Risk Patterns and Correlated Brain Activities

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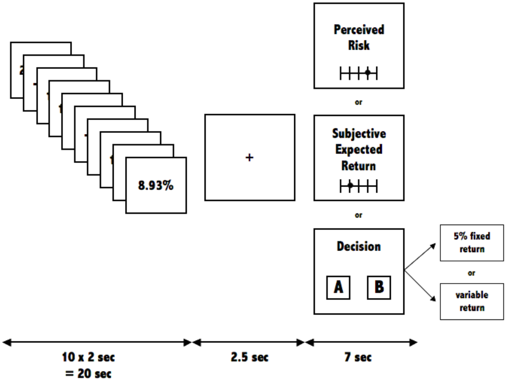
Motivation

- Which part of our brain is activated during *risky decisions* ?
- Can statistical analysis help to detect this area?
- Can we provide an *integrated* analysis of the brain?

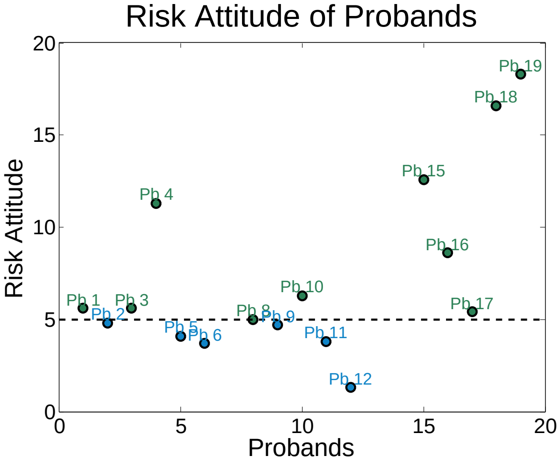

Experiment participants

- 22 volunteers (age 18-35 years), 11 females, 11 males
- no history of neurological or psychiatric diseases
- flat payment (10 EUR) $\pm$ outcome resulting from the participant's decision and modeling problems)

Risk Perception and Investment Decision



Returns Pause Decision


Risk Attitude of Probands
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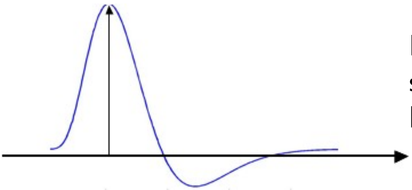
fMRI

functional Magnetic Resonance Imaging

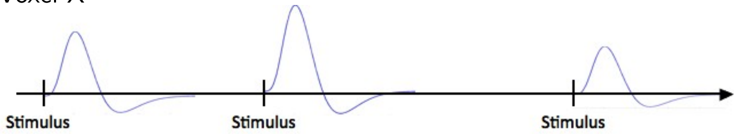
measures the oxygen level in the blood every 2-3 sec

fMRI



Is there a significant reaction to specific stimuli in the hemodynamic response?

Voxel X



Data Set

Series of 3-dim images

- each scan transformed on the resolution $2 \times 2 \times 2mm^3$
- 91 slices
- observed every 2.5 seconds
- data set: series of $T = 1360$ images with $91 \times 109 \times 91$ voxels

High-dimensional, high frequency & large data set.

fMRI methods

- existing methods to analyze these data: voxel-wise GLM
    - strong a priori hypothesis necessary
- new statistical method: DSFM
    - dimension reduction keeping the data structure
    - exploratory analysis

- Which part of our brain is activated during *risky decisions* ?
- Can statistical analysis help to detect this area?
- Is there a significant reaction to specific stimuli in the hemodynamic response?
- Can we classify the risk attitudes of probands *without* using probands' answers?

Outline

1. Motivation ✓
2. Statistical Model
3. Results vs. Proband's Behaviour
4. Conclusion
5. Future Perspectives

Panel Dynamic Semiparametric Factor Model (Panel DSFM)

$X_{t,j} = (X_{t,1}, \ldots, X_{t,J})^\top$	observable covariates defined on $\mathbb{R}^d$
$Y_{t,j} = (Y_{t,1}, \ldots, Y_{t,J})^\top$	observable random vector on $\mathbb{R}^d$
$Z_{t,j} = (Z_{t,1}, \ldots, Z_{t,L})^\top$	unobservable L-dimensional process
$(\overline{m}_0, \ldots, \overline{m}_L)$	unknown real-valued functions defined on a subset of $\mathbb{R}^d$
$\varepsilon_{t,j} \sim (0, \sigma^2_{t,j})$	errors with $\sigma^2_{t,j} < \infty$

Panel DSFM

- assume *fixed effects* α_i for individual i with $\sum_{i=1}^{n} \alpha_i = 0$
- the "average brain":

$$\overline{Y}_{t,j} = \overline{m}_0(X_{t,j}) + \sum_{l=1}^{L} \overline{Z}_{t,l}\overline{m}_l(X_{t,j}) + \varepsilon_{t,j} \,, \quad 1 \le j \le J \qquad \text{(DSFM)}$$

- individual i:

$$Y^i_{t,j} = \overline{m}_0(X_{t,j}) + \sum_{l=1}^{L} Z^i_{t,l}\overline{m}_l(X_{t,j}) + \varepsilon^i_{t,j} \qquad \text{(LS)}$$

with the general basis functions $\overline{m}_l$

Theorem

Under regularity assumptions, for $h \geq 0$

$$\frac{1}{T}\sum_{t=max[1,-h+1]}^{min[T,T-h]} \widetilde{Z}^i_{c,t}\left(\widetilde{Z}^i_{c,t+h}-\widetilde{Z}^i_{c,t}\right)^\top - \frac{1}{T}\sum_{t=max[1,-h+1]}^{min[T,T-h]} Z^i_{c,t}\left(Z^i_{c,t+h}-Z^i_{c,t}\right)^\top = \mathcal{O}_P(T^{-1/2})$$

with $Z^i_{c,t}$ & $\widetilde{Z}^i_{c,t}$ being the (rescaled) real low-dimensional time series and their estimates respectively for individual i.

Fitting fMRI Data

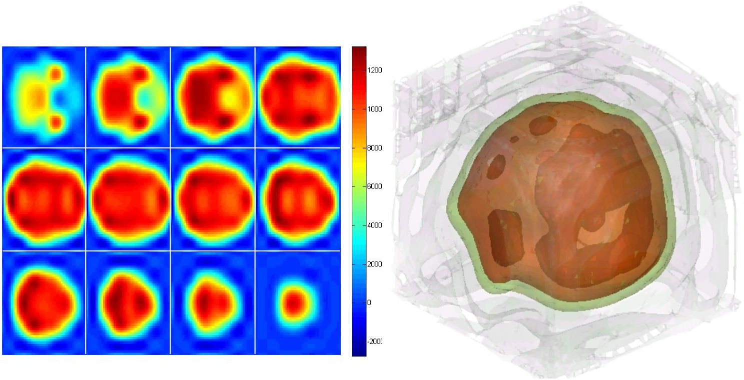
- concentrate on parts with brain scan
- reduction of the original data by taking every second slice in each direction and the first part of experiment only
- voxel's index (i_1, i_2, i_3) as covariate X_j
- BOLD signal as $Y_{t,j}$
- summary: $J = 36 \times 46 \times 46$ and $T = 722$

Estimation of DSFM

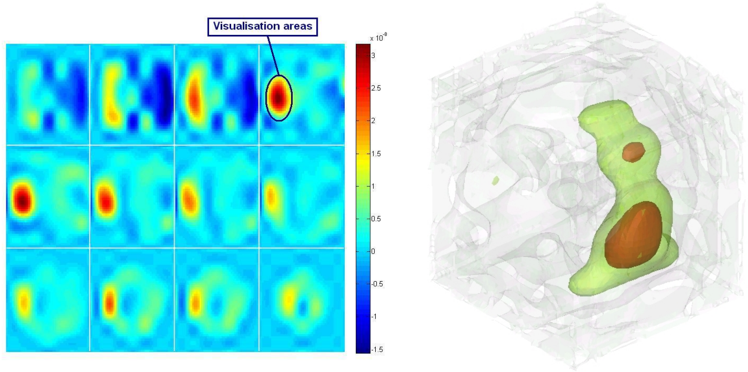
- choose $K = 7 \times 8 \times 8 = 448$ parabolic tensor B-splines to estimate $\widehat{m}$

$$1 - RV(L) = \frac{\sum_t^T \sum_j^J \{Y_{t,j} - \widehat{m}_0(X_{t,j}) - \sum_l^L \widehat{Z}_{t,l}\widehat{m}_l(X_{t,j})\}^2}{\sum_t^T \sum_j^J (Y_{t,j} - \overline{Y})^2}$$

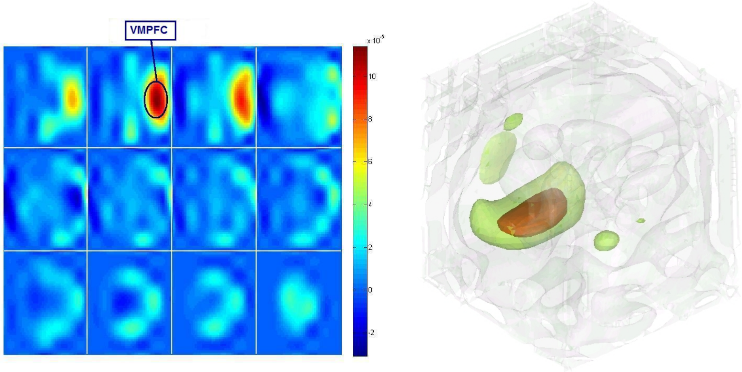
No. of factors	$L = 2$	$L = 3$	$L = 4$	$L = 5$
$1 - \overline{RV}(L)$ in %	88.85	88.88	88.91	88.94



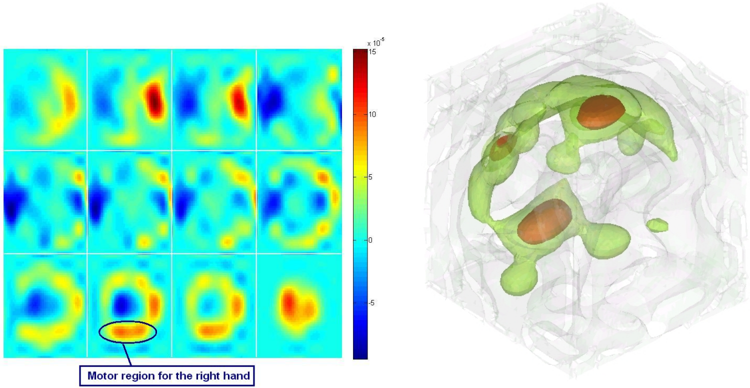
Estimated factor loading $\hat{m}_0$ with $L = 4$.



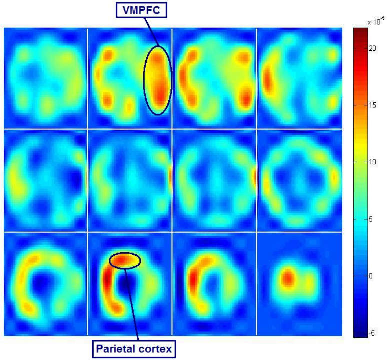
Estimated factor loading $\hat{m}_1$ with $L = 4$.

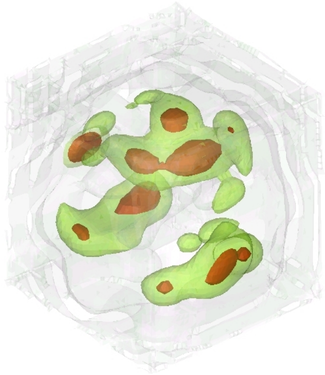


Estimated factor loading $\hat{m}_2$ with $L = 4$.
(VMPFC = Ventromedial prefrontal cortex)



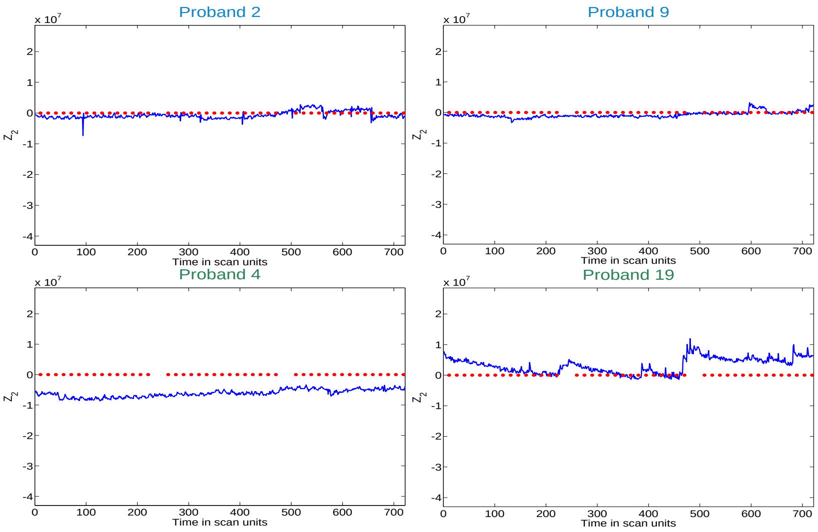
Estimated factor loading $\hat{m}_3$ with $L = 4$.



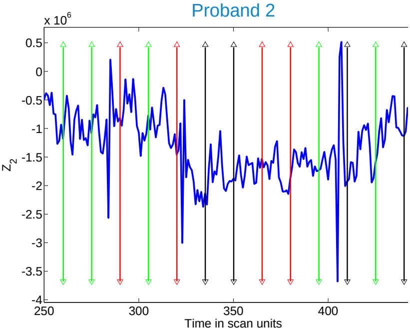


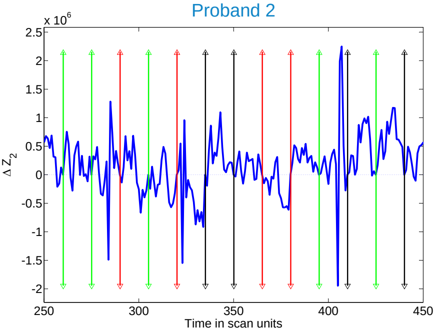
Estimated factor loading $\hat{m}_4$ with $L = 4$.

Factor $\widehat{Z}_2$



Reaction to stimuli

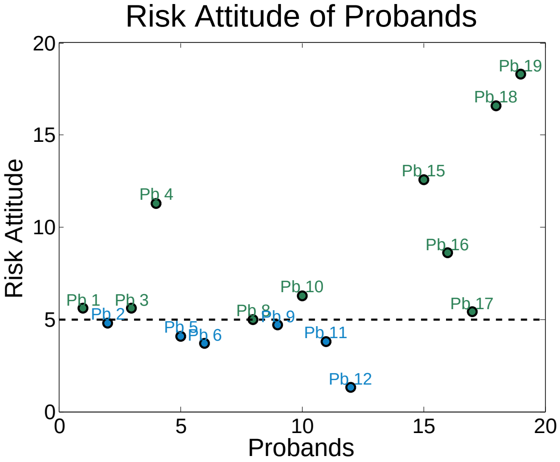




Lines correspond to the time points of judgement tasks: decision, return, risk.

Risk attitude

- modeled by the softmax function from individuals' decisions
- estimated by the Maximum Likelihood Method
- details in: Mohr, Biele, Krugel, Li & Heekeren, *Neuroimage*.(2010)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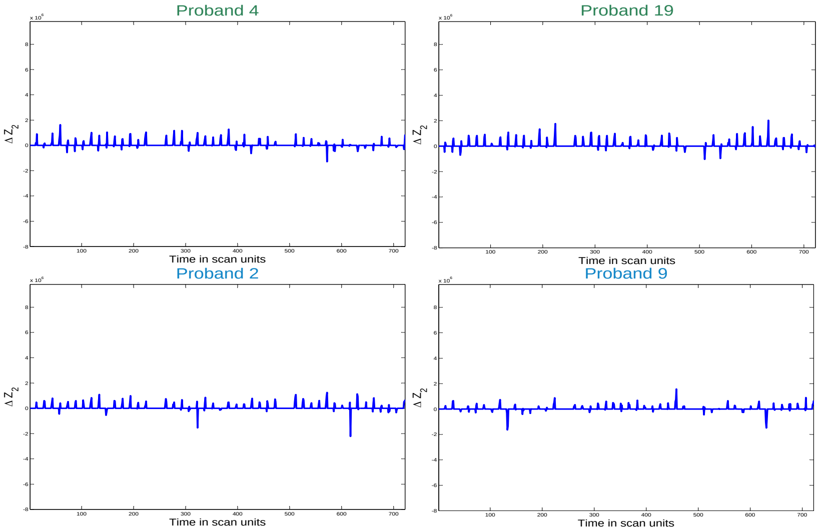
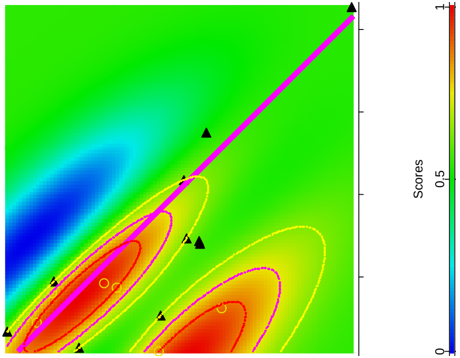
Reaction to stimuli in factor $\widehat{Z}_2$



SVM Classification Analysis

- observation: weakly (strongly) risk-averse individuals have smaller (larger) volatilities of Z_t^i inside each trial
- SVM based on:
  X_1: mean (median/upper quartile) of the 15 volatilities (of Z_t^i in each separated trial w.r.t. question type 1)
  X_2: . . . w.r.t. question type 2
  X_3: . . . w.r.t. question type 3

SVM Classification (mean of volatilities)



Classification Rates

	rate	r	C
mean	0.7500	0.250 – 0.350	20 – 90
median	0.6875	0.355 – 0.455	10 – 90
upper quartile	0.6875	0.400 – 0.550	20 – 90

The rates hold over a wide range of parameters!

Classification Rates

Mean		Estimated	
		Strongly	Weakly
Data	Strongly	0.85	0.15
	Weakly	0.42	0.58

Median		Estimated	
		Strongly	Weakly
Data	Strongly	0.90	0.11
	Weakly	0.67	0.33

Conclusion

- basis functions identify activated areas, neurological reasonable
- volatility of estimated factors show differences for individuals with different risk attitudes (2 vs. 19)
- estimated factors show similarities for probands with close risk attitudes (2 and 9)
- SVM classification analysis of measurements in Z_2 after stimulus can distinguish weakly and strongly risk-averse individuals

Future Perspectives

- Comparison with the PCA/ICA (PARAFAC) approach
- Analysis of the second part of the experiment (under assumption of independency) to "generate" larger number of subjects
- Improvement of the classification criterion
- Penalized DSFM with seasonal effects

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