CDO Surfaces Dynamics

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iTraxx over Time

Figure 1: Spreads of iTraxx tranches, Series 5, maturity 5 (left) and 10 (right) years, data from 20060407-20081103. Tranches: 1, 2, 3, 4, 5.
Figure 2: Spreads of tranches of all series observed on 20080909 (left) and 20090119 (right).
Research Goals

- Modelling the dynamics of CDO surfaces
  - spread surfaces
  - base correlation surfaces
- Applications in trading
Motivation

Dynamic Semiparametric Factor Model

Applications:


Outline

1. Motivation ✓
2. CDOs
3. DSFM
4. Empirical Study
5. Applications
6. Conclusions
Risk Transfer
**iTraxx Europe**

- A static portfolio of 125 equally weighted CDS on European entities;
- Sectors: Consumer (30), Financial (25), TMT (20), Industrials (20), Energy (20), Auto (10);
- New series of iTraxx Europe issued every 6 months (March and September) and the underlying reference entities are reconstituted;
- Maturities: 3Y, 5Y, 7Y, 10Y.
Gaussian Copula Model

Default times are modelled from the Gaussian vector \((X_1, \ldots, X_d)\): 

\[
X_i = \sqrt{\rho} Y + \sqrt{1 - \rho} Z_i,
\]

where \(Y\) (systematic risk factor), \(\{Z_i\}_{i=1}^{d}\) (idiosyncratic risk factors) are i.i.d. \(N(0, 1)\). Hence:

\[
(X_1, \ldots, X_d) \sim N(0, \Sigma),
\]

with

\[
\Sigma = \begin{pmatrix}
1 & \rho & \cdots & \rho \\
\rho & 1 & \cdots & \rho \\
\vdots & \vdots & \ddots & \vdots \\
\rho & \rho & \cdots & 1
\end{pmatrix}.
\]
Large Portfolio Framework

Assume that

- obligors have the same default probability and LGD,
- one dependence parameter $\rho$,
- $d$ very large.

Computations are simplified significantly when the portfolio loss distribution is approximated:

$$P(\tilde{L} \leq x) = \Phi \left\{ \frac{\sqrt{1 - \rho \Phi^{-1}(x)} - \Phi^{-1}(p)}{\sqrt{\rho}} \right\}.$$
Correlation’s Types

Compound correlation \( \rho(l_j, u_j), j = 1, \ldots, J \).

Figure 3: Implied correlation smile in the Gaussian one factor model, 20071022.
Correlation’s Types

Base correlation (BC) $\rho(0, u_j), j = 1, \ldots, J$.

Represent the expected loss $E\{L(l_j, u_j)\}$ as a difference:

$$E\{L(l_j, u_j)\} = E_{\rho(0, u_j)}\{L(0, u_j)\} - E_{\rho(0, l_j)}\{L(0, l_j)\}, \quad j = 2, \ldots, J.$$ 

of the expected losses of two fictive tranches $(0, u_j)$ and $(0, l_j)$.

**Bootstrapping process:** $E\{L(0, 3\%)\}$ is traded on the market,

$$E\{L(3\%, 6\%)\} = E_{\rho(0, 6\%)}\{L(0, 6\%)\} - E_{\rho(0, 3\%)}\{L(0, 3\%)\},$$

$$E\{L(6\%, 9\%)\} = E_{\rho(0, 9\%)}\{L(0, 9\%)\} - E_{\rho(0, 6\%)}\{L(0, 6\%)\}, \ldots$$
Figure 4: BC of iTraxx tranches, Series 5, maturity 5 (left) and 10 (right) years, data from 20060510-20081023. Tranches: 1, 2, 3, 4, 5.
Base Correlation Surfaces

Figure 5: Implied base correlations on day 20080909 (left) and 20090119 (right).
Dynamic Semiparametric Factor Model

\[ Y_{t,k} = m_0(X_{t,k}) + \sum_{l=1}^{L} Z_{t,l} m_l(X_{t,k}) + \varepsilon_{t,k} = Z_t^\top A \psi(X_{t,k}) + \varepsilon_{t,k} \]

- \( Y_{t,k} \): log-spreads and Z-transformed BC on day \( t \), \( t = 1, \ldots, T \)
- \( k \): intra-day numbering of BCs on day \( t \), \( k = 1, \ldots, K_t \)
- \( X_{t,k} \): two-dimensional vector of the tranche seniority and the time-to-maturity
- \( m_l \): factor functions, time invariant, nonparametric estimation
- \( Z_{t,l} \): time series, \( l = 0, \ldots, L \), dynamic behavior
- \( \psi(X_{t,k}) \): tensor B-spline basis
- \( A \): coefficient matrix
**Estimation**

Using an iterative algorithm:

\[
(\hat{Z}_t, \hat{A}) = \arg \min_{Z_t, A} \sum_{t=1}^{T} \sum_{k=1}^{K_t} \left\{ Y_{t,k} - Z_t^\top A\psi(X_{t,k}) \right\}^2
\]

Selection of \( L \), the numbers of spline knots \( R_1, R_2 \) and the orders of splines \( k_1, k_2 \) by maximising the explained variance criterion:

\[
\text{EV}(L, R_1, r_1, R_2, r_2) = 1 - \frac{\sum_{t=1}^{T} \sum_{k=1}^{K_t} \left\{ Y_{t,k} - \sum_{l=1}^{L} Z_{t,l}m_l(X_{t,k}) \right\}^2}{\sum_{t=1}^{T} \sum_{k=1}^{K_t} \left\{ Y_{t,j} - \tilde{m}_0(X_{t,k}) \right\}^2},
\]

where \( \tilde{m}_0 \) is an empirical mean surface.
DSFM without the Mean Factor

Reduce the number of factors estimated in the iterative algorithm by first subtracting the empirical mean $\tilde{m}_0$ and then fitting the DSFM:

$$Y_{t,k} = \tilde{m}_0(X_{t,k}) + \sum_{l=1}^{L} Z_{t,l} m_l(X_{t,k}) + \varepsilon_{t,k} = \tilde{m}_0(X_{t,k}) + Z_t^T A\psi(X_{t,k}) + \varepsilon_{t,k},$$

where $m_l$ are new factor functions, $l = 1, \ldots, L$. 
## Data

- Series 2-10
- Maturities 5, 7, 10Y
- 1004 days between 20050330-20090202
- 49,502 data points

<table>
<thead>
<tr>
<th>Year</th>
<th>3Y</th>
<th>5Y</th>
<th>7Y</th>
<th>10Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005</td>
<td>0</td>
<td>1478</td>
<td>715</td>
<td>1532</td>
</tr>
<tr>
<td>2006</td>
<td>181</td>
<td>3998</td>
<td>3739</td>
<td>4005</td>
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<td>2007</td>
<td>75</td>
<td>5155</td>
<td>5170</td>
<td>5172</td>
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<tr>
<td>2008</td>
<td>232</td>
<td>5904</td>
<td>5916</td>
<td>5932</td>
</tr>
<tr>
<td>2009</td>
<td>0</td>
<td>260</td>
<td>263</td>
<td>263</td>
</tr>
<tr>
<td>All</td>
<td>488</td>
<td>16740</td>
<td>15803</td>
<td>16840</td>
</tr>
</tbody>
</table>

Table 1: Number of observed values of iTraxx tranches in the period 20050330-20090202.
Data Preparation

- Convert the upfront payment quotes of the equity tranche to standard spreads using the Gaussian copula model.
- Since the data are monotone in the tranche seniority direction and positive, use log-spreads and Z-transformed-BC.

Figure 6: Daily number of curves for every surface during the period 20050330-20090202.
DSFM for Z-transformed-BC

Figure 7: Proportion of the explained variance as a function of $R_2$ (up left) with $r_2 = 2$, as a function of $r_2$ (up right) with $R_2 = 10$, as a function of $L$ (down) for $L = 1, L = 2, L = 3, r_1 = 2$ and $R_1 = 5$. 
**DSFM w/o Mean F. for Z-transformed-BC**

Figure 8: Estimated factor functions and loadings ($\hat{Z}_{t,1}$, $\hat{Z}_{t,2}$).
DSFM Estimation Results

For DSFM for both data types

- $\hat{Z}_{t,1}$ is a slope-curvature factor
- $\hat{Z}_{t,2}$ is a shift factor

<table>
<thead>
<tr>
<th>Model</th>
<th>Log-Spr</th>
<th>Z-BC</th>
</tr>
</thead>
<tbody>
<tr>
<td>DSFM</td>
<td>0.016</td>
<td>0.004</td>
</tr>
<tr>
<td>DSFM w/o mean f.</td>
<td>0.045</td>
<td>0.006</td>
</tr>
</tbody>
</table>

Table 2: Mean squared error of the in-sample fit.
DSFM without the mean factor Fit

Figure 9: In-sample fit of the models to data on 20080909 and 20090119.
Curve Trades

So, how can I make money with this?

Combine tranches of different time to maturity, see Felsenheimer et al. (2004) and Kakodkar et al. (2006):

- Flattener – sell a long-term tranche, buy a short-term tranche
  Example: sell 10Y 3-6% and buy 5Y 6-9%
  Outlook: bullish long-term, bearish short-term

- Steepener – opposite trade
JP Morgan Trading Loss, May 2012

J.P. Morgan’s flattener – bought 5Y CDX IG 9 index, sold 10Y CDX IG 9 index in a 3:1 ratio. The final loss reached $6.2 billion.
**Flattener**

Sell protection at $s_1(t_0)$ for the period $[t_0, T_1]$ and buy protection at $s_2(t_0)$ for $[t_0, T_2]$, $T_1 > T_2$. At $t_0$ for $\ell = 1, 2$:

$$\text{MTM}_\ell(t_0) = \sum_{t = t_1}^{T_\ell} \beta(t_0, t) \left[ s_\ell(t_0) \Delta t \mathbb{E}\{F_\ell(t)\} - \mathbb{E}\{L_\ell(t) - L_\ell(t - \Delta t)\} \right] = 0.$$ 

At $\tilde{t} > t_0$, the market quotes $s_\ell(\tilde{t})$ and

$$\text{MTM}_\ell(\tilde{t}) = \left\{ s_\ell(t_0) - s_\ell(\tilde{t}) \right\} \sum_{t = \tilde{t}_1}^{T_\ell} \beta(\tilde{t}, t) \Delta t \mathbb{E}\{F_\ell(t)\}.$$
Curve Trade

- A positive MTM means a positive value to the protection seller.
- If the protection seller closes the position at time $\tilde{t}$, then receives from the protection buyer $\text{MTM}_\ell(\tilde{t})$.
- Flattener-trader aims to maximize the total MTM value

$$\text{PL}(\tilde{t}) = \text{MTM}_1(\tilde{t}) - \text{MTM}_2(\tilde{t}).$$
Risk in Curve Trades

- If one buys 5Y 6-9% and sells 10Y 6-9%, then the trade is hedged for default until the maturity of the 5Y tranche. Defaults that emerge from 10Y 6-9% are covered by 5Y 6-9% till it expires.
- Series differ in the composition of the collateral.
- If one buys 5Y 6-9% and sells 10Y 3-6%, then these tranches provide protection of different portion of portfolio risk. If there is any default in 10Y 3-6%, then we must deliver a payment obligation and incur a loss.
Empirical Study

Idea
- Use DSFM to forecast spread and BC surfaces
- Calculate forecasted MTM surfaces
- Recover those tranches that maximise P&L

Remarks
- Because of many missing data and short data histories, the standard econometric methods cannot be used for the forecasting.
- Consider trades that generate no or a positive carry – the spread of the long tranche doesn’t exceed the spread of the short tranche.
- Do not account for default payments (no data of historical defaults in iTraxx), do not account for the positive carry.
Forecasting with DSFM in Rolling Windows

Let $Y_t$ be log-spreads or $Z$-transformed-BC.

- Consider a rolling window of $w = 250$.
- Estimate the DSFM$s$ using $\{Y_\nu\}_{\nu = t - w + 1}^t$ for $t = w, \ldots, T - h$.
- As a result, we get $T - w + 1$ times $\hat{m} = (\hat{m}_0, \ldots, \hat{m}_L)^\top$ and $\hat{Z}_t = (\hat{Z}_{t,0}, \ldots, \hat{Z}_{t,L})^\top$ of length $w$.
- Compute $h$-day forecast of the factor loadings using VAR.
- Due to the fixed issuing scheme, $X_{t+h,k}$ is not forecasted.
- Calculate the forecast $\hat{Y}_{t+h}$ from the forecast $\hat{Z}_{t+h}$.
- Transform $\hat{Y}_{t+h}$ suitably to get $s(t + h)$ or $\hat{\rho}(t + h)$. 
Forecasting MTM Surfaces

For predicted \( \{\hat{s}_k(t), \hat{\rho}_k(t)\} \), \( t = w + h, \ldots, T \), \( k = 1, \ldots, K_t \), compute \( \hat{\text{MTM}}_k(t) \), where the initial spread \( s_k(t_0) \) is observed on \( t_0 = t - h \).

Figure 10: MTM surfaces on 20080909 (left) and 20090119 (right) calculated using one-day spread and BC predictions obtained with the DSFM.
Transaction Costs

Calculate the ask (bid) spread by increasing (reducing) the observed spread by the following percentage:

<table>
<thead>
<tr>
<th>Maturity</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>5Y</td>
<td>1.88</td>
<td>1.78</td>
<td>2.52</td>
<td>3.77</td>
<td>6.28</td>
</tr>
<tr>
<td>7Y</td>
<td>1.49</td>
<td>1.65</td>
<td>2.31</td>
<td>2.97</td>
<td>4.87</td>
</tr>
<tr>
<td>10Y</td>
<td>1.41</td>
<td>1.66</td>
<td>1.83</td>
<td>2.52</td>
<td>4.09</td>
</tr>
</tbody>
</table>

Table 3: Average bid-ask spread excess over the mid spread as a percentage of the mid spread for Series 8 during the period 20070920-20090202.
Trading Strategies

Construct a curve trade

1. Fit and forecast the DSFM models to spreads and BC.
2. Calculate $h$-day forecasts of the MTM surfaces.
3. Recover which two tranches optimize a given strategy.

Strategies – restrict the choice to a flattener (or a steepener) with

1. a fixed tranche and fixed maturities,
2. a fixed tranche and all maturities,
3. all tranches and fixed maturities,
4. all tranches and all maturities (no restrictions),
or allow to combine flatteners and steepeners.
Backtesting

- Consider the time horizons $h = 1, 5, 20$ days.
- For the tranches that optimize a given strategy, check the corresponding historical market spreads, calculate the resulting MTM values, and the realised P&L.
# Mean of Daily Gains in Percent

<table>
<thead>
<tr>
<th>Strategy</th>
<th>DSFM 1 day</th>
<th>DSFM 1 week</th>
<th>DSFM 1 month</th>
<th>DSFM without the mean factor 1 day</th>
<th>DSFM without the mean factor 1 week</th>
<th>DSFM without the mean factor 1 month</th>
</tr>
</thead>
<tbody>
<tr>
<td>FS-AIiT-AIIM</td>
<td>0.29</td>
<td>0.10</td>
<td>0.05</td>
<td>0.30</td>
<td>0.11</td>
<td>0.04</td>
</tr>
<tr>
<td>FS-T2-AIIM</td>
<td>0.29</td>
<td>0.13</td>
<td>0.06</td>
<td>0.33</td>
<td>0.12</td>
<td>0.05</td>
</tr>
<tr>
<td>FS-T3-AIIM</td>
<td>0.19</td>
<td>0.07</td>
<td>0.03</td>
<td>0.18</td>
<td>0.07</td>
<td>0.02</td>
</tr>
<tr>
<td>FS-T4-AIIM</td>
<td>0.14</td>
<td>0.04</td>
<td>0.02</td>
<td>0.12</td>
<td>0.04</td>
<td>0.01</td>
</tr>
<tr>
<td>FS-T5-AIIM</td>
<td>0.09</td>
<td>0.04</td>
<td>0.02</td>
<td>0.08</td>
<td>0.03</td>
<td>0.01</td>
</tr>
<tr>
<td>F-T2-AIIM</td>
<td>0.30</td>
<td>0.12</td>
<td>0.06</td>
<td>0.28</td>
<td>0.12</td>
<td>0.05</td>
</tr>
<tr>
<td>F-T3-AIIM</td>
<td>0.16</td>
<td>0.06</td>
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<td>0.16</td>
<td>0.06</td>
<td>0.02</td>
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<td>0.10</td>
<td>0.03</td>
<td>0.01</td>
<td>0.10</td>
<td>0.03</td>
<td>0.01</td>
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<tr>
<td>F-T5-AIIM</td>
<td>0.09</td>
<td>0.03</td>
<td>0.01</td>
<td>0.08</td>
<td>0.03</td>
<td>0.01</td>
</tr>
<tr>
<td>S-T2-AIIM</td>
<td>0.39</td>
<td>0.15</td>
<td>0.07</td>
<td>0.45</td>
<td>0.13</td>
<td>0.05</td>
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<tr>
<td>S-T3-AIIM</td>
<td>0.27</td>
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<td>0.04</td>
<td>0.30</td>
<td>0.09</td>
<td>0.02</td>
</tr>
<tr>
<td>S-T4-AIIM</td>
<td>0.20</td>
<td>0.06</td>
<td>0.03</td>
<td>0.20</td>
<td>0.05</td>
<td>0.01</td>
</tr>
<tr>
<td>S-T5-AIIM</td>
<td>0.12</td>
<td>0.04</td>
<td>0.02</td>
<td>0.12</td>
<td>0.04</td>
<td>0.01</td>
</tr>
<tr>
<td>F-AIiT-105</td>
<td>0.20</td>
<td>0.07</td>
<td>0.03</td>
<td>0.19</td>
<td>0.06</td>
<td>0.02</td>
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<tr>
<td>F-AIiT-107</td>
<td>0.22</td>
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<td>0.03</td>
<td>0.25</td>
<td>0.08</td>
<td>0.03</td>
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<tr>
<td>F-AIiT-75</td>
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<td>0.01</td>
<td>0.14</td>
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<td>0.01</td>
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<tr>
<td>S-AIiT-510</td>
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<td>0.08</td>
<td>0.02</td>
<td>0.16</td>
<td>0.05</td>
<td>0.01</td>
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<tr>
<td>S-AIiT-710</td>
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<td>0.02</td>
<td>0.21</td>
<td>0.07</td>
<td>0.02</td>
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<tr>
<td>S-AIiT-57</td>
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<td>0.03</td>
<td>-0.01</td>
<td>0.12</td>
<td>0.03</td>
<td>-0.01</td>
</tr>
</tbody>
</table>

Table 4: Calculations based on predictions of log-spreads and Z-transformed BCs marked as LZ; based only on Z-transformed BCs marked as Z.
Investor’s Strategy

Follow a certain strategy over a year and constantly rebalance the portfolio. At $t_0$ enter an optimal (according to the DSFM) curve trade for $h$-day horizon. At $t_0 + h$ chose:

1. keep the current position for the next $h$-days,
2. close the current position and enter a new one.

Assume a margin of 10% of your notional. Every time the position is closed, add to the margin the realized P&L. If margin $\leq 0$, quit the trade.
Investor’s Strategy

Figure 11: Combined flatteners and steepeners from all tranches and all maturities. Closing profits after one year. Rebalancing after: 1 day (upper left), 1 week (upper right), 1 month (lower). Calculations based on the DSFM predictions of log-spreads and Z-transformed BCs.
Investor’s Strategy

Figure 12: Daily cumulated P&L over one year 20070614–20080529. Rebalancing after: 1 day (upper left), 1 week (upper right), 1 month (lower). Calculations based on the DSFM predictions of log-spreads and Z-transformed BCs.
Conclusions

- Investigated evolution over time of tranche spread surfaces and base correlation surfaces using the DSFM.
- Empirical study is conducted using an extensive data set of 49,502 observations of iTraxx Europe tranches in 2005-2009.
- Proposed a modification to the classic DSFM.
- Both DSFMs successfully reproduce the dynamics in data.
- Used DSFM in constructing the curve trades.
- Analysed the performance of 43 strategies that combine different positions, tranches, and maturities.
- Backtesting showed high daily gains of the resulting curve trades.
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Appendix

DSFM for Log-Spreads

Figure 13: Estimated factor functions and loadings ($\hat{Z}_{t,1}, \hat{Z}_{t,2}$).
DSFM without the Mean Factor for Log-Spreads

Figure 14: Estimated factor functions and loadings ($\hat{Z}_{t,1}$, $\hat{Z}_{t,2}$).
DSFM for Z-transformed-BC

Figure 15: Estimated factor functions and loadings ($\hat{Z}_{t,1}$, $\hat{Z}_{t,2}$).