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Valuing cryptocurrencies: Three easy pieces

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Valuing cryptocurrencies:
Three easy pieces

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Abstract

This paper surveys the capacity of simple macroeconomic models – "three easy pieces" – to account for persistent and positive valuations of privately issued assets based on the blockchain. Each of these three models – transactions demand for a means of payment, consumption-based capital asset pricing, and search and matching – highlights important aspects of digital payments. The mutual interference of these jointly produced features may impede widespread use of cryptocurrencies until technological innovations have been developed to separate them.

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1 Introduction

"Strange though I think it, I must ratify it."[1]

The publication of the Nakamoto protocol describing a permissionless, proof-of-work ledger currency based on the blockchain was hardly noticed in 2009, but heralded one of the most profound revolutions in transactions technology since the invention of negotiable checks, and possibly since the appearance of paper money in 11th century Song dynasty China or in 17th century Europe. A positive market valuation for private digital assets seems beyond doubt – at yearend 2020 at $800b, by mid-April more than $2t – yet is one that fluctuates significantly, falling to below 1.5t in June 2021. Positive valuation of cryptocurrencies is hardly guaranteed; of more than four and a half thousand tracked by coinmarketcap.com and more than 8000 issued in total, only about 2350 had positive market value.

What is this innovation really worth? What can macroeconomics and macroeconomic models say about cryptocurrency (CC) valuation? As a medium of exchange, they derive value in terms of goods over which they command and the convenience in transactions they provide. As "outside assets," they are a vehicle for wealth-holding, yet do not represent claims on other economic agents, central banks or governments but rather tokens in a collectively monitored record-keeping ledger system.[2] They are not perfectly correlated with each other, nor with returns on other asset classes. Like other financial assets, they can help insure against consumption risk. Finally, cryptocurrencies solve the double coincidence of wants, both over time and across states of the world, just as David Hume saw money as "none of the wheels of commerce" but rather as a facilitator of trade. Surges of new coin issuance and positive, rapidly growing valuation reflect a discovery process in which novel transactions technologies emerge that displace or dominate others. The economics of CC entry and market valuation are underresearched dimensions of digital finance that now involve complex forms of future payment in the form of "smart contracts" or conditional payments (e.g. Weber and Staples, 2021). The issuance of new CC embodies matching of transactions technologies with use cases in a rapidly evolving market with both high fixed development costs and signficant rewards for success.

[1]The Emperor in Goethe's *Faust* (Part II) commenting on the introduction of unbacked paper money and its effect on economic activity in the empire (*"So sehr michs wundert, muss ichs gelten lassen."* Transl. Louis MacNeice).

[2]For more details on this useful distinction see Brunnermeier et al. (2019).

This paper surveys the capacity of simple macroeconomic models – "three easy pieces" – to account for persistent, positive valuations of privately issued currencies based on the blockchain: Transactions demand for a means of payment, the consumption-based capital asset pricing model, and search and matching. While each highlights an important aspect of digital payments. none is sufficiently rich to capture all dimensions of cryptocurrency valuation adequately. I conclude that because these jointly produced features interfere with each other, a lasting expansion of the use and utility of cryptocurrencies is likely to require new technological innovations that can separate these features.

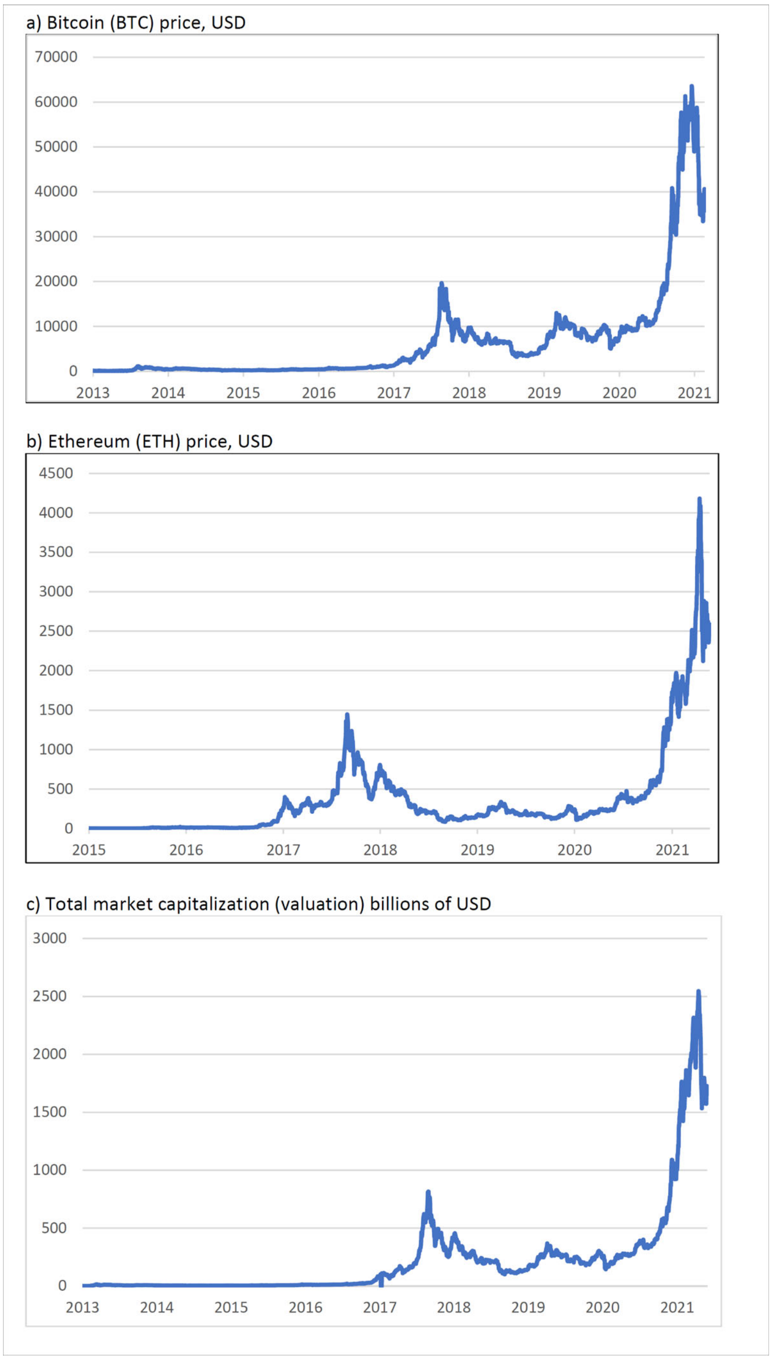
2 A few facts

Since the opening of the Mount Gox exchange on July 18, 2010, the dollar price of one Bitcoin (BTC) rose from \$0.07 to \$20,000 in mid-December 2017, after which it crashed to less than \$3500 in the subsequent year. In the following three years it recovered spectacularly, reaching \$45,000 at yearend 2020 and rising above \$60,000 in mid April 2021. In that same month, the total market value of all crytocurrencies reached \$2,3t (6% and 3% of the world's narrow money supply and GDP respectively); at 5% of the total capitalization of the largest publicly traded US firms and more than half of world's financial sector, cryptocurrencies (CC) are now an asset class, a vehicle for finance, and an alternative to the issue of traded equity.[3] Figure 1 plots the price of BTC, its nearest competitor Ethereum (ETH), as well as market capitalization (total market value) since 2013. Despite significant fluctuations, the persistent, positive valuation of this digital asset as well as several thousand others is prima facie evidence that the market can produce and accept private money (Hayek 1976, 1990). Following Jevons (1875), it serves to various degrees as a medium of exchange (MOE), unit of account (UOA), store of value (SOV), and standard of deferred payment (SDP).

CC functionality as money is predicated on convertibility into goods and services without loss. Because thick markets with high volume offer ready trading opportunities, current prices reflect expectations of future illiquidity, loss of acceptance, or value impairment; the resulting volatility is likely to

[3]US courts and regulators are currently deciding if cryptocurrency issue is equivalent to an IPO. The vaunted hedge fund Blackrock has applied for SEC approval for Bitcoin investments, considering cryptocurrencies a valid component of a well-diversified portfolio, (https://www.cnbc.com/2021/02/17/blackrock-has-started-to-dabble-in-bitcoin-says-rick-rieder.html).

Figure 1: Bitcoin and Ethereum prices (USD) and total CC market capitalization (bn.USD), 2013-2021 (%)



Source: https://www.coingecko.com

Table 1: Average daily returns and key correlations, selected CC and major stock indexes 2015-2021

Oct 2015-Mar 2021					**Correlation coefficient with:**		
	Mean	*Std. dev.*	*Skewness*	*Kurtosis*	*Dow Jones*	*Wilshire*	*CRIX*
Bitcoin	0.35	3.98	-0.02	8.59	-0.04	-0.04	0.93
Ethereum	0.57	6.02	0.37	5.12	-0.03	-0.03	0.66
Litecoin	0.36	5.75	1.80	18.03	-0.03	-0.03	0.68
Ripple	0.47	7.56	5.26	76.22	0.00	0.01	0.42
Bitcoin Cash	0.27	8.30	4.14	67.10	-0.04	-0.04	0.57
Tether	0.01	1.06	-4.17	196.81	0.01	0.01	-0.02
Dow Jones	0.05	1.22	-0.70	24.52	1.00	0.97	-0.05
Wilshire	0.06	1.19	-0.92	20.75	0.97	1.00	-0.05
CRIX	0.37	3.97	-0.44	7.27	-0.05	-0.05	1.00
					**n=1977*		
Oct 2015-Dec 2017							
	Mean	*Std. dev.*	*Skewness*	*Kurtosis*	*Dow Jones*	*Wilshire*	*CRIX*
Bitcoin	0.58	3.97	0.62	9.20	-0.02	-0.01	0.91
Ethereum	1.09	7.05	0.68	3.49	-0.02	-0.01	0.47
Litecoin	0.71	6.21	3.13	26.60	0.00	0.00	0.50
Ripple	1.07	9.25	6.36	76.37	0.01	0.01	0.28
Bitcoin Cash	1.93	16.87	3.79	30.24	-0.13	-0.04	0.15
Tether	0.00	0.67	0.62	24.27	0.00	0.01	-0.02
Dow Jones	0.07	0.66	-0.36	3.02	1.00	0.95	-0.03
Wilshire	0.06	0.70	-0.37	2.96	0.95	1.00	-0.02
CRIX	0.65	3.72	0.01	5.80	-0.03	-0.02	1.00
					**n=822*		
Jan 2018-Mar 2021							
	Mean	*Std. dev.*	*Skewness*	*Kurtosis*	*Dow Jones*	*Wilshire*	*CRIX*
Bitcoin	0.18	3.97	-0.47	8.08	-0.06	-0.06	0.95
Ethereum	0.19	5.13	-0.44	6.15	-0.04	-0.04	0.86
Litecoin	0.12	5.39	0.29	5.88	-0.05	-0.04	0.82
Ripple	0.04	6.06	1.16	14.91	-0.01	0.01	0.57
Bitcoin Cash	0.06	6.36	0.67	9.72	-0.05	-0.05	0.78
Tether	0.01	1.26	-4.27	164.58	0.01	0.01	-0.02
Dow Jones	0.04	1.50	-0.60	17.63	1.00	0.97	-0.06
Wilshire	0.06	1.43	-0.86	15.82	0.97	1.00	-0.06
CRIX	0.17	4.12	-0.66	7.79	-0.06	-0.06	1.00
					**n=1154*		

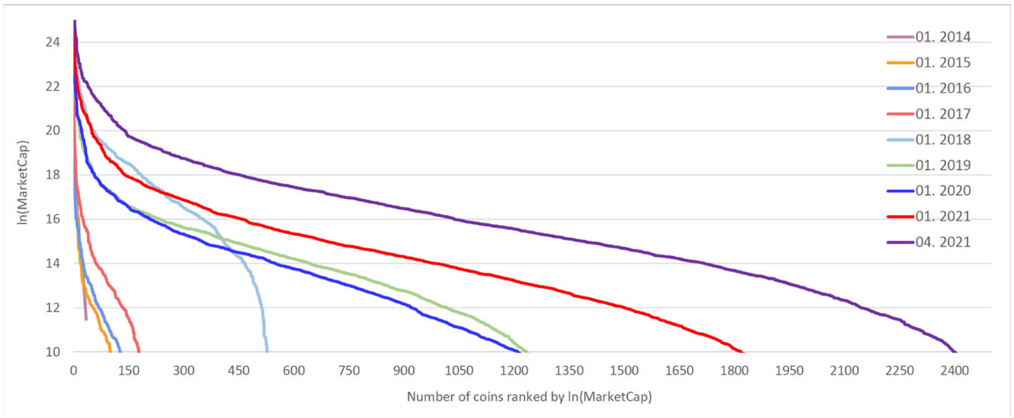
Notes : BTC Cash begins in 2017. CRIX is a dynamic index of cryptocurrency prices: see https://thecrix.de/

Source: https://www.coingecko.com

attenuate CC value as a MOE. Table 1 displays key statistics on mean daily returns in US dollars as well as summary statistics of the same including correlation coefficients with three indexes: Dow Jones 500, the S&P 1200, and the CRIX index (thecrix.de) in the period 2015-2021. While CC yields (capital gains in terms of fiat currencies) have been strikingly high, they have been accompanied by outsize variability.

The bulk of CC market value is concentrated in a few issues; at the end of April 2021, the top five coins accounted for about 74%, and the top 10 for 84% of total market capitalization. Bitcoin alone accounts for less than 50%, down from more than 2/3 in 2019. At the end of June 2021, there were 4819 cryptocurrencies listed by coinmarketcap.com; of these, 2520 had positive market value (coinmarketcap.com, accessed 28.06.2021). Figure 2 plots the log market capitalization by descending order from a scrapable website (coingekko.com). Most theoretical models of valuation would predict

Figure 2. Market capitalization of CC by rank, 2014-2021



Source: https://www.coingecko.com

that fungibility of CC will affect their valuation, as will be explored in Section 4.

3 Easy piece #1: Transactions medium

Simple models can deliver the most important insights, so it makes sense to start with a basic monetary model that can predict positive CC holdings despite dominance by interest-bearing alternatives. The first "easy piece" follows Blanchard (1979), Flood and Garber (1980), Sargent (1979, Ch.6), Blanchard and Fischer (1989, Ch.5) or Alogoskoufis (2019, Ch.22), a discrete-time variant of Cagan's (1956) classic workhorse of monetary economics.[4] In this partial equilibrium model, a single cryptocurrency is part of a broader demand for liquidity, mostly satisfied by banknotes and sight deposits at banks.

[4]Cagan (1956) studied the implications of the demand for money under conditions of a rapidly changing price level. Samuelson (1958) showed that worthless fiat money has positive value as transactions medium between saving and dissaving generations. Blanchard (1979) explored forward and backward solutions of difference equations implied by an arbitrage condition. Biais et al. (2018) model CC use in the context of an overlapping generations model with transactions benefits and costs.

3.1 CC as transactions medium: Demand

For simplicity, consider a single, circulating CC (or a set of perfectly substitutable ones) in exogenous supply that serve as a perfect substitute for fiat money in the form of cash and checking accounts at banks for a small but fixed subset of transactions. This demand derives either directly from utility maximization (money in the utility function) or indirectly via a cash-in-advance constraint.[5] The single CC offers liquidity services for normal payments as well as a transactions medium for capital flight, illegal or covert activity, or tax evasion. Start with demand for total liquidity in the absence of CC, say for cash and checkable bank accounts denominated in Euros, following Blanchard and Fischer (1989):

$$m_t^D - p_t = x_t - \eta(r_t + E_t p_{t+1} - p_t) \tag{1}$$

where m_t^D, p_t and x_t denote the logarithms of the stock of money, the price level and real transactions demand in period t, respectively. The constant η is the semi-elasticity of money demand with respect to the opportunity cost of holding it, the sum of real interest rate r_t and expected inflation $E_t p_{t+1} - p_t$; x_t is normalized to have a unit elastic effect on real money demand. The left-hand side $m_t^D - p_t$ is the logarithm of money demand expressed in terms of goods and services. Monetary neutrality is imposed a priori, as a proportional increase in nominal variables m_t, p_t, and $E_t p_{t+1}$ leaves the *real demand* for the transactions medium unchanged.

The demand for the alternative medium of exchange, say for Bitcoin (BTC), derives from some convenience and ease in use relative to standard fiat money. For simplicity, it yields all transactions benefits of conventional money up to a deterministic or stochastic loss of convenience yield $\tau_t \geq 0$. Agents hold a small, constant fraction Φ of total real transactions balances M_t/P_t in the form of BTC as $P_t^C C_t^D$, where P_t^C and C_t^D are price of BTC in terms of dominant monetary unit (Euros) and quantity of BTC demanded.[6]

[5] Sargent (1979) and Blanchard (1979) rationalized this as a loglinearization of Cagan's (1956) demand for money or an equilibrium condition in Samuelson's (1958) model of fiat money. See Brunnermeier and Niepelt (2019) for a more modern formulation.

[6] Consider a general model in which demand for liquidity L^D is a positive function of economic activity, and a negative function of the nominal interest rate. Liquidity itself is an Armington aggregator of conventional money M (checkable and savings deposits, zero interest money market funds accounts) and BTC (C):

$$L^D = \left[\alpha M^{\frac{1}{1-\rho}} + (1-\alpha) C^{\frac{1}{1-\rho}}\right]^{1-\rho}$$

where $1 > \alpha > 0$, and $\rho \geq 1$ is meant to capture substitutability of the two forms of

Think of τ_t as a periodic, recurrent, time-varying, and possibly stochastic impediment to liquidity, bid-ask spread, tax, or a transactions fee that reduces the "convenience yield" or services giving rise to BTC demand, rather than a permanent loss of value *per se*, and assume that τ_t is "not too large" so $\ln(1-\tau_t) \approx -\tau_t$. In a world without other frictions, the demand for the CC in natural logarithms satisfies:

$$c_t^D + p_t^C = \phi + m_t - p_t$$

if it provides the full set of liquidity services; for $\tau_t > 0$ it delivers $(1-\tau_t)$ of these services, or in logarithms $\ln\left[(1-\tau_t)X_t\right] \approx x_t - \tau_t$. Under these conditions, the derived demand for CC obeys

$$c_t^D + p_t^C = \phi + x_t - \tau_t - \eta\left[r_t + E_t p_{t+1} - p_t - \left(E_t p_{t+1}^C - p_t^C\right)\right] \tag{2}$$

and depends positively on net liquidity services $x_t - \tau_t$ and negatively on the expected real rate of return and the expected rate of fiat money inflation, adjusted by expected rate of appreciation of CC in the numeraire currency.

3.2 CC supply: An algorithmic rule

The logarithm of CC supply follows a deterministic and exogenous autoregressive rule:

$$c_t = g(x_t)(\overline{c} - c_{t-1})$$

where $g(x_t)$ determines the expansion path of CC supply, as a function of the distance of last period's value from the upper bound $(\overline{c} - c_{t-1})$, where g is a continuous, nondecreasing, and weakly concave function of transactions fundamentals x_t: $1 > g > 0$, $g' \geq 0$, $g'' \leq 0$, with nonzero derivatives a result of mining or information from an external oracle. This supply rule implies an algorithmic commitment to a maximal money supply as in Fernández-Villaverde and Sanches (2019). CC supply growth then follows

$$\Delta c_t = \left[g(x_t) - g(x_{t-1})\right](\overline{c} - c_{t-1}) - g(x_{t-1})\Delta c_{t-1},$$

meaning that as long as $g(x_t)$ is difference-stationary, the log of CC supply is difference-stationary. I will assume the following simple deterministic rule $g(x_t) = \gamma$ with $0 < \gamma < 1$:

$$\begin{aligned} c_{t+1} &= c_t + \gamma(\overline{c} - c_t) \\ &= (1-\gamma)\, c_t + \gamma\overline{c}. \end{aligned}$$

money in providing liquidity. The case in the text corresponds to $\rho \to \infty$ and α close to 1, implying $\Phi = \alpha/(1-\alpha)$.

This rule resembles the one followed by Bitcoin (see e.g., Tschorsch and Scheuermann 2021), with an upper limit of 21 million BTC and a decreasing rate of supply growth that is more dependent on calendar time rather than mining activity.

3.3 Equilibrium

Market equilibrium is defined as equality of demand and supply: $c_t^D = c_t^S = c_t \quad \forall t$, so combined with an exogenous supply process and money demand, p_t^C is a linear combination of today's supply (exogenous), real demand, and tomorrow's expected price level, a first-order expectational difference equation in p_t^C:

$$p_t^C = \frac{1}{1+\eta}\left[\phi + x_t - \tau_t - \eta(r_t + E_t\pi_{t+1}) - c_t\right] + \frac{\eta}{1+\eta}E_t p_{t+1}^C, \tag{3}$$

where $\pi_{t+1} \equiv p_{t+1} - p_t$ is the exogenous inflation rate in Euros (purchasing power in terms of goods and services). The price of Bitcoin depends only on current and future expected values of CC demand and supply determinants, as well as expected future price. As agents use the model to form expectations, (3) can be iterated forward recursively to period $t+T$, applying the law of iterated expectations:

$$\begin{aligned} p_t^C = \frac{1}{1+\eta}\sum_{i=0}^{T}\left(\frac{\eta}{1+\eta}\right)^i E_t\left[\phi + x_{t+i} - \tau_{t+i} - \eta(r_{t+i} + E_t\pi_{t+i+1}) - c_{t+i}\right] \\ + \frac{1}{1+\eta}\left(\frac{\eta}{1+\eta}\right)^{T+1} E_t p_{t+T+1}^C. \end{aligned} \tag{4}$$

The price of CC in terms of the fiat currency is a weighted average of "fundamentals" that depends only on current and expected determinants of supply and demand taken to period $T+1$, and the expectation of an undetermined future price in period $T+1$.[7] Imposing $\lim_{T\to\infty}\left(\frac{\eta}{1+\eta}\right)^{T+1} E_t p_{t+T+1}^C = 0$ on (4)

[7]Technically, I impose $\lim_{T\to\infty}\sum_{i=0}^{T}\left(\frac{\eta}{1+\eta}\right)^i E_t\left[\phi + x_{t+i} - \tau_{t+i} - \eta(r_{t+i} + E_t\pi_{t+i+1}) - c_{t+i}\right] < \infty$, which would hold if the variables are covariance stationary or cointegrated of order 1 with cointegrating vector implied by the model.

yields the *fundamental* solution

$$p_t^C = \frac{1}{1+\eta}\sum_{i=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^i E_t\left[\phi + x_{t+i} - \tau_{t+i} - \eta(r_{t+i} + E_t\pi_{t+i+1}) - c_{t+i}\right] \tag{5}$$

that satisfies (3). The sum of two solutions to a difference equation is also a solution, so any deviation from the fundamental price path (5) that also solves (3) can be added, yielding the complete set of solutions

$$\begin{aligned} p_t^C &= \frac{1}{1+\eta}\sum_{i=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^i E_t\left[\phi + x_{t+i} - \tau_{t+i} - \eta(r_{t+i} + E_t\pi_{t+i+1})\right] \\ &\quad -\frac{1}{1+\eta}\sum_{i=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^i E_t c_{t+i} \\ &\quad + b_t \end{aligned} \tag{6}$$

where b_t grows at rate η^{-1} per period in expectation, i.e. obeys

$$E_t b_{t+1} = \left(1 + \frac{1}{\eta}\right) b_t. \tag{7}$$

Equation (6) decomposes the CC price into three parts:

1) **a demand fundamental** driven by present and expected future convenience or other liquidity services net of opportunity cost in each period $t \geq 0$;

2) **a supply fundamental** given by the algorithmic, preprogrammed trajectory of CC issuance ($E_t\left[c_{t+i}\right]$) for $i \geq 0$);

3) **a bubble component** that obeys $E_t b_{t+1} = \left(1 + \frac{1}{\eta}\right) b_t$, which is *defined* as the deviation of the observed price from its fundamental, driven solely by expectations of future appreciation and orthogonal to rationally expected demand and supply fundamentals.

The sum of the first two components comprises the **fundamental solution**, the expectation of a geometrically weighted sum of future determinants of BTC demand and supply, where the discount factor is an increasing function of η. If $b_t > 0$, the third component represents the **nonfundamental solution** and depends solely on b_t, and grows at expected rate $g \equiv \frac{1}{\eta}$.

3.4 Analysis: Fundamental component

I focus first on the fundamental determinants of p_t^C and set $b_t = 0$, for $t = 0, 1, ...$ Given an assumed supply algorithm (3), the fundamental solution of the BTC price is[8]

$$\begin{aligned} p_t^C &= \frac{1}{1+\eta}\sum_{i=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^i E_t\left[\phi + x_{t+i} - \tau_{t+i} - \eta(r_{t+i} + E_t\pi_{t+i+1})\right] \\ &\quad -\overline{c} \ + \frac{1}{1+\eta\gamma}\left(\overline{c} - c_t\right). \end{aligned} \tag{8}$$

The model's central prediction is a dominant role for expected *future*, rather than current, fundamentals. Past fundamentals matter only to the extent they help predict future fundamentals. The greater the interest elasticity of BTC demand, the greater the relative role of future fundamentals in determining today's price. As supply is exogenous, the model attributes a stubbornly high BTC price today to persistent expectations of future net demand fundamentals. It depends *positively* on present and expected future transactions services x_{t+i}, *negatively* on transactions impediments τ_{t+i}, and *negatively* on the path of future inflation π_{t+i} and real interest rates r_{t+i}. From the supply perspective, CC valuation depends *negatively* on the upper bound of coin volume ($\overline{c}$) but *positively* on the remaining supply gap $\overline{c} - c_t$. Since the former is fixed and the latter is always positive, the cryptocurrency price should be declining over time, ceteris paribus, in anticipation of rising future supply implied by the supply gap $\overline{c} - c_t$. While the former is hard-wired into the price, the latter will depend on transpired calendar time since the launching of the protocol.

Second, the algorithmic nature of CC sharply limits the role of expected future supply fluctuations on current value (one exception would be a fork, a rare event). To date, the most prevalent protocols increase expected hash time in periods of high demand and automatically limit the supply response to high mining activity. This a priori knowledge represents an exclusion restriction of interest for econometric investigation, because c_t and $\overline{c}$ are sufficient statistics for CC supply. More generally forms of $g(x_t)$ with potential dependence on state variables and in particular, the state of the economy would weaken this conclusion.

[8] See Appendix for details.

3.5 Analysis: Bubble component

The trajectories in Figure 1 are suggestive of asset bubbles seen elsewhere in the past half-millennium. Santos and Woodford (1997) and Brunnermeier (2008) define bubbles as a deviation of asset price from a level implied by a forward-looking arbitrage condition based only on well-defined expected payoffs of the asset. One familiar example of a bubble that fulfills (6) is $b_t = \kappa\left(1+\eta^{-1}\right)^t$ with $\kappa > 0$ an arbitrary positive constant. Deterministic bubbles of this type imply implausibly high asset prices relative to the size of the economy.[9]

One way out, first proposed by Blanchard (1979), Blanchard and Watson (1982), Weil (1987) and later by Evans (1991), allows for probabilistic termination.[10] Suppose b_t is a function of a two-state Markov chain $\{P, \pi_0\}$ with standard properties and π_0 is the density of the initial state ($t = 0$). Let the state 1 be "no-bubble" and state 2 be the "bubble" state, with transitions given by $P = \begin{bmatrix} 1-p & p \\ q & 1-q \end{bmatrix}$ where p is the probability of the bubble starting, and q is the probability of an existing bubble bursting. The value of the bubble in different states is:

– $b_{t+1} = 0$ with probability $1-p$ if in state 1 (the "no-bubble state") and stays in state 1;

– $b_{t+1} = \varepsilon > 0$ with probability p if a bubble initiates (the state changes from 1 to 2), where ε is a either a constant or an iid draw from a time-invariant distribution;

– $b_{t+1} = (1+g)b_t$ with probability $1-q$ if in state 2 (the "bubble state") and remains in state 2;

– $b_{t+1} = 0$ with probability q if the bubble terminates (bursts) and the state switches from 2 to 1.

For simplicity I assume the initial bubble value conditional on inception is

[9] Negative bubbles with commonly, rationally expected and deterministically falling prices can be ruled out a priori.

[10] Blanchard and Watson (1983) augment the bubble component B_t with a periodic white noise shock that displaces the bubble each period. For convenience I omit this term.

a constant ϵ, but more generally could be an iid random variable. Conditional on existence, the bubble grows at rate $\eta^{-1} - 1$ in expected value. If g is the expected growth rate of the bubble conditional on continuation, then $\frac{1}{\eta} = (1-q)\,g$ or $g = \frac{1}{\eta(1-q)}$.[11] For $\eta = 5$, $q = 0.1$, $g = 1.05/5/(.95) \approx 20\%$. If one leads the last equation and takes conditional expectations of the change in p^C conditional on the bubble's existence, the expected rates of CC capital gain, approximated as first-differences in log prices, are:

In state 1 (no-bubble state):

$$\begin{aligned} E_t \Delta p^C_{t+1} &= \frac{1}{1+\eta} \sum_{i=1}^{\infty} \left(\frac{\eta}{1+\eta} \right)^i E_t \left[\Delta x_{t+i} - \Delta \tau_{t+i} - \eta \left(\Delta r_{t+i} + \Delta \pi_{t+i} \right) \right] \\ &\quad - \frac{1}{1+\eta\gamma} \Delta c_t + p\varepsilon \end{aligned}$$

In state 2 (bubble state):

$$\begin{aligned} E_t \Delta p^C_{t+1} &= \frac{1}{1+\eta} \sum_{i=1}^{\infty} \left(\frac{\eta}{1+\eta} \right)^i E_t \left[\Delta x_{t+i} - \Delta \tau_{t+i} - \eta \left(\Delta r_{t+i} + \Delta \pi_{t+i} \right) \right] \\ &\quad - \frac{1}{1+\eta\gamma} \Delta c_t + \frac{1}{\eta\,(1-q)} \end{aligned}$$

The bubble-termination probability q and the realized path of p^C conditional on the bubble state are assumed to be time invariant, but need not be (Blanchard 1979). When they are not, there is an infinity of possible price paths and time-dependent growth rates of the bubble component associated with different assumptions on q. Evans (1991) uses this argument to question the validity of econometric tests for bubbles based on stationarity. Jovanovic (2007, 2013) notes that some bubbles for durable goods with consumption value may exist forever.

3.6 Discussion

Stochastic bubbles have stronger theoretical foundations than its deterministic counterpart, yet macroeconomists are generally unwilling to give bubbles a free pass without further restrictions on the economic environment. In a general equilibrium of frictionless asset markets with infinitely-lived,

[11] This is an approximation. Equating $1+g$ and $\frac{1+\eta}{\eta(1-q)}$ yields $g = \frac{1+\eta}{\eta(1-q)} - 1 = \frac{1+\eta-\eta(1-q)}{\eta(1-q)} = \frac{1+\eta q}{\eta(1-q)}$. For small values of ηq the approximation is acceptable.

risk-neutral agents with unlimited access to funding at the riskless interest rate, bubbles should not arise. Abreu and Brunnermeier (2003) stress heterogeneity of expectations among agents for the continuation of stochastic bubbles despite profitable arbitrage strategies, building on the insight of DeLong et al. (1988) of limited trading capacity against mispricing. In the former, bubbles can result from a lack of coordination among rational, risk-neutral, wealth-constrained traders.[12] Scheinkman (2013) has pointed out that bubbles involve implicit options to sell the asset to more optimistic traders in the future. He also notes that frictions relevant for long positions are lower than those for short sales, and that asset price collapses are often preceded by increases in supply - which are ruled out by standard CC supply algorithms. In the latter, waves of irrational traders ("stupid money") can move prices persistently away from fundamentals, and risk-averse arbitrageurs are reluctant or insufficiently funded to bet against departures from rational pricing.[13] The model abstracts from risk aversion that might imply higher and more volatile effective discount rates applied to future payoffs. As Brunnermeier and Niepelt (2019) have shown, these effective stochastic discount factors may involve occasionally binding liquidity constraints, a factor that may contribute to increasing volatility and bubble formation.

A more subtle point relates to the difficult distinction between bubbles and apparently nonstationary movements of fundamentals. Rapidly rising valuations can reflect "speculation" or beauty contest motives, but might also reflect optimistic expectations regarding future fundamentals that are difficult to orthogonalize from rumors and sentiment, and, in particular, may themselves be nonstationary. The fundamental usefulness of a cryptocurrency could be related to the yet-unrealized evolution of networks and switching costs as well as scale economies and winner-take-all phenomena. Besides being driven purely by expectations of continuation, another interpretation is that unobservable expectations - sentiment - are increasing at a rapid rate. This is only the appearance of a bubble, because the "sentiment" itself relates to fundamentals. While equation (6) makes a clean

[12]Recent events, however (e.g. GameStop, Reddit threads) may cast doubt on the claim that coordination is sufficient to preclude the emergence of bubbles.

[13]Keynes is apocryphally credited with saying "markets are irrational longer than I can stay liquid"; yet he did write that "[i]nvestment based on genuine long-term expectation is so difficult today as to be scarcely practicable. He who attempts it must surely lead much more laborious days and run greater risks than he who tries to guess better than the crowd how the crowd will behave; and given equal intelligence, he may make more disasterous mistakes" (Keynes 1936, p.157).

distinction, it is difficult to identify bubbles in practice (Evans 1991). Price movements reflect nonstationary behavior of the underlying probability distribution governing fundamentals as well as "sentiment" regarding others' willingness to buy, rational or not. Timmermann (1994) and Jovanovic (2013) cite feedback effects running from price to fundamentals that could confound standard tests, inducing multiple equilibria and path dependence. Barbarino and Jovanovic (2007) argue that sudden, bubble-like price movements may reflect changing rational assessments of capacity and demand limits. Blanchard and Watson (1988) cite the example of an impending war. Miao (2014) shows that financial frictions allow bubbles to relax collateral constraints, with positive effects on capital formation. A CC bubble could thus divert resources to the digital finance sector, leading to more innovative and productive payment technologies, as discussed below in Section 5. Distinguishing between bubbles and fundamentals may not be meaningful or even possible.

As shown by Santos and Woodford (1997), the conditions for bubbles in general equilibrium are fragile; a key limitation of transactions-based monetary models is their partial equilibrium nature. The model sketched above, for example, implies that digital assets have an unconditional expected value that deviates from rationally expected fundamentals *even prior to issuance.* For the asset considered above with algorithmic supply given by (3), given that $\pi_0 = \left[\begin{array}{cc} \pi_{01} & \pi_{02} \end{array} \right]'$, the unconditional expectation or "ICO value" of the asset must exceed its fundamental value, as it includes a rational assessment of entering the bubble state:

$$\begin{aligned} E\left[p_0^C\right] &= \frac{1}{1+\eta}\sum_{i=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^i E_t\left[\phi + x_{t+i} - \tau_{t+i} - \eta(r_{t+i} + E_t\pi_{t+i+1})\right] \\ &\quad -\overline{c} \; + \frac{1}{1+\eta\gamma}\left(\overline{c} - c_t\right) + \pi_{02}\varepsilon \qquad (9) \end{aligned}$$

This pricing outcome could explain why ICOs appear chronically overvalued in practice.[14] This chronic mispricing requires a justification from the perspective of general equilibrium.

[14] This *must* be the case, since if the probability of entering a bubbly state were zero, the process would be nonergodic! The model presented in Blanchard and Fischer (1987) and elsewhere ignores this feature, which in fact predicts that the unconditional value of this bubbly asset is always greater than the fundamental – the mere possibility of a bubble is enough to raise the expected value of the rate of growth in the asset's value.

4 Easy Piece #2: Consumption-CAPM

The analysis of the last section arose from a pure transactions demand for CC as an alternative medium of exchange. Combined with an arbitrage condition under rational expectations, this partial equilibrium analysis cannot rule out the existence of bubbles, or their stochastic emergence. This theory also assumed risk neutrality, which is unnecessary. Because money is also a store of value, any reasonable theory of CC holding must include a wealth-holding motive, a means of moving consumption across time in a world in which risk matters to agents. Standard macro models imply motives to hold CC when alternative risky assets already exist in the form of the market portfolio. Following Lucas (1978), agents hold assets to hedge against fluctuating consumption in an uncertain world. For more detailed treatment on consumption-CAPM, see Campbell (1999) or Cochrane (2005).

4.1 The basic C-CAPM portfolio decision problem

Consider a representative household that has no labor income, but owns wealth at the beginning of period 0 equal to A_0 measured in terms of the consumption good. At the beginning of each period $t = 0, 1, ...$, it expends accumulated wealth A_t on current consumption C_t and K assets $k = 1...K$, with portfolio weights ω_{kt} and $\sum_{k=1}^{K} \omega_{kt} = 1$. This represents an *intratemporal* constraint in all periods $t = 0, 1, ...$, even if some ω_{kt} are chosen to be negative.[15] Each asset held in t yields an exogenous random net rate of return $R_{k,t+1}$ paid at the beginning of period $t + 1$ that is not in the information set in period t. These securities *do not* span the space of states of the world. There are no transactions costs, although each asset k may face exogenous, random periodic yield loss τ_{kt+1}, exogenous regulatory and other technical impediments to conversion into consumption. Initially, I set τ_{kt+1} to zero in all periods for all assets. The period-by-period budget constraint in each period $t = 0, 1, ...$ is

$$A_{t+1} = \left[\sum_{k=1}^{K} \omega_{kt}(1 + R_{kt+1}) \right] (A_t - C_t)$$

[15] In general equilibrium, all assets are in non-negative net supply.

or since $\sum_{k=1}^{K} \omega_{kt} = 1$

$$A_{t+1} = \left[R_{1t+1} + \sum_{k=2}^{K} \omega_{kt} (R_{kt+1} - R_{1t+1}) \right] (A_t - C_t) \,. \tag{10}$$

Think of R_{1t+1} as a reference rate of return that may or may not be risk-free. For any $k = 1, ..., K$, some outside authority or agency rules out "Ponzi schemes" in any period $t \geq 0$ *a priori* that

$$\lim_{T \to \infty} E_t \left[\left(\prod_{i=0}^{T} (1 + R_{k,t+i})^{-1} \right) A_{t+T} \right] = 0 \tag{11}$$

The household has standard time-separable preferences and maximizes expected utility, solving

$$\max_{\substack{\{C_t\}_{t=0}^{\infty} \\ \{A_{t+1}\}_{t=0}^{\infty} \\ \{\omega_{kt}\}_{t=0}^{\infty} \ \forall k=1...K}} \quad E_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_t) \right] \tag{12}$$

where the periodic utility function U satisfies $U' > 0, U'' < 0$; the constant β implies a pure discrete-time discount rate of $\rho = \beta^{-1} - 1$. Given A_0, the maximization is constrained by (10) for all $t \geq 0$.

In the Appendix the solution to the household's problem is presented in detail. The first order conditions mandate selection of C_t and $\{\omega_{kt}\}_{k=2}^{K}$ in each period so that for the reference asset ($k = 1$):

$$U'(C_t) = \beta E_t \left[(1 + R_{1t+1}) U'(C_{t+1}) \right] \tag{13}$$

for $k = 2, ..., K$:

$$E_t \left[(R_{kt+1} - R_{1t+1}) U'(C_{t+1}) \right] = 0. \tag{14}$$

and wealth obeys (10) for $t = 0, 1, ...$ These three equations constitute a valuation or pricing model for financial assets and CC in particular. The first two can be rewritten as

$$1 = E_t \left[\left(1 + R_{1t+1} + \sum_{k=2}^{K} \omega_{kt} (R_{kt+1} - R_{1t+1}) \right) M_{t+1} \right] \tag{15}$$

where $M_{t+1} \equiv \frac{\beta U'(C_{t+1})}{U'(C_t)}$ is the stochastic discount factor or pricing kernel (e.g. Campbell 1999, Cochrane 2005), the marginal rate of substitution of future for present consumption. If the rate of return R_{1t+1} is known and equals r_t (1 unit invested in t pays $1 + r_t$ with probability 1 in $t + 1$), (14) implies

$$1 + r_t = \frac{1}{M_{t+1}} \tag{16}$$

and the riskless asset pays the ex-post marginal rate of substitution. Because the ordering of securities is arbitrary in the third equation, for any two assets $i, j \subset \{1, 2, ..., K\}$

$$E_t \left[M_{t+1} R_{it+1}\right] = E_t \left[M_{t+1} R_{jt+1}\right]. \tag{17}$$

and for each $k = 2, ..., K$

$$E_t R_{kt+1} - r_t = -cov(M_{t+1}, R_{kt+1}).$$

implying that the excess expected return paid by asset k is negatively related to its covariance with the intertemporal rate of substitution. Assets that pay high returns when consumption is scarce (the marginal rate of substitution is high) will bear a low yield (i.e. their value/price will be high).[16]

4.2 CC Asset Pricing

Combine the fundamental pricing relation (17) for the riskless rate with the market price of the kth asset P_{kt}, $P_{1t} = 1$ and $R_{kt} \equiv \frac{(1-\tau_{kt+1})P_{kt+1}}{P_{kt}}$, where P_{kt+1} and τ_{kt+1} denote the gross payoff (ask price) and asset-specific stochastic trading friction/tax/loss respectively, the relevant valuation equation for the household is

$$P_{kt} = E_t \left[M_{t+1} P_{kt+1}(1 - \tau_{kt+1})\right]. \tag{18}$$

[16]Equations (10), (16), and (17) are applicable even if agents have different asset holdings, different information sets, or different levels of rationality (Cochrane 2005). They could describe a general equilibrium in which all assets are held by a representative agent and apply the logic to the "market portfolio", delivering a market price of risk and a Sharpe ratio. Such a complete description of the equilibrium is unnecessary for what follows. The maximization problem posed in (12) is rationale for a more general model of the pricing kernel for the decisive (marginal) investor. Lacking a complete theoretical characterization of the marginal rate of intertemporal substitution for that investor, empirical investigation directs us to agnostic macroeconomic conditioning variables for asset pricing, e.g. the log consumption to wealth ratio studied by Lettau and Ludvigson (2001). For an application to CC see Chen and Vinogradov (2020).

Using fundamental relations for covariances[17], it follows that

$$\begin{aligned} P_{kt} &= E_t M_{t+1} E_t P_{kt+1} E_t (1-\tau_{kt+1}) \\ &\quad - E_t M_{t+1} cov_t(P_{kt+1}, \tau_{kt+1}) \\ &\quad + cov_t(M_{t+1}, P_{kt+1}(1-\tau_{kt+1})). \end{aligned} \tag{19}$$

where cov_t denotes covariance conditional on information in time t. Equation (19) is a pricing or valuation equation for CC that can be summarized as follows. Ceteris paribus, the price of asset k is higher:

- the higher the expected value of the expected future price $E_t P_{kt+1}$;
- the higher the expected value of net payoff (including conversion impairment or liquidity loss) $E_t(1-\tau_{kt+1})$;
- the higher the expected value of the pricing kernel $E_t M_{t+1}$;
- the higher the covariance between the pricing kernel M_{t+1} (the marginal rate of substitution) and the net future return $(1-\tau_{t+1})P_{kt+1}$;
- the lower the covariance between future price and stochastic conversion impairment.

The pricing equation (19) offers insight into the shape of curves displayed in Figure 2, because it gives rise to a well-defined function of CC valuation as a function of the joint asset distribution of net payoffs and the pricing kernel, and thus a theoretic account of descriptive "capital distribution curves" used to describe equity valuations (e.g., Karatzas and Fernholz 2009). Height and slope are a function of the covariance of net payoff with the pricing kernel across existing coins. For two coins of equal correlation with the marginal rate of substitution, the one with lower expected transaction costs and higher liquidity will command a higher market value.

Consider an application of the pricing formula under the instructive (but unrealistic) case of conditionally and jointly log-normal, homoskedastic covariates. Log normality applied to the pricing formula (18) implies:

$$\begin{aligned} \ln P_{kt} &= \ln E_t \left[M_{t+1} P_{kt+1} (1-\tau_{kt+1}) \right] \\ &= E_t \left[\ln(M_{t+1} P_{kt+1}(1-\tau_{kt+1})) \right] \\ &\quad + 0.5 Var \left[\ln(M_{t+1} P_{kt+1}(1-\tau_{kt+1})) \right]. \end{aligned} \tag{20}$$

Using notation $p_{kt+1} = \ln(P_{kt+1})$, $m_{t+1} = \ln(M_{t+1})$, the approximation

[17] For random variables $X, Y,$ and Z, $E[XYZ] = E[X]E[YZ] + cov(X, YZ) = E[X]\left[E[Y]E[Z] + cov(Y,Z)\right] + cov(X, YZ) = E[X]E[Y]E[Z] + E[X]cov(Y,Z) + cov(X, YZ)$.

$\ln(1-\tau_{kt+1}) \approx \tau_{kt+1}$ and that

$$E_t \begin{bmatrix} p_{kt+1} \\ m_{t+1} \\ \tau_{kt+1} \end{bmatrix} = \begin{bmatrix} E_t p_{kt+1} \\ E_t m_{t+1} \\ E_t \tau_{kt+1} \end{bmatrix} \tag{21}$$

$$Var_t \begin{bmatrix} p_{kt+1} \\ m_{t+1} \\ \tau_{kt+1} \end{bmatrix} = \begin{bmatrix} \sigma_p^2 & \sigma_{mp} & \sigma_{\tau p} \\ \sigma_{mp} & \sigma_m^2 & \sigma_{\tau m} \\ \sigma_{\tau p} & \sigma_{\tau m} & \sigma_\tau^2 \end{bmatrix},$$

it follows that

$$\begin{aligned} p_{kt} &= E_t m_{t+1} + E_t p_{kt+1} - E_t \tau_{kt+1} \\ &\quad +0.5 Var_t \left[m_{t+1} + p_{kt+1} - \tau_{kt+1}\right] \\ &= E_t m_{t+1} + E_t p_{kt+1} - E_t \tau_{kt+1} \\ &\quad +0.5 \left[\sigma_p^2 + \sigma_m^2 + \sigma_\tau^2 + 2\sigma_{mp} - 2\sigma_{\tau p} - 2\sigma_{\tau m}\right]. \end{aligned} \tag{22}$$

Holding conditional variances constant, (22) implies that the current CC price p_{kt} is higher, the greater the expected value of the asset pricing kernel (the lower the rate of intertemporal substitution), the higher the expected future market price of the asset, and the lower the expectation of value impairment in the next period.[18] The payoff covariances also play a decisive role. In particular, p_{kt} is higher:
- the higher its covariance with the stochastic discount factor m_{t+1};
- the lower its covariance with the stochastic value impairment τ_{kt+1}; and the
- the lower the covariance of the stochastic value impairment τ_{kt+1} with the stochastic discount factor m_{t+1}.

Figure 3 displays time-varying estimates of correlations of daily realized returns on Bitcoin, Ethereum, and the CRIX index 2014-2021 with two major stock indexes, computed from a rolling data window of 180 previous days.[19] It reveals not only a longer-term trend of increasingly negative

[18]These variances exert a positive influence on the current price as a result of Jensen's inequality (see e.g. Campbell 1999, p.1247).

[19]Dow Jones is a price-weighted measure of 30 U.S. blue-chip companies, covering all industries except transportation and utilities. S&P1200 covers 30 countries and 7 regions.and includes over 70% of the world market capitalization representing all major sectors. CRIX (https://thecrix.de/) is a Laspeyres Index of CC prices with weights that are updated monthly according to market value share.

correlation of CC returns with the "market", but also a strong persistence in swings of those correlations over that period. This belies a low overall correlation reported in Table 1, suggesting that conditional homoskedasticity not an inappropriate. As with stock returns, temporal variation of locally measured volatility of CC returns reveal significant unconditional heteroskedasticity.

5 Easy piece #3: Search and matching

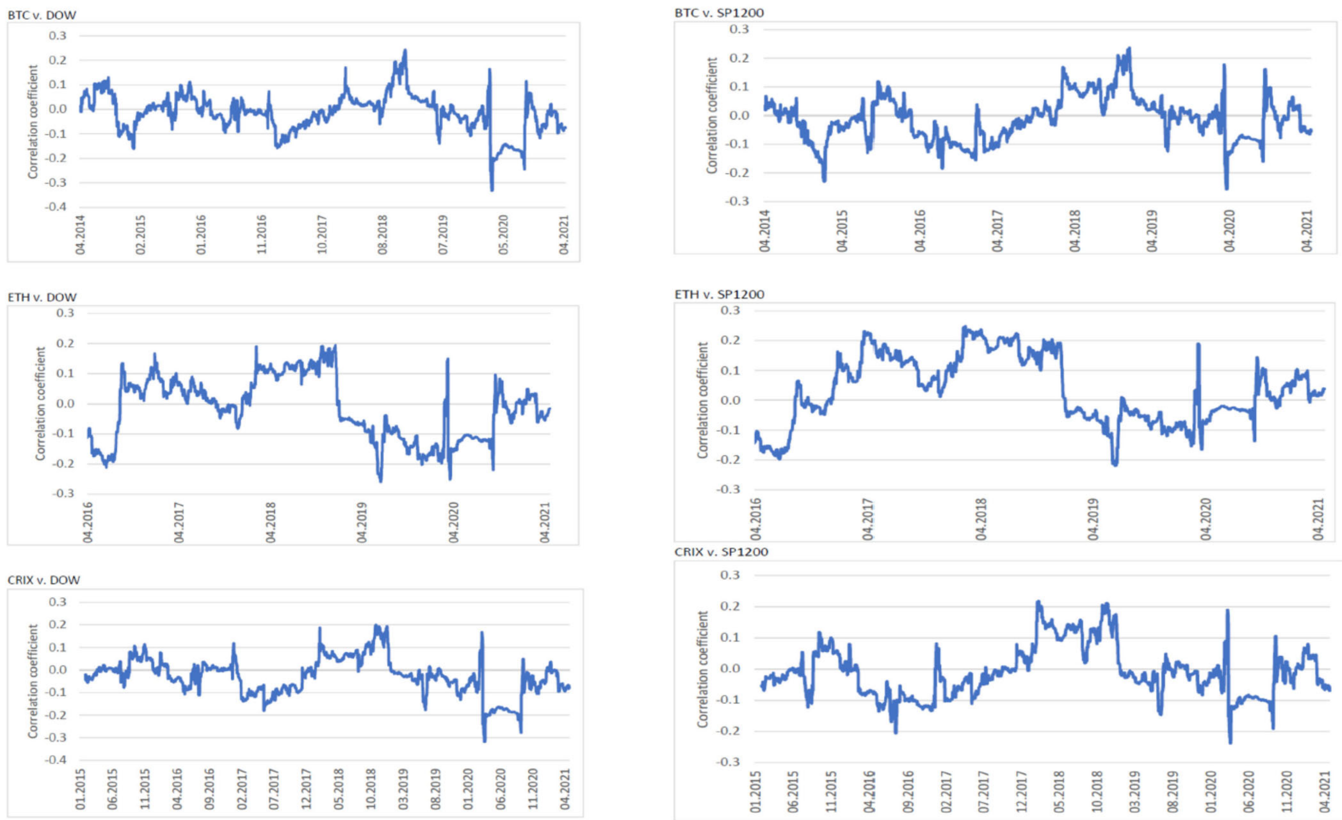
The third "easy piece" considers privately issued currencies as a solution to a matching problem between suppliers of transactions technologies and demanders for applications, that is, productive uses of those technologies. Figure 4 plots the number of coins listed on coingecko.com with positive market valuation since April 2013.[20] It reveals a slowly increasing trend punctuated by two surges of new CC issuance: One in late 2017 and a second in mid-2020. The second, which has not yet abated at the time of writing, is probably associated with new prospects of smart contracts on the Ethereum platform. In the form of digital finance and smart contracts in general, CC implement Jevon's fourth function of money – a standard of deferred payment, or conditional payment, if one is willing to interpret the notion more broadly. Because new needs arise and evolve over time and states of the world, their demand and supply are endogenous. A model of this extensive margin is needed.[21]

From this perspective, questions related to market entry and extensive margin of CC can be explored with a modified labor market model of search and matching pioneered by Mortensen and Pissarides (1994, henceforth "MP model") and summarized in Chapter 2 of Pissarides (2000). Coins arise from a match of posted technological applications (of mass v) with an unsatisfied potential user (of mass u). The former corresponds to job vacancies in the MP model, while the latter finds its analogue in the unemployed worker –

[20] Positive market valuation of the outstanding coin issue at the midnight bid price of the respective day; data were obtained from scraping coingecko.com.

[21] Almosova (2017, 2018) and Fernández-Villaverde and Sanches (2019) study price stability in a general equilibrium with a fixed number of competitive privately issued currencies under different assumptions regarding the production costs of money. They focus on money as a means of payment, taking the extensive margin as given and do not address innovation in payment systems; in the much simpler model here, I ignore monetary effects (e.g. on the average price level) and stress instead the idiosyncratic features of CC that can give rise to coin issuance.

Figure 3: Rolling correlation daily returns with Dow Jones and S&P1200 Global Indexes, 2014-2021



Source: https://www.coingecko.com

a transaction demand to be performed or satisfied. Examples of the latter are capital flight from a particular country or region, automatic execution of particular trades, tax avoidance or evasion, laundering of gains from illicit activities, paying extortionists, as well as conventional demand for a stable currency under unstable macroeconomic conditions, or one that offers particular features such as automated interest payments, execution of smart contracts, administration of loan collateral, tied transactions, or a Tobin tax. A particular match between technology application and the user employing it gives rise to joint flow productivity $x \in [0, 1]$, assuming the value 1 initially and changing according to a Poisson process, i.e. time between productivity changes following an exponential distribution with a constant exogenous arrival rate λ. A new productivity "arrival" x is a random variable with time-invariant cdf $G(x)$, possibly reflecting the number of users or external effects deriving from other coins.

A particular CC has value as long as the net joint surplus generated by a matched idea or technology (supply) with an application (demand) is positive, otherwise the CC it is abandoned. Let r be the exogenous interest rate; the steady-state capitalized value of the joint surplus $S(x)$, or the value of the technology-user match given the current productivity of a coin x – that is, the value of transactions thus enabled – follows the following value continuation equation, derived in the Appendix:

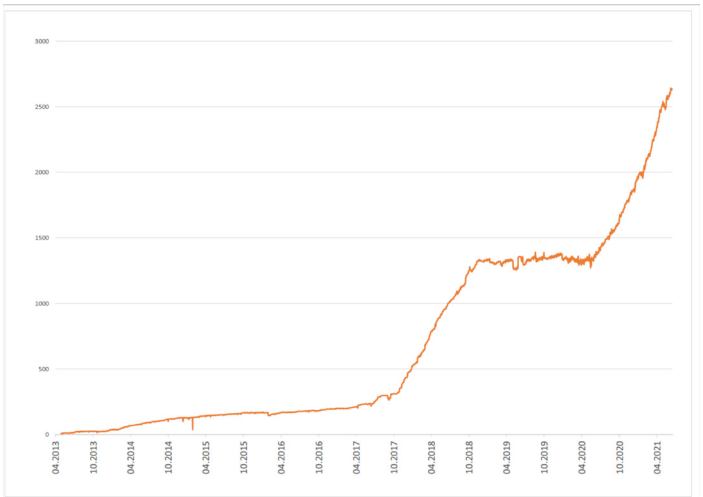
$$(r + \lambda)\, S(x) = x + \lambda \int_{R}^{1} S(s) dG(s) - rU - rV. \tag{23}$$

As long as current productivity x exceeds the reservation value R, the match's surplus value is positive and the match continues. An incidence of a new value of x with density dG is associated with a new surplus. The fallback reservation values to the respective parties in the Nash bargain are rU and rV; hence the flow surplus is calculated net of the respective opportunity costs. The determination of $S(x)$ is an essential element of the search and matching perspective of cryptocurrency valuation.

5.1 Surplus sharing

Surplus is due to frictions that arise when finding trading partners and is positive when searchers for applications ("appliers") and entrepreneurs ("suppliers") of transaction technology applications meet. Because initial productivity is maximal ($x = 1$), initial matches are never rejected in this model. Matching occurs via random search according to a constant returns

Figure 4: Coin issues with positive market value 2013-2020



Source: https://www.coingecko.com

matching technology. Define $\theta = \frac{v}{u}$ as a measure of market tightness, the ratio of suppliers to appliers; under constant returns, θ is a sufficient condition for the effectiveness of search for both sides of the market. Potential suppliers of technological applications meet potential appliers of technology at rate $q(\theta)$ per unit time with $q' < 0$, while appliers meet suppliers at rate $\theta q(\theta)$ with $\frac{d(\theta q(\theta))}{d\theta} > 0$. Match surplus is split according to a simple modified Nash bargaining rule with respective shares μ and $1-\mu$, so periodic returns are $\mu S(x)$ to appliers and $(1-\mu)S(x)$ to suppliers. The value of search to an "applier" searching for a transactions technology U follows the arbitrage equation

$$rU = z + \theta q(\theta)\mu \left[S(1) - U\right].$$

$S(1)-U$ is the capital gain implied by the match above and beyond the value of the alternative of continued search. z stands for the periodic flow utility of an "applier" with an unsatisfied demand.[22] Similarly, a "supplier" is an entrepreneur with an idea in play or technology on the drawing board in search of an application has continuation value V governed by the arbitrage

[22] In the labor search literature, this corresponds to the value of leisure, home production, or the value of unemployment benefits.

condition

$$rV = -c + q(\theta)(1-\mu)\left[S(1) - V - F\right]$$

where c represents periodic search costs paid by suppliers and F stands for fixed costs associated with implementing the technology when the match is found.

5.2 Quantities and margins

This model focuses on the extensive margin of CC use as the number of existing *matched* (satisfied) demands/applications with technologies at any point in time. As such, it is a stand-in for cumulated coins or tokens in use with positive surplus value $S(x) > 0$. The gross flow of new matches at any point in time corresponds to successful new currency launches (ICOs) and is given by $q(\theta)v = \theta q(\theta)u$. The intensive margin is not modeled explicitly, but could be proxy for the productivity of the transactions technology x (e.g. the fraction of the market that profitably uses it). Idiosyncratic productivity x described in the previous section determines the joint surplus of the match between technological application and users. The mass of employed (matched) technologies ("active coins") is $1-u$ and u represents the mass of unmatched searchers/appliers. The latter evolves as the net difference per unit time between inflows and outflows of unmatched applications: $\frac{du}{dt} = \lambda G(R)(1-u) - \theta q(\theta)u$. In the steady state, the stock of searching appliers is $u = \frac{\lambda G(R)}{\lambda G(R)+\theta q(\theta)}$ and the mass of active coins is $n = 1-u = \frac{\theta q(\theta)}{\lambda G(R)+\theta q(\theta)}$. By inspection, a higher productivity threshold R raises the number of unmatched appliers and reduces the number of active, positively valued coins. *Ceteris paribus*, an increase in θ, the relative abundance of technological ideas in search of appliers - increases the number of coins in use and reduces the mass of appliers.

5.3 Free entry and the coin creation condition

The hallmark of this class of search and matching models is a free entry condition on at least one side of the market. Free entry of new payment technologies with positive capital value at the extensive margin implies dissipation of rents. Applied to the entry of new technologies (suppliers) looking for unsatisfied demanders of applications (appliers), this condition $V = 0$

completes the description of the equilibrium. It implies:

$$\frac{c}{q(\theta)} = (1-\mu)\left[\frac{1-R}{r+\lambda} - F\right], \tag{24}$$

a downward-sloping relationship in (R, θ) space.[23] This *coin creation condition* (CCC) describes values of θ and R consistent with free entry of suppliers of new technologies when the capitalized value of an unmatched new idea has been driven to zero ($V = 0$). For fixed values of the model parameters, it is characterized as the downward sloping curve in (R, θ) space in Figure 5. Ceteris paribus, a higher level of market tightness (a relatively greater abundance of ideas in search of applications) is consistent with zero valuation of a new application only if the match is sufficiently robust, i.e. if the threshold productivity R is sufficiently low.

The equilibrium value of search U by unsatisfied demanders is

$$rU = z + \theta q(\theta)\mu\left[\frac{1-R}{r+\lambda} - F\right],$$

and is a positive function of flow utility in search z, the probability of match $\theta q(\theta)$ and thus market tightness θ, and the demander's share of surplus μ; it is a negative function of the threshold productivity R, the interest rate r, the incidence rate of productivity shocks λ, and the fixed cost of creating the coin F. It can be combined with the surplus sharing condition, yielding

$$rU = z + \frac{\mu c\theta}{1-\mu}, \tag{25}$$

fixing the reservation level of utility obtained by an unmatched applier in search for a supplier as a linear function of market tightness.

5.4 Coin abandonment condition

In this highly stylized model, heterogeneity of coins is captured by productivity x and their valuation by capitalized valuation conditional on x, $S(x)$. Technology user-use matches have zero value at $S(R) = 0$; because surplus S is strictly increasing, matches are mutually abandoned if $x < R$ or $S(x) < 0$, and the associated use case would revert to search for another

[23] Because c and q are positive, the right hand side must be positive, and thus meaningful equilibria can only exist for $F(r+\lambda) < 1$.

technology that yields at least rU, the reservation flow defined by (25):

$$z + \frac{\mu c\theta}{1-\mu} = R + \frac{\lambda}{r+\lambda}\int_R^1 (s-R)dG(s) \tag{26}$$

This *coin abandonment condition* (CAC) describes a second relation between threshold productivity (R) and the relative abundance of suppliers of payment technologies to uses/demanders (θ). It is represented by the upward-sloping curve in Figure 5. The higher R is, the more fragile is the match (the greater the likelihood that it will become obsolete); a higher value of the abandonment threshold is only consistent with a higher scarcity of potential appliers of technology (or a surfeit of suppliers of potential applications).

5.5 Equilibrium

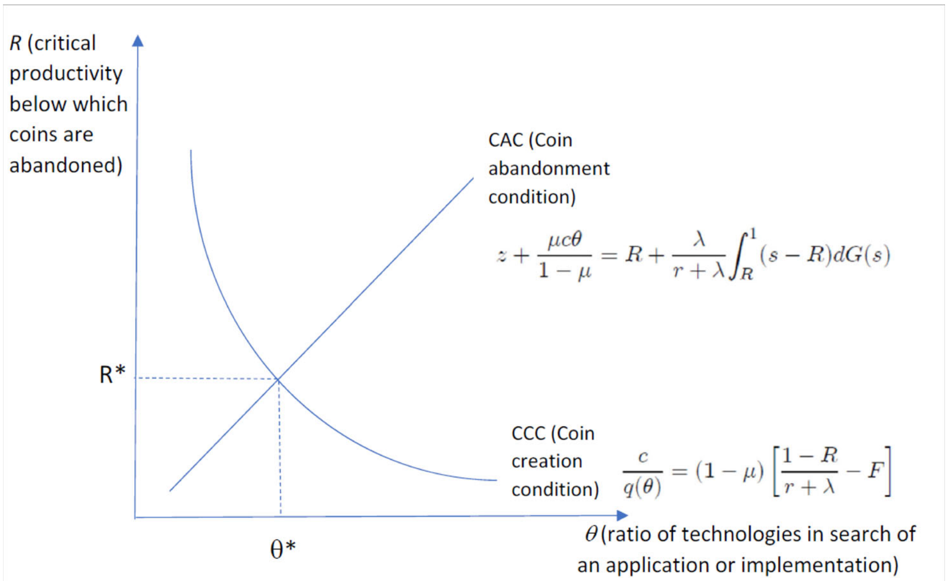
The solution to the CC valuation problem is a reservation productivity value R and a level of market tightness $\theta = \frac{v}{u}$ summarizing relative scarcity of applications to users. An equilibrium is defined as intersection of CAC and CCC conditions (26) and (24) in Figure 5, yielding an equilibrium level of tightness θ^* and the flow productivity threshold R^*. The former is a relative quantity indicator, while the latter is the threshold value of periodic productivity of the marginally useful coin.[24] Equilibrium values θ^* and R^* in turn deliver $n = 1-u$, the mass of coins in active use, the surplus value $S(x)$ active coins at any point of time (with $x > R^*$), as well as other interesting quantities. The search and matching angle on cryptocurrencies yields several insights summarized below.

Extensive margin. The mass or number of coins with positive value is endogenous and equal to

$$n \equiv 1-u = \frac{\theta q(\theta)}{\lambda G(R) + \theta q(\theta)}$$

[24] The determination of the relative abundance of technologies on offer (suppliers) relative to searcher for applications (appliers) can be represented in a two-dimensional diagram that plots the two endogenous variables R and θ as the intersection of the downward-sloping CCC curve given by (24) and the upward-sloping CAC curve given by (26). This diagram yields the usual comparative static insights of the effects of changes in exogenous parameters of the model.

Figure 5: Equilibrium in the cryptocurrency market as a matching processs



The extensive margin is a positive function of θ (the ratio of suppliers to appliers) and a negative function of the "fragility" of the marginal match R. Straightforward comparative statics analysis shows in turn that θ and R are functions of model parameters, z, c, λ, μ, F, and r, as well as the functions $q(.)$ and $G(.)$.

Valuation. The search and matching model implies a value of total coin issuance outstanding, under the assumption that $G(.)$ characterizes the steady-state cross-section of coin productivities. The aggregate "capitalization" – value of all the positively valued coins – is an integral of the valuation $S(.)$ over the range of viable match productivities $z \in [R, 1]$, weighted by the probability of their occurrences:

$$\int_R^1 S(s)dG(s) = \int_R^1 \left(\frac{s-R}{r+\lambda}\right) dG(s).$$

The shape of this value distribution will be affected by $G(.)$ but also by possible transformations of it (see extensions below). The function $S(.)$ describes the surplus value of each matched and surviving technology. In a competitive asset market, this corresponds to the market valuation of that technology.

Option value. A third implication of the model is an *option value* conferred on "dormant" coins with low productivity. As noted above, only a fraction of CC issued have positive "market capitalization", with a more than 2/3 of all coins issue falling into disuse. The theory predicts a range of current productivities will be observed that are strictly less than the flow opportunity costs of search for appliers rU, reflecting the potential incidence of a favorable future realization to the application/user pair. To see this, rewrite the CAC (26) as

$$R = z + \frac{\mu c\theta}{1-\mu} - \frac{\lambda}{r+\lambda}\int_R^1 (s-R)dG(s),$$

showing that the abandonment threshold R equals the best alternative available to appliers (the value of further search) *less* an integral term representing expected capital gain of a new realization of productivity that falls in the acceptance range $[R, 1]$. Intuitively, this option value is lower, the higher equilibrium value R (the more fragile matches are). Holding $z + \frac{\mu c\theta}{1-\mu}$ constant, the option will also be lower relative to a counterfactual coin with a productivity distribution $\widetilde{G}(.)$ that is stochastically dominated by $G(.)$.[25]

5.6 Extensions

The search and matching model approach to coin valuation can be extended in a number of directions. The "participation margin" of potential appliers could be endogenized like invidual workers in the MP model (Pissarides 2000, Chapter 7). Similarly, the distribution of match productivity could incorporate endogenous search effort on the side of technology suppliers in equilibrium. Conditional on entry, the intensive margin, *how much* of each coin is issued, was suppressed, but could be modeled to depend on cost and benefits of services provided algorithmically by coin contracts. Explicit modeling this intensive margin is likely to render match productivity a function of scale economies of the application. Alternatively, spillover effects across coin issue could transform individual into effective productivity via some increasing function and ultimately affect $S(.)$. Finally, the role of institutions such as intellectual property law could affect μ with effects on equilibrium R and θ, while fixed and periodic costs (F and c) of technology suppliers might admit external effects and potential roles for policy.

6 Concluding remarks

The correction of the CC market following its peak valuation in April 2021 of about a third represented \$1t or about 1.2% of world GDP, making cryptocurrencies a macroeconomic phenomenon. This paper offers three "easy pieces" for understanding digital assets as a medium of exchange, a store of value, and an open system for matching payment tasks with technologies to implement them. The three études are meant to help understand the dimensions of CC valuation and are necessarily oversimplifications of a complex reality. The first piece stresses the MOE functionality and the first-order role of expectations as a central source of fluctuations in CC valuation. The robust negative role of fiat currency inflation as well as real rates of return alert us to the close proximity of CC to money and its inherent unattractiveness when nominal interest rates are high; a rise in rates will have an

[25] $G(s)$ stochastically dominates $\widetilde{G}(s)$ if $G(s) \leq \widetilde{G}(s)$ $\forall s$ and $G(s) < \widetilde{G}(s)$ for some $s \in [0,1]$. If $\widetilde{G}(s) = \phi(s)G(s)$ and $\phi(s) < 1$, it is straightforward to show that, all other things equal, $R^G > R^{\widetilde{G}}$, i.e. the option value associated with $\widetilde{G}(s)$ is higher.

outsize negative impact on the valuation of digital assets, ceteris paribus. Similarly, a rise in the expected future utility of CC in executing payments will have immediate positive impact on current valuations. Formally, the space for bubbles is large, but also for expectations of nonstationary evolution of transaction technologies. Because resulting fluctuations are imperfectly correlated with other asset returns, they naturally give rise to SOV functionality, which rationalizes the second piece. Empirically, CC returns are increasingly negatively correlated with broad asset indexes, suggesting a *premium* (risk discount), at least in the brief period we can evaluate. The third piece demonstrates the role of currency competition in CC valuation. If scale diseconomies set in or ease of transactions declines, entry of new technologies will become profitable and displace old ones.

Seen through these different lenses, cryptocurrency valuation remains a conundrum, however, and the reader who has made it this far may be disappointed by my tentative conclusions. Each model succeeds and fails in its own way. Jevon's functions of money are jointly produced and interfere with each other, and the strength of these competing uses varies over time. CC are money and as such will be valued for their acceptance by others and for the liquidity convenience they provide. They are Hume's lubricant of the wheels of commerce, not only for goods and services, but also for financial transactions, including the overcoming of trading frictions, securing and transfer of collateral, market discovery, and facilitation of competitive entry and experimentation. Shifting present and expected future demand for these uses are causal for additional fluctuations that can be influenced both by fundamentals, based on the inherent utility they offer, and "sentiments" or market mood about others' expectations of CC prices rising at a rate that makes them competitive with more conventional yield-bearing assets. If price dynamics are grounded in sentiments of regarding probabilities of nonstationary behavior of productivity, they are fundamental; they are rather based on castles-in-the-sky rumors, "pump-and-dump" and selling into a rising market, they are likely to be a bubble. Curiously, the very volatility generated by these expectations provide a potential hedge against consumption risk and render CC a candidate component of a well-diversified portfolio. While CC and digital assets may resemble elaborate Ponzi schemes, they do offer new and unbundled financial services that are likely to make them a permanent fixture of the financial landscape, and not necessarily, as Warren Buffett asserted, "rat-poison squared."[26]

[26]https://www.cnbc.com/2018/05/05/warren-buffett-says-bitcoin-is-probably-rat-

Paradoxically, value stability is essential for sustainable MOE functionality, yet volatile expectations surrounding future adoption and use of CC will attenuate that value, while enhancing the hedge against consumption fluctuations across time and states of nature. The fundamental value of any CC can only be sustained by future utility in transactions, and the maintenance of transactions ledgers cannot become so costly that competing versions of money emerge and degrade or debase existing ones. Ultimately, tensions between these features are driven by common but conflicting roles of sentiment or general market mood, and by fundamental and non-fundamental factors. Keynes noted in Chapter 12 of the *General Theory*: "If I may be allowed to appropriate the term speculation for the activity of forecasting the psychology of the market, and the term enterprise for the activity of forecasting the prospective yield of assets over their whole life, it is by no means always the case that speculation predominates over enterprise." The lasting impact of CC and digital finance may ultimately lie in the use of technology to separate those competing functions of money.

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8 Appendix

8.1 Appendix #1: Demand, supply and equilibrium in a market for a MOE

Log demand for CC in period t, c_t^D, can be written as

$$\begin{aligned} p_t^C &= \phi + x_t - \tau_t - c_t^D - \eta(r_t + E_t p_{t+1} - p_t - \left(E_t p_{t+1}^C - p_t^C\right)) \\ &= \frac{1}{1+\eta}\left[\phi + x_t - \tau_t - c_t^D - \eta(r_t + E_t p_{t+1} - p_t)\right] + \frac{\eta}{1+\eta} E_t p_{t+1}^C \end{aligned} \tag{27}$$

Log supply of CC takes the form $c_t = g(x_t)(\overline{c} - c_{t-1})$ with implied first difference

$$\begin{aligned} \Delta c_t &= g(x_t)(\overline{c} - c_{t-1}) - g(x_{t-1})(\overline{c} - c_{t-2}) \\ &= g(x_t)\left(\overline{c} - c_{t-1}\right) - g(x_{t-1})\overline{c} + g(x_{t-1})c_{t-2} \\ &= g(x_t)\left(\overline{c} - c_{t-1}\right) - g(x_{t-1})\overline{c} + g(x_{t-1})\left(\overline{c} - c_{t-1}\right) \\ &\quad - g(x_{t-1})\left(\overline{c} - c_{t-1}\right) + g(x_{t-1})c_{t-2} \\ &= \left[g(x_t) - g(x_{t-1})\right]\left(\overline{c} - c_{t-1}\right) - g(x_{t-1})\left[c_{t-1} - c_{t-2}\right] \\ &= \left[g(x_t) - g(x_{t-1})\right]\left(\overline{c} - c_{t-1}\right) - g(x_{t-1})\Delta c_{t-1}, \end{aligned}$$

meaning that as long as $g(x_t)$ is difference stationary, the log of CC supply is difference-stationary, here, an AR(1) process. Under the form of g assumed in the main text, (3), it follows that

$$\begin{aligned} c_{t+\tau} &= (1-\gamma)^\tau c_t + \left[\sum_{i=0}^{\tau-1} (1-\gamma)^i\right]\gamma\overline{c} \\ &= (1-\gamma)^\tau c_t + \left[1 - (1-\gamma)^\tau\right]\overline{c} \\ &= \overline{c} - (1-\gamma)^\tau\left(\overline{c} - c_t\right) \end{aligned}$$

Demand equals supply in equilibrium, $c^D_{tt+\tau} = c_{t+\tau}$ for $\tau = 0, 1, ...$ so

$$\begin{aligned} p_t^C &= \frac{1}{1+\eta}\sum_{i=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^i E_t\left[\phi + x_{t+i} - \tau_{t+i} - \eta(r_{t+i} + E_t\pi_{t+i+1})\right] \\ &\quad -\frac{1}{1+\eta}\sum_{i=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^i \left[\overline{c} - (1-\gamma)^i(\overline{c} - c_t)\right] \end{aligned} \tag{28}$$

or

$$p_t^C = \frac{1}{1+\eta}\sum_{i=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^i E_t\left[\phi + x_{t+i} - \tau_{t+i} - \eta(r_{t+i} + E_t\pi_{t+i+1})\right] \tag{29}$$

$$-\frac{1}{1+\eta}\sum_{i=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^i \overline{c} + \frac{1}{1+\eta}\sum_{i=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^i (1-\gamma)^i(\overline{c} - c_t) \tag{30}$$

$$= \frac{1}{1+\eta}\sum_{i=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^i E_t\left[\phi + x_{t+i} - \tau_{t+i} - \eta(r_{t+i} + E_t\pi_{t+i+1})\right] \tag{31}$$

$$-\overline{c} + \frac{\overline{c} - c_t}{1+\eta}\sum_{i=0}^{\infty}\left(\frac{\eta(1-\gamma)}{1+\eta}\right)^i \tag{32}$$

$$= \frac{1}{1+\eta}\sum_{i=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^i E_t\left[\phi + x_{t+i} - \tau_{t+i} - \eta(r_{t+i} + E_t\pi_{t+i+1})\right] \tag{33}$$

$$-\overline{c} + \left(\frac{1}{1+\eta\gamma}\right)(\overline{c} - c_t) \tag{34}$$

which is equation (8) in the main text.

8.2 Appendix #2: C-CAPM

Assuming covariance-stationarity of returns and concavity of U is sufficient to write (12) in Bellman form, a time-invariant and unique value function $V(A_t)$ is the solution to the functional equation

$$V(A_t) = \max_{\substack{C_t,\ A_{t+1},\\ \{\omega_{kt}\}_{k=2}^K}} U(C_t) + \beta E_t V(A_{t+1}) \tag{35}$$

subject to (10) and imposing $\omega_{1t} = 1 - \sum_{k=2}^K \omega_{kt}\ \forall t$. First-order conditions are:
For C_t :

$$U'(C_t) = \beta E_t\left[\left(1 + R_{1t+1} + \sum_{k=2}^K \omega_{kt}(R_{kt+1} - R_{1t+1})\right)V'(A_{t+1})\right] \tag{36}$$

For A_{t+1} :

$$E_t\left[(1+R_{1t+1})V'(A_{t+1})\right]=0 \tag{37}$$

For ω_{kt}, $k=2,...K$:

$$E_t\left[(R_{kt+1}-R_{1t+1})V'(A_{t+1})\right]=0 \tag{38}$$

for all $t\geq 0$. The Benveniste-Scheinkman condition for optimality

$$V'(A_t)=\beta E_t\left[\left(1+R_{1t+1}+\sum_{k=2}^{K}\omega_{kt}(R_{kt+1}-R_{1t+1})\right)V'(A_{t+1})\right] \tag{39}$$

implies $U'(C_t)=V'(A_t), U'(C_{t+1})=V'(A_{t+1})$ and for the total portfolio

$$U'(C_t)=\beta E_t\left[\left(1+R_{1t+1}+\sum_{k=2}^{K}\omega_{kt}(R_{kt+1}-R_{1t+1})\right)U'(C_{t+1})\right] \tag{40}$$

as well as for the reference asset ($k=1$):

$$U'(C_t)=\beta E_t\left[(1+R_{1t+1})U'(C_{t+1})\right] \tag{41}$$

so for $k=2,...K$:

$$E_t\left[(R_{kt+1}-R_{1t+1})U'(C_{t+1})\right]=0. \tag{42}$$

8.3 Appendix #3: Search and matching

The value of a "technology match" between an applier and a supplier at productivity x, $S(x)$, is characterized by the following Bellman equation:

$$rS(x)=x+\lambda\int_0^1\max\left[S(s)-U-V,0\right]dG(s)-r(U+V).$$

The time-invariance of $G(s)$ implies that offers are never hoarded, so $S'(x)>0$ and the reservation property holds:

$$\begin{aligned}
rS(x) &= x+\lambda\int_R^1\left[S(s)-S(x)\right]dG(s)-\lambda G(R)S(x)-r(U+V)\\
&= x+\lambda\int_R^1 S(s)dG(s)-\lambda S(x)-r(U+V)\\
(r+\lambda)\,S(x) &= x+\lambda\int_R^1 S(s)dG(s)-r(U+V)
\end{aligned}$$

This expression can be derived in an alternative way. Let $J(x)$ and $W(x)$ the capital value of a matched technology and satisfied demand in a match and let U and V be the respective capital values of unmatched technology and unsatisfied demand. Then the continuation values of J and W must obey

$$\begin{aligned} rW(x) &= w + \lambda \int_R^1 W(s)dG(s) - \lambda W(x) + \lambda G(R)U \\ rJ(x) &= x - w + \lambda \int_R^1 J(s)dG(s) - \lambda J(x) + \lambda G(R)V \end{aligned}$$

Subtract the fallback flow surplus $r(U+V)$ from the sum of these equations:

$$\begin{aligned} rW(x) + rJ(x) - rU - rV &= x + \lambda \int_R^1 [J(s) + W(s)]\, dG(s) - \lambda [J(x) + W(x)] \\ &\quad + \lambda G(R)(U+V) - r(U+V) \\ rS(x) &= x + \lambda \int_R^1 [J(s) + W(s)]\, dG(s) - \lambda [J(x) + W(x)] \\ &\quad + \lambda G(R)(U+V) - r(U+V) \\ &\quad - \lambda(1-G(R))(U+V) + \lambda(1-G(R))(U+V) \\ rS(x) &= x + \lambda \int_R^1 S(s)dG(s) - \lambda S(x) - r(U+V) \\ (r+\lambda) S(x) &= x + \lambda \int_R^1 S(s)dG(s) - r(U+V) \end{aligned}$$

For new matches, $x = 1$, so the value of search for an "applier" searching for a transactions technology obeys

$$rU = z + \theta q(\theta)\mu [S(1) - U - V] \tag{43}$$

and the value of a technology in search of an application to a "supplier" is given by

$$rV = -c + q(\theta)(1-\mu)[S(1) - V - F].$$

Free entry implies $V = 0$ and thus

$$c = q(\theta)(1-\mu)(S(1) - F). \tag{44}$$

The *value of the surplus* is derived from (23) imposing $V = 0$:

$$(r+\lambda) S(x) = x + \lambda \int_R^1 S(z)dG(z) - rU \tag{45}$$

and this also holds for the threshold $x = R$, $S(R) = 0$ so we have

$$0 = R + \lambda \int_R^1 S(z)dG(z) - rU.$$

Subtracting the last from the previous equation results in:

$$S(x) = \frac{x - R}{r + \lambda}$$

so $S(1) = \frac{1-R}{r+\lambda}$. Combining this with (44) jointly imply the *coin creation condition* (24):

$$\frac{c}{q(\theta)} = (1 - \mu) \left[\frac{1 - R}{r + \lambda} - F \right]. \tag{46}$$

In an equilibrium with $V = 0$, the value of productivity R for which both participants in the match derive no surplus and agree to dissolve the match is given by $S(R) = 0$.

$$0 = R + \lambda \int_R^1 S(s)dG(s) - rU. \tag{47}$$

The steady state equilibrium value of search U follows from (43) with

$$rU = z + \theta q(\theta)\mu \left[\frac{1 - R}{r + \lambda} - F \right],$$

which combined with the vacancy valuation condition

$$rV = -c + q(\theta)(1 - \mu) \left[\frac{1 - R}{r + \lambda} - V - F \right]$$

and the free entry condition $V = 0$ yields the reservation level of search for appliers:

$$rU = z + \frac{\mu c \theta}{1 - \mu},$$

which for $x = R$ results in the *coin abandonment condition* (26) in the text:

$$z + \frac{\mu c \theta}{1 - \mu} = R + \lambda \int_R^1 S(s)dG(s). \tag{48}$$

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