IRTG 1792 Discussion Paper 2018-004

Pricing Cryptocurrency options: the case of CRIX and Bitcoin

Cathy YH Chen *
Wolfgang Karl Härdle *
Ai Jun Hou *
Weining Wang *

* Humboldt-Universität zu Berlin, Germany

This research was supported by the Deutsche Forschungsgemeinschaft through the International Research Training Group 1792 "High Dimensional Nonstationary Time Series".

http://irtg1792.hu-berlin.de
ISSN 2568-5619

International Research Training Group 1792

Pricing cryptocurrency options: The case of Bitcoin and CRIX

Ai Jun Hou,* Weining Wang,† Cathy Y.H. Chen,‡ Wolfgang Karl Härdle§

June 11, 2019

Abstract

Cryptocurrencies, especially Bitcoin (BTC), which comprise a new revolutionary asset class, have drawn extraordinary worldwide attention. The characteristics of the cryptocurrency/BTC include a high level of speculation, extreme volatility and price discontinuity. In this paper, we propose a pricing mechanism based on a stochastic volatility with correlated jump (SVCJ) model and compare it to a flexible co-jump model by Bandi and Renò (2016) allowing for non-affine structure. The calibration results of both models confirm the impact of jumps and co-jumps on options obtained via simulation and an analysis of the implied volatility curve. We show that a sizeable proportion of price jumps are significantly and contemporaneously anti-correlated with jumps in volatility. Our study comprises pioneering research on pricing BTC options. We show how proposed pricing mechanism underscores the importance of jumps in the cryptocurrency derivatives markets.

Key Words: CRIX, Bitcoin, Cryptocurrency, SVCJ, Option pricing, Jumps

JEL Codes: C32, C58, C52

Acknowledgement: We thank Bandi and Renò (2016) for providing us with the codes used

*Stockholm Business School, Stockholm University.

†Department of Economics, City University of London. Ladislaus von Bortkiewicz Chair of Statistics, Humboldt-Universität zu Berlin.

‡Adam Smith Business School, University of Glasgow.

§School of Business and Economics, Ladislaus von Bortkiewicz Chair of Statistics, Humboldt-Universität zu Berlin. Sim Kee Boon Institute, Singapore Management University, Charles University in Prague.

to implement the non-affine co-jumps model in this study. We acknowledge the helpful comments from the editor and two anonymous referees. We thank Xiaohao Ji for help with data processing. This research is supported by the Deutsche Forschungsgemeinschaft through the International Research Training Group 1792 "High Dimensional Nonstationary Time Series". (http://irtg1792.hu-berlin.de). In addition, it has been funded by the Natural Science Foundation of China (fund number 71528008). Ai Jun Hou acknowledges the financial support from the Jan Wallender and Tom Hedelius Foundation of Handelsbanken (P2019-0264).

1 Introduction

Bitcoin (BTC), the online decentralized digital currency and crypto-based payment system, has garnered much attention and interest worldwide since it was first introduced in 2009. The rapidly growing research on BTC structures and operations (Becker et al., 2013; Segendorf, 2014; Dwyer, 2015), economics (Kroll et al., 2013) and financial stability (Ali, 2014; Badev, 2014; ECB, 2015) indicates a prominent role for digital currency assets in contemporary financial markets. Discussions on alternative monetary systems can be found, for example, in Rogojam (2014) and Weber (2016). An analysis of the legal issues involved in using Bitcoin can be found in Elwell et al. (2013).

The purpose of this paper is first to characterize the dynamics of BTC with a popular option-pricing model, i.e., the stochastic volatility with correlated jump (SVCJ) model, and then apply the model to price forthcoming BTC options. Several studies have suggested econometric methods to model BTC prices, including cross-sectional regression models involving the majority of traded cryptocurrencies and multivariate time-series models for the dynamics. For example, Hayes (2017) performs a regression using a cross-section dataset consisting of 66 traded digital currencies to understand the price driver of cryptocurrencies. Kristoufek (2013) proposes a bivariate Vector-AutoRegression (VAR) model for the weekly log returns of Bitcoin prices. Bouoiyour (2019) investigates the long and short-run relationships between BTC prices and other related variables using an autoregressive distributed lag model.

We start our econometric analysis of the BTC price by using the ARIMA (Autoregressive Integrated Moving Average) and GARCH (generalized autoregressive conditional heteroskedasticity) models. Then we move on to discover the necessity of incorporating jumps into stochastic volatility models. This investigation is motivated by the fact that the unique feature of jumps presented in the BTC prices is ignored in most existing studies when modeling the BTC dynamics. One exception is Scaillet et al. (2018), who show that jumps are much more frequent in the BTC market than, for example, in the US equity market, as shown in Bajgrowicz et al. (2015). This implies that jumps should be considered when modeling BTC prices and options. We, therefore, employ a model that incorporates jumps in returns and the stochastic volatility processes: the SVCJ model introduced by Duffie et al. (2000). Numerous empirical studies have applied the SVCJ model in different markets. For example, Eraker et al. (2003) and Eraker (2004) use the SVCJ model to describe equity market returns and calibrate equity option pricing. They find strong evidence for jumps in returns and volatility in the US equity market. Cosma et al. (2018) develop a methodology to estimate a jump-diffusion model to price American options or other path-dependent options. To show the importance of modeling jumps in the BTC returns and variance dynamics, we compare our SVCJ estimate to simpler versions such as Bates (2000) (SVJ hereafter) and the stochastic volatility (SV) model.

Furthermore, research on the BTC derivative markets has been very limited despite its necessity. The markets for BTC futures and options traded on an unregulated exchange platform (i.e., Deribit) are rapidly increasing. The CME (Chicago Mercantile Exchange) Group, the world's leading and most diverse derivatives marketplace, launched BTC futures based on the CME CF Bitcoin Reference Rate (BRR) on 18 December 2017. Pricing BTC derivatives (e.g., options) brings new challenges. Unlike classic financial markets, the BTC market has a unique market microstructure created by a set of opaque, unregulated, decentralized and highly speculation-driven markets. Conventional financial or economic theories may fail to embrace this new asset class. The pricing of contingent claims may not be easily adapted for the crypto market since the frequent appearance of jumps in addition to stochastic volatility behaviour makes the markets incomplete. This may imply that a non-affine specification introducing nonlinearities into the drift and diffusion term of the variance process would provide more flexibility when capturing the sudden jumps in returns and variance than an affine specification. Studies such as Bakshi and Ou-Yang (2006) and Chourdakis (2011), among others, propose a non-affine structure for the volatility process. There is also a strand of literature in which the nonparametric model setup is used to analyze whether jumps in returns and variance are important model components (Aït-Sahalia, 2010; Barndroff-Nielsen, 2006). These studies suggest that jumps and non-affine structures are potentially important model components.

Recently, Bandi and Renò (2016) (BR hereafter) propose a price and variance co-jump model that generalizes the SVCJ model to capture the possible nonlinearity in the parameters of the returns and variance processes. The BR model embraces independent and correlated jumps and allows for the nonparametric parameter structure. The estimation uses high-frequency data. We also apply this model to describe the dynamics of BTC. Our option analysis depends on the experimental simulation analysis based on the results estimated from the SVCJ and BR models because we do not have the true option market data. We find that affine models are sufficient for modeling the dynamics of BTC through our estimation of the nonparametric BR model.

We summarize our main empirical findings as follows. *First*, as in the existing literature, the results from the SVCJ and BR models indicate that jumps are present in the returns and variance processes and adding jumps to the returns and volatility improves the model fit. *Second*, in contrast to existing studies that commonly report a negative leverage effect, we find that the correlation between the returns and volatility is significantly positive in the SVCJ model, and we cannot find significantly negative relations between risk and return in the BR model. This implies that increasing prices are not associated with a decrease in volatility. This is consistent with the "inverse leverage effect" in the commodity markets reported in Schwartz and Trolled (2009). The positive relationship between risk and volatility is reflected in the option prices simulated based on the parameters estimated from the SVCJ model.

Third, we find that the jump size in the returns and variance of BTC is anti-correlated. The parameter estimates of the jump size (ρ_j) from both the SVCJ and BR models are negative (though the SVCJ estimate is insignificant). It is worth noting that the correlation between the price jump size and the volatility jump size turns out to be significant with a negative coefficient with high-frequency data, while tending to be insignificant for the SVCJ fitting using daily prices. This finding is in line with existing studies of the stock market from Eraker (2004), Duffie et al. (2000) and Bandi and Renò (2016), among others. For example, Bandi and Renò (2016) report an anti-correlation with the nonaffine structure. Eraker (2004) finds a negative correlation between jump size only when augmenting return data with options data, and the negative correlation between co-jump size being identified of the implied volatility smirk. Using high-frequency data, Jocod and Todorov (2009) and Todorov and Tauchen (2010) also reported that the large jump size of prices and volatility is strongly anti-correlated.

Finally, we observe that the BTC option prices share properties similar to those observed in other markets. For instance, we find that the option prices simulated from the SVCJ and BR models decrease in moneyness from in-the-money (ITM) to out-of-the-money (OTM). The option prices increase with the time to maturity and the volatility level. Moreover, the option price level is prominently dominated by the level of volatility and therefore greatly affected by jumps in the volatility processes. The results from the implied volatility (IV) plots indicate that adding jumps in returns increases the slope of the IV curves. The greater steepness of the IV curve can be reinforced by the presence of jumps in volatility. The presence of co-jumps enlarges the IV smile further. As evidenced from the IVs curve, options with a short time to maturity are more sensitive to jumps and co-jumps. For the model robustness check, we replicate the entire analysis for the CRyptocurrency IndeX (CRIX), a market portfolio comprising several cryptocurrencies. This additional investigation is beneficial from the robustness point of view.

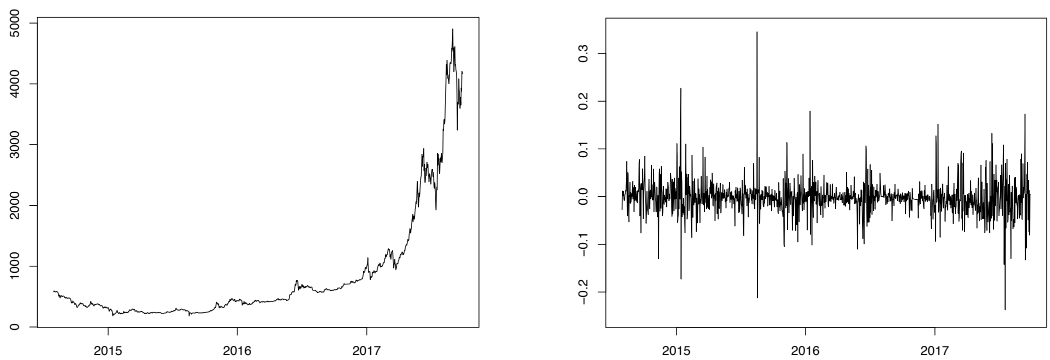
To summarize our contributions, we are the first paper to thoroughly investigate the BTC returns and return variances with advanced option-pricing models, i.e., the SVCJ and BR models. Our results have practical relevance in terms of model selection for characterizing the BTC dynamics. We document the necessity of incorporating jumps in the returns and volatility processes of BTC, and we find that jumps play a critical role in the option prices. Our approach is directly applicable to pricing in the forthcoming BTC option market. Our results are also important for policymakers to design appropriate regulations for the BTC market and for investors to create appropriate risk-management and portfolio-selection strategies.

The paper is organized as follows. Section 2 presents results of the econometric analysis of BTC. Section 3 studies the BTC return and variance dynamics with the SV, SVJ and SVCJ models. Fitting of the BR model is investigated in Section 4. Section 5 implements the option-pricing exercises. Section 6 documents an examination of the CRIX, while Section 7 concludes the study.

2 The BTC dynamic

The dynamics of BTC prices depicted in Figure 1 give us a glimpse of its evolution. A dramatic surge was observed after March 2017 due to widespread interest in cryptocurrencies (CCs). The subsequent drop in June was caused by a sequence of political interventions. Indeed, several governmental announcements of bans on initial coin offerings (ICOs), for example, have spurred intensive movements on CC markets.

Figure 1: BTC Prices and Returns



Notes: This figure graphs the BTC daily price (left panel) from 31/07/2014 to 29/09/2017 and BTC returns (right panel). The returns (R_t) are calculated as $R_t = ln(P_t) - ln(P_{t-1})$, where P_t is the BTC price at time t.

Figure 1 indicates that BTC prices do not behave like conventional stock prices. One records ex-

Table 1: Estimation result of ARIMA(2,0,2)

	Bitcoin	
Coefficients	Estimate	Standard deviation
intercept c	0.002	0.001
a_1	-0.867	0.304
a_2	-0.596	0.177
b_1	0.868	0.321
b_2	0.539	0.190

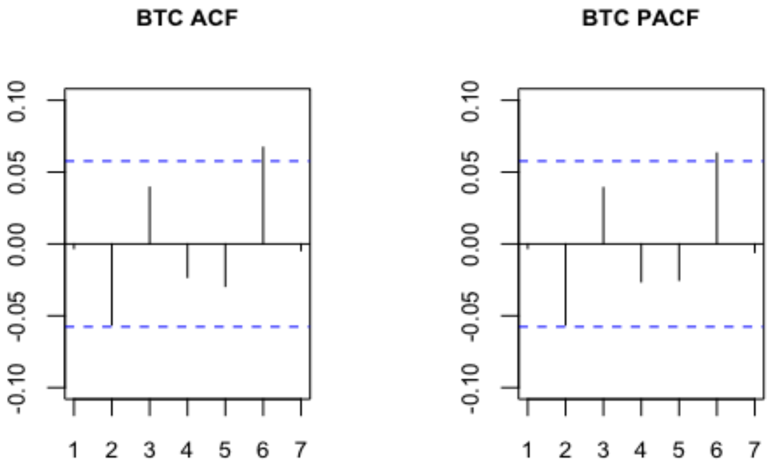
Notes: This table reports the parameter estimated from ARIMA (2,0,2) with BTC daily returns calculated as the log-first difference based on the prices from 31/07/2014 to 29/09/2017. The residual distributions are assumed to be Gaussian. The maximized likelihood value is 2231.7. The AIC and BIC are -4451.4 and -4415.74, respectively.

tremely high volatility and scattered spikes. These prices are far from being stationary. The differentiation and detrending, or change point detection are required. After an inspection through the ACF and PACF plot in Figure 2, we start with an ARIMA(p, d, q) model,

$$a(L)\Delta y_t = b_L \varepsilon_t \tag{1}$$

where y_t is the variable of interest, $\Delta y_t = y_t - y_{t-1}$, L is the lag operator and ε_t a stationary error term. Model selection criteria such as AIC or BIC indicates that the ARIMA$(2, 0, 2)$ is the model of choice. The parameters estimated from the ARIMA(2,0,2) are reported in Table 1. The significant negative signs in a_1 and a_2 indicate an overreaction, that is, a promising positive return today leads to a return reversal in the following two days or vice versa. Hence, the CC markets tend to overreact to good or bad news, and this overreaction can be corrected in the following two days. An ARIMA model for the CC assets, therefore, suggests predictability due to an "overreaction". The Ljung-Box test confirms that there is no serial dependence in the residuals based on the ARIMA$(2, 0, 2)$ specification. The details of these numerical computations are available in the quantlets used. Note that the squared residuals carry incremental information that is addressed in the following GARCH analysis.

Figure 2: ACF and PACF of BTC



Notes: This figure plots the ACF and PACF for BTC returns. The returns are the log-first difference calculated based on the price from 31/07/2014 to 29/09/2017. The x-axis plots the lags, and the y-axis plots the ACF and PACF values.

2.1 GARCH Model

The GARCH model reflects the changes in the conditional volatility of the underlying asset in a parsimonious way. Duan (1995) develops a GARCH option-pricing model in the context of the continuously compounded GARCH return process. A similar approach is taken in Heston and Nandi (2000). The volatility properties of digital currency assets have been studied in a vast amount of literature that applies GARCH-type methods (Hotz-Behofsits et al., 2018; Chu et al., 2017; Chan et al., 2017; Conrad et al., 2018).

Let us start with a GARCH-type model for characterizing the conditional variance process of BTC. The ARIMA-t-GARCH model with t-distributed innovations used to capture fat tails is as follows:

$$a(L)\Delta y_t = b_L \varepsilon_t \tag{2}$$

$$\varepsilon_t = Z_t \sigma_t, \quad Z_t \sim t(\nu)$$

$$\sigma_t^2 = \omega + \beta_1 \sigma_{t-1}^2 + \alpha_1 \varepsilon_{t-1}^2 \tag{3}$$

where σ_t^2 represents the conditional variance of the process at time t and $t(\nu)$ refers to the zero-mean t distribution with ν degrees of freedom. The choice of the t-distribution rather than the Gaussian distribution is supported by Hotz-Behofsits et al. (2018) and Chan et al. (2017).

The covariance stationarity constraint $\alpha_1 + \beta_1 < 1$ is imposed. As shown in Table 2, the β_1 estimate from BTC indicates a persistence in the variance process, but its value is relatively smaller than those estimated from the stock index returns (see Franke et al. (2019)). Typically, the persistence-of-volatility estimates are very near to one, showing that conditional models for stock index returns are very close to being integrated. By comparison, BTC places a relatively higher weight on the α_1 coefficient and relatively lower weight on the β_1 to imply a less-smooth volatility process and striking disturbances from the innovation term. This may further imply that the innovation is not pure white noise and can occasionally be contaminated by the presence of jumps.

In addition to the property of leptokurtosis, the leverage effect is commonly observed in practice. According to a large body of literature, starting with Engle and Ng (1993), the leverage effect refers to an asymmetric volatility response given a negative or positive shock. The leverage effect is captured by the exponential GARCH (EGARCH) model by Nelson (1991),

$$
\begin{aligned}
\varepsilon_t &= Z_t \sigma_t \\
Z_t &\sim t(\nu) \\
\log(\sigma_t^2) &= \omega + \sum_{i=1}^{p} \beta_i \log(\sigma_{t-i}^2) + \sum_{j=1}^{q} g_j\left(Z_{t-j}\right) \qquad (4)
\end{aligned}
$$

where $g_j\left(Z_t\right) = \alpha_j Z_t + \phi_j(|Z_{t-j}| - \mathsf{E}|Z_{t-j}|)$ with $j = 1, 2, \ldots, q$. When $\phi_j = 0$, we have the logarithmic GARCH (LGARCH) model from Geweke (1986) and Pantula (1986). To accommodate the asymmetric relation between stock returns and volatility changes, the value of $g_j\left(Z_t\right)$ must be a function of the magnitude and the sign of Z_t. Over the range of $0 < Z_t < \infty$, $g_j\left(Z_t\right)$ is linear in Z_t with slope $\alpha_j + \phi_j$, and over the range $-\infty < Z_t \leq 0$, $g_j\left(Z_t\right)$ is linear in Z_t with slope $\alpha_j - \phi_j$.

Table 2: Estimated coefficients of t-GARCH(1,1)

Coefficients	Estimates	Robust std	t value
BTC			
ω	$3.92e-05$	$1.49e-05$	2.63
α_1	$2.28e-01$	$4.46e-02$	5.12
β_1	$7.70e-01$	$5.13e-02$	14.98
ν	$3.64e+00$	$4.08e-01$	8.91

Notes: This table reports the estimated parameters from the t-GARCH(1,1) model. The robust version of standard errors (robust std) are based on the method of White (1982).

Table 3: Estimated coefficients of t-EGARCH(1,1) model

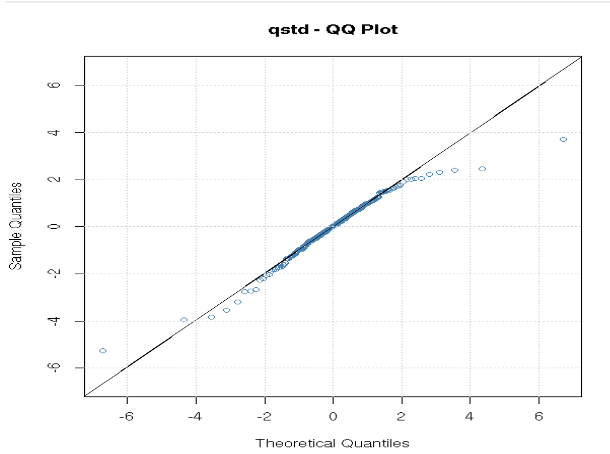
Coefficients	Estimates	Robust std	t value
BTC			
ω	$3.84e-05$	$1.47e-05$	2.61
α_1	$1.05e-03$	$5.10e-02$	0.98
β_1	$9.52e-01$	$1.54e-02$	61.73
ϕ_1	$4.16e-01$	$6.64e-02$	6.25
ν	$3.26e+00$	$4.16e-01$	7.82

Notes: This table reports the estimated parameters from the t-EGARCH(1,1) model. The robust version of standard errors (robust std) are based on the method of White (1982).

The estimation results based on the ARIMA(2,0,2)-t-EGARCH(1,1) model are reported in Table 3. The estimated α_1 is no longer significant, showing a vanished sign effect. However, a significant positive value of ϕ_1 indicates that the magnitude effect represented by $\phi_1(|Z_{t-1}| - \mathsf{E}|Z_{t-1}|)$ plays a bigger role in the innovation in $\log(\sigma_t^2)$.

We compare the model performances between two types of GARCH models through information criteria, and a t-EGARCH(1,1) model is suggested. Note that, as shown in Figure 3, the QQ plots demonstrate a deviation from the student-t. In Chen et al. (2017), GARCH and variants such as t-GARCH, EGARCH have been reported, and, while they are seen to fit the dynamics of BTC nicely, they still could not handle the extreme tails in the residual distribution. Equipped with these findings and taking into account the occasional interventions, we opt for the models with jumps for better characterization of CC dynamics. The presence of jumps is indeed more likely in this decentralized, unregulated and illiquid market. Numerous political interventions also suggest the introduction of the jump component into a pricing model.

Figure 3: The QQ plot for BTC based on the residuals of t-GARCH(1,1) model



3 SVCJ: affine specification

Although we have fitted a variety of financial econometrics models to the BTC price, there still is evidence of nonstationarity and fat tails in the residuals. The political interventions and influential media comments in the past and the hype created by sudden price moves motivates us to consider more flexible and richer SV models with jumps. Here we begin with an affine type of specification, and switch to a more general non-affine setting in Section 4. We focus the analysis on BTC and then present the results for CRIX in Section 6 and the appendix as a robustness check.

3.1 Models

In order to calibrate the BTC dynamics with the SV and SVCJ models regarding returns and volatility, we employ the continuous time model of Duffie et al. (2000) that encompasses the standard jump diffusion and the SV with jumps in returns only (SVJ) model of Bates (1996). More precisely, let $\{S_t\}$ be the price process, $\{d \log S_t\}$ the log returns and $\{V_t\}$ be the volatility process. The SVCJ dynamics are as follows:

$$d \log S_t = \mu dt + \sqrt{V_t} dW_t^{(S)} + Z_t^y dN_t \tag{5}$$

$$dV_t = \kappa(\theta - V_t)dt + \sigma_V \sqrt{V_t} dW_t^{(V)} + Z_t^v dN_t \tag{6}$$

$$\mathsf{Cov}(dW_t^{(S)}, dW_t^{(V)}) = \rho dt \tag{7}$$

$$\mathrm{P}(dN_t = 1) = \lambda dt. \tag{8}$$

Like in the Cox-Ingersoll-Ross model, κ and θ are the mean reversion rate and mean reversion level, respectively. $W^{(S)}$ and $W^{(V)}$ are two correlated standard Brownian motions with correlation denoted as ρ. N_t is a pure jump process with a constant mean jump-arrival rate λ. The random jump sizes are Z_t^y and Z_t^v. Since the jump-driving Poisson process is the same in both (5), (6), the jump sizes can be correlated. The random jump size Z_t^y conditional on Z_t^v, is assumed to have a Gaussian distribution with a mean of $\mu_y + \rho_j Z_t^v$ and standard deviation set to σ_y. The jump in volatility Z_t^v is assumed to follow an exponential distribution with mean μ_v:

$$Z_t^y | Z_t^v \sim \mathrm{N}(\mu_y + \rho_j Z_t^v, \sigma_y^2); \quad Z_t^v \sim \exp(\mu_v). \tag{9}$$

The correlation ρ between the diffusion terms is introduced to capture the possible leverage effects between returns and volatility. The media and hype interventions may be correlated as well. The correlation term ρ_j takes care of that. The SV process $\sqrt{V}_t$ is modelled as a square root process. With no jumps in the volatility, the parameter θ is the long-run mean of V_t, and the process reverts to this level at a speed governed by the parameter κ. The parameter σ_V is referred to as the volatility of volatility, and it measures the variance responsiveness to diffusive volatility shocks. In the absence of jumps, the parameter μ measures the expected log-return.

It is a rich model since it also covers SV and SVJ approaches. If we set $Z_t^v = 0$ in (9), then jumps are only present in prices, we obtain the SVJ model of Bates (1996). Taking $\lambda = 0$ such that jumps are not present, the model reduces to the pure SV model originally proposed by Heston (1993). If we set $\kappa = \theta = \sigma_V = 0$ and define $Z_t^v = 0$, the model reduces to the pure jump diffusion introduced in Merton (1976).

3.2 Estimation: Markov Chain Monte Carlo (MCMC)

Several studies propose different methods to estimate option prices (or diffusion process). For example, Singleton and Umantsev (2002), Pan (2002) and others develop procedures based on the general method of moment (GMM) that exploit the known characteristic functions of affine models. Calibration of observed option prices with the help of a Fourier transformation has been advocated by Duffie et al. (2000). However, methods based on simulation have also been employed for the analysis of options pricing. These methods include the method of simulated moments of Duffie and Singleton (1993), the indirect inference methods of Gourieroux et al. (1993) and the efficient method of moment (EMM) method of Gallant and Tauchen (1996). The generality of simulation-based methods offers obvious advantages. For example, Jacquier et al. (1994) who propose a method for estimating discrete-time SV models from returns data. Their works show that MCMC is particularly well suited to deal with SV models. Jacquier et al. (2004) extend this idea into multivariate models. Eraker et al. (2003) and Eraker (2004) have also developed the MCMC-based estimation of jump-diffusion models using equity returns data. Eraker et al. (2003) identify several advantages of using the MCMC approach over other estimation models because MCMC methods are computationally efficient and the estimating is more flexible when using simulations. The MCMC method also provides more accurate estimates of latent volatility, jump sizes, jump times, etc. A general discussion and review of the MCMC estimation of continuous-time models can be found in Johannes and Polson (2009)

Here, we calibrate the SVCJ model using the MCMC method. Doing this allows for a wide class of numerical fitting procedures that can be steered by a variation of the priors. Given that there are no BTC options yet, the MCMC method is more flexible in calibrating the stochastic variance jumps and thus able to reflect the market price of risk (Franke et al. (2019). The empirical calibration is based on the following Euler discretization:

$$
\begin{aligned}
Y_t &= \mu + \sqrt{V_{t-1}}\varepsilon_t^y + Z_t^y J_t && (10)\\
V_t &= \alpha + \beta V_{t-1} + \sigma_V \sqrt{V_{t-1}}\varepsilon_t^v + Z_t^v J_t, && (11)
\end{aligned}
$$

where $Y_{t+1} = \log(S_{t+1}/S_t)$ is the log return, $\alpha = \kappa\theta$, $\beta = 1 - \kappa$ and ε_t^y, ε_t^v are the $\mathrm{N}(0, 1)$ variables with correlation ρ. J_t is a Bernoulli random variable with $\mathrm{p}(J_t = 1) = \lambda$ and the jump sizes Z_t^y and Z_t^v are distributed as specified in (9). The daily data sample from 01/08/2014 to 29/09/2017 is used to estimate the model. All returns are in decimal form.

Now we give a brief description on how to calibrate the SVCJ model with MCMC (see also Johannes and Polson (2009), Tsay (2005) and Asgharian and Bengtsson (2006) for more details). We define the parameter vector as $\Theta = \{\mu, \mu_y, \sigma_y, \lambda, \alpha, \beta, \sigma_v, \rho, \rho_j, \mu_v\}$ and $X_t = \{V_t, Z_t^y, Z_t^v, J_t\}$ as the latent variance, jump sizes and jump. Recall that Y_t is the log-returns.

The MCMC method treats all components of Θ and $X \stackrel{\text{def}}{=} \{X_t\}_{t=1,..,T}$ as random variables. The fundamental quantity is the joint pdf $p(\Theta, X|Y)$ of parameters and latent variables conditioned on data using the Bayes formula:

$$
\mathrm{p}(\Theta, X|Y) = \mathrm{p}(Y|\Theta, X)\,\mathrm{p}(X|\Theta)\,\mathrm{p}(\Theta). \tag{12}
$$

The Bayes formula can be decomposed into three factors: $\mathrm{p}(Y|\Theta, X)$, the likelihood of the data, $\mathrm{p}(X|\Theta)$ the prior of the latent variables conditioned on the parameters and $\mathrm{p}(\Theta)$ the prior of the parameters. The prior distribution $\mathrm{p}(\Theta)$ has to be specified beforehand and is part of the model specification. In comfortable settings, the posterior variation of the parameters, given the data, is robust with respect to the prior. We will touch on this point again when we display our empirical results.

The posterior is typically not available in closed form, and therefore simulation is used to obtain random draws from it. This is done by generating a sequence of draws, $\{\Theta^{(i)}, X_t^{(i)}\}_{i=1}^N$ which form a Markov chain whose equilibrium distribution equals the posterior distribution. The point estimates of parameters and latent variables are then taken from their sample means.

We use the same priors specified in Asgharian and Nossman (2011), who estimate a large group of international equity market returns with jump-diffusion models using the MCMC method. Their results have proved that the following priors make the posterior mean a wide range of possible realistic estimates: $\mu \sim \mathrm{N}(0, 25), (\alpha, \beta) \sim \mathrm{N}(0_{2\times1}, \mathbf{I}_{2\times2}), \sigma_2^V \sim \mathrm{IG}(2.5, 0.1), \mu_y \sim \mathrm{N}(0, 100), \sigma_2^y \sim \mathrm{IG}(10, 40), \rho \sim \mathrm{U}(-1, 1), \rho_j \sim \mathrm{N}(0, 0.5), \mu_V \sim \mathrm{IG}(10, 20)$ (Inverse Gaussian) and $\lambda \sim \mathrm{Be}(2, 40)$ (Beta Distribution). The full posterior distributions of the parameters and the latent-state variables can be found in Asgharian and Nossman (2011) and Asgharian and Bengtsson (2006). We have varied the variance of the priors and found stable outcomes, i.e., the reported mean of the posterior that is taken as an estimate of Θ is quite robust relative to changes in variance of the prior distributions. The posterior for all parameters except σ_V and ρ are all conjugate (meaning that the posterior distribution is of the same type of distribution as the prior but with different parameters). The posterior for J_t is a Bernoulli distribution. The jump sizes Z_t^y and Z_t^v follow a posterior normal distribution and a truncated normal distribution, respectively. Hence, it is straightforward to obtain draws for the joint distribution of J_t, Z_t^y and Z_t^v . However, the posteriors for ρ, σ_2^V and V_t are nonstandard distributions and must be sampled using the Metropolis-Hastings algorithm. We use the random-walk method for ρ and V_t , and independence sampling for σ_V^2. For the estimation of posterior moments, we perform 5000 iteratations, and in order to reduce the impact of the starting values we allow for a burn-in for the first 1000 simulations.

The SVCJ model is known for being able to disentangle returns related to sudden unexpected jumps from large diffusive returns caused by periods of high volatility. For the BTC situation that we consider here, we are particularly interested in linking the latent historical jump times to news and known interventions. The estimates $\hat{J}_t \stackrel{\text{def}}{=} (1/N) \sum_{i=1}^{N} J_t^i$ (where N is the total number of iterations and i refers to each draw) indicate the posterior probability that there is a jump at time t. Unlike the "true" vector of jump times, it will not be a vector of ones and zero. Following Johannes et al. (1999), we assert that a jump has occured on a specific date t if the estimated jump probability is sufficiently large, that is, greater than an approporiately chosen threshold value:

$$\tilde{J}_t = 1\{\hat{J}_t > \zeta\}, \quad t = 1, 2, ..., T \tag{13}$$

In our empirical study, we choose ζ so that the number of inferred jump times divided by the number of observations is approximately equal to the estimate of λ.

The SVCJ model is estimated with the daily BTC prices from 31/07/2014 to 29/09/2017. We first calculate returns (R_t) as the log difference between BTC prices, i.e., $R_t = ln(P_t) - ln(P_{t-1})$, where P_t is the BTC price at time t. Then we use returns to estimate the SVCJ model. The parameter estimates (mean and variance of the posterior) of the SVCJ, SVJ and SV models for BTC are presented in Table 4. The estimate of μ is positive. The correlation between returns and volatility ρ is significant and positive. This is remarkable and worth noting since it is different from a negative leverage effect observed over a sequence of studies in stock markets (see, e.g., Eraker (2004)). The effect is named the "inverse leverage effect" and has been discovered in commodity markets (see Schwartz and Trolled (2009)). In other words, the "inverse leverage effect" (associated with a positive ρ) implies that increasing prices are associated with increasing volatility. The reason for this positive relationship between risk and returns might be due to BTC prices being different from conventional stock prices. In general, currencies can be seen as standard economic goods that are priced by the interaction of supply and demand on the market. These prices are driven by the macroeconomic variables of an issuing country or institution (or entity) such as GDP, interest rates or inflation. As there are no macroeconomic fundamentals for digital currencies, the supply function is fixed (if the currency amount is fixed) or it evolves according to some publicly known algorithms, which is the case for the BTC market. The demand side of the market is not driven by an expected macroeconomic development of the underlying economy (as there is none). It is driven only by the expected profits of holding the currency and selling it later (as there are no profits from simply holding the currency due to digital currencies not bearing interest). The fundamental segment of the market is completely missing due to the fact that there are no fundamentals allowing for the setting of a "fair" price.

The digital currency price is thus driven solely by the investor faith in the perpetual growth, and it is not informative but rather driven by emotion and sentiment. This is the "noise trader" behavior described by Kyle (1985) and DeLong et al. (1990). Such investors, with no access to inside information, irrationally act on noise as if it were information that would give them an edge. This positive leverage effect has been also reported by a few studies on the Chinese stock market. For example, Hou (2013) has reported that, due to the lack of institutional investors, the trading values of the Shanghai and Shenzhen stock markets are completely generated by individual investors who have no access to inside information and irrationally act on noise. This causes good news (positive shocks perceived by investors) to the market returns affecting variances more than the bad news (negative shocks).

Moreover, the estimates for the SV model are much less extreme than for the SVJ and SV models. More precisely, the volatility of variance σ_v is substantially reduced from 0.017 (SV) to 0.011 (SVJ) and 0.008 (SVCJ). The mean of the jump size of the volatility μ_v is significant and positive. The jump intensity λ is also significant. The jump correlation ρ_j is negative but insignificant, which parallels the results of Eraker et al. (2003) and Chernov et al. (2003) for stock price dynamics. This effect might be due to the fact that even with a long data history, jumps are rare events. (See the estimate of ρ_j from the non-affine specifications in Section 4. We decided to leave the comparison between the SVCJ and non-affine models, and the relevant discussion for that section.) In summary, the SVCJ model fits the data well by a smaller MSE than the SVJ and SV models.

Figure 4 shows the estimated jump in returns (first row) and the jumps in volatility (middle row) together with the estimated volatility (last row). One sees that jumps occur frequently for the returns and volatility. Apparently, the jumps in the volatility process are much larger and more frequent than for the returns. Figure 5 presents the in-sample fitted volatility processes for the SVCJ and SVJ models, respectively. It is not hard to see that both models lead to a similar overall pattern for the volatility process, though the SVCJ model produces sharper peaks for BTC.

Table 4: BTC parameters for SVCJ, SVJ and SV models

	$SVCJ$	SVJ	SV
μ	0.041	0.029	0.030
	[0.022, 0.060]	[0.011, 0.046]	[0.014, 0.046]
μ_y	-0.084	-0.562	-
	[-0.837, 0.670]	[-1.280, 0.155]	-
σ_y	2.155	2.685	-
	[1.142, 3.168]	[1.519,3.850]	-
λ	0.041	0.029	-
	[0.025, 0.056]	[0.019, 0.047]	-
α	0.010	0.010	0.009
	[0.008, 0.012]	[0.006, 0.015]	[0.006, 0.012]
β	-0.132	-0.116	-0.033
	[-0.151 -0.114]	[-0.137 -0.094]	[-0.052 -0.013]
ρ	0.407	0.321	0.169
	[0.232, 0.583]	[0.225, 0.417]	[0.066, 0.271]
σ_v	0.008	0.011	0.017
	[0.007 0.010]	[0.007 0.014]	[0.014 0.021]
ρ_j	-0.573	-	-
	[-1.832, 0.685]	-	-
μ_v	0.620	-	-
	[0.426, 0.813]	-	-
MSE	0.735	0.757	0.763

Notes: This table reports posterior means and 95% finite sample credibility intervals (in square brackets) for parameters of the SVCJ, SVJ, and SV models. All parameters are estimated using BTC daily returns calculated as the log-first difference based on the prices from 31/07/2014 to 29/09/2017.

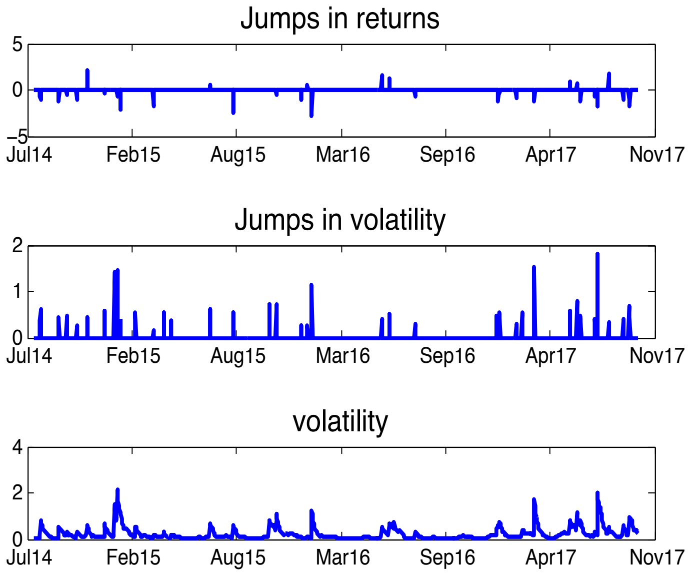
A useful model diagnosis is to examine the standardized residuals obtained from the discrete model, which estimates,

$$\varepsilon_t^y = \frac{Y_t - \mu - Z_t^y J_t}{\sqrt{V_{t-1}}} \sim N(0,1). \tag{14}$$

Once these standardized residuals are calculated based on the estimated parameters, they should, according to Equation (14), be approximately normally distributed. Figures 6 shows the QQ plots of the standardized residuals from the fitting of different models. From these diagnostics, it is evident that the GARCH and even the SV models are misspecified. For the SVJ and SVCJ models, the normal plot diagnostics are substantially improved. However, it is apparent that the SVCJ model is the preferred choice.

Finally Figure 7 graphs the 2.5th, 25th, 75th and 97.5th percentiles of the 5000 simulated prices paths of Bitcoin for each horizon up to 30 days based on the parameters reported in Table 4 for the SVCJ, SVJ, and SV models. The blue (red) colour line shows the $2.5th$ ($25th$) and

Figure 4: Jumps estimated in returns and volatility from the SVCJ model

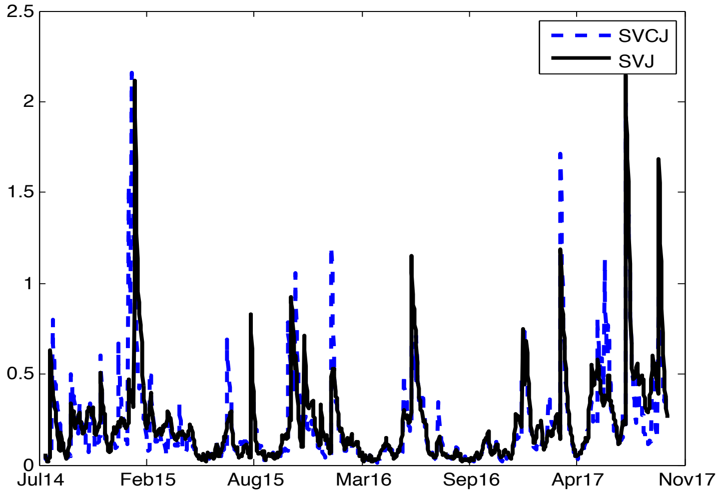


Notes: This figure graphs the estimated jumps in returns and volatility from the SVCJ model. The model is estimated using BTC daily returns calculated as the log-first difference based on the prices from 31/07/2014 to 29/09/2017. The first-, second-, and third-subfigures plot jumps in returns, jumps in volatility and the estimated volatility, respectively.

$97.5th$ ($75th$) interval of the simulated BTC prices. The blue (red) line with a sign of $\circ$, $\diamond$, $*$ shows the $2.5th$ ($25th$) and $97.5th$ ($75th$) forecast intervals for the SVCJ, SVJ, and SV models, respectively.

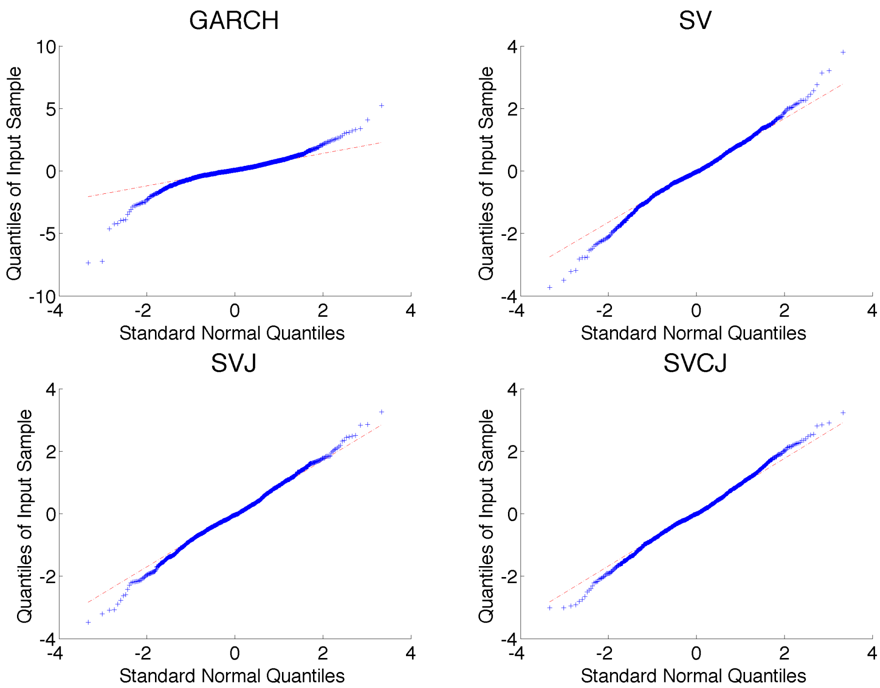
Visual inspection of Figure 7 suggests that among the three models considered, the confidence intervals of BTC prices generated by the SVCJ model are narrower, particularly at the 2.5% and 97.5% levels. In other words, the SVCJ model predicts a narrower confidence band of extreme BTC prices than the SVJ and SV models. This implies that the SVJ and SV models produce larger upper tails, i.e., these models overestimate the average BTC price compared to the prices predicted by the SVCJ model. On the other hand, the larger lower tails of the SVJ and SV models would imply that BTC would be underpriced compared to the SVCJ model.

Figure 5: Estimated volatility from the SVCJ and SVJ models



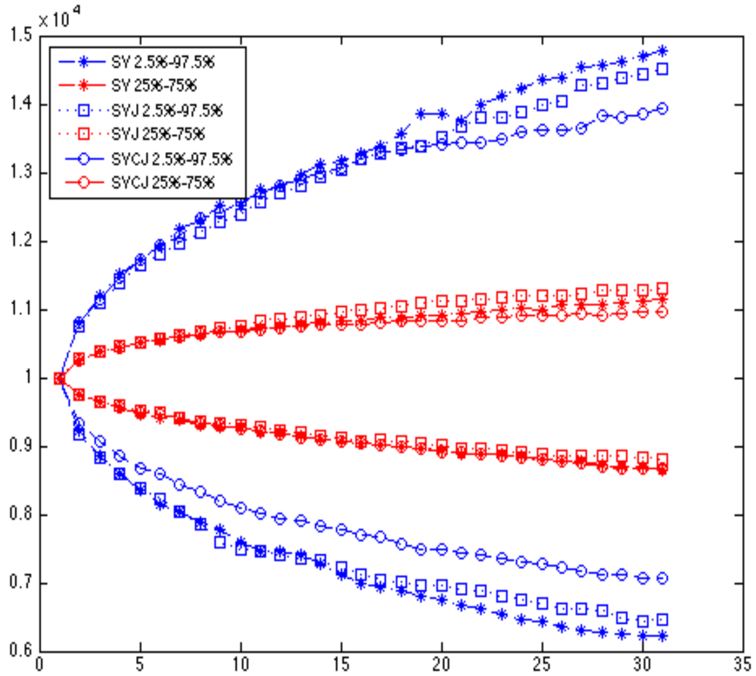
Notes: This figure plots the estimated volatility from the SVCJ (dotted blue) and SVJ (solid black) models. All models are estimated using BTC daily returns calculated as the log-first difference based on the prices from 31/07/2014 to 29/09/2017.

Figure 6: QQ plots for the SVCJ, SVJ and SV models



Notes: This figure graphs the QQ plots versus standard normal for fitted standardized residuals from the SVCJ, SVJ and SV models using BTC daily returns calculated as the log-first difference based on the prices from 31/07/2014 to 29/09/2017. We also include the QQ plot for the GARCH model using the same sample period.

Figure 7: Predicted confidence intervals of simulated observations for the SVCJ, SVJ and SV models



Notes: This figure graphs the confidence intervals of simulated BTC price paths up to 30 days based on the parameters reported in Table 4 for the SVCJ, SVJ and SV models. The blue (red) line shows $2.5th$ $(25th)$ and $97.5th$ $(75th)$ percentiles of the simulated BTC prices. The blue (red) line with a sign of $\circ$, $\diamond$, $*$ plots the $2.5th$ $(25th)$ and $97.5th$ $(75th)$ percentile for the SVCJ, SVJ and SV models, respectively.

4 SV model with jumps: non-Affine specification

4.1 Nonlinearity on return-volatility co-jumps

Imposing an affine structure in the stochastic process as documented in Section 3 may produce a specification error. Defining p_t and σ_t as the price and volatility process, respectively, following the notation of BR, we therefore consider the BR non-affine jump-diffusion model:

$$\begin{aligned} d\log(p_t) &= \mu(\sigma_t)dt + \sigma_t\{\rho(\sigma_t)dW_t^1 + \sqrt{1-\rho^2(\sigma_t)}dW_t^2\} \\ &\quad + c_{r,t}^J dJ_r + c_{r,t}^{JJ} dJ_{r,\sigma}, \\ d\xi(\sigma_t^2) &= m(\sigma_t)dt + \Lambda(\sigma_t)dW_t^1 + c_{\sigma,t}^J dJ_\sigma + c_{\sigma,t}^{JJ} dJ_{r,\sigma}, \end{aligned} \tag{15}$$

where $\xi(\cdot)$ is an increasingly monotonic function (we will choose it as $\log(\cdot)$ in the following discussions), $W = \{W^1, W^2\}$ is a bivariate standard Brownian motion vector and $J = \{J_r, J_\sigma, J_{r,\sigma}\}$ is a vector of mutually independent Poisson processes with state-dependent intensities, which are denoted as $\lambda_r(\sigma_t)$, $\lambda_\sigma(\sigma_t)$ and $\lambda_{r,\sigma}(\sigma_t)$, respectively. Thus we allow for common and independent jumps in the system. The Poisson processes are also assumed to be independent from the Brownian motion. In order to guarantee the existence and uniqueness of a solution to the system, it is assumed by BR that functions $\mu(\cdot)$, $m(\cdot)$, $\Lambda(\cdot)$, $\lambda_r(\cdot)$, $\lambda_\sigma(\cdot)$, $\lambda_{r,\sigma}(\cdot)$ and $\rho(\cdot)$ meet certain mild regularity conditions.

The BR model is estimated through the GMM method via estimated infinitesimal cross-moments. We assume the distribution of the jumps to be normal, i.e. $(c_{r,t}^J, c_{\sigma,t}^J) \sim \mathsf{N}(\mu^J, \Sigma^J)$ and $(c_{r,t}^{JJ}, c_{\sigma,t}^{JJ}) \sim \mathsf{N}(\mu^{JJ}, \Sigma^{JJ})$, with

$$\begin{aligned} \mu^J &= \begin{bmatrix} \mu_{J,r} \\ \mu_{J,\sigma} \end{bmatrix}, \quad \mu^{JJ} = \begin{bmatrix} \mu_{JJ,r} \\ \mu_{JJ,\sigma} \end{bmatrix}, \\ \Sigma^J &= \begin{bmatrix} \sigma_{J,r}^2 & 0 \\ 0 & \sigma_{J,\sigma}^2 \end{bmatrix}, \quad \Sigma^{JJ} = \begin{bmatrix} \sigma_{JJ,r}^2 & \rho_J \sigma_{JJ,r}\sigma_{JJ,\sigma} \\ \rho_J \sigma_{JJ,r}\sigma_{JJ,\sigma} & \sigma_{JJ,\sigma}^2 \end{bmatrix}. \end{aligned} \tag{16}$$

For any $p_1 \geq p_2 \geq 0$, the generic infinitesimal cross-moment of order p_1 and p_2 is defined as:

$$\theta_{p_1,p_2}(\sigma) = \lim_{\Delta \to 0} \frac{1}{\Delta} \mathsf{E}\{[\log(p_{t+\Delta}) - \log(p_t)]^{p_1}[\log(\sigma^2_{t+\Delta}) - \log(\sigma^2_t)]^{p_2} | \sigma_t = \sigma\}. \tag{17}$$

In particular $\theta_{p_1,0}$ helps identify features of the price process, and θ_{0,p_2} helps to identify those of the variance process, while the genuine cross-moments with $p_1 \geq p_2 \geq 1$ are required to identify the common parameter shared by the two processes ρ, $\lambda_{r,\sigma}$ and ρ_J.

To conduct the GMM estimation in BR, we first need to nonparametrically estimate the cross-moments that are in theory function(al)s of parameter (functions) of interest. The cross-moments are estimated via the nonparametric kernel method. In particular, denote the day index as $t = 1, ..., T$ and the equispaced time index as $i = 1, ..., N$ within each day. Denote $r_{t,i,k}$ as the high-frequency log returns for day t, knot i and minute k. We define the closing logarithmic prices as $\log(p_{t,i})$ and logarithmic spot variance estimates as

$$\hat{\sigma}^2_{t,i} = \frac{T}{T-1-n_j} \zeta_1^{-2} \sum_{k=2}^{T} |r_{t,i,k}||r_{t,i,k-1}| \mathbb{1}_{\{|r_{t,i,k}| \leq \theta_{t,i,k}\}} \mathbb{1}_{\{|r_{t,i,k-1}| \leq \theta_{t,i,k-1}\}}, \tag{18}$$

where $\zeta_1 \approx 0.7979$, $\theta_{t,i,k}$ is a suitable threshold, and n_j is the number of returns whose absolute value is greater than $\theta_{t,i,k}$. Then the generic cross-moment estimator $\hat{\theta}_{p_1,p_2}(\sigma)$ is defined as

$$\hat{\theta}_{p_1,p_2}(\sigma) = \frac{\sum_{t=1}^{T-1} \sum_{i=1}^{N} K(\frac{\hat{\sigma}_{t,i}-\sigma}{h})[\log(p_{t+1,i}) - \log(p_{t,i})]^{p_1}[\log(\hat{\sigma}^2_{t+1,i}) - \log(\hat{\sigma}^2_{t,i})]^{p_2}}{\Delta \sum_{t=1}^{T} \sum_{i=1}^{N} K(\frac{\hat{\sigma}_{t,i}-\sigma}{h})} \tag{19}$$

where $K(\cdot)$ is a kernel function and h is the bandwidth. Finally, with the estimated cross-moments, one can estimate the parameters of interest via the GMM method.

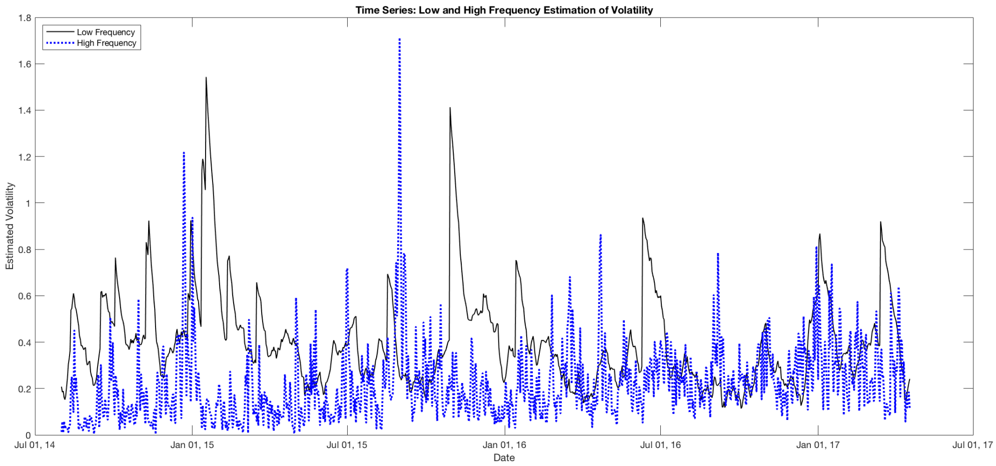
4.2 Estimation of and correspondence between two models

In this section, we fit the BR model using high-frequency data and discuss the comparison with the fitting of the SVCJ model. We collect high-frequency BTC prices from Bloomberg. The price data range is from 31/07/2014 to 29/07/2017, and we collect raw data at a frequency of

60 seconds 24 hours a day. Following Section 5.1 of BR, we aggregate the logarithm returns of Bitcoin over a 60-minute time range, namely $r_{t,i,k} = \log p_{t,i,k} - \log p_{t,i,k-1}$, with $k = 1, \cdots, 60$. In addition, we also obtain the spot variance estimates for each day t and each knot i by applying the jump robust threshold bipower variation estimator as in Equation (18).

To compare the data of the high-frequency aggregated volatility and the daily Bitcoin volatility, we plot the averaged daily spot volatility from the high-frequency data and the daily spot volatility estimates from the SVCJ model together as in Figure 8. We observe that the two sequences sometimes peak at a different time point despite that the general pattern agrees.

Figure 8: The averaged daily spot volatility and the daily spot volatility estimates.



Notes: This figure plots the averaged daily spot volatility from the high-frequency data (dotted line in blue) and the daily spot volatility estimates (solid line in black)

In Table 5, we show the full model estimation results. We note that in an unreported study we find that the nonparametric estimation suggests a parametric fit would be sufficient for the data. Therefore only the parametric fit results are shown. The drift parameter μ_r is estimated to be small and insignificant. The linear mean reversions, which can be seen as m_0 and m_1, are both negative. However they are both insignificant. The volatility of volatility Λ is estimated to be very significant with a value of 0.6766. The averaged number of independent jumps in volatility is estimated at an annual rate of $0.0519 * 252$, which is around 13. The estimated number of co-jumps is around $0.0584 * 252 \approx 17$. The mean of the independent variance jumps is significant at a level of -0.2783. $\mu_{JJ,r,0}$ is small (-0.0187) and negative, and $\mu_{JJ,r,1}$ is 0.1265.

Both parameters are insignificant at the 95% level of confidence. We do not see an obvious tendency for the jumps to be downward, as observed in Bandi and Renò (2016).

We find that the leverage ρ_0 is estimated to be negative, i.e., -0.1485, though insignificant. The leverage would increase with an increasing volatility level as ρ_1 is estimated to be significant and with a value of 0.9292. The standard deviation of the jumps in return $\sigma_{J,r}$ is estimated to be significant with the value of 0.6890. When fitting a nonlinear structure to the standard deviation of the common price jumps, the parameters $\sigma_{JJ,r,1}$ and $\sigma_{JJ,r,2}$ are both significant. The standard deviation of jumps in volatility $\sigma_{J,\sigma}$ is estimated to be 0.8619 with significance. The standard deviation of the common volatility jump $\sigma_{JJ,\sigma}$ is estimated to be insignificant. Notably, the correlation of jumps ρ_J is estimated to be negative and significant with a value of -0.5257, which is in line with BR. This negative and significant co-jump size correlation is discovered by Duffie et al. (2000), who conclude that the price and the volatility jump sizes are "nearly perfectly anti-correlated". Eraker (2004) finds a statistically significant correlation between the jump sizes only when employing option data in addition to stock returns data. Bandi and Renò (2016) also report a "nearly perfect anti-correlation" of -1.

As we use different assumptions than Bandi and Renò (2016), we can only find a subset of parameter correspondence. A direct comparison of some important parameters can be found in Table 6. We can see that for λ and $\lambda_{r,\sigma}$ the parameters agree in their significance and magnitude. For the volatility of the volatility parameter, both models indicate that it is significant. The drift in price is estimated by both models to be positive although they disagree in significance. In term of correlation of jumps, we find that our SVCJ model gives negative insignificant results while Bandi and Renò (2016) also give negative but significant result.

5 Option pricing

In the previous sections, we have shown that the SVCJ and the BR models can well describe the returns dynamics of BTC. In this section, we discuss the option prices underlying the BTC

Table 5: BR parametric estimates and their 95% confidence intervals.

	no cojumps	no ind. jumps	full model
μ_r	0.0021 [-0.1939, 0.1981]	0.0027 [-0.1933, 0.1987]	0.0082 [-0.0444, 0.0608]
ρ_0	0.0044 [-0.1150, 0.1237]	-0.0148 [-0.1401, 0.1105]	-0.1485 [-0.4851, 0.1882]
ρ_1	-0.3744 [-0.8513, 0.1025]	-0.2237 [-0.7088, 0.2614]	0.9292 [0.5884, 1.2699]
m_0	-0.0500 [-0.1275, 0.0275]	-0.0500 [-0.1275, 0.0275]	-0.0495 [-0.1475, 0.0485]
m_1	-0.0168 [-0.2128, 0.1792]	-0.0125 [-0.2085, 0.1835]	-0.0600 [-0.2560, 0.1360]
Λ	0.7634 [0.5674, 0.9594]	0.7853 [0.5893, 0.9813]	0.6766 [0.6570, 0.6963]
$\mu_{J,r}$	0.1577 [0.0372, 0.2782]	0 -	2.5486 [2.3526, 2.7446]
$\mu_{JJ,r,0}$	0 -	-0.0804 [-0.5383, 0.3774]	-0.0187 [-0.1085, 0.0711]
$\mu_{JJ,r,1}$	0 -	0.0192 [-0.6850, 0.7234]	0.1265 [-0.4183, 0.6713]
$\sigma_{J,r}$	0.6801 [0.5453, 0.8148]	0 -	0.6890 [0.4930, 0.8850]
$\sigma_{JJ,r,0}$	0 -	0.0864 [-0.3242, 0.4971]	0.0043 [-0.5459, 0.5544]
$\sigma_{JJ,r,1}$	0 -	1.8713 [1.8436, 1.8991]	1.2159 [1.0199, 1.4119]
$\sigma_{JJ,r,2}$	0 -	2.6521 [2.5377, 2.7664]	3.9590 [3.7630, 4.1550]
$\mu_{J,\sigma}$	-0.5000 [-0.5364,-0.4636]	0 -	-0.2783 [-0.4992, -0.0574]
$\mu_{JJ,\sigma}$	0 -	-1.9181 [-2.0805,-1.7557]	-0.4927 [-0.6429, -0.3425]
$\sigma_{J,\sigma}$	0.7945 [0.7379, 0.8511]	0 -	0.8619 [0.7237, 1.0001]
$\sigma_{JJ,\sigma}$	0 -	1.0705 [0.8716, 1.2693]	0.0717 [-0.0767, 0.2202]
ρ_J	0 -	-1.0000 [-1.4648, -0.5351]	-0.5257 [-0.7217, -0.3297]
λ_r	0.0002 [-0.1958, 0.1962]	0 -	0.0000 [-0.1960, 0.1960]
λ_σ	0.0700 [0.0504, 0.0896]	0 -	0.0519 [0.0323, 0.0715]
$\lambda_{r,\sigma}$	0 -	0.0060 [-0.0136, 0.0256]	0.0584 [0.0564, 0.0603]

Notes: This table reports the parameter estimates of the model specified in Equation (15) using the intra-daily BTC returns. For each parameter, we reports the estimate and the corresponding 95% finite sample credibility intervals in parentheses. The full model is shown in the forth column, and the second and third columns report the same model with the restriction of no co-jumps and no independent jumps, respectively.

with the SVCJ and BR models.

Table 6: Correspondences between estimated parameters

Hou et.al., (2019)		Bandi and Renò (2016)	
λ	0.041^{**}	$\lambda_{r,\sigma}$	0.0583^{**}
σ_v	0.008^{**}	Λ	0.6766^{**}
μ	0.041^{**}	μ_r	0.0082
ρ_j	-0.573	ρ_J	-0.5257^{**}

5.1 BTC options

After we fix the SVCJ parameters, we advance with a numerical technique called Crude Monte Carlo (CMC) to approximate the BTC option prices. Derivative securities such as futures and options are priced under a probability measure Q commonly referred to as the "risk neutral" or martingale measure. Since our purpose is to explore the impact of model choice on option prices, we follow Eraker et al. (2003) and set the risk premia to zero. This choice can be disputed, but for the lack of existence of the officially traded options a justifiable path to pricing BTC contingent claims. Suppose we have an option with a payoff at time of maturity T as $C(T)$, and typically for call option $C(T) = (S_T - K)^+$. The price of this option at time t is denoted as

$$\mathsf{E}_Q[\exp\{-r(T-t)\}C(T)|\mathcal{F}_t], \tag{20}$$

where $\mathcal{F}_t$ is a set that represents information up to time t. We approximate the European option prices of BTC using the CMC technique. The CMC simulation is done for 10000 iterations to approximate the option price using the parameters reported in Table 4 for the SVCJ, SVJ, and SV models. Since no BTC option market exists yet, we do not have real market option prices for comparison. Thus, we chose July 2017 randomly as the experimental month in our option-pricing simulation analysis. Throughout our entire analysis of option pricing, the moneyness for strike K and S at t is defined to be K/S_t. The pricing formula is a function of moneyness and time to maturity $\tau = (T-t)$ where T is the maturity day.

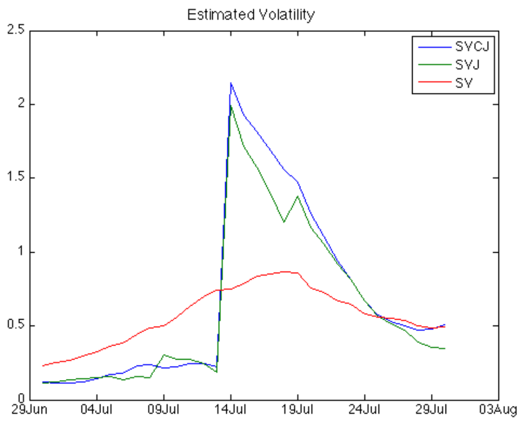
In Figure 9, we plot the simulated volatility of various models based on the parameters reported in Table 4 for the month of July 2017. It can be seen from this figure that the approximated volatility on 15 July, 2017, had a large jump (there was a large increase observed on 15 July, 2017, in the BTC historical prices). The sudden jump is perfectly captured by the SVCJ and SVJ models, while the SV model cannot characterize the volatility as well as the other two models. Assuming a BTC spot price St = 2250, the estimated BTC call option prices across moneyness and time to maturity on July 17, 2017, obtained using the SVCJ model[1] are presented in Table 7. We see that, for example, a call option on BTC with the strike $K = 1250$ and time to maturity of 90 days would be traded at 1157.95 on 17 July, 2017. It is obvious from Table 7 that the BTC option prices increase with time to maturity and decrease across moneyness from ITM to OTM. This is consistent with option prices in the real world and also observed in the equity option markets.

To further understand how the option price changes with respect to changes in time to maturity and moneyness for different models, we show in Figure 10 the one-dimensional contour plot of the option prices surface across time to maturity and moneyness estimated from the SVCJ, SVJ and SV models for the month of July 2017. When examining moneyness, the time to maturity is fixed at 30 days, and when looking at the time to maturity, moneyness is fixed at at-the-money (ATM). We can see from the contour plot that the relationship between the option price and the time to maturity or moneyness varies over time for all three models. However, one easily seen pattern is that the approximated option price is higher when the volatility is higher, i.e., the colour of the contour plot is brighter. This is especially the case for options varying across time to maturity. The SVCJ model has the most volatile pattern among the three models. This figure conveys the homogeneous message as we can see from Figure 9 in the volatility plots. For example, for the BTC price, we see a drastic change in the contour structure on, e.g., 15 July, 2017 as the price suddenly drops from 2232.65 USD on $15/07/2017$ to 1993.26 USD. The sudden drop in price should be attributed to the big jump in volatility shown in Figure 9, and we can also observe this jump on 15 July in Figure 10.

Figure 11 displays the BTC call option price differences between the estimated option price from the SVCJ and SVJ models with respect to changes in moneyness and across time to maturity for July 2017. It is not hard to see that the pattern is similar to the fitted volatility shown

[1]We have also calculated option prices for the SVJ and SV models. These results are available upon requests

Figure 9: Estimated volatility of BTC for July 2017: BTC



Notes: This figure plots the estimated volatility of the SVCJ, SVJ and SV models. The volatility is approximated based on the parameters reported in Table 4 for the month of July 2017. The x-axis notes the dates in July 2017. The blue/green/red line plots the volatility from the SVCJ, SVJ and SV models.

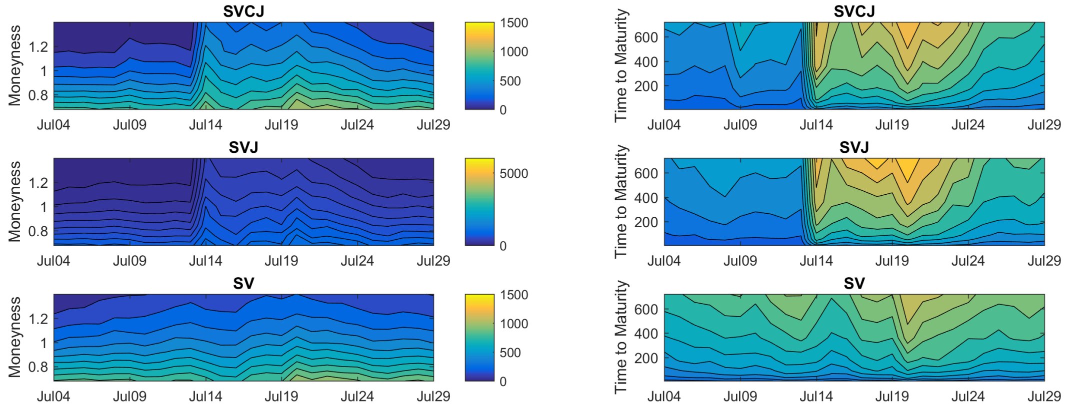
in Figure 9. Therefore, the price differences between the SVCJ and SVJ models are mainly caused by the jumps in the volatility process and the volatility level, which reflects the necessity of adopting an SVCJ model in practice.

5.2 BTC implied volatility smiles

It is well known that stochastic volatility determines excess kurtosis in the conditional distribution of returns. The excess kurtosis causes symmetrically higher implied Black Scholes volatility when strikes are away from the current prices, e.g., the level of moneyness is away from the ATM level. This phenomenon is called the "volatility smile". It is well documented in the existing literature that the effect is stronger for short and medium maturity options than for long maturity options for which the conditional returns are closer to normal (Das and Sundaram (1999)). The presence of co-jumps, and the negative correlation between the presence of co-jumps sizes yield additional sources of skewness in the conditional distribution of stock returns (Bandi and Renò (2016)).

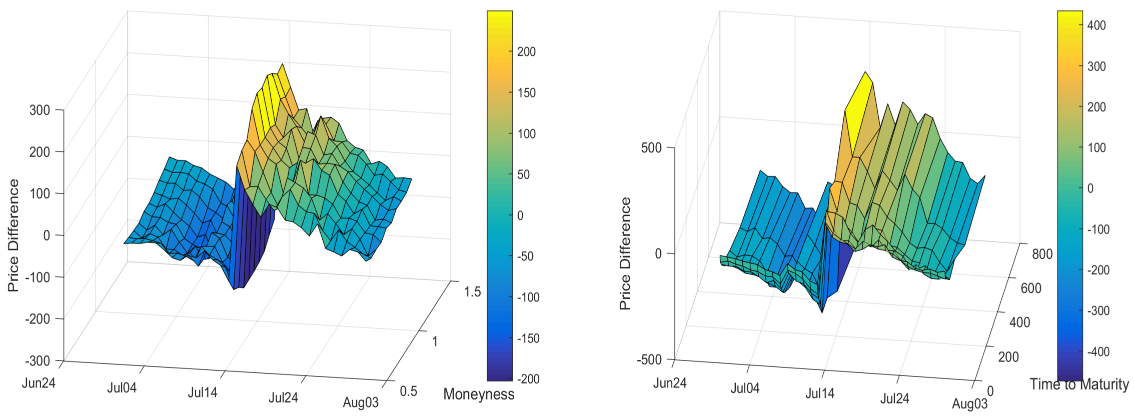
To further examine the option-pricing property of BTC, we approximate the implied Black Scholes volatility from various models for different degrees of moneyness (strike/spot) and different times to maturity. First, the European call option prices are simulated using the model

Figure 10: Call option prices across moneyness and time to maturity: BTC



Notes: This figure graphs the call option prices surface counterplot across different moneyness and different times to maturities for the month of July 2017, as shown in the right-hand side labels. When looking at moneyness, the time to maturity is fixed at 30 days, and when looking at the time to maturity, moneyness is ATM. The colour in the graph represents the price level; the brighter the colour, the higher the price.

Figure 11: Call option price differences between thr SVJ and SVCJ models: BTC



Notes: This figure plots the option price differences between the SVCJ and SVJ models for July 2017. When looking at moneyness, the time to maturity is fixed at 30 days, and when looking at the time to maturity, moneyness is ATM. The colour in the graph represents the price difference level; the brighter the colour, the larger the difference between the price from the SVCJ and SVJ models.

Table 7: Call option price of BTC on $17/07/2017$ from the SVCJ model

$K\backslash\tau$	1	7	30	60	90	180	360	720
1250.00	1069.18	1017.81	1099.87	1125.90	1157.95	1248.98	1361.04	1365.96
1350.00	959.02	959.02	1006.02	1066.67	1094.08	1224.48	1302.60	1316.03
1450.00	885.20	860.15	929.32	995.45	1046.89	1099.35	1258.83	1438.90
1550.00	802.38	791.34	901.27	950.34	1015.76	1114.94	1192.24	1332.08
1650.00	707.97	739.10	825.07	882.17	902.32	1062.17	1175.59	1282.36
1750.00	625.86	678.22	786.88	856.72	896.56	962.79	1192.61	1338.49
1850.00	552.26	618.94	697.11	785.62	862.83	897.74	1110.36	1289.51
1950.00	502.28	545.58	663.47	740.32	819.72	903.60	1052.09	1229.45
2050.00	425.46	511.28	629.14	741.65	772.51	905.30	1027.76	1193.43
2150.00	358.30	460.57	597.44	683.55	740.64	870.66	1036.76	1164.23
2250.00	302.88	408.62	543.02	633.31	720.57	872.42	938.68	1051.71
2350.00	265.91	378.10	492.86	594.01	651.03	783.37	887.62	1064.33
2450.00	211.26	347.79	470.85	580.30	657.43	761.39	940.90	1085.75
2550.00	193.69	304.13	437.06	547.15	608.36	766.19	914.62	1101.72
2650.00	156.38	266.64	421.86	518.27	571.42	719.92	827.17	992.20
2750.00	136.24	247.38	397.92	484.70	556.31	651.86	863.10	1066.75
2850.00	135.28	228.47	345.42	465.75	541.61	672.76	788.25	955.97
2950.00	100.02	202.57	341.11	413.75	488.15	627.52	780.53	917.27
3050.00	103.45	179.93	313.83	424.23	496.05	619.88	758.99	911.33
3150.00	82.59	162.72	290.90	371.20	450.85	593.10	752.88	888.89
3250.00	72.93	140.40	273.97	358.26	442.91	571.96	726.49	933.57

Notes: This table reports the approximated call option prices at different time to maturity τ and strike prices K the SVCJ model on 17/07/2017 based on the parameters reported in Table 4. The numbers in the first row are the time to maturity. The numbers in the first column are the strike prices. The spot BTC price is assumed to be 2250.

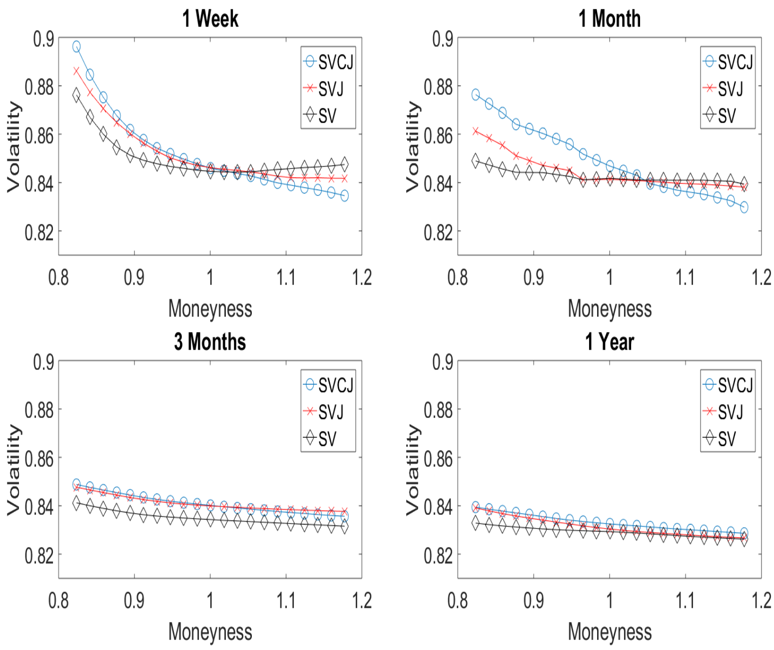
parameters reported in Table 4 for the SVCJ, SVJ and SV models. Then the volatility from various models is implied from the Black Scholes model based on the options approximated from different models. We consider four times to maturity: one week, one month, three months and one year. We report the implied volatility surface as a function of moneyness and time to maturity. The results indicate that jumps in returns and volatility include important differences in the shape of the implied volatility (IV) curves, especially for the short maturities options.

Figure 12 shows the IV curves for the SVCJ, SVJ and SV models for four different maturities and across moneyness. It can be seen from Figure 12, that adding jumps in returns steepens the slope of the IV curves. Jumps in volatility further steepen the IV curves. For short maturity options, the difference between the SVCJ, SVJ and SV models for far ITM options is quite large, with the SVCJ model giving the sharpest skew among the three models. The difference between the SVCJ and SV volatility is approximately 2-3% for up to one month. It is interesting to note that all three models have a one-side volatility skew, implying that ITM options prices are higher than the OTM options. This could be due to the skewness in the conditional distribution of BTC returns (Das and Sundaram (1999)) and/or that the negative co-jump size yields an additional source of skewness (Bandi and Renò (2016)). As time to maturity increases, the volatility curve flattens for all models. According to Das and Sundaram (1999), jumps in returns result in a discrete mixture of normal distributions for returns, which easily generates unconditional and conditional nonnormalities over short frequencies such as daily or weekly. Over longer intervals, e.g., more than a month, a central-limit effect results in decreases in the amount of excess and kurtosis. Indeed, diffusive stochastic volatility models may generate very flat curves, such as a flat BTC IV for three-month and one-year times to maturity.

However, for the SVCJ model, the curve flattens at a slightly higher level. The implied volatility of the SVJ model is closer to the SVCJ model than the SV model. The difference between the SVCJ, SVJ and SV models becomes larger with short time to maturity options, i.e., the one-week and one-month times to maturity. Similar results have been documented in other studies in which these models have been applied to equity index data. Eraker et al. (2003), Eraker (2004) and Duffie et al. (2000) find that jumps in returns and variance are important in capturing systematic variations in Black Scholes volatility. In general, although the BTC market has the unique feature of having more jumps, which makes it different from other mature markets (e.g., equity), the option prices and the IV from the affine models generally follow the conventional characteristics reported from other option markets.

We have also estimated the BR IVs with the same time to maturity and moneyness used for the SVCJ IVs. We simulate the option prices using the model parameter reported in column 4 of Table 5. We distinguish the case of ρ_J, which is set to be a model-fitted parameter from the SVCJ fit or to be zero, i.e., the IV surface corresponds to a case with a correlation between jump sizes equaling -0.5257 or a correlation between jump sizes equaling to zero. The IVs as a function of moneyness from the BR model are plotted in Figure 13. We can see that the IVs of the BR model agree with the SVCJ model. We see a one-side volatility skew, implying that the ITM call option prices are higher than the OTM call options. However, due to the significantly negative jump-size correlation ρ_J, the slope of the IVs from the BR full model is much steeper than the BR model with a case of uncorrelated jump sizes. The impact of the negative jump size correlation is much stronger for short time to maturity options, i.e., the one-week and one-month times to maturity. This is mentioned in the results of Duffie et al. (2000) as well, who find

Figure 12: The IV for the BTC market: the SVCJ, SVJ and SV models



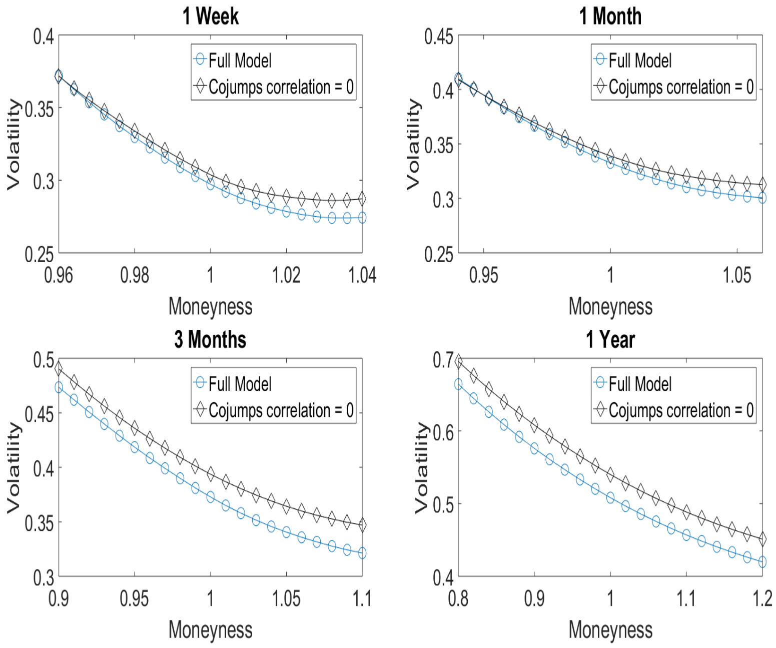
Notes: This figure plots the Black Scholes IV for the BTC market based on the SVCJ, SVJ and SV models. The x-axis shows moneyness and the y-axis shows the IV. Four times to maturity have been considered: one week, one month, three months and one year. The lines with $\circ$, $*$, $\diamond$ plots the IVs of the SVCJ, SVJ and SV models, respectively.

a superior fit of the IV smirk when calibrating a more negative correlation between jump sizes. Similarly, Eraker (2004) finds a statistically significant correlation between jump size only when employing option data in addition to returns data. Bandi and Renò (2016) also shows that anti-correlated jump sizes are a fundamental property of prices and volatility. However, the use of high-frequency data is sufficient to reveal this property with no further need for option data.

6 Robustness check using the CRyptocurrency IndeX (CRIX)

The CRyptocurrency IndeX, a value-weighted cryptocurrency market index with an endogenously determined number of constituents using some statistical criteria, is created by Trimborn and Härdle (2018). It is constructed to track the entire cryptocurrency market performance as closely as possible. The representativity and the tracking performance can be assured as CRIX considers a frequently changing market structure. The reallocation of the CRIX happens on a

Figure 13: The IV for the BTC market: BR model

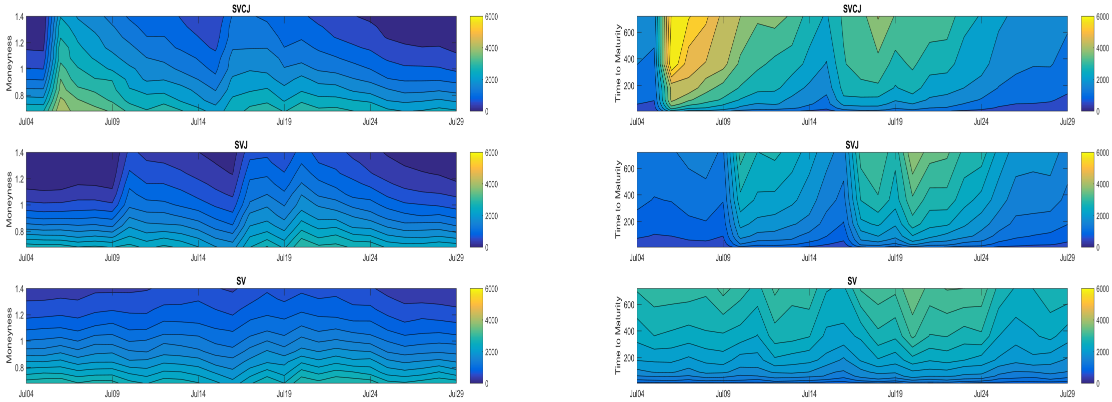


Notes: This figure plots the implied Black Scholes volatility for the BTC option prices based on the BR model. The x-axis shows moneyness, and the y-axis shows the IV. Four times to maturity have been considered: one week, one month, three months and one year. The IVs are based on the simulated option prices using the model parameters reported in Table 5. The full model uses parameters from column 4 of Table 5. A co-jumps correlation of 0 means that ρ_J is set to zero while the other parameters remain the same as in the full model.

monthly and quarterly basis (see Trimborn and Härdle (2018) and `thecrix.de` for details). CRIX has been widely investigated in the pioneering research on cryptocurrencies, including by Hafner (2018), Chen and Hafner (2019) and da Gama Silva et al. (2019). Here we perform an analysis of CRIX since this additional investigation is beneficial from a robustness point of view. Analogous to the investigation of BTC dynamics in Section 2, the econometric treatment for CRIX is implemented and reported. In brief, we summarize our major findings here. (See the appendix for the relevant figures and tables.) Firstly, we find that the shape estimates in the ARIMA-t-GARCH model along with the QQ plots shown in the appendix indicate a fatter tail in the return distribution of CRIX compared to that of BTC. Some constituents, typically altcoins (alternatives to bitcoin), in the CRIX may behave more extremely than BTC does. This indicates that the CRIX makes itself riskier than BTC as altcoins (smaller capitalization) are typically riskier than BTC (bigger capitalization). We can parallel this inference to the size effect of stocks. The calibration of the SVCJ model in Table 8 conveys a similar configuration. The mean jump size of the CRIX volatility process is reported as 0.709, which is relatively higher than 0.620 for BTC shown in Table 4.

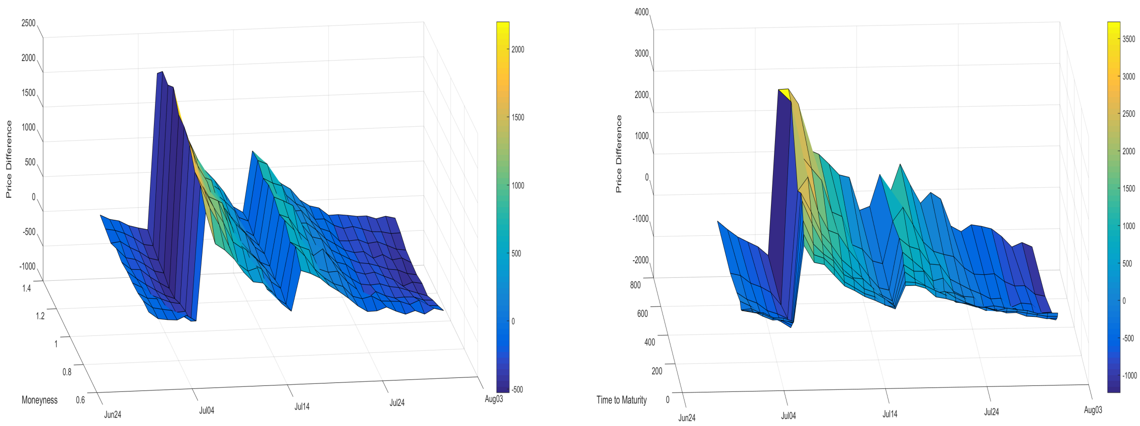
In addition, we report a pricing analysis of CRIX options. Considering a class of SV models, the fitted call option prices during July 2017 across moneyness and times to maturity are shown in Figure 14. Needless to say, a salient rise in the approximated volatility is clearly evident on July 17, 2017. The sudden jump is perfectly captured by the SVCJ and SVJ models but not by the SV model. More specifically, Figure 15 displays the price difference between the estimated option price from the SVJ, SV and SVCJ models with respect to changes in moneyness and across time to maturity. The price gap between the SVJ and SVCJ models is attributed to the presence of jumps in the volatility process and the level of volatility, which again is in favor of the SVCJ model applied to the crypto market index. In general, we confirm the consistency between BTC and the CRIX.

Figure 14: Call option prices across moneyness and time to maturity: CRIX



Notes: This figure graphs the call option prices surface counterplot across moneyness and time to maturities for the month of July 2017 for CRIX. When looking at moneyness, the time to maturity is fixed at 30 days, and when looking at the time to maturity, moneyness is ATM. The colour in the graph represents the price level; the brighter the colour, the higher the price.

Figure 15: Call option price differences between the SVJ and SVCJ models: CRIX



Notes: This figure plots the CRIX call option price differences between the SVCJ and SVJ modesl for July 2017. When looking at moneyness, the time to maturity is fixed at 30 days, and when looking at time to maturity, the moneyness is at-the-money. The colour in the graph represents the price difference level; the brighter the colour, the larger the difference between the price from the SVCJ and SVJ models.

Table 8: Parameters for the SVCJ, SVJ and SV models: CRIX

	$SVCJ$	SVJ	SV
μ	0.042	0.0437	0.017
	[0.030, 0.054]	[0.027, 0.061]	[0.000 0.034]
μ_y	-0.0492	-0.515	-
	[-0.777, 0.678]	[-1.110, 0.079]	-
σ_y	2.061	2.851	-
	[1.214, 2.907]	[1.349, 4.354]	-
λ	0.0515	0.035	-
	[0.038, 0.065]	[0.017, 0.052]	-
α	0.0102	0.026	0.010
	[0.009, 0.012]	[-0.012 0.063]	[0.007 0.012]
β	-0.188	-0.240	-0.038
	[-0.205, -0.170]	[-0.383, -0.096]	[-0.056 -0.020]
ρ	0.275	0.214	0.003
	[0.140, 0.409]	[0.014, 0.415]	[-0.130 0.136]
σ_v	0.007	0.016	0.018
	[0.005, 0.009]	[-0.001, 0.033]	[0.014 0.022]
ρ_j	-0.210	-	-
	[-0.924, 0.503]	-	-
μ_v	0.709	-	-
	[0.535, 0.883]	-	-
MSE	0.673	0.707	0.736

Notes: The table reports posterior means and 95% credibility intervals (in square brackets) for the parameters of the SVCJ, SVJ and SV models. All parameters are estimated using CRIX daily returns calculated as the log difference based on the prices from 31/07/2014 to 29/09/2017.

7 Conclusion

"The Internet is among the few things that humans have built that they do not truly understand" according to Schmidt and Cohen (2017). Cryptocurrency, a kind of innovative internet-based asset, brings new challenges but also new ways of thinking for economists, cliometricians and financial specialists. Unlike classic financial markets, the BTC market has a unique market microstructure created by a set of opaque, unregulated, decentralized and highly speculation driven markets.

This study provides a way of pricing crytocurrency derivatives using advanced option-pricing models such as the SVCJ and BR models. We find that in general, the SVCJ model performs as well as the non-affine BR model. Consistent with the evidence from the equity market, we find that the correlation between the jump sizes in returns and the volatility process is anti-correlated. The jump-size correlation is statistically (marginally) negative in the BR (SVCJ) model. Deviating from the equity market, we cannot obtain a significant negative "leverage effect" parameter ρ, which implies a nonnegative relation between returns and volatility. The reason for this relationship might be that BTC is different from the conventional stock market, not only because the BTC market is highly unregulated but also due to the fact that the BTC price is not informative (as there are no fundamentals allowing the BTC market to set a "fair" price) and is driven by emotion and sentiment. This speculative behaviour can be explained by the "noise trader" theory from Kyle (1985). The positive relation might result from the fact that BTC investors irrationally act on noise as if it were information that would give them an edge.

We find that option prices are very much driven by jumps in the returns and volatility processes and co-jumps between the returns and volatility. This can be seen from the shape of the IV curves. This study provides a grounding base, or an anchor, for future studies which aim to price cryptocurrency derivatives. This study provides useful information for establishing an options market for BTC in the near future.

Coefficients	Estimates	robust std	t value
CRIX			
ω	$4.93e-05$	$2.69e-05$	1.83
α_1	$2.23e-01$	$4.28e-02$	5.45
β_1	$7.76e-01$	$5.62e-02$	13.81
ν	$3.10e+00$	$2.19e-01$	14.15

Table 9: Estimated coefficients of t-GARCH(1,1) model

Coefficients	Estimates	robust std	t value
CRIX			
ω	$4.93e-05$	$2.69e-05$	1.83
α_1	$5.58e-02$	$4.34e-02$	1.34
β_1	$9.62e-01$	$1.38e-02$	69.43
ϕ_1	$5.36e-01$	$1.39e-01$	3.85
ν	$2.42e+00$	$2.42e-01$	10.02

Table 10: Estimated coefficients of t-EGARCH(1,1) model

8 Appendix: Collected CRIX results

This appendix presents the empirical results of CRIX covering (1) the econometric analysis of its dynamics shown in Tables 9 and 10 (2) jumps in returns and volatility from the SVCJ model shown in Figure 16 and (3) the estimated volatility from the SVCJ and SVJ models shown in Figure 17. In general, a general consistency can be found between the CRIX and BTC.

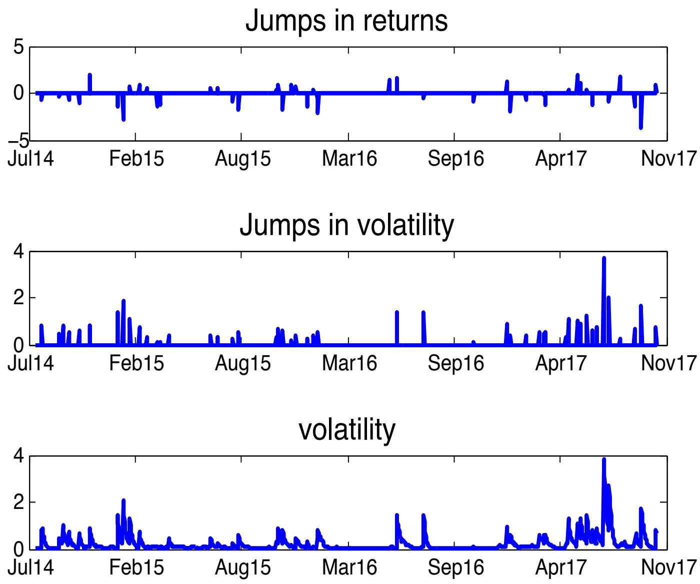


Figure 16: Jumps estimated in returns and volatility from the SVCJ model: CRIX

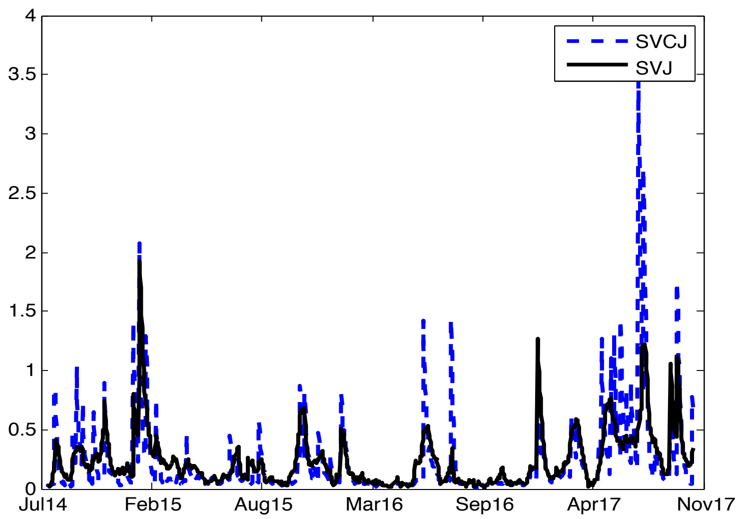


Figure 17: Estimated volatility from the SVCJ and SVJ models: CRIX

References

Aït-Sahalia, Y., J. J. (2010). Is brownian motion necessary to model high-frequency data? *The Annals of Statistics*, 38:3093–3128.

Ali, R., B. J. C. R. S. J. (2014). The economics of digital currencies. *Bank of England Quarterly Bulletin*, Q3(2):276–287.

Asgharian, H. and Bengtsson, C. (2006). Jump spillover in international equity markets. *Journal of Financial Econometrics*, 4(2):167–203.

Asgharian, H. and Nossman, M. (2011). Risk contagion among international stock markets. *Journal of International Money and Finance*, 30(1):22–38.

Badev, A., C. M. (2014). Bitcoin: Technical background and data analysis. *Federal Reserve Board, Washington D.C.*, n. 2014(104).

Bajgrowicz, P., Scaillet, O., and Treccani, A. (2015). Jumps in high-frequency data: Spurious detections, dynamics, and news. *Management Science*, 62(8):2198–2217.

Bakshi, G., J. N. and Ou-Yang, H. (2006). Do stock prices and volatility jump? reconciling evidence from spot and option prices. *Journal of Financial Econometrics*, LIX(4):1–30.

Bandi, F. M. and Renò, R. (2016). Price and volatility co-jumps. *Journal of Financial Economics*, 119(1):107–146.

Barndroff-Nielsen, O.E., S. N. (2006). Econometrics of testing for jumps in financial economics using bipower variation. *Journal of Fianncial Econometrics*, 4:1–30.

Bates, D. S. (1996). Jumps and stochastic volatility: Exchange rate processes implicit in Deutsche mark options. *The Review of Financial Studies*, 9(1):69–107.

Bates, D. S. (2000). Post-'87 crash fears in the S & P 500 futures option market. *Journal of Econometrics*, 94(1-2):181–238.

Becker, J., Breuker, D., Heide, T., Holler, J., Rauer, H. P., and Böhme, R. (2013). Can we afford integrity by proof-of-work? Scenarios inspired by the bitcoin currency. In *The Economics of Information Security and Privacy*, pages 135–156.

Bouoiyour, J., S. R. T. A. (2019). Is bitcoin business income or speculative foolery? New ideas through an improved frequency domain analysis. *Annals of Financial Economics*.

Chan, S., Chu, J., Nadarajah, S., and Osterrieder, J. (2017). A statistical analysis of cryptocurrencies. *Journal of Risk and Financial Management*, 10(2):12.

Chen, C. Y.-H. and Hafner, C. M. (2019). Sentiment-induced bubbles in the cryptocurrency market. *Journal of Risk and Financial Management*, 12(2):53.

Chen, S., Chen, C. Y.-H., Härdle, W. K., Lee, T., and Ong, B. (2017). A first econometric analysis of the crix family. *Handbook of Digital Finance and Financial Inclusion: Cryptocurrency, FinTech, InsurTech, and Regulation*.

Chernov, M., Gallant, A. R., Ghysels, E., and Tauchen, G. (2003). Alternative models for stock price dynamics. *Journal of Econometrics*, 116(1):225–257.

Chourdakis, K. Dotsis, G. (2011). Maximium likelihood estimation of non-affine volatility processes. *Journal of Empirical Finance*, 18:533–545.

Chu, J., Chan, S., Nadarajah, S., and Osterrieder, J. (2017). GARCH modelling of cryptocurrencies. *Journal of Risk and Financial Management*, 10(4):17.

Conrad, C., Custovic, A., and Ghysels, E. (2018). Long-and short-term cryptocurrency volatility components: A GARCH-MIDAS analysis. *Journal of Risk and Financial Management*, 11(2):23.

Cosma, A., G. s., Pederzoli, P., and Scaillet, O. (2018). Early exercise decision in American options with dividends, stochastic volatility, and jumps. *Journal of Financial and Quantitative Analysis*, pages 1–56.

da Gama Silva, P. V. J., Klotzle, M. C., Pinto, A. C. F., and Gomes, L. L. (2019). Herding behavior and contagion in the cryptocurrency market. *Journal of Behavioral and Experimental Finance*, 22:41–50.

Das, S. and Sundaram (1999). Of smiles and smirks: A term structure perspective. *Journal of Financial and Quantitative Analysis*, 34(2):211–239.

DeLong, J., Shleifer, A., Summers, L., and Waldmann, R. (1990). Noise trader risk in financial markets. *Journal of Political Economy*, 98:703–738.

Duan, J.-C. (1995). The GARCH option pricing model. *Mathematical Finance*, 5(1):13–32.

Duffie, D., Pan, J., and Singleton, K. (2000). Transform analysis and asset pricing for affine jump-diffusions. *Econometrica*, 68(6):1343–1376.

Duffie, D. and Singleton, K. (1993). Simulated moments estimation of markov models of.

Dwyer, G. P. (2015). The economics of bitcoin and similar private digital currencies. *Journal of Financial Stability*, 17:81–91.

ECB (2015). Virtual currency schemes- a further analysis. *ECB report*, February.

Elwell, C. K., Murphy, M. M., Seitzinger, M. V., and Murphy, E. V. (2013). Bitcoin: questions, answers, and analysis of legal issues.

Engle, R. F. and Ng, V. K. (1993). Measuring and testing the impact of news on volatility. *The Journal of Finance*, 48(5):1749–1778.

Eraker, B. (2004). Do stock prices and volatility jump? Reconciling evidence from spot and option prices. *The Journal of Finance*, 59(3):1367–1403.

Eraker, B., Johannes, M., and Polson, N. (2003). The impact of jumps in volatility and returns. *The Journal of Finance*, 58(3):1269–1300.

Franke, J., Härdle, W. K., and Hafner, C. M. (2019). *Statistics of Financial Markets: An Introduction 4th ed.* Springer Verlag Heidelberg.

Gallant, A. and Tauchen, G. (1996). Which moments to match? *Econometric Theory*, 12:657–681.

Geweke, J. (1986). Modelling the persistence of conditional variances: A comment. *Econometric Reviews*, 5(1):57–61.

Gourieroux, C., Monfort, A., and Renault, E. (1993). Indirect inference. *Journal of Applied Econometrics*, 8(S1):S85–S118.

Hafner, M. C. (2018). Testing for bubbles in cryptocurrencies with time-varying volatility. *Journal of Financial Econometrics*, forthcoming.

Hayes, A. S. (2017). Cryptocurrency value formation: An empirical study leading to a cost of production model for valuing bitcoin. *Telematics and Informatics*, 34(7):1308–1321.

Heston, S. L. (1993). A closed-form solution for options with stochastic volatility with applications to bond and currency options. *Review of Financial Studies*, 6(2):327–343.

Heston, S. L. and Nandi, S. (2000). A closed-form GARCH option valuation model. *Review of Financial Studies*, 13(3):585–625.

Hotz-Behofsits, C., Huber, F., and Zörner, T. O. (2018). Predicting crypto-currencies using sparse non-gaussian state space models. *Journal of Forecasting*, 37(6):627–640.

Hou, A. J. (2013). Asymmetry effects of shocks in chinese stock markets volatility: A generalized additive nonparametric approach. *Journal of International Financial Markets, Institutions & Money*, 23:12–32.

Jacquier, E., Polson, N., and Rossi, P. (1994). Baysian analysis of stochastic volatility models. *Journal of Business and Economic Statistics*, 12:371–389.

Jacquier, E., Polson, N., and Rossi, P. (2004). Baysian analysis of stochastic volatility models with fat-tails and correlated errors. *Journal of Econometrics*, 122:185–212.

Jocod, J. and Todorov, V. (2009). Testing for common arrivals of jumps for discretely observed multidimensional processes. *The Annals of Statistics*, 37:1792–1838.

Johannes, M. and Polson, N. (2009). Mcmc methods for continuous-time financial econometrics. *Handbook of Financial Econometrics*, 2(1).

Johannes, M., Rohit, K., and Polson, N. (1999). State dependent jump models: How do u.s. equity markets jump? *working paper, University of Chicago*.

Kristoufek, L. (2013). Bitcoin meets google trends and wikipedia: Quantifying the relationship between phenomena of the internet era. *Scientific Reports*, 3.

Kroll, J. A., Davey, I. C., and Felten, E. W. (2013). The economics of bitcoin mining, or bitcoin in the presence of adversaries. In *Proceedings of WEIS*, volume 2013, page 11.

Kyle, A. (1985). Continuous auctions and insider trading. *Econometrica*, 53:1315–1336.

Merton, R. C. (1976). Option pricing when underlying stock returns are discontinuous. *Journal of Financial Economics*, 3(1-2):125–144.

Nelson, D. B. (1991). Conditional heteroskedasticity in asset returns: A new approach. *Econometrica: Journal of the Econometric Society*, pages 347–370.

Pan, J. (2002). The jump-risk premia implicit in options: Evidence from an integrated time-series study. *Journal of Financial Economics*, 63(1):3–50.

Pantula, S. G. (1986). Comment. *Econometric Reviews*, 5(1):71–74.

Rogojam, A., B. L. (2014). The issue of competing currencies. Case study-bitcoin. *Theoretical and Applied Economics*, 21.

Scaillet, O., Treccani, A., and Trevisan, C. (2018). High-frequency jump analysis of the bitcoin market. *Journal of Financial Econometrics, in print*.

Schmidt, E. and Cohen, J. (2017). *The New Digital Age Reshaping the Future of People, Nations and Business*. Google.

Schwartz, E. and Trolled, A. (2009). Unspanned stochastic volatility and the pricing of commodity derivatives. *Review of Financial Studies*, 22(1):4423—4461.

Segendorf, B. (2014). What is Bitcoin? *Sveriges Riksbank Economic Review*, 2:71–87.

Singleton, K. J. and Umantsev, L. (2002). Pricing coupon-bond options and swaptions in affine term structure models. *Mathematical Finance*, 12(4):427–446.

Todorov, V. and Tauchen, G. (2010). Activity signature functions for high-frequency data analysis. *Journal of Econometrics*, 54:125–138.

Trimborn, S. and Härdle, W. K. (2018). CRIX: an index for cryptocurrencies. *Journal of Empirical Finance*, 49:107–122.

Tsay, R. S. (2005). *Analysis of financial time series*, volume 543. John Wiley & Sons.

Weber, B. (2016). Bitcoin and the legitimacy crisis of money. *Cambridge Journal of Economics,*, 40.

White, H. (1982). Maximum likelihood estimation of misspecified models. *Econometrica: Journal of the Econometric Society*, pages 1–25.

IRTG 1792 Discussion Paper Series 2018

For a complete list of Discussion Papers published, please visit irtg1792.hu-berlin.de.

001 "Data Driven Value-at-Risk Forecasting using a SVR-GARCH-KDE Hybrid" by Marius Lux, Wolfgang Karl Härdle and Stefan Lessmann, January 2018.

002 "Nonparametric Variable Selection and Its Application to Additive Models" by Zheng-Hui Feng, Lu Lin, Ruo-Qing Zhu And Li-Xing Zhu, January 2018.

003 "Systemic Risk in Global Volatility Spillover Networks: Evidence from Option-implied Volatility Indices " by Zihui Yang and Yinggang Zhou, January 2018.

004 "Pricing Cryptocurrency options: the case of CRIX and Bitcoin" by Cathy YH Chen, Wolfgang Karl Härdle, Ai Jun Hou and Weining Wang, January 2018.

IRTG 1792, Spandauer Straße 1, D-10178 Berlin
http://irtg1792.hu-berlin.de

This research was supported by the Deutsche Forschungsgemeinschaft through the IRTG 1792.