

SFB 649 Discussion Paper 2017-021

## The systemic risk of central SIFIs

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This research was supported by the Deutsche  
Forschungsgemeinschaft through the SFB 649 "Economic Risk".

<http://sfb649.wiwi.hu-berlin.de>  
ISSN 1860-5664

SFB 649, Humboldt-Universität zu Berlin  
Spandauer Straße 1, D-10178 Berlin



SFB 649 ECONOMIC RISK BERLIN

# The systemic risk of central SIFIs\*

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August 22, 2017

## Abstract

Systemic risk quantification in the current literature is concentrated on market-based methods such as CoVaR(Adrian and Brunnermeier (2016)). Although it is easily implemented, the interactions among the variables of interest and their joint distribution are less addressed. To quantify systemic risk in a system-wide perspective, we propose a network-based factor copula approach to study systemic risk in a network of systemically important financial institutions (SIFIs). The factor copula model offers a variety of dependencies/tail dependencies conditional on the chosen factor; thus constructing conditional network. Given the network, we identify the most “connected” SIFI as the central SIFI, and demonstrate that its systemic risk exceeds that of non-central SIFIs. Our identification of central SIFIs shows a coincidence with the bucket approach proposed by the Basel Committee on Banking Supervision, but places more emphasis on modeling the interplay among SIFIs in order to generate system-wide quantifications. The network defined by the tail dependence matrix is preferable to that defined by the Pearson correlation matrix since it confirms that the identified central SIFI through it severely impacts the system. This study contributes to quantifying and ranking the systemic importance of SIFIs.

**Key words:** factor copula, network, Value-at-Risk, tail dependence, eigenvector centrality

**JEL Classification:** C00, C14, C50, C58

## 1 Introduction

Systemic risk is a very important aspect of economic risk and played a significant role in the financial crisis of 2008. It continues to be an extremely relevant topic today. An important

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\*Financial support from the Deutsche Forschungsgemeinschaft via IRTG 1792 “High Dimensional Non Stationary Time Series”, Humboldt-Universität zu Berlin, is gratefully acknowledged.

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question is how systemic risk can be quantified. The notion of systemic risk and a macroprudential approach, relevant to financial stability and the functioning of financial markets, has gained significant attention from regulators, financial analysts and academic researchers.

The Financial Stability Board (FSB) together with the Basel Committee on Banking Supervision (BCBS) developed a methodology to select SIFIs and attribute them to categories (“buckets”). Depending on the buckets, additional common equity loss absorbency is prescribed in terms of a percentage of risk-weighted assets. This methodology is the so-called “indicator-based measurement approach” which is based on a number of indicators postulated to capture the level of global systemic importance. Among them are such indicators as bank size measured by total exposures, interconnectedness, substitutability and complexity, see Basel Committee on Banking Supervision (2013). Though these indicators are important, they do not necessarily reflect the global scope of the bank’s operations and may suffer from arbitrary weight assignment. Moreover, this approach is relatively qualitative and is limited to reflecting the fact that financial institutions vary widely and by occasion. It may not completely capture the scope of risk nor reflect the degree of risk carried by SIFIs over time.

The quantitative approach is therefore proposed as a necessary supplement. The question is *how to quantify systemic risk*. Market-based approaches, which rely more on public market data, are relatively prevalent due to availability. The interdependencies, playing the main role in systemic risk, can be inferred from market data. The aggregate risk measures such as Value-at-Risk (VaR), Expected Shortfall (ES), Marginal Expected Shortfall (MES) and Conditional VaR (CoVaR) are used to quantify systemic risk (see Diebold and Yilmaz (2014); Girardi and Tolga Ergün (2013); Banulescu and Dumitrescu (2015); Acharya et al. (2017)). However, the majority of them stand for a pairwise case; thus, it may be difficult to justify systemic risk from a system-wide perspective, which requires an explicit multivariate framework. The network-based approach is therefore proposed for this reason, and it is rigorous in theory and readily implemented in practice.

This study proposes a network-based factor copula approach which combines the network-based and market-based methods. In this framework, we begin our analysis by probing into a bank–bank network to sketch the type and strength of connectedness. Using the factor copula model to build

up the *conditional connectivity*, we then apply the market-based approach to quantify systemic risk given the predefined network. The central SIFI can be chosen as the one with a relatively higher impact on the system, which is also the main concept of MES or CoVaR. However, to implement the MES or the CoVaR method, one may rely on a built-in index comprised of a group of institutions. Extending the pairwise case to a high-dimensional tail risk spillover is unclear in the framework based on CoVaR or MES. Alternatively, the multivariate Gaussian framework is presumed, but it is very restrictive as it imposes neither tail nor asymmetric dependence, leading to underestimation of the risk of a financial system.

The factor copula model, a general conditional independence model developed by Krupskii and Joe (2013), provides a wide range of dependence types and joint distributions. In particular, dependence in the variables of interest can be explained by a few risk factors. Secondly, the number of parameters in the correlation or tail dependence matrix can be dramatically reduced. The main idea is that instead of directly defining the dependence structure between the variables of interest, one can map the variables into manageable factors and define a dependence structure through these factors.

The SIFI that contributes most to systemic risk should be the one creating higher connectivity, and can be named the “central SIFI”. This central SIFI systemically impacts the remaining SIFIs, leading to widespread distress or triggering broader contagion. Adrian and Brunnermeier (2016) point out that some institutions are *individually* systemic since they are so interconnected and generate negative risk spillover effects on others, while other smaller institutions may be systemic as a herd. In view of its role in connectedness, the central SIFI could be regarded as the factor in the factor copula model. The resulting correlation/tail dependence matrix is therefore defined through any SIFI in response to the central SIFI, which reduces the number of dependence parameters in the correlation matrix from  $\mathcal{O}(d^2)$  to  $\mathcal{O}(d)$ . The remaining SIFIs conditional on this central SIFI are conditionally independent, which permits us to establish a  $d$ -dimensional distribution by means of bivariate linking copulas. The double- $t$  factor copula with better goodness of fit is chosen in this study for the construction of theoretical tail dependence matrices and estimation.

Given the theoretical tail dependence matrices implied by the double- $t$  factor copula model, one

can identify the central nodes and quantify the VaR of the portfolio (PVaR) comprising other non-central SIFIs conditional on the central SIFI. This PVaR estimate is an ideal measure of systemic risk triggered by a particular SIFI, and one can therefore rank the systemic importance for each SIFI by comparing the magnitude of PVaR estimates of them. In a factor copula framework, the systemic relevance of SIFIs can be decisively determined by the overall tail risk they spread to other SIFIs. The central SIFI, due to its interconnectedness, is more likely to spill over its distress to other SIFIs. As a result, the tail risk of an individual SIFI or a group of SIFIs contingent on the failure of a few major SIFIs should be more severe than the tail risk incurred by non-central institutions. Such central SIFI identification can be also useful when stress-testing using individual bank failure as a starting point. The application of the proposed framework to stress test the fragility of the system conditional on the stress of the central SIFI is demonstrated.

We contribute to a growing body of literature in several aspects. First, the existing studies construct a synthetic index or system used to represent a group of institutions or state variables. By doing so, the spillover in high-dimensional data can be boiled down to a bivariate case ( $i$  versus system). The network-based factor copula is not bound to this situation, but is able to explicitly model the joint distribution of non- $i$  SIFIs conditional on  $i$ th SIFI in order to quantify the risk impact to its  $d - 1$  counterparties. Second, the distributional assumption behind the CoVaR framework is Gaussian for its analytical tractability. As pointed out by Adrian and Brunnermeier (2016) the Gaussian setting results in a neat analytical solution, but its tail properties are less desirable than those of more general distribution specifications. The factor copula model is proposed for this reason, so that the marginal distributions and copula function both can be freely chosen, constituting a more realistic joint distribution in the end. Third, we propose three types of dependence structures, and make use of them to define the networks and the central SIFIs. We show that the network defined by the copula-implied tail dependence matrix can permit the central SIFI to be identified through it as the one triggering higher tail risk and distress.

The outline of this study is as follows: in Section 2, we construct the network of SIFIs to imply their dependencies and in Section 3, we introduce the factor copula theory and theoretical tail dependence derived from it. The estimation technique is also documented. In Section 4, we

propose a network-based factor copula approach used to estimate the PVaR values and perform a stress test conditional on the identified central SIFIs. The empirical findings and discussions are provided in Section 5. Section 6 concludes with suggestions for further research.

## 2 Network analysis of SIFIs

### 2.1 Measures of dependence

Dependence of random variables can be defined via a variety of aspects such as symmetric versus asymmetric, linear versus nonlinear or tail versus entire distribution. It can be empirically measured or model-implied. Here we discuss several prevalent methods in constructing pairwise dependence.

The Pearson correlation coefficient is a popular measure of linear association between random variables. Given random observations  $x_{it}$  and  $x_{jt}$ ,  $t = 1, \dots, T$ ,  $T$  is a time horizon, the Pearson correlation coefficient  $\rho_{ij}^P$  is defined as follows:

$$\rho_{ij}^P \stackrel{\text{def}}{=} \frac{\sum_{t=1}^T (x_{it} - \bar{x}_i)(x_{jt} - \bar{x}_j)}{\sqrt{\sum_{t=1}^T (x_{it} - \bar{x}_i)^2} \sqrt{\sum_{t=1}^T (x_{jt} - \bar{x}_j)^2}} \quad (1)$$

Statistical dependence is determined through joint distributions. Of particular interest are extreme or tail dependencies, because they allow measuring the level of risk in the financial markets during market crashes more efficiently than association measures. Copula functions are flexible and efficient instruments which allow setting a wide range of dependency between random variables with various marginals.

Given  $d$  dimensions, a copula is a  $d$ -dimensional joint distribution with  $U[0, 1]$ -uniform marginals. According to the Sklar's theorem, if  $C$  is a copula and  $F_{X_1}, \dots, F_{X_d}$  are continuous marginal distributions of  $X_1, \dots, X_d$ , then one can uniquely construct a joint distribution  $F(x_1, \dots, x_d) = C\{F_{X_1}(x_1), \dots, F_{X_d}(x_d)\}$ . Extreme or tail dependence can be explicitly defined given a specific

copula. These measures gauge the strength of dependence in the tails of a bivariate distribution. To be precise, the coefficients of lower and upper tail dependence are defined as follows:

$$\begin{aligned}\Lambda_{ij}^L &\stackrel{\text{def}}{=} \lim_{q \rightarrow 0^+} \mathbf{P}(X_j \leq F_j^{-1}(q) | X_i \leq F_i^{-1}(q)), \\ &= \lim_{q \rightarrow 0^+} \frac{\mathbf{P}(X_i \leq F_i^{-1}(q), X_j \leq F_j^{-1}(q))}{q},\end{aligned}\tag{2}$$

$$\begin{aligned}\Lambda_{ij}^U &\stackrel{\text{def}}{=} \lim_{q \rightarrow 1^-} \mathbf{P}(X_j > F_j^{-1}(q) | X_i > F_i^{-1}(q)), \\ &= \lim_{q \rightarrow 1^-} \frac{\mathbf{P}(X_i > F_i^{-1}(q), X_j > F_j^{-1}(q))}{1 - q}.\end{aligned}\tag{3}$$

Alternatively, as proposed by Schmidt and Stadtmüller (2006), tail dependence can be estimated by means of empirical tail copulas. This allows to estimate tail dependence coefficients in a non-parametric setting. The marginal distributions are modelled using empirical distribution functions to avoid misspecification due to possible wrong parametric fit of the marginal distributions. The non-parametric estimators for (2), (3) are written as follows:

$$\widehat{\Lambda}_{ij}^L \approx \frac{1}{k} \sum_{t=1}^T \mathbf{I}_{\{R_i^{(t)} \leq kx_i, R_j^{(t)} \leq kx_j\}},\tag{4}$$

$$\widehat{\Lambda}_{ij}^U \approx \frac{1}{k} \sum_{t=1}^T \mathbf{I}_{\{R_i^{(t)} > T - kx_i, R_j^{(t)} > T - kx_j\}}.\tag{5}$$

where  $R_i, R_j$  are denoted as  $T \times 1$  vectors of ranks of  $x_{it}, x_{jt}$ . The parameter  $k \in \{1, \dots, T\}$  (threshold) is chosen via a plateau-finding algorithm which corresponds to balancing bias and variance. For the asymptotic results to hold, it is assumed that  $k = k(T) \rightarrow \infty$  and  $k/T \rightarrow 0$  as  $T \rightarrow \infty$ . The estimators are shown to have asymptotically normal distribution under both known and unknown marginal distributions. The details can be found in Schmidt and Stadtmüller (2006).

The economic rationale behind choosing either correlation or tail dependence matrices is subject to the risk being addressed. Correlation-based dependence accounts for variance risk while tail dependence aims to capture tail risk. It is advisable to build up a dependence matrix from tail dependence coefficients  $\Lambda_{ij}^L, \Lambda_{ij}^U$  or the empirical counterparts  $\widehat{\Lambda}_{ij}^L, \widehat{\Lambda}_{ij}^U$  for lower and upper

tail dependence. Similar emphasis can be found in Adrian and Brunnermeier (2016). Once interdependence between financial variables has been defined, further analysis is necessary to determine the network structure of the underlying system. Combined with the centrality approach, tail risk network analysis can provide valuable insights into extreme risk connective structure on a systemic scale.

## 2.2 The description of SIFIs and their interdependencies

Thirty global SIFIs listed and updated by FSB in November 2015 are ideal samples to study systemic risk in a network framework. For this study, we disregard two SIFIs, Agricultural Bank of China and Banque Populaire CE, due to their relatively shorter data periods, and use the remaining 28 SIFIs in the period 1 January 2007 to 31 December 2014. In Table 1, we list the names of the SIFIs with the corresponding indices and symbols assigned in this research, and summarize the bank-specific attributes such as debt ratio, firm size, country where the headquarters are located and the buckets assigned by BCBS. Debt ratio, a ratio of total debt to total assets, captures the fragility of a bank, while the size – as total assets – serves as a proxy for the bank being too big to fail. The bucket in the last column is defined in Table 2 of the Basel Committee document *Global systemically important banks: updated assessment methodology and the higher loss absorbency requirement*, July 2013, which is designed to reduce the moral hazard problems and systemic risk by requiring additional common equity loss absorbency as a percentage of risk-weighted assets from 3.5% (Bucket 5), 2.5% (Bucket 4), 2.0% (Bucket 3), 1.5% (Bucket 2) to 1% (Bucket 1).

Through (4), we calculate empirical lower tail dependence matrices  $\widehat{\Lambda}_{ij}^L$  for 28 SIFIs and show them in Figure 1. Each panel plot in this figure depicts the empirical lower tail dependence in a particular calendar year given its daily return data collected from Datastream. Consistent with Chen et al. (2017), one can observe the tail dependence that appeared in a geographic location e.g. a cluster in the U.S., the U.K., China, Europe and Japan. The yellow squares are generated by geographic dependencies, and the tail dependencies on a geographic basis are about 0.6. During



the European debt crisis in 2011-2012, the European SIFIs and the U.S. SIFIs by group exhibit stronger tail dependence with more yellow color distribution.

### 2.3 Adjacency matrix construction

To study systemic risk in a network framework, we need a convenient mathematical representation of a network. Graph theory is very useful to represent and visualize complexity of interactions between network elements. A graph is composed of a number of nodes/vertices and the edges between nodes. In this study, each node represents a particular SIFI, while the edge between two nodes indicates their dependence. The representation is achieved via an adjacency matrix. The adjacency matrix  $A$  with elements  $a_{ij}$  for a simple undirected graph is defined as follows:

$$A = \begin{cases} a_{ij} & \text{if there's an edge between nodes } i \text{ and } j \\ 0 & \text{otherwise,} \end{cases} \quad (6)$$

where  $a_{ij}$  determine the weights of edges between  $i, j = 1, \dots, N$ . For an unweighted network (all edges bear the same weight), all  $a_{ij} = 1$ .

The adjacency matrix can be constructed via the aforementioned dependence matrix, that is, the Pearson correlation matrix, the empirical tail dependence matrix or the tail dependence matrix implied by the factor copula model. Transforming dependence matrix into a binary adjacency matrix is analogous to statistical shrinkage techniques used to select the relevant variables into the system. The statistical rationale is that the network of SIFIs is very likely to be sparse, see Bluhm et al. (2016), which means that some edges are statistically relevant but some are not. It is not advisable to take all pairwise dependencies into account if their dependencies are not beyond a certain threshold. An observation is also made by Chen et al. (2017) and Barigozzi and Brownlees (2016). The adjacency network structure in this study is based on binary weights representing the statistically significant links between the nodes, with one (zero) used to represent a strong (weak) dependence.

The method of Ng (2006) proposes a breakpoint analysis framework to partition the order depen-

dencies into two groups. Through a uniform spacings analysis, the problem of testing cross-section correlation/dependency is turned into a problem of testing uniformity and non-stationarity. A subset of nonzero dependencies can be determined by minimizing a sum of square residuals.

The idea of uniform spacings can be generalized to any dependence matrix as long as its elements can be assumed  $U[0, 1]$ -distributed. To be precise, given a  $N \times N$  dependence matrix  $p_{ij}$ ,  $i, j = 1, \dots, N$ , breakpoint determination is achieved via several steps:

1. Sort cross-sectional dependencies into an ordered vector  $\mathbf{p} = (p_1, p_2, \dots, p_n)$ , where  $p_1$  is the smallest one and  $p_n$  is the largest one,  $n = N(N - 1)/2$ ,
2. Perform a uniform transformation of  $\mathbf{p}$  via the standard normal cdf:

$$\mathbf{\Phi} = \left( \Phi(\sqrt{T}|p_1|), \Phi(\sqrt{T}|p_2|), \dots, \Phi(\sqrt{T}|p_n|) \right), \quad (7)$$

3. Calculate the spacings  $\Delta\phi_j = \Phi(\sqrt{T}|p_j|) - \Phi(\sqrt{T}|p_{j-1}|)$ ,
4. Perform the optimization

$$\hat{\theta} = \arg \min_{\theta \in \mathbb{R}} f_n(\theta), \quad (8)$$

where  $f_n(\theta) = \sum_{j=1}^{\lceil \theta n \rceil} (\Delta\phi_j - \mu_s)^2 + \sum_{j=\lceil \theta n \rceil+1}^n (\Delta\phi_j - \mu_L)^2$  and  $\mu_s = \frac{1}{\lceil \theta n \rceil} \sum_{j=1}^{\lceil \theta n \rceil} \Delta\phi_j$ ,  $\mu_L = \frac{1}{\lceil n(1-\theta) \rceil} \sum_{j=\lceil \theta n \rceil+1}^n \Delta\phi_j$ ,  $\lceil \theta n \rceil$  is the integer part of  $\theta n$ .

As a consequence,  $\lceil \hat{\theta} n \rceil$  yields an optimal break location achieving a minimum total sum of variances from two subgroups to make the dependencies in the given group as homogeneous as possible.

Figure 2 shows the resulting adjacency matrices based on the empirical tail dependence matrices in Figure 1. The adjacency matrices vary over time. In 2007 and 2008, the Chinese SIFIs (nodes 13-15) are relatively isolated from the U.S. SIFIs (nodes 1-8); however, they turn to connect with world as of 2009. We find a lower degree of adjacency between Japanese SIFIs and others, implying that weak tail dependence might be attributed to the relatively conservative lending policies launched in Japan.

## 3 Factor copulas

### 3.1 General theory

Copulas in general are flexible tools for modelling multivariate distributions which allow for the separate modelling of marginal distributions and the dependence structure. Sklar's theorem postulates that every multivariate distribution can be represented via the corresponding marginal distributions and a copula. This property allows construction of a wide range of dependence structures for random variables which are converted to  $U(0,1)$ -uniform ones. This is done to guarantee that a copula has uniform univariate marginal distributions.

Factor copula models go a step further from other copula types to address the issue of high dimensionality and polynomial-time complexity in copula parameter estimation. Given  $d$  marginal distributions, usual copula constructions (e.g., direct multivariate copulas, vines) involve estimating  $\mathcal{O}(d^2)$  parameters. Factor copulas allow for parameter estimation to be done in linear time: for instance, compared to vine pair-copula models, they reduce the number of parameters to be estimated to  $\mathcal{O}(d)$ , see Krupskii and Joe (2013).

A general multivariate factor copula model assumes a linear dependence structure of  $d$  observed variables  $Z$  and  $p$  conditional factors  $W$ :

$$Z_j = \theta_{j|1}W_1 + \dots + \theta_{j|p}W_p + \psi_j\varepsilon_j, \quad j = 1, \dots, d. \quad (9)$$

where  $1 \leq p < d$ . In a one factor case, the representation (9) assumes the form:

$$Z_j = \theta_{j|1}W + \sqrt{1 - \theta_{j|1}^2}\varepsilon_j, \quad j = 1, \dots, d. \quad (10)$$

In the factor copula model, the copula-dependent uniform random variables  $u_j \stackrel{\text{def}}{=} F_{Z_j}(z_j)$ ,  $j = 1, \dots, d$ , obtained from the marginal transformation of  $Z_j$  in  $Z \stackrel{\text{def}}{=} (Z_1, \dots, Z_d)^T$  are assumed to be conditionally independent given variable  $V \stackrel{\text{def}}{=} F_W(w)$ . The factor copula expression is

then derived via the *mixture families* approach. Assume  $p = 1$  (one-factor case), define  $U \stackrel{\text{def}}{=} (U_1, \dots, U_d)^T$ ,  $V$ , all  $U(0, 1)$ , i.i.d., then:

$$\begin{aligned}
C_v(u_1, \dots, u_d) &= F(Z_1, \dots, Z_d) \\
&= \int_D F_{Z|V}(z|v) dF_V(v) \\
&= \int_D \prod_{j=1}^d F_{Z_j|V}(z_j|v) dF_V(v) \\
&= \int_D \prod_{j=1}^d C_{F_Z(Z_j)|V}(F_Z(z_j)|v) dv \\
&= \int_D \prod_{j=1}^d C_{U_j|V}(u_j|v) dv, \tag{11}
\end{aligned}$$

denotes a one factor copula with conditionally independent marginals  $U_1, \dots, U_d$ , given variable  $V$ ; here  $D \stackrel{\text{def}}{=} [0, 1]$ , the first and fourth equality come from Sklar theorem and uniformity, the third one from the independence assumption. Any conditional independence model given  $V$  can be expressed in this form after uniform transformation. The dependence structure of  $U$  is then defined through conditional distributions modeled by a sequence of bivariate copulas that link variables  $U_j$  to variable  $V$ .

The expression (10) allows to generate different dependence structures given the distributions of  $W$  and  $\varepsilon$ . Oh and Patton (2015) demonstrate the flexibility of the class of factor copulas by choosing marginals as normal,  $t$  and Skew- $t$  distributions to accommodate possible dependencies. However, the copulas with asymmetric and tail dependence such as double- $t$  and Skew- $t$ - $t$  factor copula normally do not have closed form. In a simple example with  $W = \Phi^{-1}(v)$  and  $\varepsilon$  both being  $\mathbf{N}(0, 1)$ , the resulting copula is Gaussian. It follows that (see Appendix 9.1)

$$C_{U_j|V}(u_j|v) = \Phi \left( \frac{\Phi^{-1}(u_j) - \theta_{j|1} \Phi^{-1}(v)}{\sqrt{1 - \theta_{j|1}^2}} \right). \tag{12}$$

The resulting expression for (11) is then

$$C_w(u_1, \dots, u_d) = \int_D \prod_{j=1}^d \left\{ (\Phi^{-1}(u_j) - \theta_{j|1} w) / \sqrt{1 - \theta_{j|1}^2} \right\} \varphi(w) dw. \tag{13}$$

In general, the conditional independence formulated by the factor model, given independent uniformly distributed random variables  $V = v$ ,  $U_j$ , takes the form

$$C_{U_j|V}(u_j|v) = F_{\varepsilon_j} \left( \frac{F_{Z_j}^{-1}(u_j) - \theta_{j|1} F_W^{-1}(v)}{\sqrt{1 - \theta_{j|1}^2}} \right), \quad (14)$$

Here  $W$ ,  $\varepsilon_j$  can have arbitrary continuous distributions, the distribution  $F_{Z_j}$  is obtained from the convolution of  $\theta_{j|1}W$  and  $\sqrt{1 - \theta_{j|1}^2}\varepsilon_j$ , according to the form of (10).

### 3.2 Factor copula under particular distributions

Specific variations of (14) are obtained by using parametric continuous distributions for the factor  $W$  and the idiosyncratic shock  $\varepsilon_j$ , respectively. Some common examples, see McNeil et al. (2015), include the so-called double- $t$  and the double- $GH$  copulas where both  $W$  and  $\varepsilon_j$  follow univariate  $t_\nu$  and generalized hyperbolic ( $GH$ ) distributions, respectively. If one utilizes the representation (11)-(14) for parameter estimation, one has to numerically compute  $F_{Z_j}$  (via convolution) and its inverse at a particular point in every iteration. This makes the computation prohibitively slow and the model impossible to use for practical purposes.

One can address this problem by using distributions which possess stability under convolution as well as fit financial data well. Among such distributions we find the family of stable distributions which, for specific values of their parameters, asymptotically exhibit power law behaviour in the tails (heavy-tailed distributions). Stable distributions can give a better fit to financial data in many cases compared to that of distributions with exponentially decaying tails, see Nolan (2014). On the other hand, these distributions may overestimate extreme risks and can be computationally intensive in parameter estimation.

An alternative is the class of  $GH$  distributions which are closed under convolution given certain constraints on their parameters. As was shown in previous research by Borak et al. (2010), statistical tests such as Kolmogorov and Anderson-Darling goodness-of-fit statistics show that two subclasses of the  $GH$  distribution, the hyperbolic and the normal inverse Gaussian ( $NIG$ )

distributions provide the best model for financial data. The double-*NIG* copula approach was applied by Kalemanova et al. (2007) for synthetic CDO pricing. Explicit dependence of the parameters of the convolution distribution  $F_{Z_j}$  on the factor loading parameter  $\theta_{j|1}$  had to be introduced in order to perform the convolution.

A drawback of *GH* distributions is that they allow for nonzero tail dependence in the factor copula framework only under restrictive assumptions. To be precise, Hammerstein (2016) concluded that asymmetric *GH* distributions have their coefficients of tail dependence either 0 or 1 while the symmetric ones have zero tail dependence. If one decides to use the *GH* class instead, one relies on imposing restrictive assumptions on the parameters.

### 3.3 Tail dependence for factor copulas

For a factor copula represented by a linear structure (10) the tail dependence coefficients in (2) and (3) can be derived in explicit form. Although factor copulas generally lack a closed-form density, using extreme value theory the analytical expression for the implied tail dependence can be therefore achieved. The implied tail dependence from factor copulas is the “conditional tail dependence”, that is, it is derived given the factor  $W$ . Conditioning on the chosen factor, we can define a  $d$ -dimensional tail dependence matrix in a conditional fashion and compare it with the unconditional one. The choice of factor together with the selected copula distribution determine the resulting tail dependence matrix.

**Proposition 3.3.1.** Let the factor copula be generated by the linear factor structure (10). Also let  $F_W$  and  $F_{\varepsilon_j}$  have regularly varying tails with a common tail index  $\alpha > 0$  so that  $\mathbb{P}(W < -s) = \mathbb{P}(W > s) = A_W s^{-\alpha}$ ,  $\mathbb{P}(\varepsilon_j < -s) = \mathbb{P}(\varepsilon_j > s) = A_\varepsilon s^{-\alpha}$  as  $s \rightarrow \infty$ ,  $A_W > 0$ ,  $A_\varepsilon > 0$ .

Then it follows that

$$\Lambda_{ij}^L = \frac{A_W \theta_{i|1}^\alpha}{A_W \theta_{i|1}^\alpha + A_\varepsilon (1 - \theta_{i|1}^2)^{\alpha/2}} \quad (15)$$

if the following conditions hold:  $A_W \theta_{i|1}^\alpha \theta_{j|1}^\alpha + A_\varepsilon (1 - \theta_{i|1}^2)^{\alpha/2} \theta_{j|1}^\alpha > A_W \theta_{j|1}^\alpha \theta_{i|1}^\alpha + A_\varepsilon (1 - \theta_{j|1}^2)^{\alpha/2} \theta_{i|1}^\alpha$  and simultaneously  $\theta_{i|1} < \theta_{j|1}$  or  $A_W \theta_{i|1}^\alpha \theta_{j|1}^\alpha + A_\varepsilon (1 - \theta_{i|1}^2)^{\alpha/2} \theta_{j|1}^\alpha < A_W \theta_{j|1}^\alpha \theta_{i|1}^\alpha + A_\varepsilon (1 - \theta_{j|1}^2)^{\alpha/2} \theta_{i|1}^\alpha$

and simultaneously  $\theta_{i|1} > \theta_{j|1}$ . On the other hand, it holds that

$$\Lambda_{ij}^L = \frac{A_W \theta_{j|1}^\alpha}{A_W \theta_{j|1}^\alpha + A_\varepsilon (1 - \theta_{j|1}^2)^{\alpha/2}} \quad (16)$$

if the following conditions hold:  $A_W \theta_{i|1}^\alpha \theta_{j|1}^\alpha + A_\varepsilon (1 - \theta_{i|1}^2)^{\alpha/2} \theta_{j|1}^\alpha < A_W \theta_{j|1}^\alpha \theta_{i|1}^\alpha + A_\varepsilon (1 - \theta_{j|1}^2)^{\alpha/2} \theta_{i|1}^\alpha$   
and simultaneously  $\theta_{i|1} < \theta_{j|1}$  or  $A_W \theta_{i|1}^\alpha \theta_{j|1}^\alpha + A_\varepsilon (1 - \theta_{i|1}^2)^{\alpha/2} \theta_{j|1}^\alpha > A_W \theta_{j|1}^\alpha \theta_{i|1}^\alpha + A_\varepsilon (1 - \theta_{j|1}^2)^{\alpha/2} \theta_{i|1}^\alpha$   
and simultaneously  $\theta_{i|1} > \theta_{j|1}$ .

*Proof.* See Appendix 9.2. □

**Proposition 3.3.2.** Let the factor copula be generated by the linear factor structure (10). Also let  $F_W$  and  $F_{\varepsilon_j}$  be  $t(\mu, \sigma, \nu)$  and  $t(\nu)$ , then

$$A_W = \frac{(\nu \sigma^2)^{\frac{\nu+1}{2}}}{\nu^{3/2} \sigma B(\nu/2, 1/2)}, \quad (17)$$

where  $B(\cdot, \cdot)$  is the beta function,  $\nu$  is degree of freedom.

*Proof.* See Appendix 9.3. □

Through Eqs.(15), (16)and (17), one can derive the resulting theoretical tail dependence matrix conditional on  $W$  in an application of double- $t$  factor copula.

### 3.4 Copula parameter estimation

Estimation of copula parameters with likelihood methods often involves quantities which do not have closed form, therefore one has to use approximative numerical methods. The likelihood function for factor copula can be derived via direct differentiation of the integrand in (11). Alternatively, one can proceed by differentiating an absolutely continuous joint distribution function  $F_{\mathbf{Z}}$  with strictly increasing, continuous marginal distribution functions  $F_{Z_1}, \dots, F_{Z_d}$ , which generates

an implicit copula  $C(u_1, \dots, u_d)$  with the corresponding density, see McNeil et al. (2015).

$$c(u_1, \dots, u_d; \theta) = \frac{f_{\mathbf{Z}}(F_{Z_1}^{-1}(u_1), \dots, F_{Z_d}^{-1}(u_d))}{f_{Z_1}(F_{Z_1}^{-1}(u_1)) \cdot \dots \cdot f_{Z_d}(F_{Z_d}^{-1}(u_d))}, \quad (18)$$

where  $f_{\mathbf{Z}}$  is the joint density of  $Z_1, \dots, Z_d$ ;  $F_{Z_j}$ ,  $f_{Z_j}$ ,  $j = 1, \dots, d$  are the marginal distribution and density of  $Z_j$ , respectively. Referring (11)-(14), it can be shown that  $f_{\mathbf{Z}}$ ,  $F_{Z_j}$  and  $f_{Z_j}$  take the following forms:

$$\begin{aligned} f_{\mathbf{Z}}(Z_1, \dots, Z_d) &= \int_0^1 \prod_{j=1}^d f_{\varepsilon_j} \left( \frac{Z_j - \theta_{j|1} F_W^{-1}(v)}{\sqrt{1 - \theta_{j|1}^2}} \right) dv, \\ F_{Z_j}(z_j) &= \int_0^1 F_{\varepsilon_j} \left( \frac{z_j - \theta_{j|1} F_W^{-1}(v)}{\sqrt{1 - \theta_{j|1}^2}} \right) dv, \\ f_{Z_j}(z_j) &= \int_0^1 f_{\varepsilon_j} \left( \frac{z_j - \theta_{j|1} F_W^{-1}(v)}{\sqrt{1 - \theta_{j|1}^2}} \right) dv. \end{aligned}$$

Plugging the above specified joint density, distribution and marginal distribution into (18), an implied copula density is therefore obtained and consequently used for a log-likelihood representation. One-dimensional numerical integration is performed to determine the integral on the interval  $[0, 1]$  in (18). Krupskii and Joe (2013) implement the Gauss-Legendre quadrature for numerical integration and optimization for maximum likelihood. A quadrature rule approximates the following definite integral on a suitable domain  $D$ :

$$\int_D f(x) dx \approx \sum_{k=1}^q \omega_k f(x_k), \quad (19)$$

where  $q$  is the number of quadrature points,  $x_k$  are the quadrature points or nodes and  $\omega_k$  are the quadrature weights. The expressions for  $\omega_k$  for different quadrature rules can be found, e.g., in Abramowitz and Stegun (1965). According to Joe (2015), the number of quadrature points  $q$  around 20-30 per dimension is usually adequate for the maximum likelihood estimate to be numerically stable. We use  $q = 21$  in the empirical study below. The parameter vector  $\theta$  of the



joint density  $c(u_1, \dots, u_d; \theta)$  can then be estimated using maximum likelihood expressed as

$$L(u_1, \dots, u_d; \theta) = \prod_{t=1}^T c(u_{1,t}, \dots, u_{d,t}; \theta) \quad (20)$$

The inverse distribution  $F_{Z_j}^{-1}$  relies also on numerical computation. It would be computationally expensive to determine this quantity in each iteration of the likelihood optimization, therefore we use an approximative numerical method. First, two grids in the intervals  $[0, 1]$  and  $[-1, 1]$  for  $u$  and  $\theta_{j|1}$ , respectively, are created. Then, given a pair of values  $(u, \theta_{j|1})$ , the value of  $F_{Z_j}^{-1}(u; \theta_{j|1})$  can be determined via a root-searching algorithm for the problem  $F_{Z_j}(x; \theta_{j|1}) - u = 0$  by solving for  $x$ . Given a 2-dimensional rectilinear grid of  $F_{Z_j}^{-1}(u; \theta_{j|1})$  values, one can perform bilinear interpolation to determine the values of  $F_{Z_j}^{-1}$  in each MLE iteration. The matrix  $F_{Z_j}^{-1}(u; \theta_{j|1})$  is computed only once prior to estimation, which significantly saves computational effort.

## 4 A network-based factor copula approach

To quantify the systemic risk caused by a particular SIFI, one can estimate the tail risk in the system conditional on SIFI  $i$  being in stress. It is worth noting that the tail risk in the system is estimated through a joint distribution specified by a factor copula framework. Ranking the estimated tail risks conditional on each SIFI achieves the goal of ranking the systemic importance among SIFIs, which determines the corresponding required level of additional loss absorbency. A network-based factor copula approach is therefore proposed for this application. It is implemented via a two-stage procedure: in the first stage we perform centrality analysis to identify the SIFIs which happen to be the central nodes; at the second stage we implement the factor copula model conditional on the identified central SIFIs to estimate the tail risk in the system and perform stress tests to central SIFIs.

## 4.1 Eigenvector centrality

Eigenvector centrality analysis is one of means to identify the most “important” vertices (nodes) in networks. In this study, the identified central nodes potentially contribute most to overall systemic risk.

Using the adjacency matrix of a network (graph), we can track the neighbours for each node  $\nu_i$ . Let  $\gamma(\nu_i)$  denote node centrality and define the centrality of a node proportional to the sum of its neighbours’ centralities:

$$\gamma(\nu_i) \stackrel{\text{def}}{=} \frac{1}{\lambda} \sum_{j=1}^N a_{j,i} \gamma(\nu_j) \quad (21)$$

where  $a_{j,i}$  are the elements of the adjacency matrix  $A$  defined in (6) and  $\lambda$  is a fixed constant. Letting  $\Gamma = (\gamma(\nu_1), \gamma(\nu_2), \dots, \gamma(\nu_N))^T$  as the centrality vectors for all nodes, we can restate the above equation as

$$\lambda \Gamma = A \Gamma \quad (22)$$

Eq. (22) indicates that  $\Gamma$  is an eigenvector of  $A$ , and  $\lambda$  is the corresponding eigenvalue. In fact, if we choose to impose a positivity constraint on the centralities’ vector  $\Gamma$ , this is the largest eigenvalue of the adjacency matrix  $A$ , and the corresponding eigenvector is the vector of network centralities. The central nodes can be selected by ranking the elements in the selected eigenvector.

As is intuitively seen from the definition (21), the eigenvector centrality measure assigns more importance to the nodes which have either many connections to other nodes or to the nodes which are themselves important.

## 4.2 Identification of central SIFIs

The question for which nodes can be considered as central nodes is subject to the underlying network structure being constructed. The identified central nodes contribute to tail risk in the system if the underlying adjacency matrix is defined through the tail dependence matrix, whereas it may simply account for variance risk if the correlation/covariance matrix is investigated. The

identification can be carried out through various dependence structures, namely the Pearson correlation matrix ( $\rho_{ij}^P$ ), the empirical tail dependence matrix ( $\widehat{\Lambda}_{ij}^L$ ) and the theoretical tail dependence matrix implied by the factor copula model ( $\Lambda_{ij}^L$ ). We then undertake the following investigations:

1. Eigenvector centrality analysis based on the  $A$  matrix defined by  $\rho_{ij}^P$  in (1)
2. Eigenvector centrality analysis based on the  $A$  matrix defined by  $\widehat{\Lambda}_{ij}^L$  in (4)
3. Singular value norm of  $\Lambda_{ij}^L$  implied by the double- $t$  factor copula model, refer to Proposition 3.3.1 and 3.3.2

The first and the second investigation are based on the eigenvector centrality analysis to make use of the adjacency matrix defined by the Pearson correlation and the empirical tail dependence, respectively. Note that a breakpoint technique by Ng (2006) is applied to convert a dependence matrix into a binary one. The third investigation uses the singular value norm as a measure of systemic risk. This is motivated by the fact that the node it is conditioned upon, is omitted from the analysis; therefore, complete eigenvector centrality analysis is not feasible.

The singular value norm of  $A$  matrix determines the “magnitude” of  $A$ ; in this study it measures the degree of systemic risk caused by the degree of “connectedness” in the financial system which is generated, e.g., by extreme tail dependence or statistical association.

The (largest) singular value norm of  $A$  is defined as:

$$\begin{aligned} \|A\| &\stackrel{\text{def}}{=} \max_x \|Ax\|_2 \\ \text{s.t. } &\|x\|_2 = 1 \end{aligned}$$

The solution can be derived as:

$$\|A\| = \sqrt{\lambda_{\max}}, \tag{23}$$

where  $\lambda_{\max}$  is the largest eigenvalue of the positive semidefinite matrix  $A^\top A$ . The norm of matrix  $A$  is the *maximum singular value* of  $A$ , which is also the square root of the largest eigenvalue of

$A^\top A$ . The central SIFI can therefore be identified if the singular value norm conditional on it is the largest one in the financial system.

### 4.3 Central SIFI as conditioning variable

To quantify institution  $i$ 's systemic risk for the extent to which it can endanger the system due to its tail event in its return distribution, we suggest controlling for the systematic risk in the return distribution and considering only the idiosyncratic part. By doing so, we then get a clear systemic risk measure exclusively triggered by ‘‘firm-specific risk’’. Furthermore, we control for the GARCH effect in the firm-specific return. To be more specific, the mean equation controls for the market effect, while the variance equation details the volatility evolution of univariate variables in a GARCH(1,1) framework. Student's  $t$  innovation is assumed for the firm-specific residual return.

$$R_{j,t} = a_j + b_j R_{M,t} + \varepsilon_{j,t}, \quad j = 1, \dots, d. \quad (24)$$

$$h_{j,t}^2 = \omega_j + \beta_j h_{j,t-1}^2 + \alpha_j \varepsilon_{j,t-1}^2 \quad (25)$$

$$Z_{j,t} = \sqrt{\frac{\nu_j}{h_{j,t}^2(\nu_j - 2)}} \varepsilon_{j,t} \sim t_{\nu_j} \quad (26)$$

where  $R_{j,t}$  is the stock return series of institution  $j$ ,  $R_{M,t}$  is the MSCI world market index return series collected from Datastream, and the residual  $Z_{j,t}$  is the standardized residual return series controlled for world market return and the GARCH effect. Nevertheless, the dependence among  $\mathbf{Z} = (Z_1, \dots, Z_d)$  may not be necessarily mutually independent, especially when systemic risk emerges in the system.

The central nodes obtained from section 4.2 are the financial institutions with a higher degree of connectedness to the rest of the SIFIs. In this regard, they can be perceived as factor in the factor copula model in the distributional sense. That is, if we control for the network effect of these institutions, which may induce systemic risk, we achieve approximate conditional independence in the network.

Then the factor representation in (10) assumes the following form:

$$Z_j = \theta_{j|i} Z_i + \sqrt{1 - \theta_{j|i}^2} \varepsilon_j, \quad j = 1, \dots, i-1, i+1, \dots, N, \quad (27)$$

where  $Z_j$  are residual return series of “non-central“ SIFIs from (26),  $N$  is the total number of SIFIs,  $i$  is the central node index. The corresponding expression for the copula in (14) is then:

$$C_{U_j|Z_i}(u_j|z_i) = F_{\varepsilon_j} \left( \frac{F_{Z_j}^{-1}(u_j) - \theta_{j|i} z_i}{\sqrt{1 - \theta_{j|i}^2}} \right), \quad (28)$$

#### 4.4 Choice of distribution

We limit our attention to the double- $t$  factor copula in this study for the following reasons: (i) it fits financial data reasonably well, see Hull and White (2004); (ii) it allows for the construction of analytical tail dependence coefficients, see Section 3.3; (iii) in Section 4.3, the systemic risk being emphasized is the risk triggered by “firm-specific risk” after controlling for market risk. The firm-specific risk is likely to be distributed as Student- $t$  as suggested by Oh and Patton (2016), while the market factor is Skew- $t$  distributed. Archimedean copulas, however, allow for tail dependence but usually have only one or two parameters to characterize the dependence between all variables, which presumes a relatively homogeneous dependence and is not so favorable for a high-dimension application.

We compare our proposed factor copula models with two alternative elliptical factor copulas e.g. Gaussian and skewed- $t$ - $t$  in terms of goodness-of-fit measured by the Akaike information criterion (AIC) which results from maximum likelihood estimation:

- Gaussian factor copula:  $Z_i$  and  $\varepsilon_j$  are chosen as  $\mathbf{N}(\mu, \sigma)$  and  $\mathbf{N}(0, 1)$ , respectively;
- Double- $t$  factor copula:  $Z_i$  and  $\varepsilon_j$  are chosen as  $t(\mu, \sigma, \nu)$  and  $t(\nu)$ , respectively;
- Skewed- $t$ - $t$  factor copula:  $Z_i$  and  $\varepsilon_j$  are chosen as the skewed- $t$  distribution by Hansen (1994) and  $t(\nu)$ , respectively.

As shown in Table 2, these results demonstrate that, judging by average AIC values over estimates under different SIFIs as conditioning factors, the choice of double- $t$  model is supported. It is chosen in 5 out of 8 years, including financial crisis periods 2007-2008 and 2011-2012. In other calendar years it yields the AIC values as comparable as those from the skewed- $t$ - $t$  model. The skewed- $t$ - $t$  model is selected in 2009-2010 while the Gaussian configuration can be accepted only in 2013. The choice of the double- $t$  factor copula model is supported in terms of goodness-of-fit analysis, and the double- $t$  specification especially possesses desirable analytical properties in the tail measures and relative parsimony. We therefore make use of it for the empirical application in the next section.

## 5 Empirical results

### 5.1 Estimates of factor loading $\theta_{j|i}$

Figure 3 shows the estimates of  $\theta_{j|i}$  in each calendar year. In 2007, the European SIFIs(nodes 16-24) are broadly connected with each conditioning node lying on the x-axis. In 2008 and 2009, the  $\theta_{j|i}$  estimates of U.S. SIFIs conditional on the SIFIs in the U.S. ( $i = 1, \dots, 8$ ) or outside the U.S ( $i = 9, \dots, 28$ ) are generally above 0.5. Similar findings can be seen in the European debt crisis during 2011-2012. The principal investigation is to search for the node  $i$  (in x-axis) showing widespread connectedness with the remaining  $j$  nodes (in y-axis), which is observed by the greater values of  $\theta_{j|i}$ . Taking 2012 as an example, one can observe that the system becomes more connected (it has more yellow grids) if we set State Street (SST, node 7), Wells Fargo (WFC, node 8) or even HSBC (node 11) as the conditioning nodes. In fact, the centrality analysis through the singular value norm of copula-implied tail dependence matrix identifies these three SIFIs as central nodes which potentially trigger a system-wide tail risk and endanger the function of the banking system.

More importantly, with the  $\theta_{j|i}$  estimates, the theoretical tail dependence matrix implied by the *double- $t$*  factor copula defined in (15), (16) and (17) is therefore derived and shown in Figure 4. As

an example, in 2012, conditional on HSBC for its central role, the copula-implied tail dependencies in European(nodes 16-21) and British(nodes 9-11) SIFIs are overwhelmingly profound; however, this is not the case if conditional on an arbitrary non-central node such as Mitsubishi UFJ (MTU). In this case, one cannot observe any tail dependence between the British and European SIFIs. Similar findings can be seen in 2007 in Figure 4. Conditional on central node NDA (node 23), a strong tail dependence between the U.S. SIFIs and the remaining SIFIs is obvious, whereas it becomes invisible conditional on a non-central node such as CCB(node 15).

## 5.2 Portfolio VaR, stress test and network analysis

Given a particular central SIFI  $Z_i$ , the systemic risk in the group of the non-central SIFIs  $Z_j$  is then quantified by the factor-copula-based Portfolio Value-at-Risk (PVaR) and the portfolio return conditional on the stress of  $Z_i$ . Both systemic risk measures are estimated according to the following algorithm:

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**Algorithm 1** Factor copula PVaR calculation and stress test

---

- 1: Perform univariate GARCH filtering to get  $Z_i$  and  $Z_j$ .
  - 2: Derive uniform marginals  $u_j$  for each  $Z_j$  and  $v$  for  $Z_i$  via marginal cdf transformation.
  - 3: Estimate copula parameters  $\theta_{j|i}$  in (27) by maximum likelihood (see Eq. 20) .
  - 4: Generate copula-dependent random numbers given the estimates  $\hat{\theta}_{j|i}$  (see Algorithm 2).
  - 5: Perform GARCH simulation of dependent residuals and calculate the PVaR as 5% or 1%-quantile of the simulated portfolio returns
  - 6: Perform a stress test given  $Z_i$ 's stress
- 

Generation of copula-dependent random numbers given the estimated factor copula parameters  $\hat{\theta}_{j|i}$  is an essential step for PVaR calculation in Algorithm 1. A straightforward procedure can be applied to simulate from a one-factor copula model. Given the number of simulated samples  $n_{sim}$  and a forecast horizon  $H$  for the PVaR, we pre-allocate a  $n_{sim} \cdot H \times N$  array  $\mathcal{U}$  and proceed as outlined in Algorithm 2:

The resulting row vectors  $(u_1, \dots, u_N)$  in  $\mathcal{U}$  will be a sample from the distribution  $C_{z_i}(u_1, \dots, u_N; \hat{\theta}_{j|i})$ . Copula-dependent random numbers in the second step of Algorithm 2 are determined via numeric inversion of (28) as mentioned in the previous section.

---

**Algorithm 2** One-factor copula simulation

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- 1: **for**  $i \leftarrow 1, n_{sim}$  **do**
  - 2:     Simulate  $v, p_1, \dots, p_N$  as independent  $U(0, 1)$ -distributed random numbers.
  - 3:     Compute  $u_j = C_{U_j|V}^{-1}(p_j|v; \hat{\theta}_{j|i}), j = 1, \dots, N$ .
  - 4:     Return  $(u_1, \dots, u_N)$ .
  - 5:     Store  $(u_1, \dots, u_N)$  in the  $i$ th row of  $\mathcal{U}$ .
  - 6: **end for**
- 

Given  $\mathcal{U}$ , in the last step of Algorithm 1 the autocorrelation and heteroscedasticity observed in the original residual returns are re-introduced back into the copula-dependent uniform random values for PVaR calculation.

A systemic crisis is caused by a failure of one institution and the subsequent spreading of the distress to the whole system, see Brechmann et al. (2013). In the framework of our conditional factor copula model, the distress level in the system can be measured by the expected portfolio returns conditional on the stress event of the central node. Explicitly, the stress return ( $SR_i$ ) of portfolio conditional on the tail event of institution  $i$  can be defined as follows:

$$SR_i \stackrel{\text{def}}{=} \mathbf{E}(\omega^\top Z | v = 0.01), \quad (29)$$

where  $\omega$  is a vector of portfolio weights. The weight on each SIFI is its market capitalization to account for “too-big-too-fail” issue.

Given a stressed situation happening to the central SIFI, we simulate the resulting impact on the remaining SIFIs. As long as the central node is precisely identified, a simultaneous drop in the values of the remaining SIFIs is expected. A 1% quantile of stock return distribution is a common attempt for initiating a fictitious stress scenario. One merit of the factor copula framework is that we can work directly with uniformly distributed data on this quantile level. The expectation in (29) is computed via Monte-Carlo simulations. Given distress in institution  $i$ , we simulate a widespread impact on the remaining SIFIs by drawing samples from the distribution of  $\mathbf{U}_{-i} | U_i = 0.01$ .

Our ultimate goal is to show that the central SIFIs, in comparison with non-central SIFIs, potentially trigger higher PVaR estimates and the stress return conditional on them. If this conjecture is



confirmed, for a ranking purpose, the following quantification of systemic risk is optional because the quantification based on the factor copula model certainly requires computational effort.

Portfolio Value-at-Risk calculation and its conditional stressed return are performed for each year under consecutive assumptions that every SIFI potentially drives tail risk, although we may expect that central SIFIs have more prominent impacts. Table 3 summarizes the PVaR estimates for each calendar year, while Table 4 reports the conditional stress return of the portfolio. Obviously, each SIFI triggers different magnitudes of tail risk of the portfolio consisting of the remaining SIFIs. From 2007 to 2008, the PVaR estimates, presenting the quantile value of portfolio returns controlling market risk, increase on average from 2.107% to 3.455% at the 95% level and from 3.692% to 5.648% at 99% level on a daily basis, showing an increasing systemic risk in the U.S. subprime crisis.

The results in Tables 3 and 4 demonstrate that the choice of the central nodes by the singular value norm of the copula-implied tail dependence matrix more often coincides with the choice made by centrality analysis performed on the empirical tail dependence matrix. This is reasonable as both of these measures gauge extreme rather than volatility risk captured by the dependence matrix based on the Pearson correlation coefficients. Although they coincidentally identify the central nodes, they are not completely identical in the sense of the information content of the network (empirical tail dependence vs. copula-implied dependence), or the centrality method (eigenvector centrality vs. the singular value norm). We therefore detail their results in separate subsections.

### **5.2.1 Results from empirical tail dependence-based centrality analysis**

As can be seen in Tables 3 and 4, empirical tail dependence-based centrality analysis seems capable of selecting important nodes in the network over time. Note that the central node is not exclusively unique during an investigative period, it is possible that few nodes bear very comparable centrality scores. The resulting network structures are shown in Figures 5, 6. In 2008, three financial institutions - Barclays (BCS), Standard Chartered (STAN) and BNP Paribas (BNP)- are chosen as central nodes. They are conveniently identified as a group or a “cluster”

on the network plot in Figure 5. Two of these institutions are British SIFIs and one of them is a French SIFI. The choice is reasonable, as Barclays was the bank that might have been expected to fail. It purchased the US broker/deal operations of Lehman Brothers after the latter's bankruptcy for almost \$2bn in September 2008. Furthermore, each of the three institutions chosen had wide exposure to emerging markets, including troubled assets. More specifically, BCS, STAN and BNP took over Lehman's structured products' businesses in India.

For 2012, as shown in Figure 6, STAN and HSBC are selected as central nodes, both being British SIFIs. The banks were fined \$1.9 billion and \$300 million, respectively, by US authorities for their role in financial transactions involving criminals and states under US sanctions. Both institutions have historically had a very large exposure to emerging markets. In the second quarter of 2015, almost 41% of HSBC's net operating income was generated in Asia. Together with Latin America and the Middle East and Africa, these markets generated 54.81% of the bank's income, as reported by The Banker (2015). At the same time, Asia, the Middle East and Africa provided 88% of STAN's operating income and 97% of profits in the first half of 2015.

In 2013, China Construction Bank (CCB) and ICBC are chosen as central nodes, as shown in Figure 6. That year, the Chinese ICBC moved to first place in the Banker's Top 1000 World Banks, see ICBC: the world's new largest bank (2013). China Construction Bank dislodged Citigroup from fifth place with a 15% increase in capital. HSBC was ranked fourth.

### **5.2.2 Results from factor copula-implied tail dependence-based analysis**

The proposed network-based factor copula approach generates the copula-implied tail dependence. The singular value norm based on this dependence enables us to rank the connectedness scores, and identify the more relevant ones. Methodologically, we contribute to the current literature for the methods used to quantify systemic risk, and we show how it can be built in a high-dimension domain. We shed some light on this issue and report the corresponding results. Continuing the discussion in Table 3 and 4, in 2008 we find that JP Morgan (JPM) and CITI (C) are identified as central nodes, and they indeed result in higher PVaR estimates and negatively impact, measured

by  $SR_i$ , to the global banking system. In 2011 and 2012, the period of the European debt crisis, HSBC is systemically very important due to the higher tail risk and more severe stress it brings into the system. The centrality analysis from both empirical and copula-implied tail dependence indicates that HSBC, as the central node, is very likely to spread a system-wide risk to other SIFIs and destabilize the system. Accordingly, this SIFI, in terms of its systemic importance, should be highly regulated by its risk exposure and charged for additional capital buffer.

It is worth noting that Wells Fargo (WFC) has been identified as a central node since 2012, which may reflect the fact that as of the third quarter of 2011 WFC has been the largest retail mortgage lender in the U.S., amounting to \$1.8 trillion in home mortgages (30% market share for U.S. mortgages). In October 2012, WFC was sued by U.S. federal attorney Preet Bharara over questionable mortgage deals. The consecutive identification in the case of WFC, through the network implied by the factor copula model, warns of a possible risk propagation by WFC. Recently, FSB committed its risk potential and upgraded its bucket bracket from 1 to 2, as shown in the 2016 G-SIBs list.

The methods proposed in Section 4.2 reveal a less-convergent identification for central SIFIs shown in Table 3 and 4. Using the lists of SIFIs and the corresponding bucket level reported by FSB during 2012-2014 in Table 5, we compare the performances of three identification methods along with the bucket approach proposed by the Basel Committee. With a focus on highly important institutions, we only report the top two buckets, namely Buckets 4 (2.5% additional capital buffer) and 3 (2.0% additional capital buffer). In 2012, Deutsche Bank (DB), identified by the eigenvector centrality of the Pearson correlation matrix, is allocated in Bucket 4. However, DB generates relatively lower PVaR estimates and milder stress than HSBC (also located in Bucket 4), identified by the empirical and copula-implied tail dependence matrix. The same observation in 2014 documents that JPM(in Bucket 4), identified by the copula-implied tail dependence matrix, indeed induces higher systemic risk than the risk from the SIFIs chosen by the other two methods. The last three rows of Table 3 report the average PVaR values from three methods (C for copula tail dependence; T for empirical tail dependence; P for Pearson correlation). It shows that the central node identification through copula-implied tail dependence causes higher downside risk in

the system.

In summary, the ranking based on the singular value norm of a copula-implied tail dependence matrix is more capable of identifying the SIFIs with higher systemic risk as measured by the PVaR estimates and the stress returns. It also shows a certain degree of coincidence with the bucket approach, but places more emphasis on modeling the interplay among SIFIs in order to produce a system-wide quantification. The capital buffer charge calculation based on it is supposed to be reasonable. Rather naturally, the centrality analysis based on the Pearson correlation matrix performs worse as the risk being addressed is not volatility risk but tail risk.

## 6 Conclusions

To quantify and rank the systemic importance of 28 SIFIs selected by FSB and the Basel Committee of Banking Supervision, we propose a network-based factor copula approach. In this framework, we firstly construct the copula-implied network structure and identify the central SIFIs there, then using the joint distribution defined by the factor copula model, we quantify the tail risk of the remaining SIFIs as a whole conditional on the predefined central SIFI. The factor copula is tractable in a high-dimensional estimation and flexible in terms of distributional choice, which permits researchers a system-wide investigation.

We visualize the interplay and the network among SIFIs from their dependencies defined by the Pearson correlation matrix, the empirical and the copula-implied tail dependence. The network from the Pearson correlation matrix can document variance risk but is limited for tail risk. The central SIFI based on this network is very unlikely to trigger risk contagion in the system. The network implied by the factor copula model is, however, unique because of its “conditional” nature. The network conditional on the central SIFI is exceptionally dense; others are more sparse. Using the singular-value matrix norm of the copula-implied tail dependence matrix, we show that the identified central SIFI induces the highest tail risk and severe stress in the system. Accordingly, this SIFI, in terms of its systemic importance, should be highly regulated by its risk exposure and

charged for additional capital buffer.

The framework in this study and the system risk measures based on it are completely system-wide, which are proposed to resolve an obstacle in a high-dimensional setting. The network and joint distribution of it can therefore be tackled and modeled. The application of this framework in the analysis of stress-testing is demonstrated. The network-based factor copula approach can be useful for regulators to quantify the connectedness of a network and overall tail risk conditional on a specific SIFI.

## 7 Tables

Table 1: Summary information on SIFIs

Index	SIFI	Firm Size	Debt Ratio	Bucket	Country
1	JP MORGAN CHASE (JPM)	21.506	0.261	4	U.S.
2	BANK OF AMERICA (BAC)	21.446	0.302	2	U.S.
3	BANK OF NEW YORK MELLON (BKM)	19.499	0.095	1	U.S.
4	CITIGROUP (CITI)	21.359	0.300	3	U.S.
5	GOLDMAN SACHS (GS)	20.624	0.509	2	U.S.
6	MORGAN STANLEY (MS)	20.501	0.417	2	U.S.
7	STATE STREET (SST)	19.106	0.153	1	U.S.
8	WELLS FARGO (WFC)	20.980	0.183	1	U.S.
9	ROYAL BANK OF SCTL (RBC)	21.588	0.252	1	U.K.
10	BARCLAYS (BCS)	21.604	0.286	3	U.K.
11	HSBC (HSBC)	21.682	0.127	4	U.K.
12	STANDARD CHARTERED (STAN)	20.136	0.187	1	U.K.
13	BANK OF CHINA (BOC)	21.200	0.160	1	China
14	ICBC (ICBC)	21.508	0.089	1	China
15	CHINA CON.BANK (CCB)	21.281	0.092	1	China
16	BNP PARIBAS (BNP)	21.684	0.136	3	France
17	CREDIT AGRICOLE (ACA)	21.489	0.211	1	France
18	SOCIETE GENERALE (GLE)	21.184	0.139	1	France
19	DEUTSCHE BANK (DB)	21.630	0.200	3	Germany
20	UNICREDIT (UCG)	20.929	0.360	1	Italy
21	ING GROEP (ING)	21.156	0.103	1	Netherlands
22	SANTANDER (SAN)	21.158	0.368	1	Spain
23	NORDEA BANK (NDA)	20.476	0.326	1	Sweden
24	CREDIT SUISSE GROUP (CS)	20.744	0.339	2	Switzerland
25	UBS GROUP (UBS)	21.008	0.251	1	Switzerland
26	MITSUBISHI UFJ (MTU)	21.533	0.159	2	Japan
27	MIZUHO (MFG)	21.247	0.233	1	Japan
28	SUMITOMO.MITSUI (SMFG)	21.044	0.125	1	Japan

\* Debt ratio is defined as the ratio of total debt to total assets of a bank; and bank size is the log value of total assets; denominated in US dollars.

\*\* Mean values during the sample period (2007-2014) are shown. The buckets assigned by BCBS correspond to required levels of additional common equity loss absorbency as percentage of risk-weighted assets from 3.5% (Bucket 5), 2.5%(Bucket 4), 2.0%(Bucket 3), 1.5%(Bucket 2) to 1%(Bucket 1)

Table 2: Computed AIC values for the double- $t$  factor copula for SIFIs as conditioning factors

	2007	2008	2009	2010	2011	2012	2013	2014
JPM	1651.28	2045.88	1141.86	1357.16	2028.69	1513.45	1640.60	1614.30
BAC	1706.51	2031.10	1174.55	1441.01	2055.94	1580.20	1629.78	1630.56
BKM	1636.44	2074.27	1194.11	1397.11	2072.05	1523.37	860.99	1645.34
CITI	1634.90	2063.41	1325.78	1394.93	2046.94	1556.02	1636.46	1592.45
GS	1715.00	2042.24	1284.64	1382.44	2050.40	1577.14	1638.96	1554.63
MS	1684.45	2057.85	1328.22	1444.16	2029.10	1564.48	1628.97	1681.20
SST	1686.35	2087.27	1307.09	1428.15	2069.61	1463.50	1570.56	1680.90
WFC	1651.84	2028.51	1167.38	1437.59	2066.66	1533.89	1641.15	1674.39
RBC	1595.68	1849.94	1484.51	1365.52	1853.41	1463.53	845.95	1569.47
BCS	1451.79	1877.24	1498.32	1292.15	1879.69	1446.24	854.39	1484.49
HSBC	1596.56	1945.31	1512.65	1381.76	2019.95	1524.02	852.89	1657.91
STAN	1608.93	1992.46	1525.08	1384.64	2039.55	1514.48	855.66	1631.04
BOC	1640.55	2076.90	1494.07	1398.34	2071.61	1570.16	1630.85	1640.68
ICBC	1703.39	2084.07	1174.55	1427.86	2077.28	1574.48	1628.97	1635.90
CCB	1707.12	2086.74	1178.36	1402.34	2076.61	1576.92	1614.87	1668.66
BNP	1377.52	1739.24	1497.51	1337.11	1753.46	1317.70	1345.96	1440.48
ACA	1483.89	1751.81	1514.19	1346.18	1869.66	1243.30	1418.62	1428.57
GLE	1441.02	1842.23	1506.49	1341.52	1783.64	1435.82	909.70	1375.61
DB	1574.20	1892.67	1483.93	1342.50	1879.15	1459.55	890.65	1549.03
UCG	1523.50	1875.63	1496.74	1364.46	1869.26	1371.54	1447.63	1460.33
ING	1583.65	1900.56	1478.99	1363.44	1923.17	1409.35	1053.89	1493.61
SAN	1611.87	1876.42	1508.17	1356.46	1905.50	1452.59	1047.23	1455.44
NDA	1642.83	1911.01	1511.90	1352.62	2004.55	1537.59	858.64	1615.30
CS	1544.85	1838.39	1493.00	1331.33	1934.72	1373.82	1506.82	1561.10
UBS	1552.63	1833.23	1521.49	1322.45	1948.43	1424.10	846.26	1561.43
MTU	1696.56	2055.61	1496.29	1455.03	2069.47	1559.77	1632.48	1602.83
MFG	1675.29	1960.57	1475.75	1327.70	2089.85	1546.52	1618.28	1679.91
SMFG	1702.30	2085.83	1461.46	1349.79	2070.68	1550.42	1644.11	1683.69
Average double- $t$	1610.03	1960.94	1401.32	1375.92	1983.54	1488.00	1312.55	1581.04
Average Gaussian	1497.87	1840.38	1385.13	1253.23	1789.66	1413.78	1462.90	1484.61
Average skewed- $t-t$	1608.48	1938.29	1407.89	1377.42	1969.90	1481.97	1301.56	1576.41

The last three rows show the average AIC values across SIFIs are generated by double- $t$ , Gaussian and skewed- $t-t$  factor copula, respectively. In the upper panel, the specific AIC value given a particular SIFI as conditional factor is selectively shown only for the double- $t$  case due to space constraints. The negative sign in front of AIC value has been suppressed here.

Table 3: Portfolio VaR estimates in double- $t$  copula model at 99% and 95% level conditional on each SIFI

Index(Symbol)	2007		2008		2009		2010		2011		2012		2013		2014	
	PVaR95	PVaR99	PVaR95	PVaR99	PVaR95	PVaR99	PVaR95	PVaR99	PVaR95	PVaR99	PVaR95	PVaR99	PVaR95	PVaR99	PVaR95	PVaR99
1(JPM)	1.972	3.570	3.922 <sup>C</sup>	6.813 <sup>C</sup>	4.155	7.028	2.498	3.296	2.614	4.387	2.755	5.428	2.114	3.827	3.068 <sup>C</sup>	4.825 <sup>C</sup>
2(BAC)	1.786	3.621	3.581	5.755	3.567	6.825	2.218	4.956	3.452	5.629	3.120	5.270	2.667	3.986	2.087	3.956
3(BKM)	1.802	3.822	3.448	6.225	3.363	5.254	1.950	4.014	2.821	4.569	2.742	5.088	1.923	3.848	1.831	3.970
4(CITI)	2.122	3.408	4.016 <sup>C</sup>	7.212 <sup>C</sup>	4.491	7.136	2.943	4.962	2.481	3.600	2.514	3.943	2.508	4.734	2.539 <sup>P</sup>	4.147 <sup>P</sup>
5(GS)	2.366	4.521	3.431	6.296	3.861	5.439	2.621	4.332	2.028	4.417	2.084	5.309	2.587	4.230	1.873	3.348
6(MS)	2.549	5.095	4.260	6.206	3.790	5.801	2.440	3.743	2.375	4.207	2.477	3.953	2.307	4.591	2.446	3.912
7(SST)	1.783	3.829	3.639	5.270	4.008	6.046	2.776	4.787	2.095	4.833	3.161 <sup>C</sup>	6.088 <sup>C</sup>	3.268	5.394	1.812	4.015
8(WFC)	2.689	4.870	3.893	6.603	4.868	6.853	2.747	4.836	2.297	4.672	3.477 <sup>C</sup>	6.255 <sup>C</sup>	3.331 <sup>C</sup>	5.880 <sup>C</sup>	3.218 <sup>C</sup>	4.900 <sup>C</sup>
9(RBS)	2.226	4.031	3.229	5.052	3.903	5.791	2.558	4.215	3.106	4.783	3.170	4.653	2.608	4.462	2.316	4.287
10(BCS)	2.256 <sup>P,T</sup>	3.616 <sup>P,T</sup>	2.781 <sup>T</sup>	4.317 <sup>T</sup>	4.891	6.436	2.704	4.718	2.540	3.885	2.238	3.406	2.088 <sup>P</sup>	3.461 <sup>P</sup>	2.618	4.385
11(HSBC)	1.347	3.211	2.765	4.165	3.414	5.422	1.821	3.312	2.654 <sup>T,C</sup>	5.747 <sup>T,C</sup>	3.707 <sup>T,C</sup>	5.859 <sup>T,C</sup>	2.258 <sup>T</sup>	4.040 <sup>T</sup>	2.217	3.878
12(STAN)	2.352	3.742	3.852 <sup>T</sup>	6.195 <sup>T</sup>	2.719	5.026	2.434	4.003	2.114	3.919	1.806 <sup>T</sup>	3.641 <sup>T</sup>	2.824	5.028	2.366	4.237
13(BOC)	2.455	5.029	3.327	6.344	2.470	3.499	2.645	4.272	2.430	4.710	2.053	4.143	3.107	5.151	2.051	4.075
14(ICBC)	2.212	3.820	3.235	5.307	2.411	3.986	2.413	3.761	2.287	5.170	2.913	4.303	1.683 <sup>T</sup>	3.580 <sup>T</sup>	2.114	3.791
15(CCB)	1.849	3.126	3.226	5.374	2.856	4.353	2.296	4.215	2.207	4.888	2.606	4.487	3.133 <sup>T</sup>	5.213 <sup>T</sup>	2.278	4.173
16(BNP)	2.185	3.502	4.185 <sup>T,C</sup>	6.614 <sup>T,C</sup>	3.456	5.807	3.107 <sup>T,C</sup>	5.024 <sup>T,C</sup>	3.619	5.298	2.816	4.549	2.721 <sup>P</sup>	3.877 <sup>P</sup>	2.605	4.598
17(ACA)	2.173	4.562	3.060	6.123	3.311	5.382	2.495	4.432	2.688	4.061	3.110	4.017	2.291	3.601	1.992	4.383
18(GLE)	2.124	3.277	3.628	4.929	3.107	5.246	2.899	4.325	2.488	5.115	2.322	5.417	2.388	3.239	2.852	3.857
19(DB)	1.651 <sup>P</sup>	4.352 <sup>P</sup>	3.863	6.567	3.069 <sup>P</sup>	4.363 <sup>P</sup>	2.527 <sup>P</sup>	3.704 <sup>P</sup>	3.563 <sup>P</sup>	4.767 <sup>P</sup>	3.255 <sup>P</sup>	4.589 <sup>P</sup>	2.631	3.880	2.800	4.247
20(UCG)	1.741	2.568	3.280	4.894	4.014	5.934	2.594	4.167	2.978	4.378	1.678	3.053	2.752	3.917	2.216	3.938
21(ING)	2.521	3.866	4.128	5.816	3.754	5.345	2.569	4.704	3.076	5.209	2.988	4.856	2.497	4.149	1.805	4.358
22(SAN)	1.374	2.981	3.255	5.232	3.045 <sup>C</sup>	5.043 <sup>C</sup>	3.107	4.754	2.881	4.780	3.054	5.414	2.239	3.920	2.360 <sup>T</sup>	4.741 <sup>T</sup>
23(NDA)	2.608 <sup>C</sup>	5.114 <sup>C</sup>	3.233	5.349	3.556	5.043	3.254	5.011	1.729	3.403	2.231	3.625	3.276	5.226	2.200	3.910
24(CS)	1.258	2.706	3.976	5.883	3.108	5.119	2.281	3.652	1.980	5.121	2.209	5.165	2.450	4.494	2.610	4.120
25(UBS)	1.630	3.559	2.656 <sup>P</sup>	4.683 <sup>P</sup>	3.373 <sup>T</sup>	4.990 <sup>T</sup>	2.611	4.861	2.862	4.739	3.250	4.311	2.966	4.399	2.193	4.059
26(MTU)	2.637	2.340	2.645	4.540	2.482	4.166	2.219	4.152	1.816	2.798	2.249	3.374	2.944	4.515	2.936	4.527
27(MFG)	2.699	2.872	3.192	4.426	2.523	4.436	3.086	4.579	3.322	4.906	3.416	5.190	3.074	4.536	2.213	3.953
28(SMFG)	2.575	2.357	3.027	5.961	2.990	4.215	2.623	4.270	2.024	2.726	2.359	5.969	2.288	4.439	2.911	4.601
Avg of all	2.107	3.692	3.455	5.648	3.484	5.435	2.587	4.324	2.590	4.525	2.706	4.584	2.604	4.343	2.377	4.186
Avg of C	2.608	5.114	4.041	6.880	3.045	5.043	3.107	5.024	2.654	5.747	3.448	6.067	3.133	5.880	3.143	4.862
Avg of T	2.256	3.616	3.606	5.708	3.373	4.990	3.107	5.024	2.654	5.747	2.756	4.700	2.358	4.277	2.360	4.741
Avg of P	1.953	3.984	2.656	4.683	3.069	4.363	2.527	3.704	3.563	4.767	3.255	4.589	2.404	3.668	2.539	4.147

Superscript  $P$ ,  $T$  and  $C$  represent the central nodes identified through the Pearson correlation matrix (P), the empirical tail dependence matrix (T) and the tail matrix implied by factor copula (C), respectively.



Table 4: Stress testing conditional on each SIFI

Index	2007	2008	2009	2010	2011	2012	2013	2014
1 (JPM)	-0.21	-1.78 <sup>C</sup>	0.16	-2.20	-0.12	-0.72	-0.93	-0.50 <sup>C</sup>
2 (BAC)	-0.32	-0.51	-0.97	-1.14	0.19	-0.24	-0.26	-0.11
3 (BKM)	-0.05	-0.63	-1.51	-0.73	0.00	-0.59	-0.67	-0.43
4 (CITI)	-0.45	-1.84 <sup>C</sup>	-1.08	0.91	-0.27	-0.65	-0.17	-0.34 <sup>P</sup>
5 (GS)	-0.75	-0.68	-0.74	-1.38	-0.22	-0.50	-0.94	-0.13
6 (MS)	-0.97	-0.51	0.34	-1.64	-0.59	-0.48	-0.62	-0.72
7 (SST)	-0.20	-0.95	-0.46	-0.94	-0.78	-0.67 <sup>C</sup>	-0.14	0.32
8 (WFC)	-0.64	-1.02	-0.41	-1.81	-0.56	-0.53 <sup>C</sup>	-0.40 <sup>C</sup>	-0.58 <sup>C</sup>
9 (RBC)	-1.35	-0.39	-1.18	-0.64	-0.79	-0.45	-0.06	-0.62
10(BCS)	-0.57 <sup>P,T</sup>	-0.24 <sup>T</sup>	-2.00	2.03	-1.82	-0.57	0.10	-1.02
11(HSBC)	-0.24	-0.38	-0.53	-1.12	-1.79 <sup>T,C</sup>	-0.78 <sup>T,C</sup>	-0.63 <sup>T</sup>	-0.75
12(STAN)	-0.06	-1.52 <sup>T</sup>	-0.20	0.04	-0.52	0.07 <sup>T</sup>	-0.28	0.07
13(BOC)	-0.80	-0.90	0.05	0.14	-0.79	-0.39	-0.21	-0.16
14(ICBC)	-0.41	-1.11	0.27	0.13	-0.41	0.15	-0.21 <sup>T</sup>	-0.33
15(CCB)	-0.82	-0.64	-0.25	-0.08	-1.01	-0.24	0.12 <sup>T</sup>	-0.01
16(BNP)	0.08	-1.99 <sup>T,C</sup>	-1.41	-2.39 <sup>T,C</sup>	0.02	-0.33	-0.19 <sup>P</sup>	-0.50
17(ACA)	-0.64	-1.00	-0.68	-0.44	-0.23	-0.03	-0.71	-0.74
18(GLE)	-0.37	-1.25	-1.05	-1.72	-1.36	-0.30	-1.04	-0.34
19(DB)	-0.47 <sup>P</sup>	-1.27	-0.40 <sup>P</sup>	-0.22 <sup>P</sup>	-0.73 <sup>P</sup>	-0.22 <sup>P</sup>	-0.54	-0.52
20(UCG)	-0.18	-0.56	-0.48	-0.81	-0.32	-0.44	0.10	-0.42
21(ING)	-1.28	-0.91	-0.68	-1.84	-0.05	-0.20	-0.74	-0.16
22(SAN)	-0.16	-0.49	-2.49 <sup>C</sup>	-1.02	-0.71	-0.56	0.05	-0.36 <sup>T</sup>
23(NDA)	-1.11 <sup>C</sup>	-0.37	-0.22	-1.85	-0.18	-0.20	-1.04	0.01
24(CS)	-0.12	-1.89	-0.74	-1.22	-0.38	-0.12	-0.69	0.08
25(UBS)	-0.74	-0.36 <sup>P</sup>	-0.79 <sup>T</sup>	-0.59	-1.19	-0.61	0.17	0.13
26(MTU)	-0.84	-0.24	-0.36	0.78	0.04	0.03	-0.94	-0.06
27(MFG)	-0.59	-0.27	0.52	0.52	-0.73	0.20	-0.99	-0.13
28(SMFG)	-0.61	-0.84	-0.34	-0.08	-0.80	-0.04	-0.71	-0.30

\* Superscript  $P$ ,  $T$  and  $C$  represent the central nodes identified through the Pearson correlation matrix (P), the empirical tail dependence matrix (T) and the tail matrix implied by factor copula (C), respectively

\*\* The expected portfolio return conditional on the stress of given SIFI is estimated through (29)

Table 5: List of SIFIs/G-SIBs from 2012 to 2014

Bucket	2012	2013	2014
4 (2.5%)	JPM (1) HSBC (11) CITI (4) DB (19)	JPM (1) HSBC (11)	JPM (1) HSBC (11)
3 (2.0%)	BCS (10) BNP (16)	BCS (10) BNP (16) CITI (4) DB (19)	BCS (10) BNP (16) CITI (4) DB (19)

\* The buckets assigned by BCBS correspond to required levels of additional common equity loss absorbency as percentage of risk-weighted assets from 3.5% (Bucket 5), 2.5%(Bucket 4), 2.0%(Bucket 3), 1.5%(Bucket 2) to 1%(Bucket 1)

# 8 Figures

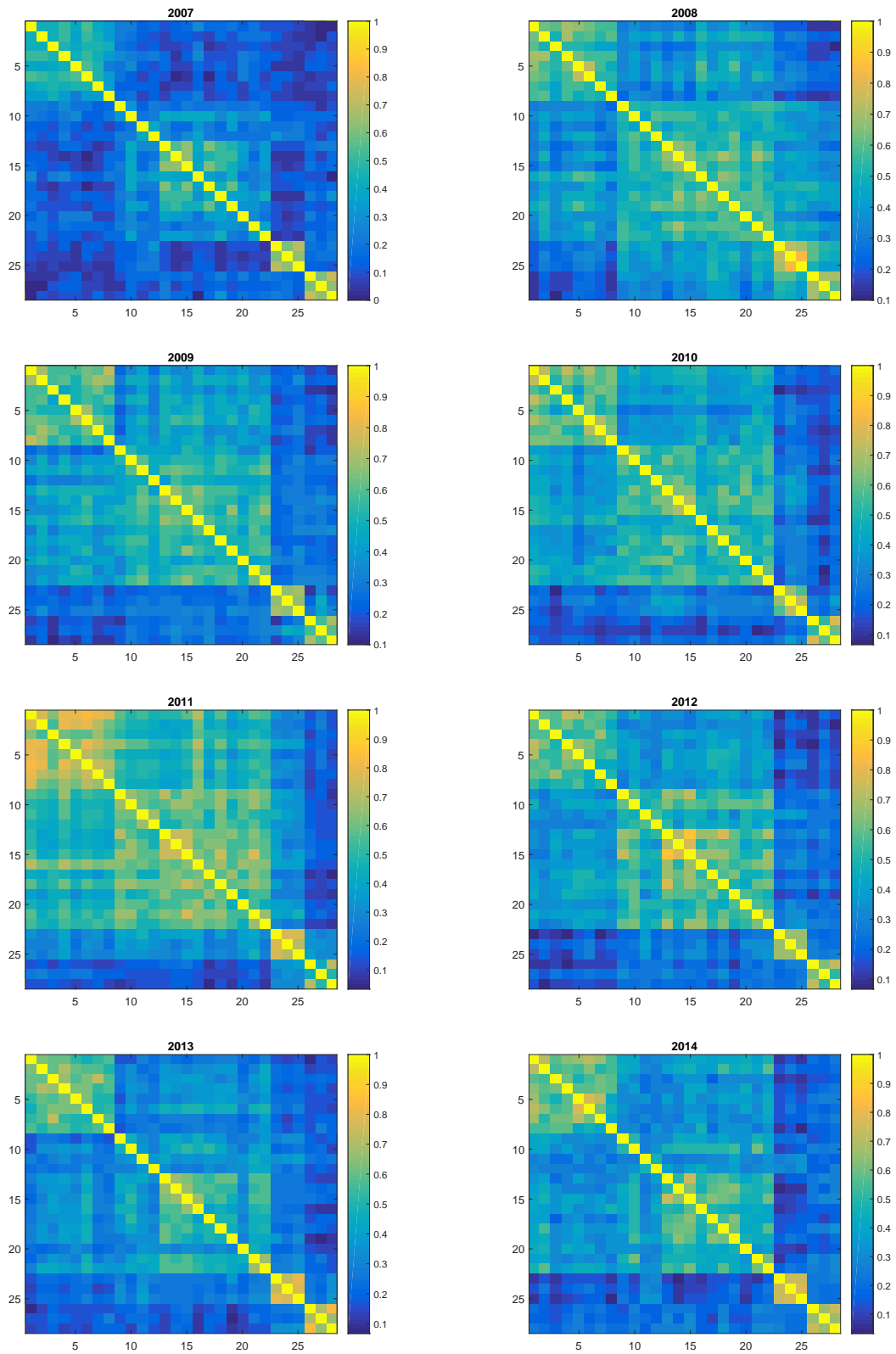


Figure 1: Empirical tail dependence matrices for 28 SIFIs

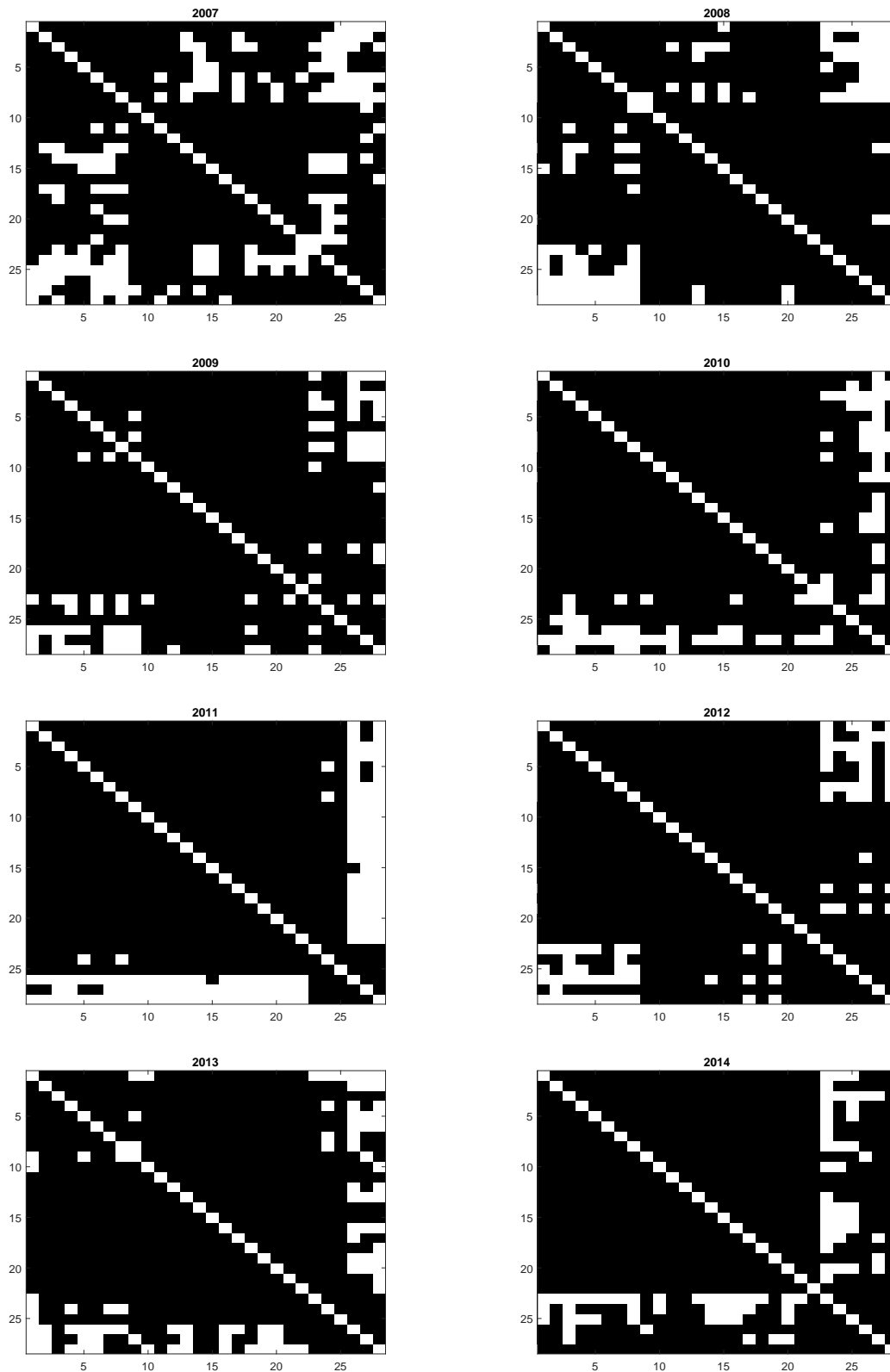


Figure 2: Binary adjacency matrices for 28 SIFIs obtained from empirical tail dependence in Figure 1 (black: 1, white: 0)

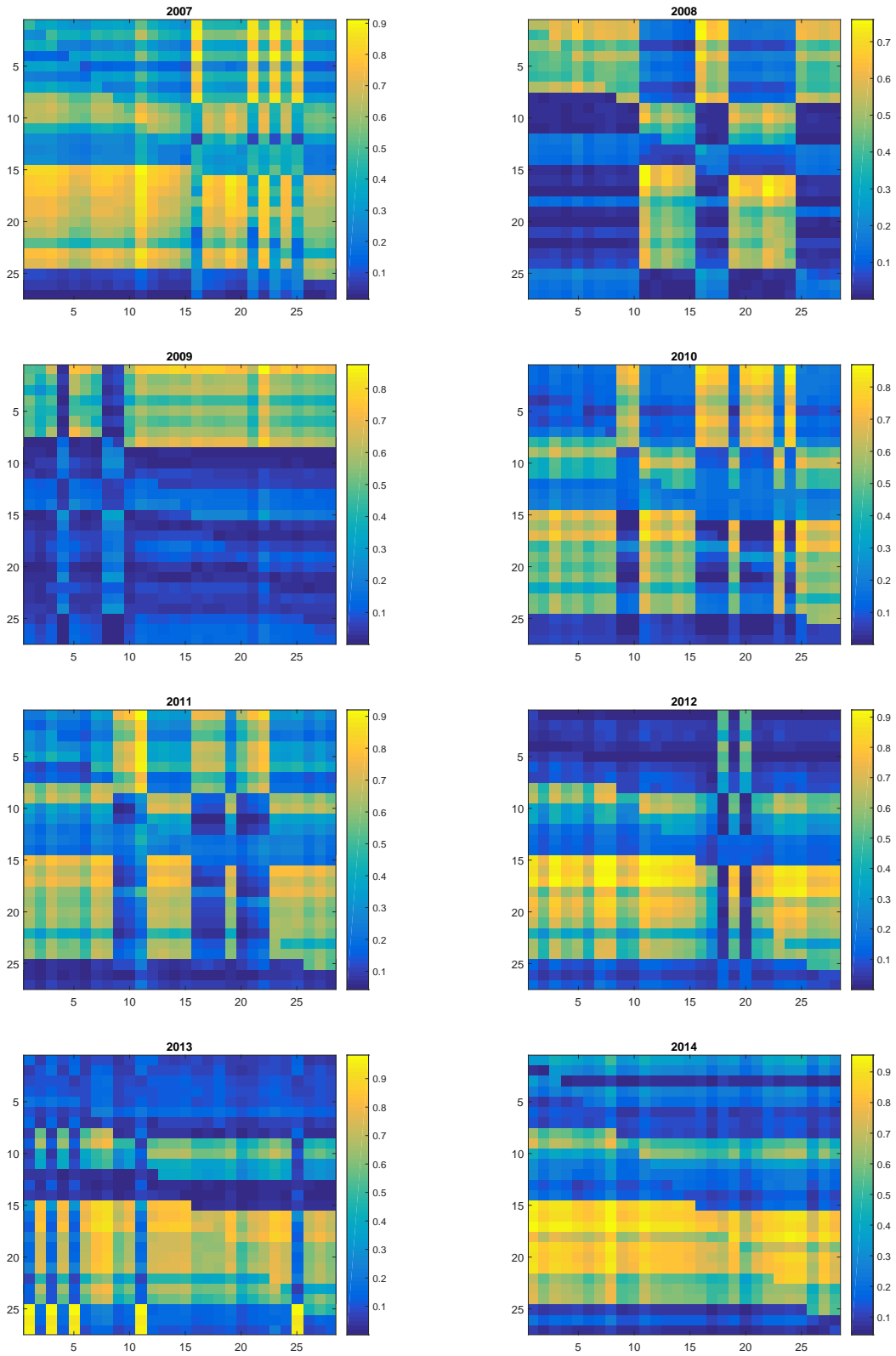


Figure 3: The estimates of  $\theta_{j|i}$  in Eq. 27 (x-axis :  $i$ , y-axis :  $j$ )

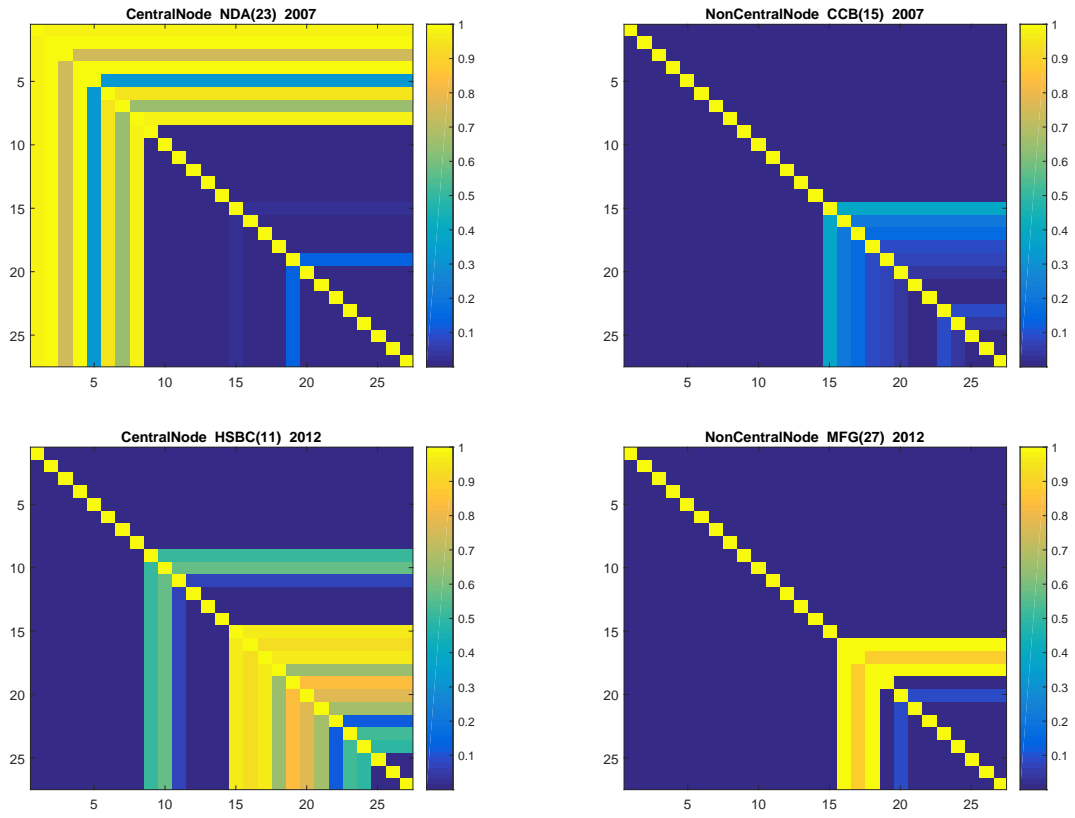


Figure 4: Factor copula implied tail dependence derived through Eqs. 15, 16 and 17

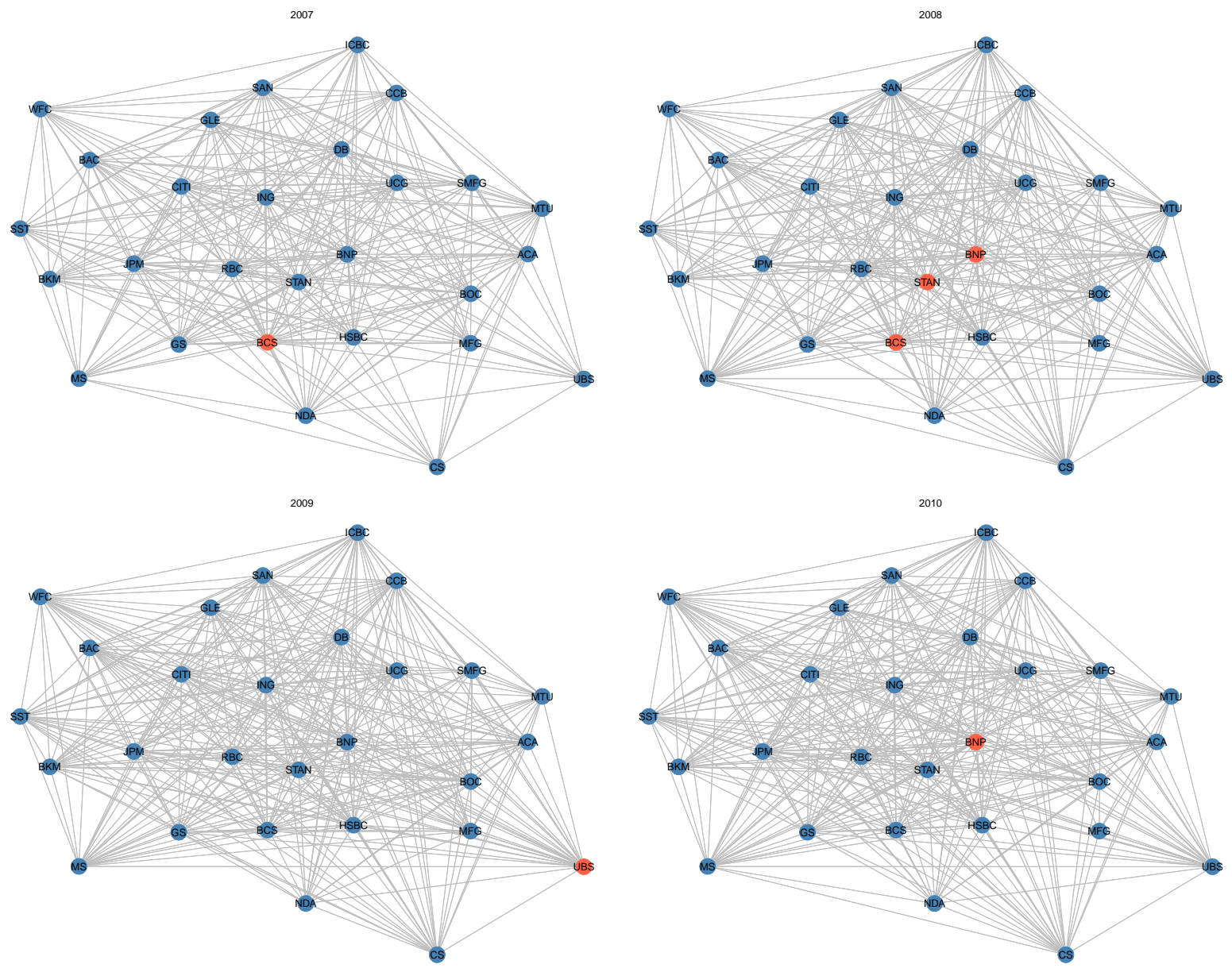


Figure 5: SIFI network structures produced by adjacency analysis on the empirical tail dependence matrix

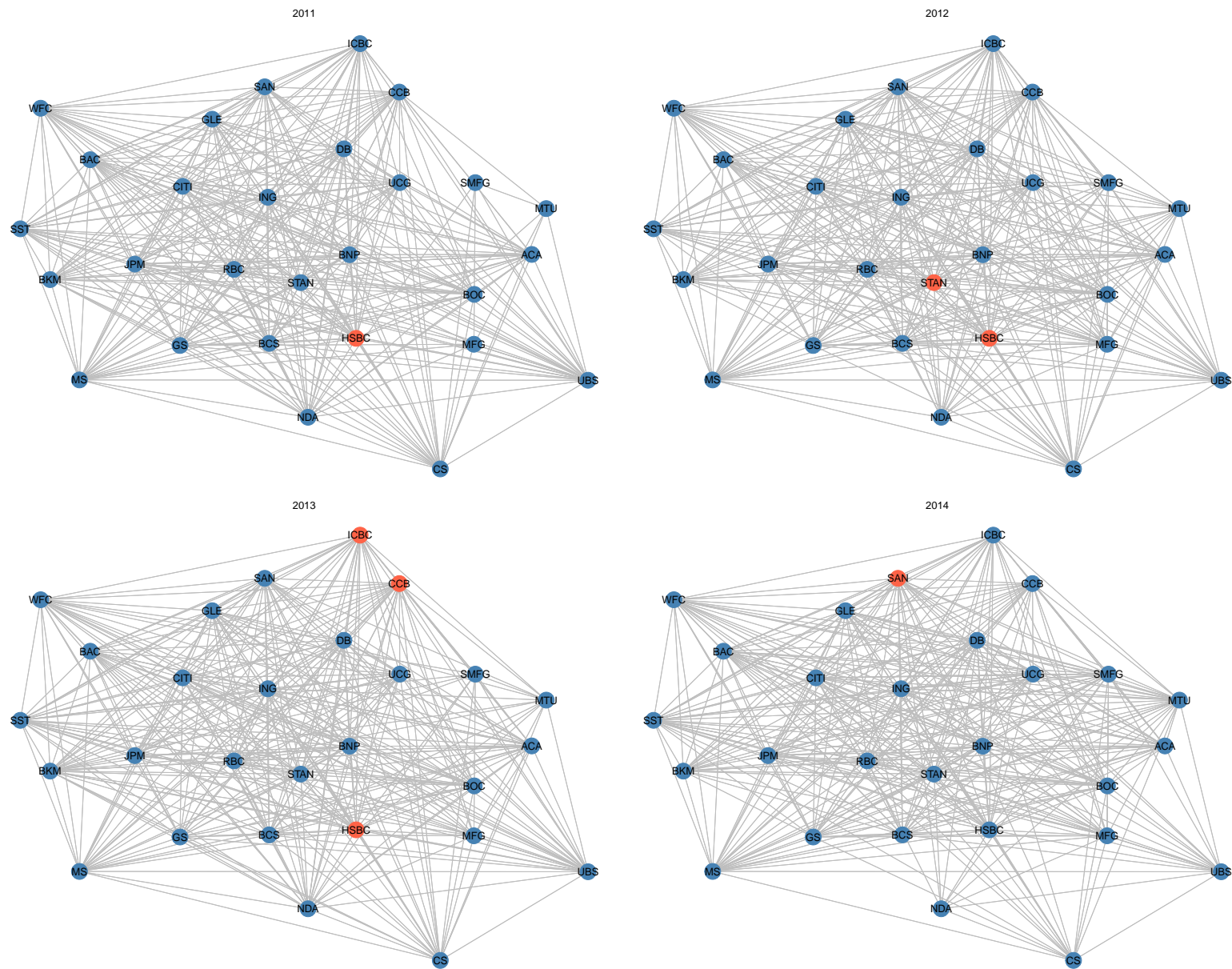


Figure 6: SIFI network structures produced by adjacency analysis on the empirical tail dependence matrix



## 9 Appendix

### 9.1 Conditional pair Gaussian copula

The expression in (12) is derived noting that  $C_{U_j|V}(u_j|v) = \partial C_{U_j,V}(u_j, v)/\partial v$ ; denoting  $\Phi_2(x, y; \rho)$  a bivariate cdf with correlation  $\rho$ , it follows then

$$C_{U_j|V}(u_j|v) = \frac{\partial C_{U_j,V}(u_j, v)}{\partial v} \quad (30)$$

$$= \frac{\partial \Phi_2(\Phi^{-1}(u_j), \Phi^{-1}(v); \alpha_{j1})}{\partial v} \quad (31)$$

$$= \frac{\partial \Phi_2(\Phi^{-1}(u_j), \Phi^{-1}(v); \alpha_{j1})}{\partial \Phi^{-1}(v)} \frac{\partial \Phi^{-1}(v)}{\partial v} \quad (32)$$

$$= \int_{-\infty}^{\Phi^{-1}(u_j)} \varphi_2(x, \Phi^{-1}(v); \alpha_{j1}) \frac{1}{\varphi(\Phi^{-1}(v))} dx \quad (33)$$

$$= \int_{-\infty}^{\Phi^{-1}(u_j)} \frac{1}{\sqrt{2\pi(1-\alpha_{j1}^2)}} \exp\left\{-\frac{(x - \alpha_{j1}\Phi^{-1}(v))^2}{2(1-\alpha_{j1}^2)}\right\} dx \quad (34)$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\Phi^{-1}(u_j) - \alpha_{j1}\Phi^{-1}(v)}{\sqrt{1-\alpha_{j1}^2}}} \exp\left\{-\frac{u^2}{2}\right\} du \quad (35)$$

$$= \Phi\left\{\frac{\Phi^{-1}(u_j) - \alpha_{j1}\Phi^{-1}(v)}{\sqrt{1-\alpha_{j1}^2}}\right\}, \quad (36)$$

where the sixth equality comes from integration by substitution. The resulting expression in (12) also can be obtained from the 1-factor correlation structure in (10).

### 9.2 Proof of Proposition 3.3.1

According to the properties of functions with regular variation, see Feller (1971), given that the tails of two variables  $W, \varepsilon_j$  are different but symmetric, then  $\mathbf{P}(W + \varepsilon_j < -s) = s^{-\alpha}(A_W + A_\varepsilon) + o(s^{-\alpha})$ , see also Hyung and de Vries (2007), Oh and Patton (2015). Then it follows:

$$\begin{aligned} \mathbf{P}(Z_j < -s) &= \mathbf{P}(\theta_{j1}W + \sqrt{1-\theta_{j1}^2}\varepsilon_j < -s) \\ &= \mathbf{P}(\theta_{j1}W < -s) + \mathbf{P}(\sqrt{1-\theta_{j1}^2}\varepsilon_j < -s) + o(s^{-\alpha}) \\ &= A_W \left(\frac{s}{\theta_{j1}}\right)^{-\alpha} + A_\varepsilon \left(\frac{s}{\sqrt{1-\theta_{j1}^2}}\right)^{-\alpha} \\ &= s^{-\alpha} \left(A_W \theta_{j1}^\alpha + A_\varepsilon (1-\theta_{j1}^2)^{\alpha/2}\right), \end{aligned} \quad (37)$$

as  $s \rightarrow \infty$ .

Consider two different dynamics of  $Z_i$  and  $Z_j$ ,  $\theta_{i|1} \neq \theta_{j|1}$ . Then, following Oh and Patton (2015), we find the link between two thresholds  $s_i$  and  $s_j$ . The relation between  $s_i/\theta_{i|1}$  and  $s_j/\theta_{j|1}$  depends if the value of the expression

$$A_W \theta_{j|1}^\alpha \theta_{i|1}^\alpha + A_\varepsilon (1 - \theta_{j|1}^2)^{\alpha/2} \theta_{i|1}^\alpha \quad (38)$$

is smaller or larger compared to the value of

$$A_W \theta_{i|1}^\alpha \theta_{j|1}^\alpha + A_\varepsilon (1 - \theta_{i|1}^2)^{\alpha/2} \theta_{j|1}^\alpha. \quad (39)$$

This follows from the observation that

$$\begin{aligned} \mathbf{P}(Z_i < -s_i, Z_j < -s_j) &\approx \mathbf{P}(\theta_{i|1} W < -s_i, \theta_{j|1} W < -s_j) + \mathcal{O}(s^{-\alpha}) \\ &\approx \mathbf{P}\left(W < -\frac{s_i}{\theta_{i|1}}, W < -\frac{s_j}{\theta_{j|1}}\right) \\ &\approx \mathbf{P}\left(W < \min\left\{-\frac{s_i}{\theta_{i|1}}, -\frac{s_j}{\theta_{j|1}}\right\}\right), \end{aligned} \quad (40)$$

as  $s \rightarrow \infty$ .

Furthermore,

$$\mathbf{P}\left(W < \min\left\{-s_i/\theta_{i|1}, -s_j/\theta_{j|1}\right\}\right) = \begin{cases} s_i^{-\alpha} A_W \theta_{i|1}^\alpha & \text{if } |s_i/\theta_{i|1}| > |s_j/\theta_{j|1}|, \\ s_j^{-\alpha} A_W \theta_{j|1}^\alpha & \text{if } |s_i/\theta_{i|1}| < |s_j/\theta_{j|1}|. \end{cases} \quad (41)$$

The condition  $|s_i/\theta_{i|1}| > |s_j/\theta_{j|1}|$  is fulfilled when simultaneously

$$A_W \theta_{i|1}^\alpha \theta_{j|1}^\alpha + A_\varepsilon (1 - \theta_{i|1}^2)^{\alpha/2} \theta_{j|1}^\alpha > A_W \theta_{j|1}^\alpha \theta_{i|1}^\alpha + A_\varepsilon (1 - \theta_{j|1}^2)^{\alpha/2} \theta_{i|1}^\alpha, \quad (42)$$

$$\theta_{i|1} < \theta_{j|1}, \quad (43)$$

or

$$A_W \theta_{i|1}^\alpha \theta_{j|1}^\alpha + A_\varepsilon (1 - \theta_{i|1}^2)^{\alpha/2} \theta_{j|1}^\alpha < A_W \theta_{j|1}^\alpha \theta_{i|1}^\alpha + A_\varepsilon (1 - \theta_{j|1}^2)^{\alpha/2} \theta_{i|1}^\alpha, \quad (44)$$

$$\theta_{i|1} > \theta_{j|1}. \quad (45)$$

On the other hand, the condition  $|s_i/\theta_{i|1}| < |s_j/\theta_{j|1}|$  is fulfilled when simultaneously

$$A_W\theta_{i|1}^\alpha\theta_{j|1}^\alpha + A_\varepsilon(1 - \theta_{i|1}^2)^{\alpha/2}\theta_{j|1}^\alpha > A_W\theta_{j|1}^\alpha\theta_{i|1}^\alpha + A_\varepsilon(1 - \theta_{j|1}^2)^{\alpha/2}\theta_{i|1}^\alpha, \quad (46)$$

$$\theta_{i|1} > \theta_{j|1}, \quad (47)$$

or

$$A_W\theta_{i|1}^\alpha\theta_{j|1}^\alpha + A_\varepsilon(1 - \theta_{i|1}^2)^{\alpha/2}\theta_{j|1}^\alpha < A_W\theta_{j|1}^\alpha\theta_{i|1}^\alpha + A_\varepsilon(1 - \theta_{j|1}^2)^{\alpha/2}\theta_{i|1}^\alpha, \quad (48)$$

$$\theta_{i|1} < \theta_{j|1}. \quad (49)$$

Combining the results from Eq.(41) and Eq.(37), the result follows.

### 9.3 Proof of Proposition 3.3.2

Given the reasoning in Oh and Patton (2015), it follows that as  $s \rightarrow -\infty$ ,

$$f_W(s) = \alpha A_W(-s)^{-\alpha-1}, \quad (50)$$

where  $f_W(s)$  is the probability density of  $W$ . Then, using the fact that the tail index  $\alpha$  equals the degrees of freedom  $\nu$  for the Student- $t$  distribution, using Mathematica, it follows that

$$\begin{aligned} A_W &= \lim_{s \rightarrow -\infty} \frac{f_W(s)}{\nu(-s)^{-\nu-1}}, \\ &= \frac{(\nu\sigma^2)^{\frac{\nu+1}{2}}}{\nu^{3/2}\sigma B(\nu/2, 1/2)}, \end{aligned}$$

where  $B(\cdot, \cdot)$  is the beta function;  $A_\varepsilon$  directly follows from  $A_W$ .

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This research was supported by the Deutsche  
Forschungsgemeinschaft through the SFB 649 "Economic Risk".

