Approximation for Overflow Moments of a Multiservice Link with Trunk Reservation[1]

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Abstract

In this paper we propose an approximation for individual overflow moments of a multiservice link with differing arrival rates, capacity requirements and mean holding times, where trunk reservation is used. The approximation can be considered as a generalization of Roberts' approximation for individual blocking probabilities of a multiservice link to higher moments. It can be computed very efficiently. Numerical results show that the approximation is accurate if the mean holding times do not greatly differ from each other.

1 Introduction

Recent developments in communication systems have led to much interest in systems where traffic of widely differing characteristics is integrated together. In the absence of controls, larger bandwidth calls can experience a much higher blocking probability than low bandwidth calls [23], [20]. Also by excessive alternative routing during periods of general overload, instability of the network and performance degradation may occur, cf. [12], [1], [21]. The trunk reservation strategy (TRS) is a very efficient and simple admission strategy for equalizing blocking probabilities or for limiting excessive alternative routing. Moreover, it is a simple fix for certain forms of instability in networks. Further, the TRS can be used to effect an almost complete priorization of the different traffic streams, while still utilizing the full capacity of the link. Various authors have considered models with TRS, cf. e.g. [1]–[4], [6], [8], [10]–[24] and the references therein.

In this paper we focus on the individual overflow moments of a multiservice link with trunk reservation, their approximation and evaluation. We consider a link of integer capacity C offered a finite number m of traffic streams indexed in the set $I := \{1, \ldots, m\}$. Calls of type $i \in I$ arrive as a Poisson stream of rate λ_i and have exponential holding times with mean $1/\mu_i$. Each such call requires an integer capacity b_i (bandwidth) and is accepted if and only if, after admittance, the link has at least $C - C_i$ remaining resource units; otherwise the call gets lost. The thresholds $C_i \in \{b_i, \ldots, C\}$, $i \in I$, are usually referred to as trunk reservation parameters. (Often the quantities $C - C_i$, $i \in I$, are also called trunk reservation parameters, cf. [4].) Without loss of generality we may assume that $C = \max(C_1, \ldots, C_m)$ and, because of possible rescaling, that the bandwidths b_i, $i \in I$, are relatively prime. The overflow traffic of type-i calls is routed to a secondary link of infinite capacity where the holding times are exponentially distributed with mean $1/\mu_i$, too. Commonly, mean and variance of the actual number of type-i overflow calls are of interest, but higher moments, too, cf. [18]. The main results of the paper are as follows: In Section 2 we give linear systems of equations for the partial factorial moments of the individual overflow streams. For network dimensioning tools, fast and numerically stable algorithms for

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the overflow moments are needed. However, this cannot be achieved for the mentioned linear systems of equations in case of a larger number of traffic types and growing capacity. Thus, in Section 3, we propose a new approximation for the overflow moments, which can be computed by recursions very fast. The approximation may be considered as a generalization of Roberts' approximation [19] for individual blocking probabilities to higher moments. In particular, it uses Roberts' approximation for the equilibrium utilization probabilities of the primary link as initialization of the recursion. In Section 4, some numerical examples are given for checking the accuracy in a variety of situations. We conjecture that the proposed approximation for the higher overflow moments can still be improved by using an improved approximation for the equilibrium utilization probabilities.

2 Linear system of equations for the overflow moments

Let $N_i(t)$ and $N_i'(t)$ denote the number of calls of type i in progress in the primary and secondary link at time t, respectively. Then the state of the primary link is given by the vector $N(t) := (N_1(t), \ldots, N_m(t))$, and $S(t) := N_1(t)b_1 + \ldots + N_m(t)b_m$ is the total amount of resource utilized by the calls in the primary link at time t. The set of allowed states Ω of $N(t)$ is determined by the bandwidths b_i and trunk reservation parameters C_i, $i \in I$, as well as by the capacity constraint $C := \max(C_1, \ldots, C_m)$: $\Omega := \{n \in \mathbb{Z}_+^m : \ (n,b) \le C, \ n_i b_i \le C_i \text{ for } i \in I\}$, where $n := (n_1, \ldots, n_m)$, $b := (b_1, \ldots, b_m)$ and $(n,b) := n_1 b_1 + \ldots + n_m b_m$. Let $p(n)$, $n \in \Omega$, and $q_0(\ell)$, $\ell \in \{0, \ldots, C\}$, be the equilibrium probabilities of the processes $N(t)$ and $S(t)$, respectively. The probability that an arriving type-i call is blocked (individual blocking probability) is given by

$$p_{b,i} = \sum_{n \in \Omega :\, (n,b) > C_i - b_i} p(n) = \sum_{\ell = C_i - b_i + 1}^{C} q_0(\ell), \quad i \in I. \tag{2.1}$$

In case of TRS a product-form solution for the equilibrium distribution $p(n)$ does not exist as one-way transitions are present in the state space destroying reversibility [9]. However, in principle the loss probabilities — and, more generally, the overflow moments — can be computed by solving the corresponding steady state equations numerically. But this becomes intractable as the number of call types and the link capacity grow, therefore approximations are of interest.

If there is no trunk reservation, i.e. $C_i = C$, $i \in I$, then the $p(n)$ are given by

$$p(n) = G^{-1}(\Omega) \prod_{i \in I} \frac{\varrho_i^{n_i}}{n_i!}, \quad n \in \Omega,$$

where $G(\Omega)$ is the normalization constant and $\varrho_i := \lambda_i/\mu_i$, $i \in I$. The utilization probabilities satisfy the following recursion and normalization condition, originally due to [19] and [7]:

$$\ell q_0(\ell) = \sum_{i \in I} \varrho_i b_i q_0(\ell - b_i), \quad \ell = 1, 2, \ldots, C, \qquad \sum_{\ell=0}^{C} q_0(\ell) = 1, \tag{2.2}$$

where $q_0(\ell) := 0$ for $\ell \in \mathbb{Z} \setminus \{0, \ldots, C\}$. More general recurrences have been derived for state dependent arrival intensities by several authors, cf. [5], [21]. The recursion (2.2) yields an efficient algorithm of complexity $O(Cm)$ for the $q_0(\ell)$ and hence also for the blocking probabilities.

Without loss of generality we will deal with the overflow stream of type-1 calls in the following. The process $(N(t), N_1'(t))$, $t \in \mathbb{R}_+$, is an irreducible Markov process with state space $\Omega \times \mathbb{Z}_+$. Let $p(n, n_1')$, $(n, n_1') \in \Omega \times \mathbb{Z}_+$, denote its equilibrium distribution,

$$f_k(n) := \sum_{n_1' \in \mathbb{Z}_+} k! \binom{n_1'}{k} p(n, n_1'), \quad n \in \Omega, \qquad f_k = \sum_{n \in \Omega} f_k(n) \tag{2.3}$$

the k-th partial factorial moment of $N_1'(t)$ on $\{N(t) = n\}$ and the k-th factorial moment of $N_1'(t)$, respectively. For $k = 0$ we have

$$f_0(n) = p(n), \quad n \in \Omega, \qquad f_0 = 1. \tag{2.4}$$

The state equations for the $p(n, n_1')$ are easily obtained. Multiplying these equations by $k!\binom{n_1'}{k}$, summing over $n_1' \in \mathbb{Z}_+$ and using $\binom{j+1}{k} = \binom{j}{k} + \binom{j}{k-1}$, $j\binom{j}{k} - j\binom{j-1}{k} = k\binom{j}{k}$, we find

$$\begin{aligned}
&\Big(\sum_{i \in I} \lambda_i \mathrm{1\!I}\{(n,b) \le C_i - b_i\} + \sum_{i \in I} n_i \mu_i + k\mu_1 \Big) f_k(n) \\
&\quad = \sum_{i \in I} \lambda_i \mathrm{1\!I}\{(n,b) \le C_i\}\, f_k(n - e_i) + \sum_{i \in I} (n_i + 1)\mu_i\, f_k(n + e_i) \\
&\qquad + k\lambda_1 \mathrm{1\!I}\{(n,b) > C_1 - b_1\} f_{k-1}(n), \quad n \in \Omega, \quad k = 0, 1, \ldots,
\end{aligned} \tag{2.5}$$

where $f_k(n) := 0$, $n \in \mathbb{Z}^m \setminus \Omega$, $k = 0, 1, \ldots$ Obviously, (2.5) even holds for $n \in \mathbb{Z}^m$. Summation over $n \in \mathbb{Z}^m$ yields

$$f_k = \varrho_1 \sum_{n \in \Omega:\, (n,b) > C_1 - b_1} f_{k-1}(n), \quad k = 1, 2, \ldots \tag{2.6}$$

In particular, for $k = 1$ we obtain $f_1 = \varrho_1 p_{b,1}$, cf. (2.1). For $k = 0$, eq. (2.5) obviously reduces to the equilibrium equations for $p(n)$, $n \in \Omega$. The $f_0(n) = p(n)$ are uniquely determined by (2.5) and the normalization condition $f_0 = 1$. For fixed $k > 0$ and given $f_{k-1}(n)$, $n \in \Omega$, from the strong row criterion it follows that the $f_k(n)$, $n \in \Omega$, are the unique solution of the linear system of equations (2.5). Thus, in view of (2.6), the factorial moments f_k can be determined by a recursive computation of the $f_j(n)$, $n \in \Omega$, for $j = 0, 1, \ldots, k-1$, in principle. But this approach becomes intractable for a larger number of call types and growing link capacity, due to numerical complexity. For up to three call types the algorithm was implemented for checking the accuracy of the proposed approximation given in the next section; for more than three call types we simulated the overflow moments.

3 Approximation for the individual overflow moments

Let

$$q_k(\ell) := \sum_{n \in \Omega:\, (n,b) = \ell} f_k(n), \quad \ell = 0, 1, \ldots, C, \quad k = 0, 1, \ldots, \tag{3.1}$$

be the k-th partial factorial moment of $N_1'(t)$ on $\{S(t) = \ell\}$ and $q_k(\ell) := 0$ for all the other integer values of ℓ and k. From (3.1), (2.3), (2.4) and (2.6) we obtain

$$\sum_{\ell=0}^{C} q_k(\ell) = f_k = \mathrm{I\!I}\{k=0\} + \varrho_1 \sum_{\ell=C_1-b_1+1}^{C} q_{k-1}(\ell), \quad k = 0, 1, 2, \ldots \tag{3.2}$$

Little's formula yields the identity

$$\sum_{\ell=0}^{C} \ell q_0(\ell) = \sum_{i\in I} \Big(\sum_{\ell=0}^{C} \mathrm{I\!I}\{\ell \le C_i - b_i\} \lambda_i b_i q_0(\ell) \Big) \frac{1}{\mu_i} = \sum_{\ell=0}^{C} \Big(\sum_{i\in I} \mathrm{I\!I}\{\ell \le C_i\} \varrho_i b_i q_0(\ell - b_i) \Big). \tag{3.3}$$

For a motivation of our approximation given below we consider the special case of $b_1 = \ldots = b_m$, $\mu_1 = \ldots = \mu_m$, i.e., of equal bandwidths b_i and equal mean holding times $1/\mu_i$. By summing the resulting simplified equations (2.5) over $n \in \mathbb{Z}^m$ with $(n, b) = \ell$ and using (3.1) after some algebra we find

$$\begin{aligned} &\Big((\ell+b_1) q_k(\ell+b_1) - \sum_{i\in I} \mathrm{I\!I}\{\ell+b_1 \le C_i\} \varrho_i b_1 q_k(\ell) \Big) - \Big(\ell q_k(\ell) - \sum_{i\in I} \mathrm{I\!I}\{\ell \le C_i\} \varrho_i b_1 q_k(\ell - b_1) \Big) \\ &\quad = k b_1 q_k(\ell) - \ k \mathrm{I\!I}\{\ell > C_1 - b_1\} \varrho_1 b_1 q_{k-1}(\ell), \quad \ell \in \mathbb{Z}, \quad k = 0, 1, 2, \ldots \end{aligned} \tag{3.4}$$

Since the r.h.s. of (3.4) vanishes for $k = 0$, we obtain the following well-known recursion for the utilization probabilities, cf. [19]:

$$\ell q_0(\ell) = \sum_{i\in I} \mathrm{I\!I}\{\ell \le C_i\} \varrho_i b_i q_0(\ell - b_i), \quad \ell = 1, 2, \ldots, C, \qquad \sum_{\ell=0}^{C} q_0(\ell) = 1. \tag{3.5}$$

Note that (2.2) (special case of complete sharing) coincides with (3.5), i.e., in both special cases (3.5) holds. In the general situation (3.5) holds in the mean only, cf. (3.3). The similarity of the two special cases encouraged Roberts [19] to propose the unique solution $\tilde{q}_0(\ell)$, $\ell = 0, \ldots, C$, of (3.5) as an approximation for the utilization probabilities $q_0(\ell)$, $\ell = 0, \ldots, C$. In view of (2.1) this provides also an approximation $\tilde{p}_{b,i}$, $i \in I$, for the individual blocking probabilities. Several authors have observed over the years that this approximation is accurate if the mean holding times do not greatly differ from each other, but that it is typically inaccurate otherwise [21]. For a modified approximation in order to improve the accuracy if the mean holding times greatly differ (e.g. in a multiservice system with video, voice and fax connections) we refer to [21], p. 79, 80. Let us return to the special case of $b_1 = \ldots = b_m$, $\mu_1 = \ldots = \mu_m$ in more detail. For $\ell \in \mathbb{Z}$ and $k = 0, 1, 2, \ldots$ let

$$r_k(\ell) := \sum_{j=0}^{\infty} \mathrm{I\!I}\{\ell - j b_1 \ge 0\} q_k(\ell - j b_1), \quad s_k(\ell) := \sum_{j=0}^{\infty} \mathrm{I\!I}\{\ell - j b_1 > C_1 - b_1\} q_{k-1}(\ell - j b_1).$$

These sequences satisfy the recursions

$$r_k(\ell) = q_k(\ell) + r_k(\ell - b_1), \quad s_k(\ell) = \mathrm{I\!I}\{\ell > C_1 - b_1\} q_{k-1}(\ell) + s_k(\ell - b_1), \tag{3.6}$$

where $r_k(\ell) = s_k(\ell) = 0$, $\ell \in \mathbb{Z} \setminus \mathbb{Z}_+$. Using (3.6) from (3.4) we obtain

$$
\begin{aligned}
&(\ell+b_1)q_k(\ell+b_1) - \sum_{i\in I} \mathrm{1\!I}\{\ell+b_1 \le C_i\}\varrho_i b_1 q_k(\ell) - kb_1 r_k(\ell) + k\varrho_1 b_1 s_k(\ell) \\
&\quad = \ell q_k(\ell) - \sum_{i\in I} \mathrm{1\!I}\{\ell \le C_i\}\varrho_i b_1 q_k(\ell-b_1) - kb_1 r_k(\ell-b_1) + k\varrho_1 b_1 s_k(\ell-b_1).
\end{aligned}
\tag{3.7}
$$

Equation (3.7) implies that the r.h.s. of (3.7) has period b_1. Thus the r.h.s. of (3.7) vanishes for $\ell \in \mathbb{Z}$ as it obviously vanishes for $\ell \in \mathbb{Z} \setminus \mathbb{Z}_+$. This together with (3.6) yields

$$
\begin{aligned}
\ell q_k(\ell) &= \sum_{i\in I} \mathrm{1\!I}\{\ell \le C_i\}\varrho_i b_1 q_k(\ell-b_1) + kb_1 r_k(\ell-b_1) - k\varrho_1 b_1 s_k(\ell-b_1) \\
&= \sum_{i\in I} \mathrm{1\!I}\{\ell \le C_i\}\varrho_i b_1 q_k(\ell-b_1) + kb_1 \sum_{j=1}^{\infty} \mathrm{1\!I}\{\ell-jb_1 \ge 0\} q_k(\ell-jb_1) \\
&\qquad - k\varrho_1 b_1 \sum_{j=1}^{\infty} \mathrm{1\!I}\{\ell-jb_1 > C_1-b_1\} q_{k-1}(\ell-jb_1), \quad \ell = 1,2,\ldots,C.
\end{aligned}
\tag{3.8}
$$

Suppose that for a fixed $k \in \mathbb{Z}_+$ the $q_{k-1}(\ell)$, $\ell = 0,1,\ldots,C$, are known. Then (3.6), (3.8), (3.2) determine uniquely $q_k(\ell)$, $\ell = 0,1,\ldots,C$. An efficient recursive algorithm of complexity $O(C(m+k)/b_1)$ for computing f_k can be derived, which is similar to those for the approximation in the general case given below, but of lower complexity.

In the general case of arbitrary bandwidths and mean holding times the following approximation $\tilde{q}_k(\ell)$ of $q_k(\ell)$ is suggested: For $\ell = 1,2,\ldots,C$ and $k = 0,1,\ldots$ let

$$
\begin{aligned}
\ell\tilde{q}_k(\ell) &:= \sum_{i\in I} \mathrm{1\!I}\{b_i \le \ell \le C_i\}\varrho_i b_i \tilde{q}_k(\ell-b_i) \\
&\quad + k\left(\mu_1 \sum_{i\in I} \mathrm{1\!I}\{\ell \ge b_i\}\varrho_i b_i \tilde{r}_k(\ell-b_i) - \lambda_1 \sum_{i\in I} \mathrm{1\!I}\{\ell \ge b_i\}\varrho_i b_i \tilde{s}_k(\ell-b_i)\right) \Big/ \left(\sum_{i\in I} \lambda_i\right),
\end{aligned}
\tag{3.9}
$$

where for $\ell = 0,1,2,\ldots,C$ and $k = 0,1,\ldots$

$$
\tilde{r}_k(\ell) := \tilde{q}_k(\ell) + \left(\sum_{i\in I} \mathrm{1\!I}\{\ell \ge b_i\}\varrho_i b_i \tilde{r}_k(\ell-b_i)\right) \Big/ \left(\sum_{i\in I} \varrho_i b_i\right),
\tag{3.10}
$$

$$
\tilde{s}_k(\ell) := \mathrm{1\!I}\{\ell > C_1-b_1\}\tilde{q}_{k-1}(\ell) + \left(\sum_{i\in I} \mathrm{1\!I}\{\ell \ge b_i\}\varrho_i b_i \tilde{s}_k(\ell-b_i)\right) \Big/ \left(\sum_{i\in I} \varrho_i b_i\right).
\tag{3.11}
$$

With $\tilde{q}_{-1}(\ell) := 0$, $\ell = 0,1,\ldots,C$, and the normalization condition corresponding to (3.2)

$$
\sum_{\ell=0}^{C} \tilde{q}_k(\ell) = \mathrm{1\!I}\{k=0\} + \varrho_1 \sum_{\ell=C_1-b_1+1}^{C} \tilde{q}_{k-1}(\ell), \quad k = 0,1,2,\ldots,
\tag{3.12}
$$

the $\tilde{q}_k(\ell)$ are uniquely determined: For $k=0$ the eqs. (3.9)–(3.12) reduce to Roberts' approximation for the utilization probabilities, cf. above. It can efficiently be computed by determining any non-trivial solution $\tilde{q}_0^{(h)}(\ell)$ of the homogeneous recursion, e.g. with initial value $\tilde{q}_0^{(h)}(0)=1$, and then normalizing it via (3.12). For obtaining $\tilde{q}_k(\ell)$ for $k>0$, first one computes any particular solution $\tilde{q}_k^{(i)}(\ell)$ of the inhomogeneous recursion (3.9)–(3.11) and any non-trivial solution $\tilde{q}_k^{(h)}(\ell)$ of the homogeneous recursion (3.9) and (3.10), where $\tilde{s}_k(\ell):=0$, $\ell=0,\ldots,C$. As initial values one can choose e.g. $\tilde{q}_k^{(i)}(0)=0$ and $\tilde{q}_k^{(h)}(0)=1$. By induction on ℓ it follows that the sequences $\tilde{q}_k^{(i)}(\ell)+t\tilde{q}_k^{(h)}(\ell)$, $\ell=0,\ldots,C$, parameterized by $t\in\mathbb{R}$ are the general solution of the inhomogeneous recursion (3.9)–(3.11). Applying (3.12) one can determine $t_k\in\mathbb{R}$ such that

$$\tilde{q}_k(\ell)=\tilde{q}_k^{(i)}(\ell)+t_k\tilde{q}_k^{(h)}(\ell),\quad \ell=0,1,\ldots,C, \tag{3.13}$$

holds. Finally, in view of $q_k(\ell)\geq 0$, $\ell=0,1,\ldots,C$, we replace $\tilde{q}_k(\ell)$ by $\max(\tilde{q}_k(\ell),0)$. Equation (3.2) suggests the following approximation $\tilde{f}_k$ for the k-th factorial overflow moment of the first overflow stream:

$$\tilde{f}_k:=\varrho_1\sum_{\ell=C_1-b_1+1}^{C}\tilde{q}_{k-1}(\ell),\quad k=1,2,\ldots \tag{3.14}$$

The resulting algorithm for $\tilde{f}_k$ is of complexity $O(Cmk)$ and, hence, very efficient compared to the numerical computation of f_k. For $k=1$ we obtain Roberts' approximation $\tilde{p}_{b,1}$ for the individual blocking probability:

$$\tilde{p}_{b,1}:=\sum_{\ell=C_1-b_1+1}^{C}\tilde{q}_0(\ell)=\tilde{f}_1/\varrho_1.$$

In case of equal bandwidths and equal mean holding times approximation (3.14) is exact.

4 Numerical examples

We consider a link with trunk reservation of capacity $C=100$ offered to $m=3$ traffic streams. In the following examples we have $\varrho_1 b_1=32$, $\varrho_2 b_2=48$, $\varrho_3 b_3=8$ and, thus, a load of 88%. In the Tables 4.1–4.4 the trunk reservation parameters are $C_1=100$, $C_2=96$ and $C_3=94$, providing a protection of the first stream. In Table 4.1 we consider the case of equal bandwidths and equal mean holding times, thus the approximation for the factorial overflow moments is exact. In Table 4.2 only the mean holding times and correspondingly the arrival rates are changed, having no influence on the approximation for the blocking probabilities. But, as the blocking probability for the protected first stream changes significantly, the approximation for the factorial overflow moments of the first stream becomes worse. The approximation is better for the other streams, which have a larger blocking probability. This may be caused by the fact that (3.5) holds in the mean, cf. (3.3). In Table 4.3 only the bandwidths and correspondingly the arrival rates are changed compared to Table 4.1. The approximation depends on the bandwidths, and the approximation for the factorial overflow moments of the first stream is much better than in case of changing the mean holding times, cf. Table 4.2. In Table 4.4 the mean holding times and the bandwidths are changed compared to Table 4.1. The approximation for the factorial overflow moments is also still better than in case of Table 4.2.

i	λ_i / μ_i	b_i / C_i	$p_{b,i}$ / $\tilde{p}_{b,i}$	f_1 / $\tilde{f}_1$	f_2 / $\tilde{f}_2$	f_3 / $\tilde{f}_3$
1	16.000000	2	0.005943	0.095092	0.055604	0.056150
	1.000000	100	0.005943	0.095092	0.055604	0.056150
2	24.000000	2	0.081394	1.953464	7.917987	43.452459
	1.000000	96	0.081394	1.953464	7.917987	43.452459
3	4.000000	2	0.149649	0.598594	0.647483	0.896254
	1.000000	94	0.149649	0.598594	0.647483	0.896254

Table 4.1. Exact approximation in case of equal bandwidths and equal mean holding times.

i	λ_i / μ_i	b_i / C_i	$p_{b,i}$ / $\tilde{p}_{b,i}$	f_1 / $\tilde{f}_1$	f_2 / $\tilde{f}_2$	f_3 / $\tilde{f}_3$
1	8.000000	2	0.002194	0.035105	0.010227	0.005485
	0.500000	100	0.005943	0.095092	0.035669	0.022455
2	24.000000	2	0.074262	1.782289	7.117383	39.163192
	1.000000	96	0.081394	1.953464	8.232954	47.146443
3	8.000000	2	0.153110	0.612440	0.880268	1.620706
	2.000000	94	0.149649	0.598594	0.853101	1.560586

Table 4.2. Approximation in case of equal bandwidths.

i	λ_i / μ_i	b_i / C_i	$p_{b,i}$ / $\tilde{p}_{b,i}$	f_1 / $\tilde{f}_1$	f_2 / $\tilde{f}_2$	f_3 / $\tilde{f}_3$
1	8.000000	4	0.034824	0.278593	0.200301	0.210952
	1.000000	100	0.038360	0.306876	0.208859	0.201556
2	24.000000	2	0.095572	2.293730	10.193891	60.140051
	1.000000	96	0.092728	2.225468	8.929272	47.021985
3	8.000000	1	0.127782	1.022253	1.907904	4.609830
	1.000000	94	0.124060	0.992481	1.679278	3.638759

Table 4.3. Approximation in case of equal mean holding times.

i	λ_i / μ_i	b_i / C_i	$p_{b,i}$ / $\tilde{p}_{b,i}$	f_1 / $\tilde{f}_1$	f_2 / $\tilde{f}_2$	f_3 / $\tilde{f}_3$
1	4.000000	4	0.022577	0.180616	0.081722	0.056188
	0.500000	100	0.038360	0.306876	0.151186	0.098403
2	24.000000	2	0.089264	2.142336	10.130069	64.917762
	1.000000	96	0.092728	2.225468	8.633632	43.722879
3	16.000000	1	0.130399	1.043189	2.750878	9.427395
	2.000000	94	0.124060	0.992481	2.069166	5.621440

Table 4.4. Approximation in the general case.

In Table 4.5 we use the parameters of Table 4.4 but without trunk reservation. Since there is no trunk reservation, the approximation for the blocking probabilities and thus for the first overflow moments is exact, but the approximation for higher overflow moments is obviously not exact.

i	λ_i μ_i	b_i C_i	$p_{b,i}$ $\tilde{p}_{b,i}$	f_1 $\tilde{f}_1$	f_2 $\tilde{f}_2$	f_3 $\tilde{f}_3$
1	4.000000 0.500000	4 100	0.097735 0.097735	0.781884 0.781884	1.092457 0.887371	2.000295 1.255612
2	24.000000 1.000000	2 100	0.046515 0.046515	1.116367 1.116367	3.341622 2.469234	14.097474 7.417658
3	16.000000 2.000000	1 100	0.022654 0.022654	0.181231 0.181231	0.154992 0.093722	0.200647 0.068983

Table 4.5. Approximation in case of a link without trunk reservation.

From the given numerical examples as well as from numerical and simulation experiments not reported here we found that the quality of the proposed approximation for the first three individual overflow moments is comparable with the quality of Roberts' approximation for the individual blocking probabilities. In particular, the approximation becomes worse if the mean holding times greatly differ from each other. However, as the stochastic dependence decreases in the case of links with a larger number of traffic streams, the proposed approximation for the factorial overflow moments is probably better in case of links with more than $m = 3$ traffic streams. Therefore and since this approximation can be computed very fast, it is suitable e.g. for dimensioning and control of networks. We conjecture that the proposed approximation for the higher overflow moments can still be improved by using an improved approximation for the equilibrium utilization probabilities as initialization of the recursion (3.9)–(3.12), cf. e.g. [21], p. 79, 80.

References

[1] Akinpelu, J.M., The overload performance of engineered networks with nonhierarchical and hierarchical routing. AT&T Bell Labs Technical Journal 63 (1984) 1261–1281.

[2] Arvidsson, Å., On the performance of a circuit switched link with priorities. IEEE J. Sel. A. Comm. 9, No. 2 (1991) 212–218.

[3] Azmoodeh, M., Macfadyen, R.N., Multi-rate call congestion: Fixed point models and trunk reservation. IEEE Eleventh UK Teletraffic Symposium. Performance Engineering in Telecommunications Networks, S. 6, (1994) 1–10.

[4] Bean, N.G., Gibbens, R.J., Zachary, S., Asymptotic analysis of single resource loss systems in heavy traffic, with applications to integrated networks. Adv. Appl. Prob. 27, No. 1 (1995) 273–292.

[5] Delbrouck, L.E.N., On the steady-state distribution in a service facility carrying mixtures of traffic with different peakedness factors and capacity requirements. IEEE Trans. on Comm., vol. COM-31, No. 11 (1983) 1209–1211.

[6] Girard, A., Blocking probability of noninteger groups with trunk reservation. IEEE Trans. on Comm., vol. COM-33, No. 2 (1985) 113–120.

[7] Kaufman, J.S., Blocking in a shared resource environment. IEEE Trans. on Comm., vol. COM-29, No. 10 (1981) 1474–1481.

[8] Kawashima, K., Trunk reservation models in telecommunications systems. Teletraffic Analysis and Computer Performance Evaluation. Eds. O.J. Boxma, J.W. Cohen, and H.C. Tijms (1986) 29–36.

[9] Kelly, F., Reversibility and stochastic networks. Wiley Chichester 1979.

[10] Kelly, F., Fixed point models of loss networks. J. Austral. Math. Soc. Ser. B 31 (1989) 204–218.

[11] Key, P.B., Optimal control and trunk reservation in loss networks. Prob. Eng. Inf. Sci. 4 (1990) 203–242.

[12] Krupp, R.S., Stabilization of alternate routing networks. Proceedings of the International Communications Conference, Philadelphia, 1982.

[13] Le Gall, F., Bernossou, J., Blocking probability for trunk reservation policy. IEEE Trans. on Comm., vol. COM-35, No. 3 (1987) 313–318.

[14] Mitra, D., Gibbens, R.J., Huang, D.B., Analysis and optimal design of aggregated-least-busy-alternative routing on symmetric loss networks with trunk reservation. Teletraffic and datatraffic in a period of change, ITC-13 (1991) 477–482.

[15] Naoumov, V., Normal-type approximation for multi-service systems with trunk reservation. Telecommunication Systems 4 (1995) 113–118.

[16] Reiman, M.I., Asymptotically optimal trunk reservation for large trunk groups. Proceedings of the 28th IEEE Conference on Decision and Control (1989) 2536–2541.

[17] Reiman, M.I., Optimal trunk reservation for a critically loaded link. Teletraffic and datatraffic in a period of change, ITC-13 (1991) 247–252.

[18] Reneby, L., Service protection and overflow in circuit switched networks. Thesis, Technical Report 107, Department of Communication Systems 1991.

[19] Roberts, J.W., Teletraffic models for the telecom 1 integrated services network. Proc. 10th Int. Teletraffic Cong., Montreal 1983.

[20] Roberts, J.W., Mocci, U., Virtamo, J. (Eds.), Broadband network teletraffic: Performance evaluation and design of broadband multiservice networks. Final report of action COST 242. Lecture notes in computer science, vol. 1155. Springer-Verlag, Berlin 1996.

[21] Ross, K.W., Multiservice loss models for broadband telecommunication networks. Springer-Verlag London Limited 1995.

[22] Songhurst, D.J., Protection against traffic overload in hierarchical networks employing alternative routing. Telecommunications Networks Planning Symposium (1980) 214–220.

[23] Tran-Gia, P., Hübner, F., An analysis of trunk reservation and grade of service balancing mechanisms in multiservice broadband networks. IFIP Workshop TC6, Modelling and Performance Evaluation of ATM Technology, La Martinique 1993.

[24] Wallin, J.F.E., Sanders, B., The calculation of overflow moments in loss systems with selective trunk reservation. Performance Evaluation 15 (1992) 195–202.