Bitcoin Pricing Kernels

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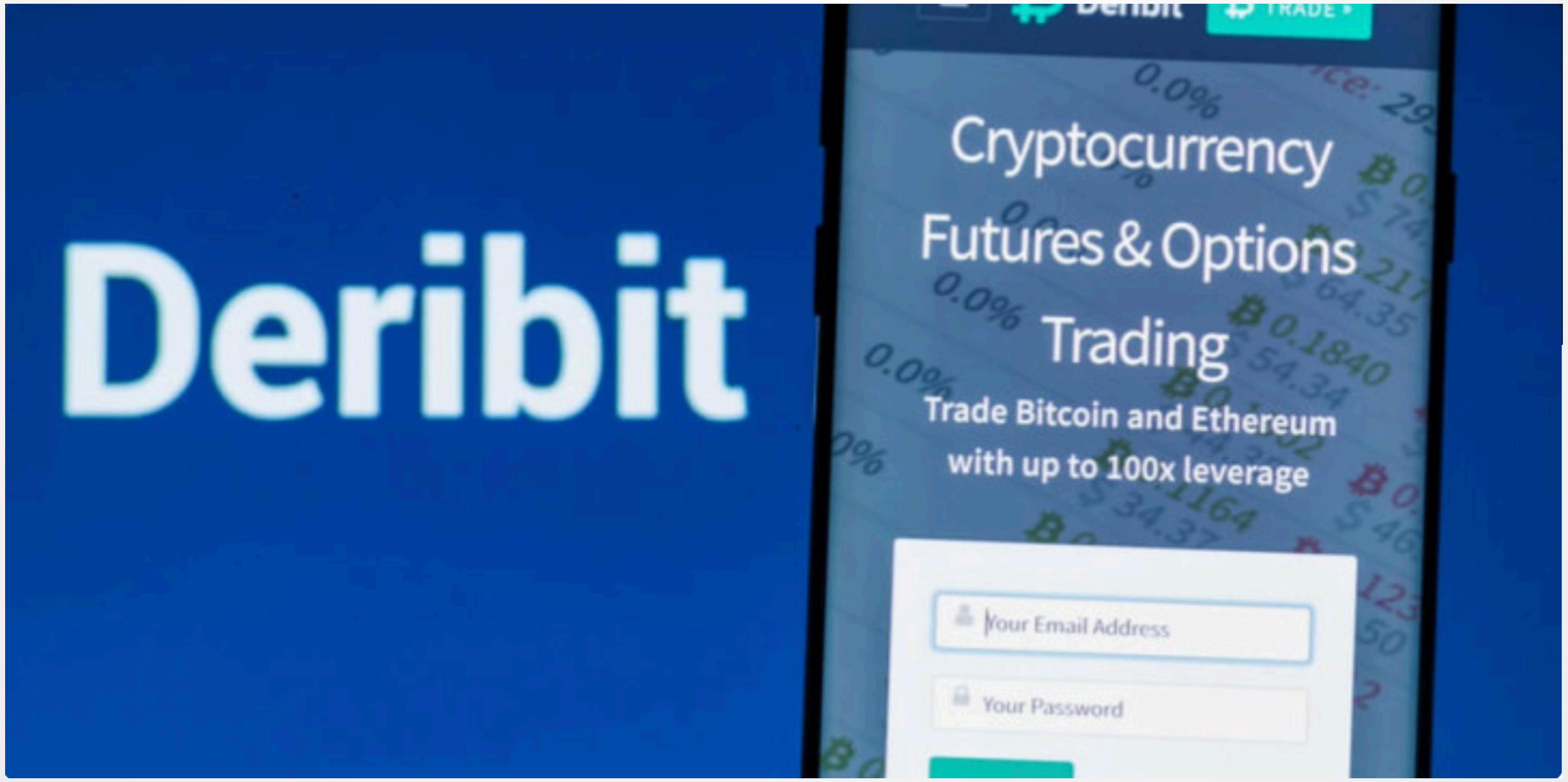
Deribit™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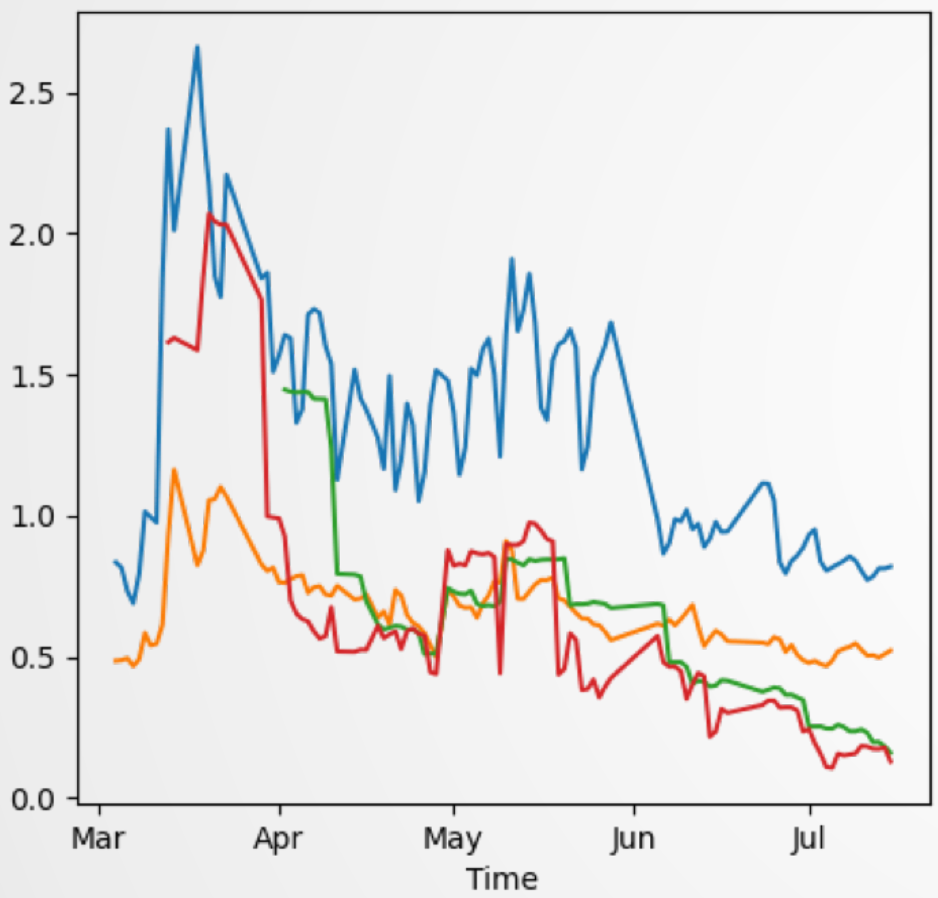
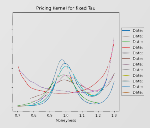
Volatile Times…

Is the market accurately pricing volatility?



Bid IV and Ask IV across all instruments and realized volatility on daily base and 21 day rolling window. Deribit’s Underlying, a Synthetic BTC USD Index.

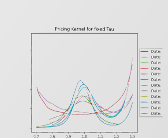


The Corona Shock

- Deribit Insurance Fund grows through liquidation orders
- Interesting data has been captured for the BRC
- Corona Shock caused spike in Option Writer defaults

Date	Bankruptcies	Insurance Balance
21 Mar 2020	28	124.18843
20 Mar 2020	90	122.13254
19 Mar 2020	53	117.49559
18 Mar 2020	44	115.75211
17 Mar 2020	101	113.70538
16 Mar 2020	38	111.83231
15 Mar 2020	9	109.75960
14 Mar 2020	40	112.11395
13 Mar 2020	391	197.60795
12 Mar 2020	1273	211.77707

Deribit Insurance Fund denoted in Bitcoin



Outline

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3. Pricing Kernels
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5. Physical Densities
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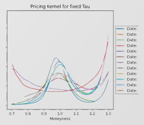
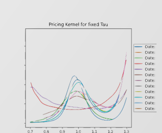


Deribit

- www.deribit.com
- Largest Bitcoin Derivatives Exchange
- Controls around 80% of the BTC Derivatives Volume

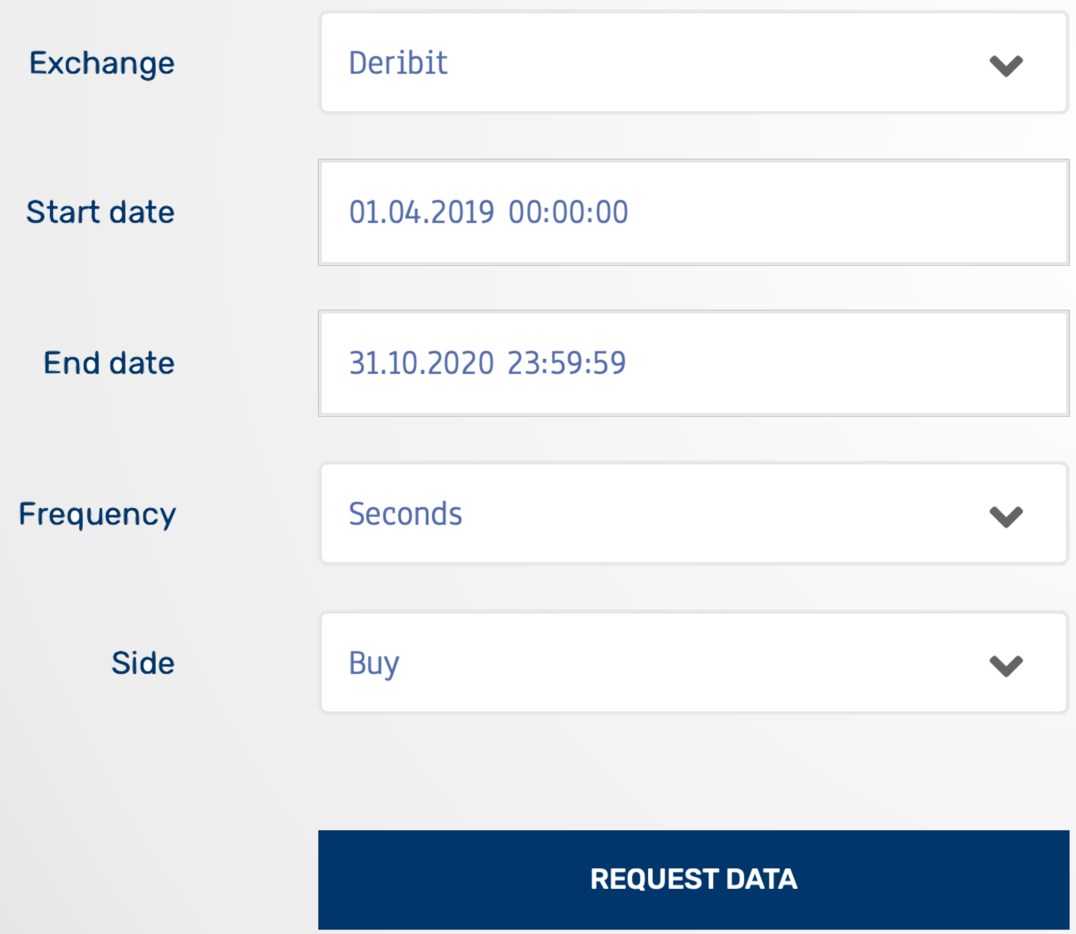
Calls — Underlying: SYN.BTC-14DEC20($19192.28) — **14 Dec 2020** — Expires In 13 hours and 3 minutes — Puts

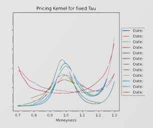
Last	Size	IV	Bid	Ask	IV	Size	Vol	Δ|Delta	Strike	Last	Size	IV	Bid	Ask	IV	Size	Vol	Δ|Delta
-	-	-	-	-	-	-	-	1.00	**16750**	0.0005	-	-	-	0.0005 $9.59	173.9%	17.5	5.0	0.00
-	-	-	-	-	-	-	-	1.00	**17000**	0.0005	-	-	-	0.0005 $9.59	157.5%	0.6	47.0	0.00
-	-	-	-	-	-	-	-	1.00	**17250**	0.0005	-	-	-	0.0005 $9.59	141.2%	1.5	33.0	0.00
-	-	-	-	-	-	-	-	1.00	**17500**	0.0005	-	-	-	0.0005 $9.59	124.9%	1.0	77.6	0.00
-	-	-	-	-	-	-	-	1.00	**17750**	0.0005	-	-	-	0.0005 $9.59	108.6%	0.5	47.8	0.00
-	-	-	-	-	-	-	-	0.99	**18000**	0.0005	-	-	-	0.0010 $19.18	105.4%	0.6	55.5	-0.01
0.0185	5.0	0.0%	0.0290 $556.58	0.0540 $1036.38	139.2%	5.0	-	0.97	**18250**	0.0005	-	-	-	0.0010 $19.19	87.3%	1.0	136.4	-0.03
0.0110	5.0	0.0%	0.0325 $623.54	0.0440 $844.18	142.7%	5.0	-	0.93	**18500**	0.0010	2.9	58.5%	0.0005 $9.59	0.0010 $19.19	68.5%	0.4	354.4	-0.07
0.0275	2.0	0.0%	0.0220 $422.25	0.0280 $537.41	88.2%	2.0	19.0	0.85	**18750**	0.0020	0.9	49.0%	0.0010 $19.19	0.0025 $47.97	66.0%	70.5	329.7	-0.15
0.0200	2.0	40.9%	0.0125 $239.90	0.0155 $297.48	63.2%	2.0	62.9	0.69	**19000**	0.0045	2.0	44.4%	0.0030 $57.56	0.0045 $86.35	55.6%	29.2	67.2	-0.31
0.0080	2.0	48.2%	0.0060 $115.15	0.0075 $143.94	58.0%	5.0	212.5	0.45	**19250**	0.0085	19.7	37.9%	0.0075 $143.94	0.0105 $201.51	57.7%	24.0	35.3	-0.55
0.0025	27.4	52.7%	0.0025 $47.97	0.0035 $67.16	61.2%	5.4	99.3	0.24	**19500**	0.0190	11.0	0.0%	0.0150 $287.87	0.0225 $431.81	83.1%	11.0	1.4	-0.76
0.0020	41.6	57.1%	0.0010 $19.19	0.0020 $38.38	70.0%	6.4	29.7	0.12	**19750**	-	0.1	0.0%	0.0215 $412.49	-	-	-	-	-0.88
0.0025	14.6	64.5%	0.0005 $9.59	0.0015 $28.78	83.1%	40.2	59.9	0.08	**20000**	-	-	-	-	-	-	-	-	-0.92
0.0020	-	-	-	0.0015 $28.78	100.4%	70.5	13.6	0.03	**20250**	-	-	-	-	-	-	-	-	-0.97
0.0015	-	-	-	0.0010 $19.18	107.7%	40.5	24.1	0.01	**20500**	-	-	-	-	-	-	-	-	-0.99



Deribit Data

- Every executed trade and order book change is saved on the BRC server
- Access with GUI via hu-berlin.de/brc
- Receive transactions via email





Pricing Kernels

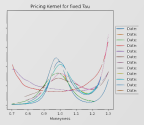
The value of a contingent claim at time t with maturity T for underlying S is

$$C_t = e^{-r\tau}\,\mathrm{E}_Q[\psi(S_T)] = e^{-r\tau}\int_{-\infty}^{\infty}\psi(S_T)q(S_T)dS_T$$

With option payoff $\Psi(S_T) = \max\{S_T - K, 0\}$, riskless rate r.

Risk-neutral pricing according to arbitrage theory

$$C_t = e^{-r\tau}\int_{-\infty}^{\infty}\psi(S_T)q(S_T)dS_T = e^{-r\tau}\int_{-\infty}^{\infty}\psi(S_T)p(S_T)\underbrace{\frac{q(S_T)}{p(S_T)}}_{Z(S_T)}dS_T$$

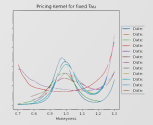


State Price Densities

- Derive the whole risk-neutral probability distribution of the Bitcoin price at maturity
- Use Rookley's method to derive SPDs
- Arbitrage-free option pricing for various instruments

$$C_t = e^{-r\tau}\,\mathrm{E}^Q[\psi(S_T)] = e^{-r_t,\tau}\int_{-\infty}^{\infty}\psi(S_T)f_t^Q(S_T)dS_T$$

$$\hat{f}(S_T) = e^{r\tau}\frac{\delta^2 C_t}{\delta K^2}\Bigg|_{K=S_t}$$



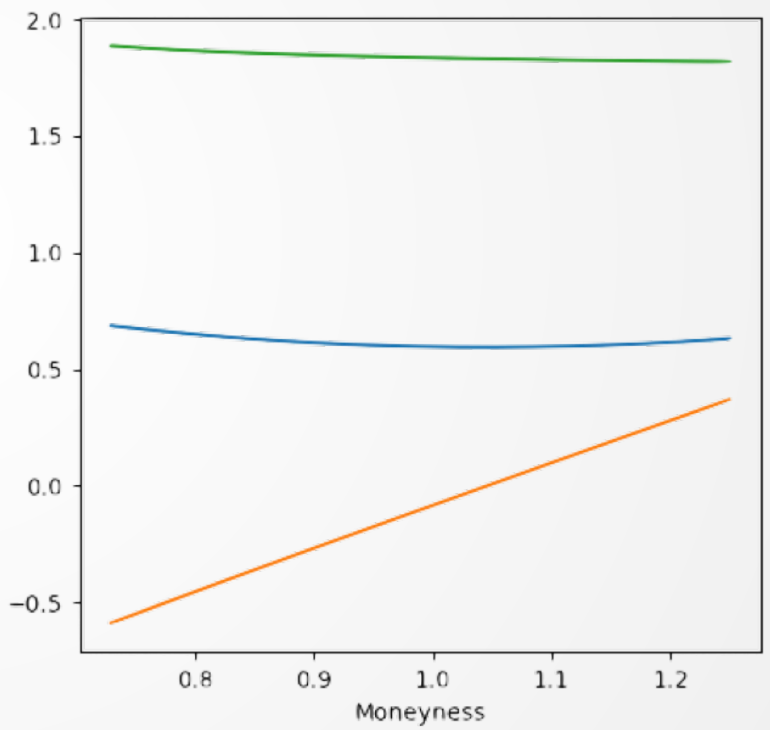
Rookley's Method

- Estimate IV surface using time to maturity and moneyness
- Use IV estimates to calculate call prices
- Second derivative of call prices enables identification of state price densities

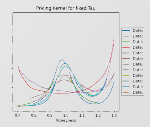
IV Decomposition

$$\sigma^{IV} = g(m,\tau) + \sigma(m,\tau)\varepsilon$$

$$\begin{aligned} g(m,\tau) = g(m_0,\tau_0) &+ \frac{\partial g}{\partial M}(m-m_0) + \frac{1}{2}\frac{\partial^2 g}{\partial M^2}(m-m_0)^2 \\ &+ \frac{\partial g}{\partial \tau}(\tau-\tau_0) + \frac{1}{2}\frac{\partial^2 g}{\partial \tau^2}(\tau-\tau_0)^2 \\ &+ \frac{\partial^2 g}{\partial M \partial \tau}(m-m_0)(\tau-\tau_0) \end{aligned}$$



Volatility Smile,
First and Second Derivative



Physical Density

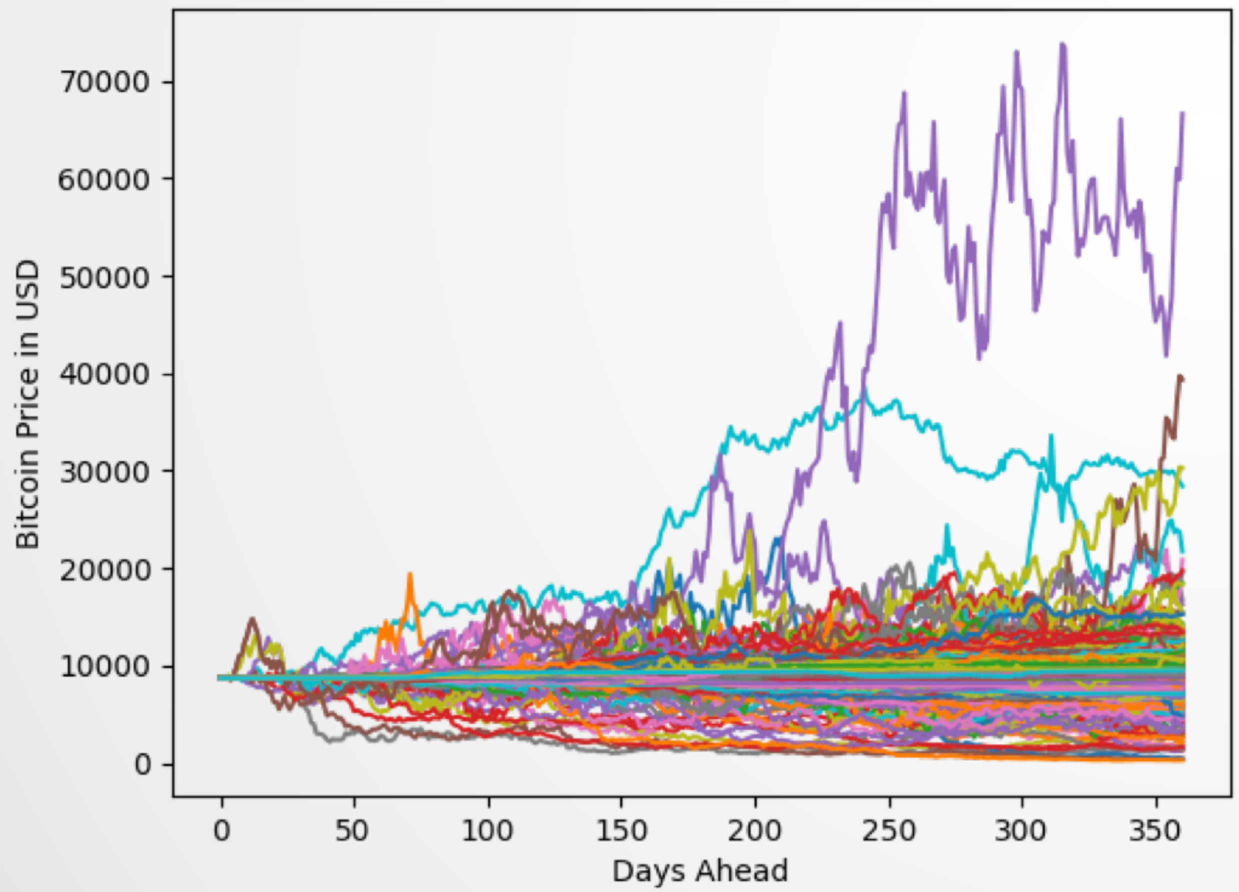
- Historical Density
- Dynamics of underlying assumed to follow SVCJ
- Alternatives: SV, SVJ, GARCH

$$\frac{dS_t}{S_t} = \mu dt \sqrt{V_t} W_t^{(s)} + Z_t^{(y)} dN_t$$

$$dV_t = \kappa(\theta - V_t)dt + \sqrt{V_t} W_t^{(v)} + Z^{(v)} dN_t$$

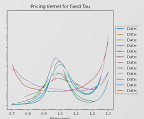
$$Cov(W^{(t)}, W^{(s)}) = \rho dt$$

$$P(dN_t = 1) = \lambda dt$$



SVCJ Simulation from 2020-03-01

		Mean	Standard Deviation
$\alpha = \kappa\tau$	α	0.0170	0.0040
$\beta = 1 - \kappa$	β	-0.0570	0.0120
Jump Arrival Rate	λ	0.0150	0.0060
Log Return w/o Jumps	μ	0.0350	0.0140
	μ_v	1.4790	0.3860
	μ_y	-0.0020	0.0430
Leverage Effect	ρ	0.0010	0.0340
	ρ_j	-0.0000	0.0200
Volatility of Volatility	σ_v	0.0230	0.0050
	σ_y	0.4100	0.0540



Shape Invariant Models

Goal: Collapse different pricing kernels into a single population curve

$$Y_{tj} = K_t(u_j) + \varepsilon_t$$

$$K_t(u_j) = \theta_{t1} K_0\left(\frac{u - \theta_{t3}}{\theta_{t2}}\right) + \theta_{t4}$$

Initialization:

1) Estimate individual regression functions using Nadaraya-Watson-Estimator

2) Set starting values for Thetas for each point in time

3) Estimate average Kernel / Reference Kernel

$$K_0^{(0)}(u) = T^{-1} \sum_{t=1}^{T} \hat{K}_t(\theta_{t2}^{(0)} u + \theta_{t3}^{(0)})$$

Loop:

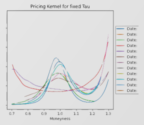
$$\arg\min_{\theta_{tj}} \int_{\mathbb{R}} \{\hat{K}_t(\theta_{t2}^{(0)} u + \theta_{t3}^{(0)}) - \theta_{t1} K_0^{r-1}(u) - \theta_{t4}\}^2 w(u) du$$

Normalize Parameters

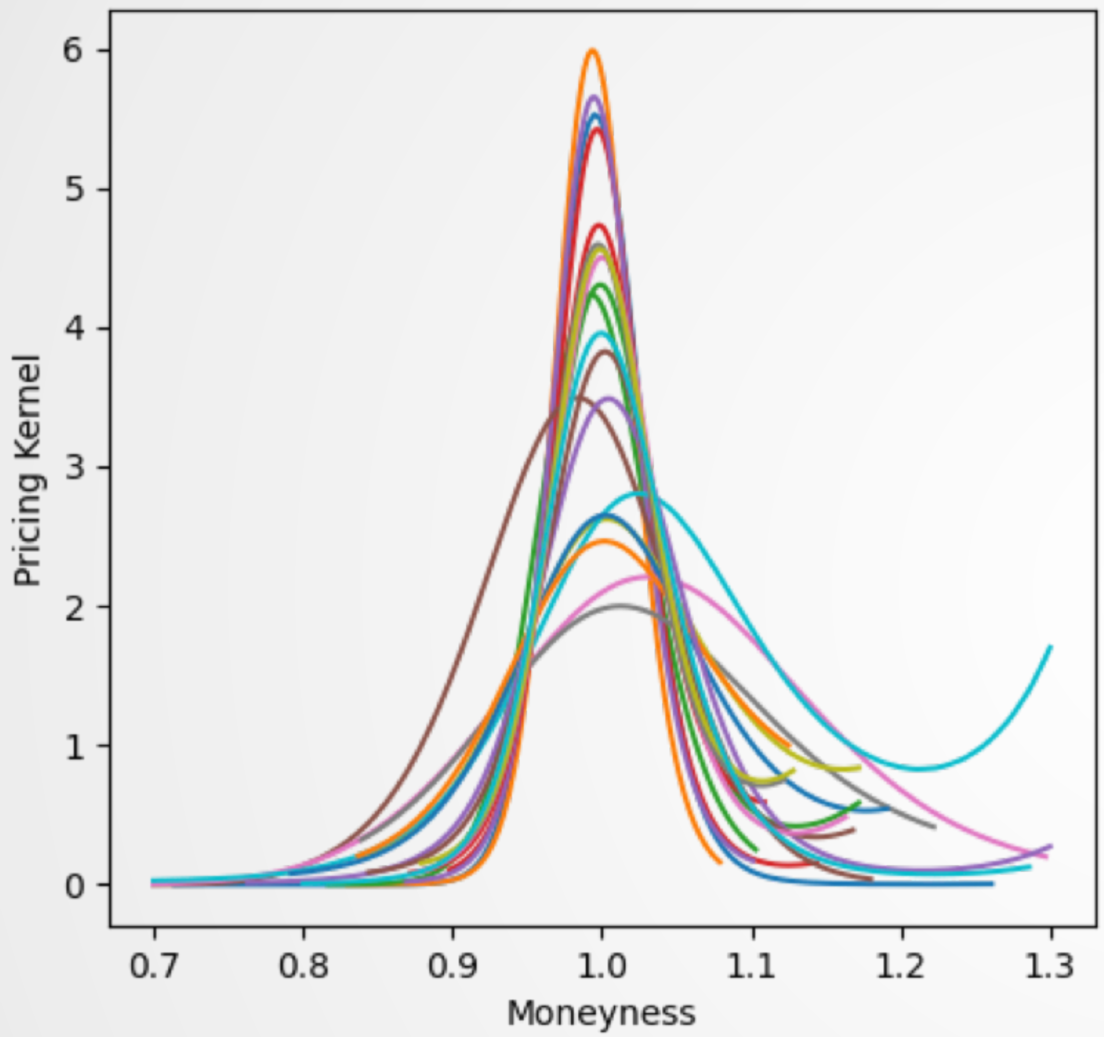
$$\theta_{tj}^r \leftarrow \frac{\theta_{tj}^r}{\sum_t \theta_{tj}^r} \qquad \theta_{tj}^r \leftarrow \theta_{tj}^r - T^{-1} \sum_{t=1}^{T} \theta_{tj}^r$$

Update Reference Kernel

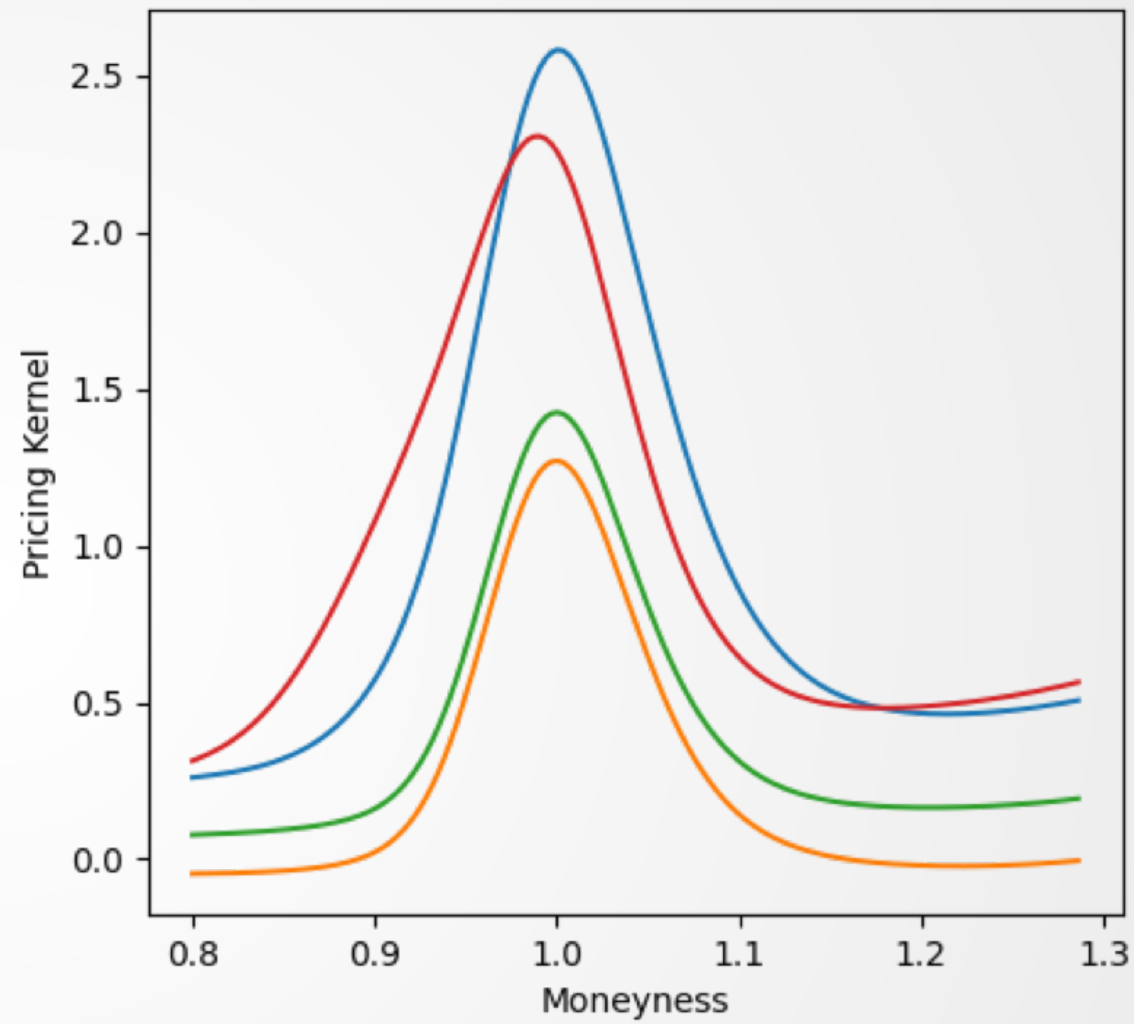
$$K_0^{(r)}(u) = T^{-1} \sum_{t=1}^{T} \hat{K}_t(\theta_{t2}^{(r)} u + \theta_{t3}^{(r)})$$



Shape Invariant Pricing Kernels

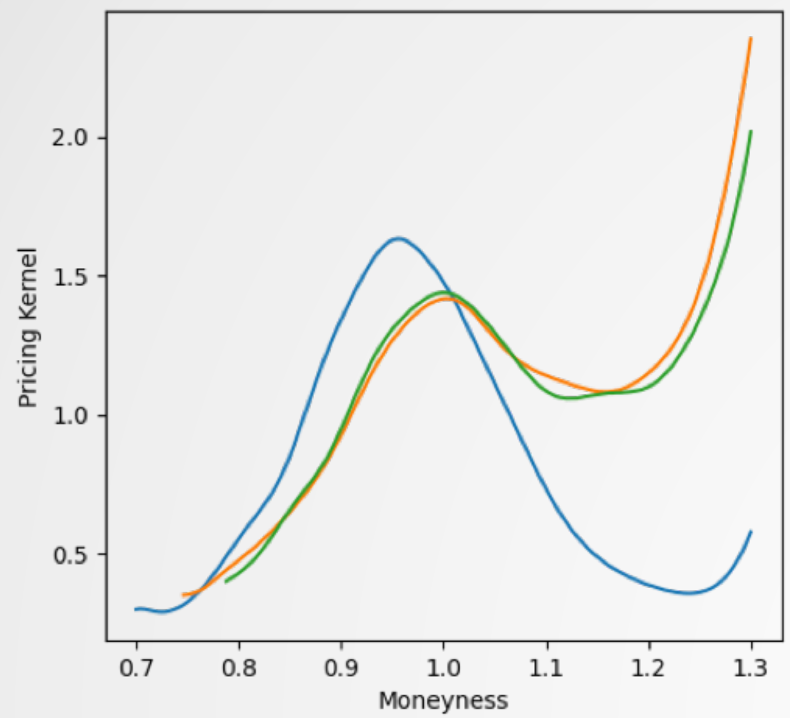


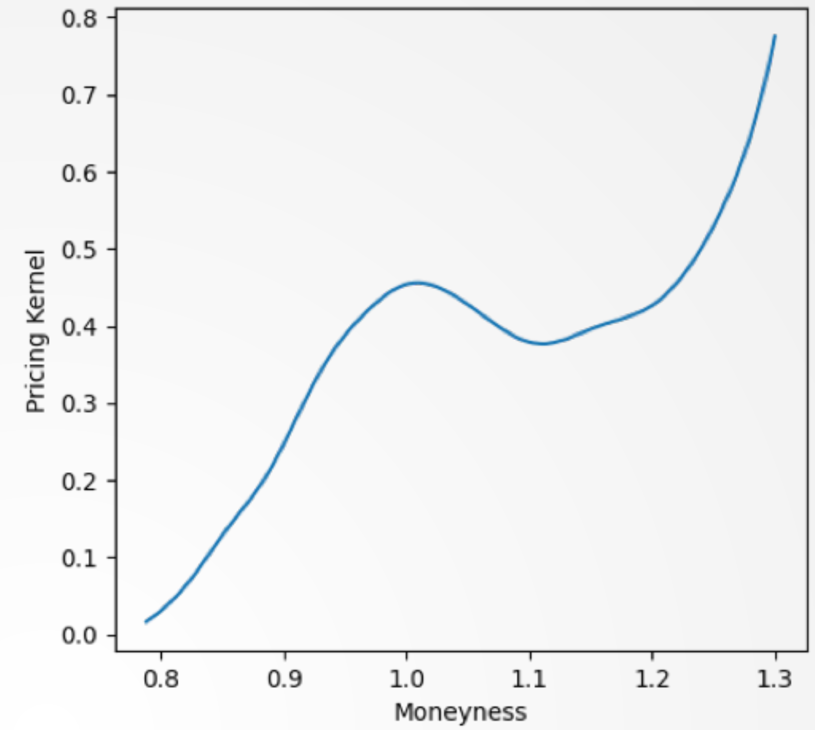
Pricing Kernels for Instruments with one day until maturity.

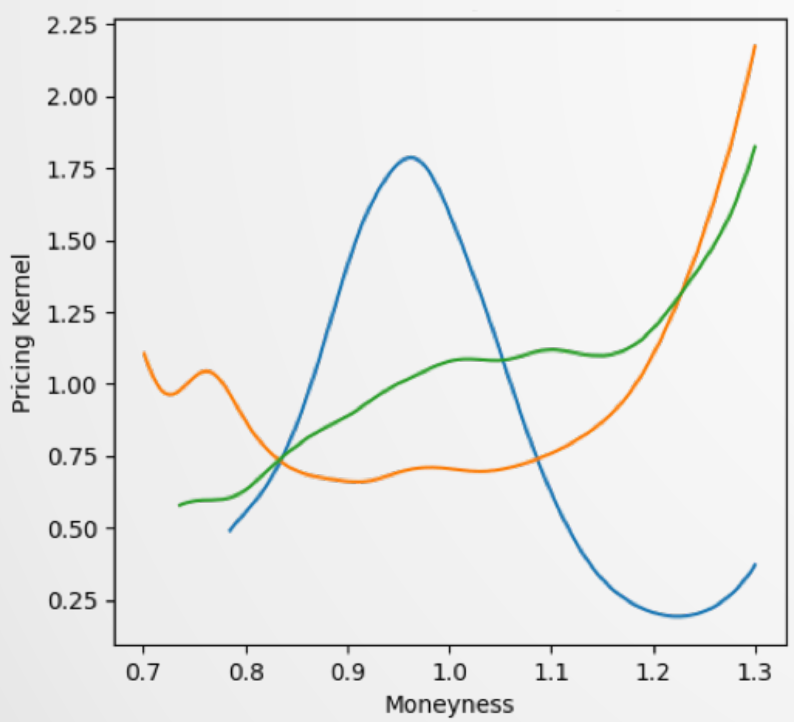


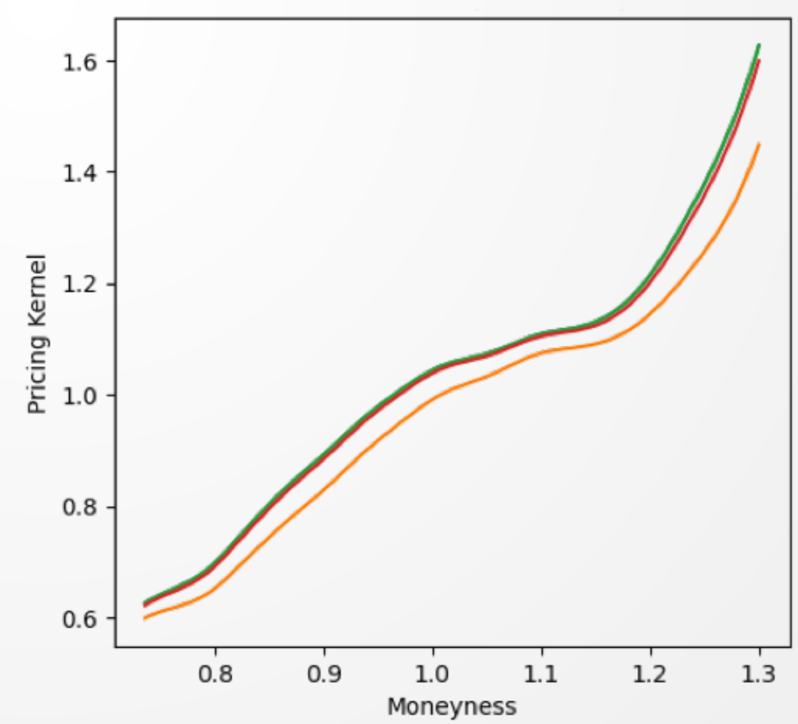
Shape Invariant Pricing Kernels across first, second, third and fourth iteration.

Pricing Kernels vs. Shape Invariant Pricing Kernels







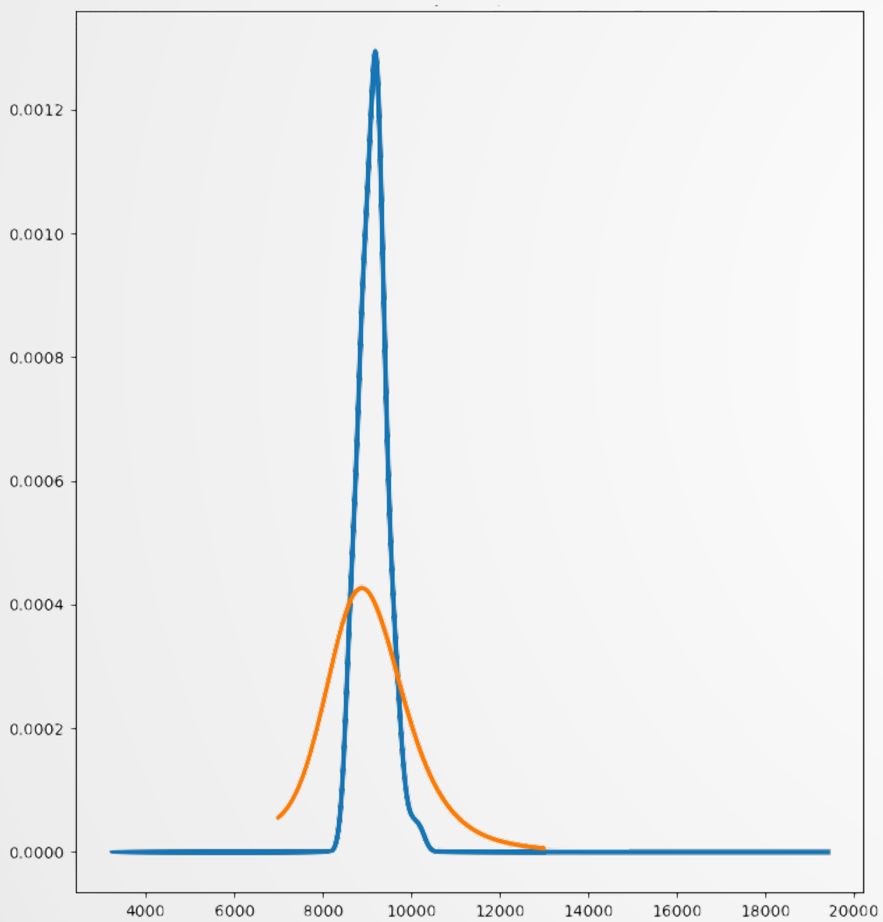


Maturity on 2020-03-20, 2020-04-10, 2020-04-17.

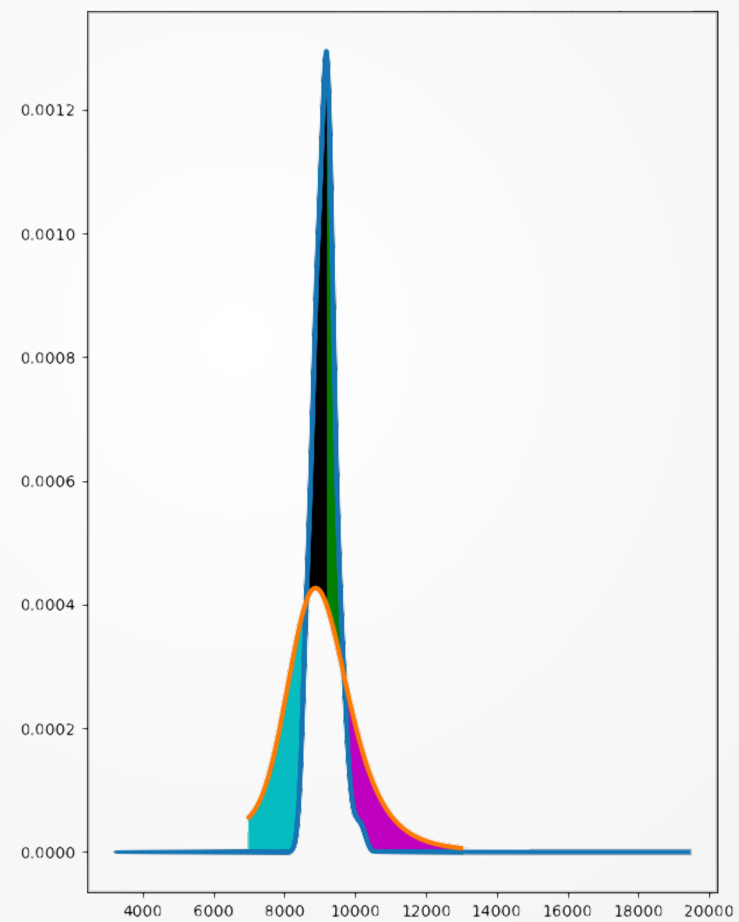
10 Days until Maturity in first row. 14 Days in second row.

Trading on Density Deviations

- Idea: If SPD deviates from PD, investors are mispricing certain events
- Correct mispricing by shorting overvalued contracts and longing undervalued ones
- Trade a call and a put spread and hold until maturity

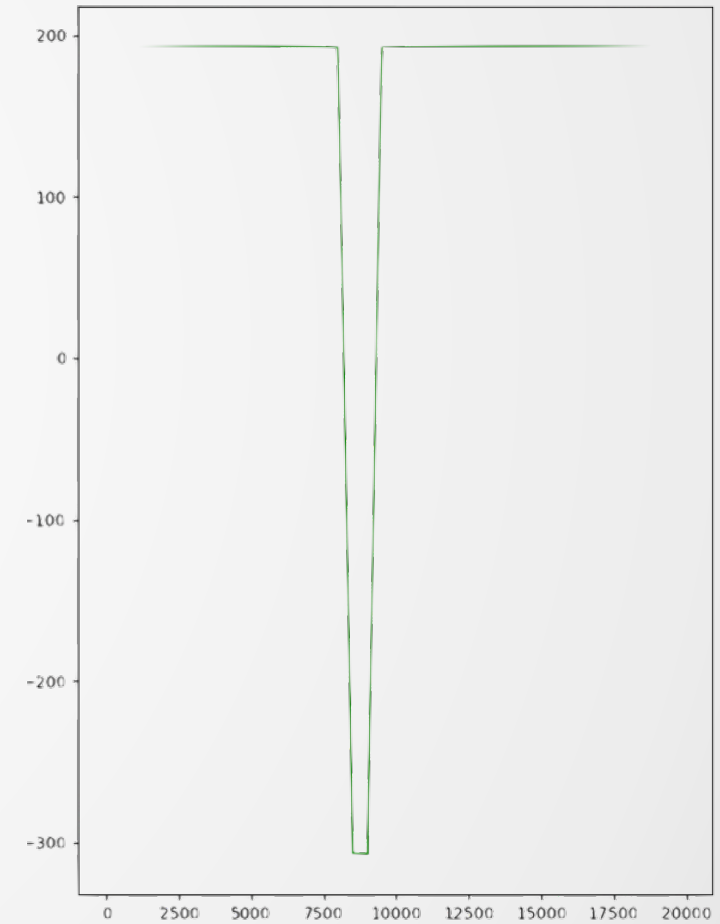


State Price Density vs Physical Density. Observed on 2020-03-06.



Trading Strategy

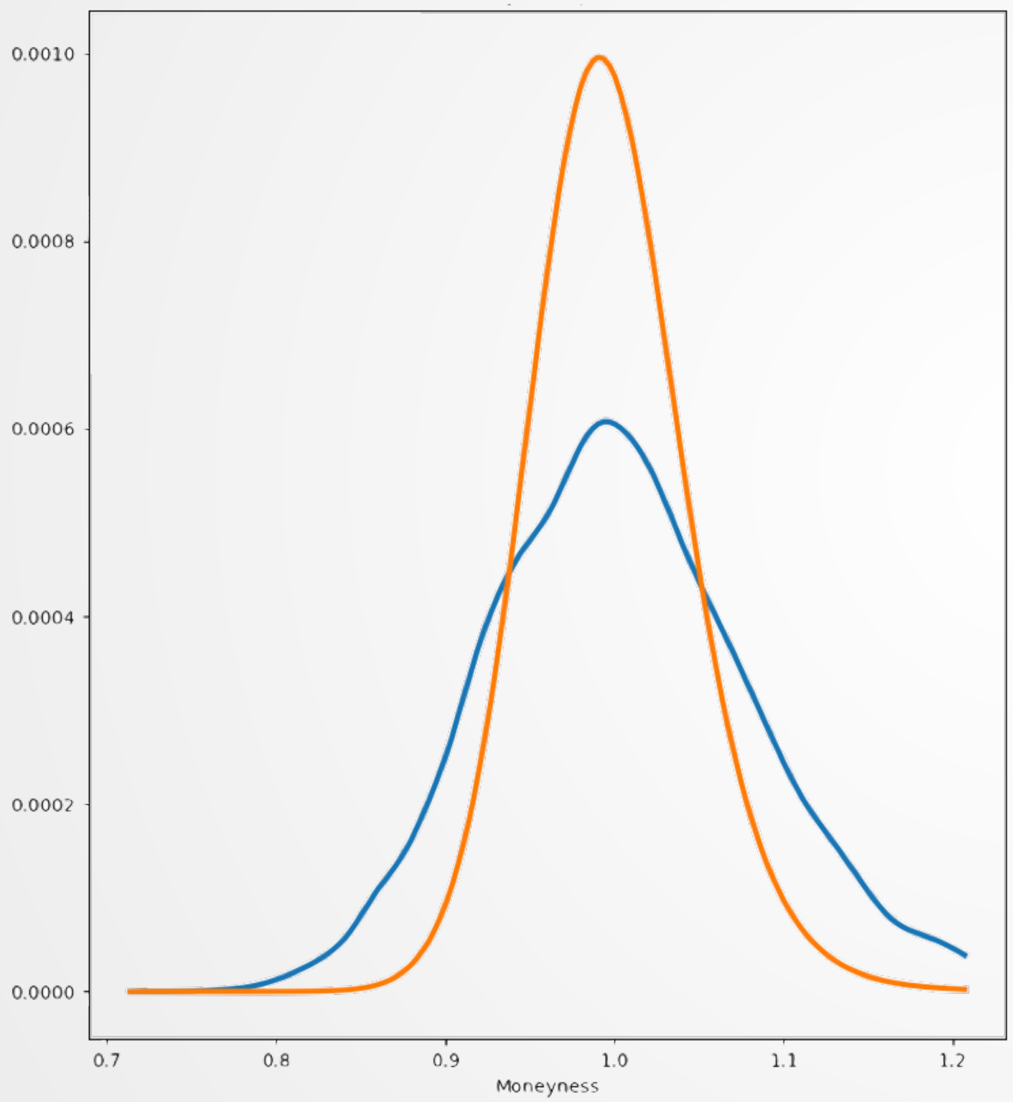
Long Put, Short Put, Long Call, Short Call.



Portfolio Payoff

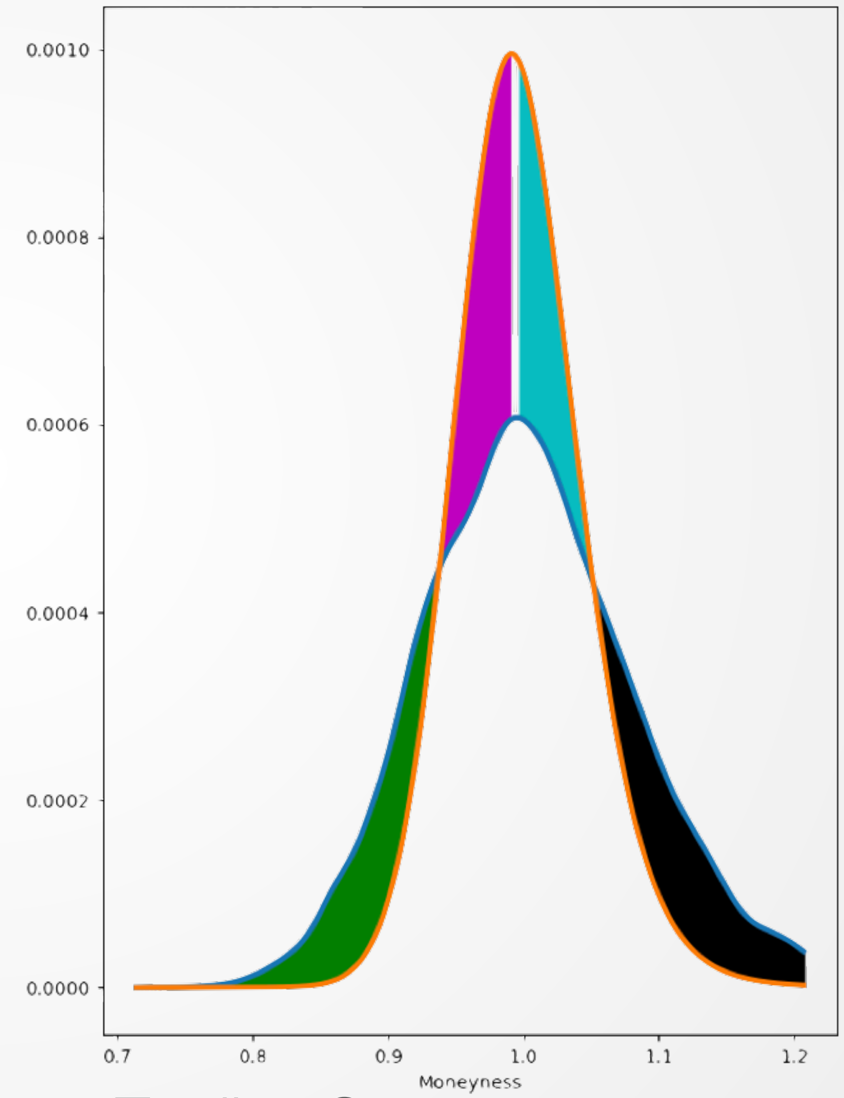
Pricing Kernel before the Corona Shock

Compare pricing of 2-Days-until-Maturity instruments before and after the Corona Shock: Market was underestimating volatility.



State Price Density vs Physical Density.

Observed on 2020-03-04.

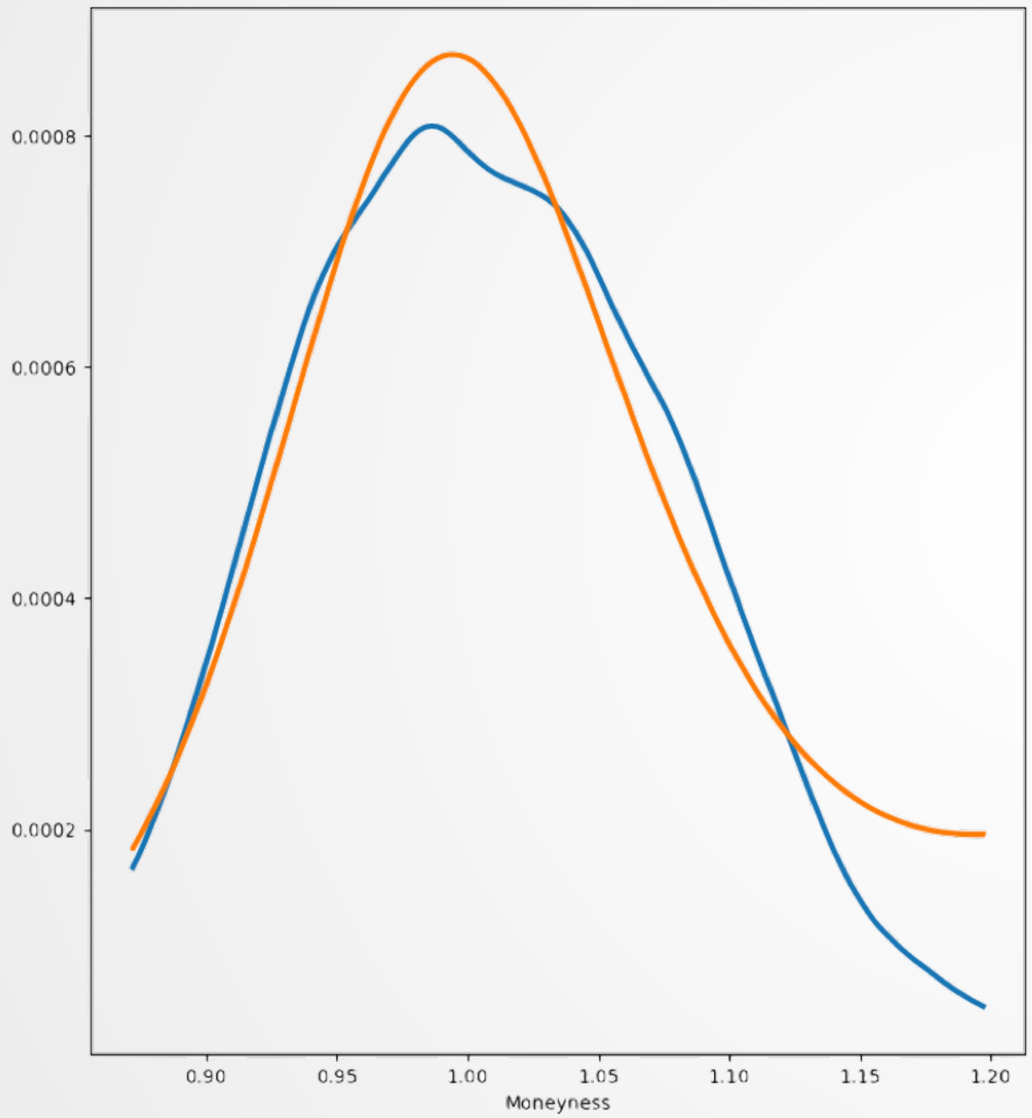


Trading Strategy

Long Put, Short Put, Long Call, Short Call.

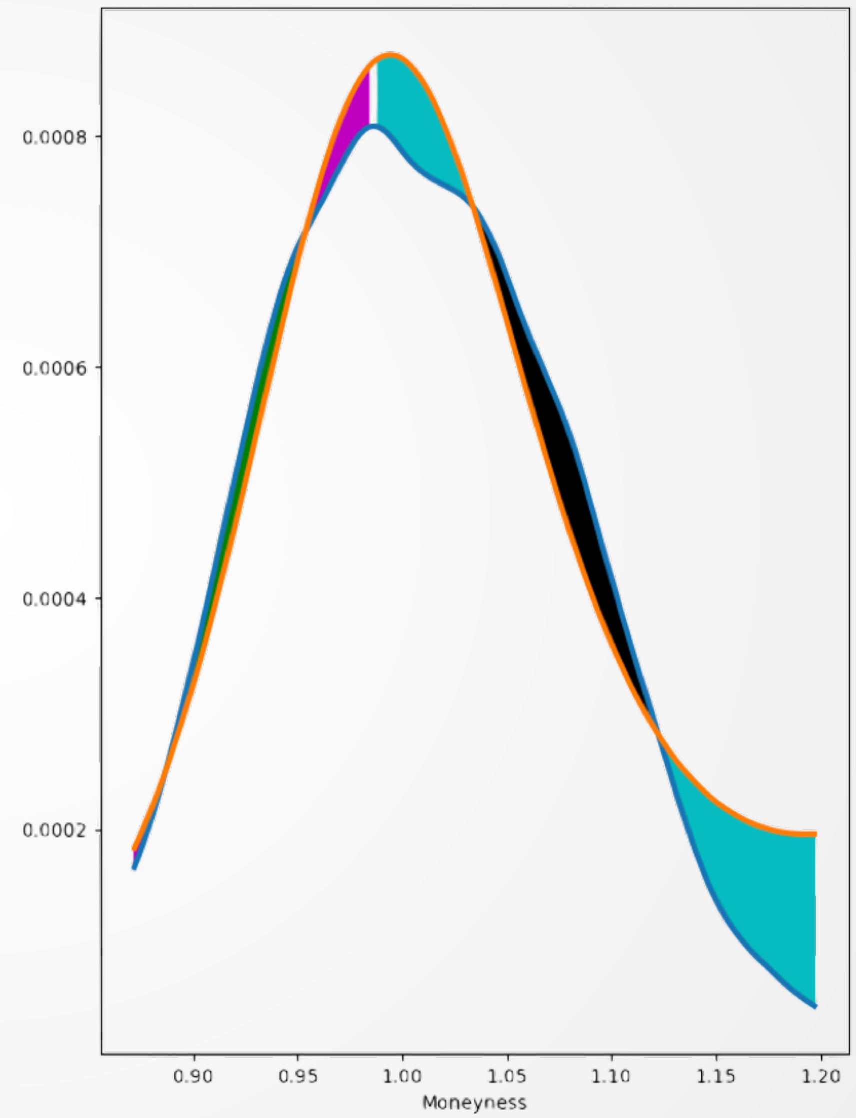
Pricing Kernels adjust quickly

Market adjusted to our SVCJ simulation after the Corona Shock.



State Price Density vs Physical Density.

Observed on 2020-03-21.

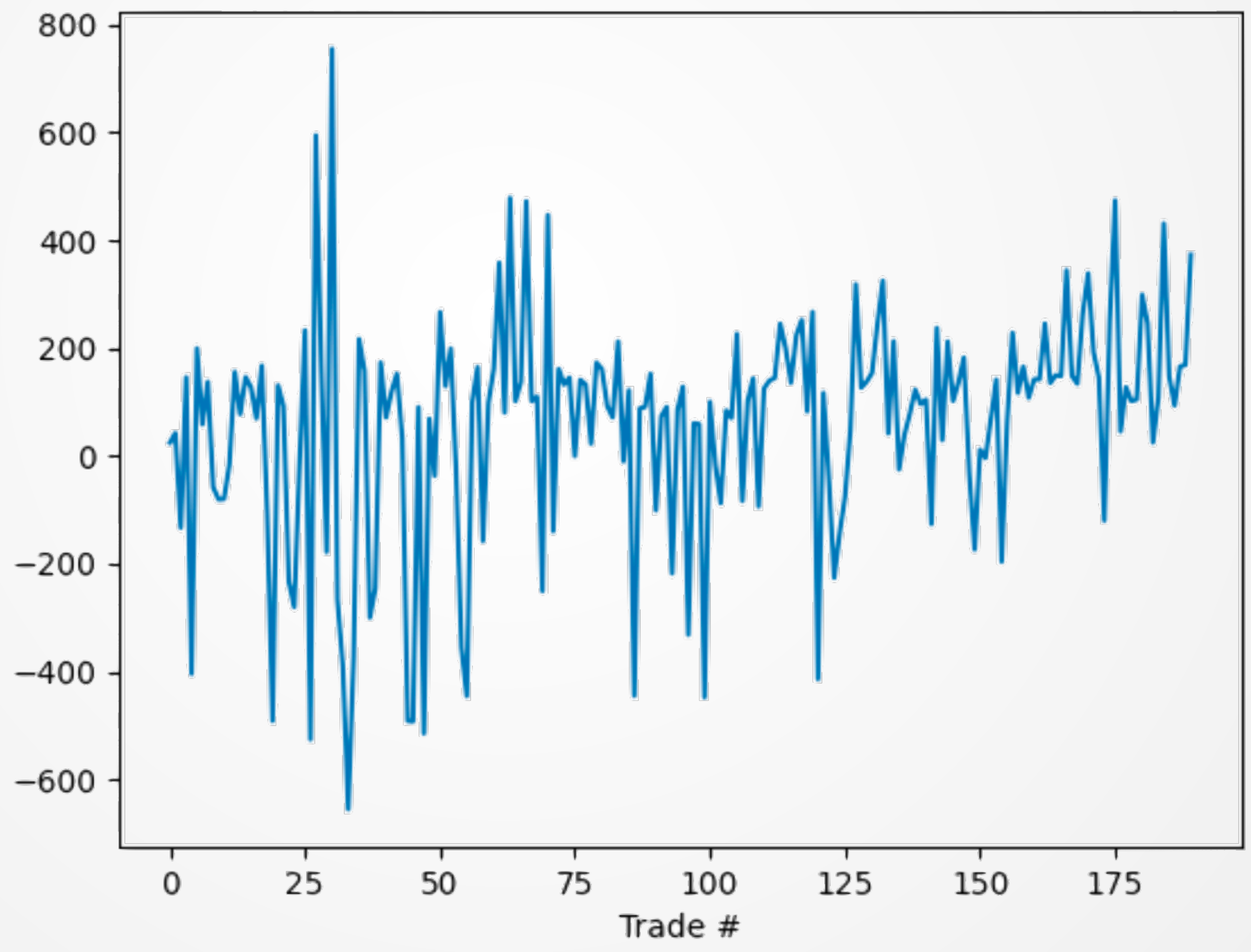


Trading Strategy

Long Put, Short Put, Long Call, Short Call.

Trading Performance

- Trading Strategy is profitable
- Sharpe Ratio is only 0.22
- Too much directional risk



Daily Trading Returns

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