On Investment Consumption Modeling with Jump Process Extensions for Productive Sectors

Tina Engler

Institute of Mathematics, Martin-Luther-University Halle-Wittenberg

We consider the optimal investment and consumption behavior of an agricultural sector that generates income from productive capital and interest payments, and that has expenditures due to interest payments for debt. The productivity of capital, the interest rate and the capital depreciation rate are uncertain and modeled as solutions of stochastic differential equations. We adopt the modeling ideas of an international debt and finance model by Fleming and Stein [1], and generalize it to a stochastic depreciation of capital stock. Moreover, we assume a discontinuous evolution of the productivity of capital, the interest rate and the capital depreciation rate. The uncertainties are modeled with the help of Wiener processes and Poisson processes. Accordingly, the investor's net wealth follows a jump diffusion stochastic differential equation. The problem consists of maximizing the sum of the running utility of consumption and the utility of wealth at terminal time T and it is formulated as a stochastic optimal control problem with a jump diffusion state equation. A stochastic maximum principle is applied to the case of HARA utility functions. We obtain an adapted solution of the corresponding backward stochastic adjoint equation. From the maximum condition we can deduce the optimal investment and consumption rate in an explicit form. Finally, the optimal consumption rate depends on a solution of an ordinary differential equation.

Reference:

[1] Fleming, W. H. and Stein, J. L. (1999). *A Stochastic Optimal Control Approach to International Finance and Foreign Debt.* CESifo Working Paper No. 204, Munich.