Localising temperature risk

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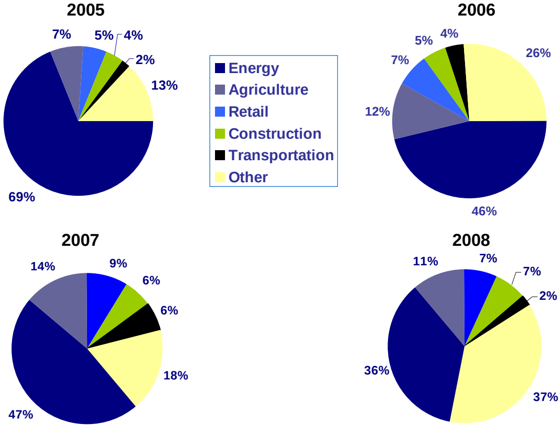
Weather

Top 5 sectors in need of financial instruments to hedge weather risk, PwC survey for WRMA:



Weather

- Influences our daily lives and choices
- Impact on corporate revenues and earnings
- Meteorological institutions: business activity is weather dependent
  - British Met Office: daily beer consumption gain 10% if temperature increases by 3° C
  - If temperature in Chicago is less than 0° C consumption of orange juice declines 10% on average
- Cash flow fluctuations, Perez-Gonzalez and Yun (2010)

Examples

- Natural gas company suffers negative impact in mild winter
- Construction companies buy weather derivatives (rain period)
- Cloth retailers sell fewer clothes in hot summer
- Salmon fishery suffer losses by increase of sea temperatures
- Ice cream producers hedge against cold summers
- Disney World (rain period)

What are Weather Derivatives (WD)?

Hedge weather related risk exposures

- Payments based on weather related measurements
- Underlying: temperature, rainfall, wind, snow, frost

Chicago Mercantile Exchange (CME)

- Monthly/seasonal/weekly temperature Futures/Options
- 24 US, 6 Canadian, 9 European, 3 Australian, 3 Asian cities
- From 2.2 billion USD in 2004 to 15 billion USD through March 2009

Weather Derivatives

Temperature CME products

- HDD$(\tau_1,\tau_2)=\int_{\tau_1}^{\tau_2}\max(18^{\circ}\mathrm{C}-T_t,0)dt$
- CDD$(\tau_1,\tau_2)=\int_{\tau_1}^{\tau_2}\max(T_t-18^{\circ}\mathrm{C},0)dt$
- CAT$(\tau_1,\tau_2)=\int_{\tau_1}^{\tau_2}T_t dt$, where $T_t=\frac{T_{t,max}+T_{t,min}}{2}$
- AAT$(\tau_1,\tau_2)=\int_{\tau_1}^{\tau_2}\widetilde{T}_t dt$, where $\widetilde{T}_t=\frac{1}{24}\int_1^{24}T_{t_i}dt_i$ and T_{t_i} denotes the temperature of hour t_i, (also referred to as C24AT index).

HDD-CDD parity:
CDD(τ_1,τ_2)-HDD(τ_1,τ_2)=CAT(τ_1,τ_2)-c$(\tau_2-\tau_1)$

Algorithm

Econometrics

$$T_t$$
$$\downarrow$$
$$X_t = T_t - \Lambda_t$$
$$\downarrow$$
$$X_t = \beta^\top X_{t-L} + \varepsilon_t, \varepsilon_t = \sigma_t e_t$$
$$\downarrow$$
$$e_t = \frac{\hat{X}_t}{\hat{\sigma}_t} \sim \mathrm{N}(0,1)$$

Fin. Mathematics.

$$CAR(p)$$
$$\downarrow$$
$$F_{CAT(t,\tau_1,\tau_2)} = \mathrm{E}^{Q_\lambda}\left[g(\Lambda_t, \mathbf{X}_t, \lambda_t)\right]$$
$$\downarrow$$
$$MPR : \lambda_t$$

- How to smooth the seasonal mean & variance curve?
- How close are the residuals to $N(0,1)$?
- How to infer the market price of weather risk?
- How to price no CME listed cities?

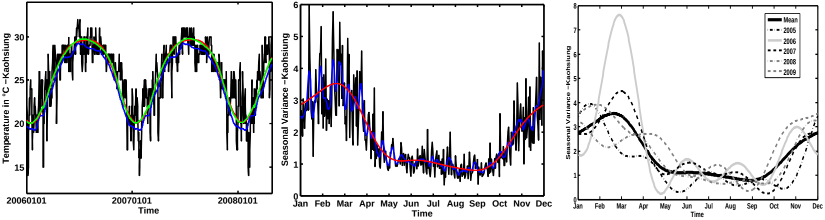


Figure 1: Kaohsiung daily average temperature, seasonal mean (left) & seasonal variation function (middle) with a Fourier truncated, the corrected Fourier and local linear Seasonal variation over years (right).

Outline

1. Motivation ✓
2. Weather Dynamics
3. Stochastic Pricing
4. Localising temperature risk
5. Conclusion

AAT Index

Can we make money?

WD type	Trading date	Measurement Period			Realised T_t
	t	τ_1	τ_2	CME[1]	$I^2_{(\tau_1,\tau_2)}$
Berlin-CAT	20070316	20070501	20070531	457.00	494.20
		20070601	20070630	529.00	574.30
		20070701	20070731	616.00	583.00
Tokyo-AAT	20081027	20090401	20090430	592.00	479.00
		20090501	20090531	682.00	623.00
		20090601	20090630	818.00	679.00

Table 1: Berlin and Tokyo contracts listed at CME. Source: Bloomberg. CME[1] WD Futures listed on CME, $I^2_{(\tau_1,\tau_2)}$ index values computed from the realized temperature data.

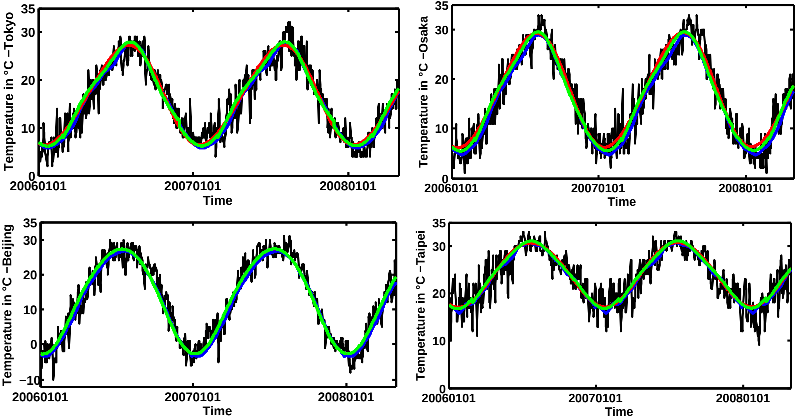


Figure 2: The Fourier truncated, the corrected Fourier and the the local linear seasonal component for daily average temperatures.

Go to details

AR(p): $X_t = \sum_{l=1}^{L} \beta_l X_{t-l} + \varepsilon_t, \varepsilon_t = \sigma_t e_t$

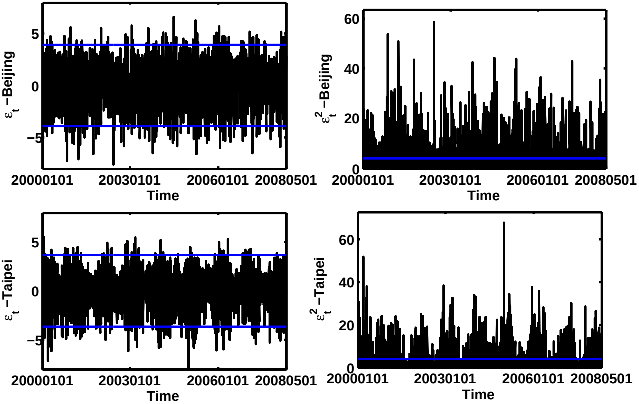


Figure 3: (square) Residuals $\hat{\varepsilon}_t$ (left), $\hat{\varepsilon}_t^2$ (right). No rejection of H_0 that residuals are uncorrelated at 0% significance level, (Li-McLeod Portmanteau test)

Go to details

ACF of (Squared) Residuals after Correcting Seasonal Volatility

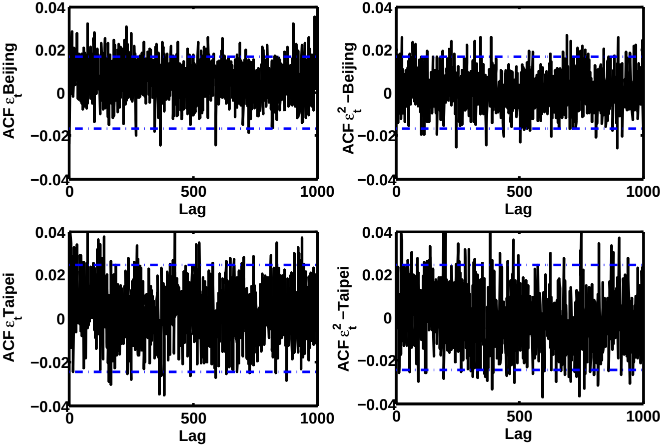


Figure 4: (Left) Right: ACF for temperature (squared) residuals $\frac{\hat{\varepsilon}_t}{\hat{\sigma}_{t,LLR}}$

Residuals $\left(\frac{\hat{\varepsilon}_t}{\hat{\sigma}_t}\right)$ become normal

City		JB	Kurt	Skew	KS	AD
Berlin	$\frac{\hat{\varepsilon}_t}{\hat{\sigma}_{t,FTSG}}$	304.77	3.54	-0.08	0.01	7.65
	$\frac{\hat{\varepsilon}_t}{\hat{\sigma}_{t,LLR}}$	279.06	3.52	-0.08	0.01	7.29
Kaohsiung	$\frac{\hat{\varepsilon}_t}{\hat{\sigma}_{t,FTSG}}$	2753.00	4.68	-0.71	0.06	79.93
	$\frac{\hat{\varepsilon}_t}{\hat{\sigma}_{t,LLR}}$	2252.50	4.52	-0.64	0.06	79.18
Tokyo	$\frac{\hat{\varepsilon}_t}{\hat{\sigma}_{t,FTSG}}$	133.26	3.44	-0.10	0.02	8.06
	$\frac{\hat{\varepsilon}_t}{\hat{\sigma}_{t,LLR}}$	148.08	3.44	-0.13	0.02	10.31

Table 2: Skewness, kurtosis, Jarque Bera (JB), Kolmogorov Smirnov (KS) and Anderson Darling (AD) test statistics (365 days). Critical values JB: 5%(5.99), 1%(9.21), KS: 5%(0.07), 1%(0.08), AD: 5%(2.49),1% (3.85)

Stochastic Pricing

The process $X_t = T_t - \Lambda_t$ can be seen as a discretization of a continuous-time process AR(L) (CAR(L)): Ornstein-Uhlenbeck process $\mathbf{X}_t \in \mathbb{R}^L$:

$$d\mathbf{X}_t = \mathbf{A}\mathbf{X}_t dt + \mathbf{e}_L \sigma_t dB_t$$

$\mathbf{e}_l$: lth unit vector in $\mathbb{R}^L$ for $l = 1, ...L$, $\sigma_t > 0$, $\mathbf{A}$: $(L \times L)$-matrix

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & \dots & \dots & 0 & 1 \\ -\alpha_L & -\alpha_{L-1} & \dots & & -\alpha_1 \end{pmatrix}$$

Go to proof details

CAT Futures

For $0 \leq t \leq \tau_1 < \tau_2$, the future Cumulative Average Temperature:

$$\begin{aligned} F_{CAT(t,\tau_1,\tau_2)} &= \mathsf{E}^{Q_\lambda}\left[\int_{\tau_1}^{\tau_2} T_s ds | \mathcal{F}_t\right] \\ &= \int_{\tau_1}^{\tau_2} \Lambda_u du + \mathbf{a}_{t,\tau_1,\tau_2}\mathbf{X}_t + \int_t^{\tau_1} \lambda_u \sigma_u \mathbf{a}_{t,\tau_1,\tau_2}\mathbf{e}_L du \\ &+ \int_{\tau_1}^{\tau_2} \lambda_u \sigma_u \mathbf{e}_1^\top \mathbf{A}^{-1}\left[\exp\left\{\mathbf{A}(\tau_2 - u)\right\} - I_L\right]\mathbf{e}_L du \end{aligned} \quad (1)$$

with $\mathbf{a}_{t,\tau_1,\tau_2} = \mathbf{e}_1^\top \mathbf{A}^{-1}\left[\exp\left\{\mathbf{A}(\tau_2 - t)\right\} - \exp\left\{\mathbf{A}(\tau_1 - t)\right\}\right]$, $I_L : L \times L$ identity matrix, λ_u MPR inferred from data, Benth et al. (2007), Härdle & López Cabrera (2011). Λ_u, σ_u to be localised.

Local Temperature Risk

Normality of ε_t requires estimating the function $\theta(t) = \{\Lambda_t, \sigma_t^2\}$ with $t = 1 \cdots 365$ days, $j = 0 \cdots J$ years. Recall:

$$
\begin{aligned}
X_{365j+t} &= T_{t,j} - \Lambda_t, \\
X_{365j+t} &= \sum_{l=1}^{L} \beta_{lj} X_{365j+t-l} + \varepsilon_{t,j}, \\
\varepsilon_{t,j} &= \sigma_t e_{t,j}, \\
e_{t,j} &\sim \mathrm{N}(0,1), i.i.d. \qquad (2)
\end{aligned}
$$

Adaptation Scale

Fix $s \in 1, 2, \ldots, 365$, sequence of ordered weights:
$W_s^{(k)} = (w(s,1,h_k), w(s,2,h_k), \ldots, w(s,365,h_k))^\top$.
Define $w(s,t,h_k) = K_{h_k}(s-t), (h_1 < h_2 < \ldots < h_K)$.
Local likelihood:

$$
\begin{aligned}
\hat{\varepsilon}_{365j+t} &= X_{365j+t} - \sum_{l=1}^{L} \hat{\beta}_l X_{365j+t-l} \\
\tilde{\theta}_k(s) &\stackrel{\text{def}}{=} \arg\min_{\theta\in\Theta} -L\{W^k(s),\theta\} \\
&= \arg\min_{\theta\in\Theta} \sum_{t=1}^{365}\sum_{j=0}^{J} \{\log(2\pi\theta)/2 + \hat{\varepsilon}_{t,j}^2/2\theta\} w(s,t,h_k),
\end{aligned}
$$

Construct the adaptive $\hat{\theta}_k(s)$ on the base of $\tilde{\theta}_k(s)$.

Local Temperature Residuals

$$\tilde{\theta}_k(s) = \sum_{t,j} \hat{\varepsilon}^2_{t,j} w(s,t,h_k) / \sum_{t,j} w(s,t,h_k)$$
$$= \sum_{t} \hat{\varepsilon}^2_{t} w(s,t,h_k) / \sum_{t} w(s,t,h_k)$$
$$\textit{with} \qquad \hat{\varepsilon}^2_t \stackrel{\text{def}}{=} (J+1)^{-1} \sum_{j=0}^{J} \hat{\varepsilon}^2_{t,j}.$$

To avoid the boundary problem, use mirrored observations: assume $h_K < 365/2$, then the observation set is extended to $\hat{\varepsilon}^2_{-364}, \hat{\varepsilon}^2_{-363}, \ldots, \hat{\varepsilon}^2_{0}, \hat{\varepsilon}^2_{1}, \ldots, \hat{\varepsilon}^2_{730}$, where

$$\hat{\varepsilon}^2_t \stackrel{\text{def}}{=} \hat{\varepsilon}^2_{365+t}, -364 \le t \le 0, \hat{\varepsilon}^2_t \stackrel{\text{def}}{=} \hat{\varepsilon}^2_{t-365}, 366 \le t \le 730$$

Parametric Exponential Bounds

$$\begin{aligned} L(W^k, \tilde{\theta}_k, \theta^*) &\stackrel{\text{def}}{=} N_k \mathcal{K}(W^k, \tilde{\theta}_k, \theta^*) \\ &= -\{\log(\tilde{\theta}_k/\theta^*) + 1 - \theta^*/\tilde{\theta}_k\}/2, \end{aligned} \quad (3)$$

where $\mathcal{K}\{W^k, \tilde{\theta}_k, \theta^*\}$ is the Kullback Leibler divergence between $\tilde{\theta}_k$ and θ^* and $N_k = \Sigma_{t=1}^{365} w(s,t,h_k) \times J$. For any $\mathfrak{z} > 0$,

$$\begin{aligned} \mathrm{P}_{\theta^*}\{L(W^k, \tilde{\theta}_k, \theta^*) > \mathfrak{z}\} &\leq 2\exp(-\mathfrak{z}) \\ \mathsf{E}_{\theta^*} L(W^k, \tilde{\theta}_k, \theta^*)^r &\leq \mathfrak{r}_r \end{aligned} \quad (4)$$

where $\mathfrak{r}_r = 2r \int_{\mathfrak{z} \geq 0} \mathfrak{z}^{r-1} \exp(-\mathfrak{z}) d\mathfrak{z}$.

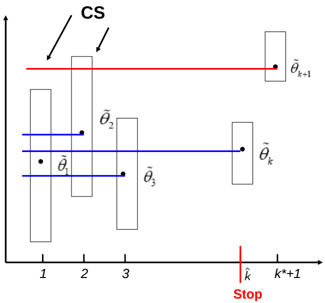
LMS Procedure

Construct an estimate $\hat{\theta} = \hat{\theta}(s)$, on the base of $\tilde{\theta}_1(s), \tilde{\theta}_2(s), \ldots, \tilde{\theta}_K(s)$.

- Start with $\hat{\theta}_1 = \tilde{\theta}_1$.
- For $k \geq 2$, $\tilde{\theta}_k$ is accepted and $\hat{\theta}_k = \tilde{\theta}_k$ if $\tilde{\theta}_{k-1}$ was accepted and

$$L(W^k, \tilde{\theta}_\ell, \tilde{\theta}_k) \leq \mathfrak{z}_\ell, \ell = 1, \ldots, k-1$$

  $\hat{\theta}_k$ is the the latest accepted estimate after the first k steps.



Propagation Condition

A bound for the risk associated with first kind error:

$$\mathsf{E}_{\theta^*} \frac{|L(W^k, \tilde{\theta}_k, \hat{\theta}_k)|^r}{\mathfrak{r}_r} \leq \alpha \tag{5}$$

where $k = 1, \ldots, K$ and $\mathfrak{r}_r$ is the parametric risk bound.

Sequential Choice of Critical Values

- Consider first only $\mathfrak{z}_1$ letting $\mathfrak{z}_2 = \ldots = \mathfrak{z}_{K-1} = \infty$. Leads to the estimates $\hat{\theta}_k(\mathfrak{z}_1)$ for $k = 2, \ldots, K$.
- The value $\mathfrak{z}_1$ is selected as the minimal one for which

$$\sup_{\theta^*} \mathsf{E}_{\theta^*} \frac{|L\{W^k, \tilde{\theta}_k, \hat{\theta}_k(\mathfrak{z}_1)\}|^r}{\mathfrak{r}_r} \leq \frac{\alpha}{K-1}, k = 2, \ldots, K.$$

- Set $\mathfrak{z}_{k+1} = \ldots = \mathfrak{z}_{K-1} = \infty$ and fix $\mathfrak{z}_k$ lead the set of parameters $\mathfrak{z}_1, \ldots, \mathfrak{z}_k, \infty, \ldots, \infty$ and the estimates $\hat{\theta}_m(\mathfrak{z}_1, \ldots, \mathfrak{z}_k)$ for $m = k+1, \ldots, K$. Select $\mathfrak{z}_k$ s.t.

$$\sup_{\theta^*} \mathsf{E}_{\theta^*} \frac{|L\{W^k, \tilde{\theta}_m, \hat{\theta}_m(\mathfrak{z}_1, \mathfrak{z}_2, \ldots, \mathfrak{z}_k)\}|^r}{\mathfrak{r}_r} \leq \frac{k\alpha}{K-1},$$

$m = k+1, \ldots, K$.

Critical Values

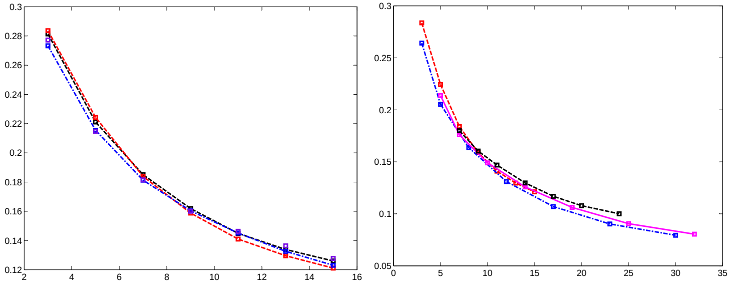


Figure 5: Simulated CV with $\theta^* = 1$, $r = 0.5$, $MC = 5000$ with $\alpha = 0.3$, 0.5, 0.8 (left), with different bandwidth sequences (right).

Bound for Critical Values

Suppose $0 < \mu \leq h_{k-1}/h_k \leq \mu_0 < 1$.

Let $\theta(.) = \theta^*$, for all $t \in (0, 365)$. There is a constant $a_0 > 0$ depending on r and μ_0, μ, s.t.

$$\mathfrak{z}_k = a_0 \log K + 2 \log(nh_k/\alpha) + 2r \log(h_K/h_k)$$

ensures the propagation condition,

Small Modeling Bias (SMB) Condition

$$\begin{aligned}\Delta(W^k,\theta) &= \sum_{t=1}^{365} \mathcal{K}\{W^k,\theta(t),\theta\}\mathbf{1}\{w(s,t,h_k)>0\} \\ &\leq \Delta, \forall k < k^* \end{aligned} \quad (6)$$

k^* is the maximum k satisfying the SMB condition.

Propagation Property:
For any estimate $\tilde{\theta}_k$ and θ satisfying SMB, it holds:

$$\mathsf{E}_{\theta(.)} \log\{1 + |L(W^k,\tilde{\theta}_k,\theta)|^r/\mathfrak{r}_r\} \leq \Delta + 1$$

Stability Property

The attained quality of estimation during "propagation" can not get lost at further steps.

$$L(W^{k^*}, \tilde{\theta}_{k^*}, \hat{\theta}_{\hat{k}}) \mathbf{1}\{\hat{k} > k^*\} \leq \mathfrak{z}_{k^*}$$

$\hat{\theta}_{\hat{k}}$ delivers at least the same accuracy of estimation as the "oracle" $\tilde{\theta}_{k^*}$

Oracle Property

Theorem

Let $\Delta(W^k, \theta) \leq \Delta$ *for some* $\theta \in \Theta$ *and* $k \leq k^*$. *Then*

$$
\begin{aligned}
E_{\theta(.)} \log \left\{ 1 + \frac{|L(W^{k^*}, \tilde{\theta}_{k^*}, \theta)|^r}{\mathfrak{r}_r} \right\} &\leq \Delta + 1 \\
E_{\theta(.)} \log \left\{ 1 + \frac{|L(W^{k^*}, \tilde{\theta}_{k^*}, \hat{\theta})|^r}{\mathfrak{r}_r} \right\} &\leq \Delta + \alpha + \log\{1 + \frac{\mathfrak{z}_{k^*}}{\mathfrak{r}_r}\}
\end{aligned}
$$

with $\hat{\theta} = \hat{\theta}_{\hat{k}}$.

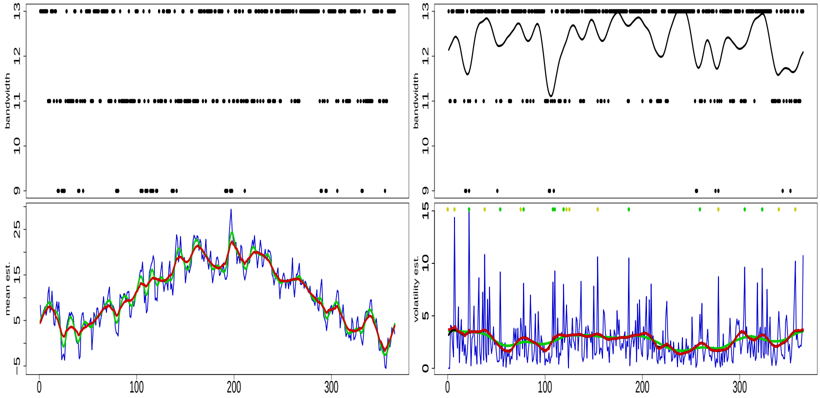


Figure 6: Estimation of mean 2007 (left) and variance 20020101-20071231 (right) for Berlin. Bandwidths sequences (upper), fixed bandwidth curve; adaptive bandwidth curve; smoothed adaptive bandwidth for squared residuals with dots for negative and positive outliers (bottom panel), $\alpha = 0.3$, $r = 0.5$

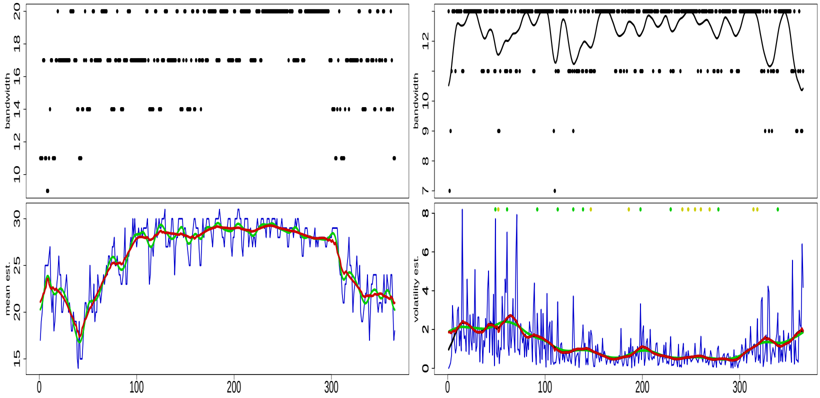


Figure 7: Estimation of mean 2008 (left) and variance 20030101-20081231 (right) for Kaohsiung. Bandwidths sequences (upper), fixed bandwidth curve; adaptive bandwidth curve; smoothed adaptive bandwidth for squared residuals with dots for negative and positive outliers (bottom panel), $\alpha = 0.3$, $r = 0.5$

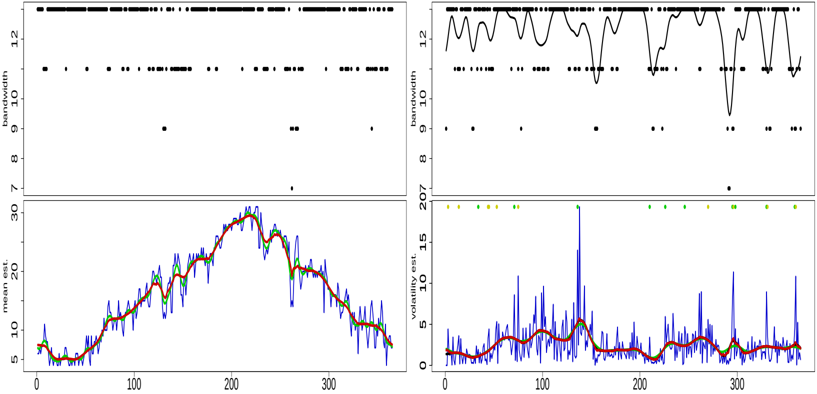
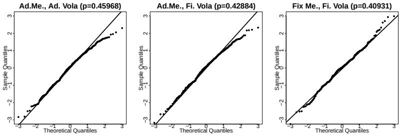
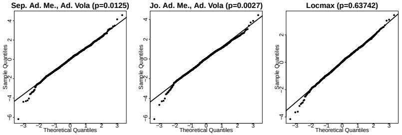


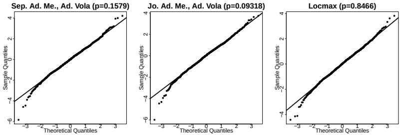
Figure 8: Estimation of mean 2008 (left) and variance 20030101-20081231 (right) for Tokyo. Bandwidths sequences (upper), fixed bandwidth curve; adaptive bandwidth curve; smoothed adaptive bandwidth for squared residuals with dots for negative and positive outliers (bottom panel), $\alpha = 0.3$, $r = 0.5$



(a) QQ-plot Berlin 1 year (2007)



(b) QQ-plot Berlin 3 years (2004-2007)



(c) QQ-plot Berlin 5 years (2002-2007)

Normalized Residuals

city	year	bandwidth	KS	JB	AD
Berlin	2007	FB	0.4288	0.0387	0.0215
	2007	AB	0.4596	0.0185	0.0076
	2007	ABS	0.4818	0.0185	0.0076
Kaohsiung	2008	FB	0.0533	7.9e-07	2.4e-06
	2008	AB	0.0540	8.3e-05	7.1e-06
	2008	ABS	0.0543	8.3e-05	7.1e-06
Tokyo	2008	FB	0.3299	0.1130	0.0229
	2008	AB	0.5743	0.1327	0.0470
	2008	ABS	0.5544	0.1327	0.0470

Table 3: P-value of normality tests for fixed bandwidth curve (FB), adaptive bandwidth curve (AB), adaptive smoothed bandwidth curve (ABS).

Iterative approach, $\theta(t) = \{\Lambda_t, \sigma_t^2\}$

Step 1. Estimate $\hat{\beta}$ in an initial Λ_t^0 using a truncated Fourier series or any other deterministic function;

Step 2. For fixed $\hat{\Lambda}_{s,\nu} = \{\hat{\Lambda}'_{s,\nu}, \hat{\Lambda}''_{s,\nu}\}^\top, s = \{1, \ldots, 365\}$ from last step ν, and fixed $\hat{\beta}$, get $\hat{\sigma}^2_{s,\nu+1}$ by

$$\begin{aligned} \hat{\sigma}^2_{s,\nu+1} &= \arg\min_{\sigma^2} \sum_{t=1}^{365} \sum_{j=0}^{J} [\{ T_{365j+t} - \hat{\Lambda}'_{s,\nu} - \hat{\Lambda}''_{s,\nu}(t-s) \\ &- \sum_{l=1}^{L} \hat{\beta}_l X_{365j+t-l}\}^2 / 2\sigma^2 + \log(2\pi\sigma^2)/2] w(s,t,h'_k); \end{aligned}$$

Iterative approach

Step 3. For fixed $\hat{\sigma}^2_{s,\nu+1}$ and $\hat{\beta}$, we estimate $\hat{\Lambda}_{s,\nu+1}, s = \{1, \dots, 365\}$ again by a local adaptive procedure:

$$\hat{\Lambda}_{s,\nu+1} = \arg\min_{\{\Lambda', \Lambda''\}^\top} \sum_{t=1}^{365} \sum_{j=0}^{J} \Big\{ T_{365j+t} - \Lambda' - \Lambda''(t-s) - \sum_{l=1}^{L} \hat{\beta}_l X_{365j+t-l} \Big\}^2 w(s,t,h'_k)/2\hat{\sigma}^2_{s,\nu+1},$$

where $\{h'_1, h'_2, h'_3, \dots, h'_{K'}\}$ is a sequence of bandwidths;

Step 4. Repeat steps 2 and 3 till both $|\hat{\Lambda}_{t,\nu+1} - \hat{\Lambda}_{t,\nu}| < \pi_1$ and $|\hat{\sigma}^2_{t,\nu+1} - \hat{\sigma}^2_{t,\nu}| < \pi_2$ for some constants π_1 and π_2.

Aggregated approach

Let $\hat{\theta}^j(t)$ the localised observation at time t of year j, the aggregated local function is given by:

$$\hat{\theta}_\omega(t) = \sum_{j=1}^{J} \omega_j \hat{\theta}^j(t) \tag{7}$$

$\arg\min_{\omega} \sum_{j=1}^{J} \sum_{t=1}^{365} \{\hat{\theta}_\omega(t) - \hat{\theta}_j^o(t)\}^2$ s.t. $\sum_{j=1}^{J} \omega_j = 1; \omega_j > 0$, $\hat{\theta}_j^o$ defined as:

1. (Locave), $\hat{\theta}_j^o(t) = J^{-1} \sum_{j=1}^{J} \hat{\sigma}_j^2(t)$
2. (Locsep) $\hat{\theta}_j^o(t) = \hat{\sigma}_j^2(t)$
3. (Locmax) one of above two approaches with maximised p-values of KS-test over year.

city		2 years			3 years			5 years		
		KS	JB	AD	KS	JB	AD	KS	JB	AD
Berlin	AB	6.9e-05	0.0002	0.3148	0.0124	1.0e-10	0.0531	0.1578	2.6e-14	0.0178
	FB	4.1e-05	8.3e-08	0.0758	0.0123	5.4e-15	0.0232	0.1416	9.3e-13	0.0189
	Locave	0.7775	0.0413	0.3250	0.5355	0.0005	0.1515	0.7566	1.0e-06	0.0504
	Locsep	0.7775	0.0413	0.3250	0.5355	0.0005	0.1515	0.7801	9.3e-07	0.0532
	Locmax	0.8304	0.0919	0.2946	0.6374	0.0001	0.1329	0.8465	4.8e-08	0.0400
Kaohsiung	AB	5.1e-09	0.0000	2.5e-08	5.1e-05	0.0000	1.0e-11	0.0020	0.0000	1.4e-17
	FB	2.1e-09	1.0e-14	4.3e-10	6.4e-05	0.0000	2.8e-12	0.0019	0.0000	8.6e-18
	Locave	0.0165	1.6e-08	3.0e-09	0.0131	1.1e-14	1.8e-11	0.0004	0.0000	1.0e-16
	Locsep	0.0165	1.6e-08	3.0e-09	0.0131	1.1e-14	1.8e-11	0.0003	0.0000	1.0e-16
	Locmax	0.0302	1.5e-08	2.9e-09	0.0163	6.7e-15	1.1e-11	0.0046	0.0000	9.2e-17
Tokyo	AB	2.3e-05	5.0e-07	0.0005	0.0471	5.6e-09	8.9e-05	0.3357	7.6e-13	9.5e-06
	FB	9.2e-05	7.2e-13	2.2e-05	0.0045	5.5e-10	0.0001	0.2642	4.6e-13	1.0e-05
	Locave	0.1075	0.0271	0.0026	0.1065	0.0002	0.0006	0.0620	9.2e-07	5.5e-05
	Locsep	0.1075	0.0271	0.0026	0.1065	0.0002	0.0006	0.0619	9.1e-07	5.5e-05
	Locmax	0.4641	0.0269	0.0029	0.4898	0.0003	0.0009	0.4086	3.1e-07	5.9e-05

Table 4: *P*-values of normality tests for fixed bandwidth curve (FB), adaptive bandwidth curve (AB) and aggregated approaches for Berlin (20020101-20071231), Kaohsiung (20030101-20081231), Tokyo (20030101-20081231).

Can we make money?

Trading date	MP		Future Prices $F_{(t,\tau_1,\tau_2,\lambda_t,\hat{\theta})}$					Real. T_t	
t	τ_1	τ_2	CME	$\lambda_t = 0$	$\lambda_t = \lambda$ Locave	$\lambda_t = \lambda$ Locsep	$\lambda_t = \lambda$ Locmax	$I_{(\tau_1,\tau_2)}$	Strategy
Berlin-CAT									
20070316	20070501	20070531	457.00	502.11	454.91	454.22	455.38	494.20	-45.11(P)
20070316	20070601	20070630	529.00	571.78	634.76	634.05	633.76	574.30	-42.78(P)
20070316	20070701	20070731	616.00	591.56	686.76	683.69	684.31	583.00	24.44(C)
Tokyo-AAT									
20081027	20090401	20090430	592.00	483.18	533.27	533.26	554.19	479.00	108.82(C)
20081027	20090501	20090531	682.00	511.07	696.31	696.32	684.99	623.00	170.93(C)
20081027	20090601	20090630	818.00	628.24	835.50	835.51	843.42	679.00	189.76(C)

Table 5: Weather contracts listed at CME. (Source: Bloomberg). $\hat{F}_{t,\tau_1,\tau_2,\lambda,\theta}$ estimated prices with MPR (λ_t) under different localisation schemes ($\hat{\theta}$ under Locave, Locsep, Locmax for Berlin (20020101-20061231), Tokyo (20030101-20081231)), Strategy (CME- $\hat{F}_{t,\tau_1,\tau_2,\lambda=0}$), P(Put), C(Call), MP(Measurement Period)

Strategy

Type	Trading date	MP		Real. T_t			
	t	τ_1	τ_2	CME	$I_{(\tau_1,\tau_2)}$	Difference	Payoff
Berlin-CAT	20070316	20070501	20070531	457.00	494.20	37.20	744EUR
Berlin-CAT	20070316	20070601	20070630	529.00	574.30	45.30	906EUR
Berlin-CAT	20070316	20070701	20070731	616.00	583.00	33.00	606EUR
Tokyo-AAT	20081027	20090401	20090430	592.00	479.00	113.00	28,2500YEN
Tokyo-AAT	20081027	20090501	20090531	682.00	623.00	59.00	147,500YEN
Tokyo-AAT	20081027	20090601	20090630	818.00	679.00	139.00	347,500YEN

Table 6: Weather contracts listed at CME. $I_{(\tau_1,\tau_2)}$ realized temperatures. 1 CAT index point (= 20 EUR per contract), 0.01 AAT index point (= 25 YEN per contract). P(Put), C(Call), MP(Measurement Period)

Conclusions and further work

- Temperature risk stochastics closer to Wiener process when applying adaptive methods
- Better estimates of Λ_t and σ_t lead to fair price -> pure MPR

References

F.E Benth and W. K. Härdle and B. López Cabrera (2011)
Pricing of Asian Temperature Risk
to appear in Statistical Tools for Finance and Insurance (Cizek, Härdle and Weron, eds.), Springer Verlag Heidelberg

W. K. Härdle and B. López Cabrera
Implied market price of weather risk
Invitated resubmission to J. Applied Mathematical Finance, 20101002

F.E Benth and J.S. Benth and S. Koekebakker (2007)
Putting a price on temperature
Scandinavian Journal of Statistics 34: 746-767

References

F.E Benth, W.K. Härdle, B. López Cabrera (2010)
On the sign of the risk premium and the market price of weather risk
Working Paper

P.J. Brockwell
Continuous time ARMA Process
Handbook of Statistics 19: 248-276, 2001

F. Perez-Gonzalez and H.
Risk Management and Firm Value: Evidence from Weather Derivatives
Available at SSRN: http://ssrn.com/abstract=1357385

Localising temperature risk

温度风险

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Appendix A

Li-McLeod Portmanteau Test– modified Portmanteau test statistic Q_L to check the uncorrelatedness of the residuals:

$$Q_L = n \sum_{k=1}^{L} r_k^2(\hat{\varepsilon}) + \frac{L(L+1)}{2n},$$

where r_k, $k = 1, \dots, L$ are values of residuals ACF up to the first L lags and n is the sample size. Then,

$$Q_L \sim \chi^2_{(L-p-q)}$$

Q_L is χ^2distributed on $(L - p - q)$ degrees of freedom where p,q denote AR and MA order respectively and L is a given value of considered lags.

Appendix

Consider 2 prob. measures P & Q. Assume that $\frac{dQ}{dP}|_{\mathcal{F}_t} = Z_t > 0$ is a positive Martingale. By *Ito's* Lemma, then:

$$\begin{aligned} Z_t &= \exp\{\log(Z_t)\} \\ &= \exp\left\{\int_0^t (Z_s)^{-1} dZ_s - \frac{1}{2}\int_0^t (Z_s)^{-2} d < Z, Z >_s\right\} \end{aligned} \quad (8)$$

Let $dZ_s = Z_s \cdot \theta_s \cdot dB_s$, then:

$$Z_t = \exp\left(\int_0^t \theta_s dB_s - \frac{1}{2}\int_0^t \theta_s^2 ds\right) \quad (9)$$

Appendix B

Let B_t, Z_t be Martingales under P, then by Girsanov theorem:

$$
\begin{aligned}
B_t^{\theta} &= B_t - \int_0^t (Z_s)^{-1} d < Z, B >_s \\
&= B_t - \int_0^t (Z_s)^{-1} d < \int_0^s \theta_u Z_u dB_u, B_s > \\
&= B_t - \int_0^t (Z_s)^{-1} \theta_s Z_s d < B_s, B_s > \\
&= B_t - \int_0^t \theta_s ds \qquad (10)
\end{aligned}
$$

is a Martingale unter Q.

Black-Scholes Model

Asset price follows:

$$dS_t = \mu S_t dt + \sigma_t S_t dB_t$$

Note that S_t is not a Martingale unter P, but it is under Q!

Explicit dynamics:

$$\begin{aligned} S_t &= S_0 + \int_0^t \mu S_s ds + \int_0^t \sigma_s S_s dB_s \\ &= S_0 + \int_0^t \mu S_s ds + \int_0^t \sigma_s S_s dB_s^\theta + \int_0^t \theta_s \sigma_s S_s ds \\ &= S_0 + \int_0^t S_s(\mu + \theta_s \sigma_s) ds + \int_0^t \sigma_s S_s dB_s^\theta \end{aligned} \tag{11}$$

Market price of Risk and Risk Premium

By the no arbitrage condition, the risk free interest rate r should be equal to the drift $\mu + \theta_s \sigma_s$, so that:

$$\theta_s = \frac{r - \mu}{\sigma_s} \tag{12}$$

In practice:
$B_t^\theta = B_t - \int_0^t \left(\frac{\mu - r}{\sigma_s}\right) ds$ is a Martingale under Q and then $e^{-rt} S_t$ is also a Martingale.

Under risk taking, the risk premium is defined as:

$$r + \Delta$$

$\mathbf{X}_t$ can be written as a Continuous-time AR(p) (CAR(p)):

For $p = 1$,

$$dX_{1t} = -\alpha_1 X_{1t} dt + \sigma_t dB_t$$

For $p = 2$,

$$\begin{aligned} X_{1(t+2)} &\approx (2 - \alpha_1) X_{1(t+1)} \\ &+ (\alpha_1 - \alpha_2 - 1) X_{1t} + \sigma_t (B_{t-1} - B_t) \end{aligned}$$

For $p = 3$,

$$\begin{aligned} X_{1(t+3)} &\approx (3 - \alpha_1) X_{1(t+2)} + (2\alpha_1 - \alpha_2 - 3) X_{1(t+1)} \\ &+ (-\alpha_1 + \alpha_2 - \alpha_3 + 1) X_{1t} + \sigma_t (B_{t-1} - B_t) \end{aligned}$$

Proof $CAR(3) \approx AR(3)$

Let

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\alpha_3 & -\alpha_2 & -\alpha_1 \end{pmatrix}$$

- use $B_{t+1} - B_t = \varepsilon_t$
- assume a time step of length one $dt = 1$
- substitute iteratively into X_1 dynamics

Proof $CAR(3) \approx AR(3)$:

$$
\begin{aligned}
X_{1(t+1)} - X_{1(t)} &= X_{2(t)}dt \\
X_{2(t+1)} - X_{2(t)} &= X_{3(t)}dt \\
X_{3(t+1)} - X_{3(t)} &= -\alpha_1 X_{1(t)}dt - \alpha_2 X_{2(t)}dt - \alpha_3 X_{3(t)}dt + \sigma_t \varepsilon_t \\
X_{1(t+2)} - X_{1(t+1)} &= X_{2(t+1)}dt \\
X_{2(t+2)} - X_{2(t+1)} &= X_{3(t+1)}dt \\
X_{3(t+2)} - X_{3(t+1)} &= -\alpha_1 X_{1(t+1)}dt - \alpha_2 X_{2(t+1)}dt \\
&\quad -\alpha_3 X_{3(t+1)}dt + \sigma_{t+1}\varepsilon_{t+1} \\
X_{1(t+3)} - X_{1(t+2)} &= X_{2(t+2)}dt \\
X_{2(t+3)} - X_{2(t+2)} &= X_{3(t+2)}dt \\
X_{3(t+3)} - X_{3(t+2)} &= -\alpha_1 X_{1(t+2)}dt - \alpha_2 X_{2(t+2)}dt \\
&\quad -\alpha_3 X_{3(t+2)}dt + \sigma_{t+2}\varepsilon_{t+2}
\end{aligned}
$$

return

Temperature: $T_t = X_t + \Lambda_t$ Seasonal function with trend:

$$\hat{\Lambda}_t = a + bt + \sum_{l=1}^{L} \hat{c}_l \cdot \cos\left\{\frac{2\pi l(t - \hat{d}_i)}{l \cdot 365}\right\}$$
$$+ \mathcal{I}(t \in \omega) \cdot \sum_{i=l}^{p} \hat{c}_i \cdot \cos\left\{\frac{2\pi(i-4)(t - \hat{d}_i)}{i \cdot 365}\right\} \qquad (13)$$

$\hat{a}$: average temperature, $\hat{b}$: global Warming. $\mathcal{I}(t \in \omega)$ an indicator for Dec., Jan. and Feb

City	Period	$\hat{a}$	$\hat{b}$	$\hat{c}_1$	$\hat{d}_1$
Tokyo	19730101-20081231	15.76	7.82e-05	10.35	-149.53
Osaka	19730101-20081231	15.54	1.28e-04	11.50	-150.54
Beijing	19730101-20081231	11.97	1.18e-04	14.91	-165.51
Taipei	19920101-20090806	23.21	1.68e-03	6.78	-154.02

Table 7: Seasonality estimates of daily average temperatures in Asia. All coefficients are nonzero at 1% significance level. Data source: Bloomberg

City(Period)	$\hat{a}$	$\hat{b}$	$\hat{c}_1$	$\hat{d}_1$	$\hat{c}_2$	$\hat{d}_2$	$\hat{c}_3$	$\hat{d}_3$
Tokyo								
(730101-081231)	15.7415	0.0001	8.9171	-162.3055	-2.5521	-7.8982	-0.7155	-15.0956
(730101-821231)	15.8109	0.0001	9.2855	-162.6268	-1.9157	-16.4305	-0.5907	-13.4789
(830101-921231)	15.4391	0.0004	9.4022	-162.5191	-2.0254	-4.8526	-0.8139	-19.4540
(930101-021231)	16.4284	0.0001	8.8176	-162.2136	-2.1893	-17.7745	-0.7846	-22.2583
(030101-081231)	16.4567	0.0001	8.5504	-162.0298	-2.3157	-18.3324	-0.6843	-16.5381
Taipei								
(920101-081231)	23.2176	0.0002	1.9631	-164.3980	-4.8706	-58.6301	-0.2720	39.1141
(920101-011231)	23.1664	0.0002	3.8249	-150.6678	-2.8830	-68.2588	0.2956	-41.7035
(010101-081231)	24.1295	-0.0001	1.8507	-149.1935	-5.1123	-67.5773	-0.3150	22.2777
Osaka								
(730101-081231)	15.2335	0.0002	10.0908	-162.3713	-2.5653	-7.5691	-0.6510	-19.4638
(730101-821231)	15.9515	-0.0001	9.7442	-162.5119	-2.1081	-17.9337	-0.5307	-18.9390
(830101-921231)	15.7093	0.0003	10.1021	-162.4248	-2.1532	-10.7612	-0.7994	-24.9429
(930101-021231)	16.1309	0.0003	10.3051	-162.4181	-2.0813	-21.9060	-0.7437	-27.1593
(030101-081231)	16.9726	0.0002	10.5863	-162.4215	-2.1401	-14.3879	-0.8138	-17.0385
Kaohsiung								
(730101-081231)	24.2289	0.0001	0.9157	-145.6337	-4.0603	-78.1426	-1.0505	10.6041
(730101-821231)	24.4413	0.0001	2.1112	-129.1218	-3.3887	-91.1782	-0.8733	20.0342
(830101-921231)	25.0616	0.0003	2.0181	-135.0527	-2.8400	-89.3952	-1.0128	20.4010
(930101-021231)	25.3227	0.0003	3.9154	-165.7407	-0.7405	-51.4230	-1.1056	19.7340
Beijing								
(730101-081231)	11.8904	0.0001	14.9504	-165.2552	0.0787	-12.8697	-1.2707	4.2333
(730101-821231)	11.5074	0.0003	14.8772	-165.7679	0.6253	15.8090	-1.2349	1.8530
(830101-921231)	12.4606	0.0002	14.9616	-165.7041	0.5327	14.3488	-1.2630	4.8809
(930101-021231)	13.6641	-0.0003	14.8970	-166.1435	0.9412	16.9291	-1.1874	-4.5596
(030101-081231)	12.8731	0.0003	14.9057	-165.9098	0.7266	16.5906	-1.5323	1.8984

Table 8: Seasonality estimates $\hat{\lambda}_t$ of daily average temperatures in Asia. All coefficients are nonzero at 1% significance level. Data source: Bloomberg.

▸ return

AR(p) $\to$ CAR(p)

City	ADF	KPSS	AR(3)			CAR(3)				
	$\hat{\tau}$	$\hat{k}$	β_1	β_2	β_3	α_1	α_2	α_3	$\tilde{\lambda}_1$	$\tilde{\lambda}_{2,3}$
Portland	-45.13+	0.05*	0.86	-0.22	0.08	2.13	1.48	0.26	-0.27	-0.93
Atlanta	-55.55+	0.21***	0.96	-0.38	0.13	2.03	1.46	0.28	-0.30	-0.86
New York	-56.88+	0.08*	0.76	-0.23	0.11	2.23	1.69	0.34	-0.32	-0.95
Houston	-38.17+	0.05*	0.90	-0.39	0.15	2.09	1.57	0.33	-0.33	-0.87
Berlin	-40.94+	0.13**	0.91	-0.20	0.07	2.08	1.37	0.20	-0.21	-0.93
Essen	-23.87+	0.11*	0.93	-0.21	0.11	2.06	1.34	0.16	-0.16	-0.95
Tokyo	-25.93+	0.06*	0.64	-0.07	0.06	2.35	1.79	0.37	-0.33	-1.01
Osaka	-18.65+	0.09*	0.73	-0.14	0.06	2.26	1.68	0.34	-0.33	-0.96
Beijing	-30.75+	0.16***	0.72	-0.07	0.05	2.27	1.63	0.29	-0.27	-1.00
Kaohsiung	-37.96+	0.05*	0.73	-0.08	0.04	2.26	1.60	0.29	-0.45	-0.92
Taipei	-32.82+	0.09*	0.79	-0.22	0.06	2.20	1.63	0.36	-0.40	-0.90

Table 9: ADF and KPSS-Statistics, coefficients of $AR(3)$, $CAR(3)$ and eigenvalues $\lambda_{1,2,3}$, for the daily average temperatures time series. + 0.01 critical values, * 0.1 critical value, **0.05 critical value (0.14), ***0.01 critical value. Historical data: 19470101-20091210.

return to AR