Portfolio Credit Risk Contribution

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The Concept of Central Counterparty (CCP)

Central Counterparty interposes itself between counterparties and becomes the buyer to every seller and the seller to every buyer.

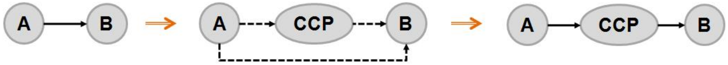


Figure 1: Legal process of replacing original OTC contract to central counterparty (novation)

Risk Reserve Architecture of CCP

Membership Requirements	e.g. minimum requirement of rating, minimum capital requirement,…
Variation margin	Margin based on daily changes in market value of the cleared product
Initial margin	Margin based on potential future exposure (via stress test, e.g. largest 5 days decline)
Default Funds	Funds based on loss given default of single largest clearing member or simultaneous defaults of second and third largest

Credit Default Swap Spread

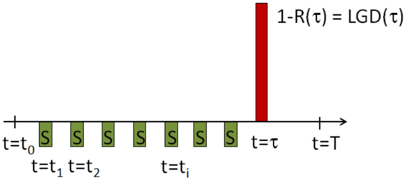


Figure 2: Cash-flow structure of CDS until credit event in τ

$$S_{t_0} = \frac{\mathsf{E}^Q_{t_0}\left[\exp\left\{-\int_{t_0}^{\tau} r(\ell)d\ell\right\}\{1 - R(\tau)\}\,\mathbf{1}_{\{\tau<T\}}\right]}{\mathsf{E}^Q_{t_0}\left[\sum_{i=1}^{n}\exp\left\{-\int_{t_0}^{t_i} r(\ell)d\ell\right\}\mathbf{1}_{\{\tau>t_i\}}\right]}$$

Risk measures

- Value at Risk (VaR)

$$\mathrm{VaR}_{t+d}^{\alpha} = \inf\{x \in \mathbb{R} \ : \ \mathrm{P}(X_{t+d} \le x \mid \mathcal{F}_t) \ge \alpha\} \qquad (1)$$

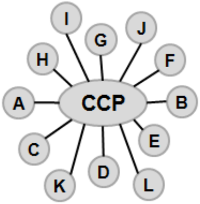
  where X_t denotes the spread returns.

- Expected shortfall (ES)

$$\mathrm{ES}_{t+d}^{\alpha} = -\mathrm{E}\left(X_{t+d} \mid X_{t+d} \le \mathrm{VaR}_{t+d}^{\alpha}\right) \qquad (2)$$

  Expected shortfall is the expected return given the return exceeds its $\mathrm{VaR}_{t+d}^{\alpha}$ value.

CCP's credit risk contribution from each member?



Question: What is the portfolio's credit risk and credit risk contribution of each constituent?

Objectives

- VaR and CoVaR calculation under consideration of CDS spreads as indicator for credit risk and market variables
- Indirect spillover effect via CoVaR calculation under consideration of market variables
- Semi-parametric quantile regression modelling
- Significance of market variables in predicting CDS spreads

Outline

1. Motivation $\checkmark$
2. Linear quantile regression
3. Partial linear quantile regression
4. CDS spreads data
5. Linear versus nonparametric quantile regression model
6. Research outlook

CoVaR

- Adrian & Brunnermeier (2011): linear quantile regressions

$$X_{i,t} = \alpha_i + \gamma_i^\top M_{t-1} + \varepsilon_{i,t},$$
$$X_{j,t} = \alpha_{j|i} + \beta_{j|i} X_{i,t} + \gamma_{j|i}^\top M_{t-1} + \varepsilon_{j,t}.$$

M_t: state variables. $F^{-1}_{\varepsilon_{i,t}}(\tau|M_{t-1}) = 0$ and $F^{-1}_{\varepsilon_{j,t}}(\tau|M_{t-1}, X_{i,t}) = 0$.

$$\widehat{VaR}_{i,t} = \hat{\alpha}_i + \hat{\gamma}_i^\top M_{t-1},$$
$$\widehat{CoVaR}_{j|i,t} = \hat{\alpha}_{j|i} + \hat{\beta}_{j|i} \widehat{VaR}_{i,t} + \hat{\gamma}_{j|i}^\top M_{t-1}.$$

Quantile Regression in Risk Calibration

- Chao, Härdle & Wang (2013): partial linear quantile regression:

$$X_{i,t} = \alpha_i + \gamma_i^\top M_{t-1} + \varepsilon_{i,t};$$
$$X_{j,t} = \tilde{\alpha}_{j|i} + \tilde{\beta}_{j|i}^\top M_{t-1} + l_{j|i}(X_{i,t}) + \varepsilon_{j,t}.$$

l: a general function. M_t: state variables. $F^{-1}_{\varepsilon_{i,t}}(\tau|M_{t-1}) = 0$ and $F^{-1}_{\varepsilon_{j,t}}(\tau|M_{t-1}, X_{i,t}) = 0$.

$$\widehat{VaR}_{i,t} = \hat{\alpha}_i + \hat{\gamma}_i^\top M_{t-1},$$
$$\widehat{CoVaR}_{j|i,t} = \hat{\alpha}_{j|i} + \hat{\gamma}_{j|i}^\top M_{t-1} + \hat{l}_{j|i}(\widehat{VaR}_{i,t}).$$

State variables

M_t: 7 state variables suggested by AB and further extension:

1. VIX
2. Short term liquidity spread
3. Change in the 3M T-bill rate
4. Change in the slope of the yield curve
5. Change in the credit spread between 10 years BAA-rated bonds and the T-bond rate
6. S&P500 returns
7. Dow Jones U.S. Real Estate index returns
8. Constituent's specific stock returns
9. Constituent's specific stock volatility returns

Data:

- G14 FI: daily CDS spreads of 14 biggest derivative dealers
- Overall data period: Sept 2002 - Dec 2011 ($N = 2228$)
- Segregation into three sub-periods
  - pre-crisis: Sept 2002 - June 2007
  - crisis: July 2007 - March 2009
  - post-crisis: April 2009 - Dec 2011

Characteristics of CDS spreads data

G14 FI	2002-2011			
	min	*max*	μ	σ
Citi	7.44	665.53	104.41	121.34
CS	8.40	265.30	62.53	52.67
GS	18.75	545.14	95.53	89.43
JPM	11.45	232.30	61.24	43.04
MS	17.83	1239.99	123.11	137.64

G14 FI	pre-crisis		crisis		post-crisis	
	μ	σ	μ	σ	μ	σ
Citi	20.01	10.89	172.08	136.65	215.82	104.23
CS	25.64	23.25	101.20	58.23	106.38	35.45
GS	33.48	13.38	176.76	114.48	160.72	66.89
JPM	31.42	18.58	96.84	45.88	94.40	32.05
MS	33.88	14.03	256.01	206.95	208.31	92.22

QR of MS daily spread returns and VIX

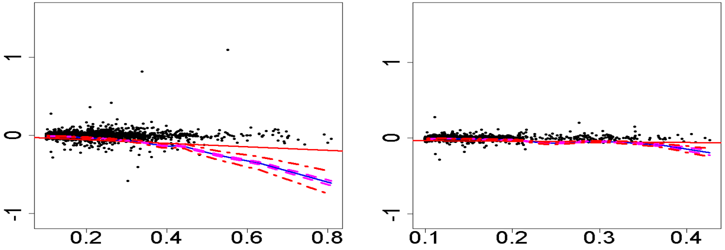


Figure 3: Left: overall period; right: pre-crisis period; y-axis = MS spread returns; x-axis = VIX. Locally linear quantile estimation. Linear quantile regression line. 95% asymptotic CB, dash: bootstrap CB.

QR of MS daily spread returns and VIX

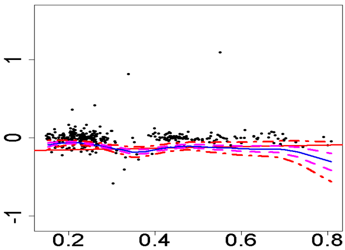


Figure 4: Left: crisis period; right: post-crisis period; y-axis = MS spread returns; x-axis = VIX. Locally linear quantile estimation. Linear quantile regression line. 95% asymptotic CB, dash: bootstrap CB.

QR of Morgan Stanley daily spread returns and daily stock returns

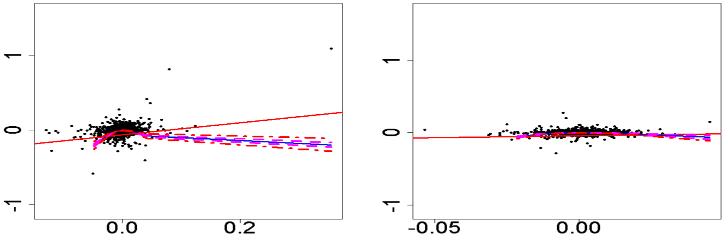


Figure 5: Left: overall period; right: pre-crisis period; y-axis = MS spread return; x-axis = MS stock return. Locally linear quantile estimation. Linear quantile regression line. 95% asymptotic CB, dash: bootstrap CB.

QR of Morgan Stanley daily spread returns and daily stock returns

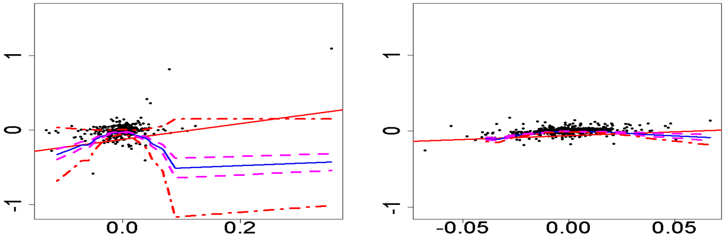


Figure 6: Left: crisis period; right: post-crisis period; y-axis = MS spread returns; x-axis = MS stock return. Locally linear quantile estimation. Linear quantile regression line. 95% asymptotic CB, dash: bootstrap CB.

Confidence band violation area

Area between confidence band and linear regression line:

$$D = \int_{-\infty}^{0} \{|y_o(x) - l(x)|\} \mathbf{1}\{y_o(x) < l(x)\} dx + \int_{-\infty}^{0} \{|y_u(x) - l(x)|\} \mathbf{1}\{y_u(x) > l(x)\} dx.$$

Area calculated for Morgan Stanley quantile regression:

State variables	2002-2011	pre-crisis	crisis	post-crisis
VIX	0.051	0.010	0.053	0.841
diffRepoTB3M	0.148	0.001	1.863	0.001
diffTB3M	0.156	1.339	14.368	0.001
diffSlopeYieldCurve	0.160	0.004	0.074	0.008
diffCDSSpread	0.003	0.003	0.003	0.004
diffEquityReturn	13.507	0.781	7.113	9.008
diffRealEstateReturn	3.557	1.092	1.251	60.255
Stock returns	37.102	0.001	0.007	0.004

Preliminary Conclusion

- Dependence behaviour during overall period differs from sub-period analysis
- Dependence between spread and market variables seems to increase in crisis period
- Low significance for non-linearity for some state variables in certain period

Next steps

- Significance test based on confidence band violation area
- Additive model (Dynamic SPM) for VaR and CoVaR calculation

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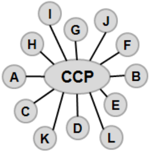

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Partial Linear Model (PLM)

- The partial linearity observation implies:

$$X_{i,t} = \alpha_i + \gamma_i^\top M_{t-1} + \varepsilon_{i,t};$$
$$X_{j,t} = \tilde{\beta}_{j|i}^\top M_{t-1} + l_{j|i}(X_{i,t}) + \varepsilon_{j,t}. \tag{3}$$

l: a general function. M_t: state variables. $F^{-1}_{\varepsilon_{i,t}}(\tau|M_{t-1}) = 0$ and $F^{-1}_{\varepsilon_{j,t}}(\tau|M_{t-1}, X_{i,t}) = 0$.

- Advantages
  - Capturing nonlinear asset dependence
  - Avoid curse of dimensionality

Estimation of Partial Linear Model

- PLM model: Liang, Härdle and Carroll (1999) and Härdle, Ritov and Song (2012)

$$Y_t = \beta^\top M_{t-1} + l(X_t) + \varepsilon_t.$$

- Consider $[0,1]$ (standard rank space). Dividing $[0,1]$ into a_n equally divided subintervals I_{nt}, $a_n \uparrow \infty$. On each subinterval, $l(\cdot)$ is roughly constant.

Estimation of PLM QR

1. Linear element β:

$$\hat{\beta} =$$

$$\operatorname*{argmin}_{\beta} \min_{l_1,\ldots,l_{a_n}} \sum_{t=1}^{n} \rho_\tau \left\{ Y_t - \beta^\top M_{t-1} - \sum_{m=1}^{a_n} l_m \mathbf{1}(X_t \in I_{nt}) \right\}$$

2. Nonlinear element $l(\cdot)$: With data $\{(X_t, Y_t - \hat{\beta}^\top M_{t-1})\}_{t=1}^{n}$, applying LLQR.

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