Downside risk and stock returns: An empirical analysis of the long-run and short-run dynamics from the G-7 Countries

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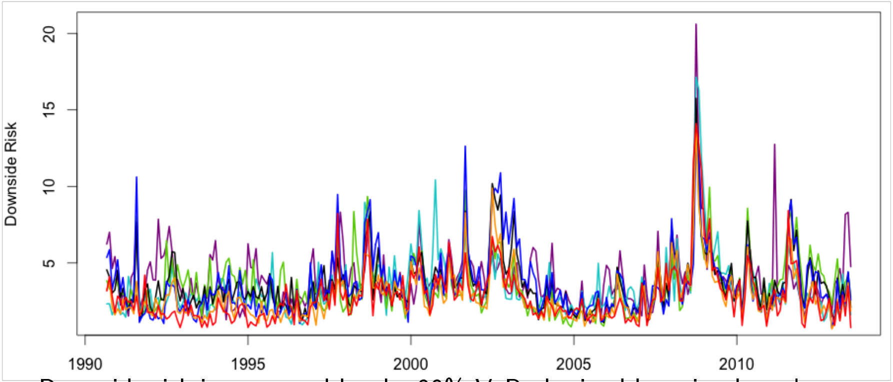
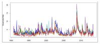
Figures/CHUlog

Time variations of downside risk in G7

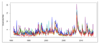


Downside risk is measured by the 99% VaR obtained by using kernel smoothing for the empirical distribution and then bootstrapping from the kernel density estimator. The kernel density has been applied in this figure.



Research questions for downside risk and returns

- Why VaR?
- The outbreak of the 2008 worldwide financial crisis
- What is the information content of VaR?
- VaR vs. higher moments of return of cdf (Cornish-Fisher expansion)
- Long memory of volatility and VaR
- Cross market effects of volatility/VaR
- Tradeoff hypothesis supported?
- Leverage effect/volatility feedback supported?
- Long-run or Short-run effect?
- Price discovery of US VaR vs. domestic VaR



VaR as a representative risk measure

VaR and higher moments

- The fourth-order Cornish-Fisher approximation at α%-quantile q_α

$$q_\alpha = z_\alpha + (z_\alpha^2 - 1)\frac{S}{6} + (z_\alpha^3 - 3z_\alpha)\frac{K}{24} - (2z_\alpha^3 - 5z_\alpha)\frac{S^2}{36}$$

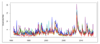
  where z_α is the α-quantile value from the standard normal distribution, and S and K are skewness and excess kurtosis, respectively

- VaR at the confidence level (1 - α)

$$\begin{aligned} V_{1-\alpha} &= -\sigma q_\alpha \\ &= -\sigma \left(z_\alpha + \left(z_\alpha^2 - 1\right) \frac{S}{6} + \left(2z_\alpha^3 - 3z_\alpha\right) \frac{K}{24} - \left(2z_\alpha^3 - 5z_\alpha\right) \frac{S^2}{36} \right) \end{aligned}$$

Evidence of long memory of second and higher moments

- Stock return variance are persistent - Ding, Engle and Granger (1993) discovered a long-memory property in stock market return.
- Second moment of conditional variance series displays a persistent phenomenon - Jondeau and Rockinger (2003), Bandi and Perron (2006), and Bollerslev et al. (2013)
- The downside risk series exhibits a long memory - Caporin (2008) and Kinateder and Wagner (2014)

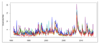


Downside risk-return in a global setting

- Is U.S. market a main driving force for the G7 dynamic risk-return relation?
- Variance and higher moments tend to co-move, whereas univariate market approach is limited.
- In the long run, a cointegration relation appears w.r.t higher moments/VaR

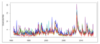
Outlines

1. Motivation ✓
2. Data and estimate Value-at-Risk (VaR)
3. FCVAR model
4. Empirical results (long-run vs. short-run)
5. Information content of VaR
6. Price discovery (US vs. G6?)
7. Conclusion



2. Data and estimating downside risk

- Stock price index: dividend-adjusted stock index in *Datastream* labeled as *TOTMK*
- Countries: The United Kingdom (UK), Germany (GM), France (FR), Italy (IT), the United States (US), Canada (CA), and Japan (JP)
- Sample period: September 1990 through July 2013
- Frequency: Use daily data to construct monthly VaR
- Value at Risk (VaR): nonparametric density estimation by kernel smoothing the empirical distribution
  - Apply Gaussian kernel $K_h = exp(-u^2/2)/\sqrt{2\pi}$
  - Estimate an integrated kernel density estimator



3. The fractionally cointegrated vector autoregression model

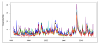
The FCVAR$_d$(p)

$$\Delta^d z_t = \alpha(\mu' + \beta' L_d z_t) + \sum_{s=1}^{p} \Gamma_s L_d^s \Delta^d z_t + \varepsilon t, \qquad t = 1, \ldots, T$$

$z_t \equiv (V_{i,t}, V_{j,t}, r_{i,t}, r_{j,t})'$ denote a 4 $\times$ 1 vector process process comprising two downside risks, $V_{i,t}$ and $V_{j,t}$ and two stock return variables $r_{i,t}$ and $r_{j,t}$, assuming $d = b$

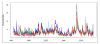
$$\begin{pmatrix} \Delta^d V_{i,t} \\ \Delta^d V_{j,t} \\ \Delta^d r_{i,t} \\ \Delta^d r_{j,t} \end{pmatrix} = \alpha\mu' + \overbrace{\begin{pmatrix} \tilde{\beta}\alpha_{11} & \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \tilde{\beta}\alpha_{21} & \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \tilde{\beta}\alpha_{31} & \alpha_{31} & \alpha_{32} & \alpha_{33} \\ \tilde{\beta}\alpha_{41} & \alpha_{41} & \alpha_{42} & \alpha_{43} \end{pmatrix} \begin{pmatrix} L_d V_{i,t} \\ L_d V_{j,t} \\ L_d r_{i,t} \\ L_d r_{j,t} \end{pmatrix}}^{\text{Long run component}}$$

$$+ \overbrace{\begin{pmatrix} \Gamma_{1,11} & \Gamma_{1,12} & \Gamma_{1,13} & \Gamma_{1,14} \\ \Gamma_{1,21} & \Gamma_{1,22} & \Gamma_{1,23} & \Gamma_{1,24} \\ \Gamma_{1,31} & \Gamma_{1,32} & \Gamma_{1,33} & \Gamma_{1,34} \\ \Gamma_{1,41} & \Gamma_{1,42} & \Gamma_{1,43} & \Gamma_{1,44} \end{pmatrix} \begin{pmatrix} L_d \Delta^d V_{i,t} \\ L_d \Delta^d V_{j,t} \\ L_d \Delta^d r_{i,t} \\ L_d \Delta^d r_{j,t} \end{pmatrix}}^{\text{Short run component}} + \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \\ \varepsilon_{4,t} \end{pmatrix}$$



3.1 Testable Hypothesis: risk-return tradeoff (Table 4)

Long Run risk-return tradeoff	Short run risk-return tradeoff
• $\tilde{\beta}$ = the long-run cointegrating relation between $V_{i,t}$ and $V_{j,t}$	• $\Gamma_{1,31} = 0$ tests the short run trade-off between $L_d \Delta^d V_{i,t}$ and $\Delta^d r_{i,t}$
• Cointegrating vector: $V_{j,t} + \tilde{\beta} V_{j,t} = e_t$, $\tilde{\beta} < 0$ and $e_t \sim I(0)$ (stationary vector)	• $\Gamma_{1,32} = 0$ tests the short run trade-off between $L_d \Delta^d V_{j,t}$ and $\Delta^d r_{i,t}$
• Reject the null $\alpha_{31} = 0$ and positive $\rightarrow$ the long run risk-return trade off for i using $V_{j,t} + \tilde{\beta} V_{j,t} = e_t$ or from j	• $\Gamma_{1,41} = 0$ tests the short-run downside risk from non-US $(L_d \Delta^d V_{i,t})$ $\rightarrow$ US return $(\Delta^d r_{j,t})$
• Reject the null $\alpha_{41} = 0$ and positive the US long run risk-return trade off	• $\Gamma_{1,42} = 0$ tests the short-run downside risk from the US on its own market, $L_d \Delta^d V_{ijt} \rightarrow \Delta^d r_{j,t}$



4. Empirical results (Gaussian kernel) (1/2)

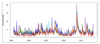
	d	$\tilde{\beta}$	$\mu'_{(3\times1)}$	$\alpha_{(4\times3)}$			$\Gamma_{s=1,(4\times4)}$				$\Gamma_{s=2,(4\times4)}$				BIC
UK	0.50	-0.88	−3.36	0.47	0.18	−0.08	−0.07	0.36	−0.02	0.05	−0.01	−0.13	−0.11	0.13	4516
	(0.06)		−3.67	(0.29)	(0.22)	(0.21)	(0.31)	(0.27)	(0.20)	(0.16)	(0.42)	(0.39)	(0.20)	(0.18)	
			0.22	−0.21	−0.33	−0.02	−0.06	0.20	0.04	−0.21	0.10	0.17	0.01	−0.10	
				(0.32)	(0.26)	(0.23)	(0.33)	(0.34)	(0.21)	(0.17)	(0.43)	(0.44)	(0.21)	(0.18)	
				3.03	2.34	−2.80	3.81	−2.98	1.62	−1.45	3.55	−2.96	1.00	−1.13	
				(1.19)	(0.94)	(0.87)	(1.29)	(1.10)	(0.82)	(0.60)	(1.46)	(1.53)	(0.73)	(0.62)	
				2.54	2.07	−0.62	2.73	−3.07	0.64	−0.95	4.64	−3.34	0.69	−1.11	
				(1.19)	(0.93)	(0.85)	(1.25)	(1.13)	(0.80)	(0.61)	(1.67)	(1.60)	(0.78)	(0.67)	
GM	0.50	-1.27	−4.29	0.38	0.15	−0.04	−0.18	0.42	−0.08	0.07	0.18	−0.10	−0.06	0.07	5041
	(0.06)		−2.81	(0.13)	(0.12)	(0.13)	(0.21)	(0.24)	(0.14)	(0.20)	(0.35)	(0.42)	(0.14)	(0.20)	
			0.08	−0.16	−0.04	−0.04	0.16	0.00	−0.04	−0.11	0.17	0.10	0.04	−0.11	
				(0.10)	(0.09)	(0.11)	(0.15)	(0.34)	(0.11)	(0.16)	(0.25)	(0.34)	(0.11)	(0.16)	
				1.12	0.43	−1.44	1.06	−0.89	0.31	−0.75	0.72	−1.83	0.09	−0.75	
				(0.53)	(0.46)	(0.55)	(0.53)	(0.90)	(0.54)	(0.79)	(1.22)	(1.63)	(0.56)	(0.79)	
				1.29	0.43	−0.31	1.12	−1.89	0.36	−1.01	2.33	−2.10	0.05	−1.01	
				(0.56)	(0.31)	(0.36)	(0.53)	(0.63)	(0.36)	(0.53)	(0.87)	(1.10)	(0.38)	(0.53)	
FR	0.48	-0.79	−2.82	0.81	−0.44	−0.22	0.17	0.18	0.09	−0.06	0.24	−0.46	0.09	−0.09	4858
	(0.06)		−2.62	(0.19)	(0.13)	(0.11)	(0.22)	(0.19)	(0.10)	(0.14)	(0.27)	(0.29)	(0.10)	(0.13)	
			0.14	−0.39	0.17	−0.04	0.48	−0.41	0.04	−0.16	0.49	−0.31	0.12	−0.18	
				(0.18)	(0.12)	(0.10)	(0.19)	(0.18)	(0.10)	(0.14)	(0.24)	(0.24)	(0.10)	(0.13)	
				0.56	0.92	−1.22	1.44	−1.32	0.13	−0.77	1.41	−1.99	−0.40	−1.19	
				(0.24)	(0.51)	(0.45)	(0.80)	(0.67)	(0.43)	(0.58)	(0.99)	(1.07)	(0.42)	(0.54)	
				0.14	0.20	−0.32	0.63	−1.25	0.39	−0.79	1.42	1.39	0.09	−0.69	
				(0.59)	(0.41)	(0.37)	(0.64)	(0.64)	(0.35)	(0.48)	(0.82)	(0.84)	(0.32)	(0.45)	

The estimated equation is: $\Delta^d z_t = \alpha(\beta' L_d z_t + \mu') + \sum_{s=1}^{p} \Gamma_s L_d^s \Delta^d z_t + \varepsilon_t$, **t=1, ..., T**

Empirical results (Gaussian kernel) (2/2)

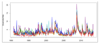
	d	$\tilde{\beta}$	$\mu'_{(3\times1)}$	$\alpha_{(4\times3)}$	$\Gamma_{s=1,(4\times4)}$	$\Gamma_{s=2,(4\times4)}$	BIC
IT	0.44 (0.05)	-0.66	$\begin{pmatrix}-3.80\\-5.47\\0.10\end{pmatrix}$	$\begin{pmatrix}0.28 & -0.05 & -0.01\\(0.08) & (0.02) & (0.06)\\0.24 & 0.01 & -0.06\\(0.07) & (0.02) & (0.05)\\0.41 & 0.05 & -0.93\\(0.17) & (0.11) & (0.27)\\0.41 & -0.23 & -0.04\\(0.20) & (0.07) & (0.18)\end{pmatrix}$	$\begin{pmatrix}-0.21 & 0.41 & 0.06 & 0.04\\(0.15) & (0.16) & (0.06) & (0.09)\\0.23 & -0.23 & 0.08 & -0.18\\(0.12) & (0.13) & (0.05) & (0.08)\\0.38 & -0.59 & -0.19 & -0.38\\(0.62) & (0.64) & (0.26) & (0.41)\\0.32 & -0.84 & 0.16 & -0.41\\(0.41) & (0.44) & (0.17) & (0.28)\end{pmatrix}$	$\begin{pmatrix}-0.10 & -0.18 & 0.08 & 0.00\\(0.21) & (0.23) & (0.06) & (0.09)\\0.25 & -0.10 & 0.11 & -0.16\\(0.18) & (0.19) & (0.09) & (0.08)\\0.75 & -0.49 & -0.20 & -0.30\\(0.90) & (0.95) & (0.25) & (0.38)\\1.13 & 1.17 & 0.06 & -0.50\\(0.56) & (0.64) & (0.17) & (0.27)\end{pmatrix}$	5110
CA	0.20 (0.03)	-1.01	$\begin{pmatrix}-2.833\\-2.371\\-1.558\end{pmatrix}$	$\begin{pmatrix}0.48 & -0.57 & -0.66\\(0.63) & (0.58) & (0.36)\\0.00 & -0.59 & 0.19\\(0.55) & (0.54) & (0.23)\\1.05 & 1.84 & -0.33\\(2.53) & (2.28) & (0.82)\\2.91 & 3.03 & -0.95\\(1.94) & (1.76) & (0.63)\end{pmatrix}$	$\begin{pmatrix}0.76 & 2.26 & -0.65 & 0.00\\(0.86) & (0.90) & (0.33) & (0.31)\\-0.24 & 2.25 & -0.17 & -0.60\\(0.75) & (0.93) & (0.25) & (0.25)\\2.66 & -3.32 & -0.90 & 2.24\\(3.42) & (3.48) & (0.92) & (1.21)\\3.94 & -4.90 & 0.86 & -0.72\\(1.59) & (2.67) & (0.70) & (0.85)\end{pmatrix}$	$\begin{pmatrix}0.18 & -0.12 & -0.06 & 0.02\\(0.35) & (0.42) & (0.14) & (0.20)\\0.14 & 0.10 & 0.04 & -0.11\\(0.25) & (0.34) & (0.11) & (0.16)\\0.57 & 1.83 & 0.09 & -0.75\\(1.22) & (1.63) & (0.56) & (0.79)\\2.33 & 1.11 & 0.18 & -1.02\\(0.87) & (1.10) & (0.34) & (0.53)\end{pmatrix}$	4852
JP	0.47	-1.455	$\begin{pmatrix}-3.39\\-2.69\\-0.23\end{pmatrix}$	$\begin{pmatrix}2.14 & -1.17 & -0.47\\(0.42) & (0.26) & (0.27)\\0.75 & -0.48 & -0.33\\(0.32) & (0.20) & (0.20)\\1.17 & -1.45 & -2.76\\(1.58) & (0.97) & (1.22)\\0.43 & 0.39 & -0.19\\(1.09) & (0.70) & (0.72)\end{pmatrix}$	$\begin{pmatrix}1.49 & -0.02 & 0.42 & -0.17\\(0.41) & (0.34) & (0.27) & (0.25)\\0.60 & -0.39 & 0.30 & -0.39\\(0.30) & (0.25) & (0.20) & (0.19)\\-1.07 & 0.24 & 1.64 & 0.86\\(1.51) & (1.25) & (1.20) & (0.92)\\0.93 & -0.78 & 0.28 & 0.14\\(1.04) & (0.87) & (0.70) & (0.68)\end{pmatrix}$	$\begin{pmatrix}1.44 & -1.05 & 0.39 & -0.16\\(0.45) & (0.54) & (0.27) & (0.27)\\0.37 & -0.16 & 0.29 & -0.29\\(0.34) & (0.39) & (0.20) & (0.21)\\-1.72 & 1.36 & 1.72 & 0.56\\(1.71) & (1.98) & (1.19) & (1.02)\\0.80 & -0.16 & 0.27 & -0.23\\(1.17) & (1.38) & (0.69) & (0.73)\end{pmatrix}$	5458

The estimated equation is: $\Delta^d z_t = \alpha(\beta' L_d z_t + \mu') + \sum_{s=1}^{p} \Gamma_s L_d^s \Delta^d z_t + \varepsilon_t$, **t=1, ..., T**



Robustness Tests

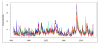
- The *double exponential kernel* to capture the fat tail.
- *Expected shortfall (ES)* is the mean losses larger than VaR, which substitutes for VaR as for the robustness.
- *Expected VaR* by using AR(1) process.
- *Control variables* are added, including *dividend yield, term spread, the detrended riskless rate, the default spread*



Robustness check using different risk measures using UK as an example (1/2)

	d	$\tilde{\beta}$	$\mu'_{(3\times1)}$	$\alpha_{(4\times3)}$			$\Gamma_{s=1,(4\times4)}$				$\Gamma_{s=2,(4\times4)}$				BIC
Panel A. Risk measure is represented by the VaR estimated from a double exponential kernel															
				0.45	0.17	−0.08	−0.08	0.32	0.00	0.03	−0.10	−0.13	−0.09	0.10	
				(0.27)	(0.20)	(0.18)	(0.29)	(0.25)	(0.17)	(0.14)	(0.38)	(0.39)	(0.17)	(0.15)	
			−3.407	−0.13	−0.27	−0.04	0.03	0.05	0.06	−0.20	0.15	0.17	0.03	−0.12	
UK	0.49	-0.89	−3.390	(0.30)	(0.23)	(0.20)	(0.31)	(0.31)	(0.18)	(0.15)	(0.40)	(0.44)	(0.18)	(0.16)	4442
	(0.06)		0.210	2.91	2.24	−2.61	3.62	−2.89	1.42	−1.34	3.76	−2.99	0.87	−1.02	
				(1.13)	(0.88)	(0.78)	(1.23)	(1.06)	(0.73)	(0.55)	(1.44)	(1.49)	(0.65)	(0.56)	
				2.47	2.01	−0.51	2.48	−2.97	0.50	−0.85	4.93	−3.44	0.58	−1.00	
				(1.14)	(0.88)	(0.76)	(1.20)	(1.09)	(0.72)	(0.56)	(1.66)	(1.56)	(0.69)	(0.60)	
Panel B. Risk measure is represented by the expected shortfall estimated from a Gaussian kernel															
				0.43	0.15	0.15	−0.14	0.43	0.00	0.03	−0.11	−0.04	−0.11	0.14	
				(0.27)	(0.21)	(0.21)	(0.30)	(0.26)	(0.22)	(0.18)	(0.40)	(0.39)	(0.22)	(0.19)	
			−4.10	−0.15	−0.28	−0.03	0.05	0.19	0.04	−0.23	0.09	0.16	0.02	−0.09	
UK	0.474	-0.89	−4.05	(0.30)	(0.23)	(0.24)	(0.31)	(0.32)	(0.22)	(0.18)	(0.40)	(0.44)	(0.22)	(0.19)	4626
	(0.06)		0.23	2.63	2.03	−2.68	3.46	−2.71	1.51	−1.36	3.40	−2.87	0.95	−1.07	
				(0.99)	(0.78)	(0.83)	(1.10)	(0.95)	(0.78)	(0.58)	(1.26)	(1.33)	(0.70)	(0.60)	
				2.21	1.81	−0.55	2.57	−2.84	0.57	−0.86	4.25	−3.14	0.65	−1.04	
				(0.99)	(0.77)	(0.81)	(1.07)	(0.98)	(0.77)	(0.59)	(1.43)	(1.38)	(0.75)	(0.64)	

The estimation is based on the equation on slide 3-1. The numbers in parentheses are standard errors.



Robustness check using different risk measures using UK as an example (2/2)

Panel C. Risk measure is represented by the expected VaR

	d	$\tilde{\beta}$	$\mu'_{(3\times1)}$	$\alpha_{(4\times3)}$	$\Gamma_{s=1,(4\times4)}$	$\Gamma_{s=2,(4\times4)}$	BIC
UK	0.46 (0.05)	-0.85	$\begin{pmatrix} -3.089 \\ -3.096 \\ 0.148 \end{pmatrix}$	$\begin{pmatrix} 0.98 & 0.36 & -0.10 \\ (0.50) & (0.40) & (0.25) \\ 0.07 & -0.29 & -0.12 \\ (0.06) & (0.48) & (0.29) \\ 5.39 & 4.82 & -2.96 \\ (2.81) & (2.27) & (1.32) \\ 5.15 & 4.87 & -1.04 \\ (2.67) & (2.60) & (1.44) \end{pmatrix}$	$\begin{pmatrix} 0.23 & 0.35 & -0.04 & 0.03 \\ (0.50) & (0.44) & (0.24) & (0.19) \\ 0.16 & 0.34 & 0.17 & -0.25 \\ (0.59) & (0.55) & (0.28) & (0.22) \\ 6.66 & -6.05 & 1.48 & -1.52 \\ (2.88) & (2.45) & (1.26) & (0.96) \\ 6.65 & -6.98 & 1.04 & -1.33 \\ (3.28) & (2.85) & (1.39) & (1.05) \end{pmatrix}$	$\begin{pmatrix} 0.67 & -0.19 & -0.20 & 0.23 \\ (0.60) & (0.59) & (0.25) & (0.22) \\ 0.69 & -0.05 & 0.07 & -0.07 \\ (0.70) & (0.69) & (0.28) & (0.24) \\ 4.96 & -4.88 & 1.32 & -1.84 \\ (3.06) & (3.15) & (1.22) & (1.05) \\ 8.62 & -7.55 & 1.41 & -2.05 \\ (3.71) & (3.77) & (1.38) & (1.18) \end{pmatrix}$	2983

Panel D. Using residuals of return to take into account control variables

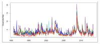
	d	$\tilde{\beta}$	$\mu'_{(3\times1)}$	$\alpha_{(4\times3)}$	$\Gamma_{s=1,(4\times4)}$	$\Gamma_{s=2,(4\times4)}$	BIC
UK	0.42 (0.04)	-0.85	$\begin{pmatrix} -3.380 \\ -3.32 \\ -0.16 \end{pmatrix}$	$\begin{pmatrix} 1.15 & -0.92 & -0.32 \\ (0.34) & (0.30) & (0.18) \\ 0.14 & 0.03 & -0.28 \\ (0.35) & (0.33) & (0.20) \\ 3.10 & 3.08 & -1.86 \\ (1.50) & (1.36) & (0.82) \\ 2.68 & 2.79 & -0.94 \\ (1.37) & (1.35) & (0.85) \end{pmatrix}$	$\begin{pmatrix} 1.96 & 1.55 & -0.78 & 0.61 \\ (1.67) & (1.49) & (0.52) & (0.66) \\ 0.05 & 2.42 & 0.98 & -1.37 \\ (2.26) & (2.08) & (0.76) & (0.99) \\ 3.56 & 2.81 & -1.68 & 1.64 \\ (5.18) & (4.64) & (1.63) & (2.05) \\ 7.26 & 6.57 & -0.02 & 1.55 \\ (5.49) & (4.92) & (1.69) & (2.13) \end{pmatrix}$	$\begin{pmatrix} 0.24 & -1.05 & 0.39 & 0.02 \\ (0.27) & (0.54) & (0.27) & (0.20) \\ 0.49 & -0.16 & 0.29 & -0.11 \\ (0.24) & (0.39) & (0.20) & (0.16) \\ 1.41 & 1.36 & 1.72 & -0.75 \\ (0.99) & (1.98) & (1.19) & (0.79) \\ 1.42 & -0.16 & 0.27 & -1.02 \\ (0.82) & (1.38) & (0.69) & (0.53) \end{pmatrix}$	2995

The estimation is based on the equation on slide 3-1. The numbers in parentheses are standard errors.

5. Downside risk, high-moment risk and ICAPM theory

$$
\begin{pmatrix} \Delta^d V_{i,t} \\ \Delta^d \sigma_{j,t} \\ \Delta^d S_{i,t} \\ \Delta^d K_{i,t} \\ \Delta^d r_{i,t} \end{pmatrix} = \alpha\mu' + \overbrace{\begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} \\ \alpha_{51} & \alpha_{52} & \alpha_{53} & \alpha_{54} \end{pmatrix} \begin{pmatrix} \tilde{\beta} & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} L_d V_{i,t} \\ L_d \sigma_{i,t} \\ L_d S_{i,t} \\ L_d K_{i,t} \\ L_d r_{i,t} \end{pmatrix}}^{\text{Long run component}}
$$

$$
+ \overbrace{\begin{pmatrix} \Gamma_{1,11} & \Gamma_{1,12} & \Gamma_{1,13} & \Gamma_{1,14} & \Gamma_{1,15} \\ \Gamma_{1,21} & \Gamma_{1,22} & \Gamma_{1,23} & \Gamma_{1,24} & \Gamma_{1,25} \\ \Gamma_{1,31} & \Gamma_{1,32} & \Gamma_{1,33} & \Gamma_{1,34} & \Gamma_{1,35} \\ \Gamma_{1,41} & \Gamma_{1,42} & \Gamma_{1,43} & \Gamma_{1,44} & \Gamma_{1,45} \\ \Gamma_{1,51} & \Gamma_{1,52} & \Gamma_{1,53} & \Gamma_{1,54} & \Gamma_{1,55} \end{pmatrix} \begin{pmatrix} L_d \Delta^d V_{i,t} \\ L_d \Delta^d \sigma_{i,t} \\ L_d \Delta^d S_{i,t} \\ L_d \Delta^d K_{i,t} \\ L_d \Delta^d r_{i,t} \end{pmatrix}}^{\text{Short run component}} + \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \\ \varepsilon_{4,t} \\ \varepsilon_{5,t} \end{pmatrix}
$$



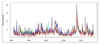
Long run information content of VaR

$$\begin{pmatrix} \Delta^d V_{i,t} \\ \Delta^d \sigma_{j,t} \\ \Delta^d S_{i,t} \\ \Delta^d K_{i,t} \\ \Delta^d r_{i,t} \end{pmatrix} = \alpha\mu' + \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} \\ \alpha_{51} & \alpha_{52} & \alpha_{53} & \alpha_{54} \end{pmatrix} \begin{pmatrix} \tilde{\beta} & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} L_d V_{i,t} \\ L_d \sigma_{i,t} \\ L_d S_{i,t} \\ L_d K_{i,t} \\ L_d r_{i,t} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \\ \varepsilon_{4,t} \\ \varepsilon_{5,t} \end{pmatrix}$$

Two underlying issues between stock returns and VaR

- What is the information content of VaR?
  Test the restrictions of
  $\alpha_{11} = 0$ (VaR and $\sigma_{i,t}$)
  $\alpha_{12} = 0$ (Skewness)
  $\alpha_{13} = 0$ (Kurtosis)

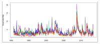
- Can stock returns be predicted based the second- and higher-moment risks?
  Test the restrictions of
  $\alpha_{51} = 0$ (VaR and $\sigma_{i,t}$)
  $\alpha_{52} = 0$ (Skewness)
  $\alpha_{53} = 0$ (Kurtosis)



Downside risk, higher-moment risk and ICAPM theory (1/2)

	d	$\tilde{\beta}$	$\mu'_{(4\times1)}$	$\alpha_{(5\times4)}$				$\Gamma_{s=1,(5\times5)}$				
UK	0.18	-0.74	−1.44	14.22	−42.78	3.42	−0.33	−16.40	52.13	−3.2	0.52	−1.64
	(0.03)		−0.5	(7.49)	(21.71)	(3.46)	(0.33)	(7.86)	(23.25)	(3.65)	(0.24)	(0.62)
			0.001	5.26	−15.65	1.53	−0.13	−5.6	17.97	−1.51	0.20	−0.52
			−3.15	(2.53)	(7.37)	(1.12)	(0.16)	(2.65)	(7.89)	(1.17)	(0.07)	(0.19)
				0.78	−2.5	−0.34	0.27	−0.81	2.63	−1.08	0.11	0.02
				(3.00)	(8.52)	(3.21)	(0.16)	(3.12)	(8.87)	(1.82)	(0.11)	(0.22)
				−1.97	3.86	3.92	−2.29	1.67	−2.03	−3.62	1.02	0.54
				(5.51)	(15.64)	(3.21)	(0.36)	(5.72)	(16.32)	(3.37)	(0.26)	(0.46)
				−5.49	−17.92	2.85	−1.43	5.55	15.16	2.27	0.02	4.23
				(2.47)	(6.94)	(1.35)	(1.16)	(2.45)	(6.97)	(1.36)	(0.84)	(1.99)
GM	0.26	-0.19	−1.62	−5.73	−14.99	−2.73	−1.3	1.74	−1.17	1.90	1.28	−0.49
	(0.02)		−0.54	(2.79)	(7.77)	(2.07)	(0.42)	(3.43)	(2.40)	(2.53)	(0.38)	(0.25)
			−0.03	−1.22	3.04	−0.80	−0.34	0.12	1.39	0.54	0.37	−0.13
			−3.18	(0.80)	(2.60)	(0.60)	(0.12)	(0.97)	(2.71)	(0.74)	(0.11)	(0.06)
				0.48	−1.19	−0.89	0.15	−0.56	2.03	−0.32	−0.06	0.13
				(1.14)	(3.21)	(0.74)	(0.16)	(1.24)	(3.51)	(0.89)	(0.15)	(0.08)
				−1.88	3.74	0.86	−2.37	−0.54	0.14	−1.32	1.76	−0.01
				(2.71)	(7.68)	(1.96)	(0.35)	(2.38)	(8.26)	(2.09)	(0.31)	(0.16)
				−10.22	−29.52	−1.73	0.79	−9.89	38.49	−0.53	−1.02	−0.17
				(5.50)	(15.45)	(7.78)	(1.68)	(11.20)	(39.19)	(0.97)	(1.49)	(0.81)

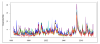
The estimation is based on the equation on slide 5-1. The numbers in parentheses are standard errors.



Downside risk, higher-moment risk and ICAPM theory (2/2)

	d	$\tilde{\beta}$	$\mu'_{(4\times1)}$	$\alpha_{(5\times4)}$				$\Gamma_{s=1,(5\times5)}$				
				−4.82	10.12	1.22	−1.30	1.05	4.56	−0.95	0.88	−11.12
				(2.04)	(16.17)	(3.37)	(0.84)	(6.84)	(9.40)	(3.66)	(0.80)	(5.12)
				−0.90	1.37	0.29	−0.32	0.17	2.21	−0.13	0.18	−3.58
			2.20	(1.88)	(5.17)	(1.07)	(0.27)	(2.16)	(6.04)	(1.15)	(0.26)	(1.62)
FR	0.17		−0.78	3.49	−9.92	−1.54	0.42	−3.20	9.13	0.08	−0.33	1.41
	(0.02)	-0.22	−0.07	(2.57)	(7.12)	(1.39)	(0.32)	(2.68)	(7.50)	(1.43)	(0.27)	(1.84)
			−2.49	−18.56	51.16	−8.51	−4.21	17.31	−48.07	8.720	3.70	10.41
				(7.03)	(19.25)	(3.62)	(0.69)	(7.19)	(19.86)	(3.72)	(0.60)	(4.70)
				−5.77	−15.60	2.96	−0.04	5.06	−13.37	2.11	−0.01	2.97
				(2.84)	(7.84)	(1.50)	(0.33)	(2.93)	(8.15)	(1.52)	(0.28)	(1.98)
				−1.22	−3.40	0.27	0.08	0.91	−0.98	0.32	−0.08	−0.27
				(0.13)	(0.38)	(0.57)	(0.06)	(0.59)	(1.69)	(0.68)	(0.18)	(0.12)
				0.25	−0.73	0.15	0.03	0.09	0.24	0.04	0.01	−0.10
			−3.77	(0.01)	(1.20)	(0.16)	(0.02)	(0.17)	(0.48)	(0.20)	(0.05)	(0.03)
US	0.42		−1.32	0.43	−1.25	−0.81	−0.01	−0.26	0.83	−0.20	0.05	0.01
	(0.06)	-0.63	0.11	(0.19)	(0.54)	(0.29)	(0.03)	(0.32)	(0.94)	(0.33)	(0.09)	(0.04)
			−3.165	−1.02	2.71	0.59	−0.22	0.46	−0.48	0.13	−0.76	0.02
				(1.26)	(3.57)	(0.75)	(0.11)	(1.22)	(3.41)	(0.80)	(0.20)	(0.10)
				−8.29	−23.10	5.82	0.26	3.47	−9.52	3.38	0.41	0.14
				(4.29)	(12.16)	(2.17)	(0.22)	(3.97)	(11.03)	(2.75)	(0.68)	(0.33)

The estimation is based on the equation on slide 5-1. The numbers in parentheses are standard errors.

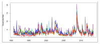


6. Relative price discovery ability

	UK	GM	FR	IT	CA	JP	Average
$\alpha_\perp^{V_i}$	36.46 %	33.52 %	31.85 %	36.80 %	5.95 %	17.78 %	27.06 %
$\alpha_\perp^{V_j}$	47.78 %	47.62 %	60.32 %	51.05 %	44.29 %	75.11 %	54.36 %
$\alpha_\perp^{r_i}$	2.32 %	1.57 %	2.77 %	3.55 %	16.01 %	2.86 %	4.85 %
$\alpha_\perp^{r_j}$	13.44 %	17.28 %	5.06 %	8.60 %	33.74 %	4.16 %	13.71%

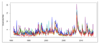
$\alpha'_\perp = (\alpha_\perp^{V_i}, \alpha_\perp^{V_j}, \alpha_\perp^{r_i}, \alpha_\perp^{r_j})$ is a vector of the orthonormal adjustment coefficients (permanent component), and governs the long-run cointegrating risk relation and long-run risk-return relation.

$\alpha_\perp^{V_j}$ is the orthonormal adjustment coefficient of US downside risk.



Conclusions

- Examine dynamic relations between stock market returns and downside risk for G7 market data.
- A fractionally cointegrated vector autoregression (FCVAR) model
- Major Findings:
  1. Downside risk cointegration
  2. Long-run positive relations and short-run tradeoff hypothesis are supported
  3. Evidence of long-run and moderate short-run leverage effects
  4. US Downside risk is more informative in the long-run dynamic process
  5. US downside risk is a leading factor in the dynamic process and contributes more in price discovery
  6. The predictability of VaR w.r.t the role of an aggregative and informative risk measures



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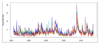

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