Computing Moments of the Exit Distribution for Diffusion Processes Using Linear Programming

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Summary: We present a new approach to the numerical computation of (higher) moments of the exit time distribution of diffusion processes. The method relies on a linear programming (LP) formulation of a process exiting from a bounded domain. The LP formulation characterizes the evolution of the stopped process through the moments of the induced occupation measure and leads to an infinite dimensional LP problem on a space of measures. Our functional method yields finite dimensional linear programs which approximate the infinite dimensional problem. An important aspect of our approach is the fact that excellent software is readily available.
We shall illustrate the method by looking at some examples. We shall specifically report on numerical computations and results related to change-point detection methods for drifted Brownian motion. Procedures for detecting changes in these models, such us CUSUM, EWMA and the Roberts-Shiryaev procedure, can be described in terms of one-dimensional diffusion processes and lead to exit time problems. Important operating characteristics for all of these procedures are the mean exit time and higher moments of the exit distributions of the diffusion processes.

1. Introduction. In this paper we describe the main ideas of a numerical method for computing the moments of exit time distributions of diffusion processes. An interesting feature of the method for the OR community is the fact that it is based on linear programming. We shall illustrate the method by looking at two examples in greater detail. We shall specifically report on numerical results related to geometric Brownian motion, a process important in mathematical finance, and to the Roberts-Shiryaev process, a process important in the context of change-point detection.
The method is a specialized version of the linear programming formulation for controlled Markov processes which, for the discrete setting, was developed by Manne [9], and extended to very general settings by e.g. Bhatt and Borkar [2], Kurtz & Stockbridge [8], Hernandez-Lerma et al. [7] and others. The key observation which underlies the LP-formulation is that the evolution of a very large class of Markov process can be described in terms of a system of equations which specify the occupation measure of the process. For controlled Markov processes this characterization of the evolution of the process leads to an infinite dimensional LP-problem, the feasible points being measures on the product space of the set of states (of the process) and the set of possible control values. For the special case of exit problems for (uncontrolled) diffusions this approach reduces to the identification of a feasible point. In the case of "no-control" the objective function of the LP problem is just an "auxiliary" tool.
The main characteristics of the method are (1) to describe (and approximate) occupation measures through its moments, (2) to characterize feasible moment sequences for exit time distributions from bounded domains by linear equations involving the generator of the process and a chosen class of test functions, and linear inequalities derived from the Hausdorff conditions for (moment) sequences of measures supported on the unit interval, the unit square, etc. (see e.g. [5] and [3]).
The paper is organized as follows. In section 2 we give some theoretical results which are the

foundation of the method. In section 3 we illustrate the method by looking at geometric Brownian motion. We compare the numerical results based on the LP-approach with the exact values. In section 4 we analyze one of the dection schemes referred to above, the Roberts-Shiryaev process, in some detail. This process is closely related to geometric Brownian motion. Thus geometric Brownian motion can be used to calibrate the numerical method for computations concerning the R-S process.

2. Theoretical results. In light of the examples to be discussed below and in order not to overburden the presentation with technicalities we restrict our discussion to exit problems of finite dimensional diffusions from a bounded interval. Due to translation and scaling properties of monomials in $\mathbb{R}^d$ we could assume w.l.o.g. the interval to be the hypercube. We shall further assume that all exit times to be considered have a finite mean value. The following result is thus an immediate consequence of the martingale characterization of diffusion processes, e.g. [4] or [11], the optional sampling theorem and the Helley-Bray theorem.

Theorem 1. *Let $(X_t)_{t\geq 0}$ be a solution of the martingale problem for a diffusion operator $\mathcal{A}$ starting at $x_0 \in \mathbb{R}^d$ at time $t_0 \geq 0$. Let τ denote the first exit time of the time-space process $(X_t, t)_{t\geq t_0}$ from a bounded "interval" $G = I \times [0, T]$, $I \subset \mathbb{R}^d$. Assume that $E[\tau] < \infty$. Then there exist a finite measure μ_0 on G^0, the interior of G, and a probability measure on ∂G, the boundary of G, such that for every function $f : \mathbb{R}^d \times \mathbb{R}^+ \to \mathbb{R}$ which is $\mathcal{C}^\infty$ on $\mathbb{R}^d$ and $\mathcal{C}^1$ on $\mathbb{R}^+$ the adjoint equation*

$$\int_{\partial G} f(x,t)d\mu_1(t,x) - f(t_0, x_0) - \int_{G^0} \left[\frac{\partial f}{\partial t} + \mathcal{A}f\right](x,t)d\mu_0(x,t) = 0 \tag{1}$$

holds. The probability measure μ_1 is the joint distribution of the exit location and the exit time of the process $(X_{t\wedge T}, t \wedge T)_{t\geq t_0}$, and μ_0 is its occupation measure.

The converse to theorem 1 was proved in [8] in a very general setting. The following result is a special case of this general theorem. We keep on using the notation introduced above.

Theorem 2. *Let μ_0 be a finite measure on G^0 and let μ_1 be a probability measure on ∂G which satisfy the adjoint equation (1). Then there exists a stochastic process $(Y_t)_{t\geq t_0}$ starting at x_0 at time t_0 which is adapted to a filtration of σ-fields $\{F_t\}$ and an $\{F_t\}$-stopping time $\tilde{\tau}$ such that for every test function f (see above)*

(i) $\quad f(Y_{t\wedge\tilde{\tau}}, t \wedge \tilde{\tau}) - \int_{t_0}^{t\wedge\tilde{\tau}} \left[\frac{\partial f}{\partial s} + \mathcal{A}f\right](Y_s, s)ds$ *in an $\{F_t\}$-martingale,*

(ii) $\quad P\Big[(Y(\tilde{\tau}), \tilde{\tau}) \in \partial G\Big] = 1,$

and

(iii) $\quad \Big\{t \mid (Y_t, t) \in \partial G, t < \tilde{\tau}\Big\}$ *has Lebesgue measure zero.*

Moreover, for each Borel subset $\Gamma_0 \subset G^0$ and $\Gamma_1 \subset \partial G$

$$E\left[\int_0^{\tilde{\tau}} I_{\Gamma_0}(Y_s, s)ds\right] = \mu_0(\Gamma_0), \tag{2}$$

$$E\Big[I_{\Gamma_1}(Y(\tilde{\tau}), \tilde{\tau})\Big] = \mu_1(\Gamma_1). \tag{3}$$

The adjoint equation (1) specifies uncountably many constraints for the two measures μ_0 and μ_1. For our numerical method we shall choose a finite set of test functions instead. To be specific, we characterize the measures μ_0 and μ_1 through their moments and therefore use monomials $f(x,t) := x_1^{n_1} \cdots x_d^{n_d} t^m$, $n_1 + \cdots + n_d = n \geq 0$, $m \geq 0$ up to a fixed order $M \in I\!N$. The reason for choosing monomials is that we are going to exploit the linear structure of the Hausdorff conditions (see below). For instance, if μ_0 is a finite measure on the unit interval and if M is chosen to be 3 we are going to work with sequences $z = (z_0, z_1, z_2, z_3)$ which satisfy the inequalities

$$\begin{array}{ll} \text{(i)} & z_i \geq 0, \quad 0 \leq i \leq 3, \\ \text{(ii)} & z_i - z_{i+1} \geq 0, \quad 0 \leq i \leq 2, \\ \text{(iii)} & z_i - 2z_{i+1} + z_{i+2} \geq 0, \quad 0 \leq i \leq 1, \\ \text{(iv)} & z_0 - 3z_1 + 3z_2 - z_3 \geq 0. \end{array} \tag{4}$$

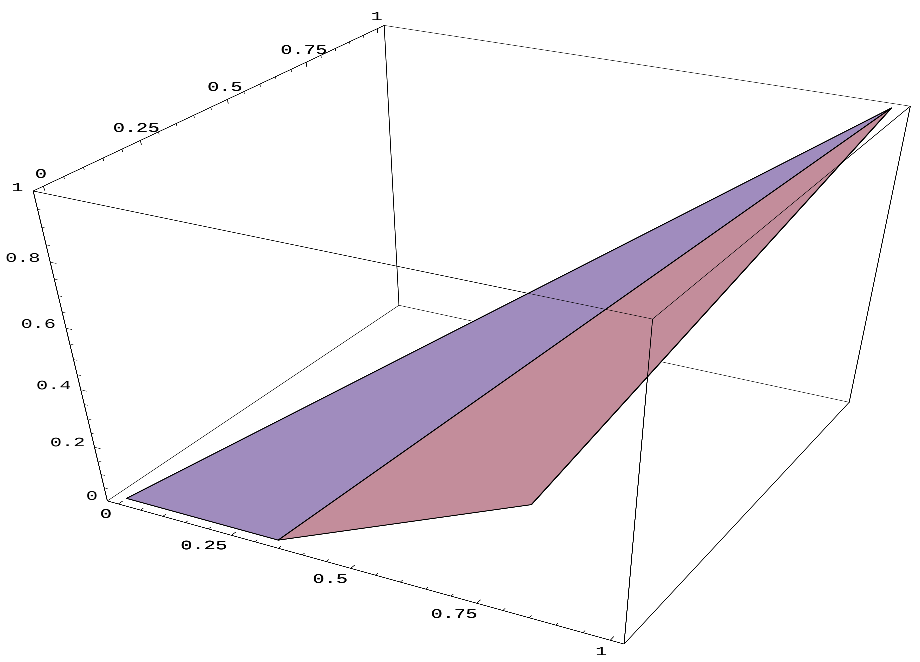

Figure 1: The set of Hausdorff sequences of order 3

Figure 1 shows the set of such admissible sequences in $I\!R^3$ if $z_0 = 1$, the case of approximating "moment sequences" of probability measures on [0,1] of order 3. This polytope has 4 corner points. Figure 2 illustrates the set of admissible sequences $(1, z_1, z_2, z_3)$ which satsify an additional linear equation, cf. the adjoint equation (1).

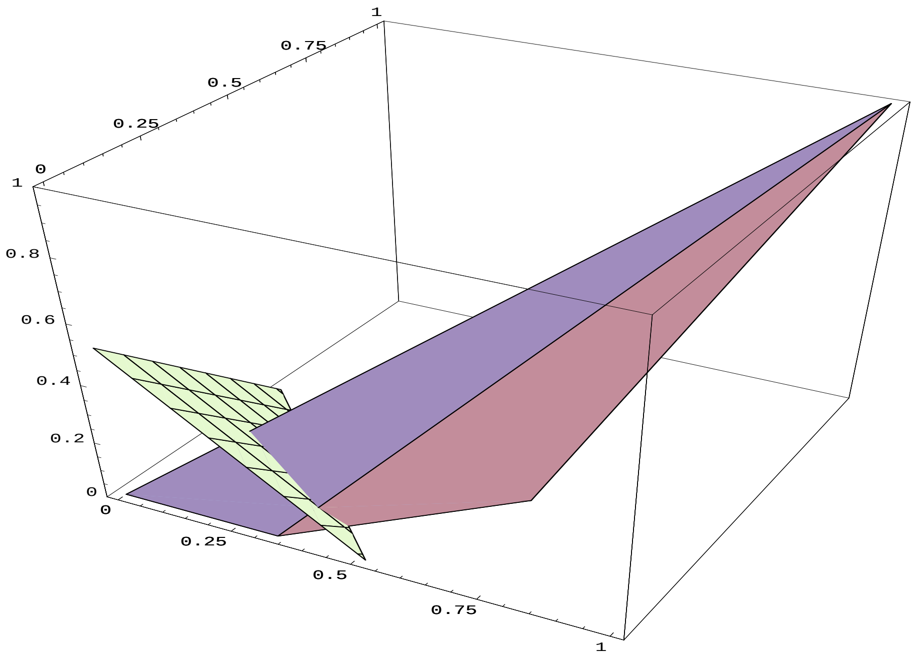

Figure 2: The set of Hausdorff sequences of order 3 with an additional equality constraint

In general, for any finite M the "finite dimensional Hausdorff conditions of order M" involve the iterated differences of a sequence $(z_0, z_1, \ldots, z_M)$ and are, in the 1-dimensional case, of the form: For every m, $0 \leq m \leq M$, and k, $0 \leq k \leq M - m$,

$$(-1)^m \Delta^m z(k) := \sum_{j=0}^{m} \binom{m}{j} (-1)^j z_{j+k} \ \geq\ 0; \tag{5}$$

see Section 3 for the 2-dimensional conditions. In the next two sections we shall illustrate how finitely many equality constraints on sequences $(z_0, z_1, \ldots, z_M)$ derived from the adjoint equation (1) together with the linear inequalities (5) provide a viable method to compute exit time distributions. To this end it is important to exploit the inversion formula for a distribution function in terms of iterated moment sequences, cf. [5] for the 1-dimensional case and [3] for the higher dimensional case. To illustrate the point, let us consider the 1-dimensional case. Given a sequence $(z_0, z_1, \ldots, z_M)$ which satisfies (5) we place the mass

$$\binom{M}{k} (-1)^{M-k} \Delta^{M-k} z(k) \tag{6}$$

at the point k/M, $0 \leq k \leq M$, and use the resulting mass distribution as an approximation to a scaled version of the exit distribution.

3. The LP-approach for geometric Brownian Motion. Let $(X_t)_{t \geq 0}$ denote geometric Brownian motion starting at x_0, i.e. (X_t) is the solution of the stochastic differential equation

$$dX_t = \alpha X_t \, dt + \sigma X_t \, dW_t, \qquad X(0) = x_0 > 0. \tag{7}$$

We shall call α the drift coefficient and σ the diffusion coefficient of this process. Geometric Brownian motion is the classical model for risky assets and is at the heart of the Black-Scholes formula, cf. eg. [1]. To compute higher moments of the exit time of $(X_t)_t$ from the interval $(0,1)$ we need to consider the generator A of the time-space process $Y_t = (X_t, t)$, $t \geq 0$, i.e. for smooth functions $f(x,t)$,

$$Af(x,t) = \frac{\partial f}{\partial t}(x,t) + \frac{\sigma^2}{2}\, x^2\, \frac{\partial^2}{\partial x^2}(x,t) + \alpha x\, \frac{\partial f}{\partial x}(x,t). \tag{8}$$

Let $\tau = \tau_{x_0}$ denote the first exit time of (X_t) from $(0,1)$ when the process starts at x_0, $0 < x_0 < 1$. We shall compare the numerical values of $E[\tau^n]$, $n \geq 0$, obtained by using the LP-approach with the exact values derived from the following analytical formula for the Laplace transform of τ (see Proposition 1 below); we have used *Mathematica* to do the necessary differentiation when computing moments, see Table 1 below.

Proposition 1. *Let $(X_t)_{t\geq 0}$ be a geometric Brownian Motion with drift parameter α and diffusion coefficient σ, $(\alpha - \sigma^2/2) > 0$, starting at x_0, $0 < x_0 < 1$, and let $\tau = \inf\{t \geq 0 \mid X_t = 1\}$. The Laplace transform of τ is given by, $\lambda > 0$,*

$$E[e^{-\lambda\tau}] = \exp\left[\ln(x_0)\left(\sqrt{\lambda A + B} - B\right)\right], \tag{9}$$

where $A = \frac{2}{\sigma^2}$ and $B = \left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)$.

Proof. Since (X_t) is a solution of the stochastic differential equation

$$dX_t = \alpha X_t dt + \sigma X_t dW_t, \quad X(0) = x_0,$$

we get

$$X_t = X_0 \exp\left[\left(\alpha - \frac{\sigma^2}{2}\right)t + \sigma W_t\right].$$

Thus, for every $\mu > 0$,

$$\left(\frac{X_t}{x_0}\right)^{-\mu} \exp\left[\mu\sigma W_t - \frac{\mu^2\sigma^2}{2}t\right] = \exp\left[-\mu\left(\alpha - \frac{\sigma^2}{2}\right)t - \frac{\mu^2\sigma^2}{2}t\right].$$

Since $E_{x_0}[\tau] < \infty$, $X_\tau = 1$ and since $\left(\exp\left[\mu\sigma W_t - \frac{\mu^2\sigma^2}{2}t\right]\right)_{t\geq 0}$ is a martingale we obtain,

$$x_0^\mu = E\left[e^{-\lambda\tau}\right],$$

where

$$\lambda := \mu\alpha - \frac{\sigma^2}{2}(\mu - \mu^2). \tag{10}$$

We obtain formula (3) when we solve Eqn. (10) for $\mu > 0$ in terms of λ.

$\diamond$

Instead of doing computations directly for X_t we shall analyze the transformed process $Z_t = (-\ln(X_t), t)$ and employ a time and space truncation technique to reduce the exit time problem for (Z_t) to one from a bounded domain. Since (X_t) solves Eqn.(1), $\xi_t := -\ln(X_t)$ equals drifted Brownian, i.e.

$$d\xi_t = -\left(\alpha - \frac{\sigma^2}{2}\right) dt + \sigma\, d(-W_t)\,,\ \xi(0) = -\log(x_0),$$

and

$$\tau_{x_0} = \tilde{\tau}_{\xi_0} \;=\; \inf\{t \mid \xi_t = 0\}\,.$$

Since (ξ_t) lives on $(0, \infty)$ we approximate $\tilde{\tau}_{\xi_0}$ by

$$\hat{\tau}^b_{\xi_0} = \inf\{\, t \mid (\xi_t) \text{ exits from } (0, b)\}\,,$$

b "large" (depending on the coefficients α and σ).
Since

$$\tau^b_{\xi_0} = \inf\left\{ t \;\middle|\; \frac{1}{b}\,\xi_t \quad \text{exits from } (0,1)\right\}$$

we shall henceforth consider the process η_t where

$$d\eta_t = -\frac{1}{b}\left(\alpha - \frac{\sigma^2}{2}\right) dt + \frac{\sigma}{b}\, d(-W_t),\ \eta(0) = -\frac{1}{b}\,\xi_0.$$

Since we need to consider the time-space process (η_t, t) we shall also truncate the time coordinate at some "large" value T. Typical values for b and T which we have used for the case $\alpha = 2$ and $\sigma^2 = 1$ are $b = 8$ and $T = 10$. The actual choice of b and T is dictated by the requirement the probability of not exiting from $(0, b) \times (0, T)$ to be "small". Using monomials of the form $x^n t^m$, n, $m \ge 0$, $x \in [0, 1]$, $t \in [0, T]$, as test functions and calling $z[n, m]$ the moments of the occupation measure of (η_t, t) on $(0, 1) \times (0, T)$, $z_0[m]$, $z_1[m]$ and $z_T[m]$ the moments of the "exit" distribution on $\{0\} \times [0, T]$, $\{1\} \times [0, T]$ and $[0, 1] \times \{T\}$, resp. the adjoint equations become, $\mu = -\frac{1}{b}\,(\alpha - \frac{\sigma^2}{2})$, $t_0 = 0$, m, $n \ge 0$,

$$\begin{aligned} 0 \;=\; & -m\, z[m-1, n] \;-\; \frac{n(n-1)\sigma^2}{2}\; z[m, n-2] \;-\; n\,\mu\, z[m, n-1] \\ & -t_0^m\, \eta_0^n \;+\; 0^n\, z_0[m] \;+\; z_1[m] \;+\; T^m\, z_T[n], \end{aligned} \tag{11}$$

where values for negative indices are defined to be zero.
To these linear equality constraints we add the linear inequality constraints due to the Hausdorff conditions for z, z_0, z_1 and z_T, viz. for all $0 \le n$, $m \le M$, $i \le M - n$, $j \le M - m$

$$\sum_{k=0}^{n}\sum_{l=0}^{m} \binom{n}{k}\binom{m}{l} (-1)^{k+l}\, \frac{z[i+k, j+l]}{T^l} \;\ge\; 0, \tag{12}$$

$$\sum_{k=0}^{n} \binom{n}{k} (-1)^k\, z_T(i+k) \;\ge\; 0, \tag{13}$$

and, $\nu \in \{0,1\}$,

$$\sum_{l=0}^{m} \binom{m}{l} (-1)^l \, \frac{z_{(\nu)}(j+l)}{T^l} \;\geq\; 0. \tag{14}$$

The (approximated) m-th moment of the exit time τ is thus given by

$$z_1(m) \;+\; z_0(m). \tag{15}$$

The following numbers for the moments of the exit time for geometric Brownian motion were obtained using the interior point solver of the *Cplex* package restricting M to 12 and using as an "artificial" objective function a scaled sum of all the variables. Similar numbers are obtained when using the dual or the primal solver of *Cplex*. Note, that the primal solver requires the largest number of iterations for these kind of problems. Since the moments of geometric Brownian motion vary over a large range depending on the choice of α, e.g. $E[\tau^{14}] \approx 3.7 \cdot 10^{21}$ if $\alpha = 1$, $\sigma^2 = 1$, and $E[\tau^{14}] \approx 3.9 \cdot 10^{-9}$ for $\alpha = 10$, $\sigma^2 = 1$, it can be seen from Table 1 that the moments computed using the LP-approach compare well with the exact values for the moments (listed in columns headed by E(xact) V(alues) below).

	$x_0 = 0.2$		$x_0 = 0.5$		$x_0 = 0.8$	
Moment	LP	EV	LP	EV	LP	EV
0	1.00000	1.000000	1.000000	1.000000	1.000000	1.000000
1	1.07296	1.072959	0.462098	0.462098	0.148762	0.148762
2	1.62811	1.628111	0.418912	0.418912	0.088247	0.088247
3	3.40601	3.406048	0.657223	0.657223	0.120955	0.120955
4	9.44191	9.443340	1.549950	1.549947	0.270741	0.270741
5	33.26220	33.300460	4.962390	4.962398	0.844981	0.844982
6	143.25000	144.073391	20.180500	20.180559	3.385860	3.385919
7	727.11000	742.695628	99.719300	99.720156	16.569500	16.572083
8	4185.14000	4456.993374	580.459000	580.470151	95.732300	95.824745

Table 1: Moments of the exit time distribution of geometric Brownian Motion with drift $\alpha = 2$, variance $\sigma^2 = 1$ and various initial values. We have used the interior point algorithm provided by Cplex, using space truncation at $b = 8$ and time truncation at $T = 10$ (for $x_0 = 0.2$), $T = 11$ (for $x_0 = 0.5$) and $T = 12$ (for $x_0 = 0.8$).

4. The LP-approach for the Roberts-Shiryaev process. The Roberts-Shiryaev process $(X_t)_{t \geq 0}$ is the solution of the stochastic differential equation

$$dX_t = (1 + 2X_t)\, dt \;+\; \sqrt{2}\, X_t \, dW_t, \;\; X(0) = X_0 = 0.$$

It is an operatimg characteristic of a detection scheme, cf. eg. [10], for the following basic continuous time model of a constant drift term embedded in Brownian noise. Let $(\xi_t)_t$ be the process

$$d\xi_t = \rho I_{\{t>\theta\}} dt \; + \; \sigma dW_t,$$

where, in the simplest case, the parameters ρ and σ are known constants. The unknown parameter θ specifies the beginning of the effect of the constant drift term ρ. Without loss of generality we may assume that $\frac{\rho^2}{2\sigma^2} = 1$. The Roberts-Shiryaev detection scheme is the procedure which sounds an alarm, indicating a change of the drift term, at the first time the statistic (X_t) hits a given threshhold value b, cf. [10]. In [6] we have used the LP method to numerically evaluate the first moment of

$$\tau = \inf \{ \, t \, | \, X_t \geq b \, \} \, .$$

Here we compute higher moments of τ. Note that the R-S process is very much like geometric BM, so we have used the latter problem, for which exact formulas for the moments are known, to calibrate our method for the former problem.
Besides the linear inequalities due to the Hausdorff conditions the adjoint equations become using time truncation at a chosen value T, $0 \leq n, m \leq M$,

$$\begin{aligned} 0 \;\; = \;\; & -m \, z[n, m-1] \; - \; n(n-1) \, z[n, m] \; - \; 2n \, z[n, m] \; - \; n \, z[n-1, m] \\ & - 0^m \cdot 0^n \; + \; b^n \, z_1[m] \; + \; T^m \, z_T[n], \end{aligned} \tag{16}$$

where $z[n, m]$ denotes the moments of the occupation measure on $(0, b) \times (0, T)$ and $z_1[m]$, $z_T[n]$ resp. the moments of measures on the boundary $\{b\} \times [0, T]$ and $[0, b] \cdot \{T\}$ resp.
Table 2 shows values for the first moments for various threshhold values b. The following formula for the first moment of τ, cf. [10],

$$E[\tau] = e^{\frac{1}{b}} \int_{\frac{1}{b}}^{\infty} \frac{e^{-x}}{x} dx$$

can be easily evaluated. This formula together with other simple estimates on higher moments of τ show that the LP approach provides accurate results for the case under consideration. Other detection schemes like the CUSUM and the EWMA procedure, cf. [10], can also be decribed in terms of one-dimensional diffusion processes. They also lead to exit time problems and can be analyzed in the same way as the Roberts-Shiryaev process.

Moment	1	2	3	4	5
0	1.000000	1.000000	1.000000	0.999998	0.999982
1	0.597304	0.933651	1.191100	1.568470	1.602940
2	0.416417	1.058040	1.773620	3.284260	3.549480
3	0.342734	1.466020	3.316280	8.926700	11.609900
4	0.334072	2.457870	7.681630	29.455300	50.678200
5	0.384111	4.909770	21.592600	110.254000	247.368000
6	0.517888	11.629800	72.314300	455.4710000	1236.630000
7	0.850480	32.633900	284.562000	2036.870000	6144.720000
8	2.135710	105.543000	1290.580000	9569.360000	30286.500000
9	5.538060	359.755000	6352.490000	46211.100000	147204.000000
10	7.096700	1134.580000	29272.400000	216009.000000	683986.000000

Table 2: Moments of the exit time from an interval $(0, b)$, $b = 1, \ldots, 5$, for the Roberts-Shiryaev process with drift value $\alpha = 2$, variance $\sigma^2 = 2$ and initial value 0. We have used a time-truncation-level of 10 and employed the dual solver of the Cplex package.

5. Implementation issues. The LP problems which arise as a result of our method are highly structured. We therefore recommend to use matrix generator languages like AMPL and GAMS to set up the linear programs. Employing such an interface greatly simplifies the testing of modifications of the basic problems. For instance, instead of only using (approximating) moment sequences as variables we have also run programs using all iterated differences as unknowns. While this dramatically increases the number of variables our test runs seem to indicate that problems with "very large" M can be handled this way.

Another issue which we addressed is the kind of solver to use. As far as packages are concerned we have tested the routines provided by CPLEX, NAG and SAS, all of which handled the problems described in sections 3 and 4 well. Due to the large number of constraints compared to the small number of variables when formulating the LP-programs using (approximating) moment sequences as variables the dual simplex algorithm requires fewer iterations than the primal one. To find a feasible point for the dual problem is easy while the first phase of the simplex algorithm for the primal problem typically requires several thousand iterations for medium size problems.

References

[1] M. Baxter and A. Rennie, *Financial calculus: An introduction to derivative pricing*, Cambridge University Press, Cambridge (1996).

[2] A. G. Bhatt and V. S. Borkar, Occupation measures for controlled Markov processes: Characterization and optimality, *Ann. Probab.*, **24** (1996), pp. 1531–1562.

[3] A. I. Dale, Two-dimensional moment problems, *Math. Scientist*, **12** (1987), pp. 21–29.

[4] S.N. Ethier and T.G. Kurtz, *Markov Processes: Characterization and Convergence*, Wiley, New York (1986).

[5] W. Feller, *An Introduction to Probability Theory and its Applications*, Vol. 2, Wiley, New York (1965).

[6] K. Helmes, S. Röhl and R. Stockbridge, Computing moments of the exit time distribution for Markov processes by linear programming, (submitted for publication).

[7] O. Hernandez-Lerma, J. C. Hennet and J. B. Lasserre, Average Cost Markov Decision Processes: Optimality Conditions, *J. Math. Anal. Appl.*, **158** (1991), pp. 396–406.

[8] T. G. Kurtz and R. H. Stockbridge, Existence of Markov Controls and Characterization of Optimal Markov Controls, to appear in *SIAM J. Control. Optim.*.

[9] A.S. Manne, Linear programming and sequential decisions, *Management Sci.*, **6** (1960), 259–267.

[10] M. S. Srivastava and Y. Wu, Dynamic samling plan in Shiryaev-Roberts procedure for detecting a change in the drift of Brownian motion, *Ann. Stat.*, **22** (1994), pp. 805–823.

[11] D. W. Stroock and S. R. S. Varadhan, *Multidimensional diffusion processes*, Springer-Verlag, Berlin (1979).