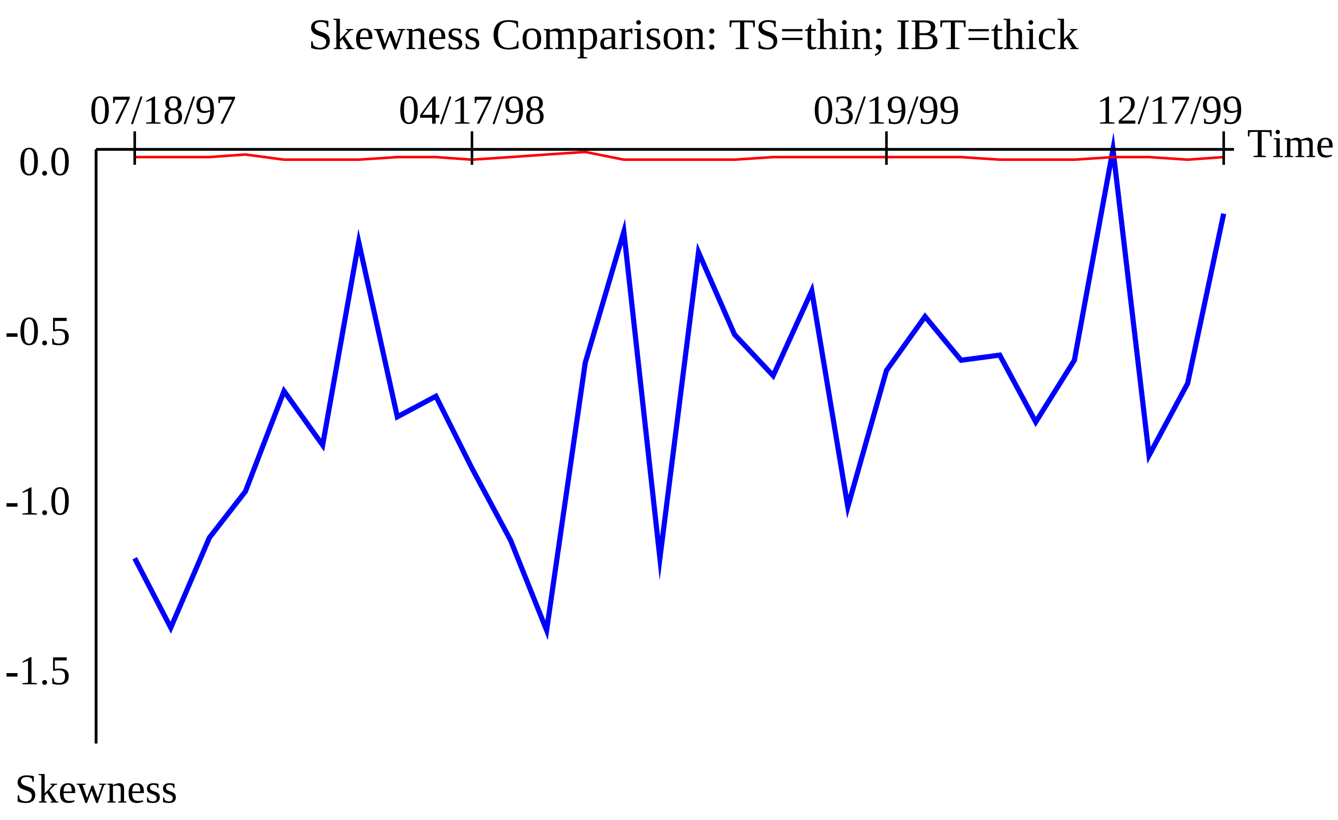
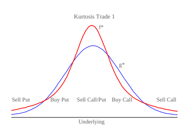



Figure 6: Comparison of Skewness of $f^*$ and $g^*$ for 30 periods.

## Kurtosis

- Implied SPD $f^*$
  - Leptokurtic in all but one period
  - Platykurtic in October 1998
- Historical SPD $g^*$
  - Systematically smaller than $3$ but very close to $3$.
- Comparison
  - Except for one period kurt($f^*$) $\gg$ kurt($g^*$)
  - October 1998: $f^*$ slightly smaller kurtosis than $g^*$



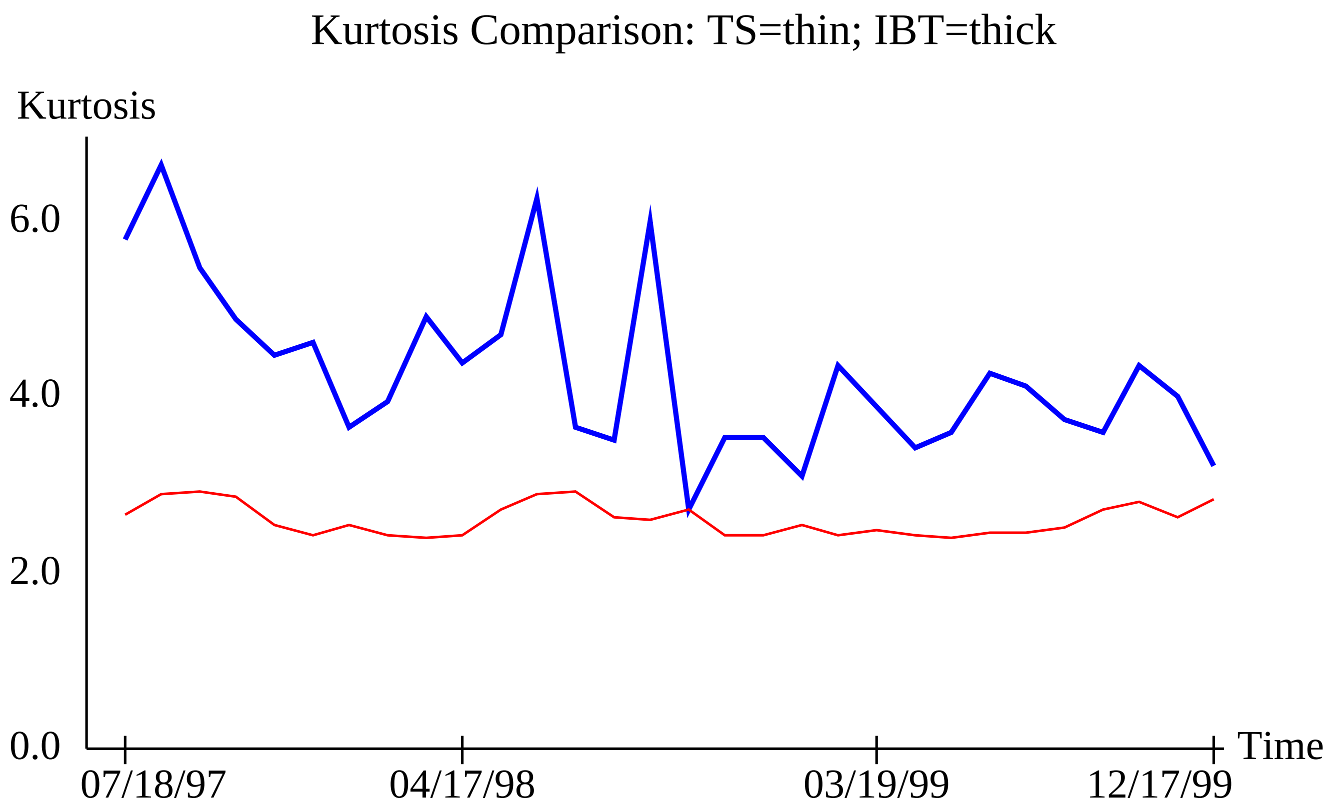



Figure 7: Comparison of Kurtosis of $f^*$ and $g^*$ for 30 periods.

# Trading Strategies

- General interest: what option (in terms of moneyness) to buy or to sell at the day at which both densities were estimated
- Exclusively European call or put options with $3$ month maturity considered
- All options are kept until expiration
- Buy/sell ONE contract of each option

Moneyness defined as $K/S_t e^{r\tau}$:

| | | | | |
|---|---|---|---|---|
| | | Moneyness(FOTM Put) | $<$ | $0.90$ |
| $0.90$ | $\leq$ | Moneyness(NOTM Put) | $<$ | $0.95$ |
| $0.95$ | $\leq$ | Moneyness(ATM Put) | $<$ | $1.00$ |
| $1.00$ | $\leq$ | Moneyness(ATM Call) | $<$ | $1.05$ |
| $1.05$ | $\leq$ | Moneyness(NOTM Call) | $<$ | $1.10$ |
| $1.10$ | $\leq$ | Moneyness(FOTM Call) | | |

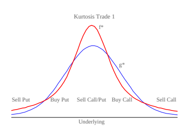
Table 1: Definitions of moneyness regions.

## Skewness Trades

| Skewness Trade 1 (S1) | Skewness Trade 2 (S2) |
|---|---|
| skew($f^*$) $<$ skew($g^*$) | skew($f^*$) $>$ skew($g^*$) |
| Sell OTM Puts | Buy OTM Puts |
| Buy OTM Calls | Sell OTM Calls |

Note: $f^*$ more skewed than $g^*$ means that skewness of $f^*$ more negative than skewness of $g^*$





**S1 Trade:** $C = e^{-r\tau} \int_0^\infty \max(S_T - K, 0) q(S_T) dS_T$

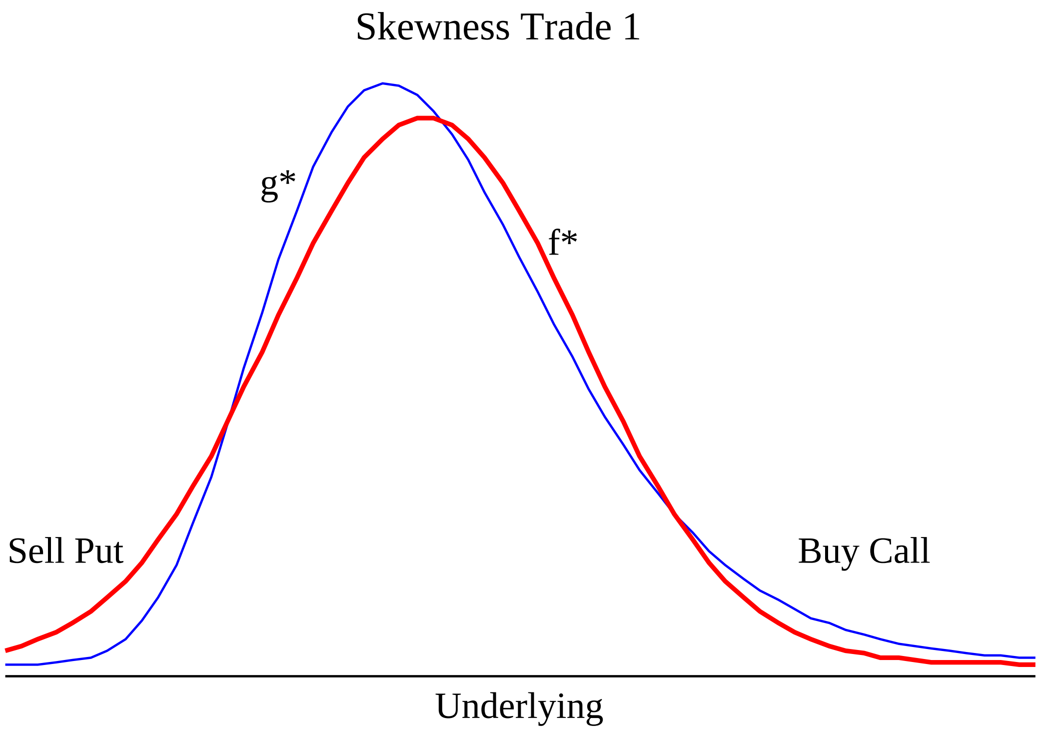
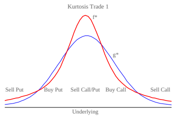



Figure 8: Skewness Trade 1





# Payoff S1 Trade

Portfolio

| Option | Moneyness |
|---|---|
| short put | 0.95 |
| long call | 1.05 |

Table 2: Table S1

Payoff function

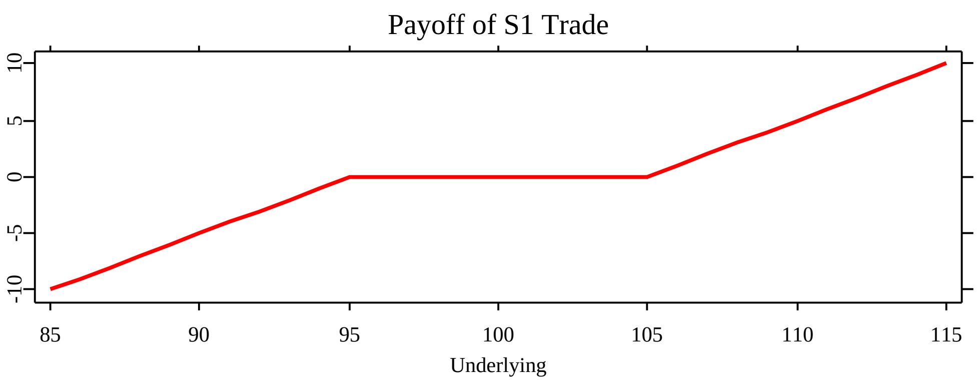
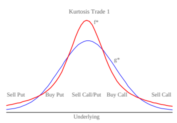



Figure 9: Payoff of S1 Trade of portfolio given in table S1.





## DAX evolution from 01/97 to 12/99

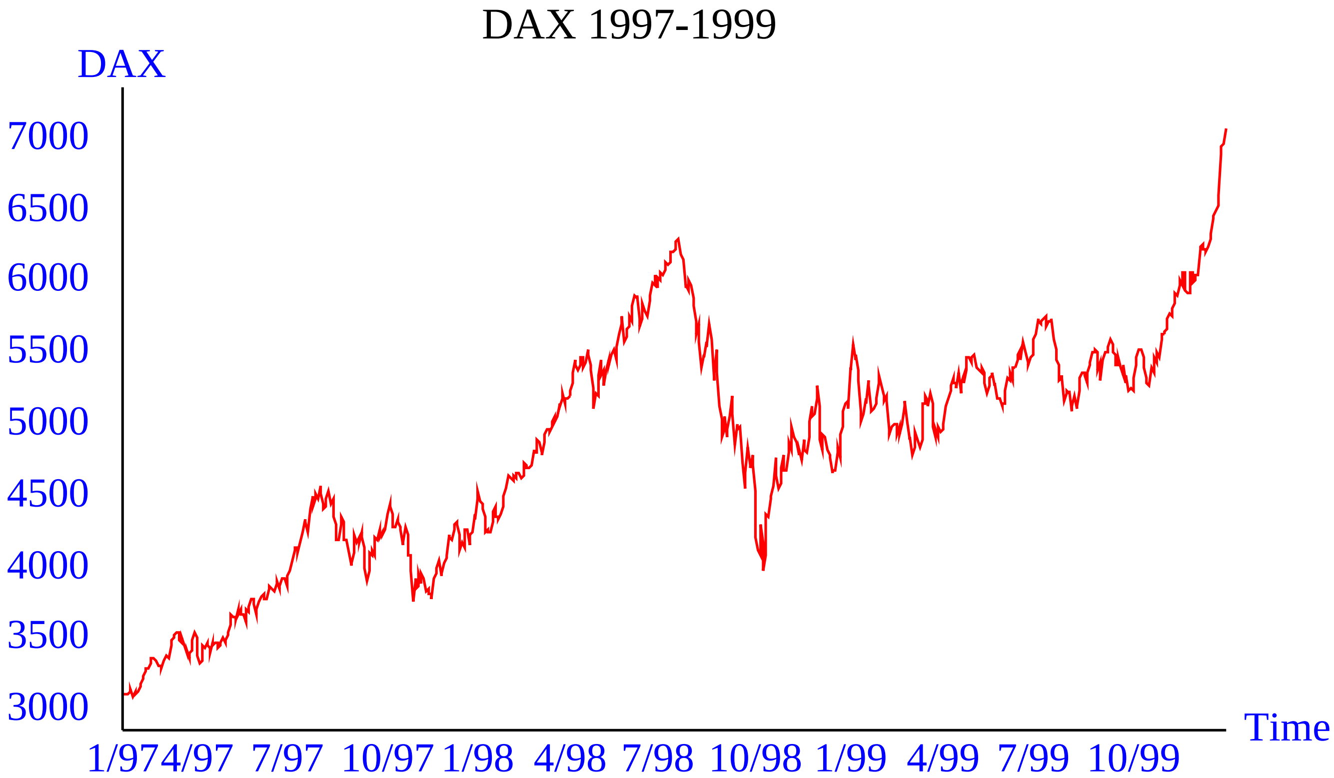
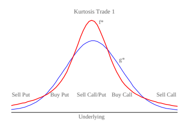



Figure 10: Evolution of DAX from 01/97 to 12/99

# Performance

- Measured in net EURO cash flows: sum of cash flows in $t = 0, t = T$
- No interest rate between these dates considered
- Strategy initiated at Monday (f.e. 4/21/97) immediately following the $3$rd Friday (f.e. 4/18/97) of each month
- Cash flow at initiation
  - Inflow generated by written options
  - Outflow generated by bought options and hypothetical $5\%$ transaction costs on prices of bought and sold options
- Cash flow in $t = T$ (with $\tau \approx 3$ months) sum of options inner values





# Performance S1–Trade (1997)

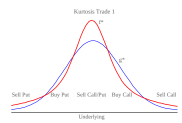
| Month | CashFlow in $t=0$ | CashFlow in $t=T$ | NetCashFlow |
|---|---|---|---|
| Apr 97 | 0.00 | 0.00 | 0.00 |
| May 97 | 0.00 | 0.00 | 0.00 |
| Jun 97 | 499.47 | 0.00 | 499.47 |
| Jul 97 | 0.00 | 0.00 | 0.00 |
| Aug 97 | 140.70 | 0.00 | 140.70 |
| Sep 97 | 70.86 | 0.00 | 70.86 |
| Oct 97 | 379.15 | 0.00 | 379.15 |
| Nov 97 | -243.08 | 916.06 | 672.99 |
| Dec 97 | 420.39 | 4196.76 | 4617.15 |

Table 3: Performance of S1 trade in 1997 with 5% transaction costs. Cash flows are measured in EUROs.

# Performance S1 Trade (1998) (2)

| Month | CashFlow in $t=0$ | CashFlow in $t=T$ | NetCashFlow |
|---|---|---|---|
| Jan 98 | 407.93 | 0.00 | 407.93 |
| Feb 98 | 110.01 | 0.00 | 110.01 |
| Mar 98 | 957.43 | 726.88 | 1684.31 |
| Apr 98 | 164.64 | 0.00 | 164.64 |
| May 98 | 0.00 | 0.00 | 0.00 |
| Jun 98 | 1570.64 | -6426.08 | -4855.44 |
| Jul 98 | 0.00 | 0.00 | 0.00 |
| Aug 98 | 483.17 | -127.14 | 356.03 |
| Sep 98 | -1813.08 | 0.00 | -1813.08 |
| Oct 98 | 0.00 | 0.00 | 0.00 |
| Nov 98 | 519.37 | 0.00 | 519.37 |
| Dec 98 | 465.57 | 0.00 | 465.57 |

Table 4: Performance of S1 trade in 1998 with 5% transaction costs. Cash flows are measured in EUROs.



# Performance S1 Trade (1999) (3)

| Month | CashFlow in $t=0$ | CashFlow in $t=T$ | NetCashFlow |
|---|---|---|---|
| Jan 99 | 562.88 | 0.00 | 562.88 |
| Feb 99 | 185.16 | 0.00 | 185.16 |
| Mar 99 | 789.97 | 0.00 | 789.97 |
| Apr 99 | 293.84 | 0.00 | 293.84 |
| May 99 | 0.00 | 0.00 | 0.00 |
| Jun 99 | 0.00 | 0.00 | 0.00 |
| Jul 99 | 0.00 | 0.00 | 0.00 |
| Aug 99 | 140.22 | 0.00 | 140.22 |
| Sep 99 | 932.62 | 3531.20 | 4463.82 |

| Sum(NCF) | Mean(NCF) | Var(NCF) | Mean/$\sqrt{\text{Var}}$ |
|---|---|---|---|
| 9855.50 | 328.52 | 2430221.51 | 0.2107 |

Table 5: Performance of S1 trade in 1999 with 5% transaction costs and simple statistics aggregating 1997, 1998, 1999. CF's in EURs.

XFGSpdTradeSkew.xpl

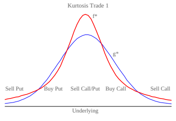
# Performance S1 Trade (4)

- NetCashFlow always positive except for portfolios initiated in June $1998$ and in September $1998$
- $28$ times moderate gains and two times large negative cash flows.
- Directional risk in $12/1997$ and $6/1998$ $\rightarrow$ large payoffs at expiration (turning points of DAX)
- Zero cash flow at initiation and at expiration $\rightarrow$ no OTM option in the database

## Kurtosis Trades

| Kurtosis Trade 1 (K1) | Kurtosis Trade 2 (K2) |
|---|---|
| kurt($f^*$) $>$ kurt($g^*$) | kurt($f^*$) $<$ kurt($g^*$) |
| Sell FOTM Puts | Buy FOTM Puts |
| Buy NOTM Puts | Sell NOTM Puts |
| Sell ATM Puts/Calls | Buy ATM Puts/Calls |
| Buy NOTM Calls | Sell NOTM Calls |
| Sell FOTM Calls | Buy FOTM Calls |





# Kurtosis Trade 1: $C = e^{-r\tau} \int_0^\infty \max(S_T - K, 0) q(S_T) dS_T$

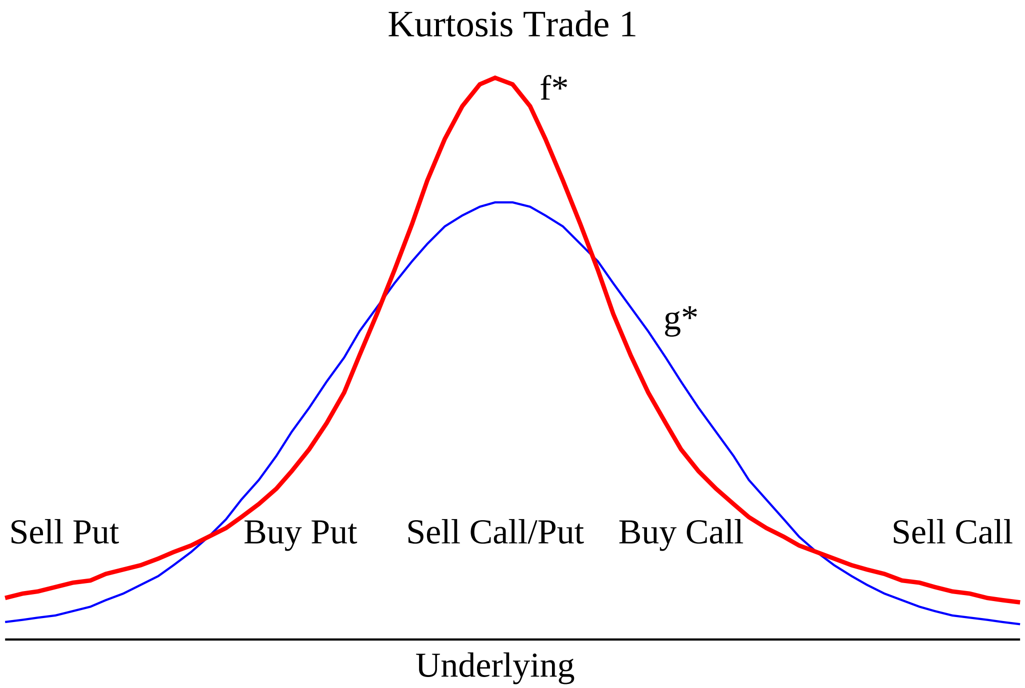



Figure 11: Kurtosis Trade 1

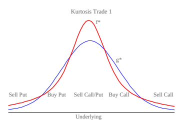
# Payoff K1 Trade

Portfolio

| Option | Moneyness |
|---|---|
| short put | $0.90$ |
| long put | $0.95$ |
| short put | $1.00$ |
| short call | $1.00$ |
| long call | $1.05$ |
| short call | $1.10$ |

Table 6: Table K1





# Payoff K1 Trade (2)

Payoff function

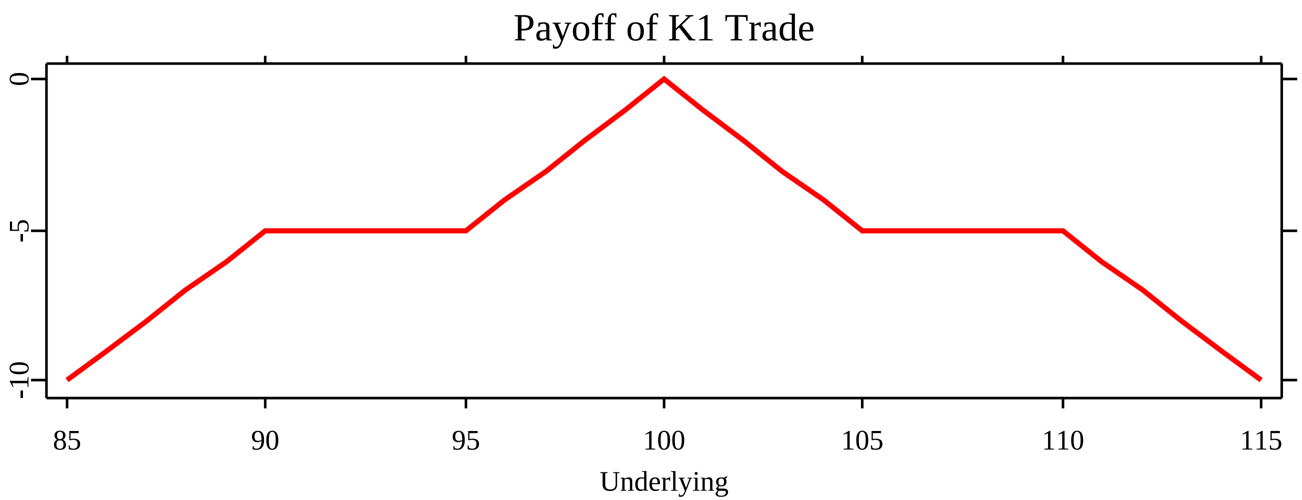



Figure 12: Kurtosis Trade 1 payoff at maturity of portfolio detailed in table K1.

# Performance K1 Trade (1997)

| Month | CashFlow in $t = 0$ | CashFlow in $t = T$ | NetCashFlow |
|---|---|---|---|
| Apr 97 | 1186.46 | -4166.64 | -2980.18 |
| May 97 | 874.10 | -2790.54 | -1916.45 |
| Jun 97 | 1070.54 | -698.36 | 372.18 |
| Jul 97 | 1488.75 | -254.20 | 1234.54 |
| Aug 97 | 1265.22 | -332.36 | 932.86 |
| Sep 97 | 2040.11 | -133.95 | 1906.16 |
| Oct 97 | 639.15 | -16.24 | 622.91 |
| Nov 97 | 1023.66 | -1516.06 | -492.40 |
| Dec 97 | 1171.24 | -2716.12 | -1544.88 |

Table 7: Performance of K1 trade in 1997 with 5% transaction costs. Cash flows are measured in EUROs.

# Performance K1 Trade (1998) (2)

| Month | CashFlow in $t = 0$ | CashFlow in $t = T$ | NetCashFlow |
|---|---|---|---|
| Jan 98 | 722.66 | -1903.26 | -1180.60 |
| Feb 98 | 1049.10 | -2757.24 | -1708.15 |
| Mar 98 | 1620.18 | -1582.66 | 37.51 |
| Apr 98 | 1606.69 | -1275.72 | 330.98 |
| May 98 | 1961.85 | -337.60 | 1624.25 |
| Jun 98 | 2448.11 | -3239.67 | -791.56 |
| Jul 98 | 0.00 | 0.00 | 0.00 |
| Aug 98 | 1690.20 | -1065.71 | 624.50 |
| Sep 98 | 4354.56 | -366.96 | 3987.61 |
| Oct 98 | 2787.59 | -1395.12 | 1392.47 |
| Nov 98 | 880.98 | -663.70 | 247.29 |
| Dec 98 | 3514.44 | -543.75 | 2970.69 |

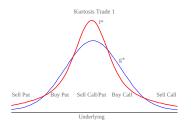
Table 8: Performance of K1 trade in 1998 with 5% transaction costs. CF's in EURs.

# Performance K1 Trade (1999) (3)

| Month | CashFlow in $t = 0$ | CashFlow in $t = T$ | NetCashFlow |
|---|---|---|---|
| Jan 99 | 1839.99 | 0.00 | 1839.99 |
| Feb 99 | 1727.85 | -761.31 | 966.53 |
| Mar 99 | 1987.80 | -685.95 | 1301.85 |
| Apr 99 | 1125.79 | -598.88 | 535.91 |
| May 99 | 1754.17 | -58.28 | 1695.89 |
| Jun 99 | 1448.94 | -484.24 | 964.70 |
| Jul 99 | 1631.62 | -2044.62 | -413.00 |
| Aug 99 | 1351.15 | -1667.91 | -316.76 |
| Sep 99 | 1636.54 | -2965.60 | -1329.05 |
| Sum(NCF) | Mean(NCF) | Var(NCF) | Mean/$\sqrt{\text{Var}}$ |
| 10915.77 | 363.86 | 2231551.71 | 0.2436 |

Table 9: Performance of K1 trade in 1999 with 5% transaction costs and simple statistics aggregating 1997, 1998, 1999. CF's in EURs.

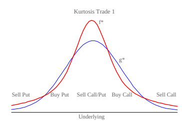
XFGSpdTradeKurt.xpl



# Performance K1–Trade (4)

- All portfolios generate a negative cash flow at expiration
- Cash flow at initiation in $t = 0$ is always positive
- Given the positive total NetCashFlow, the K1–Trade earns its profit in $t = 0$
- Payoff of portfolios set up in (a) 4/1997, 5/1997 and 11/1997 – 6/1998 relatively more negative than for portfolios (b) of 6/1997 – 10/1997 and 11/1998 – 6/1999
- DAX is moving up or down in case (a) and stays within a bounded range of quotes in case (b)





# A Word of Caution

- Only S1 and K1 Trades, no alternating use of type 1 and 2 trades
- Highly positive NetCashFlows
- Directional risk, no risk–adjusted performance measure
- Short period of time (1997 – 1999)

# A Word of Caution (2)

- Extension to historical density estimation: In Monte Carlo simulation draw random numbers from the distribution of the residuals resulting from the estimation of $\sigma$ (Härdle and Yatchew (2001)).
- Fine tuning: use distance measure to give signals when deviation in skewness or kurtosis is significant

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