Statistik II: Exercise session 6
06.01.2016, 13.01.2016

Review

- Slides: testing of Hypotheses (82-107/162)

χ^2-test X discrete, with unknown dist. $F(x)$
Observed frequencies h_i for values x_i
Hypot. distribution assumes $p_i = P(X = x_i|F_0(x))$

$$V = \sum_i \frac{(h_i - n.p_i)^2}{np_i} \sim \chi^2_{I-k-1}$$

k - number of unknown parameters, I - number of values

Exercises

Exercise 8-1 - Special refrigerators A company produces special refrigerators to conserve certain goods. The wished temperature for that type of refrigerators is -25 °C. When goods are insufficiently cooled, they go bad very easily and since the client base of the company is not big, defective products would cause the worst case - the ruin of the company. That is why the cooling performance of 100 randomly chosen produced refrigerators shall be tested on a significance level of 2.275 % in order to decide whether the production can be carried on or if constructional changes on the refrigerators need to be done. Experience shows that the cooling temperature is normally distributed with standard deviation of 2 °C.
a) What are the hypotheses for this test? Justify.
b) Formulate the underlying sample function formally and verbally and give its distribution under H_0.
c) What is the testing function and what is its distribution under H_0?
d) Determine the region of rejection.
e) Determine the value of power function if the true mean of cooling temperature is:
(i) -24.8 °C;
(ii) -25.8 °C;
(iii) -29.0 °C.
f) Sketch the power function.
g) The random sampling yielded a mean of cooling temperature of -26 °C and standard deviation of 1.5 °C.
(i) What is the test decision?
(ii) Interpret the test result in an exact way statistically as well as from the context point of view.
h) Random sampling yielded a mean of cooling temperature of -25.3 °C.
(i) What is the test decision?
(ii) Interpret the test result in an exact way statistically as well as from the context point of view.
(iii) Which mistake can be made by taking this test decision?
(iv) What is the probability that this mistake has really been made?
(v) How big is the probability to make this mistake when using this test procedure and the real is -29 °?
i) Why is it sufficient for one-sided test to consider under the null hypothesis only the case $\mu = \mu_0$?

Exercise 8-7 - Heavy-weight boxer Two boxers Jim Knockout and Bill Uppercut are both the world's best boxer. A plaster company wants to offer the world's best boer an advertisement contract worth 1 Million Euro. The Chef of this company believes that Jim Knockout is the better boxer. To proof this hypothesis statistically, the chef organizes 11 fights between both boxers. In each fight there will be winner. (Draws are excluded.)

a) Formulate the hypothesis.
b) Define the test function.
c) How is test function distributed?
d) Determine accepting and rejecting regions for this test.
e) How do you decide this test, if Jim Knockout looses 3 fights?
f) Could you have made an error in your decision? If yes, which one?
g) The company plans to publish the test result in a short and comprehensive form. Formulate this press release.

Exercise 8-11 - Coins

Three distinct coins are thrown 240 times and each time was the number of ”head” is observed. The results are summarized as follows:
0x head: 24
1x head: 108
2x head: 85
3x head: 23

Test the hypothesis that three coins are ideal. (α = 0,05)! (In an ideal coin heads and tails with are thrown with equal probability.)