Advanced Econometrics

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You have to answer 2 out of 3 problems within 90 minutes (plus 10 minutes "reading time"). If you answer all questions, only the first 2 problems will be taken into account.

You may answer in English or in German. But please stick to one language.

You find necessary underlying results and formulas in the appendix.

Do your best to write legibly. Exams or parts of exams which cannot be read with reasonable effort will not be graded.

Good luck!

Problem 1: (Pseudo) Maximum Likelihood

Define y as a random variable and $f(y;\theta)$ as a parametric density depending on a parameter vector $\theta \in \Theta$. Assume that $\partial f(y;\theta)/\partial\theta$ and $\partial^2 f(y;\theta)/\partial\theta\partial\theta'$ exist for all y and $\theta \in \Theta$. Moreover, assume that for all $\theta \in \Theta$ and all realizations of y, one can twice differentiate $\int f(y;\theta)dy$ under the integral with respect to the components of θ.

a) Show that

$$\mathrm{E}\left[\frac{\partial \ln f(y;\theta)}{\partial\theta}\right] = 0.$$

Hint: Differentiate $\int f(y;\theta)dy = 1$ *under the integral.*

b) Building on the results from a) show that

$$\mathcal{I}(\theta) = \mathrm{E}\left[-\frac{\partial^2 \ln f(y;\theta)}{\partial\theta\partial\theta'}\right], \tag{1}$$

where $\mathcal{I}(\theta) := \mathrm{E}\left[(\partial \ln f(y;\theta)/\partial\theta)(\partial \ln f(y;\theta)/\partial\theta')\right]$. How do we call $\mathcal{I}(\theta)$ and how do we call the equality in (1)?

c) Assume two distributions $f(y)$ and $f^*(y)$ and define the Kullback discrepancy as $I(f|f^*) := \mathrm{E}^*\left[\ln \frac{f^*(y)}{f(y)}\right]$. Show that

(i) $I(f|f^*) \geq 0$,

(ii) $I(f|f^*) = 0$ if and only if $f = f^*$.

Explain intuitively the idea of the Kullback discrepancy.

d) Assume that y follows a distribution belonging to the linear exponential family with $f(y;\mu(x,\theta)) = f(y;\mu) = \exp(A(\mu) + B(\mu) + C(\mu)y)$, where x is a set of (predetermined) regressors, and $\mathrm{E}_x[y] = \mathrm{E}[y|x] = \mu(x,\theta)$.

(i) Show that the expected score regarding $f(y;\mu)$ is zero only if

$$\frac{dA(\mu)}{d\mu} + \frac{dC(\mu)}{d\mu}\mu = 0.$$

(ii) Show that the information equality only holds if the variance of y is of the form $\mathrm{V}[y] = (dC(\mu)/d\mu)^{-1}$.
Hint: Use the fact that the negative Hessian can be computed as

$$E_x E_0\left[\frac{-\partial^2 \ln f(y;\mu(x,\theta_0))}{\partial\theta\partial\theta'}\right] = E_x\left[\frac{\partial\mu'}{\partial\theta}\frac{dC(\mu)}{d\mu}\frac{\partial\mu}{\partial\theta'}\right],$$

where E_x *denotes the expectation with respect to* x *and* E_0 *denotes the expectation with respect to* y *given* x.

Problem 2: Multiple-Equation GMM

Assume an M-equation model of the form

$$y_{im} = x'_{im}\theta_m + \varepsilon_{im}, \quad m = 1, 2, \ldots, M,\ i = 1, 2, \ldots, n,$$

where (y_{im}, x_{im}) follows a stationary ergodic process and x_{im} and θ_m are $(k_m \times 1)$-vectors. The error terms ε_{im} follow white noise processes. Assume a set of stationary ergodic instruments with $\mathrm{E}[w_{im}\varepsilon_{im}] = 0, \quad m = 1, 2, \ldots, M$. Moreover, assume the existence of fourth (cross-)moments of w_i and x_j, $i, j = 1, \ldots, n$.

a) Give the necessary and sufficient condition for the identification of $\theta := (\theta_1, \ldots, \theta_M)$.

b) Compute the sampling error of the multiple-equation GMM estimator based on the weighting matrix W.

c) Show that the sampling error derived in b) converges in probability to zero and state the necessary underlying assumptions.

d) Show that the single-equation GMM estimator of θ can be written as a multiple-equation GMM estimator. What does this result mean for the consistency of the single-equation GMM estimator?

e) Can the single-equation GMM estimator be as efficient as the multiple-equation GMM estimator? Justify your answer analytically.

f) Assume the parameter restriction $\theta_1 = \theta_2 = \ldots = \theta_M := \bar{\theta}$.

   (i) Derive the pooled OLS estimator of $\bar{\theta}$.

   (ii) Which orthogonality condition is exploited by the pooled OLS estimator?

   (iii) Under which condition is the pooled OLS estimator as efficient as the corresponding multiple-equation GMM estimator?

Problem 3: Essays

Write short essays on two of the following topics:

(i) HAC estimators

(ii) The Gibbs algorithm

(iii) Bayesian model comparison

(iv) Consistent pseudo-maximum likelihood estimation

(v) The accept-rejection algorithm