Perfectly competitive innovation or patent protection: What is better for growth and social welfare?

Diplom Thesis

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1 Introduction

Intentional, profit oriented inventive activity is at the basis of modern growth theory. The purposeful development of new goods and technologies is the motor for sustained economic growth. Grossman and Helpman (1991; [8]) provide various estimates from the empirical literature of growth accounting from the period 1948 to 1987, that attribute between 0.19 to 0.49 percent of annual productivity growth to R&D activity, only to go on and ask the perhaps more relevant question "What would the growth rate of output have been in the absence of any investment by firms in the creation of knowledge?" (Grossman and Helpman, 1991, p. 14; [8]).

Thus if economic growth increases the welfare of a society, an assumption that considering the evolution of European societies since the beginning of the industrial revolution seems to be reasonable, the issue of optimal resource allocation for invention should be of greatest importance. From an economists perspective, the question of optimal resource allocation naturally leads to the somewhat more precise question: Does perfect competition fail to reach the social optimum, and if so, for what reasons? Arrow (1962; [4]) identifies three potential sources for such a failure of perfect competition: indivisibilities, inappropriability, and uncertainty.

Uncertainty is a major characteristic of innovation. Both the exact characteristics and the time of arrival of an innovation cannot be determined in advance. Investments into R&D are made without knowledge of the exact outcome. This obviously involves risk. However, as Arrow points out, uncertainty in itself does not prevent perfect competition from reaching the social optimum. If one thinks of uncertainty in the way Debreu (1959; [6]) treats it, then uncertainty represents a list of commodities, potentially available in the future, that are already traded today, say in markets for commodity-options. Buyers and sellers agree on a price to be paid contingent on a specific commodity being available at a future point in time. If this commodity happens to be the outcome of an a priori uncertain production activity at this future point in time, the buyer pays the agreed price, if not, he does not pay. But real life shows, that for most commodities, such future markets do not exist. Moral hazard considerations are a central reason for competitive equilibrium to fail in the presence of uncertainty.

Innovation is the production of knowledge. New designs, formulae, or processes are the outcome of R& D. Problematically, once invented, these goods are easily copied by competitors. While the inventor bears the costs of development, his competitor might be able to quickly replicate the invention, and compete against the successful innovator. The innovator is not able to fully appropriate the economic returns of his invention, a fact that greatly mitigates his incentives for R&D, possibly driving these to a suboptimal level.

What can be done to raise the incentives for R&D in the presence of uncertainty and inappropriability? A popular potential remedy to problems of appropriability are patent rights that protect the innovator from potential competitors. However patents grant the successful innovator a monopoly and thereby introduce a new source of inefficiency into the economic system.

Furthermore, the prospect of future monopoly profits leads to the waste of economic resources in patent races (Tirole, 1988 ; [14]). However, infinitely lived patents granted on innovations are needed to guarantee long-run growth in the standard endogenous growth models of increasing product variety (Romer, 1990, [13]; Grossman and Helpman, 1991, [8]) and the model of creative destruction (Aghion and Howitt, 1992, [3]).

Is competition unambiguously bad for innovation ? From a static point of view, the opposite might indeed be true (Arrow, 1962; [4]). Suppose an incumbent monopolist and a potential competitor both have the opportunity to make an innovation, that lowers production costs. By making the innovation, the competitor would be able to enter the market, either limit price in a duopoly with the former monopolist (non-drastic innovation), or replace the monopolist (drastic innovation). Then in both cases it can be shown, that the competitor has a higher incentive to innovate. "A monopolist tends to rest on his laurels" (Tirole, 1988, p. 392; [14]). Thus in the static case, more competition might foster innovation.

But even in a dynamic analysis, to a certain degree product market competition might be favorable to innovation. In endogenous growth models with "step-by-step" innovation, where competitors first have to catch up with technological leaders before they can gain monopoly power through innovation, more competition can lead to more innovation, as the leader tries to escape from his followers in order to preserve his monopoly profits (Aghion, Harris, and Howitt, 2000; [2]). In fact, an econometric analysis of British patenting data (Aghion et al., 2002; [1]) finds an inverted U-relationship between product market competition and innovation. For low levels of competition the above mentioned escape effect dominates, for high levels the Schumpeterian threat of destruction overwhelms and deters innovation.

In the above described models, even though competition had an positive effect on innovation, in order for innovation to take place the prospect of monopolistic profits was needed. Lately, models have been developed, that are able to generate growth through purposeful inventive activities under perfect competition. Hellwig and Irmen (2000; [9]) succeed in modelling growth through intentional innovations under perfect competition by recurrence to inframarginal rents derived from U-shaped average cost curves. However, in their model any innovation is private knowledge in the first period after invention. From the second period on, the formerly private innovation is common knowledge. This spillover-effect then leads to a suboptimal equilibrium. Boldrin and Levine (2002; [5]) take another position to model innovation and thus growth under perfect competition. They claim, that an innovation cannot be copied at infinite speed. Making copies takes time, and in the beginning copying rates will have to be bounded. Eventually these rate may go to infinity, but what is crucial for competitive innovation is what happens in the first periods. They show, that even with copying rates eventually going to infinity, an innovation still has a positive price in the first period. It is the discounted profits that can be derived from making copies of the innovation, and selling them in future periods. What is crucial to the Boldrin-Levine model, is that the innovator has a protected right of first sale of his innovation. Quah (2002a, [11]; 2002b, [12]) confirms the above analysis. But he identifies two circumstances under which perfectly competitive innovation cannot take place. Firstly, if agents are allowed to take decision increasingly frequently, eventually with copying rates going to infinity, innovations no longer achieve a non-zero price in the beginning. Economic agents are allowed to switch between consumption and copying decisions too quickly. Secondly, if innovations are subject to a minimum size requirement, not all socially optimal innovations take place, and the competitive equilibrium will be suboptimal.

When competitive equilibrium fails to exists, are there alternatives to complete patent protection and monopolistic production? Michael Kremer (1999; [10]) proposes a so called "patent buy-out mechanism" to simultaneously guarantee a socially optimal level of innovation, and a certain degree of competitive production in the economy. The mechanism is as follows: Successful innovators can decide to sell their innovation in a government organized auction. With a certain probability, say γ, the patent is sold to the highest bidder in the patent auction. The winner is thus granted a monopoly over the innovation he has bought. With the residual probability $1-\gamma$, the government buys the patent and puts it into the public domain. Everybody is free to use the innovation for economic activities. Goods derived from innovations that are in the public domain are sold under competitive conditions. The original innovator is paid a mark-up over the amount the highest bidder has to pay. This mark-up is financed through taxes and is meant to raise the private value of the patent to its public value. Notice that in general the private value of a patent is inferior to its public value, as the innovator does not value future innovations based on his innovation. Furthermore does the private value include the potential capital loss through Schumpeterian distruction.

Kremer's analysis is a partial equilibrium analysis. The following model analyzes the general equilibrium consequences of Kremer's proposal in an endogenous growth model of increasing product variety. The standard model will be augmented by a flexible labor supply in order to evaluate the consequences of taxation.

Several interesting questions arise. How should the government set the mark-up over private costs? What is the optimal buy-out probability ? What will be the effect of a flexible labor supply under taxation? Finally, does the mechanism do better than perfect patent protection ?

2 The Model

In order to evaluate Kremer's patent buy-out proposal, we use the standard endogenous growth model of increasing product variety by Grossman and Helpman (1991; [8]).

The economy is populated by a continuum of infinitely lived economic agents with lifetime utility U. Agent derive utility from consumption C, and leisure $l = 1 - L$.

$$U = \int_t^\infty e^{-\rho(\tau - t)} \left[\log C(\tau) + b \log\left(1 - L(\tau)\right)\right] d\tau \tag{1}$$

The consumption index C follows the standard Dixit-Stiglitz model, where agents have a preference for variety in consumption (Dixit and Stiglitz, 1977; [7]). $0 < \alpha < 1$ specifies the degree to which different commodities x are substitutable. Higher values of α indicate a weaker preference for product variety. $n(\tau)$ designates the amount of different commodities available on the market at date τ.

$$C(\tau) = \left[\int_0^{n(\tau)} x_i(\tau)^\alpha \, di \right]^{\frac{1}{\alpha}} \tag{2}$$

The agents' utility maximization problem can be solved in two separate steps. First, the static problem of minimizing the expenditure for a given value of the consumption index $C(\tau)$ is solved. Then C, now containing the optimal shares of diversified consumption goods $x(i)$ as a function of their respective prices, is used to solve for the optimal intertemporal allocation of resources.

The static problem amounts to minimizing the total expenditure on diversified consumption goods x_i, given a level C of the overall consumption index. (For notational simplicity time subscripts are dropped for the static problem.)

$$\min_{x(i)} \int_0^n p(i)x(i) \, di \text{ s.t. } \left[\int_0^n x(i)^\alpha di \right]^{\frac{1}{\alpha}} = C \tag{3}$$

The first order condition for this constrained minimization problem is

$$\frac{x(j)^{\alpha-1}}{x(i)^{\alpha-1}} = \frac{p(j)}{p(i)}$$

$$\rightarrow x(j) = \left[\frac{p(j)}{p(i)} \right]^{\frac{1}{\alpha-1}} x(i) \tag{4}$$

Now substituting this expression back into the definition of the consumption index, and integrating over all $x(j)$s results into the demand for the good $x(i)$ as a function of C, the good's price, as well as a price aggregate.

$$x(i) = \frac{p(i)^{\frac{1}{\alpha-1}}}{\left[\int_0^n p(j)^{\frac{\alpha}{\alpha-1}}\, dj\right]^{\frac{1}{\alpha}}}\, C \tag{5}$$

Let E be the total expenditure on consumption goods, defined as $E = P\,C$. We are now able to derive the consumption good price index P.

$$E = \int_0^n p(i)x(i)\, di = \int_0^n \frac{p(i)^{\frac{\alpha}{\alpha-1}}}{\left[\int_0^n p(j)^{\frac{\alpha}{\alpha-1}}\, dj\right]^{\frac{1}{\alpha}}}\, C\, di$$

$$P = \left[\int_0^n p(i)^{\frac{\alpha}{\alpha-1}}\, di\right]^{\frac{\alpha-1}{\alpha}} \tag{6}$$

Having solved the static problem, we now turn to the problem of optimal intertemporal allocation of resources. At each moment in time economic agents are paid wages $w(\tau)$, of which they pay taxes $\theta\, w(\tau)$ to the government. They can spend the rest on aggregate consumption $C(\tau)$ at aggregate price level $P(\tau)$ or assets $a(\tau)$, that pay an interest rate $r(\tau)$. Agents take prices as given.

The following Hamiltonian expresses the dynamic optimization problem

$$H = \log C + b \log(1 - L) + \lambda((1-\theta)\, w + r\, a - c) \tag{7}$$

The first order conditions are

$$\frac{1}{C} = \lambda\, P \tag{8}$$

$$\frac{b}{1-L} = \lambda \left(1-\theta\right) w \tag{9}$$

$$\frac{\dot{\lambda}}{\lambda} = \rho - r \tag{10}$$

Taking the derivative of (8) with respect to time yields

$$\frac{\dot{E}}{E} = \frac{\dot{P}}{P} + \frac{\dot{C}}{C} = r - \rho \tag{11}$$

At each moment in time, growth in consumption expenditure equals the difference between the interest rate and the subjective discount rate.

It will prove convenient to normalize the aggregate price level P such that total consumption expenditure E equals 1, that is

$$E(\tau) = P(\tau)\, C(\tau) \equiv 1 \quad \text{for all } \tau \tag{12}$$

This normalization implies the equivalence of the interest rate and the subjective discount factor, as the growth in consumption expenditure is zero. The interest rate is thus time invariant.

$$\rho = r \tag{13}$$

2.1 The Technology

In a first step we have characterized the solution of the household's problem. We now turn to the production side of the economy. The economy consists of two production sectors, a manufacturing sector and a research sector.

The manufacturing sector produces consumption goods $x(i)$ with a linear production technology, that has as the only input factor $l(i)$.

$$x(i) = \varphi(i)\, l(i) \tag{14}$$

At each time τ, the economy produces a continuum of types of consumption goods on the interval $[0, n]$. A share γ of these goods will be produced by monopolists, who possess an infinitely lived patent on production and distribution of the goods $x(i)$. The resting share of goods, $(\ 1 - \gamma\ )$, will be produced in a perfectly competitive environment, with free entry and marginal cost pricing.

The monopolistic producers maximize profits and price their goods at a mark-up α over marginal costs $\frac{w}{\varphi(i)}$.

$$p(i)^M = \frac{w}{\varphi(i)\, \alpha} \tag{15}$$

The competitive producers price at marginal costs

$$p(j)^C = \frac{w}{\varphi(i)} \tag{16}$$

In the following we will assume all firms to have access to the same production technology with $\varphi(i) = 1$ for all i.

Due to the normalization of the aggregate price level to ensure $E = 1$, consumers' demand for consumption goods $x(i)$ (5) can be rewritten as

$$x(i) = \frac{p(i)^{\frac{1}{\alpha-1}}}{\int_0^n p(j)^{\frac{\alpha}{\alpha-1}}\,dj}$$

Given the above pricing policies of the firms, each monopolist supplies the quantity x^M, and the competitive firm supplies x^C, with

$$x^M = \frac{\alpha^{\frac{1}{1-\alpha}}}{n\,w\left[\gamma\,\alpha^{\frac{\alpha}{1-\alpha}} + 1 - \gamma\right]} \tag{17}$$

$$x^C = \frac{1}{n\,w\left[\gamma\,\alpha^{\frac{\alpha}{1-\alpha}} + 1 - \gamma\right]} \tag{18}$$

Aggregate supply of consumption goods is thus

$$X = \frac{\gamma\,\alpha^{\frac{1}{1-\alpha}} + 1 - \gamma}{w\left[\gamma\,\alpha^{\frac{\alpha}{1-\alpha}} + 1 - \gamma\right]} \tag{19}$$

Because of the special form of the production function, aggregate consumption good supply equals the total amount of labor employed in manufacturing L_m.

Each monopolist makes profits

$$\Pi^M = \frac{(1-\alpha)\,\alpha^{\frac{\alpha}{1-\alpha}}}{n(\gamma\,\alpha^{\frac{\alpha}{1-\alpha}} + 1 - \gamma)} \tag{20}$$

2.2 Innovation

The second production sector of the economy is engaged in the invention of new consumption goods. The chosen form of the production technology abstracts from important features of inventive activities, most importantly uncertainty in the size and the time of arrival of the research output. In our context, innovation is a deterministic activity, that employs labor to produce new kinds of consumption goods. In contrast to the idea of innovation as a process of creative destruction, whereby old goods and technologies are replaced by new ones, the research technology that is used here simply adds new goods to the lists of goods that have already been invented. The "old" goods are not of lower quality than the "new" ones, demand for "new" goods is not per se higher than for "old" goods. The degree of interchangeability between two different consumption goods is solely determined by the elasticity of substitution $\varepsilon = 1/(1-\alpha)$.

The total amount of goods that already exist in the economy (n) exerts a positive externality on the production cost of new goods and thereby enables sustained endogenous growth [1]. Following Romer (1990, [13]), n constitutes a proxy of the entire stock of knowledge accessible to everybody in the economy. The higher this public knowledge stock is, the more productive is the research technology of every research firm. The amount of new product designs evolves according to the research production function

$$\dot{n} = \frac{n\, L_r}{a} \tag{21}$$

where L_r is the total amount of labor employed in the research sector, and a represents a productivity parameter.

[1] Without this positive externality the economy eventually ceases to grow, as the cost of production of new product design stays constant, whereas consumers attribute a lower value to new designs as the variety of consumption goods increases.

The value v of an infinitely lived patent on a new product design equals the present discounted value of the future income stream originating from monopoly profits Π^M.

$$v(t) = \int_t^\infty e^{-[R(\tau)-R(t)]}\,\Pi^M(\tau)\,d\tau \tag{22}$$

$$\text{where } R(t) = \int_0^t r(u)\,du$$

Deriving with respect to time yields

$$r\,v = \Pi^M + \dot{v} \tag{23}$$

This "no arbitrage" condition says, that at each moment in time, the instantaneous flow of monopoly profits plus the capital gain (or loss) of the patent must equal the return of assets of size v.

2.3 The Auction

Free entry into the research sector would now lead to the condition that the price v at which a new patent could be sold needed to match the marginal cost of producing it, namely $(w\,a)/n$.

However, in the present model patents are not necessarily sold at the above price, but can also be bought out by the government at a mark-up over a price that is to be determined in an auction, a mechanism proposed by Kremer (2000, [10]).

A successful innovator is free to decide whether to be granted an infinitely lived patent on his innovation that he can sell on the market at price $(w\,a)/n$, or whether his innovation is auctioned off in a government auction. He then receives a price $\phi\, v^A$, where v^A is the outcome of the auction, and ϕ is the government mark-up, that tries to raise the private value of the patent to its public value.

Obviously, the innovator decides to take part in the auction only if his expected return from the auction is greater than or equal to the market value of his patent

$$\phi\, E(v^A) \geq v \quad (\text{ innovator's incentive constraint }). \tag{24}$$

The exact demarche of the patent buy-out mechanism is as follows. Once the innovator has decided to participate, his innovation is sold in an auction with k potential buyers bidding for it. With probability γ the highest bidder receives the patent at price v^A, which depending on the auction is his bid (first price auction) or the second highest bid (second price auction). With the residual probability $1-\gamma$, the government buys out the patent, and places it into the public domain. In both cases the innovator is paid $\phi\, v^A$. In the first case, the government pays an additional $(\phi - 1)\, v^A$ to the innovator, in the second case it bears the entire sum.

What will be the potential buyers' bids? The answer to this question clearly depends both on the buyers' valuations for the patent, and on the design of the auction.

The need for an auction to reveal the private value of a patent implies that there is some kind of uncertainty attached to this value. This uncertainty can have two sources. The first source is that different bidders may have diverging private valuations for the patent. Recall the form of the production function (14)($x(i) = \varphi(i)\, l(i)$). Assume that there are k bidders for the patent of producing the good i, each being characterized by a different potential productivity parameter $\varphi_j(i)$ with $j = 1, ..., k$. Then bidders with a higher productivity parameter will attach a higher private value to the patent [2].

But also in the case where the value of the patent is the same for all bidders, uncertainty may arise if this common value is unknown. In our case, bidders might be unable to exactly predict the technical properties of the production function for the new good. They would have to estimate the value of the patent. This gives rise to the well known problem of the winner's curse in common value auctions. If bids are an increasing function of the private estimates, then the bidder with the most optimistic estimate will win the auction. On average, the winner will suffer a loss, as his estimate of the common value will lie above the true value. In order to avoid the winner's curse, bids will have to be made conditional on the bid being the winning bid. (Wolfstetter, 1999; [15]). This bidding strategy leads to lower bids on average, and might in extreme cases even prevent trade to take place between sellers and potential buyers. However, as Kremer (1999, [10]) points out, the government mark-up, which raises the revenue of the seller above the price paid by the winning bidder, greatly mitigates the adverse selection bias of a common value auction.

In our modelling of the auction we will therefore neglect the common value problem, and focus on the case where bidders have diverging private valuations for the patent.

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[2] Observe that the monopolist's profit is $\Pi^M(i) = \left(\frac{1-\alpha}{\alpha}\right) \frac{\alpha^{\frac{1}{1-\alpha}} \varphi(i)^{\frac{\alpha}{1-\alpha}}}{\left[\gamma\, \alpha^{\frac{\alpha}{1-\alpha}} + 1 - \gamma\right]}$. Therefore $\frac{\partial \Pi^M(i)}{\partial \varphi(i)} > 0$.

Suppose the successful innovator has decided to participate in the patent auction. (Whether this is the case, actually depends on the outcome of the auction. Once we have found the auction price v^A, we will have to check, whether the innovator's incentive constraint has been satisfied in the first place.) Suppose k bidders participate in the auction. Before the auction starts, bidders draw their productivity parameter $\varphi_j(i),\ j = 1, ..., k$ from a uniform distribution on the interval $[0, 1]$. The higher the productivity parameter drawn, the higher is the bidders valuation for the patent. We assume that the patent auction is a second price auction. [3] Then obviously the bidder with the highest productivity parameter wins, and pays the bid of the second highest bidder. The expected productivity parameter of the second highest bidder is $\frac{k-1}{k+1}$. [4]. Therefore the expected price paid in the auction is

$$E(v(i)^A) = \int_t^{\infty} e^{-\rho(\tau - t)}\, \Pi^M(i;\, \varphi(i) = \frac{k-1}{k+1})(\tau)\, d\,\tau \tag{25}$$

With k sufficiently large, the expected productivity of the second highest bidder approaches 1, and consequently the price v^A approaches the valuation of the state-of-the-art producer, who has the highest possible productivity $\varphi(i) = 1$.

Will the outcome of the auction satisfy the innovator's incentive constraint? To answer this question, we need to know what the innovator's reservation

[3]Notice that a first price auction would have the same outcome. It might even be preferred to a second price auction by the government as it is less vulnerable to collusion. But for the sake of simplicity a second price auction has been chosen.

[4]The productivity parameter of the second highest bidder is the (k-1)th order statistic of a sample of k random variables drawn from a [0,1] uniform distribution:
$E[X_{(k-1)}] = \frac{k!}{(k-2)!} \int_0^1 x^{k-1}(1-x)dx = \frac{k-1}{k+1}$

price v is. It seems to be reasonable to assume, that the innovator belongs to the class of state-of-the-art producers with $\varphi(i) = 1$. He possesses the know-how to make the most productive use of his own innovation. Therefore in most auctions, the seller's reservation price will lie above the second highest bidder's valuation. For a large k, that is for a large number of participants in the auction, the expected auction outcome will get arbitrarily close to the seller's private valuation. Together with a government mark-up $\phi > 1$, the innovator's incentive constraint will be fulfilled.

In the following we will assume, that the auction outcome v^A equals the innovator's valuation v. Strictly speaking, this equality only holds for $k \to \infty$, which does not seem to be a reasonable assumption for an auction. We will have to be careful later on, when trying to evaluate the welfare properties of the government mark-up. Government mark-ups equal to or smaller than one will violate the incentive constraint (24), and thus are no possible solutions.

2.4 Balanced Growth Path

We are now able to compute a balanced growth path. The six central equations characterizing the macroeconomic equilibrium are

$$
\begin{aligned}
L &= 1 - \frac{b}{(1-\theta)\, w} \quad \text{(aggregate labor supply)} \\
X &= \frac{\gamma\, \alpha^{\frac{1}{1-\alpha}} + 1 - \gamma}{w\left[\gamma\, \alpha^{\frac{\alpha}{1-\alpha}} + 1 - \gamma\right]} \quad \text{(aggregate supply of consumption goods)} \\
\phi\, v &= \frac{w\, a}{n} \quad \text{(free entry condition in research)} \qquad (26) \\
\frac{\dot{n}}{n} &= \frac{L_r}{a} \quad \text{(research technology)}
\end{aligned}
$$

$$
\begin{aligned}
\dot{v} &= \rho\, v - \Pi^{M} \quad (\text{ no arbitrage condition }) \\
\dot{n}\,(\phi-\gamma)\,v &= \theta\, w\, L \quad (\text{ government budget constraint }) \qquad (27)
\end{aligned}
$$

The government budget constraint states, that at each moment in time the government's tax revenues must match the government's expenses for the patent buy-out mechanism. The government pays $(\phi - 1)\,v$ with probability γ, and $\phi\, v$ with probability $1-\gamma$. The instantaneous flow of new products is $\dot{n}$. Therefore, total government expenses are $\dot{n}\,(\phi-\gamma)\,v$.

Let q be the share of total labor employed in the research sector

$$
q = \frac{L_r}{L}. \tag{28}
$$

Along a balanced growth path this share is constant, that is a share $1-q$ of the total labor force works in manufacturing, the rest is engaged in research. Combining the (9),(21), (26), and (27), we can solve for the tax rate as a function of the constant share of labor employed in research

$$
\theta = \left(\frac{\phi-\gamma}{\phi}\right) q \tag{29}
$$

Furthermore we know, that the amount of labor employed in manufacturing L_m equals the aggregate supply of consumption goods X. This allows us to rewrite q as

$$
q = \frac{L-X}{L} = \frac{1 - \frac{b}{[\phi-(\phi-\gamma)q](vn)/a} - \frac{a\,(\gamma\,\alpha^{\frac{1}{1-\alpha}}+1-\gamma)}{\phi\, v\, n\left[\gamma\,\alpha^{\frac{\alpha}{1-\alpha}}+1-\gamma\right]}}{1 - \frac{b}{[\phi-(\phi-\gamma)q](vn)/a}} \tag{30}
$$

which can be solved for q as a function of the product of the endogenous variables v and n.

Now define $V \equiv 1/(v\,n)$. Deriving with respect to time we find that

$$\frac{\dot{V}}{V} = -\frac{\dot{v}}{v} - \frac{\dot{n}}{n}$$

Substituting the no arbitrage condition and the expression derived for the monopolist's profit (20) into the above equation, and defining $g \equiv \frac{\dot{n}}{n}$ as the growth rate of product variety, yields

$$\frac{\dot{V}}{V} = \left[\frac{(1-\alpha)\,\alpha^{\frac{\alpha}{1-\alpha}}}{(\gamma\,\alpha^{\frac{\alpha}{1-\alpha}} + 1 - \gamma)} \right] V - \rho - g\,. \tag{31}$$

From the research production function, aggregate labor supply, the free entry condition, as well as (30) we can express g as a function of V

$$g = \frac{1}{a} \left[1 - \frac{a\,b\,V}{[\phi - (\phi - \gamma) q(V)]} - \frac{a\,V\,(\gamma\,\alpha^{\frac{1}{1-\alpha}} + 1 - \gamma)}{\phi\,\left(\gamma\,\alpha^{\frac{\alpha}{1-\alpha}} + 1 - \gamma\right)} \right] \tag{32}$$

At each moment in time γn patents are held by monopolists. The value of their firms on the stock market is the value of their patent v. The total stock market value equals $\phi\,n\,v = \phi\,/V$. Thus $-\dot{V}/V$ represents the growth rate of the total stock market value. () and () allow us to characterize a balanced growth path under the condition that the total stock market value is constant ($\frac{\dot{V}}{V} = 0$), that is the value of patents v declines at the same rate at which product variety n increases. Figure 1 represents that solution.

Furthermore a simple argument shows, that this is the only possible growth path. Every equilibrium point has to be situated on (32), as this equation represents the technological constraint on growth. At every point above the

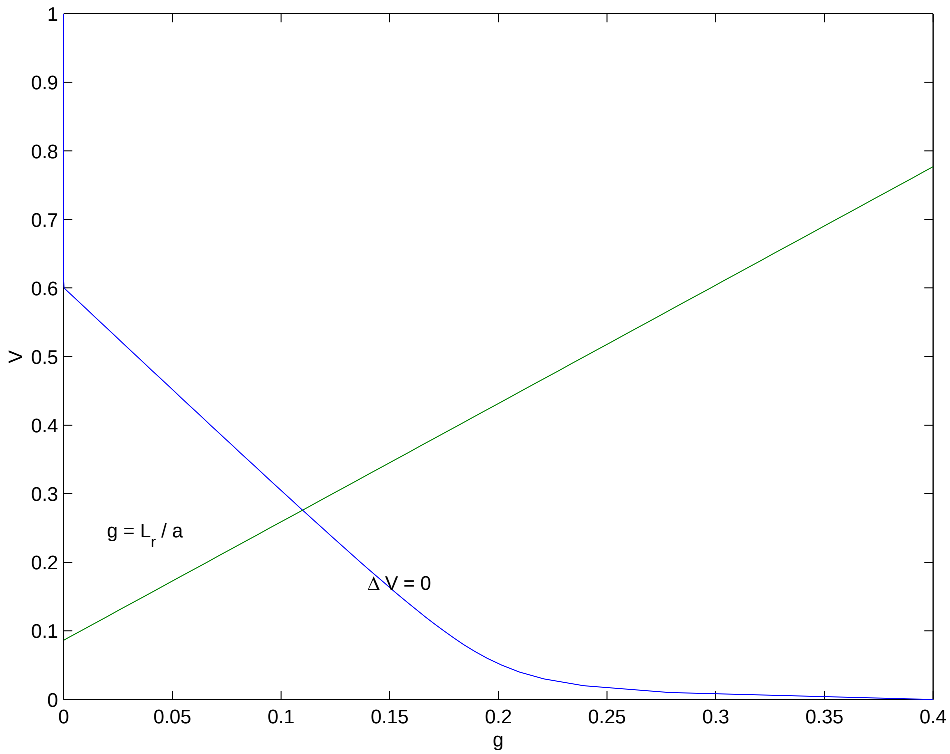

Figure 1:

$\dot{V}$-line, the differential equation (31) states, that $\dot{V} > 0$. This implies, that V goes to infinity with zero growth in patent designs. If V goes to infinity, and n stays constant, the value of patents v has to go to zero. But a constant number of design guarantees positive and non-declining monopoly profits Π^M. Therefore, given rational expectations of economic agents, v cannot go to zero, and points above $\dot{V} = 0$ cannot be rational equilibria.

Likewise, points below $\dot{V} = 0$ entail $\dot{V} < 0$. V goes to zero, as growth in patent designs approaches its maximum at $1/a$. However, observe that monopoly profits decrease with increasing product variety. Thus $v(t)$ is bounded from above by

$$v(t) = \int_t^\infty e^{-\rho(\tau - t)} \Pi^M(\tau) d\tau < \int_t^\infty e^{-\rho(\tau - t)} \Pi^M(t) d\tau = \frac{(1-\alpha)\,\alpha^{\frac{\alpha}{1-\alpha}}}{\rho n(t)(\gamma\,\alpha^{\frac{\alpha}{1-\alpha}} + 1 - \gamma)}$$

for a positive growth rate g. Obviously

$$v(t)\,n(t) < \frac{(1-\alpha)\,\alpha^{\frac{\alpha}{1-\alpha}}}{(\gamma\,\alpha^{\frac{\alpha}{1-\alpha}} + 1 - \gamma)}$$

and thereby, $V = 1/(vn)$ cannot converge to zero. Points below $\dot{V} = 0$ cannot represent balanced growth paths.

2.5 Welfare Analysis

To evaluate the welfare properties of the balanced growth path we have to solve for the social optimum. Economic agents have a preference for diversity in consumption. Therefore, given labor input L_m in manufacturing, which represents a share $1-q$ of total labor input L, and a number n of different goods in the economy, in the social optimum agents consume $x(i) = L_m/n$ of good i for all $i \,\epsilon\, [0, n]$ at all moments in time. The aggregate consumption index C thus is

$$C = \left[\int_0^n x(i)^\alpha di\right]^{\frac{1}{\alpha}} = n^{\frac{1-\alpha}{\alpha}}\,(1-q)\,L$$

The social optimum is the solution to a dynamic maximization problem in two control variables, the total labor supply L, and the relative shares devoted to manufacturing and research $1-q$ and q, and one state variable, n, the number of different consumption goods. We can set up a Hamiltonian to assess the social optimum:

$$H = \frac{1-\alpha}{\alpha} \log n + \log(1-q) + \log L + b \log(1-L) + \lambda \left(\frac{q\,L\,n}{a} \right) \tag{33}$$

The first order conditions for this problem are

$$\frac{1}{1-q} = \lambda \frac{L\,n}{a} \tag{34}$$

$$\frac{b}{1-L} = \frac{1}{L} + \lambda \frac{q\,n}{a} \tag{35}$$

$$\dot{\lambda} = \rho\,\lambda - \frac{1-\alpha}{\alpha\,n} - \lambda \frac{q\,L}{a} \tag{36}$$

Define the variable $M \equiv \lambda\,n$, representing the shadow value of total product variety. The derivative of M with respect to time is

$$\frac{\dot{M}}{M} = \frac{\dot{\lambda}}{\lambda} + g$$

Substituting the above expression into (36), and noticing that the research technology states $g = qL/a$, yields the following differential equation in M:

$$\dot{M} = \rho M - \frac{1-\alpha}{\alpha} \tag{37}$$

Optimization of economic agents imposes a transversality condition on this differential equation. The discounted shadow value of total product variety needs to converge to zero as time goes to infinity.

$$\lim_{t\to\infty} e^{-\rho t} M(t) = 0 \tag{38}$$

The only solution to (37) satisfying the transversality condition is a constant value for M, that is $\dot{M} = 0$.

$$M = \frac{1-\alpha}{\alpha\,\rho} \tag{39}$$

Thus the shadow value of an additional product design decreases with increasing product variety

$$\lambda = \frac{1-\alpha}{\alpha\,\rho\,n} \tag{40}$$

We are now able to calculate the socially optimal supply and sectoral allocation of labor.

$$L^{*} = 1 - \left(\frac{\alpha}{1-\alpha}\right)\rho\,a\,b \tag{41}$$

$$q^{*} = 1 - \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{a\,\rho}{1-a\,b\,\rho}\right) \tag{42}$$

The corresponding optimal growth rate is

$$g^{*} = \frac{q^{*}\,L^{*}}{a} \tag{43}$$

Kremer (2001, [10]) claims, that the government mark-up ϕ should be chosen such that it raises the private value of a patent v to its social value. Such a mark-up would ensure a socially optimal incentive for research.

The social value of a patent has been calculated when solving for the social optimum. It is the shadow value of product variety λ. Following Kremer, the policymaker needs to choose ϕ to satisfy

$$\phi\,v = \frac{1-\alpha}{\alpha\,\rho\,n} \tag{44}$$

Combined with the free entry condition into the research sector (26), the equalization of private and social value implies a fixed wage rate equal to

$$w^r = \frac{1-\alpha}{\alpha\, a\, \rho} \tag{45}$$

We are now able to evaluate Kremer's proposal in a general equilibrium framework with flexible labor supply. Three major problems in reaching the social optimum through government intervention arise. Firstly, the government mark-up has to be financed by taxes. These taxes distort the relative prices of consumption to leisure, and thus crowd out labor input.

Secondly, the aim to put innovations into the public domain and thereby foster competitive production and pricing of goods, introduces a new inefficiency into the economic system. In the absence of patent buy-outs all firms price monopolistically. In our framework they all set the same price, namely a mark-up over marginal costs, depending on the elasticity of substitution between any two goods. With patent buy-outs, part of the producers price at marginal costs. This leads consumers to consume too much of the competitively produced goods, and not enough of the monopolistically produced goods. Relative prices in the manufacturing sector are distorted.

Finally, the presence of competitively producing firms lowers the monopoly profits of patent owners. But lowering monopoly profits has the effect of lowering the private incentives for research. The auction price of patents v^A will fall with increasing $1-\gamma$, the share of competitively produced goods. Therefore, the introduction of a patent buy-out mechanism will partly have a negative effect on innovation. The government mark-up will have to do more than just raise the private value of patents, observed before the introduction of the mechanism, to their social value. It will also have to offset the fall in the private value of patents induced by its own introduction.

Table 1 to Table 3 summarize the numerical evaluation of Kremer's proposal.[5] In all three tables ϕ has been chosen such that the wage rate satisfies equation (45), that is the mark-up raises the private value of a patent to its social value. Table 1 and 2 treat the case of a fixed labor supply. The above mentioned crowding out effect of labor through taxes does not occur. This allows us to analyze the potentially negative effects of price distortion in the manufacturing sector. Two parameters of the model should be essential for this effect, γ and α. γ determines the share of competitively producing firms in the economy, and thus the degree of distortionary pricing through coexistence of competitive sectors and monopolists. Optimally, to minimize this distortion, the policy maker would have to set γ to an extreme value, either close to 1, or close to 0. A value close to 1 runs against the idea of the patent buy-out mechanism, as it maximizes the share of monopolistic producers. A value of γ close to 0 is equivalent to maximizing the share of competitive sectors in the economy, while keeping an infinitesimal share of monopolists, in order to guarantee the functioning of the patent auction as a revelation mechanism for the private value of patents. However, table 1 reveals the impossibility of this strategy. In fact, Kremer's proposal for setting ϕ seems to be successful in getting close to the social optimum.[6]The results of table 1 might be stunning at first sight. With increasing γ, the mark-up increases,

[5]The computer code to solve for the equilibrium values of the balanced growth path can be found in the Appendix. The parameter values have been set to $\alpha = 0.5$ (Table 1 and 3), $\gamma = 0.05$ (Table 2), $a = 3$, and $\rho = 0.05$.

[6]For the chosen parameter values, optimal values for q and g appear in the line designated "opt." in Table 1 and 3. g^* designates the optimal growth rate in Table 2

too. But less competition should normally raise monopoly profits, and thus the private value of patents. This would mean a smaller mark-up with rising γ. But monopolists produce less than competitive firms. By consequence, the higher γ, the lower the demand for labor, and thus the lower the wage level. The costs for research falls with the wage level, and thereby the private value of a patent. This effect seems to dominate the positive effect of γ on monopoly profits. Consequently, ϕ has to rise with γ.

As conjectured above, with decreasing γ we approach the socially optimal growth rate. Lowering γ lowers ϕ, but the government is constrained in its choice of ϕ. ϕ has to be bigger than 1, otherwise the innovator's incentive constraint for the patent auction is violated, and the patent buy-out mechanism breaks down. Due to the price distortions induced by the presence of monopolists in the manufacturing sector, and the government's constraint in setting γ, even in the absence of labor supply crowding out through taxes, the patent buy-out mechanism cannot achieve the first best outcome.

The second factor, that should have an influence on price distortions in the manufacturing sector is α. α determines the elasticity of substitution between different goods. The lower α is, the lower is this elasticity, and the higher is the monopoly mark-up over marginal costs. But higher mark-ups mean larger price differentials between monopolists and producers pricing at marginal costs, and thereby a larger distortion of relative prices. Table 2 contains patent auction equilibrium values and social optima for different values of α. A systematic deviation from social optimum for low values of α could not be found. Nevertheless, social optimum cannot be reached for the above mentioned reasons. Table 2 reveals two interesting (even though obvious) results. Less substitutability between goods should raise the government mark-up, as variety has a higher social value. In social optimum, a lower value of α comes together with faster growth in product variety. The second observation is, that for higher values of α, zero growth in product variety might be optimal.

Table 3 contains equilibrium values for the model with flexible labor supply. Obviously, now the taxes, that are raised to finance the patent buy-out lead to a reduction in labor supply. While in the case of a fixed labor supply, even though the social optimum could not be reached, Kremer's strategy performed reasonably well in approaching the optimum, in the presence of a flexible labor supply, growth in product variety is only half of what it should optimally be. Is the strategy of targeting the social value of a patent in the choice of ϕ still the best strategy in the presence of distortionary taxes ? Should ϕ rather be chosen such that it maximizes the growth rate of product variety? Or should the policy maker try to control the sectoral allocation of labor to the social optimum q^* ?

In order to answer these questions, we will numerically calculate the utility for different choices of γ and ϕ, and compare their welfare properties. Notice that the aggregate consumption index C can be written as a function of n, which is time dependent, and w, which along a balanced growth path was seen to be constant.[7]

[7] $w = \frac{\phi}{a\,V}$. Along a balanced growth path V is constant. Thus w is constant, too.

$$C(\tau) = n(\tau)^{\frac{1-\alpha}{\alpha}} \left[\frac{(\gamma\,\alpha^{\frac{\alpha}{1-\alpha}} + 1 - \gamma)^{\frac{1-\alpha}{\alpha}}}{w} \right]$$

Along the balanced growth path, total labor input L is also time-invariant. Thus utility U can be expressed as

$$U = \int_0^\infty e^{-\rho\tau} \left[\log C(\tau) + b \log L(\tau)\right] d\tau$$

$$= \frac{1}{\rho} \left\{ \log \left[\frac{(\gamma\,\alpha^{\frac{\alpha}{1-\alpha}} + 1 - \gamma)^{\frac{1-\alpha}{\alpha}}}{w} \right] + b \log (1-L) \right\} + \frac{1-\alpha}{\alpha} \int_0^\infty e^{-\rho\tau} \log n(\tau)\, d\tau$$

$$= \frac{1}{\rho} \left\{ \log \left[\frac{(\gamma\,\alpha^{\frac{\alpha}{1-\alpha}} + 1 - \gamma)^{\frac{1-\alpha}{\alpha}}}{w} \right] + b \log (1-L) \right\} + \frac{1-\alpha}{\alpha\,\rho} \left(n(0) + \frac{g}{\rho} \right)$$

where g represents the balanced growth rate of product variety, and $n(0)$ the economy's initial product variety.

The above expression of U allows us to compute welfare as a function of the parameter values, the equilibrium values of w, and L, as well a given $n(0)$.

Figures 2 - 5 show the results of the computations for different parameter values α, and b, and the policy variables γ, and ϕ. The y-axis represents utility U, on the x-axis appear the ϕ values, different subplots represent different levels of γ. The horizontal bar in the graphs indicates the utility level, that would be reached by the representative agent in the absence of the patent buy-out mechanism.

Except for very small values of γ, all graphs have an inverted U-shape. At first, utility increases in the government mark-up, but later on declines with increasing ϕ. For small values of ϕ the positive effect of increased growth in product variety induced by a higher mark-up dominates. For high values of ϕ

the negative effects of high tax rates and a low level of current consumption due to a high level of employment in the research sector dominate.

For a low level of α (here $\alpha = 0.1$), that is weak product market competition, agents are better off in the standard model without patent buy-outs than in the model with patent buy-outs.

Furthermore the graphs show, that for achieving the the maximum attainable utility, the policy maker has to combine both instruments ϕ, and γ in an coordinated way. For the chosen parameterization, the maximum is reached for a combination of moderate γ (around 0.05) and a mark-up the doubles the private value. Mark-ups that are set too low, or too high may eventually lead utility of the patent buy-out to be inferior to the perfect patent protection model.

3 Conclusion

The above analysis shows, that picking the right combination of mark-up and buy-out-probability is a very sophisticated choice, which, if badly performed, might even lead to agents being worse off with patent buy-outs than with total patent protection. It order to pick the right values, the policy maker needs to be very well informed about the degree of product market competition, and the workers' preferences for labor and consumption.

We have seen that the buy-out mechanism introduces three new inefficiencies into the economic system, namely relative price distortion in the leisure-labor choice through taxation, relative price distortion in the manufacturing sector due the the simultaneous presence of monopolistic, and competitive producers, and finally decline in the private incentives for innovation caused by declining monopoly profits because of increased competition in the manufacturing sector.

For a fixed labor supply, the social optimum can be approached by choosing the share of competitive firms as large as possible without violating the innovator's incentive constraint to participate in the auction, while simultaneously setting ϕ to raise the private value of a patent to the socially optimal level.

In the presence of a flexible labor supply, this policy fails to ensure closeness to the social optimum. Maximizing the share of competitive firms fails to attain the maximum level of utility.

Finally weak product market competition should not be an incentive for the policy maker to foster competition through a patent buy-out mechanism. Indeed, that the contrary is true.

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4 Appendix

4.1 Computer Code

```
global gamma alf phi a b rho LL LM
% program solving for g in patent auction model

b = 1.; % parameter for disutility of labor
alf = .5; % determines elasticity of substitution
          % 1/(1-alf) between cons. goods
a = 3; % productivity parameter of research technology
gamma = 0.05; % buy-out probability of government
rho = 0.05; % subjective discount factor
n0 = 10; % initial blueprints; needed for numerical welfare evaluation

phi_mult = 0.1:0.05:50; % patent buy-out mark-up over private value v
phi_size = size(phi_mult);

x0 = [0.5;0.3]; % starting values
gv = ones(phi_size(2),9);

for i= 1:phi_size(2)
    phi = phi_mult(i);
    gv(i,8) = phi;
    [x,fval] = fsolve(@growth,x0) % uses the function "growth"
```

```
                                % to solve for solutions of
                                % the system (29) and (30)
    gv(i,1:2) = x';             % equilibrium values of V and g
    gv(i,5) = 1 - (LM/LL);      % q; share of labor force
                                % employed in research
    gv(i,7) = ((phi - gamma)/phi)*gv(i,5);  % tax rate
    gv(i,3) = (phi/a)*(1/x(2));             % wage rate
    gv(i,4) = (1-gv(i,7))*(phi/a)*(1/x(2));
    gv(i,6) = LL;                           % total labor supply
    gv(i,9) = 1/rho*( log(((gamma*alf^(1/(1-alf)) + 1 - gamma)
              /(gv(i,3)*(gamma*alf^(alf/(1-alf)) + 1 -
              gamma)))^(1/alf))+b*log(1-LL) +
              ((1-alf)/alf)*(log(n0) + gv(i,1)/rho));   % utility
    x0 = x;  % assign equilibrium values of V and g as starting
             %values for next iteration
end


wage_kremer = (1-alf)/(alf*a*rho);  % see equation (43)


L_opt = 1 - (alf/(1-alf))*rho*a*b;  % see equation (39)
q_opt = 1 - (alf/(1-alf))*(a*rho/(1-a*b*rho)); g_opt =
        (q_opt*L_opt)/a;    % see equation (40)
g_sans = (1-alf)/(1+b)*(1/a + rho) - rho; % variety growth rate
                                    % without buy-out mechanism
```

```
V_sans = (1/(1+b))*(1/a + rho); % inverse stock market value without
q_sans = alf/((1/(a*V_sans))-b); % research labor without
L_sans = 1 - a*b*V_sans; % total labor supply without
U_sans = (1/rho)*(log(((gamma*alf^(alf/(1-alf)) +1 -
gamma)^((1-alf)/alf))./(1/(a*V_sans))) + b*log(1-L_sans)) +
(1-alf)/(rho*alf)*(n0 + g_sans/rho); % utility without



function F = growth(x) global gamma alf phi a b rho LL LM

% solves equation () for q

BB = (a*x(2)*(gamma*(alf^(1/(1-alf))) + 1 -
gamma))/(phi*(gamma*(alf^(alf/(1-alf)))+1-gamma));
AA
=(a*b*x(2)-2*phi + gamma + (phi-gamma)*BB)/(phi-gamma);
CC = (phi
- a*b*x(2) - phi*BB)/(phi-gamma);

q = -0.5*AA - sqrt((AA.^2)/4 - CC);

% q negative implies, that the research sector should
% be inactive. Therefore, q = 0 in such cases.

if q < 0
```

```
    q = 0;
end

% Calculate total labor supply LL, from first order condition ()
% LM is labor employed in manufacturing

LL = 1 - (a*b*x(2))/(phi - (phi-gamma)*q);
LM = (1-q)*LL;

% calculates for given values of g and V
% the residuum of (29) and (30)

F = [x(1) - 1/a*(LL - LM);
     x(1) - ((1-alf)*(alf^(alf/(1-alf))))/
     (gamma*(alf^(alf/(1-alf)) + 1 -gamma))*x(2) + rho ];
```

4.2 Tables and Graphics

Table 1: $b = 0$, Variation in γ

		g	q
opt.		0.28	0.85
γ	ϕ		
(0.01	0.40	0.283	0.851)
0.05	1.94	0.284	0.852
0.15	4.44	0.285	0.856
0.25	8.42	0.286	0.861
0.50	13.67	0.292	0.875
0.75	15.68	0.298	0.895

Table 2: $b = 0$, Variation in α

α	ϕ	g	q	g^*
0.1	8.42	0.328	0.984	0.328
0.3	3.13	0.312	0.937	0.312
0.6	1.57	0.259	0.777	0.258
(0.9	1.00	0.091	0.270	0)

Table 3: $b = 1$

		g	q	L
opt.		0.233	0.824	0.85
γ	ϕ			
(0.01	0.21	0.126	0.711	0.527)
0.05	1.02	0.126	0.712	0.526
0.15	2.85	0.126	0.724	0.522
0.25	4.40	0.126	0.730	0.517
0.50	7.04	0.126	0.752	0.503

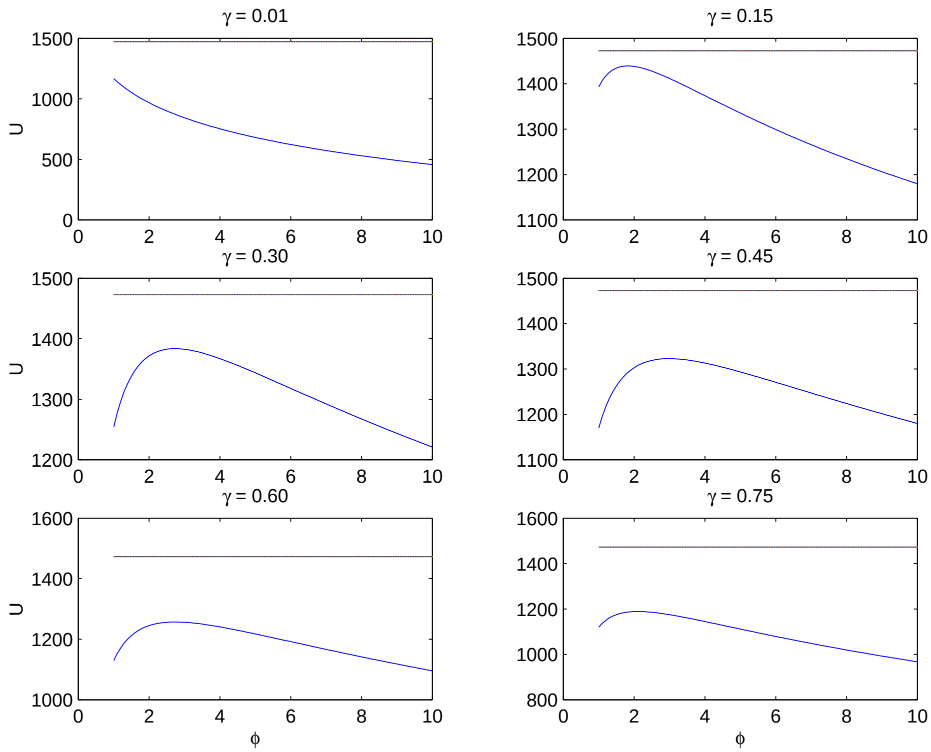

Figure 2: $\alpha = 0.1, b = 1$

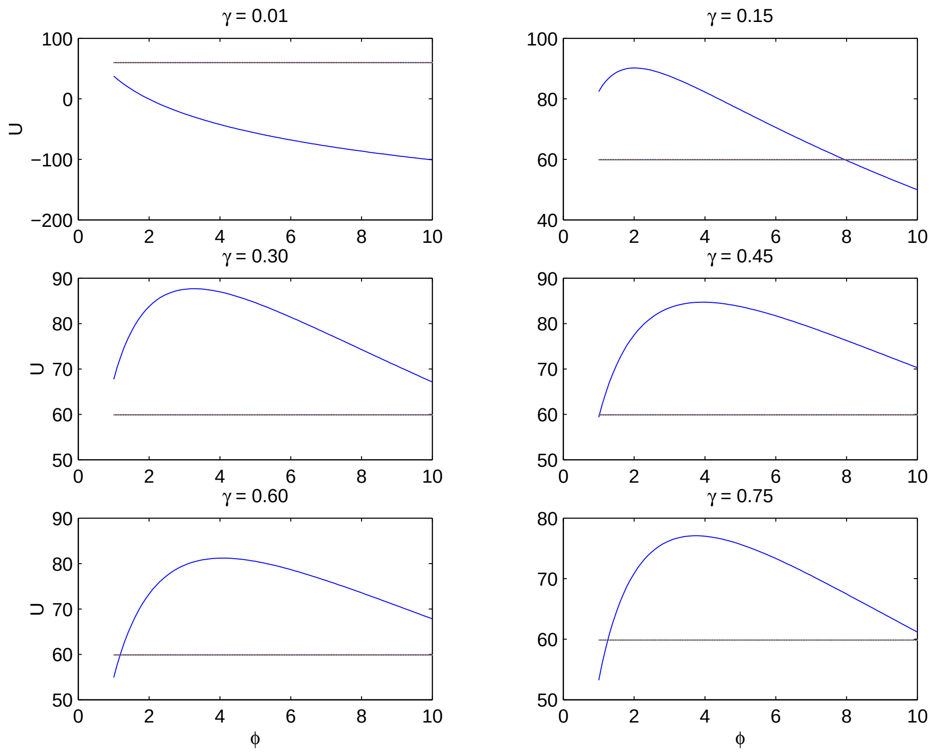

Figure 3: $\alpha = 0.5, b = 1$

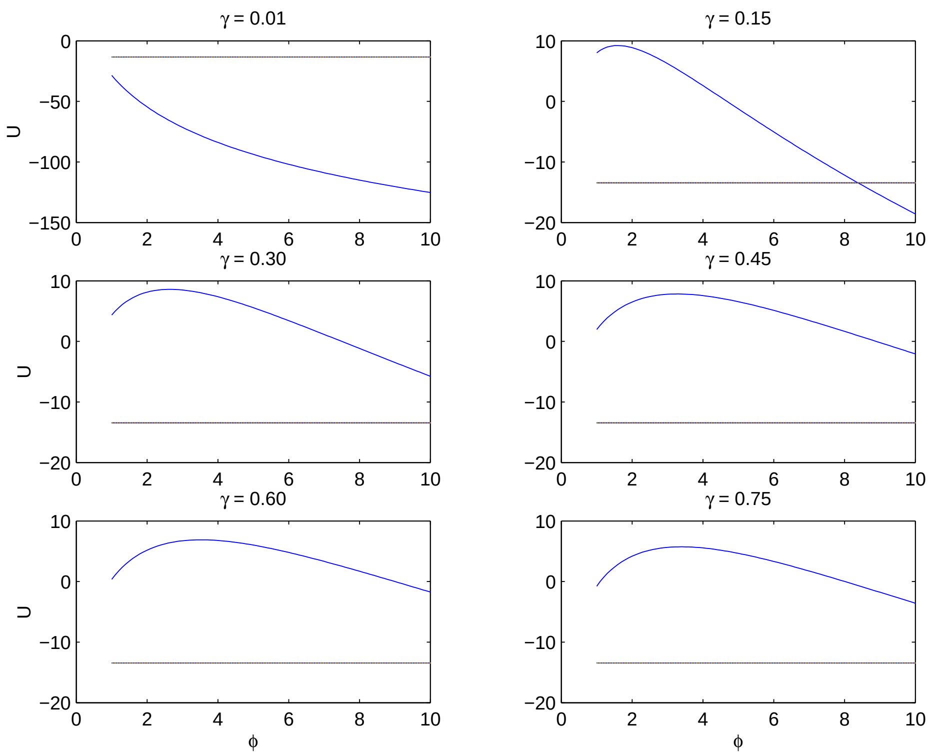

Figure 4: $\alpha = 0.7, b = 1$

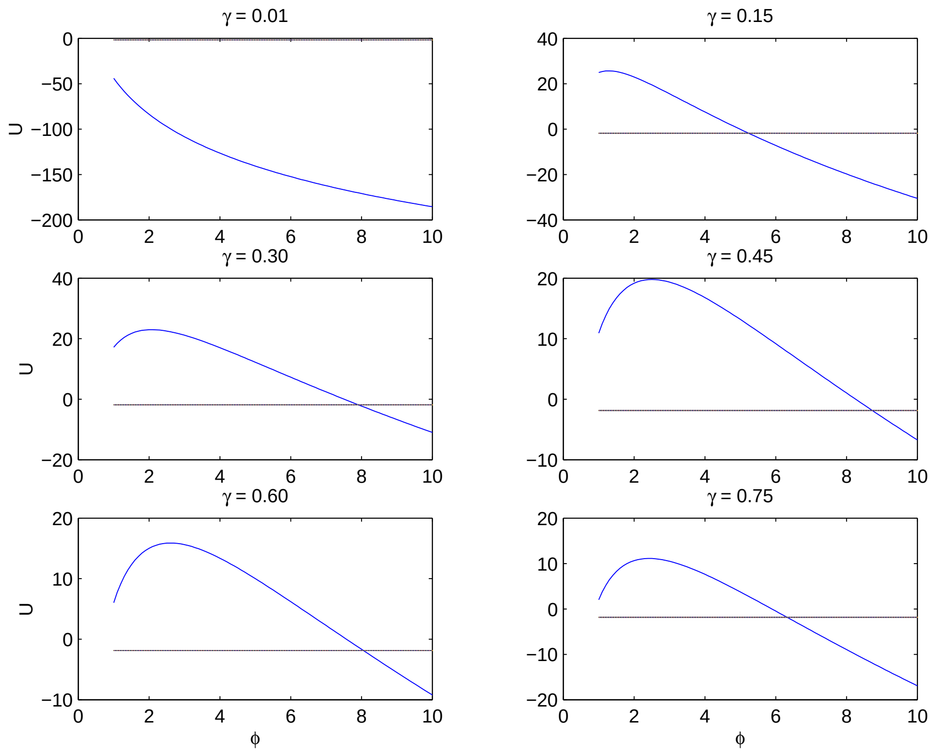

Figure 5: $\alpha = 0.5, b = 2$