Trend Detection in Car Manufacturing Processes

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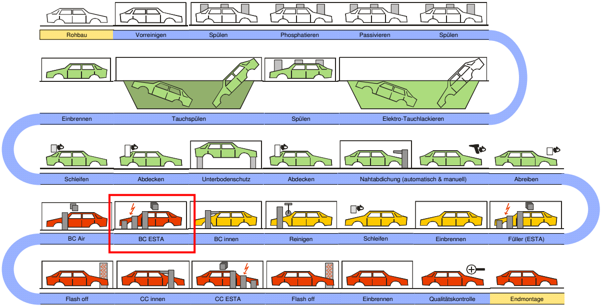
Car Production Today

- outsourcing, different suppliers involved
- painting is important cost factor
- cost(painting)>cost(body)



Problem

- early detection of trend needed
- construction of statistical tests
- comparison of test procedures

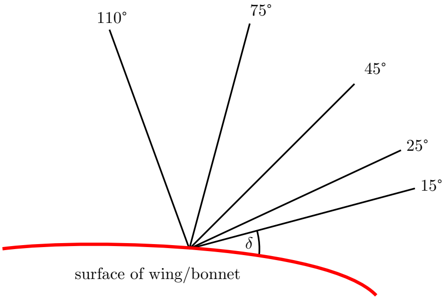
Outline

1. Motivation ✓
2. Dataset
3. Tests
4. Simulation

Dataset

- 7100 painted cars, 20 different colors
- luminance, a- and b- color axis from five angles for bonnet and wing
- explanatory variables: color lot, temperature, measuring person
- 42% silver (luminance most important component)
- customers most sensitive to changes of metallic colors

Used Angles



δ	L*	a*	b*
15°	98.94	1.58	4.43
25°	81.35	1.17	4.20
45°	50.98	0.73	2.58
75°	32.30	0.48	1.29
110°	26.11	0.21	0.87

Figure 1: Used angles and typical Lab-values for 'silver'

The Lab Colorspace

- color model proposed by International Commission on Illumination
- each color defined by a tuple of luminance and the positions between red-green (a-axis) and blue-green (b-axis)

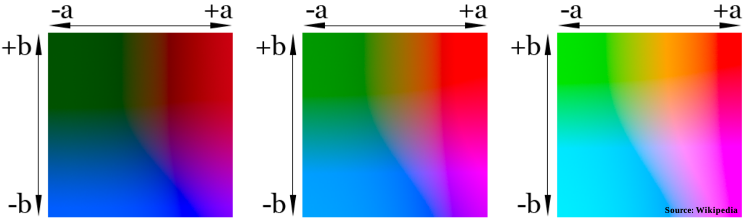


Figure 2: Color tiles for increasing values of luminance, source: Wikipedia

Data Example

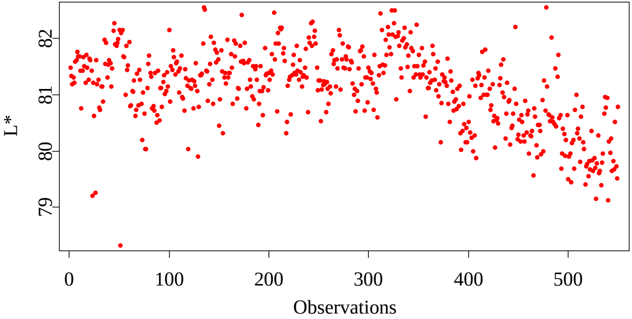


Figure 3: Sequence of L*-values for 550 'silver' observations

Proposed Test Procedures

local t-Test, 3-Slope Test
H_0: slope equal to zero

Change Point Estimation
H_0: $E(x_i) = \text{const}$

Mann-Kendall
sign-test, H_0: no trend

Local t-test

- $(\widehat{\alpha}_l, \widehat{\beta}_l) = \arg\min_{(\alpha_l,\beta_l)} \sum_{i=1}^{n} (y_i - \alpha_l - \beta_l x_i)^2 \mathrm{I}(x_i \in L_l)$
  where $L_l = [x_0 + (k-1)\cdot\Delta, x_0 + (k-1)\cdot\Delta + l], l = 1 \ldots n/l$
  and Δ indicates the step size

- no (local) linear trend, H_0: $\beta_l = 0$

- t-test statistics $t_l = \frac{\widehat{\beta}_l}{\mathrm{SE}(\widehat{\beta}_l)}$

Local t-Test

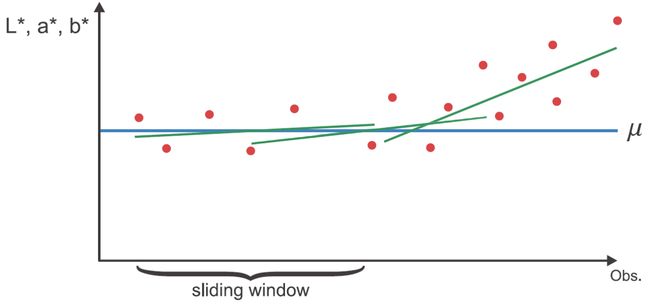


Figure 4: Schematic chart of local t-test

Change Point Estimation

$$H_0: \ \mu_1 = \mu_2 \quad H_1 : \mu_1 \neq \mu_2$$

The different μ-values are computed as $\widehat{\mu_j} = n^{-1} \sum_{i \in I_j} y_i$, the resulting test statistics is asymptotically χ^2-distributed with one degree of freedom.

$$\frac{n_1 \cdot n_2}{n_1 + n_2} \frac{(\widehat{\mu_1} - \widehat{\mu_2})^2}{\tilde{\sigma}^2} \xrightarrow{\mathcal{L}} \chi^2(1)$$

$\tilde{\sigma}^2$ calculated by non-parametric smoothing.

Non-Parametric Smoothing

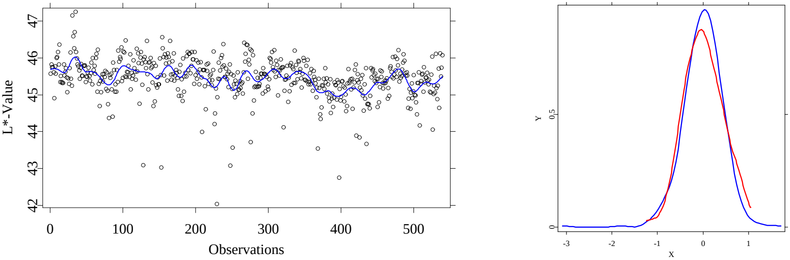


Figure 5: data with smoothed curve and distributions of residuals (blue) and fitted normal distribution pdf (red)

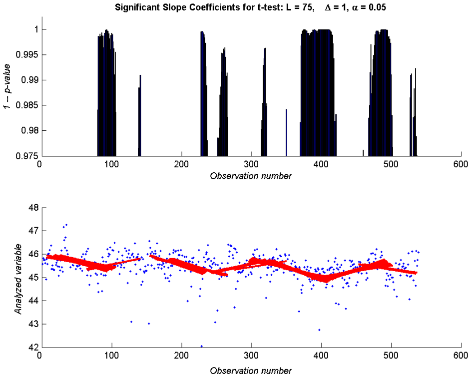
Mann-Kendall Test

$$S = \sum_{i=2}^{n} \sum_{j=1}^{i-1} \operatorname{sign}(X_i - X_j)$$

Under the null hypothesis of no trend the test statistics follows approximatively a standard normal distribution.

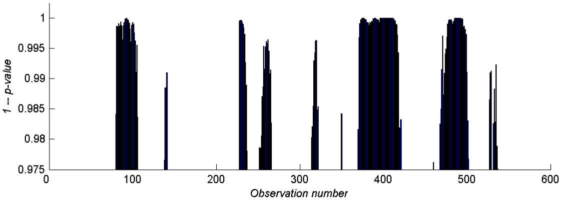
$$\frac{S}{\sqrt{\frac{n(n-1)(2n+5)}{18}}} \xrightarrow{\mathcal{L}} N(0,1)$$

Example


Significant Slope Coefficients for t-test: L = 75, Δ = 1, α = 0.05
1 -- p-value
Observation number
Analyzed variable
Observation number

Integration Measure

Idea: Look at clusters of bars

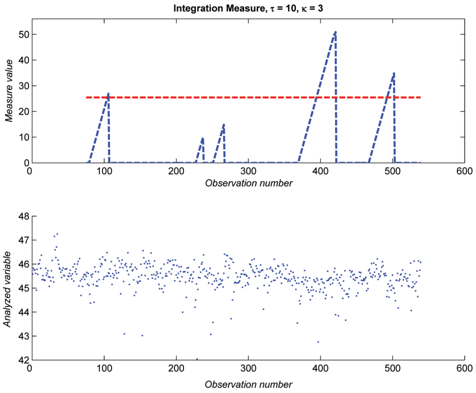


Control false alarms and gaps

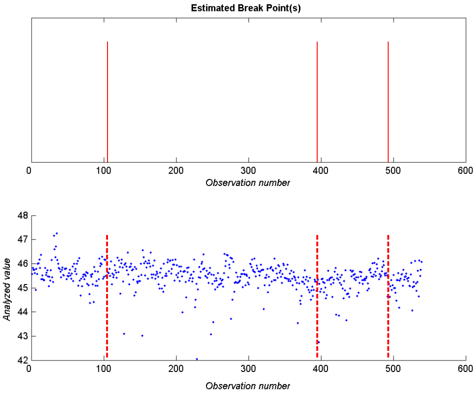
τ number of subsequent bars necessary to start one integral

κ number of allowed gaps between two groups of bars

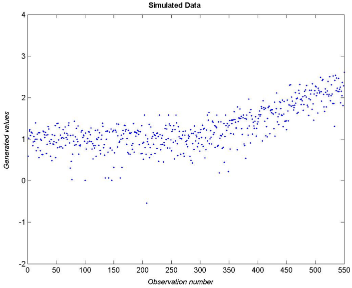
Example


Integration Measure, τ = 10, κ = 3
Measure value
0
10
20
30
40
50
0
100
200
300
400
500
600
Observation number
Analyzed variable
42
43
44
45
46
47
48
0
100
200
300
400
500
600
Observation number

Example


Estimated Break Point(s)
0
100
200
300
400
500
600
Observation number
48
47
46
45
44
43
42
Analyzed value
0
100
200
300
400
500
600
Observation number

Selection of l, τ and κ



- regression line with trend at 300 observations
- quasi-residuals calculated by non-parametric regression
- 50 simulation runs for each parameter setting

Results

	Mean delay (5 tests)						
	l	25		50		75	
	κ	3	5	3	5	3	5
τ							
3		211	211	213	213	213	213
5		131	131	132	132	132	132
10		71	71	72	72	72	72

Table 1: Mean delays of all tests for $\gamma = 2.86^{\circ}$and $\Delta = 5$

Conclusion

- choice of l, κ and τ
- comparison of tests:
  local t-test outperforms Mann-Kendall
  change point estimation performs well
- threshold value to be adjusted by further analysis

Thank You for listening!