Econometric Analysis of Financial Market Data

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You have to answer 2 out of 3 problems within 90 minutes (plus 10 minutes "reading time"). If you answer all questions, only the first 2 problems will be taken into account.

You may answer in English or in German. But please stick to one language.

Some problems contain several small sub-questions. Please give short but nevertheless precise answers.

Do your best to write legibly. Exams or parts of exams which cannot be read with reasonable effort will not be graded.

Good luck!

Problem 1: Testing for Asset Return Predictability

Table 1 shows the variance ratios and p-values of different variance ratio tests based on daily log returns, r_t.

a) Explain the fundamental idea of the variance ratio test.

b) Define the q-period log return as $r_t(q) := r_t + r_{t-1} + \ldots + r_{t-q+1}$ and denote the k-order autocorrelation by ρ_k. Show the validity of the formula

$$VR(q) := \frac{\mathrm{V}[r_t(q)]}{q\mathrm{V}[r_t]} = 1 + 2\sum_{k=1}^{q-1}\left(1 - \frac{k}{q}\right)\rho_k$$

for $k = 2$ and $k = 4$.

c) Show that

$$\frac{VR(2q)}{VR(q)} = 1 + \rho_1^{(q)},$$

where $\rho_1^{(q)}$ denotes the first-order autocorrelation for a q-period log return.

d) What can you learn from the results in Table 1 regarding the predictability of log returns measured over different time horizons?

e) Compute $VR(q)$ under the assumption of an AR(1) process for log returns, i.e.

$$r_t = c + \phi r_{t-1} + \varepsilon_t,$$

where ε_t is a white noise error term.

f) Assume that the first $q-1$ autocorrelations $\rho_1, \ldots, \rho_{q-1}$ are non-zero. Does there exist a restriction on $\rho_1, \ldots, \rho_{q-1}$ for which the variance ratio $VR(q)$ is zero anyhow? If yes, write it down. What can we learn from this result regarding the power properties of the variance ratio test?

g) The p-values shown in Table 1 are computed based on estimators for $\mathrm{V}[\hat{\rho}_k]$ which are robust and non-robust against conditional heteroscedasticity, respectively. State the corresponding null hypotheses underlying both types of p-values. How can it be explained that the corresponding p-values differ?

Problem 2: Volatility and Risk

a) Table 2 gives the estimation results of the following model for log returns r_t:

$$r_t = c + \delta\sigma_t^2 + \phi r_{t-1} + \varepsilon_t, \tag{1}$$
$$\varepsilon_t = z_t\sigma_t, \quad z_t \sim \text{i.i.d. } N(0;1), \tag{2}$$
$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2. \tag{3}$$

How do we call such a model? Motivate the specification economically and interpret the parameter estimates.

b) Figures 1 and 2 give the autocorrelograms of $\hat{z}_t$ and $\hat{z}_t^2$, respectively. Interpret the findings. What do the results imply for the goodness-of-fit of the model?

c) Table 3 shows the outcome of a test for ARCH effects. State the null hypothesis, explain the idea of the test and interpret the outcome.

d) Figures 3 and 4 show the descriptive statistics of $\hat{\varepsilon}_t$ and $\hat{z}_t$, respectively. Why does the kurtosis of $\hat{\varepsilon}_t$ exceed the kurtosis of $\hat{z}_t$? Justify your answer analytically (but without explicitly computing $\mathrm{E}[\hat{\varepsilon}_t^4]$).

e) Your colleague claims to consistently estimate the parameters of a GARCH(1,1) model of the form

$$r_t = c + \varepsilon_t,$$
$$\varepsilon_t = z_t\sigma_t, \quad z_t \text{ i.i.d. with } \mathrm{E}[z_t] = 0,\ \mathrm{V}[z_t] = 1,$$
$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

by alternatively running the ARMA(1,1) regression

$$r_t^2 = \phi_0 + \phi_1 r_{t-1}^2 + \phi_2\eta_{t-1} + \eta_t,$$

where η_t is assumed to have zero mean and to be serially uncorrelated.

(i) Prove that he is right if $c = 0$. Illustrate how to identify the GARCH parameters ω, α and β from the ARMA parameters ϕ_0, ϕ_1 and ϕ_2.

(ii) Does it also work if $c \neq 0$? Why or why not?

f) Table 4 shows the results of model (1)-(3) where $\delta = \phi = 0$ and eq. (3) is replaced by

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \gamma r_{t-1} \tag{4}$$

(i) Which effect can be captured by this specification?

(ii) Interpret the estimate of γ.

(iii) What could be a possible problem induced by specification (4)?

(iv) Suggest an alternative (G)ARCH specification which is able to capture the same effect.

Problem 3: Present Value Relations

a) Assume that the following first-order Taylor approximation for log returns, r_t, holds:

$$r_{t+1} = k + \rho p_{t+1} + (1-\rho)d_{t+1} - p_t, \tag{5}$$

where p_t denotes the log price, d_t denotes the log dividend, $\rho := 1/(1+\exp(\overline{d-p})$, $\overline{d-p}$ is the average log dividend-price ratio, and $k = -\ln(\rho) - (1-\rho)\ln(1/\rho - 1)$.

Solve (5) for p_t forward and show that the asset's fundamental value (present value), under the assumption that the transversality condition holds, is given by

$$p_t = \frac{k}{1-\rho}\left[\sum_{j=0}^{\infty} \rho^j [(1-\rho)d_{t+1+j} - r_{t+1+j}]\right]. \tag{6}$$

Interpret the resulting present value relation economically. Show that it also holds *ex ante*.

b) State the transversality condition and interpret it economically. What happens if the transversality condition does not hold?

c) Assume that conditional expectations follow an AR(1) process, i.e.,

$$\begin{aligned} \mathrm{E}_t[r_{t+1}] &= r + x_t, \\ x_t &= \phi x_{t-1} + \varepsilon_t, \end{aligned}$$

where r is a constant and ε_t is a white noise error term. Moreover, assume that log dividends follow a random walk process, i.e.

$$d_t = d_{t-1} + u_t,$$

where u_t follows a white noise error term which is independent from ε_t. Compute the present value relation (6) under these assumptions and show that the log dividend-price ratio is given by

$$d_t - p_t = \frac{x_t}{1-\rho\phi} - \frac{k-r}{1-\rho}.$$

d) Under which conditions is $d_t - p_t$ weakly stationary? What does this imply for the dynamic properties of log prices and log dividends?

e) Compute $\mathrm{V}[d_t - p_t]$ and interpret the resulting expression economically. What happens if ρ and ϕ are close to one?

Appendix

Table 1: Variance ratios $VR(q)$ for different aggregation levels q and corresponding p-values for daily log returns. Panel (2) reports p-values which are robust against conditional heteroscedasticity. Panel (3) reports p-values which are not robust against conditional heteroscedasticity.

q	(1)	(2)	(3)
	VR(q)	p-value (robust)	p-value (non-robust)
2	1.083	0.021	0.000
4	1.091	0.033	0.001
8	1.102	0.034	0.002
16	1.121	0.028	0.001

Table 2:

	Coefficient	Std. Error	z-Statistic	Prob.
DELTA	9.054311	4.736328	1.911673	0.0559
C	-0.000619	0.000509	-1.217976	0.2232
PHI	0.094731	0.017437	5.432768	0.0000
Variance Equation				
OMEGA	8.62E-05	2.71E-06	31.78359	0.0000
ALPHA	0.328816	0.025638	12.82548	0.0000

R-squared	-0.004992	Mean dependent var	0.000288
Adjusted R-squared	-0.005357	S.D. dependent var	0.011285
S.E. of regression	0.011315	Akaike info criterion	-6.241667
Sum squared resid	1.409269	Schwarz criterion	-6.238350
Log likelihood	34374.74	Durbin-Watson stat	2.089641

Figure 1: Autocorrelogram of $\hat{z}_t$.

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.009	-0.009	0.8645	0.352
		2	-0.025	-0.025	7.5912	0.022
		3	-0.011	-0.011	8.8925	0.031
		4	0.023	0.022	14.828	0.005
		5	-0.002	-0.002	14.872	0.011
		6	-0.030	-0.029	24.695	0.000
		7	0.011	0.011	26.004	0.001
		8	0.017	0.015	29.183	0.000
		9	0.007	0.007	29.689	0.000
		10	0.006	0.009	30.100	0.001
		11	0.010	0.011	31.276	0.001
		12	0.018	0.017	34.823	0.000
		13	0.003	0.004	34.918	0.001
		14	0.025	0.027	41.779	0.000
		15	0.006	0.007	42.206	0.000
		16	0.002	0.003	42.270	0.000
		17	0.000	0.001	42.270	0.001
		18	0.009	0.009	43.137	0.001
		19	-0.008	-0.009	43.874	0.001
		20	0.027	0.028	51.933	0.000

Figure 2: Autocorrelogram of $\hat{z}_t^2$.

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.024	-0.024	6.5735	0.010
		2	0.132	0.131	197.97	0.000
		3	0.138	0.147	407.53	0.000
		4	0.169	0.167	723.88	0.000
		5	0.159	0.149	1004.0	0.000
		6	0.112	0.080	1143.0	0.000
		7	0.133	0.076	1336.5	0.000
		8	0.119	0.056	1492.8	0.000
		9	0.116	0.045	1641.4	0.000
		10	0.119	0.047	1796.4	0.000
		11	0.151	0.086	2049.1	0.000
		12	0.115	0.053	2194.9	0.000
		13	0.114	0.039	2337.1	0.000
		14	0.108	0.026	2464.9	0.000
		15	0.142	0.059	2688.7	0.000
		16	0.120	0.044	2847.8	0.000
		17	0.103	0.021	2964.1	0.000
		18	0.110	0.021	3097.1	0.000
		19	0.089	-0.004	3184.2	0.000
		20	0.125	0.032	3356.9	0.000

Table 3: Test for ARCH effects.

ARCH Test:

F-statistic	99.88816	Probability	0.000000
Obs*R-squared	196.2688	Probability	0.000000

Test Equation:
Dependent Variable: STD_RESID^2
Method: Least Squares

Sample(adjusted): 5 11015
Included observations: 11011 after adjusting endpoints
White Heteroskedasticity-Consistent Standard Errors & Covariance

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.890079	0.027575	32.27821	0.0000
STD_RESID^2(-1)	-0.021232	0.006651	-3.192156	0.0014
STD_RESID^2(-2)	0.131293	0.019848	6.614778	0.0000

R-squared	0.017825	Mean dependent var	1.000159
Adjusted R-squared	0.017646	S.D. dependent var	2.318853
S.E. of regression	2.298303	Akaike info criterion	4.502491
Sum squared resid	58146.41	Schwarz criterion	4.504482
Log likelihood	-24785.47	F-statistic	99.88816
Durbin-Watson stat	2.038465	Prob(F-statistic)	0.000000

Figure 3: Descriptive statistics of $\hat{\varepsilon}_t$.

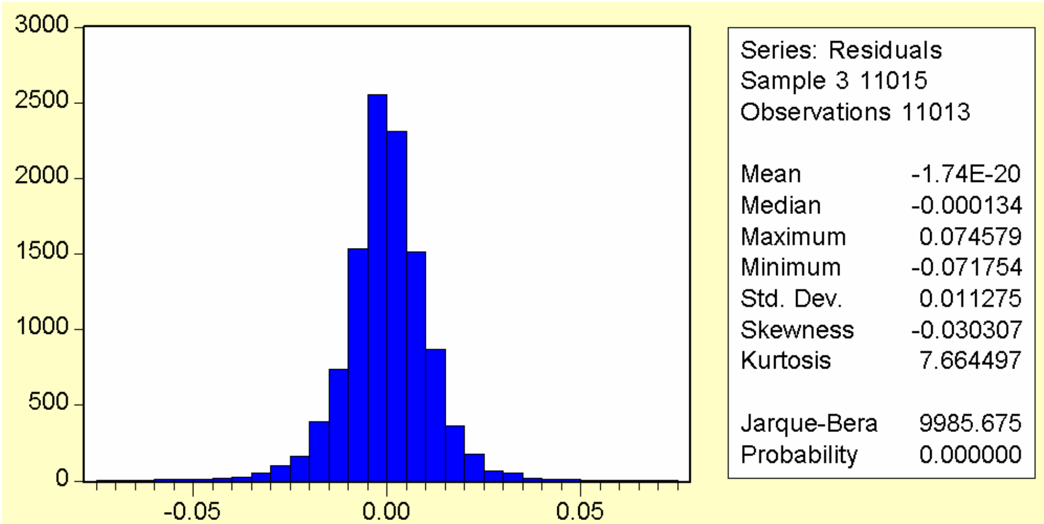

Figure 4: Descriptive statistics of $\hat{z}_t$.

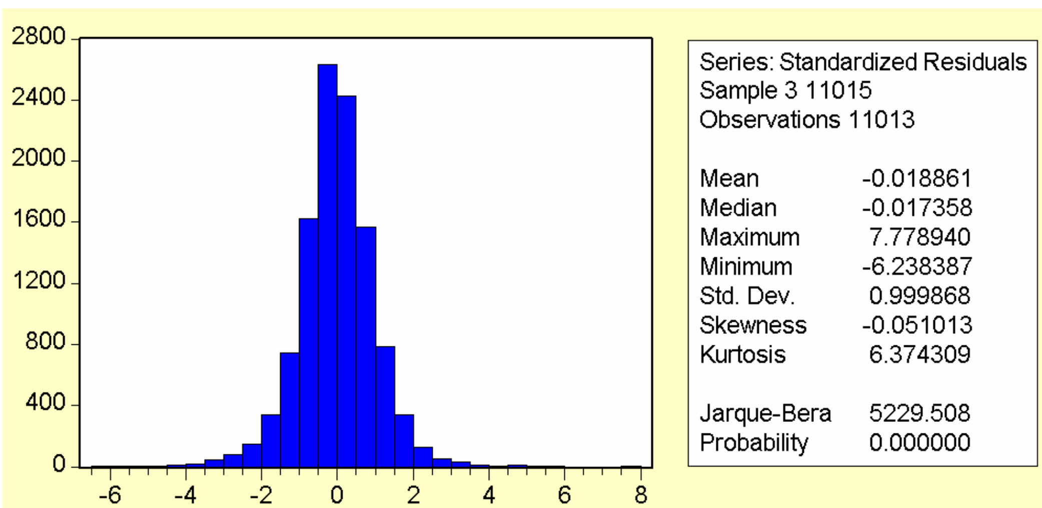

Table 4:

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000312	9.91E-05	3.150268	0.0016
Variance Equation				
OMEGA	8.83E-05	2.74E-06	32.26575	0.0000
ALPHA	0.319636	0.025883	12.34937	0.0000
GAMMA	-0.001315	0.000341	-3.859438	0.0001

R-squared	-0.000005	Mean dependent var	0.000288
Adjusted R-squared	-0.000277	S.D. dependent var	0.011285
S.E. of regression	0.011286	Akaike info criterion	-6.236052
Sum squared resid	1.402276	Schwarz criterion	-6.233398
Log likelihood	34342.82	Durbin-Watson stat	1.918515