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# **Volatility Modelling of CO2 Emission Allowance Spot Prices with Regime- Switching GARCH Models**

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# Volatility Modelling of CO<sub>2</sub> Emission Allowance Spot Prices with Regime-Switching GARCH Models

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## Abstract

We analyse the short-term spot price of European Union Allowances (EUAs), which is of particular importance in the transition of energy markets and for the development of new risk management strategies. Due to the characteristics of the price process, such as volatility persistence, breaks in the volatility process and heavy-tailed distributions, we investigate the use of Markov switching GARCH (MS-GARCH) models on daily spot market data from the second trading period of the EU ETS. Emphasis is given to short-term forecasting of prices and volatility. We find that MS-GARCH models distinguish well between two states and that the volatility processes in the states are clearly different. This finding can be explained by the EU ETS design. Our results support the use of MS-GARCH models for risk management, especially because their forecasting ability is better than other Markov switching or simple GARCH models.

*Keywords:* CO<sub>2</sub> Emission Allowances, CO<sub>2</sub> Emission Trading, Spot Price Modelling, Markov Switching GARCH Models, Volatility Forecasting

*JEL:* C53, G17, Q49, Q53, Q59

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## 1. Introduction

It is widely agreed among scientists, politicians and the broader public that the emission of greenhouse gases (GHGs) by human activity has led to an increase in the level of GHGs in the atmosphere, to global warming and climate change. These phenomena have serious impact on the environment, human beings and the economy. In response to these developments many industrialised countries agreed in the United Nations Framework Convention on Climate Change to stabilise the emission of GHGs and adopted the Kyoto protocol in 2005, thus accepting its binding obligations to reduce GHG emissions. The member states of the European

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Union<sup>1</sup> decided to fulfil their commitments jointly and implement a trading system for emission allowances, i.e. permits to emit one ton of CO<sub>2</sub> in the atmosphere, as a main mechanism to reduce emissions. The European Union Emissions Trading System (EU ETS) entered into force in 2005.

Since the introduction of the EU ETS a new market for European Union Allowances (EUAs) and their derivatives developed and with it carbon finance, a new field of applied econometrics, which investigates the behaviour of prices. The price dynamics and its determinants are of great importance for participating industries, and for sound risk management and hedging strategies of financial intermediaries as well as for policy makers who use them to evaluate the performance of the EU ETS. Furthermore, the market for EUAs is constantly growing, which makes it important for market participants to have a valid pricing model.

Having particular characteristics the EUAs should be regarded as a new class of assets (Benz and Trück, 2006), which requires new models for price forecasting. Hence, appropriate models for the spot dynamics are important for option pricing and risk management decisions. Under the cap-and-trade scheme of the EU ETS, the total number of allowances is fixed every year and thus the prices are induced by current demand. The demand is governed by shocks, such as temperature changes, the level of economic activity and energy prices as well as news releases concerning regulatory policy. All these events can alter the production of CO<sub>2</sub> and hence the short-term demand for EUAs.

Consequently, as shown in the literature, the design of an emission trading system is characterized by the price dynamics of emission permits. This leads to the question whether these features are represented in spot prices and if markets can predict them appropriately as this is the most important aspect for risk management and value-at-risk calculations. Considering the complexity of full equilibrium models, we concentrate on the performance of reduced models for emission permit prices with respect to historical spot prices.

A number of studies have focussed on the price determinants of EUA spot prices (e.g. Mansanet-Bataller et al., 2007; Alberola et al., 2007, 2008a,b; Chevallier, 2009; Hintermann, 2010; Hitzemann and Uhrig-Homburg, 2013). These studies found long-term relationships between EUA spot prices and energy prices, extreme weather events and economic activity. However, these results are sample-dependent and time-dependent, as the relationships change over time. Instead of that, we evaluate the short-term price modelling and forecasting of EUAs traded under the EU ETS by considering different variants of the underlying volatility process. For that, we do not incorporate externalities in our model and endogenise the break points by looking for models that fit longer time series.

Only a few studies investigate the stochastic behavior of short-term spot prices and provide an econometric analysis, such as Paoletta and Taschini (2008), Seifert et al. (2008), Daskalakis et al. (2009) and Benz and

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<sup>1</sup>In 2007 three non-EU members, Iceland, Liechtenstein and Norway joined the European Union Emissions Trading System

Trück (2009). The latter investigate the performance of GARCH models and Markov regime switching models and find that both approaches give satisfying results for the first period of EU ETS. GARCH volatility models with fixed parameters are too restrictive for long time series due to breaks in volatility process. We propose the use of Markov regime Switching GARCH (MS-GARCH) as a more flexible-accurate model for spot price dynamics. The MS-GARCH model as introduced by Hamilton and Susmel (1994) combines the strength of a regime switching model, which can capture breaks and non-linearities in the underlying stochastic process, with the possibility to model conditional volatility and volatility clustering. This additional volatility price component leads to a large improvement in the modeling and forecast process.

The contribution of this paper is twofold: firstly, to the best of our knowledge, this is the first paper to model EUA spot market prices from the second trading period of the EU ETS from 2008 to 2012 (Phase II) with MS-GARCH models. There is no evidence yet on how to specify appropriately reduced-form models for spot prices observed in the market. We use the specification of the MS-GARCH model of Klaassen (2002), which overcomes the issue of path-dependence in the estimation procedure. Secondly, we assess and compare the performance of the MS-GARCH models to other models within other state specifications. The results show that MS-GARCH models outperform standard approaches, describing the observed breaks and volatility clustering in the EUA spot market precisely and revealing the existence of two regimes, respectively with low and high volatility as explained by the EU ETS market characteristics. MS-GARCH models provide a better in-sample fit and density forecasts since the MS-GARCH models solve the problem of volatility persistence observed when using simple GARCH models.

The remainder of this paper is organized as follows. Section 2 gives a brief overview of the EU ETS and the EU carbon market. Section 3 presents recent literature on the modelling of EUA prices. In section 4 we describe the MS-GARCH model as well as other models and the estimation procedures. Section 5 provides an empirical analysis of EUA spot market prices (Phase II) and gives the results of the evaluation of the forecasting ability of the models under consideration. Section 6 concludes and makes suggestions for further research.

## **2. EU ETS and CO<sub>2</sub> trading**

The EU ETS system was designed using the US SO<sub>2</sub> market as a blueprint and laid down in EU Directive 2003/87/EC. Since 2005 there have been three trading periods. The first trading period, Phase I, lasted from 2005 until 2007 and served as a pilot period to test the market infrastructure. In Phase I the EUAs were freely distributed to the emitting installations. However, the liquidity in the market was low and due to oversupply

and the fact that the allowances lost their value at the end of the trading period <sup>2</sup>, prices collapsed towards the end of the trading period. Phase II, which lasted from 2008 until 2012, was the first Kyoto commitment period. Since Phase II banking and borrowing of allowances between years and trading periods is allowed, which reduces the risk of prices to collapse towards the end of the trading period (European Commission, 2012). Both in Phases I and II the allowances were distributed by the principle of grandfathering, i.e. the number of allowances a firm received were relative to the historical emission levels of its installations. The drawback of grandfathering is that it gives rents to existing firms and erects entrance barriers to new firms (Lutz et al., 2013). Therefore, in the current Phase III, which runs from 2013 until 2020, auctioning of EUAs gradually replaces free allocation. Most available empirical research uses data from this first, pilot, period. The price signals in this period were however distorted due to an oversupply of EUAs, which is why we use data from the second trading period in this study.

Replacing command-and-control regulations to control emissions, the EU ETS is a cap-and-trade system, which means that the regulator, the European Commission, fixes the total amount of emissions and allowances issued in a period. If a firm's emissions exceed the allocated volume of allowances, they can either buy allowances on the market or take abatement measures. Similarly, surplus allowances can be sold. In this way, the right to emit CO<sub>2</sub> becomes a tradable asset. The advantage of cap-and-trade system is that the marginal abatement costs are equalised among firms, independent of the initial allocation of allowances (Hintermann, 2010). Each year on April 30 firms have to surrender the number of allowances corresponding to the emissions of the previous year. If they fail to do so, the firms have to pay a penalty, 40 EUR and 100 EUR per ton CO<sub>2</sub> emitted in Phases I and II respectively, and to surrender the lacking allowances next year.

The EU ETS created a new market for CO<sub>2</sub> allowances and is now the world's largest carbon market, covering more than 11,000 installations in several sectors. Currently the system covers amongst others power plants, coke ovens, iron and steel factories, and factories producing cement, glass, lime, bricks, ceramics, pulp and paper (European Commission, 2012). The energy sector accounts for roughly half the emissions under the scheme. About half of the total CO<sub>2</sub> emissions in the EU are currently regulated by the EU ETS, while the number of installations included is still growing. Several types of transactions and derivatives have evolved. While in Phase I EUAs were mainly traded in over-the-counter (OTC) transactions, in Phase II they were traded bilaterally, in OTC transactions and on exchanges (Hintermann, 2010). There are spot, future, forward and option markets for EUAs. The spot contracts are traded on several exchanges, amongst others on Bluenext, Climex, European Energy Exchange, Green Exchange, Intercontinental exchange and Nord Pool (European Commission, 2012) of which Bluenext is the largest and most liquid exchange, covering about 70

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<sup>2</sup>Allowances issued during the first period were not transferable to the second period and hence lost their value at the end of the period. Since the second trading period, banking of allowances across periods is allowed.

per cent of total spot market transactions. Table 1 presents the total trade volume on these exchanges, which shows a steady growth both in volume and in traded value (World Bank, 2012).

Year	Number of EUA (in bn.)	Traded value (in USD bn.)
2005	0.3	7.9
2006	1.1	24.4
2007	2.1	49.1
2008	3.1	100.5
2009	6.3	118.5
2010	6.8	133.6
2011	7.9	147.8

Table 1: Total spot market trade volumes of EUAs on the six largest exchanges

Benz and Trück (2006) argue that EUAs are a new type of asset, having different characteristics than traditional stocks or commodities, as they should be considered a factor of production because the right to emit is essential for production. The prices of EUAs are, unlike the prices of stocks, which are determined by expected profits, based on expected market scarcity. It is important to point out that total supply on the market is fixed by the regulator, and that firms can influence their own demand by taking abatement measures. Furthermore, as banking was not allowed between Phase I and II, EUAs lost their value at the end of the trading period. In addition the market for EUAs is an artificial market created by the EU Directive and thus sensitive to regulatory and policy changes with a potential to influence short-term demand and supply. During Phase I and II allowances were distributed free of charge.

The price dynamics of EUAs are governed by unexpected shocks, as they depend on factors such as weather, fuel prices and economic growth. Furthermore, the supply and demand is influenced by policy changes, which cannot be forecasted precisely and create unexpected shocks. The European Commission publishes every year a report about the verified emissions under the EU ETS. This is an import signal about the demand side of the market and may create shocks, too. These particularities of the price dynamics should be incorporated in an adequate allowance pricing model.

### 3. Literature review

Since the creation of the EU ETS there has been an increasing number of studies addressing the modelling of EUA prices. The largest part of the literature concentrates on the determinants and drivers of EUA prices, concluding that there is an impact of energy prices, extreme weather events and economic activity on

allowance prices, see Mansanet-Bataller et al. (2007), Alberola et al. (2008b), Alberola et al. (2007), Alberola et al. (2008a), Chevallier (2009), Conrad et al. (2010) and Hintermann (2010). However, the relation between the allowance prices and these price fundamentals depends on the sample and period considered and changes over time. The previous studies all investigate only data from short time periods, mainly from Phase I. Several authors find structural breaks in price series of EUAs. Alberola et al. (2008b) argue that regulatory changes cause these breaks, whereas Chevallier (2009) sees changes in expectations as the main reason for them. The presence of such breaks complicates the estimation of models for long-term relationships between prices and their fundamentals and calls for endogenising these breaks into the models, such as in regime switching models.

Another strand of research concentrates on the relationship between the spot and futures market for EUAs. Using a dynamics semi-parametric factor model, Trück et al. (2012) find that the EUA market was in backwardation during Phase I, whereas during Phase II the market moved from backwardation to contango. Chevallier (2012) applies two nonlinear cointegration models, a VECM with structural shift and a threshold cointegration model, to the EUA spot and futures market. He observes that the returns of spot and futures prices correct the deviations to the long-term equilibrium, with the futures price taking the lead.

Despite the growing importance of carbon finance, few studies have focussed on the stochastic properties of daily EUA spot prices and the application of models from financial econometrics to EUA data. Exceptions are the studies of Paoletta and Taschini (2008), Seifert et al. (2008), Daskalakis et al. (2009), Benz and Trück (2009) and Hitzemann and Uhrig-Homburg (2013) which focus on the stochastic properties of daily price data and provide amongst other things evidence for conditional heteroskedasticity. Paoletta and Taschini (2008) address the unconditional tail behaviour and heteroskedasticity in the price series by applying mixed GARCH models. However, their findings are only valid for a specific period at the end of Phase I. Seifert et al. (2008) use a stochastic equilibrium model to analyse the dynamics of EUA spot prices. Their main conclusion is that a EAU pricing model should have a time- and price-dependent volatility structure. Daskalakis et al. (2009) model the effects of abolishing banking on futures prices during Phase I and develop a framework for pricing and hedging of intra-phase and inter-phase futures and options on futures. Benz and Trück (2009) use Markov switching and GARCH models for stochastic modelling of the EUA spot prices in Phase I. They find strong support for the use of both types of models to model the characteristics of the series, such as different price phases, volatility clustering, skewness and excess kurtosis. The studies addressing the stochastic properties of EUA prices are limited to data from Phase I. Due to the peculiarities of the price process in Phase I as described before, the results are possibly not generalizable to Phase II. In this paper we propose to model the log returns of the EUA spot market prices from the second trading period of the EU ETS with a MS-GARCH to model breaks in the price series with different regimes and to solve the problem of volatility persistence observed when using simple GARCH models.

Finally, there is literature on other emission allowance programs, notably on the SO<sub>2</sub> permit trading system in the United States of America. This program has already been in place since 1992. However, the findings relating to the SO<sub>2</sub> market have little relevance for modelling the CO<sub>2</sub> prices in the EU, due to the different market structure and commodity nature.

## 4. Methodology

### 4.1. Regime switching models

A way to model non-linear dynamic patterns in time series such as breaks or asymmetry is the use of regime switching models. The most popular regime-switching model is the Markov regime switching model as proposed by Hamilton (1989). It is an improvement of the random switching model proposed by Quandt (1972) in which the switching is independent over time. It also performs better than structural change models, because in the latter changes are only modelled as a reaction to identifiable exogenous changes. The Markov regime switching model allows for frequent changes at random points in time, because the regime switching process is governed by a first order Markov chain.

A regime switching model divides the time series into different phases and specifies for each phase a different underlying stochastic process. The phases are also called regimes or states. We consider the Markov regime switching model in which the state variable is a latent, unobservable variable denoted by  $s_t$ . Motivated by the data and the architecture of the ETS we restrict ourselves in this paper to models with two states, so that the state space is  $\mathcal{S} = \{1, 2\}$ . The state at time  $t$  is then a realisation of a two-state homogeneous first order Markov chain and is described by the transition probabilities  $p_{jj}$  for  $j \in \mathcal{S}$ , the probability of being in the same state as in the previous period:

$$p_{jj} = \Pr(s_t = j | s_{t-1} = j) \quad (1)$$

Because  $p_{ji} = 1 - p_{jj}$  we obtain the transition matrix  $\mathbf{P}$

$$\mathbf{P} = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} = \begin{pmatrix} p_{11} & 1 - p_{11} \\ 1 - p_{22} & p_{22} \end{pmatrix} \quad (2)$$

with  $p_{ij} = \Pr(s_t = j | s_{t-1} = i)$  denoting the probability of going from state  $i$  to state  $j$ . Due to the Markov property the current state depends only on the most recent state. Hamilton (1989) used the Markov regime switching model focussing on the mean behaviour of the variables, but the stochastic process in state  $j$ ,  $y_j$  can also be specified by other models, e.g. conditional variance models.

There are two types of uncertainty when estimating Markov switching models, the unobservable state  $s_t$  the stochastic process is in at time  $t$  and the population parameters  $\theta_j$  specifying the process in state  $j$ . Inference



on the latent state variable can only be made through the observations of  $y_t$  as  $s_t$  is not observable. The conditional probability that the process is in state  $j$  at time  $t$  is

$$\xi_{jt} = P(s_t = j | \Omega_t; \theta) \quad (3)$$

for  $j \in \mathcal{S}$ , where  $\Omega_t = \{y_t, y_{t-1}, \dots, y_1\}$  are the observations until time  $t$  and  $\theta$  is the parameter vector with the parameters specifying the stochastic process in both states and the transition probabilities. By construction,  $\sum_{j=1}^2 \xi_{jt} = 1$ . The inference on the state probabilities  $\xi_{jt}$  is performed iteratively by evaluating the density  $\eta_{jt}$  under both regimes

$$\eta_{jt} = g_j(y_t | s_t = j, \Omega_{t-1}; \theta) \quad (4)$$

where  $g_j$  is the density function of the process in state  $j$ , which depends on the specification of the model and the distribution of the error term. Knowing  $\xi_{i,t-1}$  the conditional density of observation  $y_t$  is

$$f(y_t | \Omega_{t-1}; \theta) = \sum_{i=1}^2 \sum_{j=1}^2 p_{ij} \xi_{i,t-1} \eta_{jt} \quad (5)$$

and the probability to be in state  $j$  at time  $t$  is

$$\xi_{jt} = \frac{\sum_{i=1}^2 p_{ij} \xi_{i,t-1} \eta_{jt}}{f(y_t | \Omega_{t-1}; \theta)} \quad (6)$$

This yields the conditional log likelihood of the observed data

$$\ell_{MS}(y_1, y_2, \dots, y_T | y_0; \theta) = \sum_{t=1}^T \ln f(y_t | \Omega_{t-1}; \theta) \quad (7)$$

The maximum likelihood estimator is defined by maximising (7) w.r.t.  $\theta$ . For the initialisation of  $\xi$  we use the iterative approach as suggested by Hamilton (1995) and chose  $\xi_{10} = \xi_{20} = \frac{1}{2}$ .

#### 4.2. GARCH Markov regime switching model (MS-GARCH)

The specification of the density function  $g_j$  in Equation (4) is straightforward when using a normal distribution or an Autoregressive (AR) model in the states. In case of a GARCH specification for the conditional variance we encounter a problem with the specification of the volatility. Due to the autoregressive structure of the variance, its specification is path-dependent, it depends on all the preceding unobserved state variables. Hamilton and Susmel (1994) and Cai (1994) were the first to explore Markov regime switching models with ARCH specifications in the states. However, the ARCH effects do not have the problem of path dependency.

The path dependency in the GARCH model makes evaluation of the log likelihood function intractable, as the number of paths grows exponentially with the number of observations. Gray (1996) and Klaassen (2002)

made simplifications to the GARCH model to avoid the problem of path dependency and make log likelihood estimation possible.

Klaassen (2002) uses a first-order recursive procedure for the variance specification that integrates out the path dependence by using the law of iterated expectations. Thus the variance of  $y_t$  in state  $j$  evaluated at time  $t - 1$  is described as

$$\begin{aligned} \text{Var}_{t-1}(y_t | s_t = j) &= \text{Var}_{t-1}(\varepsilon_t | s_t = j) \\ &= \alpha_{0j} + \alpha_{1j}\varepsilon_{t-1} + \beta_{1j} \mathbf{E}_{t-1} [\text{Var}_{t-2}(\varepsilon_{t-1} | s_{t-1})] \end{aligned} \quad (8)$$

Here we use the variance specification for the MS-GARCH model as in (8). The advantages of this approach are that it allows for recursive estimation of the log likelihood function and for recursive forecasting. Moreover, the regime switching GARCH model solves the problem of volatility persistence encountered in simple GARCH models.

Since there is no analytical solution for estimating Markov switching models, we use a numerical optimization algorithm in order to estimate the models. There are several numerical optimization algorithms available, such as the Newton-Raphson method or Fisher's scoring algorithms. Unfortunately in case of Markov switching models the performance of these algorithms depends on the starting values of the parameters, as they often find only local extrema. Therefore we use the Differential Evolution (DE) algorithm, which does not require the specification of starting values, but is computationally more intensive. DE makes use of arithmetic instead of logical operations and works particularly well to find the global optimum of a real-valued function of real-valued parameters (Price et al., 2005). We use the R package DEoptim, which is developed by Ardia and Mullen (2009).

### 4.3. Other models

In order to assess the performance of our forecasting approach, we compare the MS-GARCH model with different benchmark models given in the literature. In all models it is assumed that the error terms are i.i.d. random variables with zero mean and variance  $\sigma^2$ . This assumption is later verified in the empirical analysis. AR models can capture the time varying mean of a stochastic process. An AR( $p$ ) process with lag order  $p$  is defined as

$$y_t = c + \sum_{k=1}^p \phi_k y_{t-k} + \varepsilon_t \quad (9)$$

with  $\varepsilon_t$  a sequence of i.i.d. random variables with mean 0 and variance  $\sigma^2$  (Hamilton, 1994; Tsay, 2010). We also impose conditions on  $\phi_k$  for  $k \in \{1, \dots, p\}$  to ensure stationarity. When we assume that the error terms are normally distributed, i.e.  $\varepsilon_t \stackrel{i.i.d.}{\sim} \text{N}(0, \sigma^2)$ , the vector of parameters to estimate is  $\theta_{AR(p)} =$

$(c, \phi_1, \dots, \phi_p, \sigma^2)'$  and the maximum likelihood estimator is defined as maximising the log likelihood function evaluated for the observations  $y_{p+1}, \dots, y_T$ , as we need the first  $p$  observations for starting the AR process.

AR models assume homoskedasticity of the error terms. The autoregressive conditional heteroskedastic (ARCH) model of Engle (1982) was the first model to successfully provide a systemic framework to address the issue of heteroskedasticity in time series. In ARCH models the error terms are serially uncorrelated but contain higher-order dependence and can be modelled as a quadratic function of the past error terms. In practice, the ARCH model needs many lags to describe the volatility process. In order to avoid this, Bollerslev (1986) proposed a generalization of the ARCH model, the Generalised ARCH (GARCH) model, which includes the own lags of the conditional variance into the ARCH model.  $y_t$  follows a GARCH( $p, q$ ) model if

$$y_t = \sigma_t \varepsilon_t \tag{10}$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 \tag{11}$$

where  $\varepsilon_t$  is a sequence of i.i.d. random variables with zero mean and variance 1. In this case, the parameter vector to be estimated is  $\theta_{GARCH} = (\alpha_0, \dots, \alpha_p, \beta_1, \dots, \beta_q)'$ . Usually a GARCH(1,1) process suffices to capture the conditional heteroskedasticity in the series, so the parameter vector reduces to  $\theta_{GARCH(1,1)} = (\alpha_1, \beta_1)'$ . In order to ensure stationarity and a strictly positive conditional variance, the coefficients have to satisfy  $\alpha_0 > 0$ ,  $\alpha_i \geq 0$ ,  $\beta_i \geq 0$  and  $\sum_{i=1}^p \alpha_i + \sum_{j=1}^q \beta_j < 1$  (Tsay, 2010).

Assuming that the innovations are identically and independently distributed and drawn from a standard normal distribution, i.e.  $\varepsilon_t \stackrel{i.i.d.}{\sim} N(0, 1)$ , the conditional maximum likelihood estimator is defined as maximising the following conditional log-likelihood function conditional on its initial values

$$\ell_{GARCH}(y_1, y_2, \dots, y_T | \sigma_0^2, \varepsilon_0^2; \theta_{GARCH}) = -\frac{1}{2} \sum_{t=1}^T \left( \log(2\pi) + \log(\sigma_t^2) + \frac{\varepsilon_t^2}{\sigma_t^2} \right) \tag{12}$$

where  $T$  is the number of observations and  $\sigma_t^2$  as defined in (11) Zivot (2008). We use the conditional log likelihood, as the unconditional one is not known in closed form. The log likelihood is conditional on the initial values for  $\sigma_t^2$  and  $\varepsilon_t^2$ . We use the empirical variance of  $y_t$  to initialize the process as proposed by (Zivot, 2008).

The GARCH model captures the existence of volatility clustering in a more parsimonious way than the ARCH model. In fact a GARCH( $p, q$ ) model can be described as a ARCH( $\infty$ ) model (Teräsvirta, 2006). Furthermore, it can be shown that the tails of a GARCH model with normally distributed error terms are heavier than those of the normal distribution (Tsay, 2010). Heavy tails are often observed in financial time

series. The unconditional variance  $\bar{\sigma}^2$  of a GARCH model is constant and is equal to

$$\bar{\sigma}^2 = \frac{\alpha_0}{1 - \sum_{i=1}^p \alpha_i - \sum_{j=1}^q \beta_j} \quad (13)$$

The GARCH variance equation can be combined with a specification of the mean. In this paper we use a GARCH process with unconditional mean  $c$ . In this case, (10) is replaced by

$$y_t = c + \sigma_t \varepsilon_t \quad (14)$$

We also consider a specification for the conditional mean of an AR( $p$ ) process. This replaces (10) with

$$y_t = c + \sum_{k=1}^p \phi_k y_{t-k} + \sigma_t \varepsilon_t \quad (15)$$

and certain conditions on  $\phi_k$  for  $k \in \{1, \dots, p\}$  to ensure stationarity.

## 5. Empirical results

### 5.1. Data

For our empirical analysis we use daily spot market prices from Bluenext in Paris as this is the most liquid market place for spot contracts. The price is for one EUA, which gives the right to emit one ton of CO<sub>2</sub>. We use data from February 26, 2008 until November 28, 2012, which covers with 1,183 daily observations almost the whole of Phase II. For calibration of the models (in-sample fit), we use the data from the period February 26, 2008 until December 30, 2010. The data from January 3, 2011 until November 26, 2012 is used for out-of-sample evaluation of the models. The data was retrieved from Bloomberg Professional Service with ticker PNXCSPT2. We perform our analysis on log returns of the prices, which are defined as

$$y_t = \log \left( \frac{p_t}{p_{t-1}} \right) \quad (16)$$

where  $p_t$  is the daily closing price on the spot market at time  $t$ . We use log returns for our analysis in order to obtain well-behaved error terms. The top panel in Figure 1 presents a plot of the daily EUA prices and the bottom panel a plot of the daily log returns. The plot of the prices shows that the prices in phase II are, contrary to the prices in Phase I, always positive in the period under consideration and have a minimum at 6.04 EUR. The plot of the prices shows as well a decrease of the prices in 2009 and 2011, which corresponds to the effect of the economic crises in both periods. The bottom plot in Figure 1 clearly shows volatility clustering and heteroskedasticity. Especially in periods of falling prices volatility seems to be higher. This can be explained by the fact that the supply of EUAs is inelastic. When the demand decreases due to an external shock, there might rise doubt about the overall shortage of certificates on the market. In case of oversupply the EUAs could become worthless. We will confirm this observation when interpreting the regimes in the

regime switching model. Furthermore, we see an increase of volatility in the log returns between February and April of each year. This can be explained by the double bookkeeping in this period. The emitting companies received the allowances for the current year in February and had to surrender the allowances for the previous year at April 30.

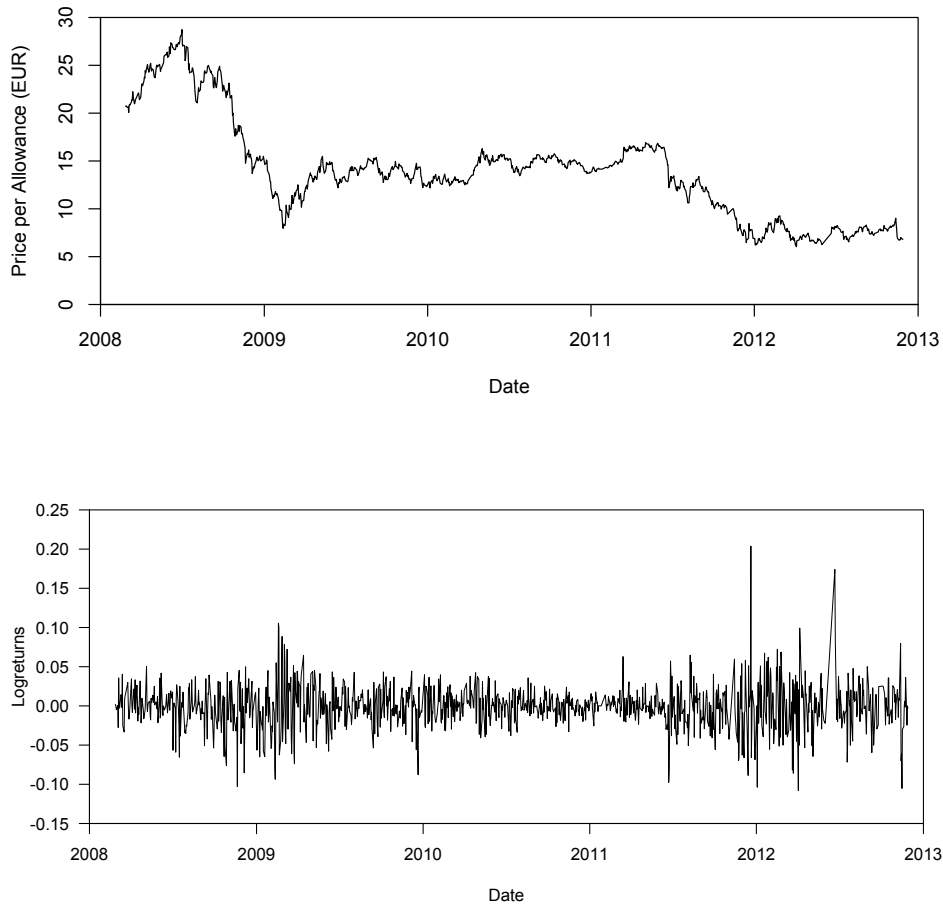


Figure 1: Daily EUA prices (top panel) and log-returns (bottom panel) from February 26, 2008 until November 28, 2012

Table 2 presents the descriptive statistics of both the spot prices and the log returns for the complete time series, the in-sample and out-of-sample period. The log returns are not significantly different from zero. Both the prices and the log returns show excess kurtosis, which means that the data is heavy-tailed. The prices are positively skewed, whereas the log returns are little skewed. The data is not normally distributed. The observed characteristics of the log return series justify the investigation into MS-GARCH models.

period	N	Mean	Min	Max	Std Dev	Skew	Kurt
Prices							
2008-2012	1183	14.016	6.040	28.730	5.071	0.76	3.32
2008-2010	725	16.273	7.960	28.730	4.581	1.09	2.99
2011-2012	458	10.433	6.040	16.930	3.505	0.61	8.44
Log returns							
2008-2012	1182	-0.0009	-0.1081	0.2038	0.0276	0.03	8.03
2008-2010	724	-0.0006	-0.1029	0.1055	0.0244	-0.20	5.02
2011-2012	458	-0.0015	-0.1081	0.2038	0.0320	0.61	8.84

Table 2: Descriptive statistics for daily prices and daily log returns

## 5.2. Results

In this section we present the results of estimating the models on the log returns in the in-sample period. We then forecast the log-returns for the out-of-sample period and finally compare the performance of the models. First we perform stationarity tests on the data. All computations were carried out in R.

Our models depend on the stationarity assumption of the time series. Therefore we apply both the Augmented Dickey Fuller (ADF) test and the Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test in order to evaluate the presence of a unit root in the log return series. Table 3 presents the test statistics, the p-values and the used lag orders for both stationarity tests. The ADF test reject the null hypothesis of a unit root process and the KPSS test accepts the null hypothesis of a stationary process. Both tests come to the same conclusion for all three periods at high significance levels.

period	ADF			KPSS		
	test statistic	p-value	lags	test statistic	p-value	lags
2008-2012	-7.437	<0.01	22	0.069	>0.1	7
2008-2010	-5.321	<0.01	20	0.108	>0.1	6
2011-2012	-4.879	<0.01	17	0.071	>0.1	4

Table 3: Results of the ADF and KPSS tests for stationarity

The next step is to test the autocorrelation structure (ACF and PACF) in the log returns. The ACF in the top panel of Figure 2 shows the presence of autocorrelation of orders 1, 2 and 4, while the sample PACF of the log returns (bottom panel of Figure 2) suggests an AR(3) process. However, according to the Akaike information criterion (AIC) (Akaike, 1973) an AR(4) process is preferred.

The parameter estimates for the fitted normal distribution and the AR model are presented in Table 4. The estimated mean and variance of the normal distribution and unconditional mean of the AR(4) model are almost the same. This indicates that the additional explanatory power of the AR model is rather limited.

Parameter	Normal	AR(4)	GARCH(1,1)	AR(4)-GARCH(1,1)
$\mu$	-0.0006	–		
$c$	–	-0.0006	-0.0003	-0.0002
$\phi_1$	–	0.0988	–	0.0031
$\phi_2$	–	-0.1391	–	-0.0696
$\phi_3$	–	0.0795	–	0.0550
$\phi_4$	–	0.0609	–	0.0199
Variance equation				
$\sigma$	0.0244	0.0240	–	–
$\alpha_0$	–	–	0.0000	0.0000
$\alpha_1$	–	–	0.0726	0.0697
$\beta_1$	–	–	0.9199	0.9214
Unconditional expectations				
$E[y_t]$	-0.0006	-0.0006	-0.0003	-0.0002
$\sigma$	0.0244	0.0240	0.0239	0.0247

Table 4: Parameter estimates of Normal, AR, GARCH(1,1) and AR(4)-GARCH(1,1) models

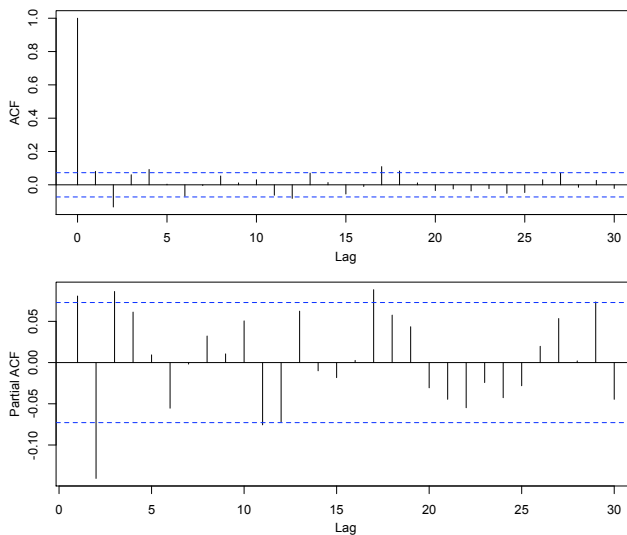


Figure 2: ACF (top panel) and PACF (bottom panel) of the log return series from February 26, 2008 until December 30, 2010

Volatility clustering or GARCH effects in the data can be detected by autocorrelation in the squared or absolute returns of the series or in the residuals of an estimated model for the mean. The top panel in Figure 3 plots the residuals of the AR(4) model and the bottom panel shows the ACF of the squared residuals. The upper plot shows a non-constant variance and the lower plot correlation in the residuals, which both are indicators for GARCH effects. To test this intuition we use the Ljung-Box or modified Q-statistic proposed by Box and Pierce (1978). The Ljung-Box test statistic for GARCH effects in squared residuals of the estimated AR(4) model is equal to 61.793 with a p-value of 3.775e-15. The test rejects the null hypothesis of white noise and confirms the presence of GARCH effects in the data.

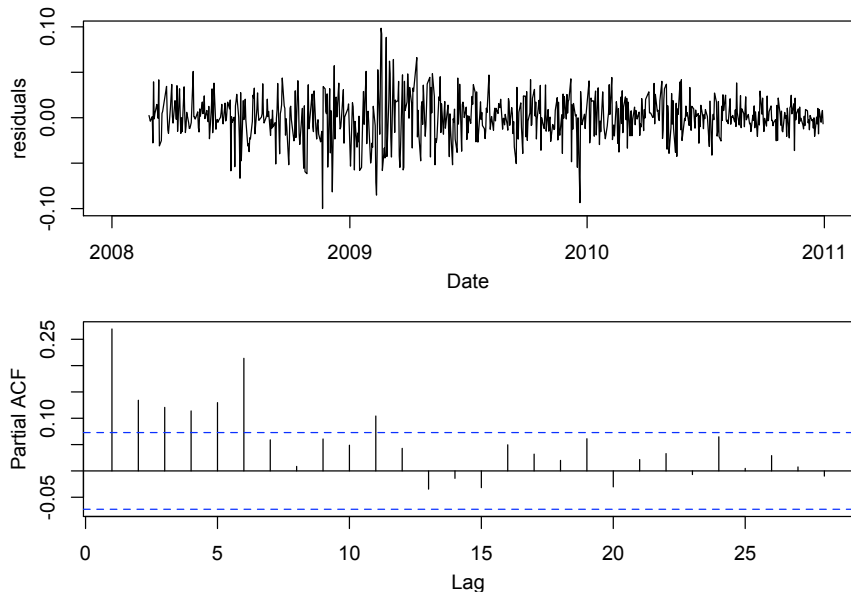


Figure 3: Residuals of the AR(4) model (upper panel) and PACF of the squared residuals of the AR(4) model (lower panel)

Table 4 shows the parameter estimates of the GARCH(1,1) and the AR(4)-GARCH(1,1) models. All parameters are significant in both models, except for  $\alpha_0$ . We also estimated higher order GARCH models, however the coefficients were not significant. For the AR-GARCH model we choose the same lag order as in the AR model estimated before. For both GARCH models the sum of the parameters  $\alpha_1$  and  $\beta_1$  is close to one and  $\alpha_0$  is small which is an indication for a high level of volatility persistence and a slow reversion to the mean. In both models we observe an unconditional mean smaller than the mean of the series. The unconditional standard deviation is close to the empirical standard deviation of the series.

Table 5 presents the in-sample parameter estimates of the Markov regime switching models with a fitted normal density (MS-Normal) and an AR(4) process (MS-AR(4)) in the regimes. In both models one state is characterised by low volatility and a positive mean ('low') and the other state is characterised by high volatility and a negative mean ('high'). The 'low' state can be interpreted as the base or normal state and the



'high' state as a period of uncertainty. This uncertainty is a result of the design of the ETS. An unexpected event, such as a drop in economic activity or regulatory announcement could reduce CO<sub>2</sub> production and thus the demand for EUAs and result in a falling price. As the supply side is fixed, there might be uncertainty on the market, whether the demand will be higher than the supply, which hence causes higher volatility.

	MS-Normal		MS-AR(4)		MS-GARCH(1,1)		MS-AR(4)-GARCH(1,1)	
Regime (i)	1 (low)	2 (high)	1 (low)	2 (high)	1 (low)	2 (high)	1 (low)	2 (high)
$\mu$	0.0014	-0.0037	-	-	-	-	-	-
$c$	-	-	0.0017	-0.0033	0.0009	-0.0042	0.0011	-0.0090
$\phi_1$	-	-	-0.0597	0.1647	-	-	-0.0339	0.3013
$\phi_2$	-	-	-0.0662	-0.1947	-	-	-0.0637	-0.2108
$\phi_3$	-	-	0.0086	0.1116	-	-	0.0261	0.1965
$\phi_4$	-	-	-0.0870	0.1078	-	-	-0.0315	0.2512
Variance equation								
$\sigma_i$	0.0161	0.0336	0.0159	0.0324	-	-	-	-
$\alpha_0$	-	-	-	-	0.0001	0.0003	0.0000	0.0002
$\alpha_1$	-	-	-	-	0.0013	0.1038	0.0078	0.1952
$\beta$	-	-	-	-	0.7166	0.7233	0.8645	0.7510
Markov estimates								
$p_{ii}$	0.9864	0.9749	0.9818	0.9698	0.9923	0.9821	0.9740	0.8818
Unconditional expectations								
$E[y_{t,i}]$	0.0014	-0.0037	0.0014	-0.0041	0.0009	-0.0042	0.0010	-0.0218
$E[\sigma_{t,i}]$	0.0161	0.0336	0.0159	0.0324	0.0136	0.0409	0.0101	0.0707
$P(s_t = i)$	0.6486	0.3514	0.6240	0.3760	0.6988	0.3012	0.8198	0.1802

Table 5: Parameter estimates of Markov switching Normal and AR(4) models

The variance in the 'high' state is in both models more than four times higher. This allows for sudden changes from low to high volatility by a regime change in the model. These changes are clearly visible in the estimated state probabilities in the upper panel of Figure 4. In the upper panel the estimated probability to be in the 'low' regime for the MS-AR(4) model is plotted. In periods of high volatility (identifiable in the lower panel of Figure 4) the probability to be in the low regime drops suddenly, which means that the probability to be in the high regime is very high as these probabilities sum up to 1. At any point in time the model assigns with high probability one of both regimes, which means that the model distinguishes well between states.

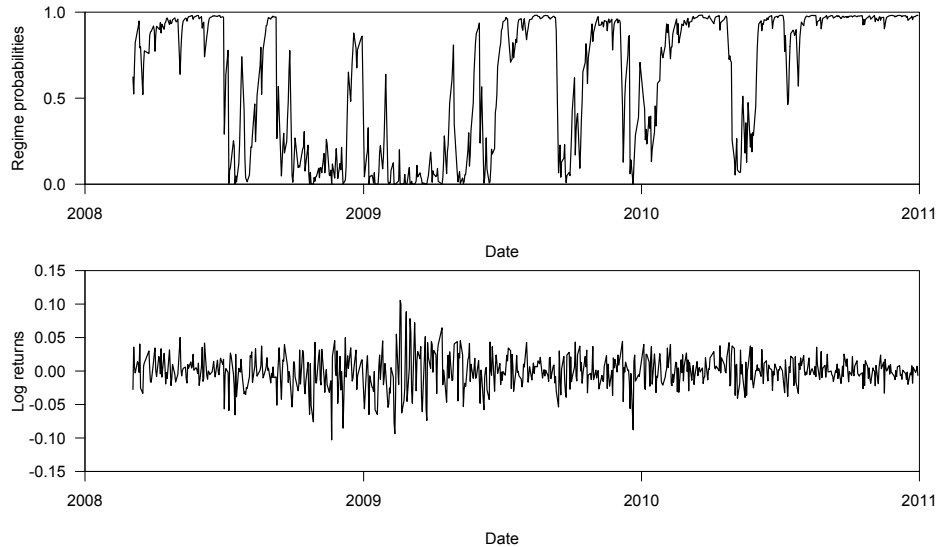


Figure 4: Estimated probabilities to be in the 'low' state for MS-AR(4) (top panel) model and log returns (bottom panel)

In both models the unconditional probability to be in the 'low' regime is much higher, 65% and 62% for respectively the MS-Normal and MS-AR(4) models. The transition probabilities to stay in the same regime are very high, close to 100% for both regimes in both models. This indicates that regime changes are rather rare. The results are similar for both models.

Table 5 also presents the estimated parameters of the MS-GARCH(1,1) and MS-AR(4)-GARCH(1,1) models. We observe the same 'low' and 'high' states as in the previous MS models. In the MS-AR(4)-GARCH(1,1) model the unconditional standard deviation in the 'high' state is even seven times higher than in the 'low' state. The transition probabilities to stay in the same state are very high for the MS-GARCH(1,1) model, which means that the number of regime switches is limited. For the MS-AR(4)-GARCH(1,1) model, we observe a lower transition probability to stay in the 'high' state, which means that this state is less stable. This is also reflected in the unconditional probability to be in the 'high' state, which is only 18%, opposed to 82% for the 'low' state. The MS-AR(4)-GARCH(1,1) model distinguishes well between the regimes. This is shown in Figure 5 which is analogous to Figure 4. Again the probability to be in the 'low' regime drops in times of high observed volatility. However, the regime selection is not as pronounced as in the case of the MS-AR(4) model.

### 5.3. Comparison of models

In order to evaluate the performance of the different models, we present several model selection criteria both for the in-sample fit and out-of-sample forecasting performance. The most natural way to compare the in-

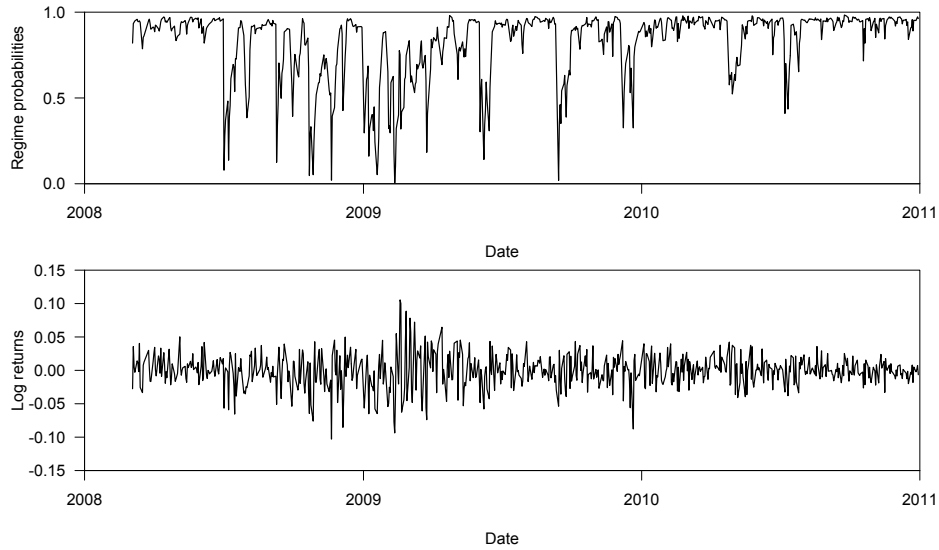


Figure 5: Estimated probabilities to be in the 'low' state for MS-AR(4)-GARCH(1,1) model (upper panel) and log returns (lower panel)

sample goodness-of-fit of the models examined is the value of the log likelihood function. We can compare these values, because all models have the same underlying distribution of the error terms and we use the same sample. The log likelihood of the MS models are naturally higher, due to the increased number of parameters. In order to account for the increased number of parameter we use the AIC criterion, which introduces a penalty term for the less parsimonious models and is defined as follows

$$\text{AIC} = -2\ell + 2k \quad (17)$$

where  $\ell$  is the value of the estimated log likelihood function and  $k$  the number of parameters in the model.

To compare the out-of-sample point forecasts of the different models we use the mean absolute error (MAE) and mean squared error (MSE). The MAE and MSE compare the actual value and the forecasted value and are respectively defined as

$$\text{MAE} = \frac{1}{h} \sum_{t=T+1}^{T+h} |\hat{y}_t - y_t| \quad (18)$$

$$\text{MSE} = \frac{1}{h} \sum_{t=T+1}^{T+h} (\hat{y}_t - y_t)^2 \quad (19)$$

where  $\hat{y}_t$  is the point forecast for time  $t$ ,  $y_t$  is the true observed value and  $h$  is the forecasting horizon.

In order to evaluate the performance of the density forecasts we perform a distributional test as described in Diebold et al. (1998). This approach is better than the comparison of confidence intervals, as this depends

on the choice of the confidence level. Assuming a normal distribution, the forecasted distribution of  $y_{t+1}$  is

$$y_{t+1} \sim N(\hat{\mu}, \hat{\sigma}^2) \quad (20)$$

where  $\hat{\mu}$  is the point forecast and  $\hat{\sigma}^2$  the forecasted variance. If this is the correct distribution with forecasted density function  $\hat{f}(y_{t+1})$  and distribution function  $\hat{F}(y_{t+1})$ , then Rosenblatt (1952) shows that  $\hat{F}(y_{t+1})$  is uniformly distributed on the interval  $[0, 1]$ . The density forecast can be evaluated by performing a distributional test for uniformity of  $\hat{F}(y_{t+1})$ , for example, the Kolmogorov-Smirnov test.

Table 6 presents measures for the in-sample goodness-of-fit of our models. According to the log likelihood value, the Markov switching models have a better fit than the standard model with the same specification. This result is confirmed by the AIC, which accounts for the parsimony of the models. The best in-sample fit has the MS-AR(4)-GARCH(1,1) model, according to the log likelihood and the AIC. Especially the Markov switching models have many parameters to estimate. The MS-GARCH models perform better than the MS models without a GARCH specification. The GARCH(1,1) and AR(4)-GARCH(1,1) models have a better in-sample fit than the MS models without GARCH specification according to the AIC. This contradicts the findings of Benz and Trück (2009), who found a similar in-sample fit for the GARCH and simple Markov switching models with using Phase I data. Finally, we notice that the autoregressive mean specification provides a better sample fit in all the models.

model	number of parameters	log likelihood	AIC
Normal	2	1651.06	-3298.11
AR(4)	6	1673.85	-3335.69
GARCH(1,1)	4	1732.45	-3456.89
AR(4)-GARCH (1,1)	8	1735.33	-3454.67
MS-Normal	6	1720.00	-3408.99
MS-AR(4)	14	1732.92	-3437.84
MS-GARCH(1,1)	10	1739.21	-3458.43
MS-AR(4)-GARCH(1,1)	18	1750.94	-3465.87

Table 6: Number of parameters, maximum log likelihood value and Akaike information criteria (AIC) for the estimated models

#### 5.4. Forecasting

We compare the forecasting performance of the previous models by performing out-of-sample forecasts. We make one-day-ahead forecasts for the period from January 3, 2011 until November 26, 2012 and compare

these forecasts with the true observed values. We use a recursive window approach in which we reestimate the model every day by using all previous data points since February 26, 2008. In this way the sample size increases when estimating and forecasting later log returns. The reestimation of the parameters is expected to improve the forecasting performance. Besides point forecasts for the log returns, we also focus on density forecasts, as these are often more relevant to risk managers. Also density forecasts allow to construct confidence intervals. We evaluate the forecasts by using the techniques described before.

The point forecasts are evaluated by calculating the average forecast error. Table 7 presents the MAE and MSE for all models. The smallest MAE is observed for the MS-AR(4)-GARCH(1,1) model, which has the second smallest MSE. The performance of the fitted normal distribution is remarkable. However, the differences in the values for MAE and MSE are small. This might be due to the short forecasting horizon. We therefore conclude that the results for the mean forecasting are without substantial differences.

model	MAE	MSE	KS	p-value KS
Normal	0.02226	0.0010263	0.4737	<2.2e-16
AR(4)	0.02244	0.0010583	0.0469	0.2657
GARCH(1,1)	0.02230	0.0010282	0.0536	0.1446
AR(4)-GARCH (1,1)	0.02231	0.0010391	0.0501	0.2005
MS-Normal	0.02234	0.0010266	0.0367	0.5695
MS-AR(4)	0.02260	0.0010407	0.0346	0.6419
MS-GARCH(1,1)	0.02232	0.0010254	0.0321	0.7314
MS-AR(4)-GARCH(1,1)	0.02229	0.0010268	0.0370	0.5592

Table 7: Mean absolute error (MAE) and mean squared error (MSE) for point forecasts and Kolmogorov-Smirnov (KS) test for density forecasts

Figures 6 plot the forecasted confidence intervals based on the normality assumption (black), the point forecasts (red) and the true values (blue). We observe smaller confidence intervals for the MS-GARCH models. Especially we see that the problem of volatility persistence is reduced by the MS-GARCH model, when comparing the confidence intervals of the MS-GARCH models with those of the GARCH models. To test this observation, we use the density test as described above. The results of the Kolmogorov-Smirnov test are presented in Table 7. The results for the MS models are much better. The best density forecasts are made with MS-GARCH(1,1) model.

Our models are based on the assumption of normality of the error terms. Figure 8 shows the kernel density plots of the standardized forecast errors. We see that the standardized forecast errors for the non-MS models seem to have heavier tails than the normal distribution. The MS models show almost normally distributed

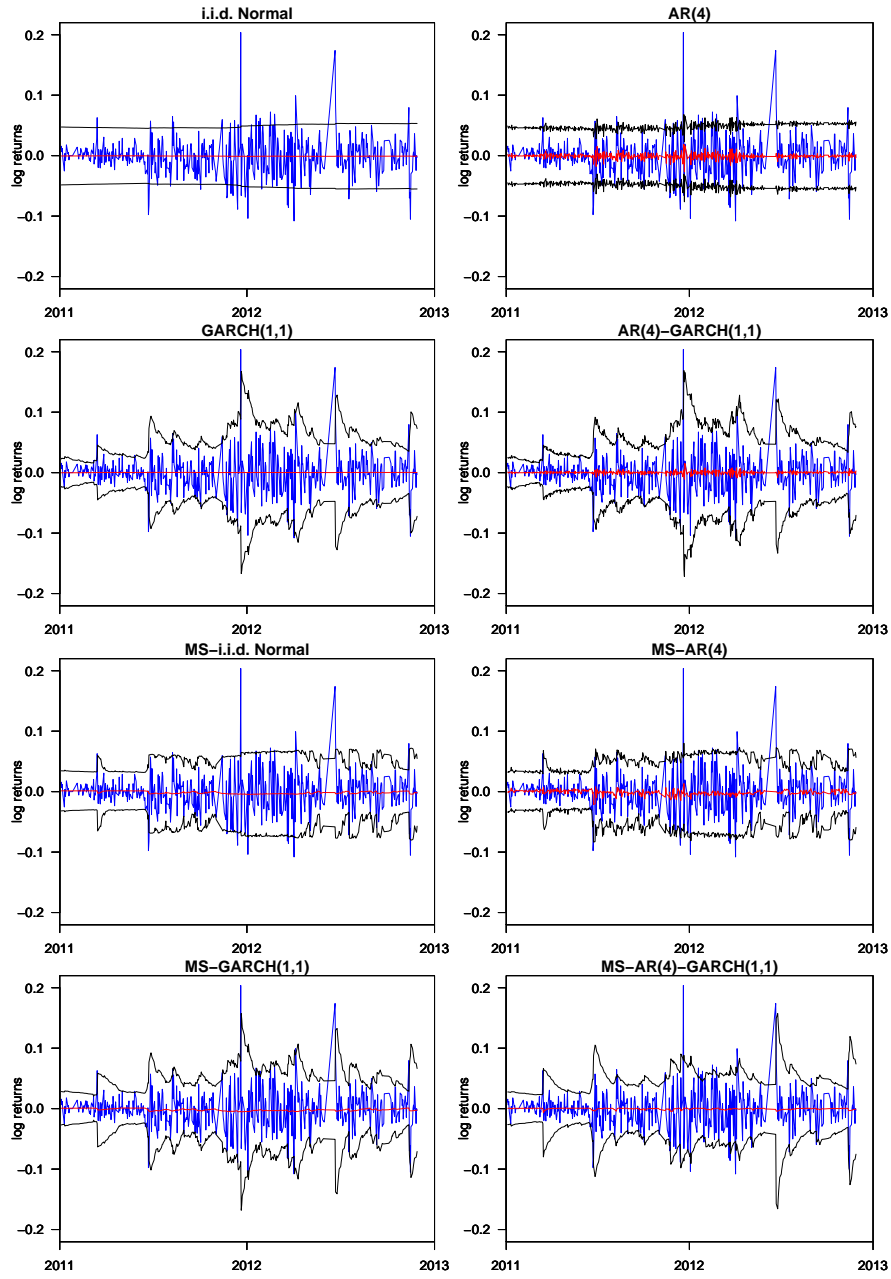


Figure 6: Forecasted confidence intervals (black), point forecasts (red) and true values (blue)

standardised forecast errors. In order to test this intuition we perform both a the Shapiro-Wilk test (Shapiro and Wilk, 1965) and a Kolmogorov-Smirnov test for normality of the standardized forecast errors. The Shapiro-Wilk test tests the null hypothesis of normality, which is rejected if the value of the test statistic is close to zero. Values close to 1 support the null hypothesis. The Kolmogorov-Smirnov test is a non-parametric test with the null hypothesis of normality. The test statistic follows the Kolmogorov distribution. The results of both tests are presented in Table 8. We do not reject the null hypothesis for any of the Markov switching models. Also for all standard models we do not reject the null hypothesis of normality.

model	SW	KS	
	test statistic	test statistic	p-value
Normal	0.935	0.0396	0.4701
AR(4)	0.933	0.0469	0.2657
GARCH(1,1)	0.936	0.0536	0.1446
AR(4)-GARCH (1,1)	0.937	0.0498	0.2067
MS-Normal	0.946	0.0367	0.5695
MS-AR(4)	0.949	0.0346	0.6419
MS-GARCH(1,1)	0.936	0.0321	0.7314
MS-AR(4)-GARCH(1,1)	0.941	0.0370	0.5592

Table 8: Results of Shapiro-Wilk (SW) and Kolmogorv-Smirnov (KS) tests for normality of the forecast errors

## 6. Conclusion

In this paper we studied the short-term spot price behaviour of EUAs, which is of particular importance in the transition of energy markets and for the development of new risk management strategies. Emphasis was given to short-term forecasting of prices and volatility. We analyse the log returns of Phase II spot market prices by investigating MS-GARCH models. The results suggest that MS-GARCH models justifies very well the feature behaviour in spot prices: volatility clustering, breaks in the volatility process and heavy-tailed distributions.

Comparing the performance of the MS-GARCH models to other models within other state specifications, suggest that MS models are clearly better than those of the non-switching models. The MS estimate two clearly different regimes, a 'low' regime with low volatility and a high mean and a 'high' regime with high volatility and a low mean, which are explained by the EU ETS market characteristics. The 'low' state can be interpreted as the base or normal state and the 'high' state as a period of uncertainty. This uncertainty is a

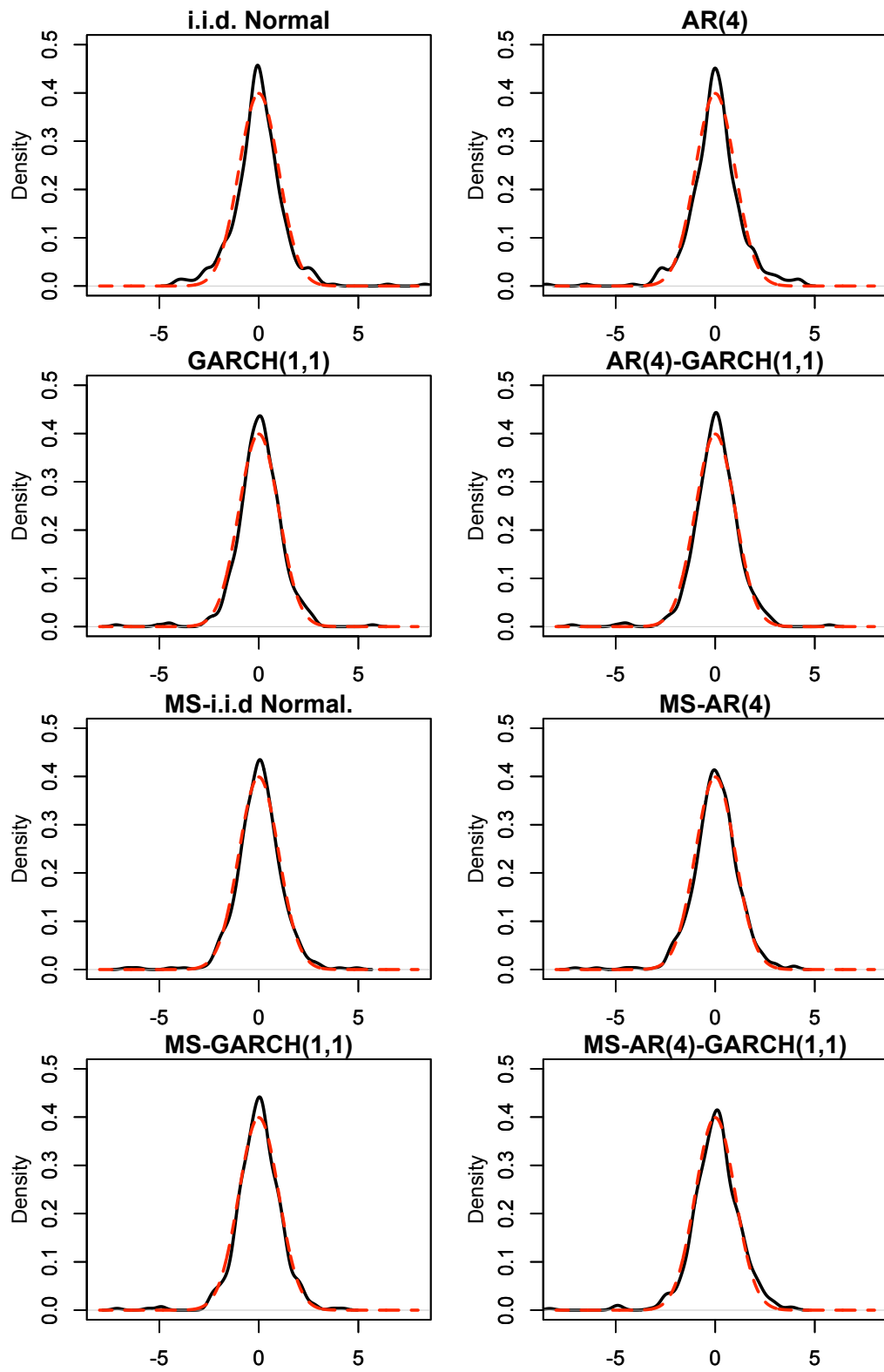


Figure 7: Kernel density plots of standardised forecast errors (black solid line) and normal densities (red dashed line)



result of the design of the EU ETS. An unexpected event, such as a drop in economic activity or regulatory announcement reduce CO<sub>2</sub> production and thus the demand for EUAs and result in a falling price. As the supply side is fixed, there might be uncertainty on the market, whether the demand will be higher than the supply, which hence causes higher volatility. The regime switching models distinguish well between the 'high' and 'low' states, indicating that the data has different states. MS-GARCH models provide a better in-sample fit, point and density forecasts since the MS-GARCH models solve the problem of volatility persistence observed when using simple GARCH models.

So far, we applied only models with normally distributed error terms. Although the modelling of fat tails is partially addressed by GARCH models, we suggest to investigate the use of other heavy-tailed distributions, such as the Student's t-distribution as suggested by Klaassen (2002) for MS-GARCH models.

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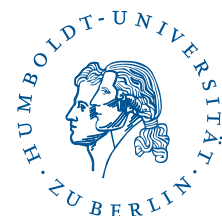
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