IRTG 1792 Discussion Paper 2020-020

Long- and Short-Run Components of Factor Betas: Implications for Stock Pricing

Hossein Asgharian *
Charlotte Christiansen *2
Ai Jun Hou *3
Weining Wang *4

* Lund University, Sweden
*2 Aarhus University, Denmark
*3 Stockholm University, Sweden
*4 University of York, UK

This research was supported by the Deutsche Forschungsgesellschaft through the International Research Training Group 1792 "High Dimensional Nonstationary Time Series".

http://irtg1792.hu-berlin.de
ISSN 2568-5619

International Research Training Group 1792

Long- and Short-Run Components of Factor Betas: Implications for Stock Pricing[*]

Hossein Asgharian, Lund University[+]

Charlotte Christiansen, Aarhus University[++]

Ai Jun Hou, Stockholm University[+++]

and

Weining Wang, University of York[++++]

September 11, 2020

[*] We are thankful for suggestions and comments from participants at the following conferences: World Finance Conference 2017 in Sardinia, Italy; New Methods for the Empirical Analysis of Financial Markets 2017 in Comillas, Spain; China Econometric Society meeting 2018; IFABS 2018 in Porto, Portugal; European Summer Meeting of the Econometric Society 2018 in Cologne, Germany; Annual SoFiE meeting 2018 in Lugano, Switzerland; FMA meeting 2018 in San Diego, USA. We are grateful for helpful comments from Federico Bandi and Oliver Linton.

Asgharian and Hou thank the *Jan Wallander* and *Tom Hedelius Foundations* for funding their research. Christiansen acknowledges financial support from the Danish Council of Independent Research (DFF 4003-00022) and CREATES (Center for Research in Econometric Analysis of Time Series) funded by the Danish National Research Foundation (DNRF78). Wang thanks the German Research Foundation (DFG WA3686 1-1) for financial support.

The authors have no conflict of interest.

[+] Hossein Asgharian, Knut Wicksell Centre for Financial Studies, Department of Economics, P.O. Box 7082, S-220 07 Lund, Sweden. Email: Hossein.Asgharian@nek.lu.se.

++ Charlotte Christiansen, DFI, CREATES, Department of Economics and Business Economics, Aarhus University, Fuglesangs Allé 4, 8210 Aarhus V, Denmark and Lund University. cchristiansen@econ.au.dk.

+++ Ai Jun Hou, Stockholm Business School, Stockholm University, SE-106 91 Stockholm, Sweden. aijun.hou@sbs.su.se.

++++ Weining Wang, University of York, UK and Humboldt University.weining.wang@york.ac.uk.

Long- and Short-Run Components of Factor Betas: Implications for Stock Pricing

September 11, 2020

Abstract: We propose a bivariate component GARCH-MIDAS model to estimate the long- and short-run components of the variances and covariances. The advantage of our model to the existing DCC-based models is that it uses the same form for both the variances and covariances and that it estimates these moments simultaneously. We apply this model to obtain long- and short-run factor betas for industry test portfolios, where the risk factors are the market, SMB, and HML portfolios. We use these betas in cross-sectional analysis of the risk premia. Among other things, we find that the risk premium related to the short-run market beta is significantly positive, irrespective of the choice of test portfolio. Further, the risk premia for the short-run betas of all the risk factors are significant outside recessions.

Keywords: long-run betas; short-run betas; risk premia; business cycles; component GARCH model; MIDAS

JEL Classifications: G12; C58; C51

1. Introduction

In this paper, we propose a new bivariate component GARCH-MIDAS model that decomposes return variances and covariances into a long-run (persistent) and a short-run (transitory) component. We use the model to obtain long- and short-run factor betas. The short-run component can be interpreted as a correction of or revision to the long-run component, due to arrival of new information (similar to the volatility model in Engle and Lee, 1999). Separating these two components gives us a better understanding of the cross-sectional relationship between risk and expected stock returns.

We apply our model to test the Fama and French (1993) three-factor model, which is one of the most widely used asset pricing models. The long- and short-run components of portfolio betas are based on the market portfolio, the small-minus-big portfolio (SMB), and the high-minus-low portfolio (HML) risk factors. The risk premia are estimated both at weekly and monthly frequency. To be able to compare our results with most of the earlier studies we use several alternative test portfolios from French's online data library. Our main analysis is based on the 30 industry portfolios, but for robustness we also consider 49 industry portfolios as well as 25 portfolios sorted on size and book-to-market value.

We contribute with a new econometric model and evaluate its theoretical properties. To the best of our knowledge, this is the first conditional model that uses the mixed data sampling (MIDAS) approach to simultaneously estimate the long- and short-run components of the variances and covariances and thereby it decomposes total betas into long- and short-run components. The long-run component of variances (covariances) depends on the historical unconditional variance (covariance). Our new additive GARCH-MIDAS model differs from the DCC-MIDAS model (see e.g. Colacito, Engle, and Ghysels, 2011; Conrad, Loch, and Ritter, 2014; Asgharian, Christiansen, and Hou, 2016) that applies the multiplicative GARCH-MIDAS volatility (from e.g. Engle, Ghysels, and Sohn, 2013; Conrad and Locch, 2015) as input. The multiplicative approach works well for univariate models, but cannot be applied directly to bivariate models because it may give ambiguous estimates of the covariance in case of negative covariance components.[1] In general, the DCC-MIDAS models assume a multiplicative form in the first step of estimating the variances and an additive form in the second step to estimate the correlation (total correlation is modelled as the sum of the long- and short run correlations). There are also several studies that use alternative models to decompose covariances into high and low frequencies. Rangel and Engle (2012) use the two-step DCC model, where the first step is the volatility model from Engle and Rangel (2008) with long- and short-run components. The second step uses the first-step residuals to model the correlations. Similarly, Bauwens, Hafner, and Pierret (2013) use a DCC model to separate long- and short-run correlations of electricity futures returns, where the long-run covariance matrix is deterministic. The advantage of our model compared to the DCC-based models is that it uses the same additive form for the variances and covariance and estimate these moments simultaneously, in a bivariate framework.

We find that data frequency matters for risk premia: none of the risk premia estimated at weekly frequency are significant, which is in contrast to the risk premia obtained at the monthly frequency. This may imply that risk premia estimated more frequently than monthly are noisy. Finally, our empirical analysis, at the monthly frequency, shows that decomposing risk across horizons may help explain the anomaly that the traditional market risk premium is not significant, as we find that

[1] In the multiplicative framework, the total covariance is the product of the long- and short-run covariances. The problem with the multiplicative framework is that the total covariance will be negative only if one of the components is negative and it will be positive if both long- and short-run covariances are negative.

the risk premium related to the short-run market beta is significantly positive.[2] This result is robust to the choice of the test portfolios. We also find that the short-run risk premia are larger in expansions than in recessions. In fact, the risk premia for short-run betas of all the risk factors are significant if we exclude recessions from our sample.

This paper is related to several strands of previous literature. Firstly, we build on conditional asset pricing models. For example, Bollerslev, Engle, and Wooldridge (1988) apply a multivariate GARCH model and define the expected return of an asset as a linear function of the conditional covariance of the asset with the market portfolio. Asgharian and Hansson (2000) and Bali (2008) use bivariate GARCH models to obtain time-varying factor betas and then use the estimated betas in monthly cross-sectional regressions to obtain corresponding risk premia. Bali and Engle (2010) use the DCC model of Engle (2002) to investigate if assets' time-varying conditional covariance with the market portfolio predicts the time-variation in the assets' expected returns. More recently, Bali, Engle, and Tang (2017) use the DCC model on individual assets to assess the predictive ability of the factor betas.

Secondly, a number of studies show that the choice of data frequency and time horizon is important for obtaining an accurate measure of risk and capturing the risk-return relationship. For example, Gilbert, Hrdlicka, Kalodimos, and Siegel (2014) show that there are large differences between high- (daily) and low-frequency (quarterly) stock betas. According to Lewellen and Nagel (2006), compounding implies that betas vary across different frequencies. Engle and Lee (1999) and Engle

---

[2] Other studies find a significant risk premium for the market beta without decomposition. However, these studies are typically based on different test portfolios than the widely used Fama and French (1993, 1997) portfolios. For example, Maio and Santa-Clara (2017) use a number of portfolios sorted on e.g. book-to-market ratio and earnings-to-price and Kim (1995) and Bali, Engle, and Tang (2017) use individual stocks. An exception is Bali (2008) that applies the bivariate GARCH model to Fama and French's (1993, 1997) test portfolios and show that the portfolios' conditional covariance with the market can predict time-variation in the expected return.

and Rangel (2008) show that models with both low- and high-frequency volatility and correlation components capture the dynamics of equity returns better than single-frequency models. Adrian and Rosenberg (2008) explore cross-sectional pricing of risk by decomposing equity-market volatility into short- and long-run components. Cenesizoglu and Reeves (2018) use a nonparametric approach and measure market beta with short-, medium-, and long-run components. The short- and medium-run components are estimated from daily returns over one- and five-year periods, and the long-run component is estimated from monthly returns over a 10-year period. Boons and Tamoni (2016) show that dividing risk into long- and short-run components helps uncover a link between risk premia and the macro economy. Andersen and Bollerslev (1997) and Calvet and Fisher (2007) argue that information in financial markets arrives at different frequencies and has different degrees of persistence. Since investors have different investment horizons, their view of systematic risk is horizon dependent, see e.g. Bansal, Dittmar, and Kiku (2009). Kamara, Korajczyk, Lou, and Sadka (2016) find that cross-sectional risk premiums vary with the return horizon. Bandi, Perron, Tamoni, and Tebaldi (2018) model market excess returns and their predictors are aggregates of uncorrelated components operating over different frequencies and they introduce a notion of scale-specific predictability.

Thirdly, several studies use the mixed data sampling (MIDAS) approach to estimate systematic risk but without decomposing it into long- and short-run betas. Gonzalez, Nave, and Rubio (2012) use the weighted average of daily returns to estimate monthly betas. Gonzalez, Nave, and Rubio (2018) define the conditional beta with two additive components, a transitory component estimated from daily returns and a long-run component based on macroeconomic state variables. Baele and Londono (2013) use Colacito, Engle, and Ghysels's (2011) DCC-MIDAS model to obtain long-run betas. They find that DCC-MIDAS betas are superior to ordinary betas in limiting the downside risk and ex-post market exposure for the minimum-variance strategy. Ghysels, Santa-Clara, and Valkanov (2005) use MIDAS volatilities to analyze the risk-return trade-off. They investigate the effects of changing the frequency of the returns in the MIDAS risk-return trade-off regressions and find that using high-frequency returns (above monthly) provides excessively noisy estimates. Therefore, they conclude that monthly returns are preferable. Ghysels, Guérin, and Marcellino (2014) continue this analysis by combining regime switching with MIDAS and consider variations across horizons.

The rest of the paper is structured as follows. Section 2 introduces the econometric framework. We present the data in section 3. In section 4, we discuss the empirical results. Finally, we conclude in section 5. An Appendix provides further details on the econometric model.

2. The component asset-pricing model

In this section, we present the new bivariate component GARCH-MIDAS model. The empirical analysis follows a two-step estimation procedure similar to Fama and MacBeth (1973). The first step entails time-series regressions to obtain total, long-, and short-run betas. In the second step, we estimate the corresponding risk premia.

2.1 First step: bivariate component GARCH model

Within the component GARCH models, there are two general approaches to distinguish short- from long-run movements: the additive approach from Engle and Lee (1999) and the multiplicative approach from Engle, Ghysels, and Sohn (2013). The multiplicative approach, despite working well in univariate models, cannot be applied to bivariate models as we may have negative covariances. We use the additive approach and extend Asgharian and Hansson's (2000) and Bali's (2008) bivariate GARCH model to a bivariate component GARCH model to decompose the total variance and covariance to a long-run (persistent) component and a short-run (transitory) component. The advantage of the proposed model compared to for instance the DCC-MIDAS model is that it provides an estimate of the covariance, instead of correlation, which is used directly to calculate betas.

We use the subscripts *s* and *t* to keep track of time, where *s* and *t* denote periods corresponding to the weekly and monthly frequency, respectively. We assume that the mean equations for the weekly excess returns for portfolio *i* ($r_{i,s,t}$) and the state variable *x* ($r_{x,s,t}$) follow a simple form where they are equal to a constant plus an error term ($\varepsilon_{i,s,t}$ and $\varepsilon_{x,s,t}$, respectively):

$$
\begin{aligned}
r_{i,s,t} &= \gamma_i + \varepsilon_{i,s,t} \\
r_{x,s,t} &= \gamma_x + \varepsilon_{x,s,t}.
\end{aligned} \tag{1}
$$

The error terms are assumed to follow normal distributions with mean zero and time dependent variances, $q_{i,s,t}$ and $q_{x,s,t}$ and covariance, $q_{ix,s,t}$.

Engle and Lee's (1999) univariate additive component GARCH model defines the total conditional variance as the sum of a long-run (persistent) component and a short-run (transitory) component where the total variance follows a GARCH(1,1) model. They replace the unconditional variance used in GARCH models by the long-run time-varying variance. The idea of an unconditional time-varying variance is also presented in, for example, Amado and Teräsvirta's (2014) model. We extend the idea in Engle and Lee (1999) to a bivariate GARCH(1,1) model to estimate each portfolio's conditional variances as well as their conditional covariances with the common factors, one factor at a time.

In the parameterization of the GARCH equation, we use the BEKK specification to reduce the number of parameters, cf. Engle and Kroner (1995). The formulation of the intercept follows Santis and Gerard (1997), while the unconditional moments (the τ's) are time varying. The total variances and covariance are modeled as:

$$q_{i,s,t} = \tau_{i,t}(1 - a_i^2 - b_i^2) + a_i^2 \varepsilon_{i,s-1,t}^2 + b_i^2 q_{i,s-1,t}$$
$$q_{x,s,t} = \tau_{x,t}(1 - a_x^2 - b_x^2) + a_x^2 \varepsilon_{x,s-1,t}^2 + b_x^2 q_{x,s-1,t} \quad (2)$$
$$q_{ix,s,t} = \tau_{ix,t}(1 - a_i a_x - b_i b_x) + a_i a_x \varepsilon_{i,s-1,t}\varepsilon_{x,s-1,t} + b_i b_x q_{ix,s-1,t} ,$$

where $\tau_{i,t}$, $\tau_{x,t}$, and $\tau_{ix,s,t}$ are the long-run variances and covariance. We use the equally weighted moving average of the past five years' observations to estimate the long-run variances and covariances. This is similar to Colacito, Engle, and Ghysels's (2011) approach to estimating long-run correlation in the DCC-MIDAS model.[3] The advantage is that the estimated long-run betas are equal to the conventional estimate of the unconditional beta. This facilitates straightforward comparisons with earlier studies:

$$\tau_{i,t} = \frac{1}{K}\sum_{k=1}^{K}(r_{i,k} - \mu_{i,K})^2$$
$$\tau_{x,t} = \frac{1}{K}\sum_{k=1}^{K}(r_{x,k} - \mu_{x,K})^2 \quad (3)$$
$$\tau_{ix,t} = \tfrac{1}{K}\textstyle\sum_{k=1}^{K}(r_{i,k} - \mu_{i,K})(r_{x,k} - \mu_{x,K}) ,$$

where *K* is the number of months within the past five years, i.e., *K*=60.[4] $r_{i,k}$ and $r_{x,k}$ are the monthly returns for portfolio *i* and state variable *x*. $\mu_{i,K}$ and $\mu_{x,K}$ are the means of the monthly returns of

[3] We have also used the more general beta-lag polynomial weighting function for describing the long-run moments, where the weighting function is estimated. However, the estimation converges to the equally weighted average in most cases. The model with the beta-lag polynomial weighing function is described in the Appendix.

[4] The five-year window with monthly returns is conventional for estimating unconditional betas (see e.g. Fama and French, 1993).

the past K months for portfolio i and state variable x, respectively. The long-run component is the average of the squared deviations of the monthly returns from their mean within the past five years. In this way, the long-run betas are identical to the conventional rolling-window betas.[5] Therefore, the short-run variance (covariance) is defined as a function of the deviation of the lagged shock and the lagged total variance (covariance) from the long-run variance (covariance).

The total betas are calculated from the estimated total covariance and variance, and the long-run betas from the estimated long-run covariance and variance:

$$\hat{\beta}_{x,i,s,t}^{total} = \frac{\hat{q}_{ix,s,t}}{\hat{q}_{x,s,t}} \qquad \text{and} \qquad \hat{\beta}_{x,i,s,t}^{long} = \frac{\hat{\tau}_{ix,s,t}}{\hat{\tau}_{x,s,t}}. \tag{4}$$

The short-run betas are the differences between the total and long-run betas.

In addition to the mixed-frequency model, we also work with a single-frequency model. In the single-frequency model, s and t are identical, i.e. either weekly or monthly frequency.

Several restrictions have been applied to ensure that the conditional variance–covariance matrix is positive definite at each s and t. The details are in the appendix, where we also discuss identification and stationarity of the model. The log-likelihood function for model estimation is also given in the appendix.

2.2 Second step: cross-sectional regressions

In our setting, the expected returns depend on both long- and short-run components of the three risk premia, stemming from the market, SMB, and HML risk factors.

[5] In the GARCH–MIDAS model, the long-run component is calculated as the weighted sum of the realized variances and covariance. We also estimate the model with realized moments based on daily data and exponential weights. The conclusions remain unaltered.

The second step concerns the Fama and MacBeth (1973) cross-sectional regressions. There is one cross-sectional regression for each period *s*. When we consider the total betas, it reads as follows.

$$R_{i,s,t} = C_{0s,t}^{\text{total}} + C_{1s,t}^{\text{total}} \beta_{M,i,s,t}^{\text{total}} + C_{2s,t}^{\text{total}} \beta_{SMB,i,s,t}^{\text{total}} + C_{3s,t}^{\text{total}} \beta_{HML,i,s,t}^{\text{total}} + \varepsilon_{i,s,t}, \text{for } i = 1, \dots, N \,. \quad (5)$$

We also do cross-sectional regressions with both short- and long-run betas and thereby obtain long- and short-run risk premia. This is new to the literature.

$$R_{i,s,t} = c_{0s,t} + c_{1,s,t}^{\text{long}} \beta_{M,i,s,t}^{\text{long}} + c_{1,s,t}^{\text{short}} \beta_{M,i,s,t}^{\text{short}} + c_{2,s,t}^{\text{long}} \beta_{SMB,i,s,t}^{\text{long}} + c_{2,s,t}^{\text{short}} \beta_{SMB,i,s,t}^{\text{short}} + \quad (6)$$

$$c_{3,s,t}^{\text{long}} \beta_{HML,i,s,t}^{\text{long}} + c_{3,s,t}^{\text{short}} \beta_{HML,i,s,t}^{\text{short}} + \varepsilon_{i,s,t}, \quad \text{for } i = 1, \dots, N \,.$$

The risk premia are the average of the estimated slope coefficients, the *c*'s. We use the time series of the estimated coefficients to investigate the properties of the factor risk premia such as whether the average coefficients are significant and, if so, whether they are positive or negative. We use Newey and West (1987) corrected standard errors, which is similar to Bali, Engle and Tang (2017). The usage of portfolios rather than individual assets as test assets help reduce the errors-in-variables problem of using estimated explanatory variables (the betas) in the second step regressions.

3. Data

Our analysis is based on the value-weighted excess log-returns for 30 industry portfolios at weekly and monthly frequency. We use market, SMB, and HML risk factors as state variables (Fama and French, 1993). The sample covers the period from 1945 to 2015 and includes several recessions such as the dotcom bubble and the recent financial crisis. For robustness, we also use 49 value weighted industry portfolios and 25 size and book-to-market double-sorted portfolios.[6] We use the NBER recession indicator to measure of the state of the macroeconomy.[7]

Table 1 shows descriptive statistics for the monthly excess returns of the 30 industry portfolios. The mean returns are significantly positive and varies from 6.0% per year ("Other") to 11.5% per year ("Smoke"). The standard deviations are relatively large, ranging from 13.3% per year ("Utilities") to 32.4% per year ("Coal"). For all industry portfolios, we observe negative skewness and positive excess kurtosis, revealing extreme negative returns.

4. Empirical results

In this section, we show the empirical results. First, we show the results regarding estimations of betas and following for the estimated risk premia. At the end, we discuss how the risk premia are related to the state of the economy.

4.1 Estimation of the bivariate component GARCH model

We use two different frequency pairs (s, t) to decompose the long- and short-run components. The long-run component, t, is at the monthly frequency and the short-run component, s, varies between weekly and monthly frequency.[8] Our benchmark model is based on the monthly-monthly (M-M) frequency. That is, the returns in equation (1), the total variance and covariance in equation (2), and the long-run variances and covariance in equation (3) are all based on monthly returns. This is a GARCH specification with time-varying unconditional moments. We use the monthly-

[6] We gratefully obtain the data from French's online data library.

[7] We obtain the NBER recession data from the NBER webpage.

[8] We also estimate the model with the monthly-daily combination. The results are similar to those with the monthly-weekly combination. For the sake of brevity, those results are not reported.

monthly approach as the base case to be able to compare our results with earlier studies since it is conventional to use a five-year moving window with monthly returns to estimate betas and the monthly frequency to estimate the cross-sectional regression. We also use an alternative specification of the component GARCH model in which we keep the long-run moments in equation (3) at the monthly frequency while changing the frequency of the bivariate variance and covariance in equation (2) and the returns in equation (1) to weekly (denoted monthly-weekly or M-W). This is a MIDAS specification.

Table 2 shows the means and standard deviations of the parameter estimates of the bivariate component GARCH model in equations (1) to (3) estimated for each of the 30 industry portfolios together with each of the three factors, one at a time, both for the monthly-weekly and monthly-monthly specifications. The parameter estimates show that the volatilities are persistent, because all the *b* coefficients are much greater than the corresponding *a* coefficients. The related standard deviations are very small, indicating that the volatility persistence holds for most of the industries. As expected, the estimated mean returns (the γ's) are larger in the monthly-monthly specification than in the monthly-weekly specification.

To illustrate the estimated betas over time, we use the financial industry ("Fin") as an example. Figure 1 shows the time series of the total and long-run betas for the monthly-monthly and monthly-weekly frequency for this industry. The long-run betas are smoother than the total betas, especially when we use the monthly-weekly frequency instead of the monthly-monthly frequency. The market betas are less variable than the SMB and HML betas at both frequencies. As expected, the estimated betas are, in general, very large during the recent financial crises, which supports the large contribution of the financial industry to the systematic risk during this period.

A number of studies examine the link between the cross-sectional dispersion of industry betas and the state of the economy. Gomes, Yaron, and Zhang (2003) find that the heterogeneity of betas across firms increases during recessions leading to increasing beta dispersion. This effect is reinforced by the countercyclical behavior of dispersion of the firms' characteristics, which is in line with the findings of Chan and Chen (1988). Similarly, Baele and Londono (2013) find that the empirical cross-sectional dispersion in industry betas increases during recessions.

We use the method in Baele and Londono (2013) to calculate the cross-sectional dispersion of the betas for each month. The cross-sectional dispersion coefficient at a given point in time is the value-weighted sum of squares of each industry's beta minus the average beta across all industries. Table 3 (top rows) shows the time-series average of the cross-sectional dispersion of the estimated betas based on the monthly-monthly frequency. The dispersions of the short-run betas are on average smaller than those of the long-run betas.

To investigate how the cross-sectional dispersion of industry betas varies across the business cycle, we regress the dispersion coefficients for all the betas on the NBER recession indicator. The regression results are reported in Table 3 (bottom rows). The dispersion of the total betas is larger in recessions than in expansions, which supports the findings of earlier studies (e.g., Gomes, Yaron, and Zhang, 2003; Baele and Londono, 2013). Interestingly, our results show that the larger cross-sectional dispersion of the total betas in recessions depends on the short-run betas, as the all the short-run dispersions are significantly larger in recessions than in expansions, while none of the long-run dispersions are significantly different in recessions and expansions. The long-run betas reflect the slow movements of the factor loadings, while the recession periods are fairly short lived.

4.3 Cross-sectional regressions

To evaluate our component GARCH model, we compare its pricing ability with that of two alternative models for estimating betas: the traditional rolling-window OLS regressions (unconditional betas) and the bivariate GARCH model. For these comparisons, we use both weekly and monthly returns. Table 4 shows all the estimated risk premia. None of the models describe the cross-sectional variation in expected returns perfectly, as all models have significant intercepts (alphas).

Panel A of Table 4 shows the total risk premia obtained from the various models, i.e. the mean of the estimated time-series coefficients from the cross-sectional regressions in equation (5). First, the table shows the estimated risk premia associated with the unconditional betas. The market and HML risk premia are not significant. The SMB risk premium is significantly positive at the monthly frequency, which is in accordance with earlier findings, whereas it is insignificant at the weekly frequency. Table 4 also shows the risk premia obtained from the conventional bivariate GARCH model. None of the risk premia are significant irrespectively of data frequency. Finally, the table shows the total risk premia related to the component GARCH model. Here the total risk premia are qualitatively similar to the unconditional risk premia, namely, that only the SMB risk premium is significant, and only so at the monthly-monthly frequency. So, if we limit our interest to total risk premia, the component GARCH model provides the same information as the traditional model. This also implies that the total risk premia results for our new model confirm previous findings.

Panel B of Table 4 shows the new risk premia of the long- and short-run components of beta. At the monthly-weekly frequency, none of the risk premia in Panels A and B of Table 4 are significant. This indicates that risk premia based on the weekly frequency is too noisy. For the monthly-monthly frequency, several of the risk premia are significantly positive. The risk premia associated with both the long- and short-run SMB betas are significant. Interestingly, the risk premium associated with the short-run market beta is also significantly positive.

To investigate if our results are robust to the choice of the test assets, we estimate our model for some alternative portfolios. First, we use 25 doubled-sorted Fama and French (1993) book-to-market and size portfolios. Second, we use a finer division into industries (49 industry portfolios). Panel C of Table 4 shows the variations in the long- and short-run risk premia for the three data sets based on monthly-monthly frequency. The risk premium of the long-run beta for HML, as expected, becomes highly significant when we use the portfolios sorted based on book-to-market and size. The risk premia related to the short-run market beta are significantly positive for all three data sets. So, this finding is not specific to the 30 industry portfolios.

4.4 Risk premia across the business cycle

In Table 5, we relate the risk premia to the state of the economy as measured by NBER recessions. More specifically, we calculate the average risk premia from the cross-sectional regressions for the entire period as well as separately for recessions and expansions. Panel A is concerned with the total risk premia and panel B with the short- and long-run risk premia. For the unconditional model, the risk premia during expansions are similar to those for the entire sample period. The values are very different in recessions, where the market risk premium is significantly negative, showing large average ex-post realized return for risky firms, i.e. firms with high market betas. For the bivariate GARCH model, the risk premia of all the factors are insignificant for all the subsamples, except the market risk premium which is significantly negative in recessions. The total betas from the component GARCH model also give significant risk premia for SMB and

HML (only at the 10% level for the latter). Overall, none of these estimations give a significantly positive risk premium for the market beta, which is consistent with findings from the previous literature.

For the component GARCH model (Panel B of Table 5), the short- and long-run SMB risk premia are significantly positive and slightly larger in expansions than for the entire sample period. The short-run market risk premium is significantly positive for the entire sample period and during expansions. The short-run market risk premium is larger in expansions than for the entire period, which is caused by the negative (and insignificant) risk premium in recessions. The negative short-run market risk premium in recessions is similar to the negative unconditional risk premium. Risk premia for both short- and long-run HML for expansions are positive and significant at the 10% level. The insignificance of the HML factor for the total period is due to important recession periods that cause a large negative realized mean return and result in an insignificant risk premium. In general, excluding recessions from our sample, i.e. only considering expansions, makes the risk premia for all the betas, except for the long-run market beta, significant and have the expected sign.

5. Conclusion

This paper proposes a new model for decomposing systematic risk into long- and short-run components and provides an important empirical application. The new bivariate component GARCH model enables us to simultaneously decompose total variances and covariances into long- and short-run variances and covariances and thereby to estimate the corresponding components of the factor betas. We model the long-run variances and covariances based on the unconditional variance and covariance of past long-run monthly returns, while the short-run variances and covariances are based on higher or same frequency data (weekly or monthly).

The main analysis is based on the 30 industry portfolios. We investigate the dynamics and determinants of market, SMB, and HML industry betas (Fama and French, 1993). We apply our component GARCH model to each factor and an industry portfolio to estimate long- and short-run variances and covariances. From these, we calculate long- and short-run betas and use them in cross-sectional regressions to estimate the long- and short-run risk premia associated with each factor.

We find that the cross-sectional dispersion in short-run betas increases in recessions. Moreover, we find that the data frequency matters for estimation of the risk premium: none of the risk premia estimated at weekly frequency are significant. At the monthly frequency, our analysis of the risk premia highlights the importance of decomposing risk across horizons. Although, the risk premia associated with both the long- and short-run SMB betas are significant, only the risk premium associated with the short-run market beta is significantly positive. The results appear to be robust to the choice of data set, at least for a finer division into industry portfolios and for portfolios based on book-to-market and size. Further, we fine that the risk premia of the short-run betas of all the risk factors are significant outside recessions.

References

Adrian, T., and J. Rosenberg (2008). Stock Returns and Volatility: Pricing the Short-Run and Long-Run Components of Market Risk. *Journal of Finance* 63, 2997-3030.

Amado, C., and T. Teräsvirta (2014). Modelling Changes in the Unconditional Variance of Long Stock Return Series. *Journal of Empirical Finance* 25, 15-35.

Andersen, T. G., and T. Bollerslev (1997). Intraday Periodicity and Volatility Persistence in Financial Markets. *Journal of Empirical Finance* 4, 115-158.

Asgharian, H., and B. Hansson (2000). Cross-Sectional Analysis of Swedish Stock Returns with Time-Varying Beta: The Swedish Stock Market 1983-96. *European Financial Management* 6, 213-233.

Asgharain, H., C., Christiansen, and A., Hou (2016), Macro-Finance Determinants of the long - Run Stock-Bond Correlations: The DCC-MIDAS Specification, *Journal of Financial Econometrics* 14, 617-642.

Baele, L., and J. M. Londono (2013). Understanding Industry Betas. *Journal of Empirical Finance* 22, 30-51.

Bali, T. G. (2008). The Intertemporal Relation between Expected Returns and Risk. Jo*urnal of Financial Economics* 87, 101-131.

Bali, T.G., and R.F. Engle (2010). The Intertemporal Capital Asset Pricing Model with Dynamic Conditional Correlations. *Journal of Monetary Economy* 57, 377-390.

Bali, T. G., Engle, R. F., and Y. Tang (2017). Dynamic Conditional Beta Is Alive and Well in the Cross Section of Daily Stock Returns. *Management Science* 63, 3760-3779.

Bandi, F. M., B. Perron, A. Tamoni, and C. Tebaldi (2018). The Scale of Predictability. *Journal of Econometrics*, in press.

Bansal, R., R. Dittmar, and D. Kiku (2009). "Cointegration and Consumption Risks in Asset Returns. *Review of Financial Studies* 22, 1343-1375.

Bauwens, L., C. M. Hafner, and D. Pierret (2013). Multivariate Volatility Modeling of Electricity Futures. *Journal of Applied Econometrics* 28, 743-761.

Bollerslev T., Engle R.F., and J.M. Wooldridge (1988). A Capital Asset Pricing Model with Time-Varying Covariances. *Journal of Political Economy* 96, 116-131.

Boons, M., and A. Tamoni (2016). Horizon-Specific Macroeconomic Risks and the Cross Section of Expected Returns. Working Paper, London School of Economics.

Boussama, F., F. Fuchs, and R. Stelzer (2011). Stationarity and Geometric Ergodicity of BEKK Multivariate GARCH Models. *Stochastic Processes and Their Applications* 121, 2331-2360.

Calvet, L. E., and A. J. Fisher (2007), Multifrequency News and Stock Returns. *Journal of Financial Economics* 86, 178-212.

Cenesizoglu, T., and J. Reeves (2018). CAPM, Components of Beta and the Cross Section of Expected Returns. *Journal of Empirical Finance*, in press.

Chan, K. C., and N.-F. Chen (1988), An Unconditional Asset-Pricing Test and the Role of Firm Size as an Instrumental Variable for Risk. *Journal of Finance* 43, 309-325.

Colacito, R., R. F. Engle, and E. Ghysels (2011). A Component Model for Dynamic Correlations. *Journal of Econometrics* 164, 45-59.

Conrad, C., K. Loch, and D. Ritter (2014). On the Macroeconomic Determinants of the Long-Term Oil-Stock Correlation. *Journal of Empirical Finance* 29, 26-40.

Conrad, C., and K. Loch (2015). Anticipating long-term stock market volatility. *Journal of Applied Econometrics* 30, 1090-1114.

Engle, R.F. (2002). Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models. *Journal of Business and Economic Statistics* 20, 339-350.

Engle, R. E., E. Ghysels, and B. Sohn (2013). Stock Market Volatility and Macroeconomic Fundamentals. *Review of Economics and Statistics* 95, 776-797.

Engle, R., and K. Kroner (1995). Multivariate Simultaneous Generalized ARCH. *Econometric Theory* 11, 122-150.

Engle, R., and G. Lee (1999). A Permanent and Transitory Component Model of Stock Return Volatility. In ed. R. F. Engle and H. White, *Cointegration, Causality, and Forecasting: A Festschrift in Honor of Clive W. J. Granger*. Oxford University Press, Oxford, England, 475-497.

Engle, R., and J. G. Rangel (2008). The Spline GARCH Model for Low Frequency Volatility and Its Global Macroeconomic Causes. *Review of Financial Studies* 21, 1187-1222.

Fama, E. F., and K. R. French (1993). Common Risk Factors in the Returns on Stocks and Bonds. *Journal of Financial Economics* 33, 3-56.

Fama, E. F., and K. R. French (1997). Industry Costs of Equity. *Journal of Financial Economics* 43, 153-193.

Fama, E. F., and J. D. MacBeth (1973). Risk, Return, and Equilibrium: Empirical Tests. *Journal of Political Economy* 81, 607-636.

Ghysels, E., P. Guérin, and M. Marcellino (2014). Regime Switches in the Risk-Return Trade-Off. *Journal of Empirical Finance* 28, 118-138.

Ghysels, E., P. Santa-Clara, and R. Valkanov (2005). There Is a Risk-Return Trade-Off After All. *Journal of Financial Economics* 76, 509-548.

Gilbert, T., C. Hrdlicka, J. Kalodimos, and S. Siegel (2014). Daily Data is Bad for Beta: Opacity and Frequency-Dependent Betas. *Review of Asset Pricing Studies* 4, 78-117.

Gonzalez, M., J. Nave, and G. Rubio (2012). The Cross Section of Expected Returns with MIDAS Betas. *Journal of Financial and Quantitative Analysis* 47, 115-135.

Gonzalez, M., J. Nave, and G. Rubio (2018). Macroeconomic Determinants of Stock Market Betas. *Journal of Empirical Finance* 45, 26-44.

Gomes, J. F., A. Yaron, and L. Zhang (2003). Equilibrium Cross Section of Returns. *Journal of Political Economy* 111, 693-732.

Kamara, A., Korajczyk, R. A., Lou, X., and R. Sadka (2016). Horizon Pricing. *Journal of Financial and Quantitative Analysis* 51, 1769-1793.

Kim, D. (1995). The Errors in the Variables Problem in the Cross-Section of Expected Stock Returns. *Journal of Finance* 50, 1605-1634.

Lewellen, J., and S. Nagel (2006). The Conditional CAPM Does Not Explain Asset-Pricing Anomalies, *Journal of Financial Economics* 82, 289-314.

Maio, P. and Santa-Clara, P. (2017). Short-Term Interest Rates and Stock Market Anomalies. *Journal of Financial and Quantitative Analysis* 52, 927-961.

Newey, W., and K. West (1987). A Simple, Positive Semi-Definite, Heteroscedastic and Autocorrelation Consistent Covariance Matrix. *Econometrica* 55, 703-708.

Pedersen, R. S., and A. Rahbek (2014). Multivariate variance targeting in the BEKK-GARCH model. *Econometrics Journal* 17, 24-55.

Rangel, J. G., and & R. F. Engle (2012). The Factor–Spline–GARCH Model for High and Low Frequency Correlations. *Journal of Business & Economic Statistics* 30, 109-124.

Santis, G. D. and B. Gerard (1997). International Asset Pricing and Portfolio Diversification. *Journal of Finance* 52, 1881-1912.

Appendix

This appendix contains technical details about the component GARCH model.

A.1. Beta-lag polynomial weighting function

In a more general version of the long-run moments than equation (3), we use the beta-lag polynomial weighting function. Here the long-run moments are:

$$\tau_{i,t} = \sum_{k=1}^{K} \varphi_k(w_1, w_2) V_{i,t-k}$$
$$\tau_{x,t} = \sum_{k=1}^{K} \varphi_k(w_1, w_2) V_{x,t-k} \qquad \text{(A.1.1)}$$
$$\tau_{ix,t} = \sum_{k=1}^{K} \varphi_k(w_1, w_2) V_{ix,t-k} \ ,$$

where

$$V_{i,t} = (r_{i,t} - \mu_{i,t})^2$$
$$V_{x,t} = (r_{x,t} - \mu_{x,t})^2 \qquad \text{(A.1.2)}$$
$$V_{ix,t} = \left(r_{i,t} - \mu_{i,t}\right)\left(r_{x,t} - \mu_{x,t}\right) .$$

$\mu_{i,t}$ and $\mu_{x,t}$ are the means of the monthly returns for *i* and *x* over five-year historical data before each *t*, and *K* is the number of periods within the five years. The long-run component is the average of the squared deviations of the monthly returns from their mean. In this way, the long-run betas are identical to the conventional rolling-window betas.

The weighting scheme is described by a beta-lag polynomial:

$$\varphi_k(w_1, w_2) = \frac{\left({}^{k}/_{K}\right)^{w_1-1}(1 - {}^{k}/_{K})^{w_2-1}}{\sum_{j=1}^{k}({}^{j}/_{K})^{w_1-1}(1 - {}^{k}/_{K})^{w_2-1}}. \qquad \text{(A.1.3)}$$

In the empirical analysis in the paper we use $w_1 = w_2 = 1$.

A.2. Likelihood function

The bivariate component GARCH model written in matrix form is as follows

$$\begin{pmatrix} r_{i,s,t} \\ r_{x,s,t} \end{pmatrix} = \begin{pmatrix} \gamma_i \\ \gamma_x \end{pmatrix} + \begin{pmatrix} \varepsilon_{i,s,t} \\ \varepsilon_{x,s,t} \end{pmatrix} \sim Q_{s,t}^{1/2} \zeta_t \tag{A.2.1}$$

$$Q_{s,t} = \begin{pmatrix} q_{i,s,t} & q_{ix,s,t} \\ q_{ix,s,t} & q_{x,s,t} \end{pmatrix}$$

$$\begin{aligned} &= \begin{pmatrix} \tau_{i,t}(1-\alpha_i^2 - b_i^2) & \tau_{ix,t}(1-\alpha_i\alpha_x - b_i b_x) \\ \tau_{ix,t}(1-\alpha_i\alpha_x - b_i b_x) & \tau_{x,t}(1-\alpha_x^2 - b_x^2) \end{pmatrix} \\ &+ \begin{pmatrix} \alpha_i & 0 \\ 0 & \alpha_x \end{pmatrix} \begin{pmatrix} \varepsilon_{i,s-1,t}^2 & \varepsilon_{i,s-1,t}\varepsilon_{x,s-1,t} \\ \varepsilon_{i,s-1,t}\varepsilon_{x,s-1,t} & \varepsilon_{x,s-1,t}^2 \end{pmatrix} \begin{pmatrix} \alpha_i & 0 \\ 0 & \alpha_x \end{pmatrix} \\ &- \begin{pmatrix} b_i & 0 \\ 0 & \alpha b_x \end{pmatrix} \begin{pmatrix} q_{i,s-1,t} & q_{ix,s-1,t} \\ q_{ix,s-1,t} & q_{x,s-1,t} \end{pmatrix} \begin{pmatrix} b_i & 0 \\ 0 & b_x \end{pmatrix}. \end{aligned} \tag{A.2.2}$$

The error terms in the return equation are assumed to be bivariate normally distributed with Q_t as the conditional variance–covariance matrix and ζ_t is an IID vector process such that $\mathrm{E}(\zeta_t \zeta_t') = \boldsymbol{I}$, where $\boldsymbol{I}$ is the identity matrix.

The log likelihood function is

$$L(\Theta) = -\frac{1}{2}\sum_{i=1}^{T}\left[\ln(2\pi) + ln\left|Q_{s,t}\right|\right] + \varepsilon_{st}' Q_{s,t}^{-1} \varepsilon_{st}. \tag{A.2.3}$$

A.3. Positive definiteness

Here, we discuss the necessary restrictions on the parameters to ensure the positive definiteness of the conditional variance–covariance matrix, $Q_{s,t}$.

Recall that the residuals from the return equations of the bivariate component GARCH model are assumed joint normal:

$$\boldsymbol{\varepsilon}_{s,t}|\mathcal{F}_{s-1,t} \sim \mathbb{N}\,(0, \boldsymbol{Q}_{s,t}), \tag{A.3.1}$$

where, without loss of generality, s and t denote periods corresponding to the higher and lower frequency, respectively, $t = 1, \dots, T$ and $s = 1, \dots, t, \dots, 2t, \dots, S$, and $S = T \times m$, where m is the block size. [.] denotes the floor function of the quotient. Clearly, $t = [s/m] + 1$ (note that in practice the block size, m, might be different as we might not have T full blocks of data.).

The returns are an exogenous p covariate and are denoted $R_t = (r_{it}, r_{xt})^T$, where i and x denote portfolios and state variables, respectively. Assume also that R_t is a component-wise stationary, ergodic, strongly mixing process with mixing coefficient $\sum_{n=1}^{\infty} \alpha_n^{1-2/\gamma} < \infty\,(\gamma > 2)$. Q_t is parameterized as being measurable to $\mathcal{F}_{s-1,t}$ and the exogenous variable R_t. $\boldsymbol{\varepsilon}_{s,t}$ is a $d \times 1$ matrix and $\boldsymbol{Q}_{s,t}$ is a $d \times d$ matrix. We consider a bivariate model, so d=2. Without loss of generality, $\boldsymbol{\varepsilon}_{s,t} = \boldsymbol{Q}_{s,t}^{1/2}\eta_{s,t}$, and $\eta_{s,t}$ is assumed to be IID bivariate Gaussian $\sim \mathbb{N}(0, I_d)$, and is independent of

$$\mathcal{F}_{s-1,t} = \sigma(\varepsilon_{s-1,t}, \varepsilon_{s-2,t}, \dots, \varepsilon_{s-1,t}, \varepsilon_{s-2,t}, \dots).$$

The bivariate component GARCH model in matrix form is given as

$$\boldsymbol{Q}_{s,t} = \boldsymbol{\tau}_t - \boldsymbol{A}'\tau_t\boldsymbol{A} - \boldsymbol{B}'\tau_t\boldsymbol{B} + \boldsymbol{A}'\boldsymbol{\varepsilon}_{s-1,t}, \boldsymbol{\varepsilon}'_{s-1,t}\boldsymbol{A} + \boldsymbol{B}'\boldsymbol{Q}_{s-1,t}\boldsymbol{B}, \tag{A.3.2}$$

where $\boldsymbol{\tau}_t$ is a $d \times d$ random variable (long-run exogenous matrix), $\boldsymbol{A}$ and $\boldsymbol{B}$ are $d \times d$ coefficient matrices and that are assumed to be real matrices. In particular, $A = (a_i, 0; 0, a_x)$, $B = (b_i, 0; 0, b_x)$ as in equation (2).

Remark The term $\boldsymbol{\tau}_t - \boldsymbol{A}\tau_t\boldsymbol{A} - \boldsymbol{B}'\tau_t\boldsymbol{B}$ resembles the variance-targeting constant term in Pedersen and Rahbek (2014). However, it is worth noting that the constant term is time varying in our case and is driven by low-frequency variables.

$\boldsymbol{\tau}_t$ is defined as follows: $\boldsymbol{\tau}_t = \boldsymbol{V}_{tK}\text{diag}(\boldsymbol{\omega})\boldsymbol{V'}_{tK} = \sum_{k=1}^{K} \boldsymbol{\omega}_k \boldsymbol{V}_{tk}\boldsymbol{V'}_{tk}$, where $\boldsymbol{V}_{tK}$ is a $d \times K$ matrix of (low-frequency) exogenous shocks, $\boldsymbol{\omega}$ is a $K \times 1$ vector, and $\text{diag}(\boldsymbol{\omega})$ is a $K \times K$ matrix with diagonal elements equal to $\boldsymbol{\omega}$. In our case, $d = 2$ and $\boldsymbol{\omega} = (\omega_1, \omega_2, \dots, \omega_K)^T$ is set to be $(\phi_1(\omega_1,\omega_2), \phi_2(\omega_1,\omega_2), \dots, \phi_K(\omega_1,\omega_2))^T$ defined in equation (A.1.3), and $\boldsymbol{V}_{tK} = (V_{t1}, \dots, V_{tk})$ with $\boldsymbol{V}_{tk} = (r_{i,t-k} - \mu_i, r_{x,t-k} - \mu_2)^T$.

To ensure $\boldsymbol{Q}_{s,t}$ is positive definite at each *s* and *t*, we first need to impose a condition to guarantee that $\boldsymbol{C}_{rt} \stackrel{\text{def}}{=} \boldsymbol{\tau}_t - \boldsymbol{A}'\boldsymbol{\tau}_t\boldsymbol{A} - \boldsymbol{B}'\boldsymbol{\tau}_t\boldsymbol{B}$ is positive definite. We define the matrix $\boldsymbol{C} = 1 - \boldsymbol{A}'1\boldsymbol{A} - \boldsymbol{B}'1\boldsymbol{B}$, where 1 is a 2 x 2 matrix of ones. Then $\boldsymbol{C} = (1 - a_i^2 - b_i^2, 1 - a_i a_x - b_i b_x; 1 - a_i a_x - b_i b_x, 1 - a_x^2 - b_x^2) \stackrel{\text{def}}{=} (c_i, c_{ix}; c_{ix}, c_x)$.

Proposition 1. *(Positive definiteness of $\boldsymbol{C}_{rt}$) If $c_i > 0, c_x > 0, c_i c_x - c_{ix}^2 > 0$, then the matrix $\boldsymbol{C}$ is positive definite, and $\boldsymbol{C}_{rt}$ is positive definite almost surely.*

Proof. Because $\boldsymbol{\tau}_t = \boldsymbol{V}_{tK}\text{diag}(\boldsymbol{\omega})\boldsymbol{V}'_{tK} = \sum_{k=1}^{K} \omega_k V_{tk} V'_{tk}$, we can define the matrix $\boldsymbol{\tau}_t = (\widetilde{\omega}_1^2, \widetilde{\omega}_{12};\ \widetilde{\omega}_{12}, \widetilde{\omega}_2^2)$ with $\widetilde{\omega}_1^2 = \sum_{k=1}^{K} \omega_k (r_{i,t-k} - \mu_i)^2$, $\widetilde{\omega}_2^2 = \sum_{k=1}^{K} \omega_k (r_{x,t-k} - \mu_x)^2$ and $\widetilde{\omega}_{12} = \sum_{k=1}^{K} \omega_k (r_{x,t-k} - \mu_x)(r_{i,t-k} - \mu_i)$. Now, we can write that $\boldsymbol{C}_{rt} = (c_i\widetilde{\omega}_1^2, c_{ix}\widetilde{\omega}_{12}; c_{ix}\widetilde{\omega}_{12}, c_x\widetilde{\omega}_2^2)$. We calculate the two eigenvalues of $\boldsymbol{C}_{rt}$. Letting $T_i = c_i\widetilde{\omega}_1^2 + c_x\widetilde{\omega}_2^2$ and $D_i = c_i\widetilde{\omega}_1^2 c_x\widetilde{\omega}_2^2 - c_{ix}^2\widetilde{\omega}_{12}^2$,

$$\lambda_1(C_{rt}) = \left(\frac{T_i}{2}\right) + \left(\frac{T_i^2}{4} - D_i\right)^{\frac{1}{2}}, \lambda_2(C_{rt}) = \left(\frac{T_i}{2}\right) + \left(\frac{T_i^2}{4} - D_i\right)^{\frac{1}{2}}.$$

As $c_i, c_x > 0$, $T_i > 0$ because $\frac{T_i^2}{4} - D_i = \frac{\left(c_i\widetilde{\omega}_1^2 + c_x\widetilde{\omega}_2^2\right)^2}{4} - c_i\widetilde{\omega}_1^2 c_x\widetilde{\omega}_2^2 + c_{ix}^2\widetilde{\omega}_{12}^2 = \frac{\left(c_i\widetilde{\omega}_1^2 + c_x\widetilde{\omega}_2^2\right)^2}{4} + c_{ix}^2\widetilde{\omega}_{12}^2 \geq 0$. Further, by the Cauchy-Schwarz inequality $\widetilde{\omega}_{12}^2 \leq \widetilde{\omega}_1^2\widetilde{\omega}_2^2$, we have $D_i \geq (c_i c_x - c_{ix}^2)\widetilde{\omega}_1^2\widetilde{\omega}_2^2$. Therefore, by $c_i c_x - c_{ix}^2 > 0$, $D_i \geq 0$. This leads to $\lambda_1(\boldsymbol{C}_{rt}), \lambda_2(\boldsymbol{C}_{rt}) \geq 0$. Moreover

$\lambda_2(\boldsymbol{C}_{rt}) = 0$ if and only if $\widetilde{\omega}_1 = 0$ or $\widetilde{\omega}_2 = 0$. As the weights ω are positive and the returns $r_{i,t-k} - \mu_i, r_{x,t-k} - \mu_x$ are continuously distributed, $\widetilde{\omega}_1 = 0$ and $\widetilde{\omega}_2 = 0$ with probability 0.

Remark If we would like to extend the model to a multidimensional case, this result can also be proved by considering $\boldsymbol{C}_{rt} = \boldsymbol{C} \circ \boldsymbol{r}_t$, where $\circ$ denotes the elementwise (Hadamard) product of two matrices. As $\boldsymbol{r}_t$ is a weighted sum of almost surely positive-definite matrices $\boldsymbol{V}_{tk}\boldsymbol{V}'_{tk}$ (symmetric and real), then by Weyl's inequality in matrix theory, the smallest eigenvalue of $\boldsymbol{r}_t$ is almost surely positive as well. Also, $\boldsymbol{C}$ is positive definite according to our conditions. Therefore, we have by the Schur product theory for the Hadamard product, as $\boldsymbol{c}_{rt}$ is the Hadamard product of $\boldsymbol{C}$ and $\boldsymbol{\tau}_t$, $\boldsymbol{C}_{rt}$ is almost surely positive definite.

Proposition 2. (Positive definiteness of $\boldsymbol{Q}_{s.t}$) Suppose that diagonal element $b_x \neq 0$ and $a_i > 0$, $b_i > 0$, $\boldsymbol{Q}_0$ is a positive definite matrix and conditions in proposition 1 hold, then $\boldsymbol{Q}_{s,t}$ is positive definite for all s.

Proof. If $\boldsymbol{C}_{rt}$ is almost surely positive definite, $\boldsymbol{B}'\boldsymbol{Q}_{s-1,t}\boldsymbol{B}$ is positive definite, and $\boldsymbol{A}'\boldsymbol{\varepsilon}_{s-1,t}\boldsymbol{\varepsilon}'_{s-1,t}\boldsymbol{A}$ is semipositive definite, we have that $\boldsymbol{Q}_{s,t}$ is positive definite. The positive definiteness of $\boldsymbol{C}_{rt}$ is addressed by proposition 1. Since $\boldsymbol{Q}_0$ is positive definite, $\boldsymbol{B}'\boldsymbol{Q}_0\boldsymbol{B}$ is positive definite, so the positive definiteness of $\boldsymbol{B}'\boldsymbol{Q}_{s,t}\boldsymbol{B}$ follows by iteration. As it can be seen that $\text{rank}\left(\boldsymbol{A}'\boldsymbol{\varepsilon}_{s-1,i}\boldsymbol{\varepsilon}'_{s-1,i}\boldsymbol{A}\right) = 1$, then it follows that $\boldsymbol{Q}_{s,t}$ is positive definite.

A.4. Stationarity

Here, we show the identifiability and stationarity results. We rewrite the model in vector form as

$$\text{vec}\{\boldsymbol{Q}_{s,t}\} = \left(I - \boldsymbol{A}^{'} \otimes \boldsymbol{A}^{'} - \boldsymbol{B}^{'} \otimes \boldsymbol{B}^{'}\right)\text{vec}\{\boldsymbol{r}_t\} + \boldsymbol{A}^{'} \otimes \boldsymbol{A}^{'}\ \text{vec}\left\{\boldsymbol{\varepsilon}_{s-1,t}\boldsymbol{\varepsilon}_{s-1,t}^{'}\right\} + \boldsymbol{B}^{'} \otimes \boldsymbol{B}^{'}\ \text{vec}\{\boldsymbol{Q}_{s-1,t}\}. \quad \text{(A.4.1)}$$

As we do not need all the elements of a symmetric matrix, we write (A.3.2) in terms of the vech operator. The vech form of the bivariate component model specified in equation (A.3.2) can be derived as

$$\text{vech}\{\boldsymbol{Q}_{s,t}\} = \widetilde{\boldsymbol{C}}\text{vech}\{\boldsymbol{r}_t\} + \widetilde{\boldsymbol{A}}\text{vech}\left\{\boldsymbol{\varepsilon}_{s-1,t}\boldsymbol{\varepsilon}_{s-1,t}^{'}\right\} + \widetilde{\boldsymbol{B}}\text{vech}\{\boldsymbol{Q}_{s-1,t}\}, \quad \text{(A.4.2)}$$

where the operator vech denotes the vectorized part of the lower diagonal elements of a symmetric matrix. $\widetilde{\boldsymbol{A}} = \text{diag}(a_i^2, a_i a_x, a_x^2)$, $\widetilde{\boldsymbol{B}} = \text{diag}(b_i^2, b_i b_x, b_x^2)$, and $\widetilde{\boldsymbol{C}} = \boldsymbol{I}_{d(d+1)/2} - \widetilde{\boldsymbol{A}} - \widetilde{\boldsymbol{B}}$.

Proposition 3. *(Identifiability) Suppose that* $a_i > 0$ *and* $b_i > 0$. *Then the parameters in equation (2) are identifiable.*

Proof. In equation (2), the coefficient attached to $\varepsilon_{i,s-1,t}^2$ is a_i^2, which is identified up to its sign, as is the b_i coefficient. The coefficient associated with $\varepsilon_{i,s-1,t}\varepsilon_{x,s-1,t}$ is $a_i a_x$. Since a_i is identified, a_x is identified as well. Similarly, b_x is identified. ■

The stationarity of the BEKK model is studied in Boussama, Fuchs, and Stelzer (2011). Next, we prove that we need to ensure that the spectral radius of $\widetilde{\boldsymbol{A}} + \widetilde{\boldsymbol{B}}$ is less than one for the stationarity of our model. In particular, this is equivalent to $\max(a_i^2, |a_i a_x|, a_x^2) + \max(b_i^2, |b_i b_x|, b_x^2) < 1$.

Proposition 4. *(Covariance Stationarity) If* $\max(a_i^2, |a_i a_x|, a_x^2) + \max(b_i^2, |b_i b_x|, b_x^2) < 1$, *the model is covariance stationary, and the stationary covariance* Σ *is of the form* $\text{vech}\{\Sigma\} = (I - \widetilde{\boldsymbol{A}} - \widetilde{\boldsymbol{B}})^{-1}\widetilde{\boldsymbol{C}}_{\tau_\infty}$.

The stationary solution of equation (2) is

$$\text{vech}\{\boldsymbol{Q}_{s,t}\} = \sum_{l=1}^{\infty} \widetilde{\boldsymbol{B}}^{l-1}\widetilde{\boldsymbol{A}}\text{vech}\{\boldsymbol{\varepsilon}_{s-l,i}\boldsymbol{\varepsilon}_{s-l,t}^{'}\} + \sum_{l=1}^{\infty} \widetilde{\boldsymbol{B}}^{l-1}\widetilde{\boldsymbol{C}}\text{vech}\{\boldsymbol{\tau}_{\lfloor (s-l)/m \rfloor+1,t}\}. \tag{A.4.3}$$

Proof. *As in the proof of proposition 2.7 in Engle and Kroner (1995), denote by* $\mathbb{E}_t$ *the conditional expectation* $\mathbb{E}(\cdot\,|\mathcal{F}_t)$, *conditioning on the information set* $\mathcal{F}_t$.

$$\begin{aligned}\mathbb{E}_{s-L}\text{vech}\left\{\boldsymbol{\varepsilon}_{s,t}\boldsymbol{\varepsilon}_{s,t}^{'}\right\} \\ &= \sum_{l=2}^{L}(\widetilde{\boldsymbol{A}} + \widetilde{\boldsymbol{B}})^{l-2}\widetilde{\boldsymbol{C}}\mathbb{E}_{s-L}\text{vech}\{\boldsymbol{\tau}_{\lfloor (s-l+1)/m \rfloor+1,t}\} \\ &+ (\widetilde{\boldsymbol{A}} + \widetilde{\boldsymbol{B}})^{L-1}\text{vech}\{\boldsymbol{Q}_{s-L+1}\}\end{aligned} \tag{A.4.4}$$

As $L \to \infty$, $(\widetilde{\boldsymbol{A}} + \widetilde{\boldsymbol{B}})^{L-1} \to 0$ *if* $\max(a_i^2, |a_i a_x|, a_x^2) + \max(b_i^2, |b_i b_x|, b_x^2) < 1$.

As we have assumed that $\{\boldsymbol{R}_{\text{t}}\}$ are element-wise strong mixing processes, the elements in $\boldsymbol{\tau}_t$ are the weighted sum of functions related to $\{\boldsymbol{R}_t\}$. Mixing series are measure preserving. It can be seen that for the blocks $b = 1, 2, \dots, \lfloor L/m \rfloor$, $\sum_{l=(b-1)m+1}^{bm}(\widetilde{\boldsymbol{A}} + \widetilde{\boldsymbol{B}})^{l-2}\widetilde{\boldsymbol{C}}\mathbb{E}_{s-L}\text{vech}\{\boldsymbol{\tau}_{\lfloor (s-l+1)/m \rfloor+1,t}\}$ will be mixing. Note that within block b, as $\text{vech}\{\boldsymbol{\tau}_{\lfloor (s-l+1)/m \rfloor+1,t}\}$ does not vary with respect to s, therefore the value $\mathbb{E}_{s-L}\text{vech}\{\boldsymbol{\tau}_{\lfloor (s-l+1)/m \rfloor+1,t}$ stays the same within a block. As long as $L/m \to \infty$, it is not hard to see that

$$\lim_{L\to\infty} \sum_{l=2}^{L}(\widetilde{\boldsymbol{A}} + \widetilde{\boldsymbol{B}})^{l-2}\widetilde{\boldsymbol{C}}\mathbb{E}_{s-L}\text{vech}\boldsymbol{\tau}_{\lfloor (s-l+1)/m \rfloor+1,t} \underset{p}{\to} (I - \widetilde{\boldsymbol{A}} - \widetilde{\boldsymbol{B}})^{-1}\widetilde{\boldsymbol{C}}_{\tau_\infty}, \tag{A.4.5}$$

where $\boldsymbol{\tau}_\infty = \mathbb{E}\text{vech}\boldsymbol{\tau}_{\lfloor (s-l+1)/m \rfloor+1}$.

Table 1: Summary statistics for excess returns of 30 industry portfolios

The table shows the yearly means, standard deviations, excess kurtosis, and skewness of the excess returns in percentage for the 30 industrial portfolios. The monthly sample covers the period from 1945 to 2015. The data are from Kenneth French's online data library. ***, **, and, * indicate significance at the 1%, 5%, and, 10% levels, respectively.

	Mean	St. dev	Excess kurtosis	Skewness
Food	8.610***	14.179	2.479***	−0.056
Beer	9.767***	18.303	4.868***	0.412***
Smoke	11.512***	19.633	2.870***	−0.065
Games	9.370***	23.862	2.519***	−0.186**
Books	7.641***	19.509	2.412***	−0.025
Hshld	8.179***	16.119	1.503***	−0.302***
Clths	8.400***	20.611	3.115***	−0.083
Hlth	10.165***	16.960	1.903***	0.066
Chems	8.006***	18.456	2.121***	−0.096
Txtls	8.865***	23.308	9.418***	0.492***
Cnstr	7.811***	19.776	2.431***	−0.210**
Steel	6.138***	23.904	2.335***	−0.240***
FabPr	7.751***	20.155	2.513***	−0.384***
ElcEq	9.906***	20.931	1.443***	−0.160*
Autos	7.768***	22.176	5.893***	0.209**
Carry	9.737***	21.213	1.360***	−0.260***
Mines	6.400***	23.920	2.220***	−0.173**
Coal	9.475***	32.400	2.714***	0.143*
Oil	9.232***	18.199	1.062***	−0.005
Util	7.096***	13.343	1.098***	−0.201**
Telcm	6.486***	14.695	1.837***	−0.174**
Servs	9.935***	21.490	1.514***	−0.151*
BusEq	9.681***	22.163	2.051***	−0.311***
Paper	8.612***	17.269	2.098***	−0.169**
Trans	7.584***	19.354	1.279***	−0.200**
Whlsl	8.294***	18.602	2.283***	−0.298***
Rtail	9.001***	17.528	2.409***	−0.222***
Meals	10.068***	20.450	2.478***	−0.405***
Fin	8.523***	17.779	1.808***	−0.405***
Other	5.959***	19.152	1.752***	−0.388***

Table 2: Parameter estimates of the component GARCH model

The table shows the means and standard deviations of the parameter estimates from the bivariate component GARCH model specified in equations (1)-(3) for the monthly-weekly (M-W) and monthly-monthly (M-M) frequencies. The estimations are based on the 30 industry portfolios and the market, small-minus-big (SMB), and high-minus-low (HML) factors. The sample covers the period from 1945 to 2015.

		γ_i		γ_x		a_i		a_x		b_i		b_x	
		Mean	Std dev	Mean	Std dev	Mean	Std dev	Mean	Std dev	Mean	Std dev	Mean	Std dev
M-W	Market	0.208	0.036	0.190	0.011	0.241	0.015	0.265	0.016	0.963	0.005	0.954	0.007
	SMB	0.201	0.038	0.012	0.005	0.290	0.047	0.262	0.015	0.942	0.021	0.949	0.008
	HML	0.209	0.032	0.062	0.004	0.251	0.023	0.272	0.006	0.959	0.008	0.958	0.002
M-M	Market	0.731	0.146	0.662	0.057	0.282	0.020	0.284	0.026	0.941	0.014	0.943	0.011
	SMB	0.693	0.150	0.008	0.031	0.284	0.030	0.319	0.021	0.939	0.020	0.880	0.037
	HML	0.803	0.192	0.305	0.057	0.289	0.028	0.313	0.016	0.927	0.028	0.939	0.014

Table 3: Cross-sectional dispersion of betas

The table shows the time-series means of the cross-sectional dispersion of the estimated total, long- and short-run betas, as well as the coefficients and *t*-values from univariate regressions of the dispersion coefficients on the NBER recession indicator. ***, **, and, * indicate significance at the 1%, 5%, and, 10% levels, respectively.

	Total			Long			Short		
	Market	SMB	HML	Market	SMB	HML	Market	SMB	HML
Mean dispersion	0.286***	0.434***	0.523***	0.271***	0.413***	0.478***	0.115***	0.180***	0.233***
Intercept	0.282***	0.431***	0.519***	0.269***	0.414***	0.480***	0.113***	0.174***	0.229***
t-value	74.553	112.830	113.111	79.995	158.618	124.801	72.901	68.935	65.254
Recession	0.031***	0.023**	0.033***	0.009	-0.010	-0.017	0.014***	0.040***	0.024**
t-value	3.084	2.211	2.730	0.981	-1.411	-1.606	3.299	5.878	2.519

Table 4: Risk premia

Panel A shows the risk premia estimated using unconditional betas, betas from a conventional bivariate GARCH model for weekly and monthly frequencies, and total betas from the bivariate component GARCH model for the monthly-weekly (M-W) and monthly-monthly (M-M) frequencies. Panel B shows the risk premia for long- and short-run betas from the bivariate component GARCH model with M-W and M-M frequencies. Panel C shows the risk premia estimated using long- and short-run betas from the bivariate component GARCH model for the M-M frequency using four different data sets as test assets. For each factor, market, small-minus-big (SMB), and high-minus-low (HML), the factor risk premium is estimated as the average of the time-series of the estimated coefficients obtained from the weekly (monthly) repeated multivariate cross-sectional regressions of the weekly (monthly) returns of the 30 industry portfolios on their factor betas. The estimations cover the period from 1950 to 2015. ***, **, and, * indicate significance at the 1%, 5%, and, 10% levels, respectively.

Panel A. Risk premia for total betas

		Intercept		Market		SMB		HML	
		Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val
Unconditional	*Weekly*	0.175***	4.11	−0.010	−0.12	0.034	0.90	0.026	0.65
	Monthly	0.858***	4.69	−0.464	−1.55	0.490***	3.01	0.114	0.71
Bivariate	*Weekly*	0.147***	4.21	0.001	0.02	−0.008	−0.27	−0.040	−1.10
GARCH	*Monthly*	0.643***	3.86	0.039	0.14	0.166	0.93	0.007	0.04
Component	*M-W*	0.156***	4.24	0.019	0.26	0.005	0.16	−0.013	−0.34
GARCH	*M-M*	0.661***	4.08	−0.020	−0.07	0.463***	2.64	0.217	1.23

Panel B. Risk premia for component GARCH betas

			Long						Short					
	Intercept		Market		SMB		HML		Market		SMB		HML	
	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val
M-W	0.150***	3.32	0.018	0.20	0.013	0.30	0.021	0.42	0.050	0.58	0.039	0.90	−0.019	−0.43
M-M	0.844***	4.79	−0.275	−0.89	0.645***	3.01	0.321	1.40	0.818**	2.00	0.669**	1.87	0.402	1.29

Panel C: Risk premia variations across data sets

			Long						Short					
	Intercept		Market		SMB		HML		Market		SMB		HML	
	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val
25 BM-Size	0.876***	4.29	−0.306	−1.05	0.371***	2.55	0.356**	2.01	0.318*	1.76	0.054	0.32	0.102	0.65
30 Industries	0.844***	4.79	−0.275	−0.89	0.645***	3.01	0.321	1.40	0.818**	2.00	0.669**	1.87	0.402	1.29
49 industries	0.467***	3.35	0.202	0.83	0.350**	2.41	0.284*	1.72	0.690***	2.60	0.470**	2.31	0.105	0.61

Table 5: Recession and risk premia

The table shows the risk premia for the entire sample and for NBER expansions and recessions. Panel A shows the risk premia using monthly unconditional betas, monthly betas from the conventional bivariate GARCH model, and the total betas from the bivariate component GARCH model with the monthly-monthly frequency. Panel B shows the risk premia associated with the short- and long-run betas from the component GARCH model with the monthly-monthly (M-M) frequency. For each factor, market, small-minus-big (SMB), and high-minus-low (HML), the factor risk premium is estimated as the average of the time-series of the estimated coefficients obtained from the monthly repeated multivariate cross-sectional regressions of the monthly returns of the 30 industry portfolios on their factor betas. The estimations cover the period from 1950 to 2015. ***, **, and, * indicate significance at the 1%, 5%, and, 10% levels, respectively.

Panel A. Risk premia for total betas

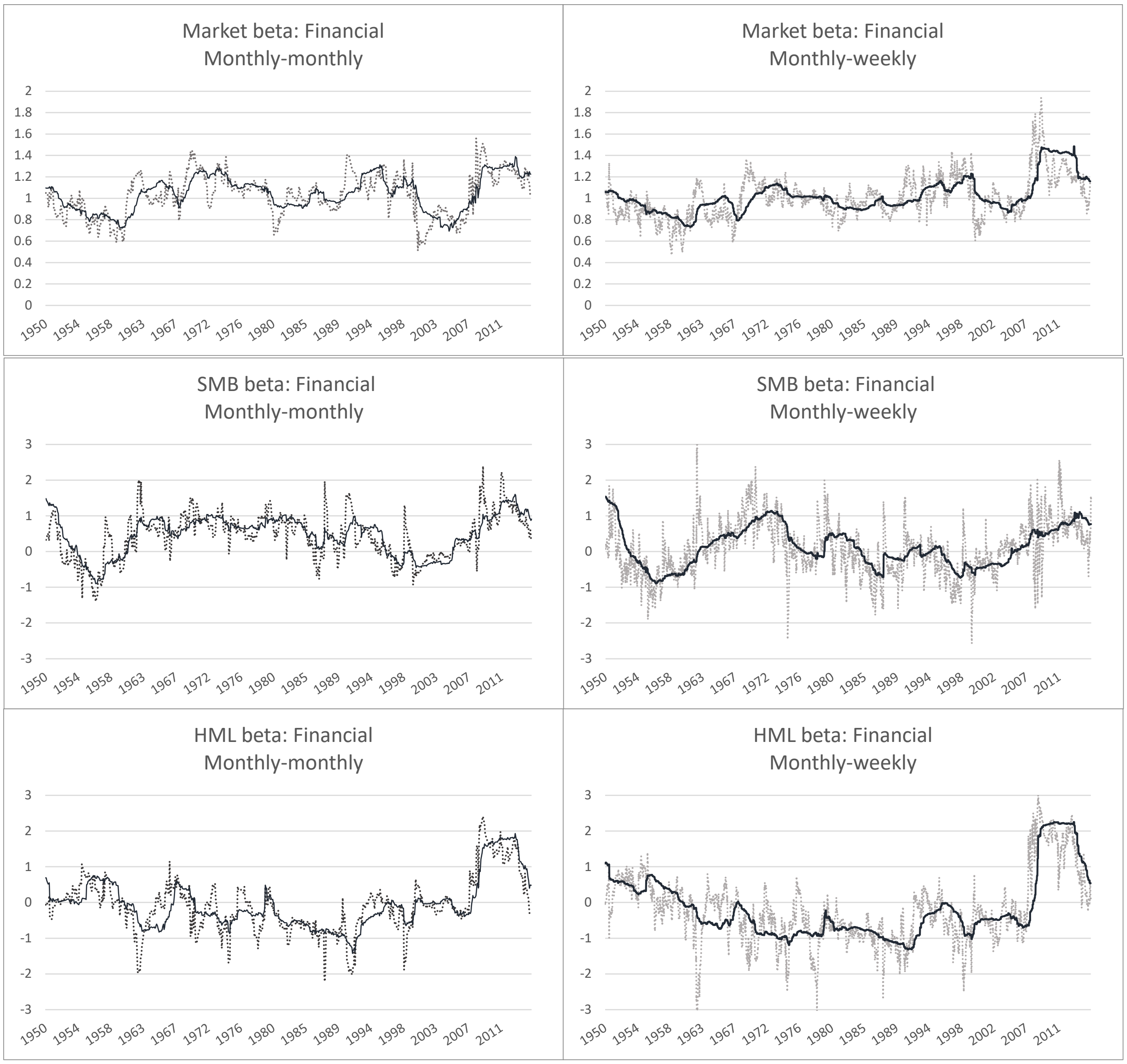
		Intercept		Market		SMB		HML	
	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val	Coef.
Unconditional	*Entire*	0.858***	4.69	−0.464	−1.55	0.490***	3.01	0.114	0.71
	Expansion	0.834***	4.378	−0.159	−0.516	0.427**	2.493	0.231	1.382
	Recession	1.004**	2.128	−2.335***	−3.070	0.876**	2.063	−0.604	−1.459
Bivariate GARCH	*Entire*	0.643***	3.86	0.039	0.14	0.166	0.93	0.007	0.04
	Expansion	0.557***	3.231	0.294	1.054	0.132	0.692	0.119	0.634
	Recession	1.174***	2.751	−1.522**	−2.206	0.375	0.795	−0.681	−1.468
Component GARCH	*Entire*	0.661***	4.08	−0.020	−0.07	0.463***	2.64	0.217	1.23
	Expansion	0.651***	3.876	0.134	0.501	0.433**	2.404	0.316*	1.782
	Recession	0.720*	1.730	−0.964	−1.452	0.647	1.451	−0.389	−0.887

Panel B. Risk premia for component GARCH betas

			Long						Short					
	Intercept		Market		SMB		HML		Market		SMB		HML	
	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val	Coef.	*t*-val
Entire	0.844***	4.79	−0.275	−0.89	0.645***	3.01	0.321	1.40	0.818**	2.00	0.669**	1.87	0.402	1.29
Expansion	0.777***	4.253	−0.011	−0.033	0.668***	3.148	0.443*	1.935	1.025**	2.369	0.753**	2.073	0.490*	1.642
Recession	1.259***	2.783	−1.897**	−2.395	0.507	0.966	−0.431	−0.760	−0.450	−0.420	0.156	0.174	−0.137	−0.185

Figure 1: Factor betas estimated by the component GARCH model

The graphs plot the total (dotted line) and long-run (solid line) market, small-minus-big (SMB), and high-minus-low (HML) betas estimated by the component GARCH model at monthly-monthly and monthly-weekly frequency for the financial industry as the test portfolio. The estimated betas are for the period from 1950 to 2015.



IRTG 1792 Discussion Paper Series 2020

For a complete list of Discussion Papers published, please visit http://irtg1792.hu-berlin.de.

001 "Estimation and Determinants of Chinese Banks' Total Factor Efficiency: A New Vision Based on Unbalanced Development of Chinese Banks and Their Overall Risk" by Shiyi Chen, Wolfgang K. Härdle, Li Wang, January 2020.

002 "Service Data Analytics and Business Intelligence" by Desheng Dang Wu, Wolfgang Karl Härdle, January 2020.

003 "Structured climate financing: valuation of CDOs on inhomogeneous asset pools" by Natalie Packham, February 2020.

004 "Factorisable Multitask Quantile Regression" by Shih-Kang Chao, Wolfgang K. Härdle, Ming Yuan, February 2020.

005 "Targeting Cutsomers Under Response-Dependent Costs" by Johannes Haupt, Stefan Lessmann, March 2020.

006 "Forex exchange rate forecasting using deep recurrent neural networks" by Alexander Jakob Dautel, Wolfgang Karl Härdle, Stefan Lessmann, Hsin-Vonn Seow, March 2020.

007 "Deep Learning application for fraud detection in financial statements" by Patricia Craja, Alisa Kim, Stefan Lessmann, May 2020.

008 "Simultaneous Inference of the Partially Linear Model with a Multivariate Unknown Function" by Kun Ho Kim, Shih-Kang Chao, Wolfgang K. Härdle, May 2020.

009 "CRIX an Index for cryptocurrencies" by Simon Trimborn, Wolfgang Karl Härdle, May 2020.

010 "Kernel Estimation: the Equivalent Spline Smoothing Method" by Wolfgang K. Härdle, Michael Nussbaum, May 2020.

011 "The Effect of Control Measures on COVID-19 Transmission and Work Resumption: International Evidence" by Lina Meng, Yinggang Zhou, Ruige Zhang, Zhen Ye, Senmao Xia, Giovanni Cerulli, Carter Casady, Wolfgang K. Härdle, May 2020.

012 "On Cointegration and Cryptocurrency Dynamics" by Georg Keilbar, Yanfen Zhang, May 2020.

013 "A Machine Learning Based Regulatory Risk Index for Cryptocurrencies" by Xinwen Ni, Wolfgang Karl Härdle, Taojun Xie, August 2020.

014 "Cross-Fitting and Averaging for Machine Learning Estimation of Heterogeneous Treatment Effects" by Daniel Jacob, August 2020.

015 "Tail-risk protection: Machine Learning meets modern Econometrics" by Bruno Spilak, Wolfgang Karl Härdle, October 2020.

016 "A data-driven P-spline smoother and the P-Spline-GARCH models" by Yuanhua Feng, Wolfgang Karl Härdle, October 2020.

017 "Using generalized estimating equations to estimate nonlinear models with spatial data" by Cuicui Lu, Weining Wang, Jeffrey M. Wooldridge, October 2020.

018 "A supreme test for periodic explosive GARCH" by Stefan Richter, Weining Wang, Wei Biao Wu, October 2020.

019 "Inference of breakpoints in high-dimensional time series" by Likai Chen, Weining Wang, Wei Biao Wu, October 2020.

IRTG 1792, Spandauer Strasse 1, D-10178 Berlin
http://irtg1792.hu-berlin.de

This research was supported by the Deutsche Forschungsgemeinschaft through the IRTG 1792.

IRTG 1792 Discussion Paper Series 2020

For a complete list of Discussion Papers published, please visit http://irtg1792.hu-berlin.de.

020 "Long- and Short-Run Components of Factor Betas: Implications for Stock Pricing" by Hossein Asgharian, Charlotte Christiansen, Ai Jun Hou, Weining Wang, October 2020.

IRTG 1792, Spandauer Strasse 1, D-10178 Berlin
http://irtg1792.hu-berlin.de

This research was supported by the Deutsche Forschungsgemeinschaft through the IRTG 1792.