Sparse Change-point model

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May 19‘ 2014

Motivation

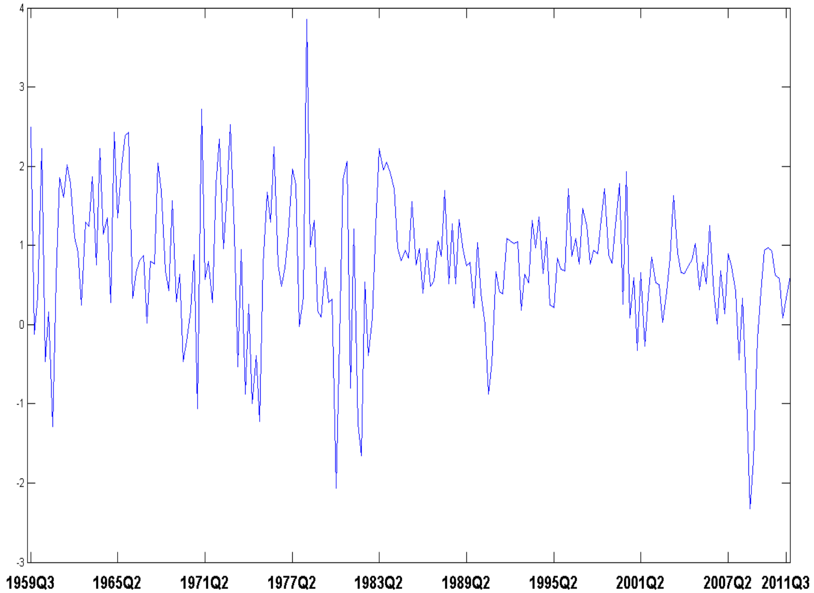
Limitations of standard CP models :

1. Classical inferences available for AR and ARCH models
   $\rightarrow$ Difficult to estimate path dependence models (CP-ARMA,CP-GARCH).
2. Optimal number of regimes computed by Marginal likelihood
   $\rightarrow$ Many useless estimations and uncontrolled penalty.
3. Each new regime increases the number of model parameters
   $\rightarrow$ Over-parametrization.
4. Forecasts based on the last regime
   $\rightarrow$ Uncertainties on parameters and inaccurate predictions.

Example

ARMA model

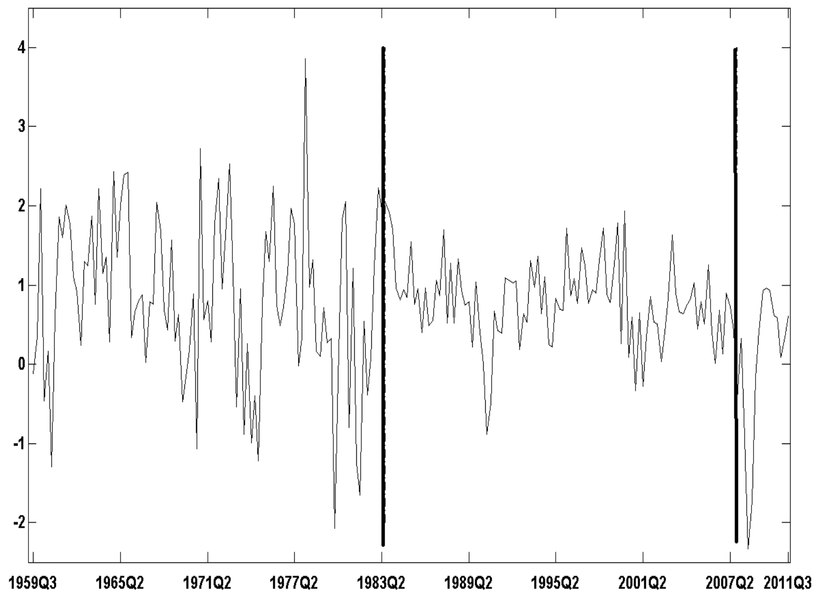
Standard



$$y_t = c + \beta y_{t-1} + \phi \epsilon_{t-1} + \epsilon_t$$

No dynamic for c, β, ϕ

Change-Point



$$y_t = c_i + \beta_i y_{t-1} + \phi_i \epsilon_{t-1} + \epsilon_t$$

A latent variable governs the dynamic of breaks

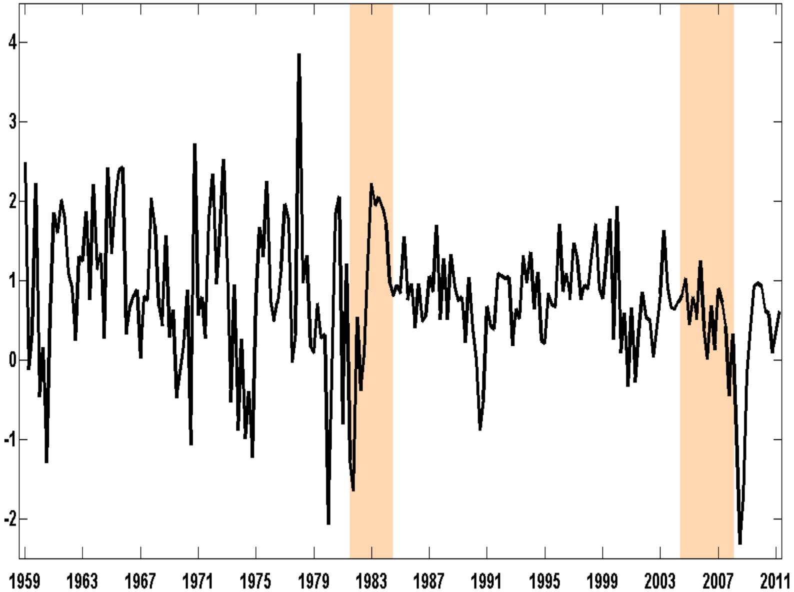
Contribution

CP models using shrinkage priors :

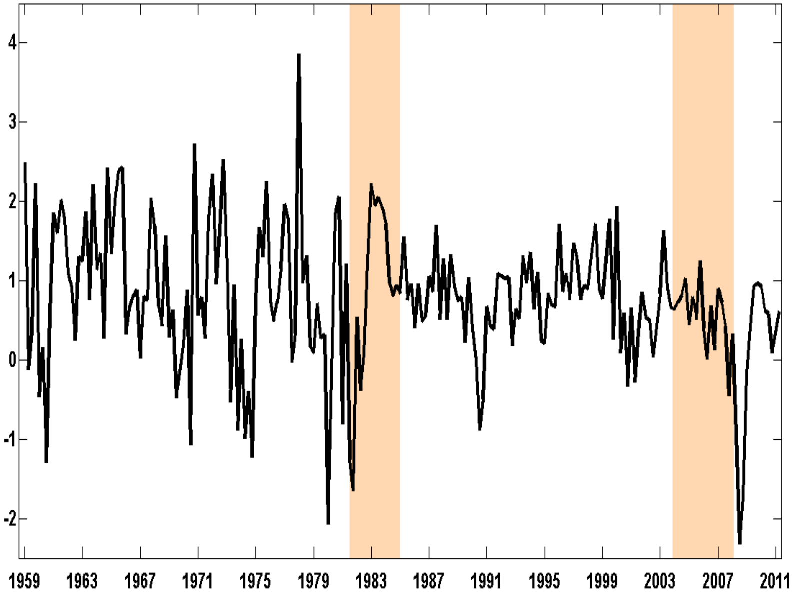
1. Adapted to estimate path dependence models (CP-ARMA,CP-GARCH).
2. Optimal number of regimes obtained in one estimation with user-specified penalty.
3. Controls the over-parametrization.
   $\rightarrow$ Only a few parameter evolves from one regime to another.
4. Very good forecast performances.

Example

CP-ARMA



CP-ARMA with Shrinkage priors



	CP-ARMA			Our CP-ARMA		
σ^2	1.11	0.25	0.53	1.11	0.27	0.27
	(0.14)	(0.04)	(0.20)	(0.15)	(0.04)	(0.04)
MA term	-0.08	-0.37	0.48	-0.34	-0.34	0.5
	(0.15)	(0.13)	(0.29)	(0.09)	(0.09)	(0.19)

Outline

The CP-ARMA model

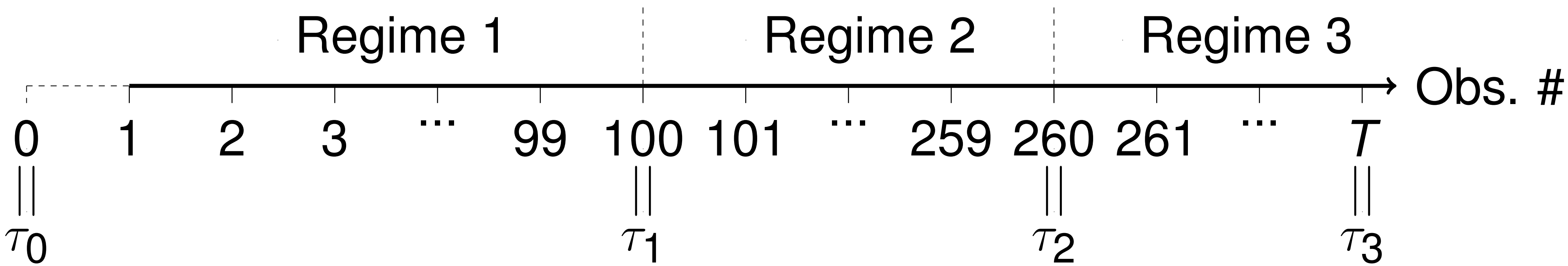
Let $Y_T = \{y_1, ..., y_T\}$ be a time series of T observations.

The CP-ARMA model defined by :

$$
\begin{aligned}
y_t &= c_1 + \beta_1 y_{t-1} + \phi_1 \epsilon_{t-1} + \epsilon_t \text{ with } \epsilon_t \sim N(0, \sigma_1^2) \text{ for } t \leq \gamma_1 \\
y_t &= c_2 + \beta_2 y_{t-1} + \phi_2 \epsilon_{t-1} + \epsilon_t \text{ with } \epsilon_t \sim N(0, \sigma_2^2) \text{ for } t \leq \gamma_2 \\
\dots & \dots \dots \\
y_t &= c_{K+1} + \beta_{K+1} y_{t-1} + \phi_{K+1} \epsilon_{t-1} + \epsilon_t \text{ for } \gamma_K < t \leq T
\end{aligned}
\tag{1}
$$

Focus on the break dates $\Gamma = (\tau_1, ..., \tau_K)'$ instead of a state vector $S_T = \{s_1, ..., s_T\}$

Let $\Theta = (c_1, \beta_1, \phi_1, \sigma_1^2, ..., c_{K+1}, \beta_{K+1}, \phi_{K+1}, \sigma_{K+1}^2)'$.



D-DREAM algorithm : specification

The MCMC scheme is

1. $\pi(\Theta|\Gamma, Y_T)$
2. $\pi(\Gamma|\Theta, Y_T)$

We use a Metropolis algorithm :

DiffeRential Adaptative Evolution Metropolis (Vrugt et al. (2009))

- DREAM automatically determines the **size** of the jump.
- DREAM automatically determines the **direction** of the jump.
- DREAM is well suited for **multi-modal** posterior distributions and for **high dimensional** parameters.
- DREAM proposal is **symmetric**.

However only suited for continuous parameters.
D-DREAM for Discrete parameters (Bauwens et al. (2011))

Outline

The CP-Model and its estimation

Shrinkage methods

Empirical applications

Conclusion

Shrinkage priors

How to determine which model parameter(s)
evolves from one regime to another ?

Testing all the possibilities and computing the Marginal likelihood

1. Too many posterior distributions to be estimated.

   $\rightarrow$ For 4 regimes and 4 parameters by regime : 256 models.
   $\rightarrow$ In the empirical example : 8 regimes (4096 models).

2. No proof that the Marginal likelihood will choose the right spec.

3. Break date parameters for each parameters : Over-parametrization.

Shrinkage priors

How to determine which model parameter(s) evolves from one regime to another ?

Keep the specification of a standard CP-ARMA but shrink the irrelevant parameters and break dates toward zero.

1. Only one estimation is required.
2. Break is identified only if it improves the likelihood.
3. Long regimes : estimators tend to their distribution without shrinkage.

Transformation of the model

For applying a shrinkage prior, the CP-ARMA model becomes :

$$y_t = (c_1 + \sum_{i=2}^{k} \Delta c_i) + (\beta_1 + \sum_{i=2}^{k} \Delta \beta_i) y_{t-1} + (\phi_1 + \sum_{i=2}^{k} \Delta \phi_i) \epsilon_{t-1}$$

$$+ \epsilon_t \text{ with } \epsilon_t \sim N(0, \sigma_1^2 + \sum_{i=2}^{k} \Delta \sigma_i^2) \text{ for } t \in]\gamma_{k-1}, \gamma_k]$$

with $\Delta c_i = c_i - c_{i-1}$.

The first regime : $\{c_1, \beta_1, \phi_1\} \sim N(0, 10 I_3)$ and $\sigma_1^2 \sim U[0, 30]$.

Shrinkage on the other parameters :

For Example : $\Delta c_i | \tau \sim N(0, \tau)$ and $\tau \sim Q$.

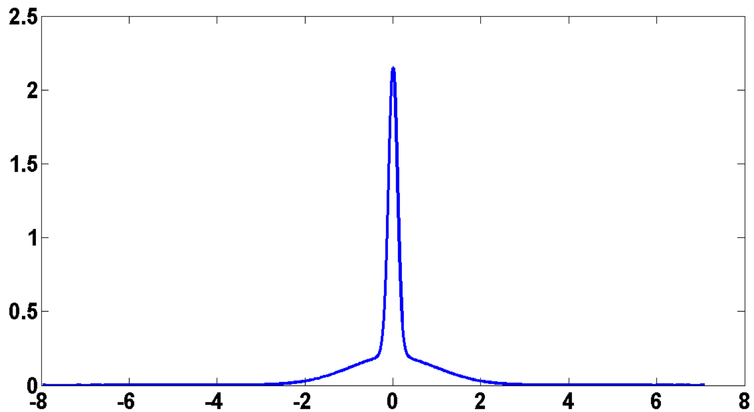
Shrinkage priors

Shrinkage is about the distribution Q : $\Delta c_i | \tau \sim N(0, \tau)$ and $\tau \sim Q$.

- The absolutely continuous spike-and-slab prior (Ishwaran and Rao (2000)) :

$\tau = \sigma^2 \kappa$ with $\sigma^{-2} \sim G(\frac{v}{2}, \frac{v}{2})$ and $\kappa | \omega = \omega \delta_{\kappa=0.00001} + (1-\omega)\delta_{\kappa=1}$.

The marginal distribution of $\Delta c_i | \omega$ is a mixture of two student distributions :

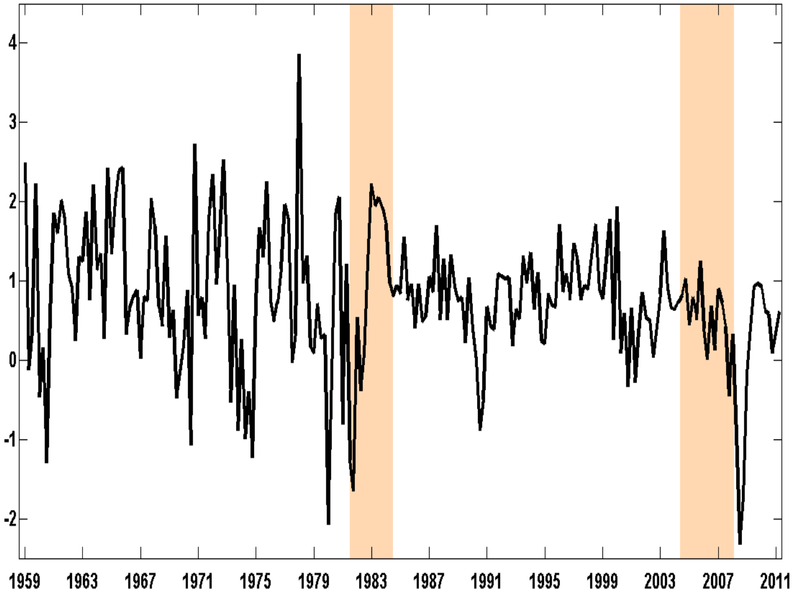


Spike and Slab marginal distribution

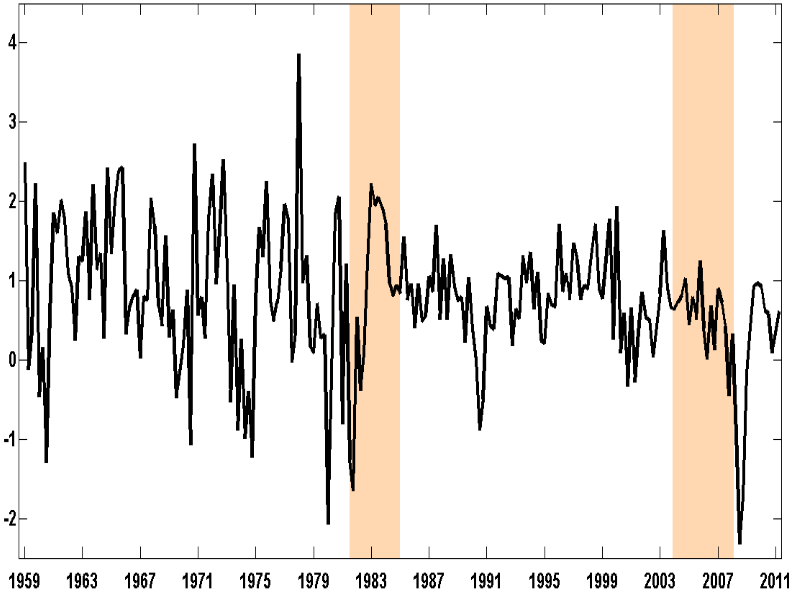
Outline

US GDP growth rate 1959-2011

CP-ARMA



CP-ARMA with Shrinkage priors



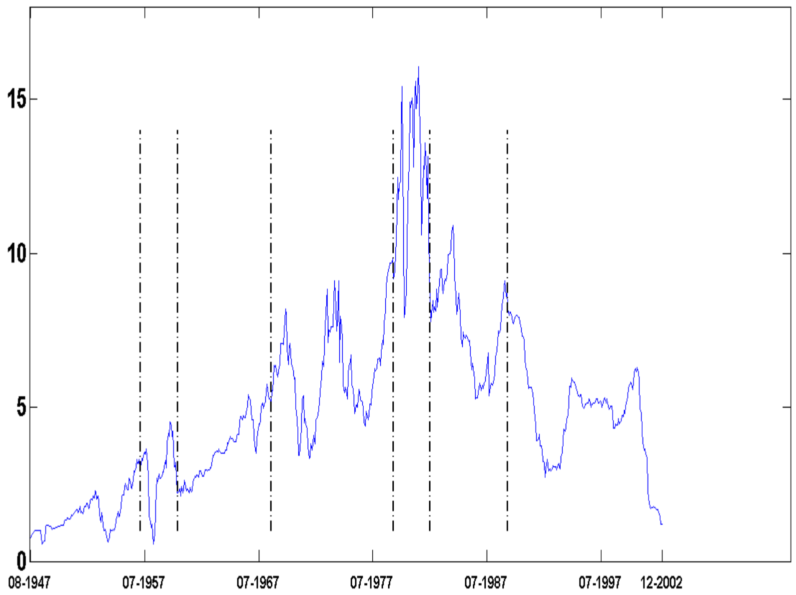
CP-ARMA with Spike and Slab

Regime	2	3
c	0.03	0.04
β	0.03	0.03
ϕ	0.03	**1.00**
σ^2	**1.00**	0.06

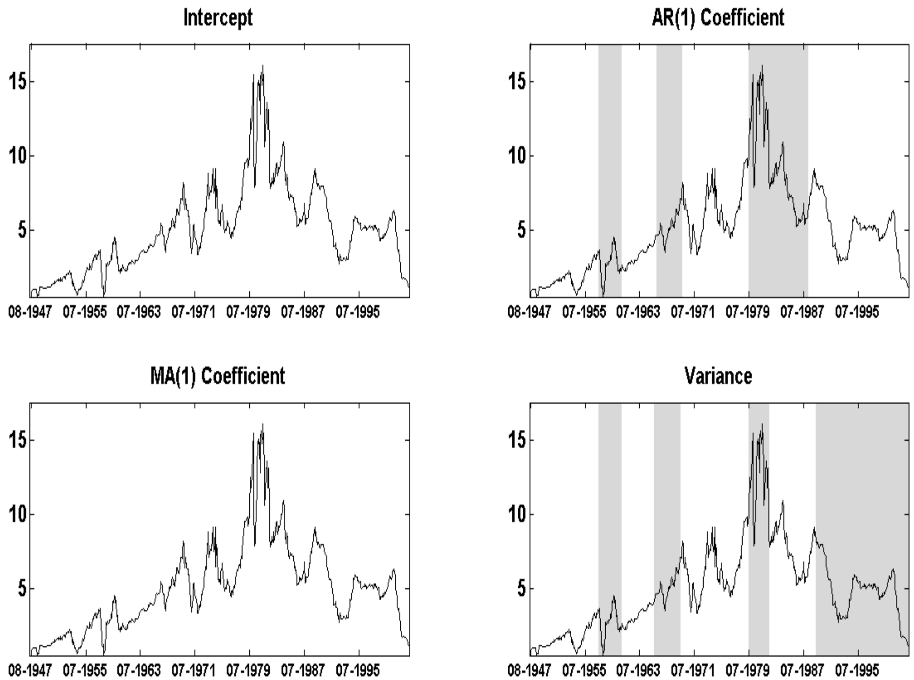
Only the variance and the MA parameter change over time

Monthly 3-Month US T-bill rate 1947-2002

Pesaran et al.



CP-ARMA with Shrinkage priors



Outline

Conclusion

$\rightarrow$ Algorithm : Inference for ARMA models with structural breaks.

- Detects the parameters that change from one regime to another.
- Shrinks all the irrelevant parameters toward zero.
- Shrinks all the irrelevant regime.
  $\rightarrow$ No need of the marginal likelihood.

$\rightarrow$ Empirical enhancements :

- Could improve the interpretation of the presence of structural breaks.
- Very good prediction performances.