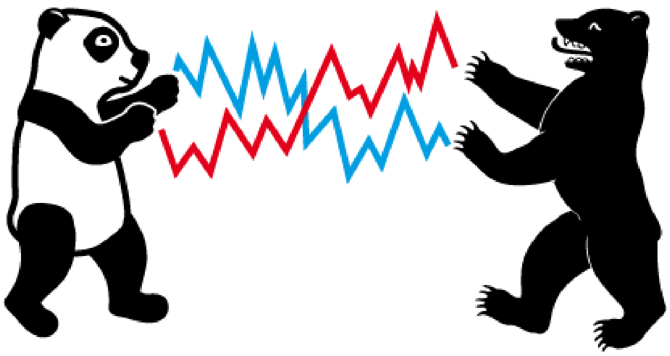
IRTG 1792 Discussion Paper 2018-058

Investing with cryptocurrencies evaluating the potential of portfolio allocation strategies

Alla Petukhina*
Simon Trimborn*
Wolfgang Karl Härdle*
Hermann Elendner*



* Humboldt-Universität zu Berlin, Germany

This research was supported by the Deutsche Forschungsgemeinschaft through the International Research Training Group 1792 "High Dimensional Nonstationary Time Series".

http://irtg1792.hu-berlin.de
ISSN 2568-5619

International Research Training Group 1792

Investing with cryptocurrencies - evaluating the potential of portfolio allocation strategies [1]

Alla Petukhina [2]
Simon Trimborn [3]
Wolfgang Karl Härdle [4]
Hermann Elendner [5]

October 24, 2018

Abstract

The market capitalization of cryptocurrencies has risen rapidly during the last few years. Despite their high volatility, this fact has spurred growing interest in cryptocurrencies as an alternative investment asset for portfolio and risk management. We characterise the effects of adding cryptocurrencies in addition to traditional assets to the set of eligible assets in portfolio management. Out-of-sample performance and diversification benefits are studied for the most popular portfolio-construction rules, including mean-variance optimization, risk-parity, and maximum-diversification strategies, as well as combined strate-

[1]Financial support from IRTG 1792 "High Dimensional Non Stationary Time Series", Humboldt-Universität zu Berlin, and NUS FRC grant R-155-000-199-114 "Augmented machine learning and network analysis with applications to cryptocurrencies and blockchains" is gratefully acknowledged.
[2]Humboldt-Universität zu Berlin, C.A.S.E. - Center for Applied Statistics and Economics, Spandauer Str. 1, 10178 Berlin, Germany, tel: +49 (0)30 2093-5748, E-Mail: petukhia@hu-berlin.de
[3]Department of Statistics & Applied Probability, National University of Singapore, Block S16, Level 7, 6 Science Drive 2, 117546 Singapore tel: +65 6516-1245, E-Mail: simon.trimborn@nus.edu.sg
[4]Humboldt-Universität zu Berlin, C.A.S.E. - Center for Applied Statistics and Economics, Spandauer str. 1, 10178 Berlin, Germany; Sim Kee Boon Institute for Financial Economics, Singapore Management University, 50 Stamford Road, Singapore 178899; Weierstrass Institute, Mohrenstr. 39 10117 Berlin, Germany, tel: +49 (0)30 2093-5630, E-Mail: haerdle@hu-berlin.de
[5]Finance Group, Humboldt-Universität zu Berlin, Dorotheenstr. 1, 10099 Berlin, Germany, E-Mail: hermann.elendner@wiwi.hu-berlin.de

gies. To account for the frequently low liquidity of cryptocurrency markets we incorporate the LIBRO method, which gives suitable liquidity constraints. Our results show that cryptocurrencies can improve the risk-return profile of portfolios. In particular, cryptocurrencies are more useful for portfolio strategies with higher target returns; they do not play a role in minimum-variance portfolios. However, a maximum-diversification strategy (maximising the Portfolio Diversification Index, PDI) draws appreciably on cryptocurrencies, and spanning tests clearly indicate that cryptocurrency returns are non-redundant additions to the investment universe.

Keywords: cryptocurrency, CRIX, investments, portfolio management, asset classes, blockchain, Bitcoin, altcoins, DLT

JEL Classification: C01, C58, G11

1 Introduction

Cryptocurrencies (CCs) exhibited remarkable performance in the past years. Driven by huge inflows of capital into the market, CCs gained strongly in market value. Recently, Initial Coin Offerings (ICOs) brought even more capital into the market by offering an easy way to bring Venture Capital into projects. Simultaneously indices like CRIX, developed by Trimborn and Härdle (2018), were introduced to capture the market evolution and provide a basis for ETFs. Driven by these developments, cryptocurrency markets became increasingly attractive for investors, who are beginning to consider CCs as a new class of alternative investments. Prior research investigated investment in Bitcoin (BTC): Brière et al. (2015) and Eisl et al. (2015) studied the performance of traditional portfolios, when BTC is added to them. They documented enhanced portfolios in terms of risk-return profiles. Klein et al. (2018) found BTC not to be the New Gold based on its time dependent behaviour. Hafner (2018) studied the time series of BTC in terms of the appearance of bubbles, while Scaillet et al. (2018) reported frequent price jumps in BTC trading. These properties imply high risk on BTC positions, requiring risk-optimized portfolios when investing into them. Due to the huge capital inflow and consequently high realised returns over the last years, altcoins (CCs other than BTC) became interesting for investors, too. Moreover, they are of interest for investors due to the diversification effect. The effect was observed by Elendner et al. (2017), who found CCs to have a low linear dependency with each other. They also found the top 10 CCs by market capitalization to have a low linear dependency with traditional assets.

Further investigating this effect, first studies focused on the effect of CCs being added to a portfolio of traditional assets. Chuen et al. (2017) investigated the performance of such a portfolio when adding CRIX, Trimborn and Härdle (2018), into them, which is equal to consider an ETF on CRIX, as so a sentiment optimized portfolio when utilizing the top CCs in CRIX. Trimborn et al. (2017) introduced LIquidity Bounded Risk-return Optimization (LIBRO) and considered including a large sample of CCs as alternative investment into a portfolio consisting of S&P100, US Bonds and Commodities. They considered Markowitz and Conditional Value-at-Risk optimized portfolios. Due to the low liquidity in the CC market compared to traditional markets, LIBRO performs overall well in this market and protects an investor from the risk of an inability to trade a CC in the necessary amounts due to low trading volume. Alessandretti et al. (2018) investigated LSTMs and decision trees as portfolio optimization methods for portfolios only consisting of CCs, finding enhanced return performance.

Several studies covered specific aspects and strategies of investing with CCs. To the best of our knowledge, it remains an open question which objective function leads to which kind of investment strategy. We intend to fill this gap by comparing a broad variety of investment strategies on portfolios including different kind of traditional assets and CCs. We consider risk-oriented, return-oriented, risk-return-oriented and combined strategies, see Table 1 for a full list of all strategies under consideration. We provide a broad study considering extending window and rolling window as approaches for the optimization of target function for the portfolio weights. We test the robustness of the results under 3 different kinds of re-allocation frequencies, daily, weekly and monthly. Furthermore, we test the performance of the strategies when using the method LIBRO of Trimborn et al. (2017) on them. To the best of our knowledge, this is the broadest study on investing with CCs conducted so far.

The paper is organized as follows. Section 2 gives an overview of the asset allocation models under consideration with the focus on interconnections between them. In Section 2.2 we explain the idea of model averaging for various investment strategies. Section 3 reviews the LIBRO strategy of Trimborn et al. (2017). In section 4 we explain the methodology for comparing the performance of models considered. Section 5 describes the dataset of portfolio components and Section 6 analyses an out-of-sample performance of all portfolio strategies with CCs and traditional assets. The results are summarized in Section 7.

The codes used to obtain the results in this paper are available via `www.quantlet.de` .

2 Description of the Asset-Allocation Models

Consider a matrix $X \in \mathbb{R}^{P \times N}$ of P - days-long dataset of N asset log-returns. In our comparative study we rely on a "moving-window" approach. Specifically, we choose an estimation window of length $K = 252$ days (i.e. one year). We investigated the performance of strategies for three rebalancing period lengths k: monthly – with $k = 21$ days, weekly – with $k = 5$ days and daily with $k = 1$ day.[6] In each rebalancing period t ($t = 1, \dots, T$, where T is a number of moving windows, defined as $T = \frac{P-K}{k}$), starting on date $K + 1$, we use the data in the previous K days to estimate the parameters required to implement a particular strategy. These estimated parameters are then used to determine the relative portfolio weights w in the portfolio of only-risky assets. We then use these weights to compute the return in rebalancing period $t + 1$. This process is continued by adding the k daily-returns for the next period in the dataset and dropping the earliest returns, until the end of the dataset is reached. The outcome of this rolling-window approach is a series of $P - K$ daily out-of-sample returns generated by each of the portfolio strategies listed in Table 1. For simplification we omit the index t for moving window or rebalancing period.

Traditional evaluation literature (e.g. DeMiguel et al. (2009), Schanbacher (2014)) considers an investor whose preferences are specified in terms of utility functions and fully described by the portfolio mean μ_P and variance σ_P. Merton (1980) demonstrated that a very long time series is required in order to receive the accurate estimates of expected returns. Taking into account the high potential for the error of expected returns' estimates, some authors, e.g. Haugen and Baker (1991), Chopra and Ziemba (1993) and Chow et al. (2011), suggested to utilize only estimates of covariance matrix as inputs for the optimization procedure. Thus, investors assume that all stocks have the same expected returns and under this strong assumption the optimal portfolio is the global minimum-variance portfolio. Minimum-variance portfolio strategy represents one of the so-called risk-based portfolios, i.e. the only input used is the estimate of the variance-covariance matrix. In this paper we consider the most popular ones: Maximum Diversification,

[6] We also tested strategies on extending window as in Trimborn et al. (2017), but as the insights are similar these results are not reported.

Risk-Parity, Minimum Variance and Minimum CVaR portfolio. In section 2.1 we describe the individual strategies from the portfolio-choice literature that we consider. Along with traditional approaches we consider a decision maker with risk preferences, specified directly in percentile terms, and portfolio construction based on higher portfolio returns distribution moments such as skewness and kurtosis. Therefore, in our comparative study we distinguish three groups of analysed individual strategies: return-oriented, risk-oriented (or risk-based, as in Clarke et al. (2013)), as well as a Maximum Sharpe ration (MV-S) - tangent portfolio rule, which can be recognised in our dimensions as risk-return oriented strategy.

Taking into account that ranking of models changes over time and motivated by fact that in many fields combination of models performs well (see e.g. Clemen (1989), Avramov (2002)) we also include to our analysis the combination of portfolio models based on bootstrap approach inspired by Schanbacher (2014) and Schanbacher (2015). The detailed methodology of combined portfolio models is discussed in section 2.2.

2.1 Asset allocation models

In this section we overview a set of models that are considered later in the empirical analysis. We discuss links between the strategies and give conditions under which they are equivalent. In general we use the "plug-in" approach, i.e. we replace moments of returns' distributions by their sample counterparts.

2.1.1 Equal weighted portfolio

The most naïve portfolio is equal weighting (EW). Investors allocate capital evenly and every asset has weight $w = 1/N$. EW is easy to implement: the portfolio manager is not required to make assumptions on the distribution of the assets' returns, DeMiguel et al. (2009). The EW portfolio is indeed a mean-variance optimal portfolio if the constituents have the same expected returns and covariances.

2.1.2 Mean-variance portfolio

Many portfolio managers rely on the Markowitz risk-return or mean-variance (MV) rule, which combines assets into an "efficient" portfolio offering risk-adjusted target returns, Härdle and Simar (2015). Essential weaknesses of MV portfolio are the normal distribution assumption of financial returns and risk measured by multiple of volatility. The drawbacks of Markowitz portfolio in terms of composition, widely discovered in the literature, are portfolio concentration, i.e. high portfolio weights are assigned to a limited subset of the full set of assets or securities, and high sensitivity to small changes in estimates of inputs - parameters μ and σ, see Jorion (1985), Simaan (1997), Kan and Zhou (2007). In the Gaussian World portfolio weights w are obtained by the solution of the following optimization problem:

$$\begin{aligned} \min_{w \in \mathbb{R}^p} \quad & \sigma_P^2(w) \stackrel{\text{def}}{=} w^\top \Sigma w \\ \text{s.t.} \quad & \mu_P(w) = r_T, \\ & w^\top 1_N = 1, \quad w_i \geq 0 \end{aligned} \tag{1}$$

where $\Sigma \stackrel{\text{def}}{=} \mathsf{E}_{t-1}\{(X-\mu)(X-\mu)^\top\}$ and $\mu \stackrel{\text{def}}{=} \mathsf{E}_{t-1}(X)$ are the sample covariance matrix and vector of mean returns respctively, $\mu_P(w) \stackrel{\text{def}}{=} w^\top \mu$, is the portfolio mean and r_T – "target" return, ranging from minimum return to maximum return to trace out an efficient frontier. E_{t-1} is the expectation operator conditional on the information set available at $t-1$.

We compare three benchmark Mean-Variance portfolio: global minimum variance portfolio ("MinVar" in Table 1), tangency portfolio ("MV-S") and portfolio with the highest in-sample return (Risk–return–max ret - "RR–max ret"). In our classification approach a risk-based decision is MinVar, which is the most averse to risk and has the lowest target portfolio return. In opposite a return-orientated RR–max ret portfolio is located on a high risk end of Markowitz efficient frontier. MV-S portfolio occupies a middle-ground between these two: it maximizes a Sharpe ratio (18), involving in this way both risk and return estimation for portfolio weights construction. We characterise MV - S as a risk-return based strategy.

2.1.3 Conditional Value-at-Risk Portfolio Optimization

A strong limitation of Markowitz based portfolio strategies is the assumption of Gaussian distributions of assets' log-returns. Well known stylized facts indicate that variance or volatility is an insufficient risk measure, leading to non-optimal portfolio composition. Chuen et al. (2017) and Elendner et al. (2017) as well as descriptive statistics of our investment universe, shown in Figure 6 and Table 8 in the Appendix 8.2, provide strong evidence of this heavy-tailed distributions of cryptocurrencies. In order to react to this fact we therefore include higher moments. More precisely we construct Mean - Conditional Value at Risk (CVaR) optimization portfolio as in Rockafellar and Uryasev (2000), Krokhmal et al. (2002). Given $\alpha < 0.05$ risk level, the CVaR optimized portfolio weights w are calculated as:

$$\min_{w \in \mathbb{R}^N} \text{CVaR}_\alpha(w), \;\; \text{s.t. } \mu_P(w) = r_T, w^\top 1_p = 1, w_i \geq 0, \tag{2}$$

$$\text{CVaR}_\alpha(w) = -\frac{1}{1-\alpha} \int\limits_{w^\top X \leq -\text{VaR}_\alpha(w)} w^\top X f(w^\top X|w) dw^\top X, \tag{3}$$

with $\frac{\partial}{\partial w^\top X} F(w^\top X|w) = f(w^\top X|w)$ the probability density function of the portfolio returns with weights w. $\text{VaR}_\alpha(w)$ is the corresponding α-quantile of the cdf, defining the loss to be expected in $(\alpha \cdot 100)\%$ of the times.

As for Mean-variance portfolio we construct an efficient frontier and compare two portfolios, one in terms of risk-orientation, one return-orientated. Since we employ a plug-in method to calculate return-orientated MinVar and MinCVaR portfolios, they exhibit an identical composition and are only invested in the riskiest asset with the highest expected return. Due to this reason we do not separate them and name this portfolio Risk-return – max return portfolio ("RR – Max ret" in Table 1).

2.1.4 Risk Parity (Equal risk contribution - ERC) portfolio

One of traditional risk-based portfolio concepts is the Risk Parity approach. The underlying idea is an adjustment of weights such that each asset has the same contribution to portfolio risk, see Qian (2006). Maillard et al. (2010) derived properties of such portfolio and renamed them to "equal-risk contributions" (ERC) instruments. The Euler decomposition of the portfolio volatility $\sigma_P(w) = \sqrt{w^\top \Sigma w}$, Härdle and Simar (2015), allows to present it in the following form:

$$\sigma_P(w) \stackrel{\text{def}}{=} \sum_{i=1}^{N} \sigma_i(w) = \sum_{i=1}^{N} w_i \frac{\partial \sigma_P(w)}{\partial w_i} \tag{4}$$

where $\frac{\partial \sigma_P(w)}{\partial w_i}$ is the marginal risk contribution and $\sigma_i(w) = w_i \frac{\partial \sigma_P(w)}{\partial w_i}$ the risk contribution of i-th asset. Finally to construct ERC portfolio one calibrates:

$$\sigma_i(w) = \frac{1}{N} \quad \forall i \tag{5}$$

ERC portfolio can be compared to EW portfolio where instead of allocating capital equally across all the assets, ERC portfolio allocates the total risk equally across the assets. Consequently, under the condition of equality of log-returns distributions variances ERC portfolio is identical to EW portfolio. ERC portfolio are comparable to MinVar portfolio, which focus on parity of marginal contributions of all assets.

2.1.5 Maximum diversification portfolio with Portfolio diversification index (PDI)

Originally Maximum diversification portfolio (MD) uses an objective function introduced in Choueifaty and Coignard (2008) that maximizes the ratio of weighted average asset volatilities to portfolio volatility or diversification ratio, see (21). In our study instead of the diversification ratio we maximize a Portfolio diversification index (PDI) proposed by Rudin and Morgan (2006). It consists in assessing a Principal Component Analysis (PCA) on weighted asset returns' covariance matrix, i.e. identifying possibly independent sources of variation. In its original form PDI does not account for the actual portfolio weights, here we incorporate weighted returns. One optimizes:

$$\max_{w \in \mathbb{R}^N} \text{PDI}_P(w), \text{ s.t. } w^\top 1_p = 1, \quad w_i \geq 0 \tag{6}$$

$$PDI_P(w) = 2\sum_{i=1}^{N} iW_i - 1, \tag{7}$$

where $W_i = \frac{\lambda_i}{\sum_{i=1}^{N} \lambda_i}$ are the normalised covariance eigenvalues λ_i in decreasing order, i.e. the relative strengths. Thus, an "ideally diversified" portfolio, i.e. in situations when all assets are perfectly uncorrelated and $W_i = 1/N$ for all i, then $PDI = N$. On the contrary a $PDI \approx 1$ indicates diversification is effectively impossible. Thus, in case of perfectly uncorrelated assets the MD portfolio will be exactly the EW portfolio. The PDI summarises the diversification of large number of securities using a single statistic, and can compare the diversification across different portfolio or time periods.

2.2 Averaging of portfolio models

Along with individual allocation models we also consider combinations of models. Every individual model experiences an estimation risk, to reduce such risk the idea of models' combination or diversification got a high attention in different areas. Model-averaging is used in forecasting, Avramov (2002). The traditional model averaging methods use information criteria - like AIC or BIC - to identify shares of models. In case of allocation models the likelihood is unknown, therefore to calculate models shares we use the loss l, which is defined as follows:

$$l(w) = w^\top \hat{\mu} - \frac{\gamma}{2} w^\top \hat{\Sigma} w. \tag{8}$$

Parameter γ reflects the investor's risk aversion with γ being large (small) for a risk-averse (risk-seeking) investor. We use two approaches to combine: Naïve averaging of the portfolio weights as well as the combination method based on a bootstrap procedure, described in Schanbacher (2014). However, to account for possible time series dependencies at a daily frequency, we apply the stationary bootstrap algorithm Politis and Romano (1994) with automatic block-length selection proposed by Politis and White (2004).

Consider a set of m asset allocation models. The corresponding portfolio weights are given by $W = (w^1, \ldots, w^m)$. Shares of individual models are $\pi = (\pi^1, \ldots, \pi^m)$, such that

$\pi^\top 1_m = 1$. The element i represents the share of the i-th model in the combination. Then the combined portfolio weight is given by:

$$w^{comb} = \sum_{i=1}^{m} \pi^i w^i \qquad (9)$$

The Naïve combination over all asset allocation models just assigns equal shares, i.e. $\pi_t^i = \frac{1}{m}$ for all $i = 1, \ldots, m$.

Alternative approach is to set share π_t^i equal to the probability that model i outperforms all other models. We apply a bootstrap method to estimate the probabilities. For every period t we generate a random sample with replacement of k returns using returns $X_{k(t-1)+1} \ldots X_{k(t-1)+1+K}$, i.e. K-long returns vectors of the $t-1$ rolling-window. We apply all m asset allocation models to these bootstrapped returns. The procedure is repeated B times. Let $s_{i,b} = 1$ if model i outperforms in terms of the loss function other models in the b-th bootstrapped sample, otherwise $s_{i,b} = 0$. The probability of model i being best, is estimated by

$$\hat{\pi}_t^i = \frac{1}{B} \sum_{b=1}^{B} s_{i,b} \qquad (10)$$

where $B = 100$ is a number of independent bootstrap samples, $s_{i,b} = 1$ if model i is the best model in the b-th sample.

3 LIBRO framework

In this section, we review the LIBRO framework for portfolio formation, introduced by Trimborn et al. (2017) . LIBRO avoids low liquidity assets to take on a too high portfolio weight by introducing weight constraints depending on liquidity.

Since liquidity does not have a unique definition, one has to decide which measure to employ. Wyss (2004) surveys a variety of liquidity measures, from which the Trading Volume (TV) was chosen as a proxy for liquidity for the CC market. Measures like bid-ask spread would be applicable too but reliable order book data for all CCs are not available since the market lacks a dominant or central exchange trading all assets. A huge advantage of TV is, that this data are available for all markets. Thus we follow Trimborn et al. (2017)

Model	Reference	Abbreviation
Equally weighted	DeMiguel et al. (2009)	EW
Risk-oriented strategies		
Mean – Var – min var	Merton (1980)	MinVar
Mean – CVaR – min risk	Rockafellar and Uryasev (2000)	MinCVaR
Equal Risk Contribution (Risk-parity)	Maillard et al. (2010)	ERC
Maximum Diversification	Rudin and Morgan (2006)	MD
Return-oriented strategies		
Risk – Return– max return	Markowitz (1952)	RR – Max ret
Risk-Return-oriented strategies		
Mean – Var – max Sharpe	Jagannathan and Ma (2003)	MV – S
Combination of models		
Naïve Combination	Schanbacher (2015)	Comb Naïve
Weight Combination	Schanbacher (2014)	Comb

Table 1: List of asset allocation models

and use TV as the liquidity measure.

TV is defined as:

$$TV_{ij} = p_{ij} \cdot q_{ij} \tag{11}$$

where p_{ij} is the closing price of asset i at date j, and q_{ij} is the volume traded at date j of asset i. The liquidity of asset i in period t can be measured using the sample median of trading volume:

$$TV_i = \frac{1}{2}(TV_{i,up} + TV_{i,lo}) \tag{12}$$

where $TV_{i,up} = TV_{i,\lceil \frac{l+1}{2} \rceil}$ and $TV_{i,lo} = TV_{i,\lfloor \frac{l+1}{2} \rfloor}$.

Define M as the total amount invested on all N assets, thus Mw_i is the market value held in asset i. Trimborn et al. (2017) formulate the constraint on the weight of asset i by:

$$Mw_i \leq TV_i \cdot f_i, \tag{13}$$

where f_i controls the speed an investor intends to clear the current position on asset i. For example, a $f_i = 0.5$ means the position on asset i can not be larger than the 50% median trading volume. It results the boundary for the weight on asset i:

$$w_i \leq \frac{TV_i \cdot f_i}{M} = \widehat{a}_i. \tag{14}$$

The beauty of this approach lies in its ease to include it into any kind of portfolio optimization method.

4 Evaluation of Portfolios' Performance

4.1 Performance measures

In order to assess performance of investment strategies over time we consider five common performance criteria widely used in literature as well as by practitioners. Performance measures were computed based on the time series of daily out-of-sample returns generated by each strategy. First, we measure the out-of-sample cumulative wealth of every strategy i.

$$W_{i,t+1} = W_t + \hat{w}_t^\top X_{t+1} \tag{15}$$

The initial portfolio wealth is $W_0 = \$1$. Cumulative wealth, while naturally of high interest to measure the performance that can be achieved over the period considered, is not sufficient to rank our allocation approaches. That is why we compute two traditional quantities to measure risk-adjusted returns: Sharpe ratio and Certainty-equivalent, as well as Adjusted Sharpe ratio to address the necessity to evaluate MinCVaR strategy and issue of non-Gaussian nature of returns' distribution.

The Sharpe ratio of strategy i is defined as the sample mean of out-of-sample excess returns (over the risk-free asset), divided by their sample standard deviation :

$$\widehat{SR}_i = \frac{\hat{\mu}_i}{\hat{\sigma}_i^2} \tag{16}$$

The Certainty Equivalent (CEQ) covers a large range of potential investors. For the case $\gamma = 1$ it is also equivalent to the close form solution of Markowitz (1952) portfolio optimization problem defined in (1).

$$\widehat{CEQ_{i,\gamma}} = \hat{\mu}_i - \frac{\gamma}{2}\hat{\sigma_i}^2 \tag{17}$$

As can be noted CEQ is equivalent to the loss function l defined in (8). Both CEQ and SR are more suitable for assessment of strategies with normally distributed returns. To address this drawback, Pezier and White (2008) proposed Adjusted Sharpe Ratio (ASR). ASR explicitly incorporates skewness and kurtosis:

$$\widehat{ASR_i} = \widehat{SR_i}\Big[1 + \Big(\frac{S}{6}\Big)\widehat{SR_i} - \Big(\frac{K}{24}\Big)\widehat{SR_i}^2\Big] \tag{18}$$

where SR is the Sharpe Ratio, S - skewness and K - excess kurtosis. Thus, the ASR accounts for the fact that investors prefer positive skewness and negative excess kurtosis, as it contains a penalty factor for negative skewness and positive excess kurtosis.

To assess potential transaction costs associated with asset rebalancing we use the turnover measure. We compute the average turnover between two consecutive rebalancing dates with the following formula:

$$Turnover_i = \frac{1}{T-K}\sum_{t=1}^{T-K}\sum_{j=1}^{N}|\hat{w}_{i,j,t+1} - \hat{w}_{i,j,t+}| \tag{19}$$

where $w_{i,j,t}$ and $w_{i,j,t+1}$ are weights assigned to the asset j for periods t and $t+1$ and $w_{i,j,t+}$ is its weight right before rebalancing at $t+1$. Thus, we account for price change over time and assume that one needs to execute trades in order to rebalance the portfolio towards the w_t target. Higher turnover leads investors to significant transaction costs, consequently the lower the Turnover of the strategy, the better it performs.

4.2 Test for the difference of performance measures for two allocation strategies

To test if strategies are significantly different from each other, we derive the p-values. The common approach by Jobson and Korkie (1981) with corrections derived in Memmel (2003) is widely used in the performance evaluation literature (e.g. in DeMiguel et al. (2009)). This test is not appropriate when returns have tails heavier than the normal distribution or are of time series nature. Instead, in our empirical study as a testing procedure we chose the Ledoit and Wolf (2008) test with the use of robust inference methods. We tested difference for both CEQ and SR. We report results for its HAC (heteroskedasticity and autocorrelation) inference version. The procedure is described in Appendix 8.1.

4.3 Measures of diversification effects

To measure allocation concentration and portfolio diversification effects we calculated three measures: Portfolio Diversification Index (PDI) as in the equation (7), Effective N and Diversification ratio. Effective N has been introduced by Strongin et al. (2000). For every asset $j = 1 \dots N$:

$$N_{Eff}(w_t) = \frac{1}{\sum_{j=1}^{N} w_{j,t}^2} \tag{20}$$

N_{Eff} varies from 1 in the case of highest concentration, i.e. portfolio entirely invested in a single asset, to N - its maximum for equally weighted portfolio. The design of effective N is related to other traditional concentration measures, e.g., the Herfindahl Index is also the sum of squared market shares to measure the amount of competition. Effective N can be interpreted as the number of equally-weighted stocks that would provide the same diversification benefits as the portfolio under consideration.

Choueifaty et al. (2011) suggested the diversification ratio, it measures the proportion of the portfolio's weighted average volatility to its overall volatility:

$$DR(w_t) = \frac{w_t^\top \sigma_t}{\sqrt{w_t^\top \Sigma_t w_t}} = \frac{w_t^\top \sigma_t}{\sigma_{P,t}(w_t)} \tag{21}$$

Thus, the diversification ratio has the form of Sharpe ratio (18), where the sum of weighted asset volatilities replaces the expected excess return. In case of perfectly correlated assets DR equals 1, contrary to the situation of "ideal diversification", i.e. perfectly uncorrelated assets, $DR = \sqrt{N}$. Thus, in our empirical study we will report results on DR^2 for two reasons. First to make it comparable to the other two used metrics and second, Choueifaty and Coignard (2008) demonstrate that for a universe of N independent risk factors, the portfolio that weighted each factor by its inverse volatility would have a DR^2 equal to N. Hence DR^2 can be viewed as a measure of the effective degrees of freedom within a given investment universe.

5 Data

5.1 Data Sample

Our empirical analysis uses daily returns on a sample of CCs and traditional assets over the period January 2015 to December 2017 (781 daily log-returns). CC prices are taken from the publicly available CRIX cryptocurrencies database (thecrix.de). We require CCs to have continuous return time-series over the chosen testing time period. Thus, our final data sample for portfolios construction includes 55 CCs.

To test the performance of each of the strategies considered in a meaningful context, our research question studies the effects of including CCs *as an addition to* classical portfolio management. Therefore, we start our investment universe with 16 traditional assets, including 5 asset classes: equity, fixed-income, fiat currencies, commodities, and real estate. Since CCs are global in nature, our traditional assets cover the 5 main geographic and economic areas. In this way, the asset space is sufficiently broad and diversified to measure the relevance of each approach over a test period, and at the same time is narrow enough to not lead to a high-dimensionality issue for covariance estimation. The full list of traditional constituents of the investment universe is provided in Table 2. Tables 8 and 7 in Appendix 8.2 report summary statistics of all portfolios' constituents considered in the empirical study.

The main characteristics of our data corresponds to the findings of the prior literature, e.g., Elendner et al. (2017), Chuen et al. (2017): CCs outperform traditional asset classes in terms of average daily realised return, have higher return volatility, means are mostly positive while the medians are mostly negative, positive movements occur less frequently

Name	Asset class
EURO STOXX 50	Equity
S&P100	Equity
NIKKEI225	Equity
FTSE100	Equity
SSE (Shanghai Stock Exchange) index	Equity
MSCI ACWI COMMODITY PRODUCERS	Commodities
GOLD	Commodities
FTSE EPRA/NAREIT DEV REITS	Real Estate
EUR/USD	Fiat currency
GBP/USD	Fiat currency
CNY/USD	Fiat currency
YEN/USD	Fiat currency
Eurozone 10Y Gov Bonds	Fixed income
UK 10Y Gov Bonds	Fixed income
USA 10Y Treasuries	Fixed income
Japan 10Y Gov Bonds	Fixed income

Table 2: List of traditional constituents of the investment universe. Source: *Bloomberg*

than negative ones, but with higher absolute values (minimal and lower deciles' absolute values are less than maximal and higher deciles' for the majority of CCs). Correlation analysis of the top 5 CCs by market capitalisation with traditional asset classes shows a high potential of CCs to improve diversification: all correlation coefficients do not exceed 0.1.

6 Empirical results

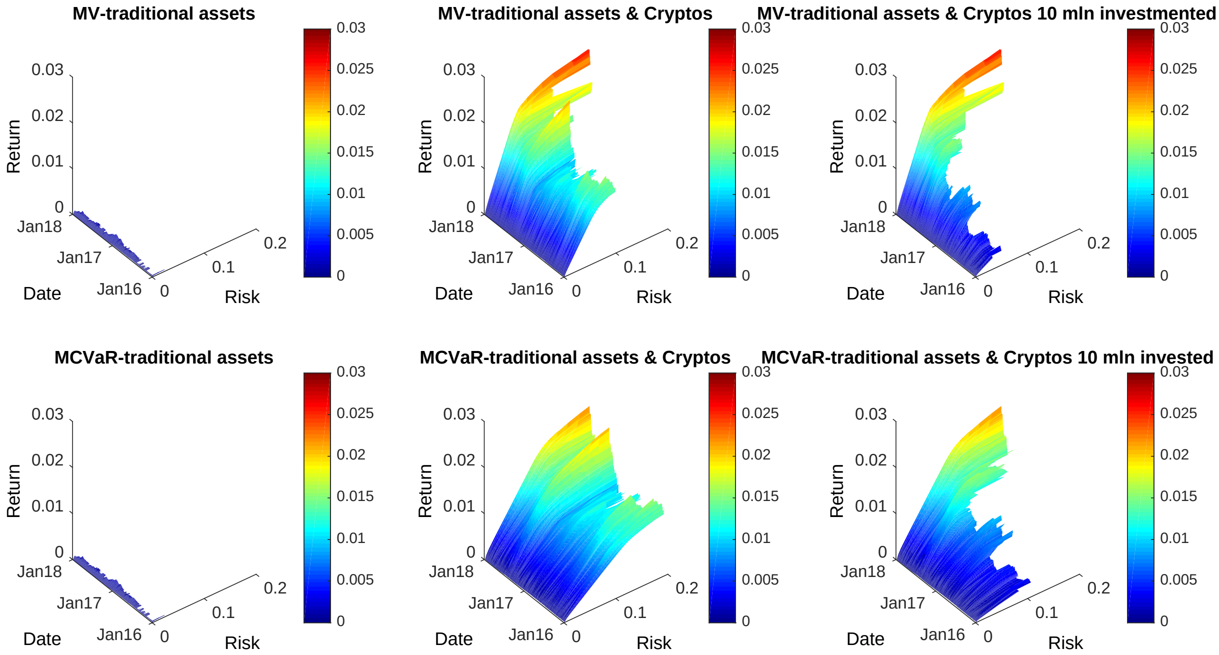


Figure 1: Efficient frontiers surfaces:

CCPEfficient_surface

6.1 Portfolios' performance analysis

In this section we discuss performance of portfolio allocation strategies in two dimensions: first, we are interested in risk-adjusted performance and second, diversification benefits generated by every method considered. In the beginning of our performance comparison we examine effects of CCs on efficient frontiers. Figure 1 plots Mean-variance and Mean-CVaR efficient frontiers of traditional assets' portfolios with and without CCs, as well as with and without liquidity constrains, built on daily basis. For both optimisation rules incorporation of CCs leads to the noticeable moves of frontiers up as well as stretching in both dimensions, i.e. under the same level of risk portfolios with CCs give the same level of returns as well as much higher returns can be reached with CCs included. Second important observation is Mean-Variance frontiers in most cases are shorter than Mean-CVaR frontiers (the same level of returns has lower variance than CVaR) what could be viewed as an evidence of underestimation of risk, measured by variance and predicted by theory. LIBRO-approach shortens frontiers in the beginning of the investment period, eliminating influence of turbulent growing CCs with insignificant trading volume. At the same time it is visible that, roughly starting from January 2017, there is almost no difference between frontiers with (LIBRO) and without constraints, which can be explained by the growth of both trading volumes and capitalisation of the entire market. To justify this visual effect we conduct two mean-variance spanning tests on each of the 55 CCs: the corrected test Huberman-Kandel (HK) (Huberman and Kandel (1987)) and the step-down test by Kan and Zhou (2012).

Table 9 in Appendix 8.3 lists only CCs with at least one test rejecting the spanning at the 10% level. The corrected HK test rejects spanning for 16 CCs including coins with the highest market capitalisation Bitcoin(BTC), Ripple (XRP), Dash (DASH) and Litecoin (LTC). The step-down test provides the information on the source for spanning rejection: F_1 tests for spanning of tangency portfolios of whereas F_2 tests for global minimum portfolios' spanning. From Table 9, F_1 test rejects spanning for 27 CCs, pointing out that tangency portfolios with CCs included are significantly different from benchmark tangency portfolio and F_2 rejects spanning only for two CCs. Thus, we can conclude that there is an evidence that MV-S portfolio can be improved by 27 from 55 CCs, but there is much weaker evidence that MinVar portfolio can be improved. This result can be supported by the dynamics of portfolios' composition presented in Figures 4 and 5 for unconstrained and LIBRO portfolios respectively. It can be noticed, that MinVar portfolios in both cases are entirely constructed from traditional assets, whereas MV-S portfolios have a CCs' component through the whole investment period.

First we examine cumulative wealth, produced by different allocation strategies. Figures 2 and 3 display dynamics of cumulative wealth with and without liquidity constraints for all nine strategies considered. As benchmarks we also plot S&P100, EW, MV-S and MinVar portfolios built only from traditional investment constituents (Traditional Assets - "TrA"). Following conclusions can be drawn: in terms of Cumulative wealth portfolios with CCs outperform or perform equally compared to all portfolios with conventional constituents, the most promising results exhibit MD with accumulated wealth 515%, RR-max ret with 470% and COMB with 354% portfolios. EW portfolio also exhibits high performance and reached 364 % of cumulative wealth. All risk-based portfolios, except MD, underperform EW portfolio. These results are relevant for both LIBRO and unconstrained approaches. LIBRO portfolios have slightly lower performance measured in accumulated wealth, yet they account for a further risk source, low liquidity. Table 3 summarises all performance indicators. The conducted t-test to compare difference between means of returns of all strategies and EW portfolio did not justify the statistical significance for COMB and RR-Max ret portfolios and confirmed it for the rest of models.

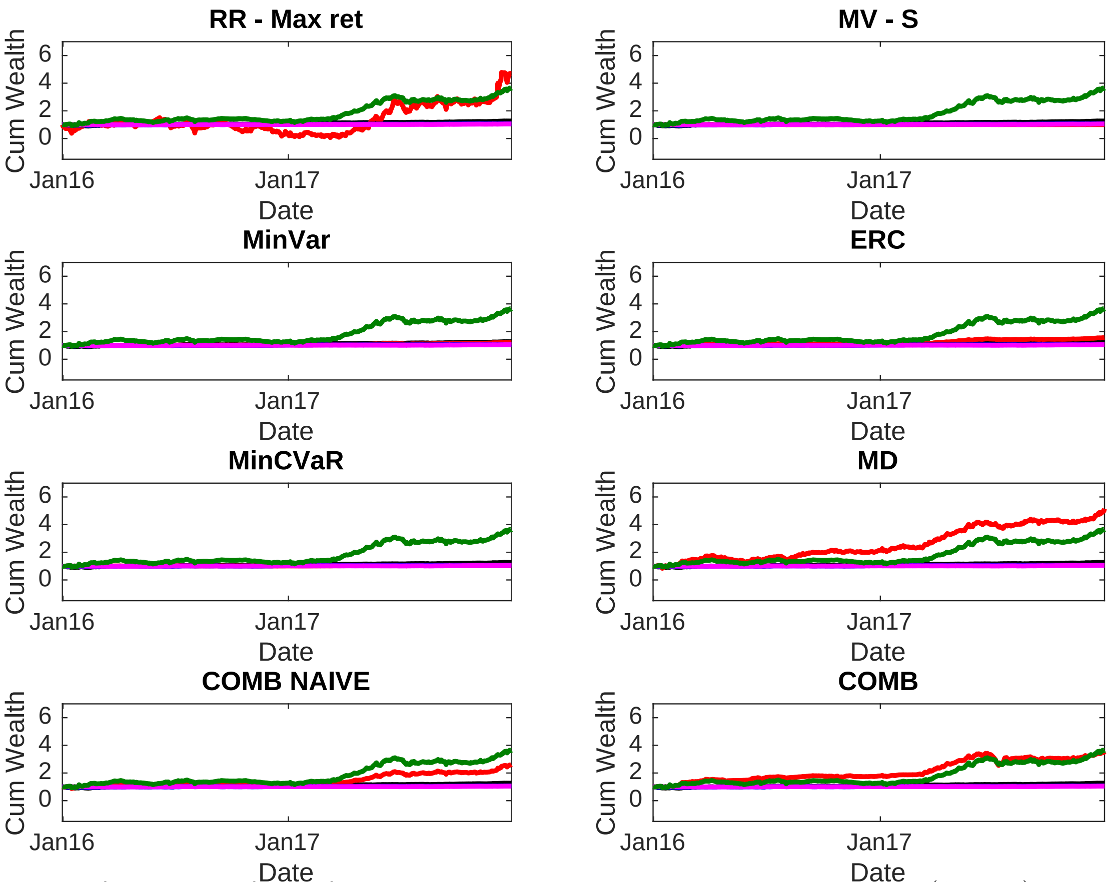


Figure 2: Performance of portfolio strategies without liquidity constraints ($l = 21$). Daily cumulative returns of portfolio strategies over the period from 2016-01-01 to 2017-12-31 with the following colour code: S&P100, EW, EW–TrA, RR-Max ret–TrA and corresponding Allocation strategy from Table 1

CCPPerformance

Further we analyse risk-adjusted performance for all portfolios. MD demonstrates not only the superior absolute but also the highest risk-adjusted performance. Thus, SR for MD portfolio is 0.158, ASR – 0.158 and CEQ is 0.007. Results also demonstrate that LIBRO approach improves risk-adjusted measures in most cases. This tendency

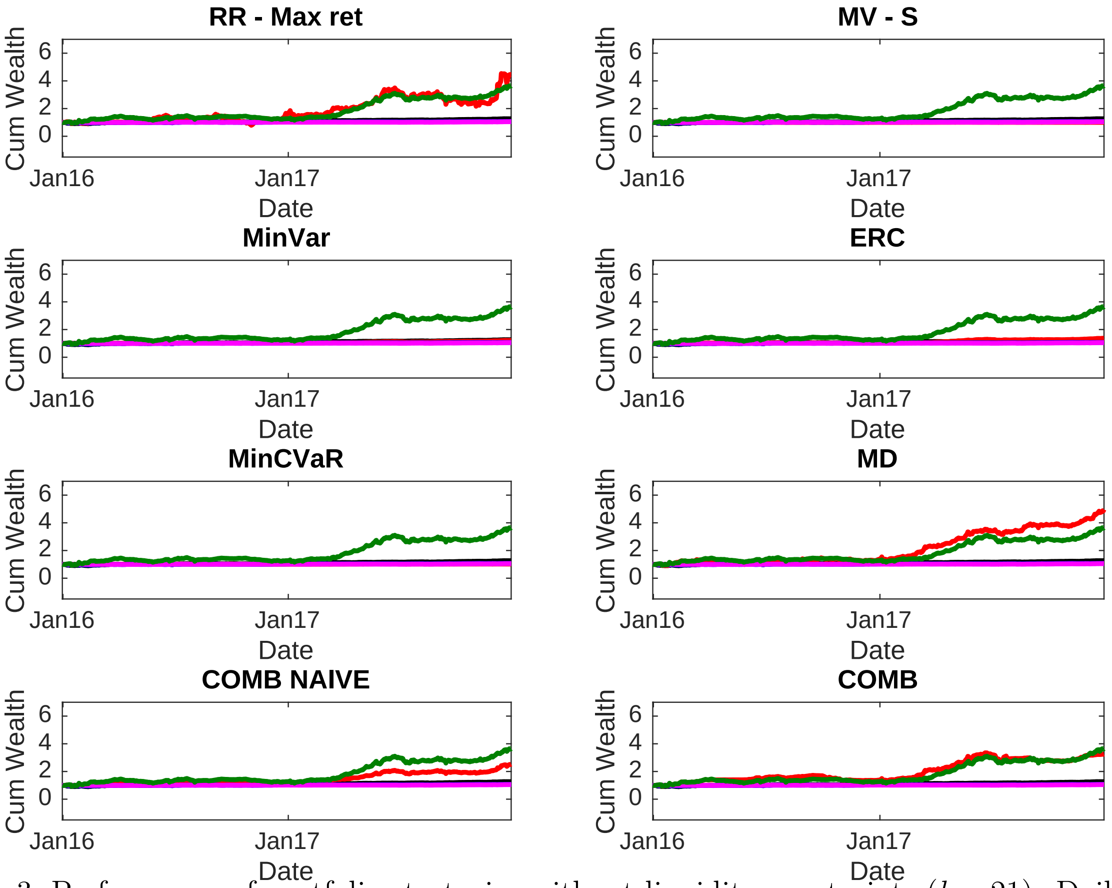


Figure 3: Performance of portfolio strategies without liquidity constraints ($l = 21$). Daily cumulative returns of portfolio strategies over the period from 2016-01-01 to 2017-12-31 with the following colour code: S&P100, EW, EW–TrA, RR-Max ret–TrA and corresponding Allocation strategy from Table 1

CCPPerformance

remains consistent for constrained portfolios as well. Table 4 analyses the significance of difference between CEQ and SR as it is described in section 4.2. Although MD, COMB and COMB NAÏVE have SR and CEQ returns higher or comparable with EW, tests do not support significance of this result. Whereas ERC portfolio has higher SR and this difference is significant. The comparison of risk-adjusted metrics for MV-S, MinVar and MinCVaR confirms the conclusions from the analysis of cumulative wealth: there is no significant difference with traditional assets' portfolios. As a robustness check we also conduct analysis for weekly and daily rebalanced portfolios, results are provided in Appendix 8.4, confirming the fairness of conclusions made. The last two columns of Table 3 compare results for portfolio Turnover. RR-Max ret has the lowest Turnovers 0.23 and 0.76 for unconstrained and LIBRO rules respectively. This outcome is not surprising,

Allocation Strategy	Portfolio performance measures: monthly rebalancing									
	CW		SR		ASR		CEQ		TURNOVER	
	No const	10 mln	No const	10 mln	No const	10 mln	No const	10 mln	No const	10 mln
Benchmark strategies										
S&P100	1.261	1.261	0.080	0.080	0.079	0.079	0.000	0.000	0.000	0.000
EW TrA	1.069	1.069	0.048	0.048	0.047	0.047	0.004	0.004	4.824	4.824
MV-S TrA	1.052	1.052	0.068	0.068	0.068	0.068	0.000	0.000	1.359	1.359
EW	3.644	3.644	0.132	0.132	0.132	0.132	0.004	0.004	1.102	1.102
Risk-oriented strategies										
MinVar	1.001	1.001	0.065	0.071	0.065	0.072	0.001	0.002	3.924	3.710
	(3.03)***	(3.03)***								
MinCVaR	1.024	1.020	0.048	0.040	0.048	0.040	0.000	0.000	5.987	4.490
	(3.02)***	(3.02)***								
ERC	1.558	1.373	0.158	0.145	0.157	0.145	0.001	0.001	1.167	1.245
	(2.85)***	(2.94)***								
MD	5.147	4.964	0.158	0.177	0.158	0.179	0.007	0.007	4.408	5.748
	(–1.85)*	(–1.71)*								
Return-oriented strategies										
RR-Max ret	4.703	4.455	0.003	0.003	0.003	0.003	0.000	0.000	0.229	0.761
	(–0.49)	(–0.45)								
Risk-Return-oriented strategies										
MV-S	1.214	1.211	0.119	0.125	0.120	0.125	0.000	0.000	3.211	2.395
	(2.96)***	(2.96)***								
Combination of models										
COMB NAÏVE	2.613	2.524	0.126	0.134	0.127	0.135	0.003	0.003	2.281	1.202
	(1.98)**	(2.05)**								
COMB	3.542	3.254	0.126	0.117	0.125	0.117	0.004	0.004	0.881	1.201
	(0.16)	(0.62)								

Table 3: Performance measures for monthly rebalancing frequency ($l = 21$): the superior results are highlighted in red. t-statistics of the difference between returns series of each strategy from EW portfolio are shown in parentheses. * - 0.1 , ** - 0.05 and *** - 0.01 levels of significance.

taking into account that RR-Max ret portfolio is the most concentrated one and consists from the one asset with the highest return, see Figures 4 and 5. Both minimum risk portfolios as well as MD have higher turnovers what prompts higher transactional trading costs. Other important findings from analysis of portfolios' compositions of allocation are following: LIBRO approach as it was expected affects significantly the portfolio weights, the most visible effect is for models with high share of CCs, namely MD and RR-Max ret, as well as in MV-S and ERC where the share of CCs decreased for the first half of the investment period. The weights distribution of COMB portfolios is not robust and changes over the investment period dramatically: from high concentration of traditional assets to high concentration of CCs, confirming the idea that no individual model outperforms its competitors permanently.

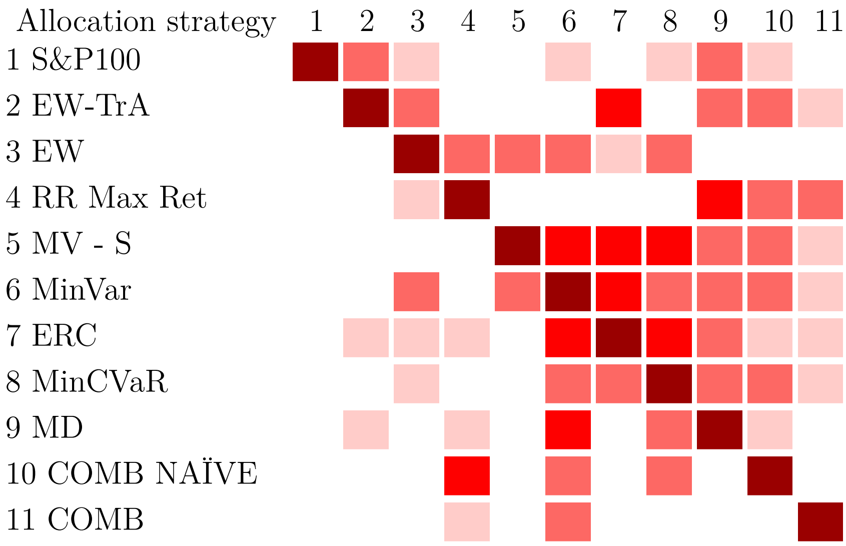


Table 4: p-value of the difference between the SR (lower triangle) and CEQ (upper triangle) of all strategies with each other with significance codes **0.01**, **0.05** and **0.1** (without liquidity constraints)

CCPTests

To shed light on the source of behaviour of portfolios' design we also compare the risk structures for all strategies, see Figures 7 and 8. Volatility structure of CCs leads to the disproportionate risk contribution relative to their capital weights: traditional assets have visibly less share in risk architecture.

6.2 Diversification analysis

In this section we investigate the diversification characteristics of allocation rules. Table 5 reports results on three chosen diversification metrics. The RR-Max ret as it was expected does not produce any diversification benefits, as it mainly consists of only one asset. Different range of values for various diversification metrics in Table 5 emphasises the fact that diversification has many aspects and its quantification depends significantly on the specified definition, consequently different measures do not always provide consistent conclusions about diversification effects of portfolios. Thus, MinCVaR characterised by best diversification benefits measured by DR^2 – 15.22 (14.90)[7], and diversification in this case defined as the lowest possible correlation across assets. Relatively similar outcomes have MinVar and ERC portfolios 13.65 (13.51) and 11.42 (12.36) respectively. MD portfolio

[7] Here and further on in parentheses we will give values of performance metric for LIBRO portfolios

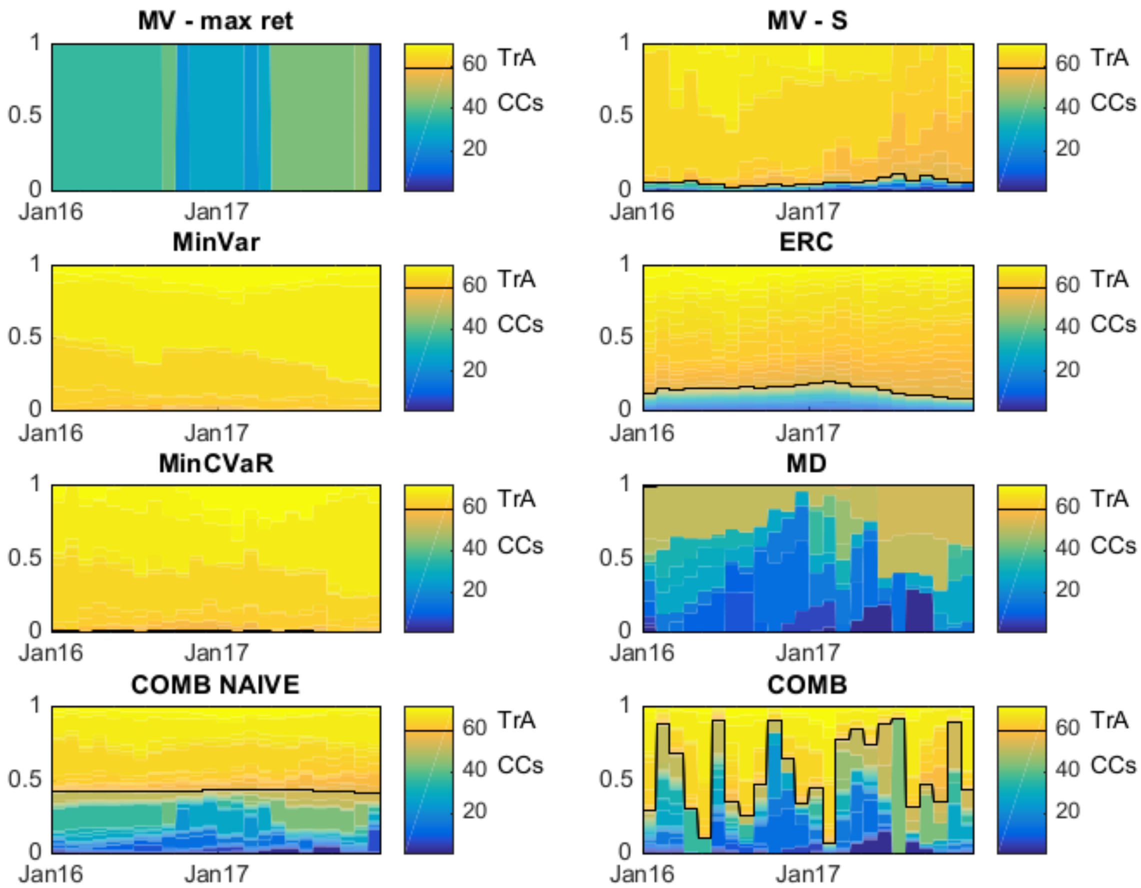


Figure 4: Change in the composition of the portfolios allocation over the period without liquidity constraints over the period from 2016-01-01 to 2017-12-31: the black line separates conventional assets - TrA (upper yellow part of the spectrum) from cryptocurrencies - CCs (lower green-blue part of the spectrum)

CCPWeights

is a special case in this type of diversification with values 2.99 (2.41), at the same time PDI 25.93 (25.90) of MD portfolio is the highest based on its objective function, emphasising outstanding diversification capabilities defined as the number of independent sources of variation in the portfolio.

The ERC portfolio is characterised by the highest Effective N 17.63, what is also a typical result (see e.g. Clarke et al. (2013)) due to ERC nature: it has all assets in solution by definition. The lowest Effective N 3.37 has MV-S portfolio from traditional assets, showing 3.37 equally-weighted stocks would provide the same diversification benefits . All other individual strategies also have Effective N ranged from 3 to 4. One more remarkable

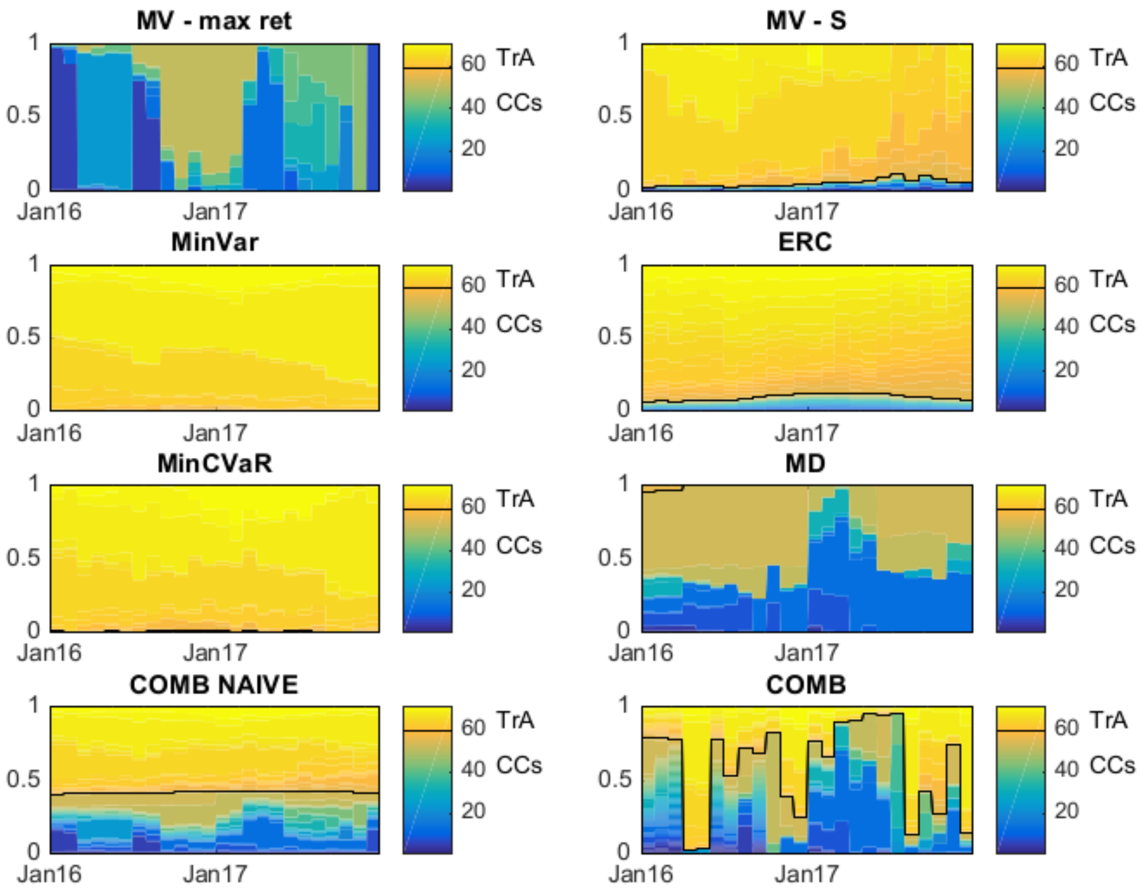


Figure 5: Change in the composition of the portfolios' allocation over the period from 2016-01-01 to 2017-12-31 with 10 mln US$ invested (LIBRO approach): the black line separates conventional assets - TrA (upper yellow part of the spectrum) from cryptocurrencies - CCs (lower green-blue part of the spectrum)

CCPWeights

result concerns combined portfolios concentration: COMB portfolio Effective N equal 8.58 for unconstrained case and 14.84 for LIBRO portfolio, COMB Naïve is over 12 for both cases. For two other metrics combined portfolios demonstrate performance, measured by DR^2 similar with MD – 3.95(4.58) for COMB Naïve and 4.09(4.26) for COMB; measured by PDI - similar to other risk-based portfolios: MinVar, MinCVaR and ERC.

Also we would like to highlight the difference of diversification effects for MV-S portfolios with and without CCs: diversification measured by DR^2 increased with incorporation of CCs from 5.7 to 9.05 (9.36), but PDI scales up more greatly, from 5.19 to 25.41 (25.45). So we can conclude that incorporation of CCs improves portfolios' diversification capabilities, especially determined as the distribution of principal portfolio variances. As it can be noticed from results liquidity constraints do not have a strong effect on diversification features of portfolios, all metrics change just slightly.

Allocation Strategy	**Portfolio diversification effects: monthly rebalancing**					
	DR^2		**Effective** N		**PDI**	
	No const	**10 mln**	**No const**	**10 mln**	**No const**	**10 mln**
Benchmark strategies						
MV - S TrA	5.70	5.70	3.37	3.37	5.19	5.19
Return oriented strategies						
RR - Max ret	1.00	1.00	1.00	1.90	1.00	1.00
Risk-oriented strategies						
MinVar	13.65	13.51	3.48	3.47	25.40	25.40
MinCVaR	15.22	14.90	4.07	4.07	25.40	25.40
ERC	11.42	12.36	17.63	14.97	25.42	25.42
MD	2.99	2.41	4.08	3.01	25.93	25.90
Risk-Return-oriented strategies						
MV -S	9.05	9.36	3.70	3.76	25.41	25.41
Combination of models						
COMB NAÏVE	3.95	4.58	12.55	12.72	25.46	25.45
COMB	4.09	4.26	8.58	14.84	24.80	25.36

* All diversification measures are calculated based on in-sample data and averaged over the period 20150101-20171130

Table 5: Measures of diversification for monthly rebalancing ($l = 21$). The superior results are highlighted in red.

7 Conclusion

This study investigates cryptocurrencies as new assets available to portfolio management. We analyze the performance of commonly used statistical asset-allocation models with a unique dataset on historical prices and trading volumes of 55 cryptocurrencies, combined with 16 traditional assets. The rules-based investment methods cover a broad spectrum of investors' objectives, from the classical Markowitz model to recent strategies, aiming to maximize portfolio diversification. Along with individual portfolio allocation strategies, in the spirit of model averaging, we also include combined strategies. The performance of portfolios is evaluated with different measures: raw investment gains in the form of cumulative raw return, risk-adjusted performance and diversification effects produced by portfolios.

We find that due to the volatility structure of cryptocurrencies, the application of traditional risk-based portfolios, such as equal-risk contribution, minimum-variance and minimum-CVaR portfolios, does not boost the performance of investments significantly. In contrast, approaches such as the maximum-return strategy (or strategies with high target returns) but also the maximum-diversification portfolio (with PDI) prompt higher expected returns via higher cryptocurrency exposure for investors. As for diversification benefits, we demonstrate an enhanced diversification in comparison with only conventional assets' portfolios. We document that various rules have different effect on portfolio diversification and result highly depends on the concept of diversification and consequently chosen measure of its quantification.

Furthermore, following the idea of model averaging and diversification across models we show that both naive and bootstrap-based combined portfolios exhibit robust high risk-adjusted returns. Portfolios with model-averaged weights achieve significantly higher performance than purely risk-oriented strategies and not significantly lower than the best performing strategies.

As robustness checks we apply the allocation rules with different rebalancing frequencies as well as with and without constraints addressing the liquidity risk of cryptocurrencies, namely the LIBRO strategy proposed by Trimborn et al. (2017). The results remain coherent across all frameworks. Further extensions can be made along three main lines: first, more involved estimators of expected returns and the covariance matrix could be employed; second, more performance measures could be used to evaluate the investment strategies' results; and third, additional portfolio-allocation strategies could be included in the comparison. In particular, factor-based APT (arbitrage price theory) models would constitute the complementary approach to statistical-optimisation techniques studied in this paper.

References

Alessandretti, L., A. ElBahrawya, L. M. Aielloc, and A. Baronchelli (2018). “Machine Learning the Cryptocurrency Market”. *arXiv:1805.08550v1 [physics.soc-ph].*

Avramov, D. (2002). “Stock return predictability and model uncertainty”. *Journal of Financial Economics* 64.3, pp. 423–458.

Brière, M., K. Oosterlinck, and A. Szafarz (2015). “Virtual currency, tangible return: Portfolio diversification with bitcoin”. *Journal of Asset Management* 16.6, pp. 365–373.

Chopra, V. K. and W. T. Ziemba (1993). “The Effect of Errors in Means, Variances, and Covariances on Optimal Portfolio Choice”. *The Journal of Portfolio Management* 19.2, pp. 6–11.

Choueifaty, Y. and Y. Coignard (2008). “Toward maximum diversification”. *Journal of Portfolio Management* 35.1, p. 40.

Choueifaty, Y., T. Froidure, and J. Reynier (2011). “Properties of the most diversified portfolio”.

Chow, T.-m., J. Hsu, V. Kalesnik, and B. Little (2011). “A survey of alternative equity index strategies”. *Financial Analysts Journal* 67.5, pp. 37–57.

Chuen, D. L. K., L. Guo, and Y. Wang (2017). “Cryptocurrency: A New Investment Opportunity?” *The Journal of Alternative Investments* 20.3, pp. 16–40.

Clarke, R., H. De Silva, and S. Thorley (2013). “Risk parity, maximum diversification, and minimum variance: An analytic perspective”. *Journal of Portfolio Management* 39.3, p. 39.

Clemen, R. T. (1989). “Combining forecasts: A review and annotated bibliography”. *International journal of forecasting* 5.4, pp. 559–583.

DeMiguel, V., L. Garlappi, and R. Uppal (2009). “Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy?” *The review of Financial studies* 22.5, pp. 1915–1953.

Eisl, A., S. M. Gasser, and K. Weinmayer (2015). *Caveat Emptor: Does Bitcoin Improve Portfolio Diversification?* SSRN Scholarly Paper ID 2408997. Rochester, NY: Social Science Research Network.

Elendner, H., S. Trimborn, B. Ong, and T. M. Lee (2017). “The Cross-Section of Crypto-Currencies as Financial Assets”. *Handbook of Digital Finance and Financial Inclusion: Cryptocurrency, FinTech, InsurTech, and Regulation*. Ed. by D. Lee Kuo Chuen and R. Deng. Vol. 1. Elsevier.

Hafner, C. M. (2018). “Testing for Bubbles in Cryptocurrencies with Time-Varying Volatility”. *Journal of Financial Econometrics*.

Härdle, W. K. and L. Simar (2015). “Applications in Finance”. *Applied Multivariate Statistical Analysis*. 4th ed. Springer, pp. 487–499.

Haugen, R. A. and N. L. Baker (1991). “The efficient market inefficiency of capitalization–weighted stock portfolios”. *The Journal of Portfolio Management* 17.3, pp. 35–40.

Huberman, G. and S. Kandel (1987). “Mean-variance spanning”. *The Journal of Finance* 42.4, pp. 873–888.

Jagannathan, R. and T. Ma (2003). “Risk Reduction in Large Portfolios: Why Imposing the Wrong Constraints Helps”. *The Journal of Finance* 58.4, pp. 1651–1683.

Jobson, J. D. and B. M. Korkie (1981). “Performance hypothesis testing with the Sharpe and Treynor measures”. *The Journal of Finance* 36.4, pp. 889–908.

Jorion, P. (1985). “International portfolio diversification with estimation risk”. *Journal of Business*, pp. 259–278.

Kan, R. and G. Zhou (2012). "Tests of Mean-Variance Spanning." *Annals of Economics & Finance* 13.1.

Kan, R. and G. Zhou (2007). "Optimal portfolio choice with parameter uncertainty". *Journal of Financial and Quantitative Analysis* 42.3, pp. 621–656.

Klein, T., H. P. Thu, and T. Walther (2018). "Bitcoin is not the New Gold–A comparison of volatility, correlation, and portfolio performance". *International Review of Financial Analysis* 59, pp. 105–116.

Krokhmal, P., J. Palmquist, and S. Uryasev (2002). "Portfolio optimization with conditional value-at-risk objective and constraints". *Journal of risk* 4, pp. 43–68.

Ledoit, O. and M. Wolf (2008). "Robust performance hypothesis testing with the Sharpe ratio". *Journal of Empirical Finance* 15.5, pp. 850–859.

Maillard, S., T. Roncalli, and J. Teïletche (2010). "The properties of equally weighted risk contribution portfolios". *The Journal of Portfolio Management* 36.4, pp. 60–70.

Markowitz, H. (1952). "Portfolio Selection". *The Journal of Finance* 7.1, pp. 77–91.

Memmel, C. (2003). "Performance hypothesis testing with the Sharpe ratio". *Finance Letters* 1, pp. 21–23.

Merton, R. C. (1980). "On estimating the expected return on the market: An exploratory investigation". *Journal of financial economics* 8.4, pp. 323–361.

Pezier, J. and A. White (2008). "The relative merits of investable hedge fund indices and of funds of hedge funds in optimal passive portfolios". *The Journal of Alternative Investments* 10.4, pp. 37–49.

Politis, D. N. and J. P. Romano (1994). "The stationary bootstrap". *Journal of the American Statistical association* 89.428, pp. 1303–1313.

Politis, D. N. and H. White (2004). “Automatic block-length selection for the dependent bootstrap”. *Econometric Reviews* 23.1, pp. 53–70.

Qian, E. E. (2006). “On the financial interpretation of risk contribution: Risk budgets do add up”. *Journal of Investment Management* 4.4, pp. 41–51.

Rockafellar, R. T. and S. Uryasev (2000). “Optimization of conditional value-at-risk”. *Journal of risk* 2, pp. 21–42.

Rudin, A. M. and J. S. Morgan (2006). “A portfolio diversification index”. *The Journal of Portfolio Management* 32.2, pp. 81–89.

Scaillet, O., A. Treccani, and C. Trevisan (2018). “High-Frequency Jump Analysis of the Bitcoin Market*”. *Journal of Financial Econometrics*.

Schanbacher, P. (2014). “Combining Portfolio Models”. *Annals of Economics and Finance* 15.2, pp. 433–455.

Schanbacher, P. (2015). “Averaging across asset allocation models”. *Jahrbücher für Nationalökonomie und Statistik* 235.1, pp. 61–81.

Simaan, Y. (1997). “Estimation risk in portfolio selection: the mean variance model versus the mean absolute deviation model”. *Management science* 43.10, pp. 1437–1446.

Strongin, S., M. Petsch, and G. Sharenow (2000). “Beating benchmarks”. *The Journal of Portfolio Management* 26.4, pp. 11–27.

Trimborn, S. and W. K. Härdle (2018). “CRIX an Index for cryptocurrencies”. *Journal of Empirical Finance* 49, pp. 107–122.

Trimborn, S., M. Li, and W. K. Härdle (2017). “Investing with Cryptocurrencies - a Liquidity Constrained Investment Approach”. *Journal of Financial Econometrics revise and resubmit 20171113*.

Wyss, R. v. (2004). “Measuring and Predicting Liquidity in the Stock Market”. *Doctoral thesis*.

8 Appendix

8.1 Test for the difference of the SR and CEQ

Ledoit and Wolf (2008)

Let $\nu = (\mu_i, \mu_j, \sigma_i, \sigma_j)$ -vector of moments of strategies i and j

- Difference of CEQ and SR

$$f_{CEQ}(\nu) = \mu_i - \frac{\gamma}{2}\sigma_i^2 - \mu_j + \frac{\gamma}{2}\sigma_j^2 \tag{22}$$

$$f_{SR}(\nu) = \frac{\mu_i}{\sigma_i} - \frac{\mu_j}{\sigma_j} \tag{23}$$

- Delta method: if $\sqrt{T-M}(\hat{\nu} - \nu) \xrightarrow{\text{d}} N(0, \Psi)$, then

$$\sqrt{T-M}(\hat{f} - f) \xrightarrow{\text{d}} N(0, \nabla' f(\nu)\Psi\nabla f(\nu)), \tag{24}$$

  where ∇f is a derivative of f.

- Standard Error for $\hat{f}$:

$$SE(\hat{f}) = \sqrt{\frac{\nabla' f(\nu)\Psi\nabla f(\nu)}{T-M}} \tag{25}$$

- Solutions for consistent estimator for $\hat{\Psi}$: HAR and Bootstrap inference

- HAR inference

$$\Psi_{T-M} = \frac{T-M}{T-M-4}\sum_{j=-T+M+1}^{T-M-1} Ker\left(\frac{j}{S_{T-M}}\right)\hat{\Gamma}_{T-M}(j) \tag{26}$$

  where $Ker(\cdot)$ is a kernel, S_{T-M} - bandwidth

- A two-sided p-value for H_0: $f = 0$

$$\hat{p} = 2\Phi\frac{|\hat{f}|}{SE(\hat{f})} \tag{27}$$

8.2 Descriptive statistics of portfolio components

	BTC	XRP	LTC	DASH	XMR
SSE	–0.03	–0.07	–0.03	0.02	0.06
NIKKEI225	–0.05	0.03	–0.04	0.01	0.01
FTSE100	0.02	0.03	0.02	0.07	0.08
SP100	0.00	0.03	0.00	0.10	0.03
SX5E	0.03	–0.01	0.01	0.06	0.06
MSCI CP	0.03	0.04	0.02	0.08	0.05
REIT	0.00	–0.01	0.00	0.00	0.04
EUR	–0.05	0.03	–0.03	0.04	–0.08
JPY	0.00	–0.02	0.01	0.07	0.07
CNY	0.00	–0.05	0.02	0.01	–0.03
GBP	–0.02	0.02	–0.05	0.00	–0.02
GOLD	0.01	0.02	0.00	0.00	–0.03
UK10Y	0.04	0.05	0.04	–0.09	0.04
JP10Y	0.01	0.04	0.02	–0.05	–0.03
US10Y	0.03	0.06	0.02	–0.05	–0.01
EU10Y	0.03	0.07	0.04	–0.10	0.05

Table 6: Correlations between CCs and conventional financial assets

Asset name	Max	P_{90}	Med	Mean	P_{10}	Min	SD
SSE	5.60	1.69	0.06	0.00	-1.36	-8.87	1.64
NIKKEI 225	7.43	1.36	0.01	0.03	-1.31	-8.25	1.28
FTSE 100	3.51	1.02	0.02	0.02	-1.01	-4.78	0.92
SP 100 - PRICE INDEX	4.19	0.95	0.00	0.03	-0.80	-4.01	0.77
SX5E	4.60	1.37	0.01	0.01	-1.29	-9.01	1.20
MSCI CP	4.66	1.27	0.00	0.01	-1.20	-5.87	1.05
REIT	3.34	1.06	0.03	0.00	-1.13	-6.71	0.93
EUR	3.02	0.69	0.00	-0.00	-0.70	-2.38	0.59
JPY	2.22	0.66	0.00	-0.01	-0.68	-3.78	0.62
CNY	1.83	0.19	0.00	0.01	-0.18	-1.20	0.20
GBP	3.00	0.71	-0.02	-0.02	-0.70	-8.40	0.67
GOLD	4.56	0.98	0.01	0.01	-0.93	-3.43	0.83
UK10Y	1.99	0.37	0.00	0.00	-0.39	-1.10	0.32
JP10Y	0.74	0.12	0.00	-0.00	-0.12	-0.54	0.12
US10Y	1.28	0.36	0.00	-0.00	-0.36	-1.11	0.30
EU10Y	0.85	0.27	0.00	-0.00	-0.26	-1.42	0.24

Table 7: Summary statistics for returns of traditional assets (in %)

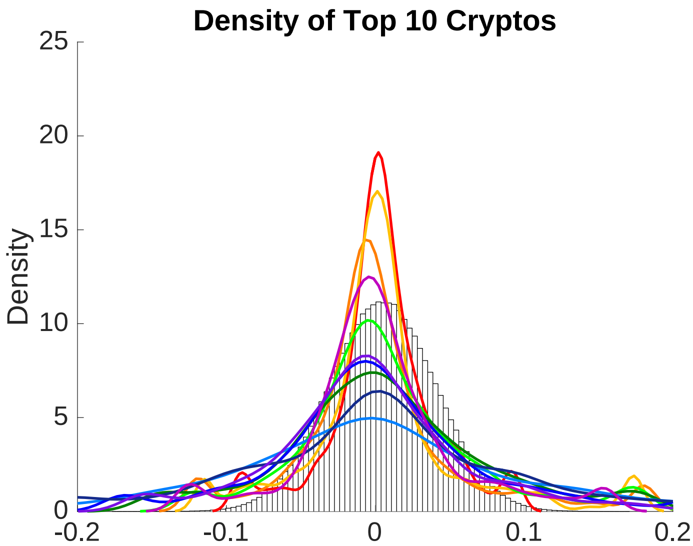


Figure 6: Density of Top-10 CCs against normal distribution (time span is 2015-01-01 to 2017-12-31).

CCPHistReturnsDensity

CC	Max	P_{90}	Med	Mean	P_{10}	Min	SD
ABY	33.78	15.23	–0.20	0.53	–13.66	–23.83	11.72
AUR	21.62	11.02	0.13	0.46	–9.72	–17.81	8.14
BCN	24.77	12.49	0.21	0.43	–10.71	–21.09	9.42
BLK	19.01	8.69	–0.17	0.28	–8.26	–16.28	7.11
BTC	9.27	5.12	0.39	0.49	–3.65	–9.06	3.57
BTCD	22.33	11.46	0.37	0.64	–9.47	–18.34	8.22
BTM	60.20	22.15	–0.04	0.73	–19.97	–43.88	18.63
BTS	21.87	9.90	–0.13	0.42	–7.48	–16.93	7.41
BURST	23.71	12.45	0.29	0.44	–10.45	–21.68	9.35
BYC	28.99	13.11	–0.05	0.29	–11.86	–23.29	10.67
CANN	28.25	12.25	0.00	0.43	–11.01	–22.99	10.12
CURE	32.11	13.32	0.00	0.42	–11.30	–23.89	10.79
DASH	17.58	7.90	0.00	0.80	–5.67	–12.29	5.95
DGB	32.28	11.74	–0.56	0.67	–9.51	–20.52	9.85
DGC	28.94	12.14	–0.20	0.20	–10.99	–24.62	10.39
DMD	20.16	10.88	0.09	0.50	–9.06	–20.38	8.15
DOGE	15.43	7.20	–0.12	0.31	–4.63	–12.58	5.32
EAC	41.30	13.87	–0.27	–0.21	–13.61	–42.72	14.22
EMC2	29.14	14.10	–0.13	0.53	–12.03	–23.80	10.76
FTC	32.69	12.89	–0.61	0.20	–11.15	–24.71	10.89
GRS	45.36	18.65	–0.27	0.87	–15.79	–28.97	14.58
HUC	32.26	16.77	–0.27	0.55	–15.21	–25.12	12.68
IOC	27.87	15.70	0.47	0.94	–12.61	–24.92	11.22
LTC	17.41	6.69	0.15	0.53	–5.28	–11.47	5.33
MAX	62.91	17.42	–0.16	0.28	–18.87	–50.89	19.16
NAV	35.63	14.88	–0.08	0.68	–12.99	–26.06	11.86
NEOS	36.55	15.20	–0.14	0.46	–13.54	–26.56	12.34
NLG	21.72	11.44	0.01	0.56	–9.41	–16.17	7.89
NMC	19.13	7.85	–0.22	0.19	–6.60	–14.09	6.34
NOTE	26.25	11.35	–0.17	0.01	–11.19	–21.40	9.60
NVC	21.69	6.62	–0.25	0.16	–7.38	–13.51	6.57
NXT	24.15	9.38	–0.43	0.32	–7.83	–15.68	7.51
POT	25.89	11.68	0.24	0.56	–10.55	–20.63	9.23
PPC	17.40	7.15	–0.14	0.23	–6.37	–14.24	6.14
QRK	40.29	12.02	–0.22	0.20	–12.20	–31.78	12.75
RBY	21.26	11.74	0.62	0.77	–9.94	–20.01	8.61
RDD	39.91	18.47	0.33	0.63	–17.28	–30.68	14.57
SLR	26.67	12.94	0.18	0.56	–12.13	–22.71	10.22
SPR	36.57	18.52	0.00	0.28	–17.22	–32.18	14.53
START	29.32	14.44	–0.64	0.20	–11.89	–21.96	10.61
SYS	30.83	13.17	–0.11	0.97	–9.74	–19.56	10.07
UNO	19.81	11.38	0.26	0.39	–9.48	–23.53	8.75
VIA	33.43	14.26	0.01	0.64	–12.86	–22.12	11.31
VRC	35.03	15.13	–0.17	0.59	–12.93	–28.39	12.16
VTC	32.14	13.26	–0.21	0.56	–11.41	–23.14	10.84
WDC	28.44	12.01	–0.53	0.17	–11.56	–23.09	10.11
XCN	48.36	18.58	–0.84	0.02	–19.74	–37.13	16.65
XCP	22.92	11.71	–0.37	0.18	–10.57	–19.65	9.11
XDN	27.28	13.07	–0.08	0.29	–11.24	–23.15	10.23
XMG	25.01	12.37	0.02	0.44	–10.58	–19.05	9.20
XMR	17.80	9.64	0.19	0.66	–7.17	–14.71	6.87
XPM	23.29	9.85	–0.17	0.21	–8.67	–19.59	8.51
XRP	18.09	6.81	–0.37	0.25	–5.39	–12.01	5.62
XST	36.42	16.23	–0.14	0.49	–14.61	–29.09	13.13
ZET	27.10	14.59	–0.31	0.04	–13.88	–25.11	11.38

Table 8: Summary statistics for returns of CCs (in %)

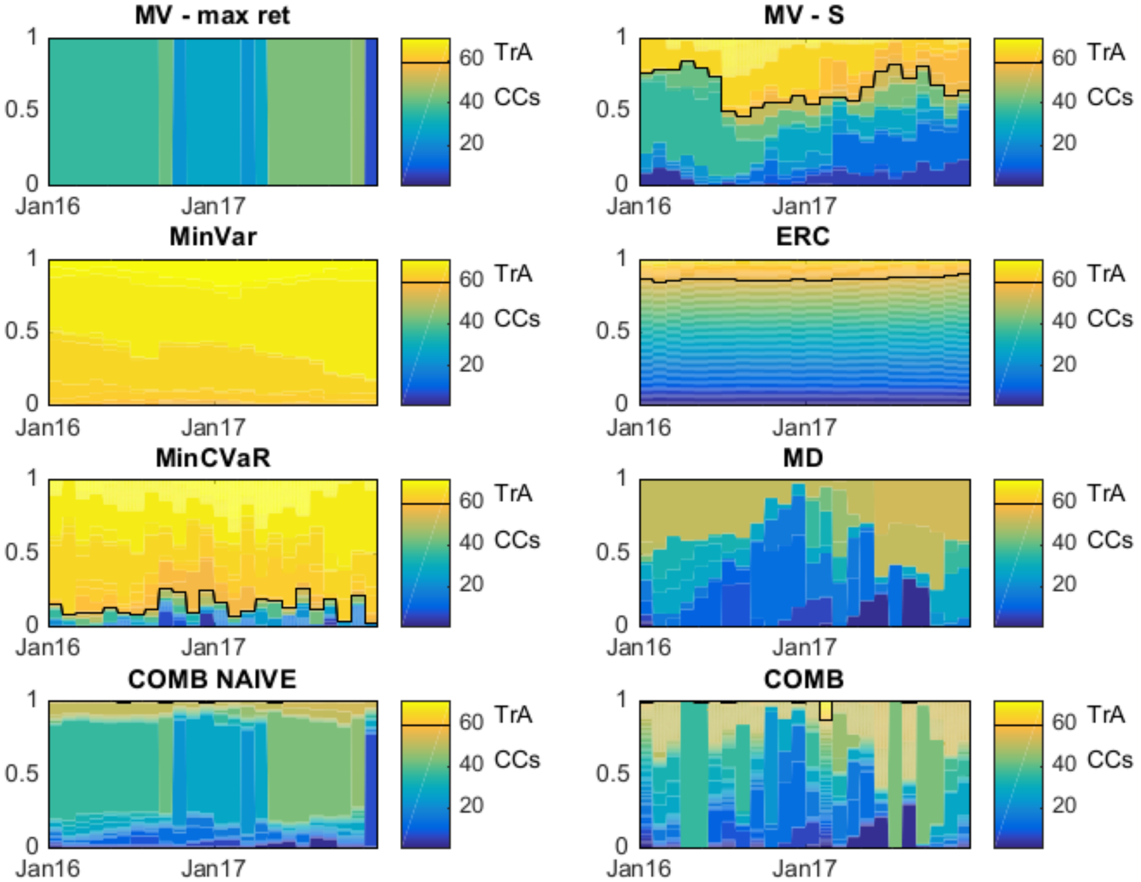


Figure 7: Change of risk contributions for portfolio strategies without liquidity constraints over the period from 2016-01-01 to 2017-12-31: the black line separates conventional assets - TrA (upper yellow part of the spectrum) from cryptocurrencies - CCs (lower green-blue part of the spectrum)

CCPRisk_contribution

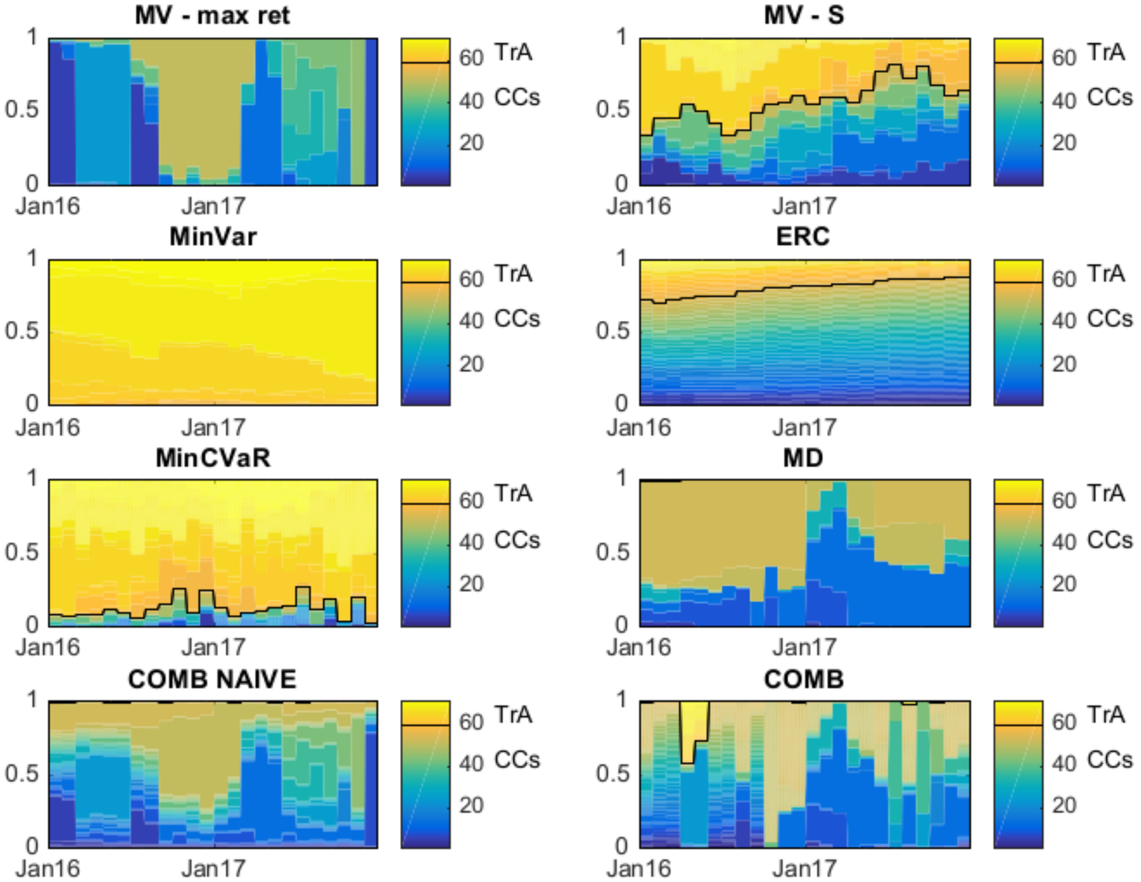


Figure 8: Change of risk contributions for portfolio strategies over the period from 2016-01-01 to 2017-12-31 with 10 mln US$ invested (LIBRO approach): the black line separates conventional assets - TrA (upper yellow part of the spectrum) from cryptocurrencies - CCs (lower green-blue part of the spectrum)

CCPRisk_contribution

8.3 Results of spanning tests of cryptocurrencies inclusion into the investment universe

Cryptocurrency	F-Test	F-Test1	F-Test2	Cryptocurrency	F-Test	F-Test1	F-Test2
aur	1.66	2.96	0.36	nav	1.69	2.88	0.49
	(0.19)	(0.09)	(0.55)		(0.19)	(0.09)	(0.48)
btc	4.97	9.41	0.52	neos	2.54	4.87	0.20
	(0.01)	(0.00)	(0.47)		(0.08)	(0.03)	(0.66)
btcd	3.77	4.74	2.75	nxt	2.43	4.70	0.16
	(0.02)	(0.03)	(0.10)		(0.09)	(0.03)	(0.69)
btm	2.04	3.21	0.87	pot	1.96	3.37	0.55
	(0.13)	(0.07)	(0.35)		(0.14)	(0.07)	(0.46)
bts	5.44	9.75	1.10	ppc	2.00	2.86	1.13
	(0.00)	(0.00)	(0.30)		(0.14)	(0.09)	(0.29)
burst	1.95	3.86	0.04	spr	3.18	2.16	4.19
	(0.14)	(0.05)	(0.84)		(0.04)	(0.14)	(0.04)
cann	3.18	3.80	2.54	sys	3.23	6.45	0.02
	(0.04)	(0.05)	(0.11)		(0.04)	(0.01)	(0.90)
dash	5.92	11.12	0.70	uno	1.50	2.86	0.14
	(0.00)	(0.00)	(0.40)		(0.23)	(0.09)	(0.71)
dgb	1.87	3.71	0.04	via	3.04	5.33	0.75
	(0.16)	(0.06)	(0.85)		(0.05)	(0.02)	(0.39)
dmd	1.80	3.59	0.00	vtc	4.81	9.06	0.54
	(0.17)	(0.06)	(0.95)		(0.01)	(0.00)	(0.46)
doge	1.76	3.28	0.23	xmg	1.92	3.70	0.15
	(0.17)	(0.07)	(0.63)		(0.15)	(0.06)	(0.70)
emc2	3.84	7.48	0.20	xmr	1.53	3.04	0.03
	(0.02)	(0.01)	(0.66)		(0.22)	(0.08)	(0.85)
ftc	2.35	3.03	1.66	xrp	3.50	6.45	0.54
	(0.10)	(0.08)	(0.20)		(0.03)	(0.01)	(0.46)
ltc	3.48	6.66	0.29	xst	3.07	3.54	2.58
	(0.03)	(0.01)	(0.59)		(0.05)	(0.06)	(0.11)

Table 9: Spanning Test for Cryptocurrencies Effect on Portfolios Constructed from Traditional Investment (p-value is given in brackets)

8.4 Results for daily and weekly rebalanced portfolios

Allocation Strategy	Portfolio performance measures: weekly rebalancing									
	CW		SR		ASR		CEQ		TURNOVER	
	No const	10 mln	No const	10 mln	No const	10 mln	No const	10 mln	No const	10 mln
Benchmark strategies										
S&P100	1.261	1.261	0.080	0.080	0.079	0.079	0.000	0.000	0.000	0.000
EW TrA	1.069	1.069	0.048	0.048	0.047	0.047	0.004	0.004	16.892	16.892
MV-S TrA	1.064	1.064	0.082	0.082	0.082	0.082	0.000	0.000	6.889	6.889
EW	3.644	3.644	0.132	0.132	0.132	0.132	0.004	0.004	5.375	5.375
Risk-oriented strategies										
MinVar	1.010	1.010	0.024	0.069	0.024	0.070	−0.001	0.002	15.841	16.510
MinCVAR	1.031	1.028	0.062	0.057	0.062	0.057	0.000	0.000	11.338	7.010
ERC	1.557	1.379	0.160	0.148	0.160	0.147	0.001	0.001	6.091	11.034
MD	4.785	4.743	0.147	0.167	0.147	0.168	0.006	0.006	21.831	52.368
Return-oriented strategies										
RR-max ret	2.011	3.331	0.025	0.025	0.025	0.025	0.000	0.000	1.081	2.930
Risk-Return-oriented strategies										
MV- S	1.282	1.267	0.162	0.162	0.162	0.162	0.001	0.001	21.121	39.072

Table 10: Performance measures for weekly rebalancing frequency ($k = 5$). The superior results are highlighted in red.

Allocation Strategy	Portfolio diversification effects: weekly rebalancing					
	DR^2		Effective N		PDI	
	No const	10 mln	No const	10 mln	No const	10 mln
Benchmark strategies						
MV - S TrA	5.63	5.63	3.38	3.38	5.15	5.15
Risk-oriented strategies						
RR - Max ret	1.00	1.00	1.84	2.74	1.00	1.00
Return-oriented strategies						
MinVar	13.70	13.62	3.47	3.47	25.29	25.29
MinCVaR	15.60	15.44	4.19	4.19	25.29	25.29
ERC	11.33	12.29	17.51	15.06	25.31	25.31
MD	2.91	2.44	3.98	3.05	25.79	25.82
Risk-Return-oriented strategies						
MV -S	9.28	9.56	3.70	3.69	25.30	25.30

* All performance measures are averages over the test out-of-sample period 20160101-20171231

Table 11: Measures of diversification for weekly rebalancing ($k = 5$). The superior results are highlighted in red.

Allocation Strategy	Portfolio performance measures: daily rebalancing									
	CW		SR		ASR		CEQ		TURNOVER	
	No const	10 mln	No const	10 mln	No const	10 mln	No const	10 mln	No const	10 mln
Benchmark strategies										
S&P100	1.070	1.261	0.080	0.080	0.079	0.079	0.000	0.000	0.000	0.000
EW Trad Assets	1.069	1.069	0.048	0.048	0.047	0.047	0.004	0.004	10.543	10.543
MV-S Trad assets	1.060	1.060	0.077	0.077	0.077	0.077	0.000	0.000	NA	NA
EW	3.644	3.644	0.132	0.132	0.132	0.132	0.004	0.004	13.781	13.781
Risk-oriented strategies										
MinVar	1.002	1.001	−0.016	0.039	−0.016	0.039	−0.008	0.000	24.544	12.537
MinCVAR	1.007	1.012	0.013	0.025	0.013	0.025	0.000	0.000	17.805	15.623
MD	4.185	4.578	0.125	0.161	0.125	0.163	0.005	0.006	53.686	53.687
ERC	1.546	1.369	0.158	0.146	0.158	0.146	0.001	0.001	15.221	12.096
Return-oriented strategies										
RR-max ret	0.069	2.408	0.005	0.002	0.005	0.002	0.000	0.000	0.244	9.943
Risk-Return-oriented strategies										
MV- S	1.235	1.231	0.134	0.139	0.134	0.139	0.000	0.000	17.484	44.729

Table 12: Performance measures for daily rebalancing frequency ($k = 1$). The superior results are highlighted in red.

Benchmark Strategy	Portfolio diversification effects: daily rebalancing					
	DR^2		Effective N		PDI	
	No const	10 mln	No const	10 mln	No const	10 mln
Benchmark strategies						
MV - S Trad Assets	5.64	5.64	3.40	3.40	5.15	5.15
Risk-oriented strategies						
RR - Max ret	1.00	1.00	1.83	2.74	18.32	24.39
Return-oriented strategies						
MinVar	13.80	13.72	3.47	3.47	25.26	25.26
ERC	11.32	12.30	17.48	15.03	25.29	25.28
MinCVaR	15.55	15.39	1.77	2.84	17.63	24.06
MD	2.91	2.43	3.99	3.04	25.77	25.79
Risk-Return-oriented strategies						
MV -S	9.29	9.57	3.74	3.75	25.28	25.28

* All performance measures are averages over the test out-of-sample period 20160101-20171231

Table 13: Measures of diversification for daily rebalancing ($k = 1$). The superior results are highlighted in red.

IRTG 1792 Discussion Paper Series 2018

For a complete list of Discussion Papers published, please visit irtg1792.hu-berlin.de.

001 "Data Driven Value-at-Risk Forecasting using a SVR-GARCH-KDE Hybrid" by Marius Lux, Wolfgang Karl Härdle and Stefan Lessmann, January 2018.
002 "Nonparametric Variable Selection and Its Application to Additive Models" by Zheng-Hui Feng, Lu Lin, Ruo-Qing Zhu asnd Li-Xing Zhu, January 2018.
003 "Systemic Risk in Global Volatility Spillover Networks: Evidence from Option-implied Volatility Indices " by Zihui Yang and Yinggang Zhou, January 2018.
004 "Pricing Cryptocurrency options: the case of CRIX and Bitcoin" by Cathy YH Chen, Wolfgang Karl Härdle, Ai Jun Hou and Weining Wang, January 2018.
005 "Testing for bubbles in cryptocurrencies with time-varying volatility" by Christian M. Hafner, January 2018.
006 "A Note on Cryptocurrencies and Currency Competition" by Anna Almosova, January 2018.
007 "Knowing me, knowing you: inventor mobility and the formation of technology-oriented alliances" by Stefan Wagner and Martin C. Goossen, February 2018.
008 "A Monetary Model of Blockchain" by Anna Almosova, February 2018.
009 "Deregulated day-ahead electricity markets in Southeast Europe: Price forecasting and comparative structural analysis" by Antanina Hryshchuk, Stefan Lessmann, February 2018.
010 "How Sensitive are Tail-related Risk Measures in a Contamination Neighbourhood?" by Wolfgang Karl Härdle, Chengxiu Ling, February 2018.
011 "How to Measure a Performance of a Collaborative Research Centre" by Alona Zharova, Janine Tellinger-Rice, Wolfgang Karl Härdle, February 2018.
012 "Targeting customers for profit: An ensemble learning framework to support marketing decision making" by Stefan Lessmann, Kristof Coussement, Koen W. De Bock, Johannes Haupt, February 2018.
013 "Improving Crime Count Forecasts Using Twitter and Taxi Data" by Lara Vomfell, Wolfgang Karl Härdle, Stefan Lessmann, February 2018.
014 "Price Discovery on Bitcoin Markets" by Paolo Pagnottoni, Dirk G. Baur, Thomas Dimpfl, March 2018.
015 "Bitcoin is not the New Gold - A Comparison of Volatility, Correlation, and Portfolio Performance" by Tony Klein, Hien Pham Thu, Thomas Walther, March 2018.
016 "Time-varying Limit Order Book Networks" by Wolfgang Karl Härdle, Shi Chen, Chong Liang, Melanie Schienle, April 2018.
017 "Regularization Approach for NetworkModeling of German EnergyMarket" by Shi Chen, Wolfgang Karl Härdle, Brenda López Cabrera, May 2018.
018 "Adaptive Nonparametric Clustering" by Kirill Efimov, Larisa Adamyan, Vladimir Spokoiny, May 2018.
019 "Lasso, knockoff and Gaussian covariates: a comparison" by Laurie Davies, May 2018.

IRTG 1792, Spandauer Straße 1, D-10178 Berlin
http://irtg1792.hu-berlin.de

This research was supported by the Deutsche Forschungsgemeinschaft through the IRTG 1792.

IRTG 1792 Discussion Paper Series 2018

For a complete list of Discussion Papers published, please visit irtg1792.hu-berlin.de.

020 "A Regime Shift Model with Nonparametric Switching Mechanism" by Haiqiang Chen, Yingxing Li, Ming Lin and Yanli Zhu, May 2018.
021 "LASSO-Driven Inference in Time and Space" by Victor Chernozhukov, Wolfgang K. Härdle, Chen Huang, Weining Wang, June 2018.
022 " Learning from Errors: The case of monetary and fiscal policy regimes" by Andreas Tryphonides, June 2018.
023 "Textual Sentiment, Option Characteristics, and Stock Return Predictability" by Cathy Yi-Hsuan Chen, Matthias R. Fengler, Wolfgang Karl Härdle, Yanchu Liu, June 2018.
024 "Bootstrap Confidence Sets For Spectral Projectors Of Sample Covariance" by A. Naumov, V. Spokoiny, V. Ulyanov, June 2018.
025 "Construction of Non-asymptotic Confidence Sets in 2 -Wasserstein Space" by Johannes Ebert, Vladimir Spokoiny, Alexandra Suvorikova, June 2018.
026 "Large ball probabilities, Gaussian comparison and anti-concentration" by Friedrich Götze, Alexey Naumov, Vladimir Spokoiny, Vladimir Ulyanov, June 2018.
027 "Bayesian inference for spectral projectors of covariance matrix" by Igor Silin, Vladimir Spokoiny, June 2018.
028 "Toolbox: Gaussian comparison on Eucledian balls" by Andzhey Koziuk, Vladimir Spokoiny, June 2018.
029 "Pointwise adaptation via stagewise aggregation of local estimates for multiclass classification" by Nikita Puchkin, Vladimir Spokoiny, June 2018.
030 "Gaussian Process Forecast with multidimensional distributional entries" by Francois Bachoc, Alexandra Suvorikova, Jean-Michel Loubes, Vladimir Spokoiny, June 2018.
031 "Instrumental variables regression" by Andzhey Koziuk, Vladimir Spokoiny, June 2018.
032 "Understanding Latent Group Structure of Cryptocurrencies Market: A Dynamic Network Perspective" by Li Guo, Yubo Tao and Wolfgang Karl Härdle, July 2018.
033 "Optimal contracts under competition when uncertainty from adverse selection and moral hazard are present" by Natalie Packham, August 2018.
034 "A factor-model approach for correlation scenarios and correlation stress-testing" by Natalie Packham and Fabian Woebbeking, August 2018.
035 "Correlation Under Stress In Normal Variance Mixture Models" by Michael Kalkbrener and Natalie Packham, August 2018.
036 "Model risk of contingent claims" by Nils Detering and Natalie Packham, August 2018.
037 "Default probabilities and default correlations under stress" by Natalie Packham, Michael Kalkbrener and Ludger Overbeck, August 2018.
038 "Tail-Risk Protection Trading Strategies" by Natalie Packham, Jochen Papenbrock, Peter Schwendner and Fabian Woebbeking, August 2018.

IRTG 1792, Spandauer Straße 1, D-10178 Berlin
http://irtg1792.hu-berlin.de

This research was supported by the Deutsche Forschungsgemeinschaft through the IRTG 1792.

IRTG 1792 Discussion Paper Series 2018

For a complete list of Discussion Papers published, please visit irtg1792.hu-berlin.de.

IRTG 1792, Spandauer Straße 1, D-10178 Berlin
http://irtg1792.hu-berlin.de

This research was supported by the Deutsche Forschungsgemeinschaft through the IRTG 1792.

IRTG 1792 Discussion Paper Series 2018

For a complete list of Discussion Papers published, please visit irtg1792.hu-berlin.de.

058 "Investing with cryptocurrencies – evaluating the potential of portfolio allocation strategies" by Alla Petukhina, Simon Trimborn, Wolfgang Karl Härdle, Hermann Elendner, October 2018.

IRTG 1792, Spandauer Straße 1, D-10178 Berlin
http://irtg1792.hu-berlin.de

This research was supported by the Deutsche Forschungsgemeinschaft through the IRTG 1792.