Econometric Analysis of Financial Market Data

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You have to answer 2 out of 3 problems within 90 minutes (plus 10 minutes "reading time"). If you answer all questions, only the first 2 problems will be taken into account.

You may answer in English or in German. But please stick to one language.

Some problems contain several small sub-questions. Please give short but nevertheless precise answers.

Do your best to write legibly. Exams or parts of exams which cannot be read with reasonable effort will not be graded.

Good luck!

Problem 1: Properties of Asset Returns

a) Assume the following process for log returns r_t:

$$r_t = z_t(\omega + \alpha r_{t-1}^2)^{1/2}, \quad |\alpha| < 1, \quad t = 1, \ldots, T, \tag{1}$$

where z_t is i.i.d. with $\mathrm{E}[z_t] = 0$ and $\mathrm{V}[z_t] = 1$. How do we call such a process?

b) Compute the unconditional mean and the unconditional variance of r_t.

c) Show that the returns r_t are uncorrelated.

Hint: Use the law of iterated expectations.

d) Compute $\mathrm{Corr}[r_t^2, r_{t-k}^2], \quad k > 0$.

e) Show that the parameters ω and α can be estimated by OLS and give the corresponding estimators. Justify why your estimators are unbiased.

f) Tables 1 and 2 show the autocorrelation functions of $\hat{z}_t$ and $\hat{z}_t^2$ based on residuals $\hat{\varepsilon}_t$ arising from an estimation of (1).

(i) What can you conclude regarding the goodness-of-fit?

(ii) In which direction would you eventually extend your model?

g) How could process (1) be modified such that it corresponds to a white noise process but not necessarily to a martingale difference sequence? Give an example.

Problem 2: Value-at-Risk

a) Let V_t be the value of an asset position at time period t. Give the formal definition of the Value-at-Risk (VaR) with respect to a change in value of V_t from t to $t+l$ for a long position and risk probability p, denoted by $VaR_t(p,l)$.

b) Assume that the one-period log returns of the underlying asset, r_t, are normally distributed, i.e. $r_t \sim$ i.i.d. $N(\mu_t, \sigma_t^2)$ and denote the cumulated l-period log return by $r_t[l] := \sum_{i=1}^{l} r_{t-l+i}$. Show that in this context the VaR defined as in a) is given by

$$VaR_t(p,l) = V_t\left(\exp(\Phi^{-1}(p)\sqrt{\mathrm{V}[r_{t+l}[l]|\mathcal{F}_t]} + \mathrm{E}[r_{t+l}[l]|\mathcal{F}_t]) - 1\right),$$

where $\Phi^{-1}(p)$ denotes the p-th quantile of the standard normal distribution and $\mathcal{F}_t$ is the information set up to period t.

c) Table 3 gives the estimation results of a Gaussian AR(1)-GARCH(1,1) model for daily log returns r_t of the form

$$
\begin{aligned}
r_t &= c + \phi r_{t-1} + \varepsilon_t, \\
\varepsilon_t &= z_t \sigma_t, \quad z_t \sim \text{i.i.d. } N(0,1), \\
\sigma_t^2 &= \omega + g(\varepsilon_{t-1}) + \beta \sigma_{t-1}^2, && (2) \\
g(\varepsilon_t) &= \alpha \varepsilon_t^2 + \gamma \varepsilon_t^2 \mathbb{1}_{\{\varepsilon_t < 0\}}. && (3)
\end{aligned}
$$

How do we call the GARCH specification defined by (2) and (3) and what is the underlying economic motivation?

d) Use the parameter estimates shown in Table 3 and compute the VaR for day $t+1$ for a risk probability of $p = 0.01$ and a long position of 100,000 Euro assuming you are at the end of day t and

(i) $r_t = r_{t-1} = 0.01$ and $\mathrm{V}[r_t|\mathcal{F}_{t-1}] = 0.3 \cdot 10^{-4}$,

(ii) $r_t = r_{t-1} = -0.01$ and $\mathrm{V}[r_t|\mathcal{F}_{t-1}] = 0.3 \cdot 10^{-4}$.

Hint: The 0.01*-quantile of the* $N(0,1)$*-distribution is given by* $\Phi^{-1}(0.01) = -2.3263$.

e) Now assume that the log returns follow the process

$$
\begin{aligned}
r_t &= z_t \sigma_t, \quad z_t \sim \text{i.i.d. } N(0,1), \\
\sigma_t^2 &= \alpha \sigma_{t-1}^2 + (1-\alpha) r_{t-1}^2, \quad 0 < \alpha < 1.
\end{aligned}
$$

(i) How do we call such a process?

(ii) Compute the VaR for a period of $l = 20$ days for a risk probability $p = 0.01$ if $\mathrm{V}[r_{t+1}|\mathcal{F}_t] = 0.3 \cdot 10^{-4}$.

Problem 3: Asset Pricing Models

a) Tables 4 and 5 show the results of two capital asset pricing model (CAPM) regressions for portfolios built on stocks with small market capitalization (Table 4) and portfolios built on stocks with large market capitalization (Table 5).

   (i) Write down the underlying CAPM regression equation.

   (ii) How would you test the CAPM based on these regressions? What can you conclude from the estimation results regarding the validity of the CAPM?

   (iii) Give a possible economic explanation why the CAPM does hold or does not hold.

b) Give an economic interpretation of the R^2's shown in Tables 4 and 5. Give a possible economic explanation why the numbers substantially differ.

c) Tables 6 and 7 show the results of an unrestricted and a restricted estimation of a system of CAPM equations for 10 size portfolios. Suggest a simple (asymptotic) test for the joint significance of the intercept terms. Give the test statistic and its asymptotic distribution.

d) You run T individual cross-sectional regressions (for $t = 1, \dots, T$), each based on N portfolios built on market capitalization,

$$Z_{it} = \gamma_{0,t} + \gamma_{1,t}\hat{\beta}_i + \varepsilon_{it}, \quad i = 1, \dots, N,$$

where Z_{it} denotes the excess return of portfolio i at t, $\hat{\beta}_i$ denotes the corresponding (pre-estimated) market beta, and ε_{it} is a white noise error term.

   (i) How would you test for the validity of the CAPM in this context?

   (ii) You find a strong and significantly positive relationship between Z_{it} and $\hat{\beta}_i$. What could be a possible effect driving this result and how could you test for it?

e) How would you test whether not only systematic risk but also idiosyncratic risk is priced? Suggest an appropriate econometric specification.

Appendix

Table 1: Autocorrelogram of $\hat{z}_t$.

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.356	0.356	1517.6	0.000
		2	0.174	0.054	1879.9	0.000
		3	0.090	0.015	1978.2	0.000
		4	0.033	-0.013	1991.0	0.000
		5	0.014	-0.001	1993.2	0.000
		6	0.003	-0.002	1993.4	0.000
		7	0.002	0.002	1993.4	0.000
		8	-0.011	-0.013	1994.8	0.000
		9	-0.004	0.004	1994.9	0.000
		10	-0.010	-0.009	1996.1	0.000
		11	-0.009	-0.002	1997.0	0.000
		12	-0.010	-0.006	1998.4	0.000
		13	-0.010	-0.004	1999.6	0.000
		14	0.005	0.013	1999.8	0.000
		15	0.013	0.011	2001.8	0.000
		16	0.023	0.015	2007.9	0.000
		17	0.010	-0.006	2009.1	0.000
		18	0.000	-0.007	2009.1	0.000
		19	0.008	0.010	2010.0	0.000
		20	0.009	0.005	2011.0	0.000

Table 2: Autocorrelogram of $\hat{z}_t^2$.

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.026	-0.026	7.9225	0.005
		2	0.092	0.092	109.86	0.000
		3	0.103	0.108	237.07	0.000
		4	0.116	0.116	399.37	0.000
		5	0.113	0.106	552.06	0.000
		6	0.116	0.100	714.31	0.000
		7	0.089	0.064	809.66	0.000
		8	0.115	0.079	969.84	0.000
		9	0.083	0.044	1052.1	0.000
		10	0.092	0.044	1153.3	0.000
		11	0.082	0.033	1234.1	0.000
		12	0.111	0.061	1382.7	0.000
		13	0.084	0.038	1467.8	0.000
		14	0.111	0.060	1615.6	0.000
		15	0.091	0.042	1714.1	0.000
		16	0.083	0.026	1796.0	0.000
		17	0.102	0.044	1920.9	0.000
		18	0.068	0.009	1977.1	0.000
		19	0.081	0.016	2056.3	0.000
		20	0.094	0.031	2162.9	0.000

Table 3: AR-GARCH regression

Dependent Variable: LOG_RETURN
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 09/27/07 Time: 16:29
Sample (adjusted): 3 11015
Included observations: 11013 after adjustments
Convergence achieved after 13 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-1)^2*(RESID(-1)<0) +
C(6)*GARCH(-1)

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000214	8.96E-05	2.393084	0.0167
AR(1)	0.093610	0.010097	9.271382	0.0000
Variance Equation				
C	1.46E-06	1.49E-07	9.811144	0.0000
RESID(-1)^2	0.058909	0.004144	14.21566	0.0000
RESID(-1)^2*(RESID(-1)<0)	0.059628	0.005857	10.17979	0.0000
GARCH(-1)	0.900416	0.004126	218.2090	0.0000

R-squared	-0.001171	Mean dependent var	0.000288
Adjusted R-squared	-0.001625	S.D. dependent var	0.011285
S.E. of regression	0.011294	Akaike info criterion	-6.473043
Sum squared resid	1.403911	Schwarz criterion	-6.469062
Log likelihood	35649.81	Hannan-Quinn criter.	-6.471702
Durbin-Watson stat	2.098238		
Inverted AR Roots	.09		

Table 4: CAPM regression for a portfolio of small stocks

Dependent Variable: Excess Returns Small Stocks
Method: Least Squares

Sample: 1 930
Included observations: 930
Newey-West HAC Standard Errors & Covariance (lag truncation=6)

	Coefficient	Std. Error	t-Statistic	Prob.
C	3.098131	0.390841	7.926829	0.0000
MARKET_Prem	0.475236	0.154032	3.085315	0.0021

R-squared	0.082166	Mean dependent var	3.403269
Adjusted R-squared	0.081177	S.D. dependent var	9.122213
S.E. of regression	8.744121	Akaike info criterion	7.176788
Sum squared resid	70954.55	Schwarz criterion	7.187187
Log likelihood	-3335.207	Hannan-Quinn criter.	7.180754
F-statistic	83.07612	Durbin-Watson stat	1.638298
Prob(F-statistic)	0.000000		

Table 5: CAPM regression for a portfolio of large stocks

Dependent Variable: Excess Returns Large Stocks
Method: Least Squares

Sample: 1 930
Included observations: 930
Newey-West HAC Standard Errors & Covariance (lag truncation=6)

	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.006653	0.030813	-0.215928	0.8291
MARKET_Prem	0.931452	0.008248	112.9335	0.0000

R-squared	0.971986	Mean dependent var	0.591409
Adjusted R-squared	0.971956	S.D. dependent var	5.198367
S.E. of regression	0.870541	Akaike info criterion	2.562745
Sum squared resid	703.2778	Schwarz criterion	2.573144
Log likelihood	-1189.677	Hannan-Quinn criter.	2.566711
F-statistic	32198.13	Durbin-Watson stat	1.898276
Prob(F-statistic)	0.000000		

Table 6: Unrestricted estimation of a system of CAPM equations of 10 size portfolios.

System: CAPM
Estimation Method: Full Information Maximum Likelihood (Marquardt)
Date: 09/27/07 Time: 16:50
Sample: 1 930
Included observations: 930
Total system (balanced) observations 9300
Convergence achieved after 1 iteration

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	3.098131	0.579502	5.346199	0.0000
C(2)	0.475236	0.048920	9.714632	0.0000
C(3)	0.128042	0.276156	0.463659	0.6429
C(4)	1.398775	0.036840	37.96843	0.0000
C(5)	0.134543	0.189170	0.711225	0.4769
C(6)	1.321413	0.026586	49.70317	0.0000
C(7)	0.127856	0.169375	0.754868	0.4503
C(8)	1.254357	0.022951	54.65373	0.0000
C(9)	0.118123	0.130767	0.903304	0.3664
C(10)	1.247444	0.020006	62.35462	0.0000
C(11)	0.085135	0.099777	0.853252	0.3935
C(12)	1.208187	0.015021	80.43448	0.0000
C(13)	0.103212	0.083923	1.229846	0.2188
C(14)	1.160204	0.012292	94.38416	0.0000
C(15)	0.070406	0.060756	1.158830	0.2465
C(16)	1.113346	0.009967	111.6999	0.0000
C(17)	0.047316	0.046323	1.021446	0.3070
C(18)	1.073763	0.006944	154.6231	0.0000
C(19)	-0.006653	0.033737	-0.197215	0.8437
C(20)	0.931452	0.006908	134.8401	0.0000

Log likelihood	-16807.43	Schwarz criterion	36.29201
Avg. log likelihood	-1.807251	Hannan-Quinn criter.	36.22768
Akaike info criterion	36.18802		
Determinant residual covariance		2359.589	

Equation: ME1_P= C(1)+C(2)*MARKET_P
Equation: ME2_P= C(3)+C(4)*MARKET_P
Equation: ME3_P= C(5)+C(6)*MARKET_P
Equation: ME4_P= C(7)+C(8)*MARKET_P
Equation: ME5_P= C(9)+C(10)*MARKET_P
Equation: ME6_P= C(11)+C(12)*MARKET_P
Equation: ME7_P= C(13)+C(14)*MARKET_P
Equation: ME8_P= C(15)+C(16)*MARKET_P
Equation: ME9_P= C(17)+C(18)*MARKET_P
Equation: ME10_P=C(19)+C(20)*MARKET_P

Table 7: Restricted estimation of a restricted system of CAPM equations of 10 size portfolios.

Estimation Method: Full Information Maximum Likelihood (Marquardt)
Date: 09/27/07 Time: 16:56
Sample: 1 930
Included observations: 930
Total system (balanced) observations 9300
Convergence achieved after 25 iterations

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.540115	0.051418	10.50442	0.0000
C(2)	1.401478	0.036028	38.90004	0.0000
C(3)	1.324246	0.025916	51.09745	0.0000
C(4)	1.257044	0.022309	56.34798	0.0000
C(5)	1.249928	0.019500	64.09969	0.0000
C(6)	1.209976	0.014675	82.45285	0.0000
C(7)	1.162371	0.012113	95.96391	0.0000
C(8)	1.114822	0.009621	115.8790	0.0000
C(9)	1.074755	0.006863	156.6058	0.0000
C(10)	0.931309	0.006785	137.2539	0.0000

Log likelihood	-16875.91	Schwarz criterion	36.36578
Avg. log likelihood	-1.814614	Hannan-Quinn criter.	36.33362
Akaike info criterion	36.31379		
Determinant residual covariance		2733.984	

Equation: ME1_P= C(1)*MARKET_P
Equation: ME2_P= C(2)*MARKET_P
Equation: ME3_P= C(3)*MARKET_P
Equation: ME4_P= C(4)*MARKET_P
Equation: ME5_P= C(5)*MARKET_P
Equation: ME6_P= C(6)*MARKET_P
Equation: ME7_P= C(7)*MARKET_P
Equation: ME8_P= C(8)*MARKET_P
Equation: ME9_P= C(9)*MARKET_P
Equation: ME10_P=C(10)*MARKET_P