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A factor-model approach for correlation scenarios and correlation stress-testing

Natalie Packham *
Fabian Woebbeking *2

* Berlin School of Economics and Law, Germany
*2 Goethe University Frankfurt, Germany

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A factor-model approach for correlation scenarios and correlation stress testing*

Natalie Packham† Fabian Woebbeking‡

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Abstract

In 2012, JPMorgan accumulated a USD 6.2 billion loss on a credit derivatives portfolio, the so-called "London Whale", partly as a consequence of de-correlations of non-perfectly correlated positions that were supposed to hedge each other. Motivated by this case, we devise a factor model for correlations that allows for scenario-based stress testing of correlations. We derive a number of analytical results related to a portfolio of homogeneous assets. Using the concept of Mahalanobis distance, we show how to identify adverse scenarios of correlation risk. In addition, we demonstrate how correlation and volatility stress tests can be combined. As an example, we apply the factor-model approach to the "London Whale" portfolio and determine the value-at-risk impact from correlation changes. Since our findings are particularly relevant for large portfolios, where even small correlation changes can have a large impact, a further application would be to stress test portfolios of central counterparties, which are of systemically relevant size.

JEL Classification: G11, G32

Keywords: Correlation stress testing, scenario selection, market risk, "London Whale"

1 Introduction

Diversification – typically captured by correlation – lies at the heart of many financial applications: a diversified portfolio is less risky than a concentrated portfolio; hedging strategies may involve only imperfectly correlated assets instead of perfect substitutes. It is well-known that correlations are not constant over time and may be strongly affected by specific events (Karolyi and Stulz, 1996; Longin and Solnik, 2001; Ang and Bekaert, 2002; Wied et al., 2012; Pu and Zhao, 2012; Adams et al., 2017). Changes in correlation may lead to potentially unexpected or unquantified losses, see e.g. LTCM (Jorion, 2000), Amaranth Advisors (Chincarini, 2007).

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†Berlin School of Economics and Law, packham@hwr-berlin.de

‡Goethe University Frankfurt, woebbeking@finance.uni-frankfurt.de

This paper develops a technique for generating correlation matrices from specific risk factor scenarios. The method allows to challenge diversification benefits in a realistic way by quantifying potential losses from correlation changes or a correlation break-down due to various scenarios. Consequently, worst-case scenarios and their impact can be identified. Quantifying these risks is particularly important if a portfolio or a hedging strategy may be adversely affected by a correlation breakdown amongst the portfolio constituents. For example, hedging strategies involving non-perfect substitutes, such as a stock portfolio and index futures for hedging, are sensitive to correlation changes and thus vulnerable to adverse correlation scenarios.

The technique borrows elements from parameterising correlation matrices in interest rate modelling, e.g. Rebonato (2002); Brigo (2002); Schoenmakers and Coffey (2003). These parameterisations have in common that the degree of correlation depends on the difference in maturity of the underlying interest rates (e.g. swap rates). In its simplest form, correlations are determined by $\mathbf{e}^{-\beta|i-j|}$, where $\beta > 0$ is a constant parameter, and i, j are maturities. This captures the stylised fact that correlations decay with increasing maturity difference.

In this paper, this approach is generalised by defining factors that characterise differences in the assets under consideration, and by parameterising correlations via "distances" capturing these differences. The parameters themselves can be calibrated for example from historical data. Scenarios are generated by varying the parameters, where an increase in a parameter captures a de-correlation related to a factor.

The method is capable of identifying the factor structure of worst case scenarios. More specifically, given the mapping of correlation risk factor to risk measure, one can find the global maximum of the risk measure and infer the corresponding risk factor scenario. As each parameter represents an economically relevant correlation risk factor, it is therefore possible to identify critical portfolio structures that might require particular attention from a risk management perspective.

Aside from the impact of a given scenario, one is also interested in the plausibility of the chosen scenarios. This can be implemented by assigning a joint probability distribution to the correlation parameters in order to define a constraint for correlation scenarios. In this paper, the constraint is specified via the so-called Mahalanobis distance, which measures the distance of a normally distributed random vector from the center of the distribution.

As correlation stress often occurs jointly with volatility shocks, we also demonstrate how to combine the two stress scenarios. To model volatility separately we assume that asset returns follow a multivariate Student t-distribution (as opposed to a normal distribution). As a t-distribution can be conveniently decomposed into a correlated normal distribution component and an inverse-gamma-distributed scaling factor, volatility stress is introduced by setting the scaling factor to a given quantile.

To demonstrate the technique, correlation stress tests are applied to the portfolio of the so-called "London Whale", a term used in the finance industry to denote a USD 6.2 billion loss in 2012 of a credit derivative portfolio run by JPMorgan. In late 2011, in an effort to reduce the risk of the position without monetising losses, the notional amount of the portfolio was increased, while relying on the ability of similar credit index positions to act as hedges for each other (JPMorgan, 2013; United-States-Senate, 2013b). Our analysis shows that correlation scenarios and stress tests reveal the high riskiness of this portfolio and thus might have led to a more appropriate risk assessment of the portfolio.[1]

A further application where correlation scenario and stress testing can reveal inherent risks is the practice of so-called "portfolio margining" in initial margin calculations of clearing houses. Here, netting of offsetting positions reduces the margin requirement. However, when positions are not perfect hedges, but only highly correlated, an adverse correlation scenario could lead to substantial margin calls, thereby increasing counterparty risk at a systematic level.

The literature on establishing correlation stress tests is scarce, even though it is well established that correlations are not constant over time and may be strongly affected by specific events (Longin and Solnik, 2001; Wied et al., 2012; Pu and Zhao, 2012). Adams et al. (2017) observe that correlations vary over time and, in addition, experience level shifts and structural breaks that occur in response to economic or financial shocks. Krishnan et al. (2009) and Mueller et al. (2017) provide empirical evidence that investors demand a correlation risk premium, which is related to the uncertainty about future correlation changes. Buraschi et al. (2010) develop a framework for inter-temporal portfolio choice that includes hedging components against correlation risk.

The prominent role of correlation in financial portfolios led to regulatory agencies calling for risk model stress tests that account for "significant shifts in correlations" (BCBS, 2006, p. 207 ff.). However, there is little literature on parametric correlation modelling, in particular related to risk-factor driven stress testing: aside from challenges of mathematical consistency in correlation modelling (correlation matrices must be positive semi-definite, see e.g. Qi and Sun (2010); Ng et al. (2014) for solutions to this problem), the specification of stressful yet plausible scenarios for correlations is far from straightforward.

The selection of plausible scenarios poses a challenge in the development of stress testing methods in general. The use of historical or hypothetical scenarios is problematic, as the probability and thus the plausibility of a scenario is typically unknown, while at the same time relevant scenarios might be neglected. In an extensive study, Alexander and Sheedy (2008a) compare various well-known models in their ability to conduct meaningful stress tests. Glasserman et al. (2015) develop an empirical likelihood approach for the selection of stress scenarios, with a focus on reverse stress testing. Kopeliovich et al. (2015) present a reverse stress testing method to determine scenarios that lead to a specified loss level. Breuer et al. (2009) and Flood and Korenko (2015) use the Mahalanobis distances to select scenarios from a multivariate distribution of risk factors.

Breuer and Csiszár (2013) extend these approaches and consider various application scenarios, amongst them stressed default correlations, which refer to the correlations of Bernoulli variables denoting the default or survival of loans or obligors. Studer (1999) considers correlation breakdowns by identifying the worst-case correlation scenario in a constrained region of

[1] Other risks specific to the "London Whale", especially concerning the size of the position relative to market size, are treated in Cont and Wagalath (2016).

P&L scenarios. However, solving the problem turns out to be intractable in the sense that it is NP-hard. Also, the likelihood or plausibility of such a correlation scenario is not known. The difference in our setting is that we model correlation itself in a parametric way and – imposing a distribution assumption on the risk factors driving correlation, e.g. calibrated from historical data – find the risk-factor scenario that produces the worst loss within a given range of plausible correlation scenarios.

The paper is structured as follows: In Section 2 we present the correlation stress testing methodology alongside analytical results for both, the stress test and scenario selection procedures. Section 3 consists of a concise review of the "London Whale" case as well as the results from correlation stress testing the credit portfolio using the methods developed in the previous section. Section 4 concludes. A detailed review of the London Whale case is provided as an online appendix at https://ssrn.com/abstract=3210536.

2 Correlation parameterisation and stress testing

2.1 Factor model

The principal idea behind the correlation parameterisation developed in this paper is to split portfolio correlations into dependence contributions associated with several risk factors. With each risk factor a parameter determining the degree of de-correlation on the overall correlation is associated. Calibrating these parameters and then adjusting them allows to translate specific economic scenarios into changes on correlations.

More precisely, let C be an $n \times n$-correlation matrix (i.e., positive semi-definite, symmetric, with entries in $[-1, 1]$, and ones on the diagonal) related to the returns of n financial instruments. In the context of the London Whale position analysed later, the entries of C are the correlations of credit index spread returns and related tranche spread returns.[2] In the London Whale case these are typically positive (with only few exceptions near zero, which are set to a small positive constant), and generally we assume that all correlations are in $(0, 1]$.

The factors that determine the correlations are denoted by $\mathbf{x} = (x^1, \ldots, x^m)'$. In the context of the London Whale position, the factors include the maturity, the index series, a dummy variable determining whether the security is investment grade or not, and others. Further choices could be factors relating to geographical regions, industries or balance sheet data. Correlations c_{ij} of securities i and j are modelled as

$$c_{ij} = \exp\left(-(\beta_1|x_i^1 - x_j^1| + \beta_2|x_i^2 - x_j^2| + \cdots + \beta_m|x_i^m - x_j^m|)\right), \quad i, j = 1, \ldots, n,$$

with $\beta_1, \ldots, \beta_m$ positive coefficients, the parameters. This is the simplest, most parsimonious functional form relating differences in the risk factors with the correlations of the securities. It implies that the greater the distance $|x_i^k - x_j^k|$, the greater the de-correlation amongst the securities i and j. If two instruments are identical in all respects, then they are assigned a correlation

[2] In this setting correlations are not implied tranche correlations, which are used for pricing, but historical spread return correlations, as would be used in risk management.

of 1. With additional information about the relationship between risk factors and correlations, other, more complex functional forms may be feasible. Similar approaches to parameterising correlation matrices are common in interest-rate modelling, see e.g. Schoenmakers and Coffey (2003); Rebonato (2004); Brigo and Mercurio (2006).

In the simple model above, given historical returns, the parameters $\beta_1, \ldots, \beta_m$ are easily determined by standard regression techniques such as OLS on the transformed correlations $-\ln(c_{ij})$. A scenario such as "the correlation between investment grade and high-yield securities decreases" is then implemented by increasing the corresponding β-parameter. With parameters calibrated on a regular basis, the parameter history can be used to obtain reasonable scenarios.

2.2 Stress testing a homogeneous portfolio

To better understand the stress testing effect we consider a stylised, homogeneous portfolio and derive closed formulas for the impact of various correlation stress test scenarios. A homogeneous portfolio reduces the number of parameters involved to a minimum and therefore allows for a general understanding of the behaviour under stress. This is similar to analysing diversification effects of a homogeneous portfolio in standard Markowitz portfolio theory.

The setup is as follows: the m risk factors are binary in the sense that they express properties that are either present or absent in a security. The number of securities is $n = 2^m$ and they exhibit all 2^m combinations of risk factor combinations (for example, one could set $(\mathbf{1}_{\{x_i^1 \neq x_j^1\}}, \ldots, \mathbf{1}_{\{x_i^m \neq x_j^m\}}) = (i-1) \oplus (j-1)$, with $(i-1) \oplus (j-1)$ the bitwise XOR operator of the binary representations of $i-1$ and $j-1$). As a consequence, no two securities are equal in terms of their risk factor exposure. The securities all have equal volatility, and the portfolio is equally-weighted. However, as a consequence of choosing binary risk factor combinations, the correlations are not homogeneous.

We assume that risk of the portfolio is measured by value-at-risk (VaR) in a *variance-covariance approach*, i.e.,

$$\mathrm{VaR}_\alpha = -\mathrm{N}_{1-\alpha} \cdot V_0 \cdot \left(\mathbf{w}^\intercal \, \mathbf{\Sigma} \, \mathbf{w}\right)^{1/2}, \tag{1}$$

where $\mathrm{N}_{1-\alpha}$ denotes the $(1-\alpha)$-quantile of the standard normal distribution, V_0 denotes the current position value, $\mathbf{w}$ is the vector of portfolio weights and $\mathbf{\Sigma}$ denotes the covariance matrix of the portfolio components' returns with entries σ^2 along the diagonal denoting the stand-alone variances of the assets. In this setting we assume that the expected return is zero, which is a reasonable assumption for short time horizons.

The normal distribution assumption can easily be generalised, e.g. to a Student t-distribution. We will use this more generalised setup in Section 2.3 when we combine correlation and volatility stress testing.

Proposition 1. *The portfolio variance is given by*

$$\mathbf{w}^\intercal \, \mathbf{\Sigma} \, \mathbf{w} = \frac{\sigma^2}{n} \prod_{k=1}^{m} \left(1 + \mathbf{e}^{-\beta_k}\right), \tag{2}$$

The average correlation amongst pairwise different asset returns is

$$\overline{\rho}(\boldsymbol{\beta}) = \frac{1}{(n-1)} \prod_{k=1}^{m} \left(1 + \mathbf{e}^{-\beta_k}\right) - \frac{1}{n-1}. \tag{3}$$

Proof. Assuming without loss of generality that the bitwise XOR operator described above defines the value of the indicator variables, the portfolio variance simplifies to

$$\begin{aligned}
\mathbf{w}^{\intercal}\,\boldsymbol{\Sigma}\,\mathbf{w} &= \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \mathbf{e}^{-\sum_{k=1}^{m} \beta_k \cdot \mathbf{1}_{\{x_i^k \neq x_j^k\}}} \sigma^2 \\
&= \frac{\sigma^2}{n^2} \sum_{i=1}^{n} \left[\sum_{j=1}^{n/2} \mathbf{e}^{-\sum_{k=1}^{m-1} \beta_k \cdot \mathbf{1}_{\{x_i^k \neq x_j^k\}} - \beta_m \cdot 0} + \sum_{j=n/2+1}^{n} \mathbf{e}^{-\sum_{k=1}^{m-1} \beta_k \cdot \mathbf{1}_{\{x_i^k \neq x_j^k\}} - \beta_m \cdot 1} \right] \\
&= \frac{\sigma^2}{n^2} \sum_{i=1}^{n} \left(1 + \mathbf{e}^{-\beta_m}\right) \cdot \sum_{j=1}^{n/2} \mathbf{e}^{-\sum_{k=1}^{m-1} \beta_k \cdot \mathbf{1}_{\{x_i^k \neq x_j^k\}}}.
\end{aligned}$$

Iterating this calculation $m = \log_2 n$ times gives

$$\mathbf{w}^{\intercal}\,\boldsymbol{\Sigma}\,\mathbf{w} = \frac{\sigma^2}{n^2} \sum_{i=1}^{n} \prod_{k=1}^{m} \left(1 + \mathbf{e}^{-\beta_k}\right) = \frac{\sigma^2}{n} \prod_{k=1}^{m} \left(1 + \mathbf{e}^{-\beta_k}\right).$$

The average correlation is given as

$$\frac{1}{n(n-1)} \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} \mathbf{e}^{-\sum_{k=1}^{m} \beta_k \cdot \mathbf{1}_{\{x_i^k \neq x_j^k\}}} = \frac{1}{n(n-1)} \left[\sum_{i=1}^{n} \sum_{j=1,}^{n} \mathbf{e}^{-\sum_{k=1}^{m} \beta_k \cdot \mathbf{1}_{\{x_i^k \neq x_j^k\}}} - \sum_{i=1}^{n} 1 \right],$$

and the claim follows from the first part of the Proposition. □

Corollary 2. *The sensitivity of the variance with respect to a single β-factor β_l is*

$$\frac{\partial \mathbf{w}^{\intercal}\boldsymbol{\Sigma}\mathbf{w}}{\partial \beta_l} = -\frac{\sigma^2}{n^2}\, \mathbf{e}^{-\beta_l} \cdot \prod_{k \neq l} \left(1 + \mathbf{e}^{-\beta_k}\right).$$

If the β-factors are homogeneous, i.e., $\beta_1 = \cdots = \beta_m = \beta$, then the overall sensitivity is

$$\frac{\partial \mathbf{w}^{\intercal}\boldsymbol{\Sigma}\mathbf{w}}{\partial \beta} = \frac{\partial \frac{\sigma^2}{n^2} \left(1 + \mathbf{e}^{-\beta}\right)^m}{\partial \beta} = -\frac{\sigma^2}{n} \frac{m \left(1 + \mathbf{e}^{-\beta}\right)^m}{1 + \mathbf{e}^{\beta}}.$$

For stress testing we associate a probability distribution with $\boldsymbol{\beta}$. This allows to formulate scenarios in probabilistic terms and determine their impact. We assume that the risk factors themselves are homogeneous, i.e., $\beta_k = \beta$, for all $k = 1, \ldots, m$, and that changes in the risk factor coefficients $\Delta\boldsymbol{\beta}$ are jointly normally distributed, each one with mean 0 and variance σ_β^2 and with correlations ρ_β.

Following e.g. Kupiec (1998), we define a stress scenario on one set of ("core") risk factors and, assuming that the given covariance matrix is unaltered by the stress scenario, set the

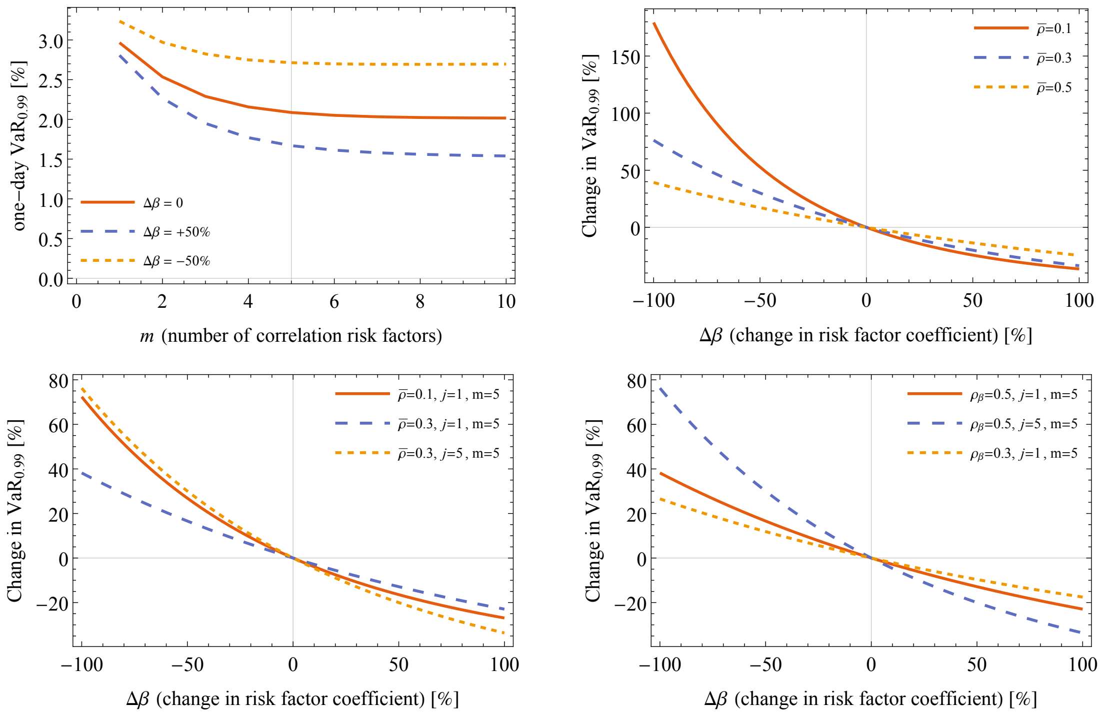

Figure 1: Top left: Portfolio VaR as a function of the number of risk factors m (cf. Proposition 1). Top right: Change in portfolio VaR as a function of $\Delta\beta$ (cf. Proposition 1), calibrated to different initial average asset correlations $\overline{\rho} \in \{0.1, 0.3, 0.5\}$. Bottom left: Change in portfolio VaR as a function of $\Delta\beta$ with peripheral risk factor changes that correlate with $\rho_\beta = 0.5$ (cf. Proposition 2). Bottom right: Change in portfolio VaR as a function of $\Delta\beta$ with correlated peripheral risk factor changes (cf. Proposition 3). All graphs show a 99% Value-at-Risk, and the initial and unstressed β is calibrated to an average asset correlation of $\overline{\rho} \approx 0.3$, unless indicated otherwise (cf. Equation (3)).

remaining ("peripheral") risk factors to their optimal estimates conditional on the scenario. Let $\boldsymbol{\beta}_s$ denote the $j < m$ core factor parameters that are stressed directly. The remaining $m-j$ peripheral risk factor parameters $\boldsymbol{\beta}_u$ are affected by the stress scenario only indirectly. Under the normal distribution setting above, it holds that the optimal estimator of $\Delta\boldsymbol{\beta}_u$ conditional on $\Delta\boldsymbol{\beta}_s$ is (e.g. Theorem §13.2 of Shiryaev, 1996):

$$\mathbb{E}(\Delta\boldsymbol{\beta}_u | \Delta\boldsymbol{\beta}_s) = \Sigma_{us}\Sigma_{ss}^{-1}\Delta\boldsymbol{\beta}_s,$$

where Σ_{us} and Σ_{ss} denote the covariance and variance matrices of $\boldsymbol{\beta}_u$ and $\boldsymbol{\beta}_s$.

Proposition 3. *The portfolio variance when j of the β-risk factors coefficients are stressed by $\Delta\beta$ is given by*

$$\mathbf{w}^\intercal\,\boldsymbol{\Sigma}\,\mathbf{w} = \frac{\sigma^2}{n}\left(1 + \mathbf{e}^{-(\beta+\Delta\beta)}\right)^j \cdot \left(1 + \mathbf{e}^{-\left(\beta + \frac{j\cdot\rho_\beta}{(j-1)\rho_\beta+1}\Delta\beta\right)}\right)^{m-j}.$$

Proof. It is easily verified that the entries of the $(m-j)\times j$-matrix $\Sigma_{us}\Sigma_{ss}^{-1}$ are $\dfrac{\rho_\beta}{(j-1)\rho_\beta+1}$. $\square$

To illustrate the impact of a correlation stress, we apply the above results to a portfolio of $n = 2^m$ assets, where each asset has an annualised volatility of $\sigma = 0.25$ and the average asset correlation in the portfolio is 0.3 unless specified differently. The risk factor coefficient β is calibrated to reflect the target asset correlation for the number of factors m, e.g. with $m = 5$ this is achieved by $\beta = 0.5204$ (cf. Equation (3)). Figure 1 shows that the portfolio VaR decreases as a function of both the number of correlation risk factors m and the risk factor coefficient β. Both results are not surprising as the increasing number of assets diversifies away idiosyncratic risk leaving only systematic risk, which in addition, is lowered by an increasing β. This result is driven by the long-only structure of the portfolio. The risk of a portfolio that contains hedging positions may behave differently, as will be observed in the London Whale example in Section 3.

The impact of changes in β to the overall portfolio VaR is shown in the top right graph of Figure 1. For this long-only portfolio the VaR decays with decreasing correlation. The lower boundary of 0 for β, i.e. a reduction of 100% in Figure 1, identifies the worst case scenario. The impact of $\Delta\beta$ is asymmetric, where the risk of correlation changes is higher than the benefit. Diversification benefits through changes in correlation are possible for diversified portfolios under non-perfect correlation. For hedging portfolios, which often rely on very high or almost perfect correlation, correlation changes are most often undesirable and considered a risk.

The steepness or sensitivity of VaR to changes in β is determined by the initial average asset correlation, the correlation between risk factors, as well as the number of factors $j \leq m$ that are initially stressed (see bottom graphs in Figure 1). In other words, an initially high average asset correlation reduces the risk of changing correlations, because the portfolio is already poorly diversified. The VaR impact when stressing a subset of j factors increases with ρ_β, which captures the dependence between correlation risk factors.

2.3 Joint stress test of correlation and volatility

It is well documented that large changes in correlation coincide with volatility shocks, e.g. (Alexander and Sheedy, 2008b; Longin and Solnik, 2001; Loretan and English, 2000). To this end, we develop a simple technique that combines both stress scenarios. The principal idea is to assume that a d-dimensional vector of asset returns $\mathbf{X}$ follows a Student t-distribution, $\mathbf{X} \sim t(\tilde{\mathbf{\Sigma}}, \nu)$, with $\nu > 2$ and with $\tilde{\Sigma}$ a matrix describing the dependence as explained further below, where we assume for simplicity that expected asset returns are zero. Then, $\mathbf{X}$ follows a normal variance mixture distribution with decomposition (cf. Chapter 6.2 of McNeil et al. (2015))

$$\mathbf{X} = \sqrt{V} \cdot A \cdot \mathbf{Z},$$

where $\mathbf{Z} \sim \mathrm{N}(0, I_k)$, i.e., $\mathbf{Z}$ is a vector of independent standard normally distributed random variables, V is independent of $\mathbf{Z}$ and $V \sim \mathrm{Ig}(1/2\,\nu, 1/2\,\nu)$, i.e., the mixing variable V follows an inverse gamma distribution, and A is a $d \times k$ matrix such that $\tilde{\mathbf{\Sigma}} = AA^T$. Because of $\mathbb{E}V = \dfrac{\nu}{\nu - 2}$, the covariance matrix of $\mathbf{X}$ is $\mathbf{\Sigma} = \dfrac{\nu}{\nu - 2}\tilde{\mathbf{\Sigma}}$ (note that the expectation and covariance matrix are defined only if $\nu > 2$). The correlation matrices of $\mathbf{X}$ and $A\mathbf{Z}$ are the same.

Under the assumption of a t-distribution, the t-VaR at level α is, cf. Equation (1)

$$\mathrm{VaR}^t_\alpha = -t_{\nu,1-\alpha} \cdot V_0 \left(\mathbf{w}^\intercal \,\tilde{\boldsymbol{\Sigma}}\, \mathbf{w}\right)^{1/2} = -t_{\nu,1-\alpha} \cdot V_0 \cdot \left(\frac{\nu-2}{\nu}\right)^{1/2} \left(\mathbf{w}^\intercal \,\boldsymbol{\Sigma}\, \mathbf{w}\right)^{1/2}. \tag{4}$$

Volatility stress at the level $\tilde{\alpha} \in [0,1]$ is introduced by setting V to the $\tilde{\alpha}$-quantile $q_{\tilde{\alpha}}$ of the $\mathrm{Ig}(1/2\,\nu, 1/2\,\nu)$ distribution. This conveniently captures that the volatility stress induced is a systematic event. Furthermore, the severity of the stress event depends on the heaviness of the tails, expressed by ν. The VaR in this scenario is determined from

$$\mathbf{P}\left(\mathbf{w}^T X \leq \mathrm{VaR}_\alpha | V = q_{\tilde{\alpha}}\right) = \mathbf{P}\left(\sqrt{q_{\tilde{\alpha}}}\mathbf{w}^T A\mathbf{Z} \leq \mathrm{VaR}_\alpha\right), \tag{5}$$

with $\mathbf{w}^T A\mathbf{Z} \sim \mathrm{N}(0, \mathbf{w}^T\tilde{\boldsymbol{\Sigma}}\mathbf{w})$. Consequently, the stressed t-VaR is derived from Equation (5) as a normal distribution VaR with the standard deviation scaled according to the fixed mixing variable contribution:

$$\mathrm{VaR}^t_{\alpha,\tilde{\alpha}} = -\mathrm{N}_{1-\alpha} \cdot V_0 \cdot \sqrt{q_{\tilde{\alpha}}}(\mathbf{w}^T\tilde{\boldsymbol{\Sigma}}\mathbf{w})^{1/2} = -\mathrm{N}_{1-\alpha} \cdot V_0 \cdot \sqrt{q_{\tilde{\alpha}}}\left(\frac{\nu-2}{\nu}\right)^{1/2} (\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w})^{1/2}. \tag{6}$$

To achieve a joint volatility and correlation stress, both methods are combined: a scaling factor determined from the quantile of the mixing variable as in Equation (6) is applied independently of a correlation scenario $\Delta\beta$ as in Proposition 3.

2.4 Stress test scenario selection

2.4.1 Mahalanobis distance

When stress testing, aside from understanding the impact of given scenarios, one is also interested in the converse question: What is the worst scenario amongst all scenarios that occur within some pre-given range? One way to specify the range is via the so-called Mahalanobis distance, which measures the distance of a realisation of a normally distributed random vector from its mean.

Recall that correlations $c_{i,j}$ are modelled as

$$c_{ij} = \exp\left(-(\beta_1|x_i^1 - x_j^1| + \beta_2|x_i^2 - x_j^2| + \cdots + \beta_k|x_i^m - x_j^m|)\right), \quad i,j = 1,\ldots,n,$$

with positive parameters $\beta_1,\ldots,\beta_m$. If $\boldsymbol{\beta} = (\beta_1,\ldots,\beta_m)^\intercal$ is a random vector with $\mathbb{E}(\boldsymbol{\beta}) = \overline{\boldsymbol{\beta}}$ and covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{\beta}}$, then the *Mahalanobis distance* is defined as

$$D(\boldsymbol{\beta}) = \left((\boldsymbol{\beta} - \overline{\boldsymbol{\beta}})^\intercal \boldsymbol{\Sigma}_{\boldsymbol{\beta}}^{-1}(\boldsymbol{\beta} - \overline{\boldsymbol{\beta}})\right)^{1/2}.$$

Furthermore, if $\boldsymbol{\beta} \sim \mathrm{N}(\overline{\boldsymbol{\beta}}, \boldsymbol{\Sigma}_{\boldsymbol{\beta}})$, then the square of the Mahalanobis distance follows a chi-squared distribution, i.e., $D^2(\boldsymbol{\beta}) \sim \chi^2(m)$.[3]

We are interested in identifying the worst-case scenario $\boldsymbol{\beta}^*$ that maximises VaR subject to a constraint on the Mahalanobis distance:

$$\boldsymbol{\beta}^* = \underset{\boldsymbol{\beta}: D^2(\boldsymbol{\beta}) \leq h}{\operatorname{argmax}} \operatorname{VaR}_\alpha(\boldsymbol{\beta}),$$

where VaR_α is given by Equation (1) with correlation matrix imposed by $\boldsymbol{\beta}$. If the parameter h in the constraint is chosen as the α^*-quantile of the $\chi^2(m)$-distribution, then $\boldsymbol{\beta}^\star$ expresses the worst correlation scenario amongst all scenarios that lie on the inner ellipsoids covering a probability of α^*. From Equation (1) it is obvious that maximising Var_α does not depend on α and is equivalent to maximising the variance. A trivial consequence is that $\boldsymbol{\beta}^*$ also maximises expected shortfall $\mathrm{ES}_\alpha = \dfrac{1}{1-\alpha}\displaystyle\int_\alpha^1 \mathrm{VaR}_u \, \mathrm{d}u$. Writing the diagonal matrix with the standard deviations as the entries on the diagonal as $\boldsymbol{\sigma} = (\operatorname{diag}(\boldsymbol{\Sigma}(\beta)))^{\frac{1}{2}}$, gives

$$\boldsymbol{\beta}^* = \underset{\boldsymbol{\beta}: D^2(\boldsymbol{\beta}) \leq h}{\operatorname{argmax}} \mathbf{w}^\intercal \boldsymbol{\Sigma}(\boldsymbol{\beta}) \mathbf{w} = \underset{\boldsymbol{\beta}: D^2(\boldsymbol{\beta}) \leq h}{\operatorname{argmax}} \mathbf{w}^\intercal (\boldsymbol{\sigma}\, \boldsymbol{C}(\boldsymbol{\beta})\, \boldsymbol{\sigma}) \mathbf{w} = \underset{\boldsymbol{\beta}: D^2(\boldsymbol{\beta}) \leq h}{\operatorname{argmax}} \sum_{i=1}^n \sum_{j=1}^n w_i\, w_j\, \sigma_i\, \sigma_j\, c_{ij}(\boldsymbol{\beta}).$$

The Lagrangian is

$$\begin{aligned}\mathcal{L} &= \boldsymbol{w}^\intercal (\boldsymbol{\sigma}\, \boldsymbol{C}(\beta)\, \boldsymbol{\sigma}) \mathbf{w} + \lambda((\boldsymbol{\beta} - \overline{\boldsymbol{\beta}})^\intercal \boldsymbol{\Sigma}_{\boldsymbol{\beta}}^{-1} (\boldsymbol{\beta} - \overline{\boldsymbol{\beta}}) - h) \\ &= \sum_{i=1}^n \sum_{j=1}^n w_i\, w_j\, \sigma_i\, \sigma_j\, c_{ij}(\boldsymbol{\beta}) + \lambda\Big(\sum_{i=1}^m \sum_{j=1}^m (\beta_i - \overline{\beta}_i)(\beta_j - \overline{\beta}_j) q_{ij} - h\Big),\end{aligned}$$

with q_{ij} the entries of $\boldsymbol{\Sigma}_{\boldsymbol{\beta}}^{-1}$.

The first-order conditions are

$$\frac{\partial}{\partial \beta_l}\mathcal{L} = -\sum_{i,j=1}^n w_i w_j \sigma_i \sigma_j \mathbf{e}^{-\sum_{k=1}^m \beta_k |x_i^k - x_j^k|} \cdot |x_i^l - x_j^l| + 2\lambda \sum_{j=1}^m (\beta_j - \overline{\beta}_j) q_{lj} = 0, \quad l = 1, \ldots, m \tag{7}$$

$$\frac{\partial}{\partial \lambda}\mathcal{L} = D^2(\boldsymbol{\beta}) - h = 0 \tag{8}$$

Assuming that all factors are indicators measuring if a property is present in both securities

[3] The approach can easily be extended to heavy-tailed distributions, by assuming that $\boldsymbol{\beta}$ follows an elliptic distribution.

or not, i.e., $|x_i^l - x_j^l| = \mathbf{1}_{\{x_i^l \neq x_j^l\}}$ gives

$$\begin{aligned}
\frac{\partial}{\partial \beta_l}\mathcal{L} &= -\mathbf{e}^{-\beta_l} \underbrace{\sum_{i,j=1}^{n} w_i w_j \sigma_i \sigma_j \mathbf{e}^{-\sum_{k=1,k\neq l}^{m} \beta_k \mathbf{1}_{\{x_i^k \neq x_j^k\}}} \cdot \mathbf{1}_{\{x_i^l \neq x_j^l\}}}_{=c_{l,1}} \\
&\quad \underbrace{+ 2\lambda\Big(\sum_{j=1,j\neq l}^{m} (\beta_j - \overline{\beta}_j) q_{lj}\Big) - 2\lambda\overline{\beta}_l q_{ll}}_{=c_{l,2}} + \underbrace{2\lambda q_{ll}}_{=c_{l,3}} \beta_l \\
&= -c_{l,1}\mathbf{e}^{-\beta_l} + c_{l,2} + c_{l,3}\beta_l = 0, \quad l = 1, \dots, k && (9) \\
\frac{\partial}{\partial \lambda}\mathcal{L} &= D^2(\boldsymbol{\beta}) - h = 0 && (10)
\end{aligned}$$

Assuming throughout that the factors are chosen in such a way that at least for one pair of securities the respective indicator is 1 implies that $c_{l,1} \neq 0$, for all $l = 1, \dots, k$.

Proposition 4. *The solutions to (9) satisfy*

$$\beta_l^* = W\left(\frac{c_{l,1}\mathbf{e}^{c_{l,2}/c_{l,3}}}{c_{l,3}}\right) - \frac{c_{l,2}}{c_{l,3}}, \quad l = 1, \dots, k,$$

where $W(z)$ is the Lambert W-function *(also called* product logarithm*), which gives the solution for w in $z = w\,\mathbf{e}^w$, $z \in \mathbb{C}$.*

Proof. For ease of notation, we omit the index l, so we show that $-c_1\,\mathbf{e}^{-\beta} + c_2 + c_3\beta = 0$, with $\beta = W\left(\frac{c_1\mathbf{e}^{c_2/c_3}}{c_3}\right) - \frac{c_2}{c_3}$. Setting $w := W\left(\frac{c_1\mathbf{e}^{c_2/c_3}}{c_3}\right)$ gives

$$-c_1\,\mathbf{e}^{-(w-c_2/c_3)} + c_2 + c_3(w - c_2/c_3) = -c_1\,\mathbf{e}^{-(w-c_2/c_3)} + c_3 w = 0,$$

which can be re-arranged to

$$-\frac{1}{w}\mathbf{e}^{-w}\,c_1\mathbf{e}^{c_2/c_3} + c_3 = 0.$$

Using that $\frac{1}{w}\mathbf{e}^{-w} = \frac{c_3}{c_1\,\mathbf{e}^{c_2/c_3}}$ yields the claim. □

2.4.2 Homogeneous portfolio analysis

To better understand the stress testing effect we consider a stylised, homogeneous portfolio as in Section 2.2 and determine the worst stress scenario that lies within a pre-specified Mahalanobis distance. As before, the m risk factors are binary and the number of securities is $n = 2^m$ comprising all 2^m risk factor combinations. The securities all have equal volatility, and the portfolio is equally-weighted. The risk factor coefficients $\boldsymbol{\beta}$ are also assumed to be homogeneous, i.e., they have identical means $\overline{\beta}$, variances σ_β^2 and correlations ρ_β.

Proposition 5. *In the homogeneous setting, the risk factor coefficients of the worst scenario* *within a given Mahalanobis distance $\sqrt{h}$ are constant, i.e., $\beta_1^* = \cdots = \beta_m^* = \beta^*$, and given by*

$$\beta^* = \overline{\beta} - \sqrt{\frac{h\sigma_\beta^2(1+(m-1)\rho_\beta)}{m}}.$$

Proof. Because of the binary risk factors, the first-order conditions (7) simplify to

$$\frac{\partial}{\partial \beta_l}\mathcal{L} = -\frac{\sigma^2}{n^2}\sum_{i,j=1}^{n} \mathbf{e}^{-\sum_{k=1}^{m}\beta_k \mathbf{1}_{\{x_i^k \neq x_j^k\}}} \cdot \mathbf{1}_{\{x_i^l \neq x_j^l\}} + 2\lambda \sum_{k=1}^{m}(\beta_k - \overline{\beta})q_{lk} = 0, \quad l = 1,\ldots,m.$$

where q_{lk} are the entries of $\mathbf{\Sigma}_\beta^{-1}$ and, because of the homogeneity of $\boldsymbol{\beta}$, $q_{11} = \cdots = q_{mm}$ and q_{lk} constant for all $l \neq k$. It is easily verified that $q_{11} = \dfrac{(m-2)\rho_\beta + 1}{(1+(m-2)\rho_\beta - (m-1)\rho_\beta^2)\sigma_\beta^2}$ and $q_{12} = -\dfrac{\rho_\beta}{(1+(m-2)\rho_\beta - (m-1)\rho_\beta^2)\sigma_\beta^2}$.

For fixed l, the number of instances where $\mathbf{1}_{\{x_i^l \neq x_j^l\}} = 1$, $i,j = 1,\ldots,n$, is $n^2/2$; in particular, this number is constant regardless of the choice of l. Whenever $\mathbf{1}_{\{x_i^l \neq x_j^l\}} = 1$, then, across all i,j, the number of terms where $\mathbf{1}_{\{x_i^k \neq x_j^k\}} = 1$ holds is equally distributed: the 2^{m-1} terms, when $k \neq l$, are the result of all combinations of $m-1$ zeros and ones. As a consequence, the sums in the first-order conditions have the same number of terms and would differ only in $\beta_1,\ldots,\beta_m$; however, because they all have the same structure it follows that $\beta_1^* = \cdots = \beta_m^* = \beta^*$. Hence, the first-order conditions reduce to one condition, which is given by

$$\frac{\partial}{\partial \beta}\mathcal{L} = -\frac{\sigma^2}{n}\sum_{k=0}^{m-1}\binom{m-1}{k}\mathbf{e}^{-\beta\cdot(1+k)} + 2\lambda(\beta - \overline{\beta})(q_{11} + (m-1)q_{12}) = 0.$$

Because all β's are equal, the worst stress scenario at a given Mahalanobis distance $\sqrt{h}$ is one of the two solutions of the quadratic equation

$$(\boldsymbol{\beta} - \overline{\boldsymbol{\beta}})^\intercal \mathbf{\Sigma}_{\boldsymbol{\beta}}^{-1}(\boldsymbol{\beta} - \overline{\boldsymbol{\beta}}) = \Big(\sum_{i=1}^{m}\sum_{j=1}^{m}(\beta_i - \overline{\beta}_i)(\beta_j - \overline{\beta}_j)q_{ij}\Big) = (mq_{11} + m(m-1)q_{12})(\beta - \overline{\beta})^2 = h.$$

Solving for β gives

$$\beta = \overline{\beta} \pm \sqrt{\frac{h}{mq_{11} + m(m-1)q_{12}}} = \overline{\beta} \pm \sqrt{\frac{h\sigma_\beta^2(1+(m-1)\rho_\beta)}{m}}.$$

The claim follows because the portfolio variance is monotone decreasing in β. □

Obviously, ceteris paribus, the portfolio risk and VaR of the worst-case scenario increase with the risk factor variance σ_β^2 and the risk factor correlation ρ_β. They decrease with increasing number of risk factors, m. However, we will see in the examples below that if the initial $\overline{\beta}$ is fitted from a given constant asset correlation matrix, then the worst-case scenario may increase with the number of risk factors as well.

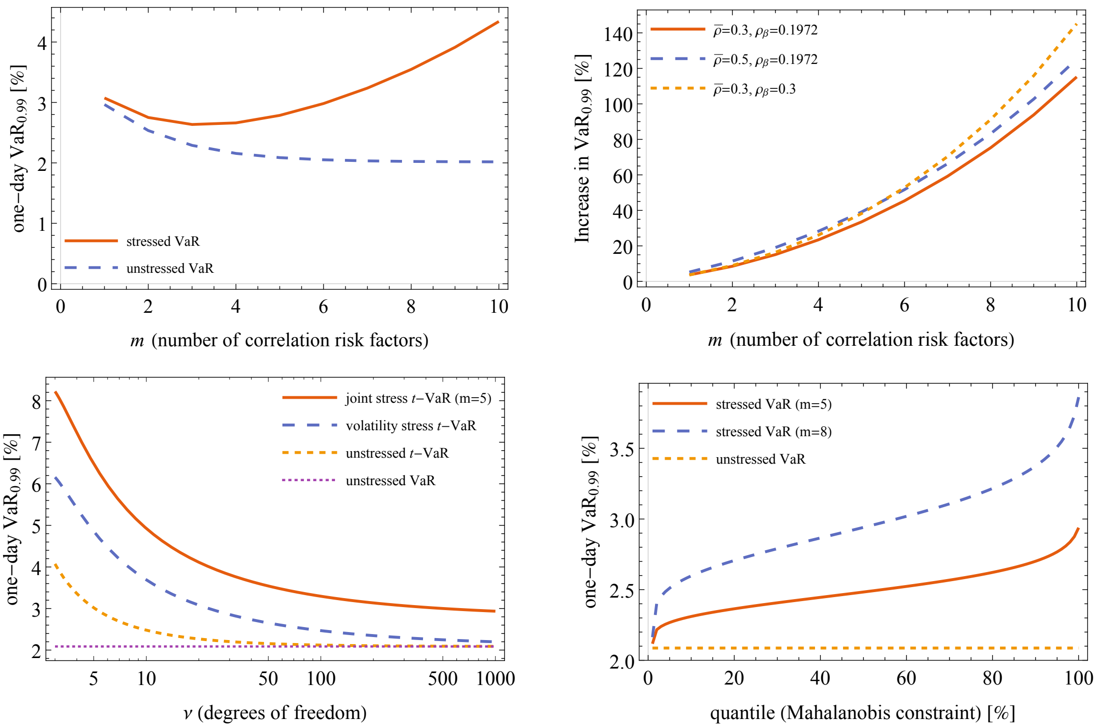

Figure 2: Worst-case (within 95% probability deviation from mean) portfolio value-at-risk (VaR). Top left: Initial and stressed one-day VaR ($\alpha = 0.99$) with average asset correlation $\overline{\rho} = 0.3$, annualised asset volatility $\sigma = 0.25$, beta coefficient correlation $\rho_\beta = 0.1972$ and beta standard deviation $\sigma_\beta = 0.1428$. Top right: Percentage increase in VaR for various parameter setups. Bottom left: Joint correlation and volatility stress test as a function of ν, with volatility stress level $\tilde{\alpha} = \alpha = 0.99$. Bottom right: VaR as a function of the Mahalanobis constraint quantile.

In this setting, we consider a portfolio where the asset returns have an average correlation of 0.3. With five β risk factor coefficients, this is achieved by $\beta = 0.5204$ (cf. Equation (3)). We set $\rho_\beta = 0.1972$ and $\sigma_\beta = 0.1428$ (these values correspond to the historical averages from the "London Whale" case described below). The 95% worst-case scenario is $\beta = 0.2361$. With an annualised asset volatility of 0.25, the initial one-day 99%-VaR of 2.09% increases by 33% to 2.79%. Figure 2 shows the worst-case portfolio variance increase as a function of the number of correlation risk factors m as well as for several parameter constellations. The bottom left graph of Figure 2 shows the impact of a joint correlation and volatility stress scenario on a one-day 99%-VaR. As laid out in Section 2.3, in addition to the correlation stress scenario, volatility is scaled to a stress level corresponding to the $\tilde{\alpha}$-quantile of a Student t-distribution, cf. Equation (6), where we have chosen $\tilde{\alpha} = \alpha = 0.99$. The volatility stress alone increases the unstressed t-VaR by up to 51%, depending on the parameter ν of the t-distribution. Stressing both correlation and volatility can add up to 102% to the unstressed t-VaR.

3 Application to the "London Whale" portfolio

3.1 The "London Whale" case

In 2012, JPMorgan Chase & Co. reported a loss of approximately USD 6.2 billion on a credit derivative portfolio that originated on the books of the comparably small Chief Investment Office (CIO) in London. This case – known as the "London Whale" – was generated by an authorised trading position, so, contrary to most other large trading losses, it cannot be attributed to fraud or unauthorised trading. Interestingly, the loss took place at one of the world's largest investment banks, widely known for its advanced risk management, e.g. as the the innovator of the widely recognised RiskMetrics and CreditMetrics frameworks (JPMorgan, 2013).

To understand JPMorgan's strategy, trading and risk management of the loss generating credit portfolio, we consolidate publicly available information on the London Whale. This section presents our findings in a very concise format, a detailed review is available at https://ssrn.com/abstract=3210536.

JPMorgan, in its function as a lender, is naturally exposed to credit risk. In mid-2011, JPM's Chief Investment Office decided to establish a short credit position via its synthetic credit portfolio (SCP), a portfolio of credit index derivatives.[4] The initial purpose of the portfolio was to act as a macro hedge that would offset naturally long credit exposure (JPMorgan, 2013, p. 26). A similar strategy and portfolio was already successfully employed during the 2008–2009 credit crisis. The decision to re-establish the portfolio was possibly influenced by the deteriorating credit environment in Europe at that time.

The portfolio was based on the two major global credit derivative index families, the CDX for the United States and the iTraxx for Europe.[5] In addition to the indices, each comprising a portfolio of 125 single-name credit default swaps (CDS), there exists a market of tranche products, similar to synthetic collateralized debt obligations, with the indices as underlyings. Both, the CDX and iTraxx provide different sub-indices, such as an Investment Grade index (IG) and a High Yield index (HY). At some point, the SCP comprised more than 120 positions, including most of the active indices and tranches. For details on the valuation of credit indices and their tranche products, we refer to Appendix B and O'Kane (2008).

The proposed trading strategy was called "Smart Short" (United-States-Senate, 2013b, p. 51), which translates into a long-short strategy where credit protection on high yield indices is financed by selling protection on investment grade indices. Hence, the upfront and flow payments can be netted while the resulting portfolio is sensitive to changes in the market spread between the two position sides.

By the end of 2011, JPMorgan's senior management assessed an improvement of the global credit environment, thus requiring less default protection. Hence the decision was made to reduce the SCP's risk weighted assets (RWA). The traders in charge estimated that a direct

[4] A short position in credit risk corresponds to buying CDS protection, i.e., the protection buyer receives default insurance in return for a fixed premium. Thus, a deteriorating credit quality benefits the protection buyer as a payout becomes more likely and hence, the position can be called a short credit risk position.

[5] The CDX and iTraxx index families are owned, managed, compiled and published by Markit Group Limited.

liquidation of their positions would cost up to USD 590 million (see internal meeting documents in United-States-Senate (2013a, Exhibit 8)). Faced by this number, the CIO management decided against a direct reduction, in favour of "managing" profit and losses (P&L) while gradually reducing RWA over time (JPMorgan, 2013, pp. 29 ff.).

A new trading strategy aimed at reducing RWA by increasing positions with opposite market sensitivity. In order to comply with stress limits, the strategy was implemented by forward spread trades, as was stated later during an interview with JPMorgan's internal task force (United-States-Senate, 2013b, p. 52). In the context of the SCP, forward spread trades meant buying protection on short maturity indices, while selling protection on longer maturities. This would hedge in the near term but generate credit exposure on the long term.

By the end of January 2012, after experiencing a loss of 50 million from the default of Kodak, the traders where faced with three objectives: stemming the year-to-date (YTD) losses on the SCP, reducing RWA and maintaining protection to prevent default losses ("Kodak moments"). All objectives were addressed simultaneously by adding more positions to the portfolio, namely, long risk positions to participate in the upward moving market, while generating carry to fund the YTD losses and short risk positions. Additionally, protection was bought to create positive P&L from Kodak type events. Therefore, the traders increased the size of both their long and short positions.

On March 23, 2012, the CIO's most senior executive ordered the traders to "put the phones down", i.e., to cease all related trading activities (United-States-Senate, 2013a, Exhibit 1i). At this point, the SCP had a net notional of about USD 157 billion (United-States-Senate, 2013a, Exhibit 1a), which was 260% up from the September 2011 net notional (and slightly more than Vietnam's 2012 GDP). The SCP's top 10 positions as of March 23, 2012 are shown in Table 1. Ceasing to trade meant, of course, that the traders could no longer influence P&L, and as a consequence the losses on the SCP sky-rocketed.

Publicly available reports by JPMorgan's internal task force (2013) and the United-States-Senate (2013b) focus on management and organizational problems, position misreporting, market manipulation, and spreadsheet-errors. This neglects that the classical risk measures employed might have been insufficient in their own right. To monitor the SCP, JPMorgan (2013) primarily used the Value-at-Risk that would be reported in its 10-K filings. Cont and Wagalath (2016) find that this risk measure was insufficient due to the size of the SCP, as it scales linearly with position size and neglects market impact. In addition, the authors state that a correlation decay, which was observable before the collapse of the portfolio, was possibly caused by the SCP's own market impact.

Aside from VaR, *credit spread widening of 10%* (CSW-10) is the second pivotal risk measure, a sensitivity measure for the profit and loss impact of a simultaneous 10% increase in credit spreads. JPMorgan's traders relied heavily on this measure to balance their portfolio in a way that offsetting positions would minimize the overall CSW-10 (JPMorgan, 2013).

However, a hedging strategy primarily based on a sensitivity measure, CSW-10 in this case, ignores that correlation amongst the portfolio components may be imperfect. Value-at-risk takes

Table 1: Top 10 Positions of the SCP as per March 23, 2012, reported in USD net notional and as percentage share of the respective market. The market's net notional is the net protection bought on an index series by net buyers (or equivalently sold by net sellers) (DTCC, 2011). The publicly available data is aggregated (net notional) and may not contain all live positions due to possible disclosure restrictions, which explains the occurrence of values in excess of 100%.

	Index					
Name	Series	Tenor	Tranche (%)	Protection	Net Notional ($)	Share (%)
CDX.IG	9	10yr	Untranched	Seller	72,772,508,000	50.19
	9	7yr	Untranched	Seller	32,783,985,000	22.61
	9	5yr	Untranched	Buyer	31,675,380,000	21.85
iTraxx.EU	9	5yr	Untranched	Seller	23,944,939,583	37.01
	9	10yr	22 – 100	Seller	21,083,785,713	22.04
	16	5yr	Untranched	Seller	19,220,289,557	64.18
CDX.IG	16	5yr	Untranched	Buyer	18,478,750,000	78.92
	9	10yr	30 – 100	Seller	18,132,248,430	50.35
	15	5yr	Untranched	Buyer	17,520,500,000	117.01
iTraxx.EU	9	10yr	Untranched	Seller	17,254,807,398	26.67
Net Total					137,517,933,681	

Data source: United-States-Senate (2013a, Exhibit 36) and DTCC (2014, Section 1, Table 7).

into account correlations, but to the best of our knowledge, potential *changes* in correlation were ignored. As the SCP was a portfolio composed of a large number of offsetting positions that are highly, but not perfectly dependent, correlation is easily seen to be a, if not *the*, crucial risk driver. A change in the correlations, for instance amongst high-yield and investment grade positions, amongst index and tranche positions, amongst CDX and itraxx positions, could easily lead to large P&L swings.

The portfolio hedging alone is a strong indicator for the correlation dependence of the SCP. Additionally, the size of the SCP and the resulting market-impact could have affected correlation. It is therefore possible that on top of the normal variation in correlation, the portfolio was exposed to more erratic changes that would be captured only by stress tests.

3.2 Correlation stress testing the "London Whale"

3.2.1 Correlation methodology

In the following, the sensitivity to various correlation scenarios of the SCP position is calculated from historical data. The analysis is based on the portfolio composition of 23 March 2012, the day when trading ceased. The historical data are provided by Markit and consist of daily CDX and itraxx spreads and tranche data (spreads, upfront payments and base correlations) of the series in place. Details on how the tranche data was transformed to credit spreads are given in Appendix B.

As the main risk factors affecting correlation the following five properties are identified: maturity, index series, investment grade (yes/no), CDX vs. itraxx, index vs. tranche. Informa-

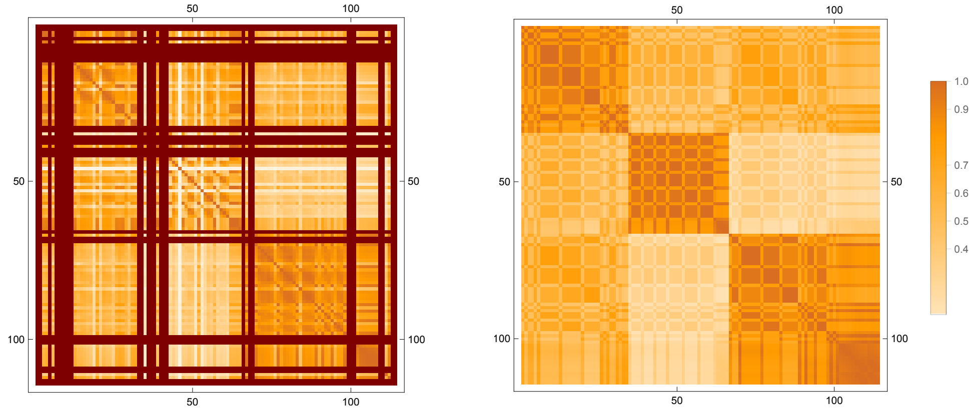

Figure 3: Correlation matrices of 23 March 2012. Left: Empirical correlation matrix; right: parameterised correlation matrix. The dark red entries are unavailable correlations due to insufficient data. The three blocks of highly correlated data consist of (from top to bottom): CDX IG, CDX HY and iTraxx securities.

tion about seniority of tranches was considered, but failed to provide useful results. Hence, the correlation c_{ij} of credit spread returns of credit derivatives indexed by i and j is given by:

$$c_{ij} = \exp\big(- (\beta_1 |\text{isCDX}_i - \text{isCDX}_j| + \beta_2 |\text{isIG}_i - \text{isIG}_j| + \beta_3 |\text{maturity}_i - \text{maturity}_j| \\ + \beta_4 |\text{series}_i - \text{series}_j| + \beta_5 |\text{isIndex}_i - \text{isIndex}_j|)\big). \quad (11)$$

In the results provided below, all distance measures are normalised to $[0, 1]$, which makes the impact of the calibrated parameters comparable.

At any point in time t, the parameters $\beta_1, \ldots, \beta_5$ are calibrated from the 250 credit spread returns preceding day t. Daily parameters are calibrated starting from 1 March 2011 through 12 April 2012. The instruments entering the calibration are the 117 instruments identified to be in the SCP portfolio on 23 March 2012. The precise set of instruments entering on each date differs slightly through time for various reasons such as maturing instruments, spread availability, etc. Figure 3 shows the empirical correlations and the calibrated correlations from Equation (11) as of 23 March 2012. The calibrated coefficients $\boldsymbol{\beta} = (0.35, 0.37, 0.21, 0.05, 0.20)^\intercal$ indicate a strong de-correlation amongst the regional property (CDX vs. itraxx) and the credit quality (investment grade vs. high-yield), a lesser de-correlation amongst maturity and amongst index vs. tranche product. The series-factor on the other hand provides a strong correlation.

The calibrated parameters for the whole time period (1 March 2011–12 April 2012) are shown in Figure 4. The chart shows that the credit quality (investment grade versus high yield) de-correlated over time, while the correlation differences driven by the region (CDX vs. itraxx) decreased. Especially in Q4 2011 and Q1 2012, when strategic decisions regarding the SCP were made, these were major drivers of correlation changes.

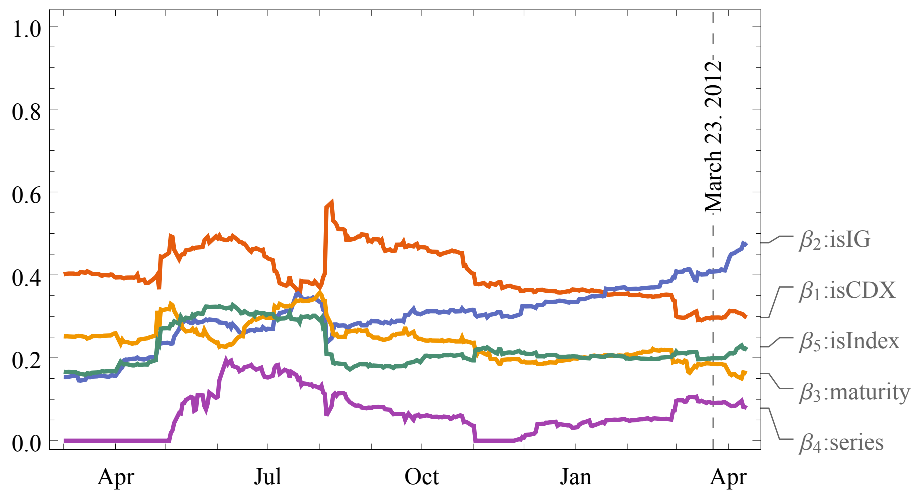

Figure 4: Coefficients associated with correlation parameterisation of CDX and itraxx positions in London Whale position; 01/03/2011–12/04/2012. All distances are normalised to $[0, 1]$ to make the coefficients comparable. Data source: Markit.

3.2.2 CDS portfolio risk

The risk of a CDS portfolio is expressed by value-at-risk (VaR) using the variance-covariance approach, i.e., the portfolio change is approximated by a first-order Taylor approximation in the credit spreads, and credit spread returns are assumed to be normally distributed. The portfolio risk is then fully captured by the portfolio variance. To simplify notation, we omit the maturity of a CDS contract and use the following notation related to CDS position i in a portfolio of n CDS positions: $s_{i,t}$ denotes the fair spread at time t, A_i is the notional amount of the position and $\mathrm{RPV01}_{i,t}$ is the risky PV01 at time t. Details, such as the calculation of $\mathrm{RPV01}_{i,t}$, are given in Appendix A.

The portfolio value is then expressed as $V_t = \sum_{i=1}^{n} V_{i,t} \approx \sum_i A_i \,\mathrm{RPV01}_{i,t}\,(s_{i,0} - s_{i,t})$, where A_i is positive for a short protection position and A_i is negative for a long protection position. The portfolio P&L ΔV is approximated by spread returns in the following way:

$$\begin{aligned}\Delta V &\approx -\sum_i A_i \,\mathrm{RPV01}_{i,t-1}\,\Delta s_i = -\sum_i A_i \,\mathrm{RPV01}_{i,t-1}\,\frac{\Delta s_i}{s_{i,t-1}}\, s_{i,t-1} \\ &= \underbrace{-\Big(\sum_j A_j \mathrm{RPV01}_{j,t-1}\, s_{j,t-1}\Big)}_{=:V_{t-1}} \cdot \sum_i w_i\, r_i,\end{aligned}$$

where $w_i = \dfrac{A_i\,\mathrm{RPV01}_{i,t-1}\, s_{i,t-1}}{\sum_j A_j \mathrm{RPV01}_{j,t-1}\, s_{j,t-1}}$ denotes the percentage weight of the position in the portfolio and $r_i = \dfrac{\Delta s_i}{s_{i,t-1}}$ denotes the spread return. For ease of notation, we write V_{t-1} for the linear approximation of the portfolio value.

Now, assuming that $\mathbf{r} = (r_1, \ldots, r_n)^\intercal \sim \mathrm{N}(0, \boldsymbol{\Sigma})$, i.e., spread returns are jointly normally distributed with expectation 0 (a reasonable assumption for small time horizons) and covariances described by the $n \times n$ matrix $\mathbf{\Sigma}$, the portfolio VaR is again given by the variance-covariance approach, see Equation (1).

3.2.3 Results

On March 23, 2012 JPMorgan's senior management ordered to cease all trading activities for the SCP. The exact portfolio composition is known only for this day from publicly available sources. To calculate risk figures for the SCP, the relevant credit index data is taken from Markit and converted as necessary via the credit valuation model in Appendix A.

The approach uses historical data to fit parameters for both the P&L distribution and the correlation model. After processing the data and excluding constituents with insufficient observations, 93 constituents with a total net notional of USD 154.34 billion remain to be included in the calculations. The unstressed delta-normal 1-day VaR at the 99% confidence level is USD 339.32 million (base case), which is about twice as high as the VaR reported by JPMorgan (2013, pp. 124 ff.). This number will be used as a benchmark for scenarios with stressed correlations.

The problem of finding the constrained global maximum of $\mathrm{VaR}_\alpha(\boldsymbol{\beta})$ is solved numerically. To ensure robustness, first and second order conditions as well as different algorithms are reviewed, including Nelder Mead, Differential Evolution, Simulated Annealing and Random Search, all with the same result. In terms of computational time, Simulated Annealing appears most efficient.

As laid out in Section 2.4.1, a plausibility constraint can be applied to the stress testing method. For this application, where correlation parameters are assumed to follow a multivariate normal distribution, this means considering only correlation scenarios that lie on or below a quantile ellipsoid, which is determined by a Mahalanobis distance. Out of the set of feasible scenarios, the one with the highest value at risk for a given quantile is reported in Table 2.

For the 99%-quantile constraint the (variance-covariance)-VaR is USD 381.08 million, which corresponds to a 12.31% increase relative to the base case. This is a substantial increase given that a daily VaR increase at the 99% level is expected to occur several times a year. The worst case, which is unconstrained with respect to the Mahalanobis distance of the parameters, produces a 1-day VaR of USD 620.96 million, which is 83.01% greater than the base case. These results ignore changes from other risk factors, such as volatility, that would typically increase in a downside scenario as well. In fact, its size makes the SCP especially vulnerable to other factors, such as market liquidity issues and resulting plausible correlation scenarios that are not reflected in historical data.

The results of a joint stress test capturing simultaneous changes in correlation and volatility are also presented in Table 2. Here, the volatility stress level $\tilde{\alpha}$ is set to the same quantile as the Mahalanobis constraint. The parameter ν, which captures the heaviness of the tails of the underlying return distribution, is determined by fitting a multivariate t-distribution to the 250 trading days of returns prior to March 23, 2012.[6] With $\nu = 13.5$, the unstressed t-VaR is already

[6] The parameter ν is fitted by a combination of matching the first two moments and maximum likelihood.

Table 2: The SCP portfolio's 1-day 99% value-at-risk for different Mahalanobis quantile constraints. Percentage changes denote the relative distance to the base VaR, i.e., the VaR under the original setup of March 23, 2012. The joint stress test captures simultaneous changes in correlation and volatility, with percentage changes referring to the base t-VaR scenario. The heaviness of the tails of the return distribution is calibrated to $\nu = 13.5$. The volatility stress level $\tilde{\alpha}$ for the joint stress test is set to the quantile in column one.

	correlation stress			joint stress	
Quantile	$\text{VaR}_{0.99}$	$t\text{-VaR}_{0.99}$	Change(%)	$t\text{-VaR}_{0.99}$	Change(%)
base case	339.32	354.98		354.98	
0.7	366.87	383.80	8.12	386.28	8.82
0.8	369.39	386.44	8.86	416.41	17.31
0.9	372.89	390.10	9.89	464.40	30.83
0.95	375.76	393.11	10.74	510.54	43.82
0.99	381.08	398.67	12.31	617.38	73.92
0.995	383.00	400.68	12.87	664.73	87.26
0.999	386.88	404.74	14.02	780.37	119.84
unconstrained*	620.96	649.62	83.00	1252.53	252.85

*Unconstrained w.r.t. correlation changes; $\tilde{\alpha}$ remains on the 0.999 level.

slightly higher than the normal VaR, i.e. USD 354.98 million instead of 339.32 million. The joint stress t-VaR is roughly USD 617 million at the 99% confidence level, which corresponds to a 73.92% increase over the unstressed t-VaR.

A correlation stress test would have enabled JPMorgan's risk management to identify key risk drivers and more appropriately assess the risks of its portfolio. Figure 5 shows box-plots for the correlation parameters as well as the parameters as of 23 March 2012, and the worst case parameters at a Mahalanobis constraint equivalent to 99%. All parameters, with the exception of β_2, which identifies whether an index is investment grade or high-yield, are stressed upwards, hence, decorrelate. The slight downward shift of β_2 can be attributed to a parameter increase prior to the stress test.

Furthermore, Cont and Wagalath (2016) report a breakdown of correlation between CDX.IG.9 and CDX.IG.10 immediately after trading was halted in March 2012, when the CIO started to sell its large positions (see CDX.IG.9 in Table 1), which "is a signature of the market impact of the CIO's trading". This structural break could not have been predicted by historical data, which shows that the actual risk – owed to the size of the portfolio – was closer to the worst case scenario than suggested by the plausibility constraints. In the unconstrained (worst-)case, the parameter β_4, capturing decorrelation between index series, appears as the major risk driver.

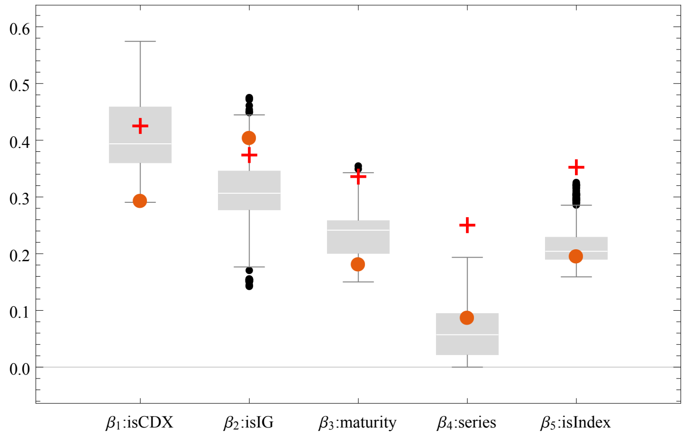

Figure 5: Box-plots of correlation parameters. Dots indicate the observed parameters as of 23.03.2012, crosses indicate the worst-case scenario under a 99%-quantile Mahalanobis distance constraint.

4 Concluding remarks and outlook

The dependence structure amongst portfolio components is of great relevance to the risk inherent in a financial portfolio, and as such, correlation stress testing provides important information about portfolio risk. The methodology developed in this paper maps risk factors to correlations, which in turn allows to specify correlation stress test scenarios in terms of risk-factor changes. In addition, worst-case risk-factor scenarios can be identified, yielding insights into the main factors driving portfolio risks and potential losses. We derive analytical results that allow for computing the value-at-risk impact both for given scenarios and for determining worst-case scenarios.

To illustrate the method in a realistic setting, the correlation stress testing methodology is applied to the case of the "London Whale". This serves as an interesting case, because – in an attempt to decrease the portfolio's riskiness – the notional amount was increased significantly by adding highly-correlated offsetting positions. Such a trading strategy is extremely vulnerable to correlation changes and therefore lends itself to illustrating the importance of correlation stress testing. Historical data suggests that, amongst the worst 1% of correlation scenarios, the 1-day 99%-VaR of the portfolio would have increased by 12% or more. Such a scenario is expected to occur 2–3 times a year. The overall worst-case correlation scenario for the portfolio entails a VaR increase of 83%. These are ceteris paribus results, isolating the effect of a correlation change and neglecting that in reality large correlation changes would typically occur jointly with volatility swings. A joint scenario where volatility and correlation are jointly stressed yields an increase of 73.92% in the 99% t-VaR at a 99%-volatility stress leveland a worst-case increase in VaR of 252.85%.

Correlation stress tests are particularly insightful on portfolios with large positions, as adverse correlation moves may be triggered from the market impact of large trades. More specifically, the ordinary co-movement of two or more assets may be disturbed by the market price impact of large trades. Hence, an appropriate risk assessment of large portfolios must take into account that a small risk footprint under conventional risk measures might be due to offsetting exposures, where only a correlation risk measure can reveal the true portfolio risk.

A further application of correlation stress testing as developed in this paper would be the analysis of central counterparties (CCP), which clear exceptionally large financial portfolios. Currently, 62% of the USD 544 trillion in notional outstanding of interest rate derivatives is cleared through CCPs (Wooldridge, 2016). As part of their mandate and to protect themselves from the default of clients, CCPs use a margining system consisting of a *variation margin* (daily mark-to-market settlement) and an *initial margin* (buffer to cover market losses following a client's default). To account for diversification benefits and to reduce clients' clearing costs, margins are typically calculated on a client's netted position.[7] The resulting margin requirements may be highly correlation-sensitive. Moreover, adverse correlation scenarios may affect many or even all clients, creating simultaneous margin calls to post additional collateral. The correlation stress testing method developed here is capable of identifying these kinds of systemic risk events. In addition, correlation stress testing on the clearing client side would provide insights on possible future margin requirements and the resulting collateral funding risk.

A Credit default swap valuation

Given an underlying entity (e.g. a sovereign or a company), a *credit default swap (CDS)* is a contract between two counterparties, the protection buyer and the protection seller, that insures the protection buyer against the loss incurred by default of the underlying entity within a fixed time interval. The protection buyer regularly pays a constant premium, the *credit spread* or *CDS spread*, which is fixed at inception, up until maturity of the CDS or the default event, whichever occurs first. This stream of payments is termed the *premium leg* of the CDS. In return, the protection seller agrees to compensate the protection buyer for the loss incurred by default of the underlying entity at the time of default in case this occurs before maturity. This constitutes the *protection leg* of the CDS. The CDS spread that makes the value of the premium leg and the protection leg equal is the *fair CDS spread*.

More precisely, let $r > 0$ denote the default-free interest rate, assumed to be constant for simplicity. Furthermore, assume that the payment at default is a fraction $(1 - R)$ of the notational amount, $R \in [0, 1)$. The probability of default of the underlying entity at time t until time T is denoted by $P(t, T)$; this probability is conditional on any information available until time t. Denote by $s(t, T)$ the fair credit spread at time t of a CDS with maturity T. Here, we follow the convention that entering into a CDS involves no initial cash-flow, that is, the market value of a CDS at inception is 0. Even though CDS are nowadays traded with an upfront payment, this still corresponds to the common quoting convention. In other words, the discounted fair values of the premium and the default legs are equal. In addition, to simplify the exposition and notation, we assume that credit spreads are paid continuously instead of quarterly. From the point of view of a protection seller, the value of a CDS contract entered at

[7] Similar offers are found for almost any financial service provider that employs a margining system, e.g. brokers and future exchanges.

time t is given via risk-neutral pricing by the value of the premium leg minus the value of the protection leg. Combining this with the fact that the initial value of the CDS is 0 gives:

$$0 = s(t,T) \underbrace{\int_t^T \mathbf{e}^{-r(u-t)} \left(1 - P(t,u)\right) \mathrm{d}u}_{=\text{RPV01}} - (1-R) \int_t^T \mathbf{e}^{-r(u-t)} P(t, \mathrm{d}u),$$

or, re-arranging for the spread,

$$\frac{s(t,T)}{1-R} = \frac{\int_t^T \mathbf{e}^{-r(u-t)} \, \mathrm{d}P(t,u)}{\int_t^T \mathbf{e}^{-r(u-t)} \left(1 - P(t,u)\right) \mathrm{d}u}. \tag{12}$$

The term RPV01 denotes the *risky present value of a basis point*. For a full derivation we refer to e.g. Chapter 6 of O'Kane (2008).

The *mark-to-market value* of an existing CDS position is expressed as the cost of unwinding the transaction by entering into an offsetting CDS position. In the following we assume a CDS contract with maturity T and notional \$1 entered at time $0 \leq t$ from the point of view of the protection seller. Conditional on no-default at time t, the value of the position at time t is

$$\begin{aligned} V_t &= s(0,T) \int_t^T \mathbf{e}^{-r(u-t)} \left(1 - P(t,u)\right) \mathrm{d}u - (1-R) \int_t^T \mathbf{e}^{-r(u-t)} P(t, \mathrm{d}u) \\ &= \left(s(0,T) - s(t,T)\right) \int_t^T \mathbf{e}^{-r(u-t)} \left(1 - P(t,u)\right) \mathrm{d}u = \left(s(0,T) - s(t,T)\right) \text{RPV01}(t,T). \end{aligned}$$

Here we have used that the values of the premium and protection legs of the time-t CDS are equal.

A simplification of the valuation occurs by assuming that, similar to a constant interest rate, the default probabilities are subject to a constant *hazard rate* $\lambda_t > 0$, i.e., $P(t,T) = 1 - \mathbf{e}^{-\lambda_t (T-t)}$, $T \geq t$. The credit spread formula (12) reduces to the so-called *credit spread triangle*,

$$\frac{s(t,T)}{1-R} = \lambda_t, \tag{13}$$

and the RPV01 is then expressed as

$$\text{RPV01}(t,T) = \int_t^T \mathbf{e}^{-(r+\lambda_t)(u-t)} \, \mathrm{d}u = \int_t^T \mathbf{e}^{-(r+s(t,T)/(1-R))(u-t)} \, \mathrm{d}u. \tag{14}$$

For value-at-risk calculations, it is useful to approximate the P&L ΔV by a first-order Taylor-approximation on the spread change $\Delta s = s(t,T) - s(t-1,T)$:[8]

$$\begin{aligned} \Delta V = V_t - V_{t-1} &\approx \frac{\partial}{\partial s} V_{t-1} \cdot \Delta s \\ &= -\text{RPV01}(t-1,T) \cdot \Delta s + \left(s(0,T) - s(t-1,T)\right) \cdot \frac{\partial}{\partial s} \text{RPV01}(t-1,T) \cdot \Delta s. \end{aligned}$$

[8]Assuming that Δt is small, we ignore the change due to time-decay.

The second term involves a product of spread changes and is therefore smaller, so we shall ignore it for ease of computations, giving

$$\Delta V \approx -\mathrm{RPV01}(t-1,T)\cdot \Delta s.$$

B Tranche spread calculation

This appendix gives a brief outline of the calculation of fair spreads of credit index tranches required for estimating the β-parameters of the "London Whale" portfolio. The calculations are based on O'Kane (2008). For tranches, the given market data consists of running spreads, upfront payments and base correlations. To make all calculations involving both index and tranche positions consistently use spread time series, the tranche data are transformed into financially equivalent fair spreads without upfront payment.

The present value of an index tranche with attachment point K_1, detachment point K_2 and maturity T is given by (cf. O'Kane, 2008, Equation (20.1)):

$$\mathrm{PV}(K_1,K_2) = U(K_1,K_2) + S(K_1,K_2)\underbrace{\int_0^T Z(t)Q(t,K_1,K_2)\,\mathrm{d}t}_{=\mathrm{RPV01}} - \int_0^T Z(t)(-\mathrm{d}Q(t,K_1,K_2)),$$

where $U(K_1,K_2)$ is the upfront spread that is paid at inception of the trade, $S(K_1,K_2)$ is the running spread that is paid regularly (continuously in the case considered), $Z(t)=\mathbf{e}^{-rt}$ is the time-t discount factor and where $Q(t,K_1,K_2)$ denotes the *tranche survival probability.* The expression $\int_0^T Z(t)Q(t,K_1,K_2)\,\mathrm{d}t$ denotes the *risky present value of a basis point (RPV01)* and $\int_0^T Z(t)(-\mathrm{d}Q(t,K_1,K_2))$ corresponds to the value of the protection leg that pays in case of default. The financially equivalent spread without an upfront payment satisfies $s = U(K_1,K_2)/\mathrm{RPV01} + S(K_1,K_2)$, so the calculation reduces to calculating the *tranche survival curve* $Q(t,K_1,K_2)$, $t\in[0,T]$.

Assuming m contiguous tranches, the survival probabilities $Q(T,0,K_1), Q(T,K_1,K_2),\ldots,$ $Q(T,K_{m-1},K_m)$ can be calculated iteratively from the expected tranche losses for each tranche (cf. O'Kane, 2008, pp. 378), which in turn are calculated from the base correlations and the index survival probability in a one-factor Gaussian latent variable model (cf. O'Kane, 2008, pp. 305–307).

Finally, the survival probabilities need to be calculated for $t\in[0,T]$. Market convention holds that the quoted index spread corresponds to a flat term structure (O'Kane, 2008, p. 190). Making the same assumption for the tranche spread s, the *hazard rate* λ entering tranche survival probabilities $Q_T=\mathbf{e}^{-\lambda T}$ is determined via the so-called *credit triangle* $\lambda = s/(1-R)$, giving tranche survival probabilities $Q_t=\mathbf{e}^{-\lambda t}$. The RPV01 is then calculated as

$$\int_0^T Z(t)Q(t,K_1,K_2)\,\mathrm{d}t = \int_0^T \mathbf{e}^{-(r+\lambda)}\,\mathrm{d}t = \frac{1-\mathbf{e}^{-(r+\lambda)T}}{r+\lambda}.$$

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