Consumption CAPM and Cross Section of Expected Returns

Master Thesis

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Presented by

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Abstract

Sharp incapability inherited in a class of Consumption CAPM models in explaining cross-section of returns on size and book to market ratio sorted portfolios was present and has been overcame by two consumption model. With conditioning information provided by consumption wealth ratio or non-housing consumption ratio, these models perform well with US data. This paper tries to verify its applicability to UK data. A similarly sorted data and an industry sorted data helps to find out, the conditioning information of non-housing consumption ratio is very rich and the empirical work within two markets is comparable. However, finite sample distribution of Fama-MacBeth procedure should be considered in further analysis.

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Chapter 1

Introduction

With first investigation to Consumption-based asset pricing theory by Lucas (1978), intertemporal substitution channel of consumption has been used to explain return on risk free asset and risky asset in a large volume of literature. In last two decades, the development of this theory has been marked as a major achievement in the field of financial economics (Campbell and Cochrane 2000). However, these model always generate disappointing empirical estimation. On time series domain, consumption based models are significantly outperformed by portfolio based CAPM models. Such a poor performance can be attributed to errors of measurement in consumption data or incorrect specification of representative agent's preference (Cochrane 1996).

Although CAPM is superior in time series predictability, it also suffers from lack of power in explaining cross sectional variation on Fama and French size and book to market sorted US data as shown in Figure 1.1. The fitted and realized average return on portfolios should lie on a 45° line. But regression on CAPM reveals the fact that it forecast different returns with similar outcome. The canonical CAPM model has also been tested, Lettau and Ludvigson (2001) report a R-square of 16%, better than CAPM of 1% but is still in short of prediction power.

To see how such a test can be made, one can derive models and find its associated information set, such as macroeconomic variables in consumption based models. As a common sense, stock returns can be predicted with a less information set than that of trader in a conditional model. It is intuitive to search for important variables along this direction. Because consumption models always inherit non-linear functional form, it is difficult to estimate directly. One solution is, although there is no acceptable evidence, to turn to linear consumption based factor pricing models with macroeconomic variables, because they are well correlated to stock returns. In this direction two papers are equally appealing both theoretically and empirically. Lettau and Ludvigson (2001) infer the power of consumption wealth ratio as conditioning variable while Piazzesi, Schneider and Tuzel (2003) develop a consumption model on separability of consumption in housing and non-housing sector. The second model perform very well in both time series and cross sectional domain with US data.

This paper in principle develops no new theories. Instead it aims to test existing models and procedure on a set of new data. The purpose is to justify these two models of consumption based CAPM with respect to their performance on United Kingdom data. In a theory-based search for a good consumption model, I find comparable empirical evidence in UK dataset. A standard second pass Fama-MacBeth (1973) procedure depicts very good cross sectional result for both models. Although the procedure has limitations and the data

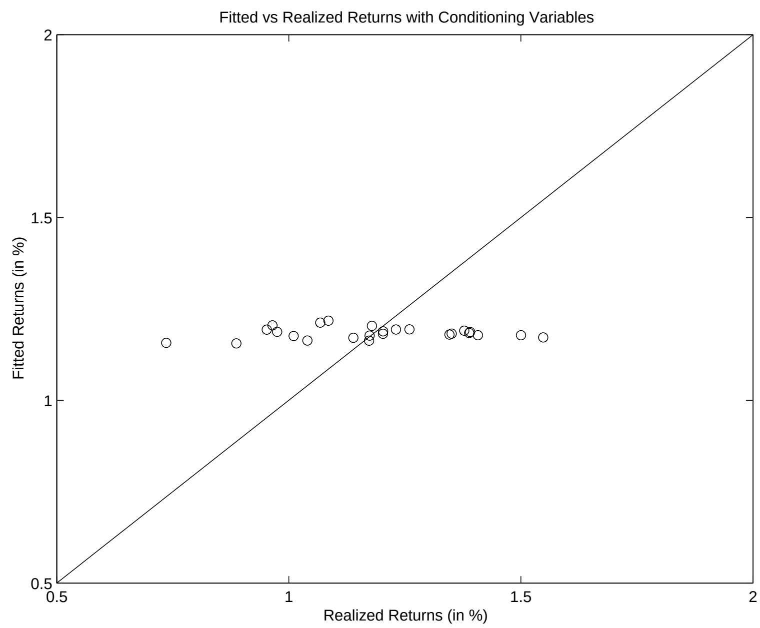

Figure 1.1: Model: US Static CAPM with Fama French 5*5 Portfolio (R-square 0.01)

collection is not perfect, the estimation can still be interpreted as a bigger success of the housing consumption model.

Rest of this paper is organized as following. Chapter 2 summarize consumption CAPM and its empirical application in last 3 decades and make some verified link between CAPM and consumption CAPM; Chapter 3 picks out the models that can explain cross sectional data in a class of factor pricing CAPM model or consumption CAPM models that implied time varying pricing kernel; Chapter 4 introduce the dataset, preparative works and empirical procedure; Chapter 5 list main results, makes some comparison to its extensions and discuss the robustness of empirical work; the last Chapter makes conclusive remarks.

Chapter 2

Literature on CCAPM

2.1 Theories of Consumption and Asset Returns

I begin the topic of theoretical CCAPM with Lucas's (1978) basic pricing equation below. A wide class of intertemporal asset pricing models can be represented as such a form. M_{t+1} denotes Stochastic Discount Factor (SDF) or pricing kernel. Return of every asset, risk free or risky, can be derived from SDF.

$$1 = E_t[M_{t+1}R_{t+1}] \text{ and } P_{t+1} = E_t[M_{t+1}X_{t+1}] \tag{2.1}$$

The SDF M_{t+1} in 2.1 for Consumption based asset pricing model can be obtained by deriving first order condition of intertemporal substitution problem of consumption. Explicitly, M_{t+1} is $\beta\frac{u'(C_{t+1})}{u'(C_t)}$, which implies risk free rate and β is discount factor. In such problem, the choice of representative agent's utility function play a very important role for its equilibrium solution. For component of consumption, it is widely accepted to use a constant relative risk aversion (CRRA) model of power utility over consumption. Relative risk aversion (RRA) is defined as $-\frac{C \cdot U''_C}{U'_C}$, as a measure for preference curvature, or degree of sensitivity to intertemporal consumption substitution. Utility of the form $U(C_t) = \frac{C^{1-\gamma}}{1-\gamma}$ yield a RRA of γ, which has important implication for further empirical investigation for earlier Consumption CAPM researchers. Such model setting delivers good implication for relation between real interest rate and consumption behavior. However, it also implies a lower Sharp Ratio. With US data of consumption and stock return, a calibration on Sharp Ratio can only hold with a CRRA bigger than 50, which is beyond economic intuition. That is called Equity Premium Puzzle. In order to explain excess return and volatility of return series, Consumption base model also leave other puzzles open, such as risk free rate puzzle and excess volatility puzzle, etc. Evidence from data motivated new theories and models to explain these anomalies. Habit Formation model is one of them. By assuming a time non-separability of utility on consumption, which is by definition caused by a "habit" in consumption, the model can simulate stock return data very similar to actual series with respect to both first and second moment properties.

However, models that succeed in explaining these puzzles are usually not easy to be explicitly estimated for the highly non-linear nature and non-separability of consumption utility among periods. GMM is a good tool in solving CCAPM empirical problems, as innovated by Hansen and Singleton (1982). However, estimate from GMM always depend on choice of instrumental variables. With different set of instruments, the estimate result might differ significantly, although in both cases you could not reject form J-statics of overidentification. When coming back to canonical Consumption CAPM models, they are rejected on US data and international data, see Hansen and Singleton (1982) and Wheatley (1988).

Two of recent papers pioneered in the direction to attention to both model implications and empirical superiority. Wachter (2002) developed an estimation strategy with GMM based on generalized habit formation model of Campbell and Cochrane (1999). Piazzesi, Schneider and Tuzel (2003) construct a housing model and estimated its parameters. Both researches achieve significant parameter estimation and convincing results of simulation.

Furthermore, Lettau and Ludvigson (2000, 2001) explain time series and cross section variation of US portfolios returns with a residual called *cay* of the shared trend among consumption (c_t), asset holding (a_t) and labor income (y_t). These updated research output, although tackling with different assumption, all perform well empirically. In next chapter, I introduce them in detail and try to work on cross sectional regression with UK data.

2.2 Empirical Consumption CAPM and Beta Representation

The focus of this part is to formulate a viable procedure to test explicitly a consumption based model with respect to its ability in explaining time series and cross section of asset returns. To deal with non-linearity and large number of parameters, GMM can be used given more moments condition provided by instrument variables is achieved. As

I stated before, many GMM estimation on consumption based CAPM model fails to deliver acceptable econometric results. In fact, GMM method can be linked to scaled beta representation of CAPM models, or factor pricing models by their joint properties of using instruments for improved estimation result. Indeed, an equivalent estimation strategies can be observed for these two methods.

2.2.1 Beta Representation of CAPM Models

Alternatively, one can write CAPM model in beta representation. This representation is a multi-factor extension to Sharpe (1964) and Lintner's (1965) asset "beta" measure for a formal description between systematic risk and firm specific risk. In other words, asset return is determined by some variables in an information set, each have a certain beta with individual asset return.

$$E[R_i] = \gamma_0 + \sum_{s=1}^{K} \gamma_s \beta_{i,s} = b'\lambda \tag{2.2}$$

In 2.2 $b'\lambda$ denote its matrix representation, where $b = \begin{pmatrix} 1 & \beta_1 & ... & \beta_K \end{pmatrix}'$ and $\lambda = \begin{pmatrix} \gamma_0 & ... & \gamma_K \end{pmatrix}$. A general consumption CAPM can be derived as a factor pricing CAPM model by writing $M_{t+1} = a + b \cdot Endo_{t+1}$ in 2.1 and rewriting for $E[R_{i,t+1}] - R_0 = \beta_{i,t} \cdot (Endo_{t+1} - R_0)$, with trivial rearrangement operation. Note: i denotes a representative asset return, R_0 is a series of return uncorrelated to $Endo_{t+1}$, there is no time subscript because of zero beta with $t+1$ information and constant a and b; $\beta_{i,t}$ represent $\frac{Cov(R_{i,t+1}, Endo_{t+1})}{Var(Endo_{t+1})}$, it's a beta coefficient between individual asset and explaining variable calculated ex ante. Endogenous variables can be more than one, derivation of equivalence for multiple factor pricing model can be proved in the same way. The unobservable endogenous variables can be market return or other contemporaneous variables that capture pricing information (Campbell, Lo and MacKinlay 1997). Therefore, empirical test for consumption CAPM can follow a similar approach with CAPM by finding a model imply or approximate linear factor relation in SDF, in a multi-factor beta or scaling variable specification. Nevertheless, there is basically distinction between CAPM and Consumption CAPM despite of the common approach. The difference is, CCAPM seeks to interpret the model economy while CAPM can describe the data in hand. However, in either case, the choice of explanatory variables should be cautionary. Otherwise the empirical research will fall into the trap of satisfactory performance over limited data available, as a result of arbitrary chosing factors.

On the contrary, another interesting argument is the equivalence between special form Consumption CAPM and CAPM model. As Cochrane (1999) shows, a consumption CAPM turns to be CAPM when taking total period of model as 2 and an economy with quadratic utility and without labor income. Three examples are provided by Cochrane (1999).

2.2.2 Unconditional and Conditional Factor Pricing Models

I leave conditioning (or scaled) model explained in detail in this section. The assumption for M_{t+1} in last subsection can be amended to time varying intercept and slope, which contain information of end of last period. $M_{t+1} = a_t + b_t \cdot Endo_{t+1}$ is now conditional linear factor model. Conditioning variable b_t can be scaled to improve test result. Hansen and Singleton (1982) firstly introduced scaled factor method to improve estimation performance for models with instrumental variables. Campbell and Cochrane (2000) even found, scaled factor can deliver far better results for cross sectional asset pricing regression given a habit formation component in the model. This argument motivates a thorough test over preferability of scaled model in Chapter 5. Notice here the scaled models can be equivalent to a GMM estimation in terms of using conditioning information. A general expression of scaled or conditioning model is:

$$M_{t+1} = \gamma_0 + \sum_{s=1}^{p} \gamma_s z_{s,t} + \sum_{s=1}^{q} \theta_s R_{s,t+1} + \sum_{s=1}^{l} \gamma_s z_{s,t} R_{s,t+1} \tag{2.3}$$

Empirically, also as show in last subsection, a cross sectional regression on:

$$E[R_{i,t+1}] = E[R_{0,t}] + b'_{z_t} f_{z_t} + b'_{R_{t+1}} f_{R_{t+1}} + b'_{z_t R} f_{z_t R_{t+1}} \tag{2.4}$$

In 2.4 b is a column vector of betas (β), f is vector of constant coefficient (λ) for a given time period. Pricing errors is reported as α_{t+1}. The unconditional model is simply:

$$E[R_{i,t+1}] = E[R_{0,t}] + b'_{R_{t+1}} f_{R_{t+1}} \tag{2.5}$$

2.2.3 Conditioning Variables

The empirical researcher can at most have a subset of information of trader's. This show the actual difficulty for economists to draw a comprehensive mapping of returns over market behavior. Here comes the problem of choosing pricing factor for unconditional or conditional models. There are basically two way to perform a search, one is statistics approach and the other is theoretical approach (Campbell, Lo and MacKinlay, 1997). And there are enough variables one can choose. But this search, for the good conditioning variables, shouldn't be like "fishing" without "explicit connection to real risk", as argued by Cochrane (1996). I specify the pricing factors for the model only based on significance to systematic risk of whole market. The doctrine I adopt in this project is: to approximate a certain candidate model, which is non-linear in most cases, to a linear unconditional or scaled factor pricing models (beta representation); to write conditional model for predicting asset return.

Chapter 3

The Models

This chapter documents several models, either CAPM or consumption CAPM, and associated regression equations. Although the focus is to investigate cross sectional asset pricing tests for consumption models, it is helpful to compare the result with CAPM models for measure of success. First of all, I want to make my result comparable to the result of Lettau and Ludvigson (2001) and Piazzesi, Schneider and Tuzel (2003). This brings me to specify following models for estimation:

3.1 Static CAPM and Fama French 3 Factor Models

Static CAPM model use contemporaneous market excess return as the only explanatory variable in its unconditional model:

$$E[R_{i,t+1}] = E[R_{0,t}] + \beta_{rm^{ex}} \lambda_{rm^{ex}_{t+1}} \tag{3.1}$$

In contrast to consumption model this simple model performs much better in time series domain. Mean-standard-deviation-frontier can explain a wide range of excess return and volatility in stock market. But as shown by application to US data, its cross sectional forecast for average expected return is very "flat". It can not explain cross sectional variation in terms of very low p-value.

Fama and French suggested important information on profitability of a firm implied by its size and book to market ratio. Empirical work shows firms with small size (market capitalization) and high book to market ratio will usually have higher expected returns. The size and book to market effect has been witnessed in many developed capital markets, including UK (Maroney and Protopapadakis 2002).

They manipulate 2 mimic portfolio, HML and SMB, namely High minus Low and Small minus Big. The idea of HML is to construct a portfolio which is long in high book to market stocks while short in low book to market ration stocks and cancel out effect of firm size; similar definition applies to SMB. They believe size and book to market effect remain persistently in firms so mimic portfolio based on such information also convey important information on market risk. The construction of HML and SMB portfolios and size, book to market sorted portfolios is introduced in Chapter 4. With HML, SMB and a market excess return, the factor pricing equation is:

$$E[R_{i,t+1}] = E[R_{0,t}] + \beta_{rm^{ex}}\lambda_{rm^{ex}_{t+1}} + \beta_{HML}\lambda_{HML_{t+1}} + \beta_{SMB}\lambda_{SMB_{t+1}} \tag{3.2}$$

3.2 Consumption CAPM and Human Capital CAPM Model

If we continue the Euler equation of canonical consumption CAPM model and assume power utility, a risk free rate associated SDF can be approximated:

$$m_{t+1} \approx \delta\left(1-\gamma \Delta c_{t+1}\right)$$

Right hand set yields only constant and expected consumption growth. Only consumption deviation have impact on risk price. Two alternatives can be employed: (1) write unconditional model on Δc_{t+1} implied by equation of SDF or (2) add a scaled instruments in addition to unconditional information of Δc_{t+1}. As Campbell and Cochrane argued, if habit persistence exists, the model become:

$$M_{t+1} \approx \delta\{1-\gamma g \lambda\left(s_t\right)-\gamma\left(\varphi-1\right)\left(s_t-\bar{s}\right)-\gamma[1+\lambda\left(s_t\right)] \Delta c_{t+1}\}$$

where utility function is $u\left(C_t, X_t\right)=\left(C_t-X_t\right)^{1-\gamma} /\left(1-\gamma\right)$ and X_t is the habit level, s_t is the log of the surplus consumption ratio, φ is persistence parameter of log surplus ratio, g as expected consumption growth and $\lambda\left(s_t\right)$ as sensitivity function specified by Campbell and Cochrane (1999). This equation reveals complicated risk price determination. While it is difficult to solve it and find implied macroeconomic variables to be involved in cross sectional estimation, as surplus consumption ratio is unobservable. However, it is justified to find some approximate and include a conditioning variable. In chapter 6 some extension to this model will be discussed. The simple consumption CAPM pricing equation is:

$$E[R_{i,t+1}]=E[R_{0,t}]+\beta_{\Delta c,t} \lambda_{\Delta c_{t+1}} \tag{3.3}$$

Roll (1977) stressed the importance of including human capital in pricing equation. Such proof has been found by Jagannathan and Wang (1996). Because human capital is unobservable, labor income can be used approximately. Lettau and Ludvigson (2001) suggest two version of Human Capital CAPM, one is unconditional and the other incorporating variation correction by a scaled factor:

$$E[R_{i,t+1}] = E[R_{0,t}] + \beta_{\Delta y}\lambda_{\Delta y_{t+1}} + \beta_{rm^{ex}}\lambda_{rm^{ex}_{t+1}} \tag{3.4}$$

$$E[R_{i,t+1}] = E[R_{0,t}] + \beta_z\lambda_{z_t} + \beta_{\Delta y}\lambda_{\Delta y_{t+1}} + \beta_{rm^{ex},t}\lambda_{rm^{ex}_{t+1}} + \beta_{z,\Delta y}\lambda_{z\cdot\Delta y_{t+1}} + \beta_{z,rm^{ex}}\lambda_{z\cdot rm^{ex}_{t+1}} \tag{3.5}$$

3.3 Lettau and Ludvigson' cay As Conditioning Variables

Theoretical foundation of Lettau and Ludvigson's approach is consumption wealth ratio's broadly confirmed ability to summarize return expectation in many models. Although the inherited information might be rich, it is, again, unobservable. In order to find an approximate for this series, they split total wealth into asset holding and labor income (for human capital). The consumption wealth ratio:

$$c_t - w_t \equiv c_t - \omega a_t - (1-\omega)\, y_t$$

c a and y denote consumption asset holding and labor income, ω is nonhuman wealth share. Estimation on this ratio can be done as:

$$cay_t \approx E_t \sum_{i=1}^{\infty} \rho_w^i \left(r_{m,t+i} - \Delta c_{t+i} \right) - (1 - \omega)\, v_t$$

Lettau and Ludvigson estimated cay with Dynamic Least Square method suggested by Stock and Watson (1993):

$$c_t = \alpha + \beta_a a_t + \beta_y y_t + \sum_{j=-k}^{k} b_{a,i} \Delta a_{t-i} + \sum_{j=-k}^{k} b_{y,i} \Delta y_{t-i} + \varepsilon_t \tag{3.6}$$

The stationary series of cay is constructed with estimation result. Lettau and Ludvigson demonstrate evidence that cay has strong foreseeability for excess returns. This series will be used as scaled variable. Instruments for 3.5 is chosen by Lettau and Ludvigson as cay they constructed above. A second application is to investigate whether it can improve cross sectional regression with only market excess return, namely, Static CAPM. The estimated pricing equation is:

$$E[R_{i,t+1}] = E[R_{0,t}] + \beta_{cay} \lambda_{cay_t} + \beta_{rm^{ex}} \lambda_{rm^{ex}_{t+1}} + \beta_{cay,rm^{ex}} \lambda_{cay \cdot rm^{ex}_{t+1}} \tag{3.7}$$

3.4 Housing Consumption Models

Based on the argument that consumers concern residential real estate in their intertemporal decision and Cochrane's (1996) finding that real estate investment growth has impact on cross section of stock returns in an investment CAPM model, Piazzesi, Schneider and Tuzel (2003) formulate housing consumption and a numeraire consumption. The preference is:

$$E\left[\sum_{t=0}^{\infty}\beta^{t}u\left(c_{t},s_{t}\right)\right] \tag{3.8}$$

$$u\left(c_{t},s_{t}\right)=\frac{\left[\left(c_{t}^{\frac{\epsilon-1}{\epsilon}}+\omega s_{t}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}\right]^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$$

where c_t and s_t denote, respectively, numeraire consumption and housing service as a result of installed housing capital in last period h_{t-1} with a coefficient η. ω is coefficient. Consumption is designed as a Constant Elasticity of Substitution between consumption in housing and non-housing goods, with a elasticity ϵ. Intertemporal substitution of total consumption coincide with canonical consumption model with an intertemporal elasticity coefficient $\sigma = \frac{1}{\gamma}$. Two kind of real assets are assumed: accumulated housing capital h_t and a single Lucas-tree asset of dividend d_t, transacted at price p_t^h and p_t^s. The flow budget constraint is then:

$$c_{t}+p_{t}^{h}h_{t}+p_{t}^{s}\theta_{t}=\left(p_{t}^{s}+d_{t}\right)\theta_{t-1}+p_{t}^{h}h_{t-1}\left(1-\delta\right)+p_{t}^{h}\overline{h}_{t}^{n} \tag{3.9}$$

Total supply of housing capital is $\overline{h}_{t}^{n}$, which is stochastic. θ_t is number of Lucas-tree securities. δ is depreciation rate of housing capital. The endowment to household is defined as dividends. The equilibrium is achieved with a solution to SDF:

$$M_{t+1}=\beta\left(\frac{c_{t+1}}{c_{t}}\right)^{-\frac{1}{\sigma}}\left(\frac{1+\omega\left(\frac{s_{t+1}}{c_{t+1}}\right)^{\frac{\epsilon-1}{\epsilon}}}{1+\omega\left(\frac{s_{t}}{c_{t}}\right)^{\frac{\epsilon-1}{\epsilon}}}\right)^{\frac{\sigma-\epsilon}{\sigma(\epsilon-1)}}$$

To make this representation helpful for our factor pricing specification, a non-housing consumption share α_t is introduced. Prior to obtaining this share, please notice the household can acquire housing service either from accumulating housing capital with lagged decision or exchange with other agents. Because this redistribution does not matter for aggregate market equilibrium of two type of real assets, I can consider this exchange separately. Assume price as q_t, the equilibrium price is defined by intratemporal substitution of two kind of consumption.

$$q_t = \frac{u_s(c_t,s_t)}{u_c(c_t,s_t)} = \omega\left(\frac{s_t}{c_t}\right)^{-\frac{1}{\epsilon}}$$

And the share of non-housing consumption after final redistribution:

$$\alpha_t \equiv \frac{c_t}{c_t + q_t s_t} = \frac{1}{1+\omega\left(\frac{s_t}{c_t}\right)^{\frac{\epsilon-1}{\epsilon}}} \tag{3.10}$$

Now the logarithm SDF can be written approximately as:

$$m_{t+1} \approx \log\beta - \frac{1}{\sigma}\Delta c_{t+1} + \frac{\epsilon-\sigma}{\sigma(\epsilon-1)}(1-\alpha_t)\Delta\log\left(\frac{\alpha_{t+1}}{1-\alpha_{t+1}}\right) \tag{3.11}$$

This representation implies that (1) changes in expenditure structure on housing and non-housing consumption decision contain as much information as consumption growth and (2) housing consumption share $(1-\alpha_t)$ can be a very useful scaling factor for a cross sectional regression on categorized portfolio. Piazzesi, Schneider and Tuzel (2003) investigate two pricing equations below.

$$E[R_{i,t+1}] = E[R_{0,t}] + \beta_{\Delta c}\lambda_{\Delta c_{t+1}} + \beta_{\Delta\log\alpha}\lambda_{\Delta\log\alpha_{t+1}} \tag{3.12}$$

$$E[R_{i,t+1}] = E[R_{0,t}] + \beta_{\Delta c}\lambda_{\Delta c_{t+1}} + \beta_{\Delta\log\alpha}\lambda_{\Delta\log\alpha_{t+1}} + \beta_{\log\alpha,\Delta c}\lambda_{\log\alpha_t\cdot\Delta c_{t+1}}$$

$$+\beta_{\log\alpha,\Delta\log\alpha_{t+1}}\lambda_{\log\alpha_t\cdot\Delta\log\alpha_{t+1}} + \beta_{\log\alpha}\lambda_{\log\alpha_t} \tag{3.13}$$

Chapter 4

Data and Econometric Methods

In this chapter, the collection of data and empirical methods of cross section regression will be introduced. The central part of data is the formation of 5 by 5 portfolio data from UK stock market. In order to do this, the approach of Liew and Vassalou (2000) is adopted. Cross section estimation is the popular Fama and MacBeth (1973) method. Some measure of goodness of fit is discussed, too. This chapter also includes some summary statistics of variables I use.

The data I collect includes: per capita consumption, per capita labor income, per capita consumption on housing, per capital household net worth for measure of wealth, T-bill rate as short interest rate, retail price index to calculate inflation and value-weighted real return of UK stock market. United States data can be obtained from website of Kenneth French, John Cochrane and Martin Lettau.

4.1 Data Manipulation

Economic data is provided by Office of National Statistics, UK on internet. I obtain consumption data by summing non-durable and service consumption. Housing consumption is house rent plus water, electricity, gas and other household expenditure. Labor income is defined as wages and salaries minus taxes on labor income. To examine a cointegration residual of consumption, labor income and household wealth, the data of household net worth is critical. However, there is only annual household net worth data for UK since 1987. I transfer the annual data to quarterly data by assuming a constant growth rate over a year. Risk free rate is the real short rate calculated from discount rate of T-bill. Gross variables and returns are deflated with inflation delivered by RPI.

Financial data is from Datastream, where a full range of financial data on company size, transaction and other characteristic description can be found. In order to construct the 5 by 5 portfolio sorted by size and book to market ratio, I collect end-month price, dividend, book to market ratio, market value of 672 FTSE listed UK companies. Among all 1600 FTSE listed UK companies, financial companies are excluded because they have large leverage, which violate the assumption of Book to Market ratio as a measure for profitability for all firms (Fama and French 1992). Rest firms are picked out if Datastream document its market data since listing date. Sorting is done in each year end from 1968 to 2001. To be sorted into group of different size and Book to market group, the firm must have a positive book to market ration at December of that year. I first sort qualified companies to 5 book to market group at quintile value. Then I sort in each group according to quintile value of market capitalization (MV). A summary of this sorting can be depicted by Table B.5.

Return of 25 sorted portfolio is the value weighted return in each month, then transferred to quarterly frequency. Sorted portfolios are named B_iS_j and $\{i, j\} = \{1, 2, 3, 4, 5\}$ according its group index for B/M and size. The summary information on size and B/M in each group is in Table A.3. In addition, I construct a group of industrial portfolios to view power of estimation on cross sectional variation among asset return. Among 64 sectors of non-financial firms, I choose 25 industries where maximal numbers of firms can be found. Table A.1 summarize the coverage of these portfolios.

With above portfolio return data I can calculate Fama and French's (1992, 1993) mimic portfolio defined as a "neutral" trading investment. I take the average of difference in high and low book to market ratio and big and small firms, given there is same firm size (big, medium or small) and same book to market ratio (high, medium or low).

$HML = \frac{1}{6}((P_{B5S1} - P_{B1S1}) + (P_{B5S3} - P_{B1S3}) + (P_{B5S5} - P_{B1S5}) + (P_{B4S1} - P_{B2S1}) + (P_{B4S3} - P_{B2S3}) + (P_{B4S5} - P_{B2S5}))$

$SMB = \frac{1}{6}((P_{B1S1} - P_{B1S5}) + (P_{B3S1} - P_{B3S5}) + (P_{B5S1} - P_{B5S5}) + (P_{B1S2} - P_{B1S4}) + (P_{B3S2} - P_{B3S4}) + (P_{B5S2} - P_{B5S4}))$

4.2 Lettau and Ludvigson's cay

In order to investigate the conditioning information in cointegration residual of consumption, labor income and household wealth, first one must see the three series' time series properties including non-stationarity and cointegration relation. These series are real, per capita by taking logarithm, as shown in Figure 4.1 for their standardized series indexed for mean=100. Table 4.1 is the unit root test of c_t, a_t and y_t.

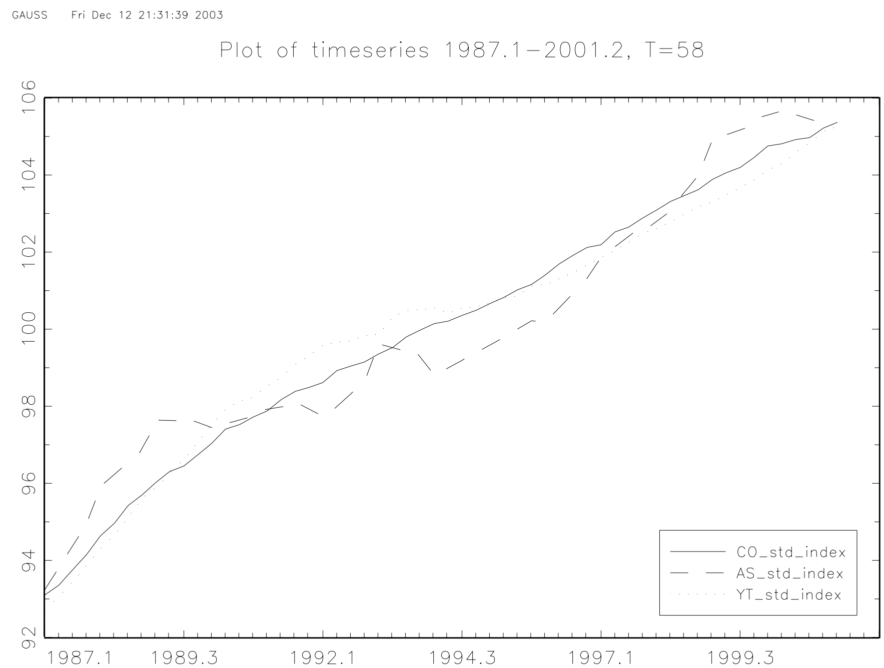

Figure 4.1: Standardized Consumption, Asset Holding and Labor Income

Table 4.1 Test for Unit Root

Variables	Lag	Test Statistics	1% Critical Value	5% Critical Value	10% Critical Value
		Augmented Dickey-Fuller Test			
c_t	3	-2.95	-3.43	-2.86*	-2.57*
a_t	4	-0.05	-3.43	-2.86	-2.57
y_t	5	-0.49	-3.43	-2.86	-2.57
		Phillips-Perron Test			
c_t	3	-4.01	-3.55*	-2.92*	-2.59*
a_t	3	-1.38	-3.55	-2.92	-2.59
y_t	3	-3.45	-3.55	-2.92*	-2.59*

Note: c_t, a_t, y_t are real per capita non durable consumption, household net worth and labor income. Optimal Number of Lags is reported by Akaike Info Criterion (AIC) provided by JMulti software. For Phillips-Perron test I use recommended Newey-West Truncation Lag of 3. Star (*) indicate a rejection of Null Hypothesis of no unit root (stationarity). Both tests are done with a intercept for each variable. The ADF test for consumption is executed on sample of 1968Q1 to 2001Q2. Otherwize the sample period is 1987Q1 through 2001Q2.

The proof of unit root in asset holding and labor income is significant in UK data. However, the non-stationarity of consumption has been rejected on 5% percent quantile for ADF test and 1% quantile of Phillips-Perron Test. This poses minor challenge for validity for applying cay framework in my dataset. One explanation is consumption, especially non-durable consumption's extremely smoothness. However, I proceed to estimation of cay and deem it one of evidence on whether cay can be generally approved.

Table 4.2 shows result of Johansen trace test on cointegration between cay elements. Although Schwartz Criterion (SC) report an optimal lag of 1, I report test for both 1 and 2 as lag because Akaike Info Criterion report a quite different lag 8. I attribute it to worse data quality of household wealth in contrast to consumption and labor income. For the number of zero cointegration equation, null hypothesis has been rejected under both lag choice. With lag 1 a cointegration rank of 1 is rejected at 5% critical value while it is accepted for lag of 2. This tantalizing result pose another challenge for my adoption of cay estimation because Lettau and Ludvigson (2001) have this single cointegration relation as their basic assumption. Undoubtedly, the cointegration relation and common trend is significant on the plot (Figure 4.1), strong econometric evidence must be fulfilled before full embracing their model.

Table 4.2 Johansen Cointegration Test

Cointegration Rank	Test Statistics	5% Critical Value	1% Critical Value
	lag=1		
0**	46.51	29.68	35.65
1*	18.26	15.41	20.04
2	2.51	3.76	6.65
	lag=2		
0**	38.90	29.68	35.65
1	13.63	15.41	20.04
2	2.09	3.76	6.65

Note: This table is the result of Johansen Trace Test for Cointegration. I choose lag as 1 or 2 for cointegration equation between c_t, a_t, y_t. *(**) denotes rejection of the hypothesis at the 5%(1%) level. In this test, only intercept is included. Sample period is 1987Q1 to 2001Q2.

The estimation and investigation above reveal some relatively "good" prerequisite, although not "strong" proof, for me to estimate cay for UK dataset. There are two alternatives to perform such an operation. First I use Stock and Watson's (1993) Dynamic Least Square method 3.6, as recommended by Lettau and Ludvigson. Then I estimate a (V)ECM equation of c_t, a_t, y_t, given there is a cointegration rank of among variables. For first method (DLS), lag number is chosen to be 2 by SC and AIC, and parameter estimation and their t-statistics with OLS is:

$$c_t = -0.15 + 0.28a_t + 0.75y_t$$
$$(-1.76) \quad (4.75) \quad (7.23)$$

These parameters are significant different from zero according to 1% critical value, expect for the constant term has a p-value of 0.08. With this result I construct the cay as $\widehat{cay}_{DLS} \equiv c_t - 0.28a_t - 0.75y_t + 0.15$. Because there is a cointegration equation among c_t, a_t, y_t, a Vector Error Correlation Model can better represent deviations of their shared trend.

$$\Delta Y_t = \Pi Y_{t-1} + \sum_{s=1}^{p} \Gamma_s \Delta Y_{t-s} + U_t \tag{4.1}$$

In 4.1 $Y_t = [c_t \; a_t \; y_t]'$ and U_t is vector of errors. This model can be written as:

$$\begin{bmatrix} \Delta c_t \\ \Delta a_t \\ \Delta y_t \end{bmatrix} = \begin{bmatrix} \alpha_{1,1} \\ \alpha_{2,1} \\ \alpha_{3,1} \end{bmatrix} \cdot [1 \; \beta_a \; \beta_y] \cdot \begin{bmatrix} c_{t-1} \\ a_{t-1} \\ y_{t-1} \end{bmatrix} + \sum_{s=1}^{p} \begin{bmatrix} \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{bmatrix}_s \cdot \begin{bmatrix} \Delta c_{t-1} \\ \Delta a_{t-1} \\ \Delta y_{t-1} \end{bmatrix} + \begin{bmatrix} u_{1,1} \\ u_{2,1} \\ u_{3,1} \end{bmatrix}$$

For a lag p of 1, the first equation in this model is estimated with EVIEWS as:

$$\Delta c_t = -0.02 \cdot (c_{t-1} - 0.52a_{t-1} - 0.39y_{t-1}) + 0.33\Delta c_{t-1} + 0.05\Delta a_{t-1} + 0.45\Delta y_{t-1}$$

$$(-0.70) \qquad (6.19) \quad (3.48) \quad (2.43) \quad (1.37) \quad (2.88)$$

The estimation result is significant according to t-statistics of parameters. Again, this method deliver another measure for cay as: $\widehat{cay}_{VECM} \equiv c_t - 0.52a_t - 0.39y_t$. In order to make two series of cay to be comparable, I remove intercept from first estimation. The two $\widehat{cay}$ are compared in Figure 4.2. In the regression later, I use cay from a VECM with no intercept, because it can replicate the work by Lettau and Ludvigson by improving cross sectional evidence in some models.

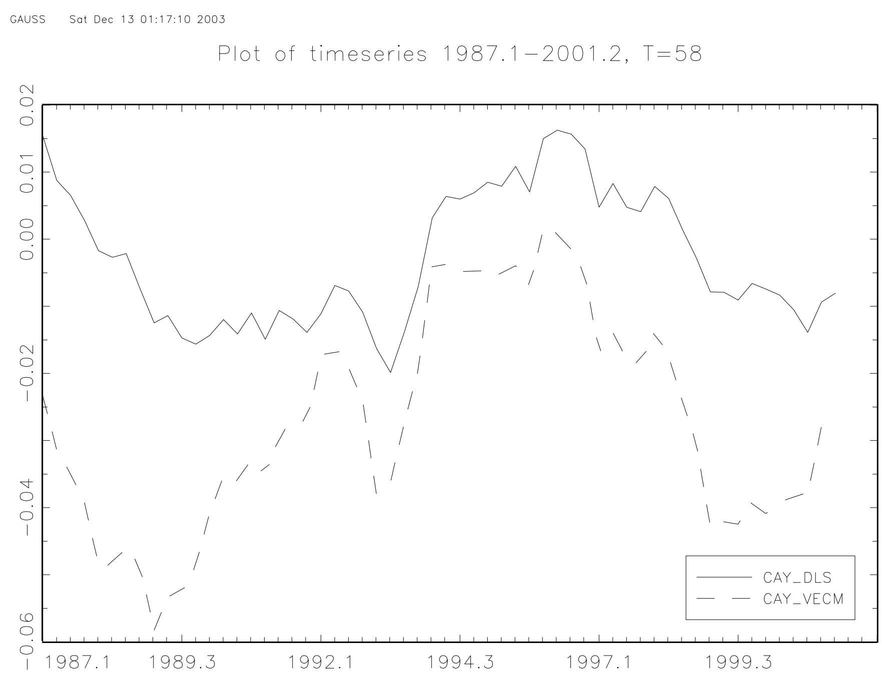

Figure 4.2: Lettau and Ludvigson's cay Estimated with DLS and VECM

4.3 Summary Statistics

Table 4.3 report the descriptive statistics and cross-correlation information of most data series, both explanatory variables and returns used in this project.

Table 4.3 Summary Statistics

	Δc_{t+1}	Δy_{t+1}	$r_{m,t+1}$	$r_{f,t+1}$	$ln\Delta\alpha_{t+1}$	$\widehat{cay}_{VECM,t+1}$
mean	0.71	0.66	3.30	1.68	-11.60	-2.70
std Dev	0.33	0.41	8.57	0.60	0.48	1.60
			Cross-correlation			
Δc_{t+1}	1.00	0.50	0.05	0.30	0.50	-0.31
Δy_{t+1}		1.00	-0.08	0.54	0.74	-0.66
$r_{m,t+1}$			1.00	0.02	-0.03	0.02
$r_{f,t+1}$				1.00	0.73	-0.49
$ln\Delta\alpha_{t+1}$					1.00	-0.77
$\widehat{cay}_{VECM,t+1}$						1.00
$\widehat{cay}_{DLS,t+1}$	0.13	-0.32	0.10	-0.40	-0.33	0.71
$r_{FF3,t+1}$	0.01	-0.16	0.77	-0.01	-0.14	-0.04

Note: First two rows report mean and standard deviation in percent of each variable. Second block reports contemporaneous cross-correlation. The variables are: Δc_{t+1} and Δy_{t+1} are real non-durable consumption and labor income growth; $r_{m,t+1}$ is real market return based on MSCI index; $r_{f,t+1}$ is real risk free rate based on Treasury bill rate; $ln\Delta\alpha_{t+1}$ is log non-housing consumption share; $\widehat{cay}_{VECM,t+1}$ and $\widehat{cay}_{DLS,t+1}$ are cay series calculated from VECM and DLS regression; $r_{FF3,t+1}$ is equal weighted real return on 5*5 UK Fama French portfolio. The sample period is 1987Q1 to 2001Q2.

4.4 Econometric Methods

An appealing way to test cross sectional predictability of variables in a factor pricing model is second-pass Fama-MacBeth procedure. It is very easy to proceed. The idea is to estimate, for each member of the cross section, the pricing betas from time series. Then use the estimated betas, together with a constant term to regress on each cross sectional realization the coefficient on betas. One can take a rolling estimated beta and assume smooth change in factor betas. I adopt the simple method of one constant beta in full sample.

$$E[R_{i,t}] = b_i'\lambda_t + \alpha_{i,t} \tag{4.2}$$

To estimate 4.2 (or 2.2) we obtain the estimation of b by time series OLS regression of portfolio returns on explanatory, either contemporaneous unconditional variables or conditioning instruments, variables.

$$\widehat{b}_i = (z_i' z_i)^{-1} z_i' R_i \quad i = 1, ..., N$$

On each time period of the whole time series, Fama and MacBeth recommend to regress $E[R_t]$ on constant and beta to obtain $\widehat{\lambda}_t$ and take time series average:

$$\widehat{\lambda} = \frac{1}{T}\sum_{t=1}^{T}\widehat{\lambda}_t \text{ and } \widehat{\alpha}_i = \frac{1}{T}\sum_{t=1}^{T}\widehat{\alpha}_{i,t}$$

Standard deviation of $\widehat{\lambda}$ and pricing error $\widehat{\alpha}_i$ is the sampling error:

$$\sigma^2\left(\widehat{\lambda}\right) = \frac{1}{T^2}\sum_{t=1}^{T}\left(\widehat{\lambda}_t - \widehat{\lambda}\right)^2 \text{ and } \sigma^2\left(\widehat{\alpha}_i\right) = \frac{1}{T^2}\sum_{t=1}^{T}\left(\widehat{\alpha}_{i,t} - \widehat{\alpha}_i\right)^2$$

Significance of parameter in λ is given by the t-statistics:

$$t_{k,\widehat{\lambda}} = \frac{\widehat{\lambda}_k}{\widehat{\sigma}_{\widehat{\lambda}_k}} \sim t\left(T-1\right) \quad k = 1, ..., K$$

Shanken (1992) suggest a corrected measure for t-statistics above because the precision of risk price λ estimation is overstated with "errors-in-variables". However, Jagannathan and Wang (1998) defend Fama-MacBeth t-statistics by posing the prerequisite of weak autoregressive conditional heteroscedasticity in the data of asset returns. In the appendix, I show ARCH LM (Lagrangian Multiplier) test on each return series and find less support to ARCH effect in the data. So for the t-statistics I use only above method.

The average pricing errors for each portfolio return are calculated to be another measure of success. A formal test is presented by Cochrane (1999):

$$\widehat{\alpha}' cov\left(\widehat{\alpha}\right)^{-1}\widehat{\alpha} \sim \chi^2\left(K-1\right)$$

Chapter 5

Empirical Findings and Further Discussion

For each candidate model, regression results of parameter and R-square are depicted in Table B.1 and Figure 5.1 through 5.8. Sample period includes first quarter of 1987 through second quarter of 2001. Model 1 through 8 correspond to 3.1, 3.3, 3.2, 3.4, 3.5, 3.7, 3.12, 3.13, respectively. Note in the plot of fitted versus realized expected returns I use the notation of "conditioning variable" loosely, but for unconditional model the variation within groups come from contemporaneous explanatory variables.

5.1 UK Regression Results

The most striking finding is the high R-square and excellent fit of some models. The realized versus fitted average expected return plots for each models are shown in Figure 5.1 through 5.8. The "flat" beta forecast ability is weaker than we see in US application.

By switching to using consumption growth for unconditional model, the R-square increase from 0.5175 to 0.9881. Estimation on human capital CAPM model show equally strong predictability of labor income growth. Indeed, as consumption and labor income have a common trend and their growth is highly and positively correlated, the good performance is not a surprise. So far I have conducted 3 estimation, but the R-square is almost approaching unity. One might question the estimation procedure I proposed. But I attribute it to the portfolio I construct or problem of errors-in-variables. Because the same procedure has been applied to US data and moderate R-square has been found.

Fame and French (1992, 1993) 3 factor model is as successful in explaining cross sectional variation in these returns as simple CCAPM and human capital CAPM do. As a benchmark for historical research on US data, further discussion on improvement by scaling factors can be compared.

Turning to scaled factor models I found the model with Lettau and Ludvigson's lagged cay as conditioning variable does improve the human capital model. But it is difficult to tell the degree of improvement in terms of R-square, from 0.9931 to 0.9984. By contrast I found the best model is the scaled housing model with a R-square of 1, which means a perfect fit. Conditioning information contained by non-housing consumption share is very rich in forecast stock excess returns. When incorporate this instrument in pricing equations. The information conveyed by consumption growth is decreasing, as compared to previous simple CCAPM model. Indeed, even if I test the model with only non-housing consumption share: $E[R_{i,t+1}] = E[R_{0,t}] + \beta_{\log \alpha}\lambda_{\log \alpha_t}$ or $E[R_{i,t+1}] = E[R_{0,t}] + \beta_{\log \alpha}\lambda_{\log \alpha_{t+1}}$, both unconditional model and conditional yield very appealing result, where R-square is

0.9999 and all pair of fitted-realized return point rest on 45° straight line. A comparison can be made for Lettau and Ludvigson's cay with non-housing share by fitting the same models $E[R_{i,t+1}] = E[R_{0,t}] + \beta_{cay}\lambda_{cay_t}$ or $E[R_{i,t+1}] = E[R_{0,t}] + \beta_{cay}\lambda_{cay_{t+1}}$. These two estimation yield a R-square near 0.99 for UK data.

There is much more promising result in terms of parameter values and its associated t-statistics. Risk price for consumption growth is significant, in terms of t-statistics, in all models except for housing model model scaled on non-housing consumption share. The sign of risk price for consumption growth, labor income and market return are always positive and consistent with theory. As a complementary measure of success average pricing errors is shown in Table B.3. The test for null hypothesis of joint zero pricing error only reject static CAPM.

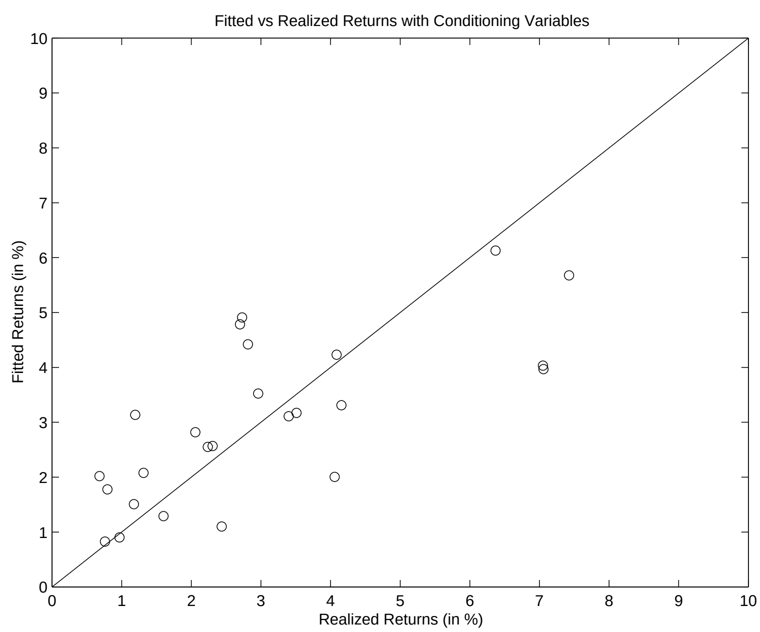

Figure 5.1: Model: Static CAPM with Fama French 5*5 Portfolio

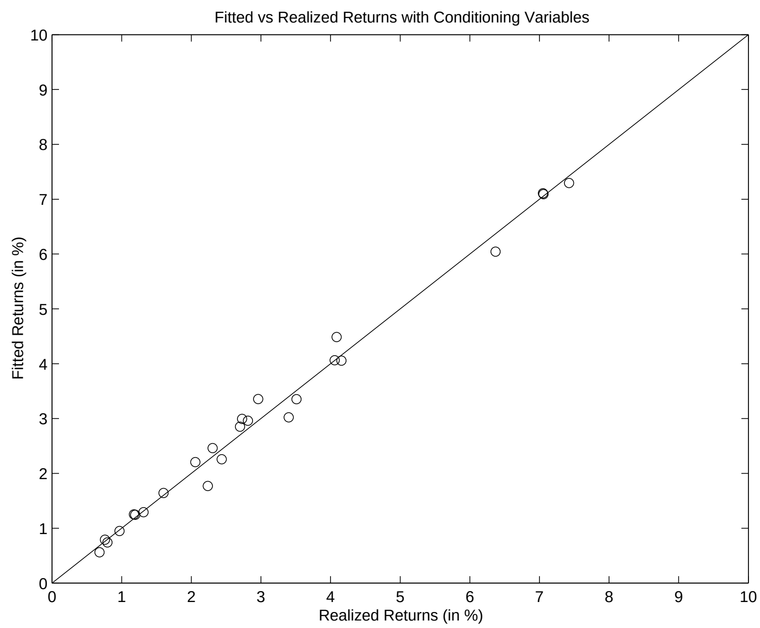

Figure 5.2: Model: Simple Consumption CAPM with Fama French 5*5 Portfolio

5.2 Comparison with US Results

To build a comparative basis, I also plot fitted-realized returns for Model 1, Model 5, Model 6 and Model 4. With this data set, the improvement from conditioning variable is much significant. In summary, the results between two dataset is comparable: improvement posed by scaled factors, coincidence on sign of risk price. Together with result from Lettau and Ludvigson (2001), some difference include: UK result have better result on CAPM, not only in terms of fit, but UK data deliver a positive and significant risk price of market return, which is consistent with CAPM theory, but in US data it is negative and insignificant from zero. The constant term in two set of regression differ a lot. In UK data the constant is all insignificant except for static CAPM, but US data yield mostly significant not zero constant. In this sense UK data is fitted in models better because the theory implies the

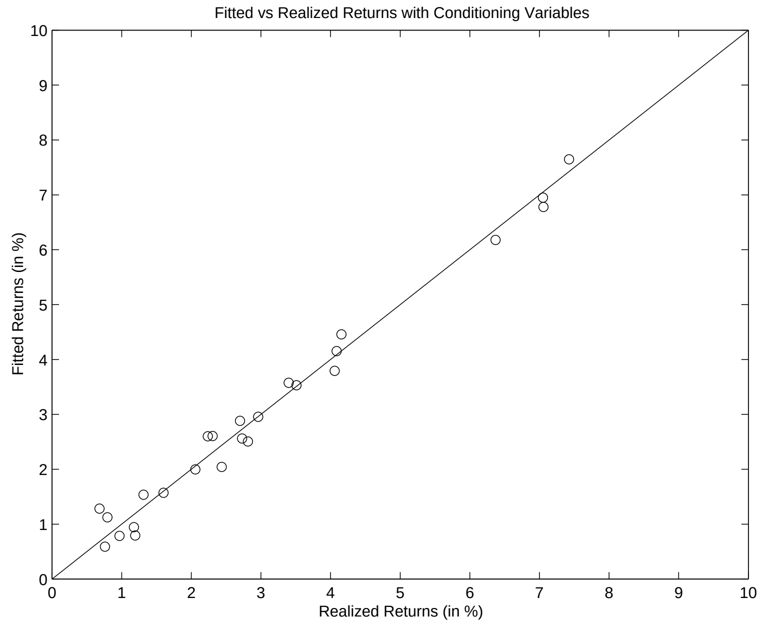

Figure 5.3: Model: Fama French 3 Factor Model with Fama French 5*5 Portfolio

intercept in this cross sectional regression should be zero (Piazzesi, Schneider and Tuzel 2003).

5.3 Results with Industrial Portfolios Returns

In order to test the general application of the candidate models and implication of conditioning variables, I construct a set of industrial portfolio from the same source. I pick out 25 industrial sector to form series of value weighted return. The estimation result is shown in Appendix B (Table B.2). In contrast to size and book to market portfolios, this dataset generate similar parameter estimation and conclusive R-square. And it also generate a group of more moderate R-squares. The Fama-French 3 factor model perform worse because information on firm characteristic measure is less in this industry based

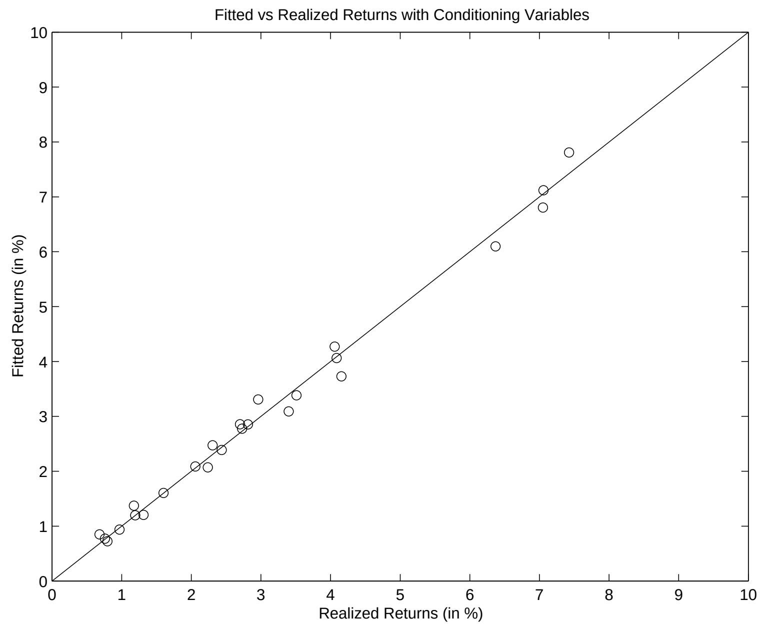

Figure 5.4: Model: Human Capital CAPM with Fama French 5*5 Portfolio

sorting. However, there important implication delivered by this set of regression is: cay has less conditioning information in this dataset than that of non-housing share. As a complete estimation the average pricing errors and fitted-realized return plot can be found in Table B.4 of appendix B.

5.4 Discussion on Empirical Methods

This project has several weak components, which is open to discussion, resulting from data and Fama-MacBeth procedure. First is the construction of *cay*, and then the "superior" properties of most explanatory variables. The lack of quarterly data on household asset holding pose the first obstacle on estimation of *cay*. My procedure to assume a constant growth rate in a year is not able to generate enough deviation in a_t. An improvement might

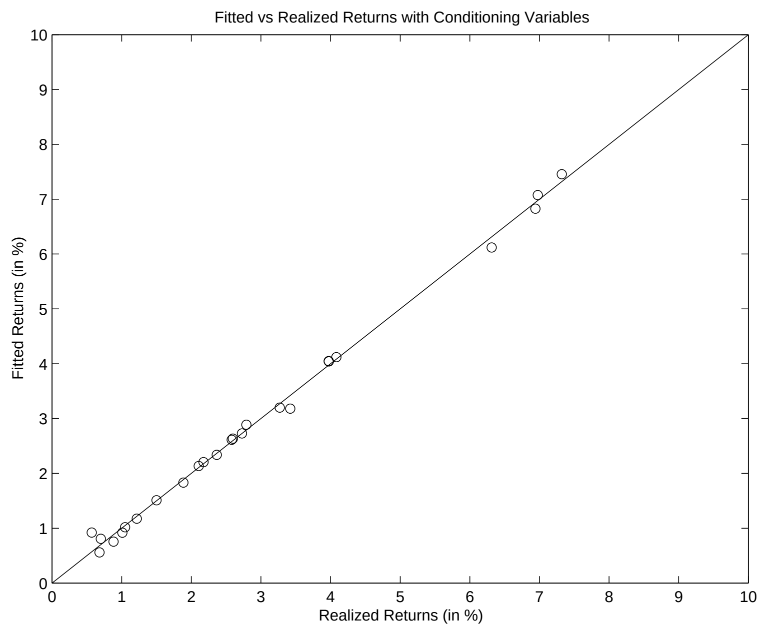

Figure 5.5: Model: Scaled CAPM on cay (Fama French 5*5 Portfolio)

be achieved by use deviation information in year of components of asset holding, such as currency asset, deposit and financial asset. However I haven't found data on quarterly non-financial asset. Still the improvement can not be promising. One can also challenge construction of *cay* by investigating less volatile consumption. In fact, as I increase length of consumption series, I can see more evidence of a unit root.

The errors-in-variables (EIV) happens in Fama-MacBeth procedure because the market beta, which might be changing through time, is not observable. In whatever procedure to obtain it only the dataset it self can be used (Campbell, Lo and MacKinlay 1997). The sorted data should have less EIV effect after weighting a group of individual asset (Fama and MacBeth 1973). One explanation might be, in the early period there is less firms, then in some groups the number of firms is two low to construct good grouping return

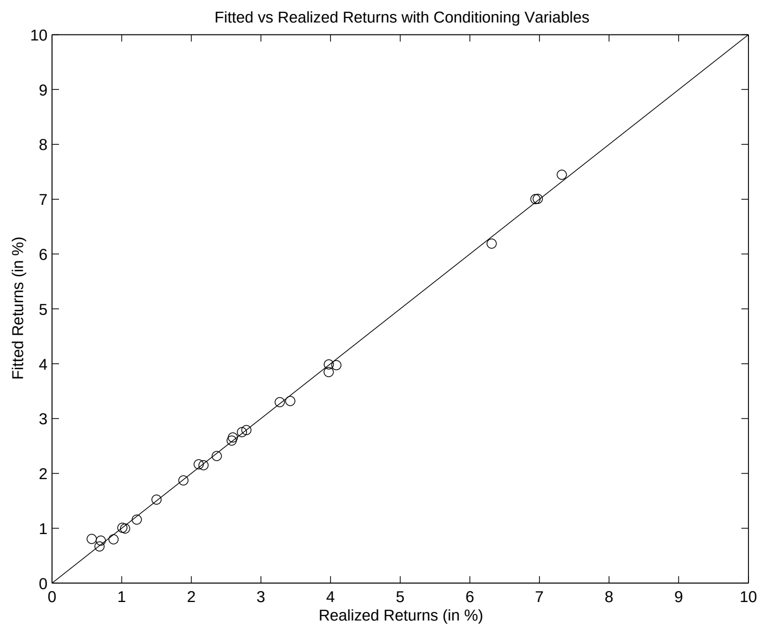

Figure 5.6: Model: Scaled Human Capital CAPM (Fama French 5*5 Portfolio)

series. One evidence is the more moderate result from the industrial portfolios. Shanken (1992) proved under EIV, the errors of estimated beta approaches zero with increasing T and risk price estimation is consistent. But as I have only 58 observation. The procedure does have EIV problem in this respect. However, the procedure can still deliver important information even when such finite sample property is ignored. The result is conclusive on most questions early part asked.

To summarize this point, I believe results of this empirical work is robust although further improvement is foreseeable.

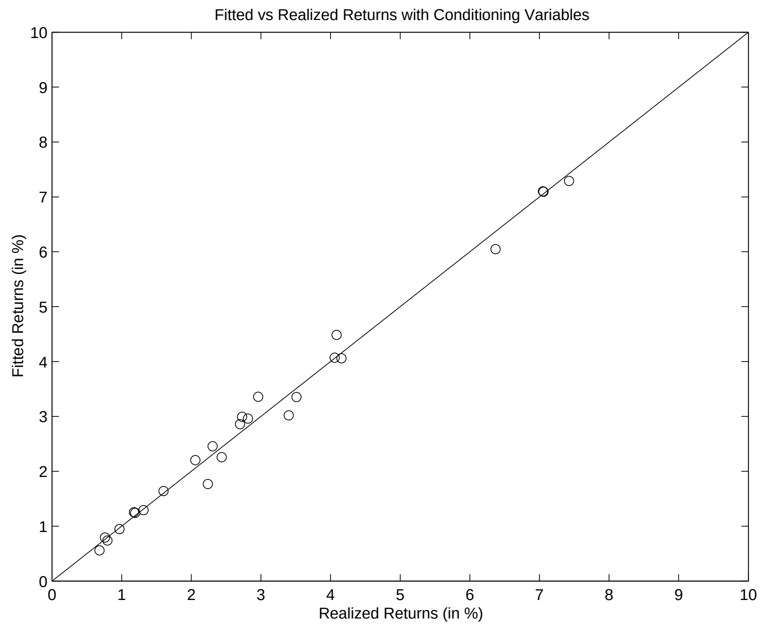

Figure 5.7: Model: Unscaled Housing CAPM (Fama French 5*5 Portfolio)

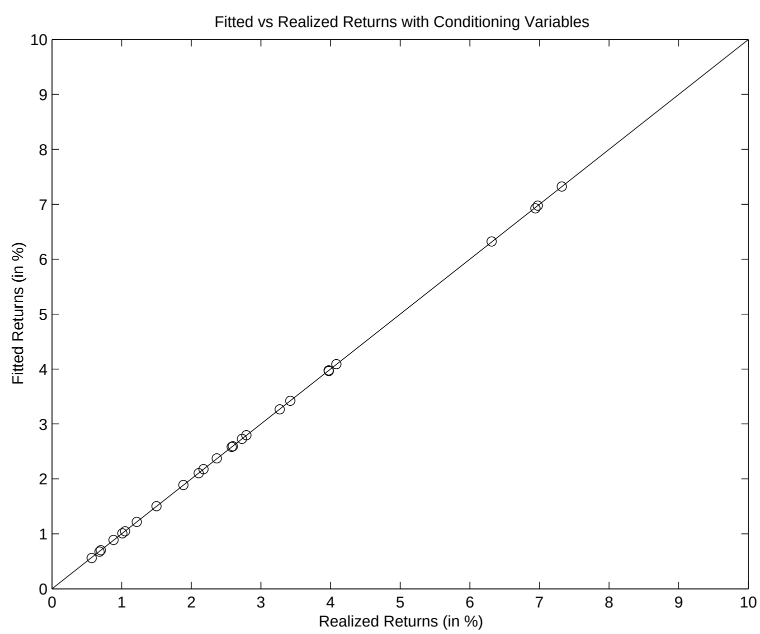

Figure 5.8: Model: Scaled Housing CAPM with Fama French 5*5 Portfolio

Chapter 6

Conclusion

This thesis investigates a wide class of, mainly two, CAPM or consumption CAPM model with respect to their ability in explaining cross sectional variation on returns to size and book to market ratio sorted UK portfolios. Conditioning information is widely found in many macroeconomic variables. Although there is no theoretical foundation, some consumption factor pricing model can still yield appealing forecasting results. A formal way is to solve for time-varying stochastic discount factor and approximate it as a factor model, especially scaled factor model when non-linearity prevails.

I construct a dataset of size and book to market ratio sorted portfolios on UK FTSE data. Together with macroeconomic data I test 8 candidate unconditional and conditional models and some extension. Regression over UK stock market data approves general conclusion of Lettau and Ludvigson (2001) and Piazzesi, Schneider and Tuzel (2003). In these models, consumption wealth ratio and non-housing consumption ratio, respectively, play very important role in rendering portfolio returns in the cross section. To support above findings, I further introduce a group of industrial portfolios and found lower R-square but more appealing result for interpretation.

A *cay* representation has, but not very significant, predictability in cross sectional domain. Improvement can be achieve by estimating, or of course finding an exact quarterly series of household asset holding. There might be problem of errors-in-variables in the procedure I use, so finite example properties of Fama-MacBeth (1973) should be taken into consideration in further research on this topic.

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Chapter 7

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Appendix A

A.1 FTSE Industrial Portfolios

Table A.1: Selected Industry Portfolios

#	FTSE Code	Industry	Number of Firms
1	581	Business Support Services	68
2	977	Software	40
3	972	Computer Services	32
4	547	Publishing & Printing	31
5	267	Engineering - General	31
6	528	Retailers - Soft Goods	23
7	137	Other Construction	22
8	538	Leisure Facilities	20
9	118	Chemicals - Speciality	9
10	253	Electronic Equipment	18
11	134	House Building	17
12	435	Food Processors	17
13	539	Restaurants, Pubs & Breweries	16
14	446	Medical Equipment & Supplies	16
15	132	Building & Construction Materials	16
16	526	Retailers - Hardlines	15
17	596	Rail, Road & Freight	15
18	542	Television, Radio and Filmed Entertainment	15
19	73	Oil & Gas - Exploration & Production	14
20	583	Education, Business Training & Employment Agencies	14
21	341	Clothing & Footwear	13
22	630	Food & Drug Retailers	13
23	545	Media Agencies	12
24	527	Retailers - Multi Department	11
25	318	Vehicle Distribution	10

Note: I compare all 84 industries in all 676 FTSE listed firms and choose 25 industries to construct a industrial portfolio. This portfolio covers 508 firms and range from 1987Q1 to 2001Q2. The returns in constructed industrial portfolio are value weighted quarterly real returns within each industry.

A.2 ARCH LM Test of Portfolio Returns

Autoregressive Conditional Heteroscedasticity can appear in many high frequency financial data, as introduced by Engel (1982). For residual of an equation, an ARCH LM test can be implemented to test the Null that there is no ARCH effect in residual:

$$e_t^2 = \beta_0 + (\sum_{s=1}^{p} \beta_s e_{t-s}^2) + v_t$$

I simply treat the portfolio return as residual above and test the hypothesis that to a lag p there is no ARCH effect in this single series. Up to a lag p, this test statistics will have a χ^2 asymptotic distribution with a degree of freedom p. I choose the optimal lag specified by Akaike info Criterion for each portfolio and report the test statistics and p-value in the table A.2 below. There are some portfolio for which all information criterion report an optimal lag as zero, in that case I use 1 for lag, indicated by 1* in the table.

Table A.2 Test for ARCH in Portfolio Return

portfolio	lag	test stat	p	portfolio	lag	test stat	p
B1S1	6	6.66	.35	B3S4	7	3.42	.84
B1S2	1*	0.18	.67	B3S5	1*	0.17	.68
B1S3	1*	0.01	.91	B4S1	1*	1.81	.18
B1S4	1*	0.02	.88	B4S2	7	13.6	.06
B1S5	1	4.31	.04	B4S3	7	4.66	.70
B2S1	1	0.07	.80	B4S4	1*	1.37	.24
B2S2	1	0.57	.45	B4S5	1*	0.79	.37
B2S3	1*	0.60	.44	B5S1	1*	0.25	.62
B2S4	1*	0.21	.64	B5S2	2	1.16	.56
B2S5	7	4.43	.73	B5S3	5	4.01	.55
B3S1	2	1.33	.25	B5S4	6	1.64	.95
B3S2	1*	0.00	.96	B5S5	5	3.38	.64
B3S3	1*	0.35	.55				

Among all the portfolio, ARCH effect can only be observed on B1S5 and B4S2 with a 5% and 10% quantile. The less ARCH effect in this dataset might lie in the quarterly frequency and short sample size. Given this examination, I omit the Shanken (1992) corrected t-statistics and p-value in model estimation of chapter 5. Figure A.1 is average return plot of four delegate portfolios with biggest (or smallest) size and highest (or lowest) book to market ratio: B1S1, B1S5, B5S1 and B5S5.

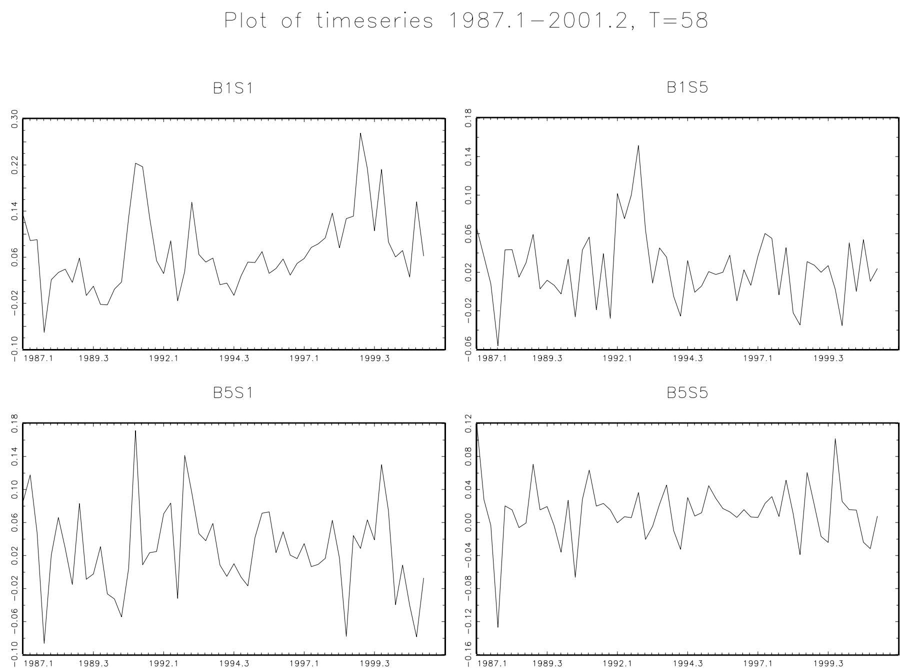

Figure A.1: Plots of Average Return of 4 Sample UK Sorted Portfolios

A.3 Comparable Regression Results with US Data

(Figure A.2 through A.5)

A.4 Cross Sectional Evidence for Industrial Portfolios

(Figure A.6 through A.13)

A.5 Average Size and Book to Market Ratio of Each Groups

(Table A.3)

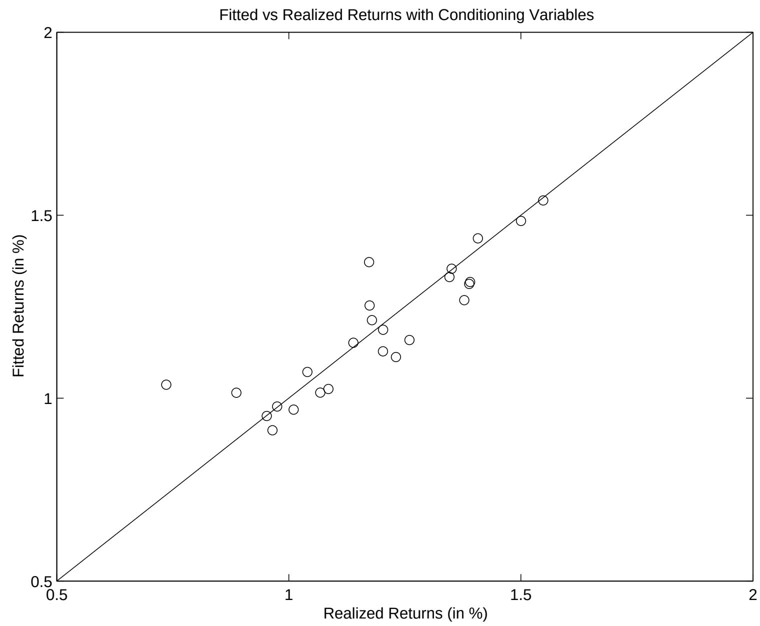

Figure A.2: Model: US Simple Consumption CAPM (R-square 0.58)

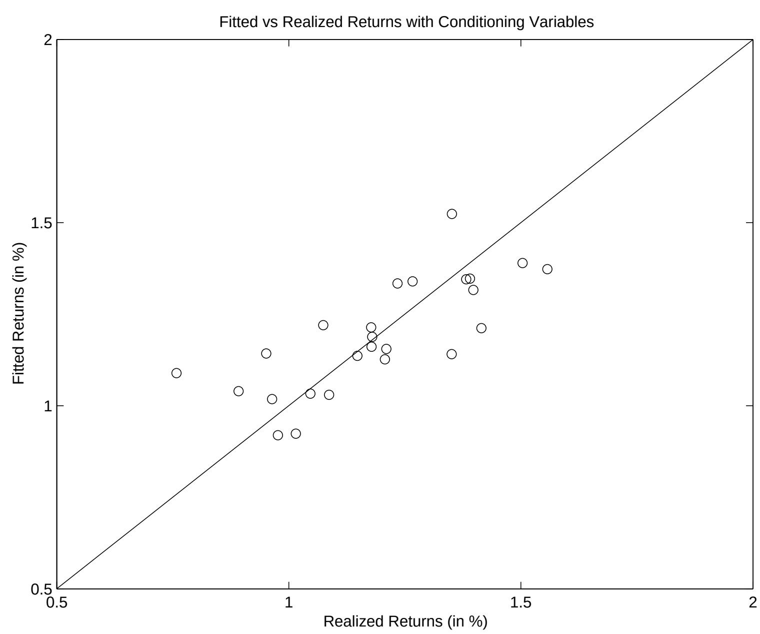

Figure A.3: Model: US Scaled CAPM on cay (R-square 0.58)

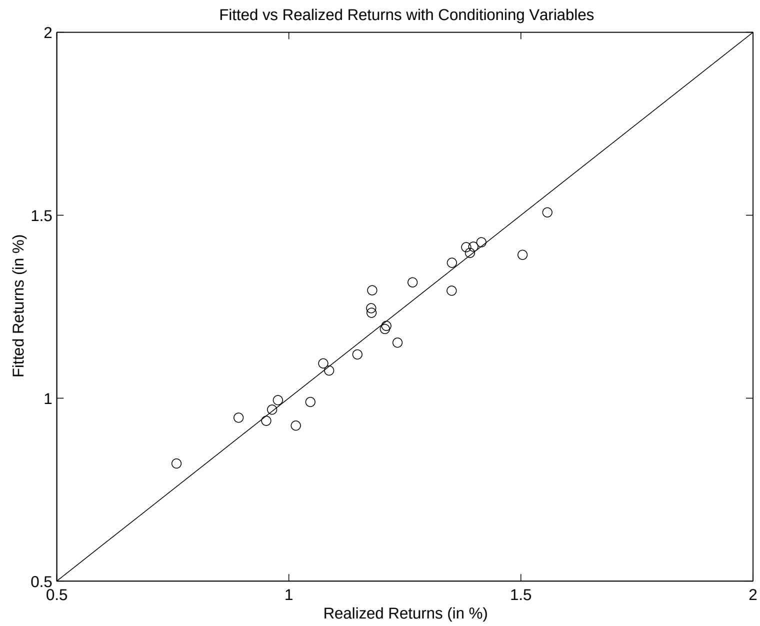

Figure A.4: Model: US Scaled Human Capital CAPM (R-square 0.93)

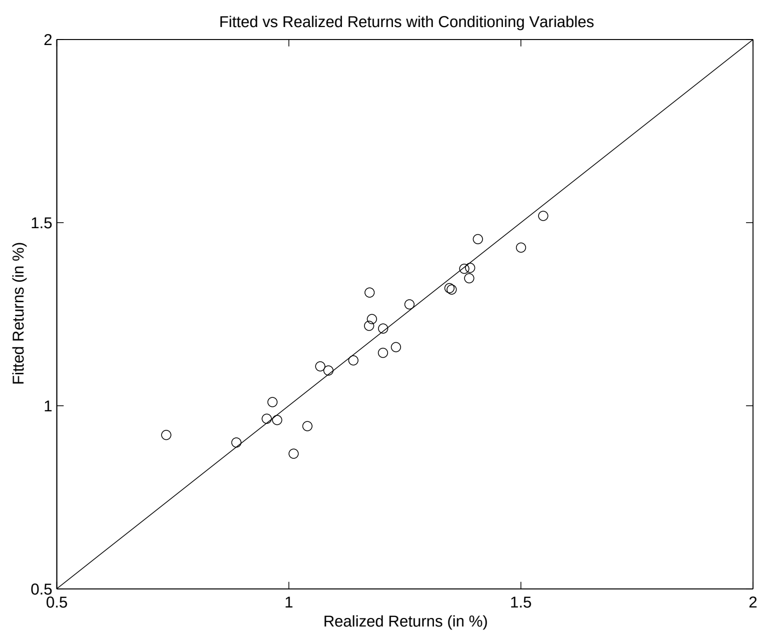

Figure A.5: Model: US Human Capital CAPM (R-square 0.87)

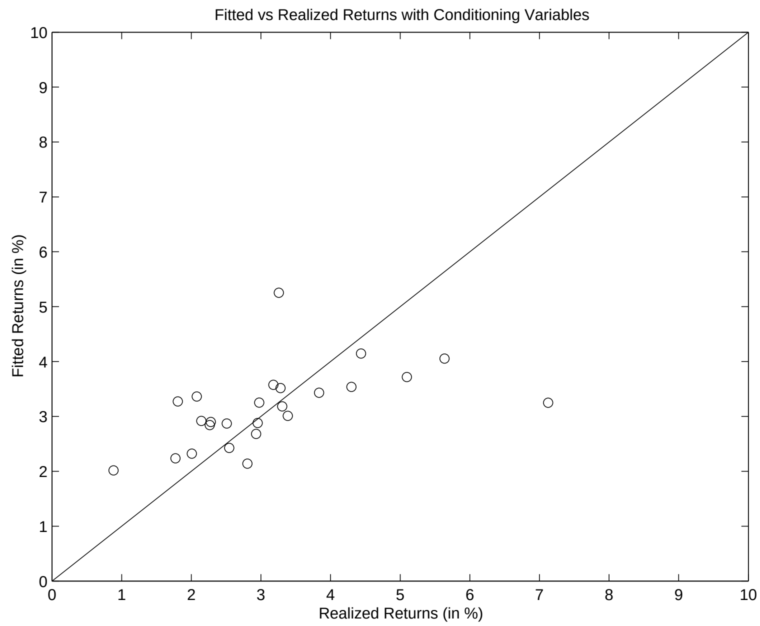

Figure A.6: Model: Static CAPM with Industrial Sector Sorted Data

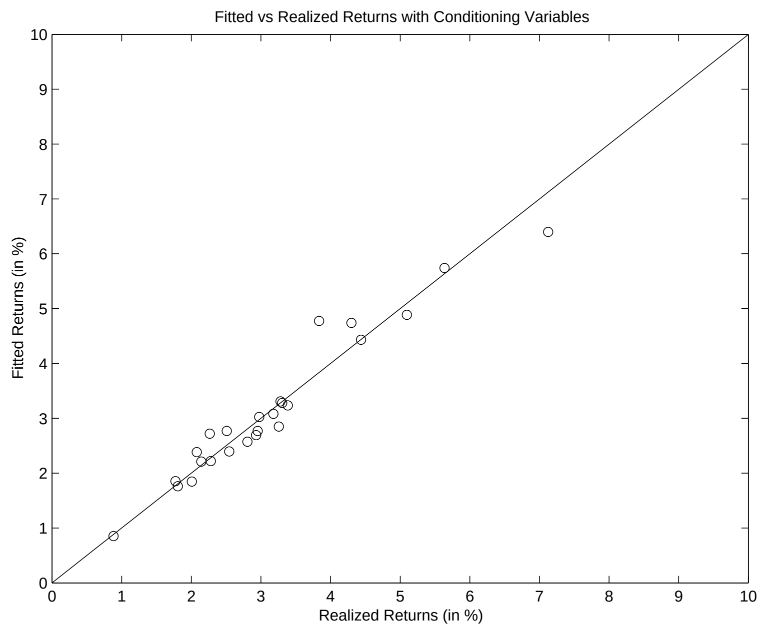

Figure A.7: Model: Simple Consumption CAPM with Industrial Sector Sorted Data

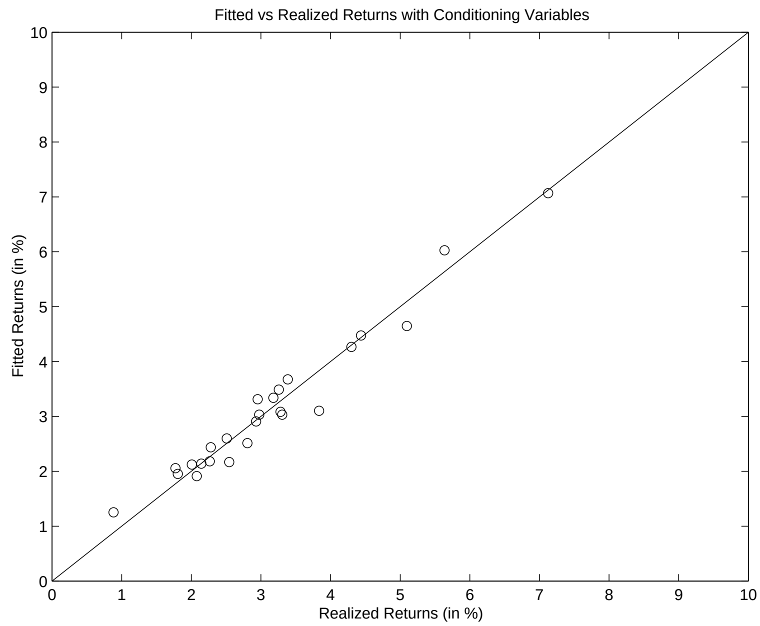

Figure A.8: Model: Fama French 3 Factor Model with Industrial Sector Sorted Data

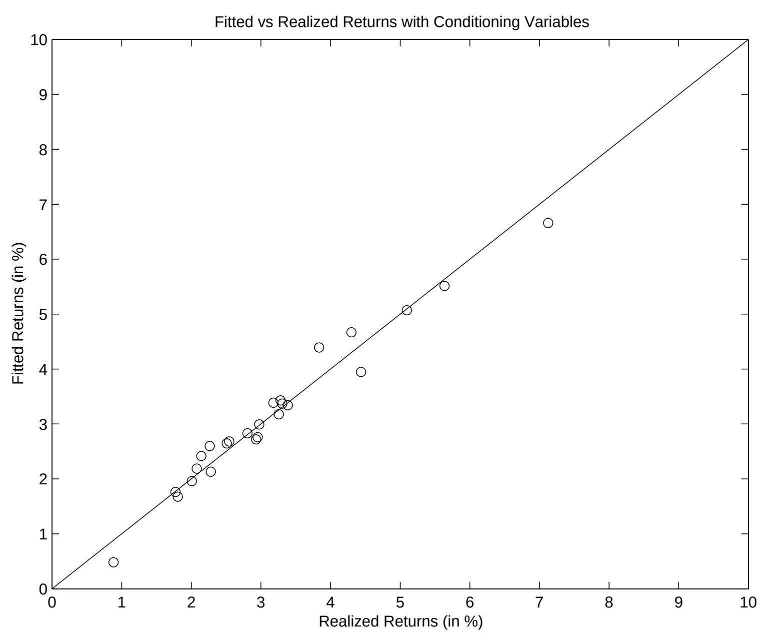

Figure A.9: Model: Human Capital CAPM with Industrial Sector Sorted Data

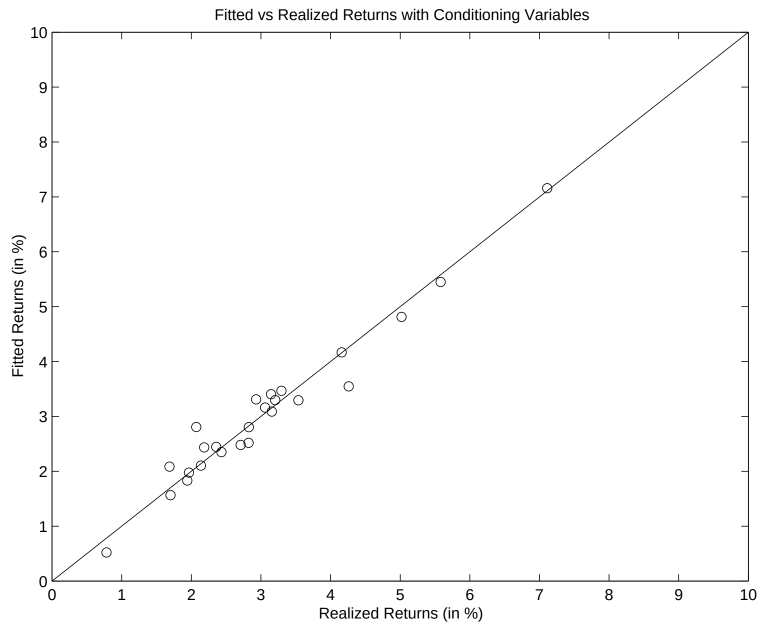

Figure A.10: Model: Scaled CAPM with Industrial Sector Sorted Data

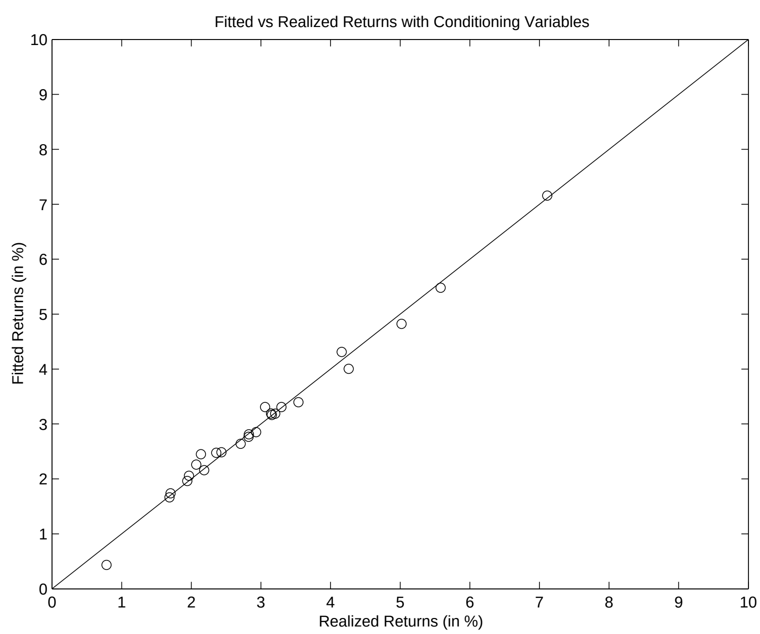

Figure A.11: Model: Scaled Human Capital CAPM with Industrial Sector Sorted Data

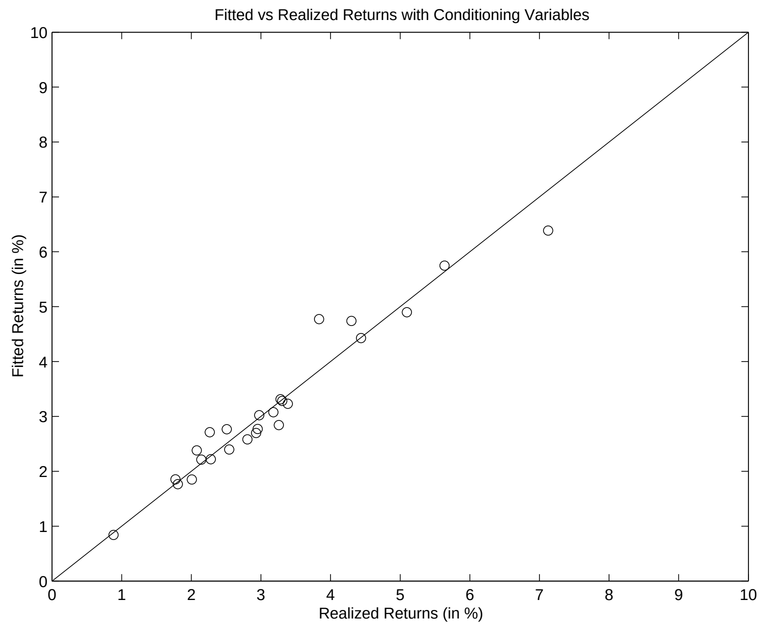

Figure A.12: Model: Unscaled Housing CAPM with Industrial Sector Sorted Data

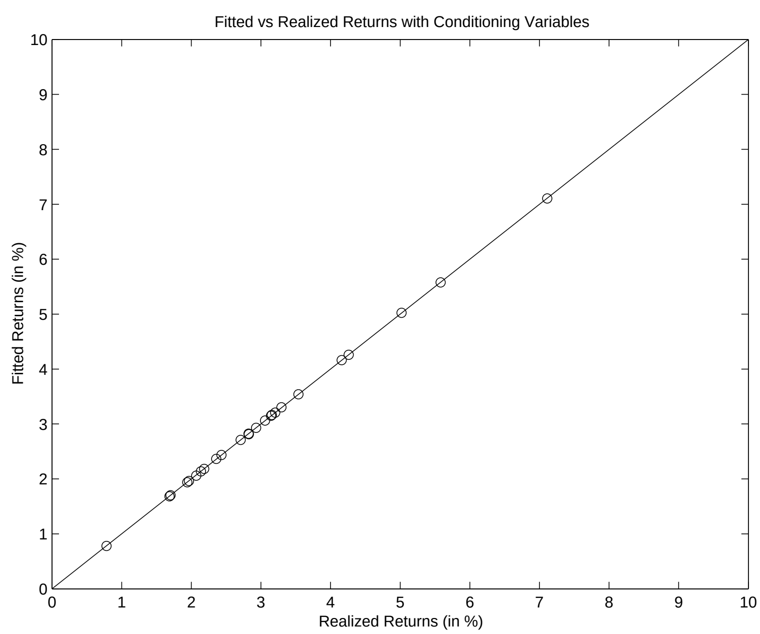

Figure A.13: Model: Scaled Housing CAPM with Industrial Sector Sorted Data

Appendix B

B.1 Main Estimation Results

(Table B.1 and Table B.2)

B.2 Average Pricing Errors

(Table B.3 and Table B.4)

Table A.3 Summaries of Size and Book To Market Ratio Firms 1968 - 2001

	Average Size in Each Size Group						Average B/M in Each B/M Group				
Year	Amount	Group 1	Group 2	Group 3	Group 4	Group5	Group 1	Group 2	Group 3	Group 4	Group 5
1968	114	2.68	8.19	18.76	55.41	464.65	0.84	1.30	1.79	2.48	4.39
1969	119	2.60	8.17	15.93	47.40	368.96	0.72	1.10	1.49	1.98	3.95
1970	136	2.00	4.89	12.69	38.75	334.59	0.63	1.02	1.29	1.83	3.91
1971	187	1.52	3.95	10.41	38.27	312.71	0.71	1.23	1.72	2.45	5.12
1972	215	1.30	3.53	9.93	34.85	322.15	0.79	1.27	1.70	2.32	5.15
1973	225	0.77	1.82	4.68	20.44	232.97	0.54	0.79	1.03	1.34	2.91
1974	233	0.34	0.85	2.33	9.63	113.79	0.23	0.33	0.44	0.58	1.15
1975	233	0.96	2.44	9.56	32.45	274.62	0.34	0.58	0.78	1.04	1.92
1976	233	0.79	2.34	7.12	30.27	313.69	0.29	0.47	0.63	0.84	1.31
1977	234	1.63	4.28	10.81	46.36	398.41	0.39	0.64	0.88	1.18	1.91
1978	236	1.50	3.92	12.10	45.90	409.61	0.43	0.65	0.83	1.09	1.93
1979	237	1.37	4.11	11.95	49.06	456.41	0.36	0.54	0.73	1.00	1.97
1980	240	1.72	5.98	17.59	63.35	540.76	0.29	0.46	0.68	1.02	2.17
1981	243	1.90	7.03	17.64	64.28	570.71	0.35	0.52	0.73	1.06	2.11
1982	250	1.99	6.86	18.24	77.73	706.32	0.32	0.57	0.86	1.24	2.73
1983	260	2.62	8.15	26.29	102.04	908.80	0.43	0.72	1.02	1.51	3.46
1984	277	3.36	12.51	39.65	135.90	1094.40	0.54	0.81	1.13	1.76	5.33
1985	289	4.46	15.88	44.71	167.90	1519.21	0.65	0.96	1.39	2.06	7.02
1986	309	5.25	19.59	56.87	221.08	1933.69	0.78	1.27	1.82	2.50	5.96
1987	332	7.23	23.43	63.15	216.40	2092.99	0.91	1.50	1.99	2.62	7.70
1988	350	7.66	26.17	67.78	224.91	2189.80	0.90	1.46	1.92	2.59	6.68

1989	385	7.86	23.82	63.58	236.66	2881.29	0.88	1.42	1.96	2.72	7.17
1990	416	4.84	19.20	51.94	195.74	2445.47	0.60	1.03	1.49	2.19	5.37
1991	417	6.21	29.17	64.16	210.02	2900.41	0.59	1.12	1.69	2.48	6.84
1992	421	9.50	38.88	91.49	327.28	3443.75	0.53	1.14	1.84	2.70	12.81
1993	437	14.16	44.34	114.72	382.88	4145.62	0.83	1.58	2.29	3.38	16.34
1994	455	12.32	42.20	106.66	330.61	3857.86	0.83	1.49	2.13	3.14	13.77
1995	481	17.09	53.70	120.41	369.11	4188.32	0.84	1.56	2.26	3.48	27.23
1996	487	18.58	61.67	139.20	426.07	4514.55	0.89	1.61	2.49	3.91	14.67
1997	518	18.92	61.48	145.10	401.98	5067.70	0.89	1.58	2.53	4.20	30.47
1998	527	16.88	56.31	128.30	390.39	6204.17	0.63	1.25	2.04	3.67	36.55
1999	533	25.65	80.53	175.22	502.28	8534.98	0.73	1.39	2.59	5.78	210.05
2000	535	27.29	81.22	159.90	427.18	7866.30	0.69	1.30	2.31	4.85	42.70
2001	558	20.44	64.00	127.39	375.14	6864.87	0.66	1.26	2.22	3.74	23.14

Table B.1: Estimation Results with FamaMacBeth Method on Fama French 5*5 Sorted Return

		Factors								Scaled Factors				R^2
Model	Constant	$\Delta \ln c_{t+1}$	$\Delta \ln y_{t+1}$	$\Delta \ln \alpha_{t+1}$	$\ln \alpha_t$	R_{msci}	$\widehat{cay}_t$	SMB	HML	$R_{msci} \cdot \widehat{cay}_t$	$\Delta \ln y_{t+1} \cdot \widehat{cay}_t$	$\ln \alpha_t \cdot \Delta \ln c_{t+1}$	$\ln \alpha_t \cdot \Delta \ln \alpha_{t+1}$	$(\overline{R}^2)$
1	-5.90					23.57								0.5175
	(-2.13)					(8.52)								[0.5089]
2	-0.14	0.90												0.9881
	(-0.34)	(2.18)												[0.9879]
3	-0.53					3.02		2.32	-2.81					0.9831
	(-0.22)					(1.24)		(0.95)	(-1.15)					[0.9822]
4	-0.12		0.96			1.95								0.9915
	(-0.04)		(0.33)			(0.68)								[0.9912]
5	-0.55					2.88	-3.35			-0.06				0.9966
	(-0.21)					(1.09)	(-1.27)			(-0.02)				[0.9964]
6	-0.10		0.71			1.78	-2.79			-0.03	-0.02			0.9984
	(-0.03)		(0.22)			(0.55	(-0.86)			(-0.01)	(-0.01)			[0.9983]
7	-0.15	0.90		0.00										0.9882
	(-0.37)	(2.19)		(0.00)										[0.9877]
8	-0.00	0.1		-0.01	-11.16							-0.08	0.00	1.0000
	(-0.00)	(0.65)		(-0.01)	(-10.70)							(-0.07)	(0.00)	[1.0000]

Note: Model 1 is static CAPM model; Model 2 is simple Consumption CAPM model; Model 3 is Fama and French 3 factor Model; Model 4 is Human Capital CAPM model; Model 5 is Lettau and Ludvigson's $\widehat{cay}$ model with scaling market excess return; Model 6 is Lettau and Ludvigson's $\widehat{cay}$ model with scaling market excess return and Human Capital growth; Model 7 is unscaled housing model; Model 8 is scaled housing model. Coefficient is set to be in percent for sake of comparision. T-statistics is shown in parenthesis, the two side 5% quantile is ± 1.67. R^2 is reported in last column, with a degree of freedom adjusted R squre in block parenthesis. R_{msci} refers to market excess return from MSCI. The sample period is 1987Q1 to 2001Q2.

Table B.2: Estimation Results with FamaMacBeth Method on Industrial Sector Sorted Return

		Factors								Scaled Factors				R^2
Model	Constant	$\Delta \ln c_{t+1}$	$\Delta \ln y_{t+1}$	$\Delta \ln \alpha_{t+1}$	$\ln \alpha_t$	R_{msci}	$\widehat{cay}_t$	SMB	HML	$R_{msci} \cdot \widehat{cay}_t$	$\Delta \ln y_{t+1} \cdot \widehat{cay}_t$	$\ln \alpha_t \cdot \Delta \ln c_{t+1}$	$\ln \alpha_t \cdot \Delta \ln \alpha_{t+1}$	$(\overline{R}^2)$
1	-0.57					23.12								0.2745
	(0.23)					(2.67)								[0.2615]
2	-0.07	0.85												0.9448
	(-0.13)	(1.46)												[0.9438]
3	1.14					-0.96		2.42	-3.02					0.9581
	(0.43)					(-0.37)		(0.92)	(-1.15)					[0.9557]
4	-0.37		0.98			1.85								0.9656
	(-0.15)		(0.39)			(0.74)								[0.9644]
5	-0.46					2.05	-3.49			-0.04				0.9571
	(-0.18)					(0.81)	(-1.38)			(-0.01)				[0.9547]
6	0.07		0.73			1.39	-2.79			-0.02	-0.02			0.9884
	(0.03)		(0.30)			(0.56)	(-1.12)			(-0.01)	(-0.01)			[0.9872]
7	-0.05	0.85		0.00										0.9448
	(-0.11)	(1.43)		(-0.02)										[0.9428]
8	-0.01	0.7		-0.01	-11.65							-0.08	0.00	1.0000
	(-0.00)	(0.34)		(-0.00)	(-5.71)							(-0.04)	(0.00)	[1.0000]

Note: Model 1 is static CAPM model; Model 2 is simple Consumption CAPM model; Model 3 is Fama and French 3 factor Model; Model 4 is Human Capital CAPM model; Model 5 is Lettau and Ludvigson's $\widehat{cay}$ model with scaling market excess return; Model 6 is Lettau and Ludvigson's $\widehat{cay}$ model with scaling market excess return and Human Capital growth; Model 7 is unscaled housing model; Model 8 is scaled housing model. Coefficient is set to be in percent for sake of comparision. T-statistics is shown in parenthesis, the two side 5% quantile is ±1.67. R^2 is reported in last column, with a degree of freedom adjusted R squre in block parenthesis. R_{msci} refers to market excess return from MSCI. The sample period is 1987Q1 to 2001Q2.

Table B.3 Average Pricing Errors with UK Size and B/M Data

Model	1	2	3	4	5	6	7	8
B1S1	3.02	-0.14	0.10	0.20	0.11	-0.06	-0.11	0.01
B1S2	3.09	0.08	0.28	-0.04	-0.10	-0.03	0.07	0.00
B1S3	0.24	0.40	0.19	0.33	0.20	0.12	0.38	-0.01
B1S4	0.29	0.37	-0.18	0.27	0.07	-0.03	0.38	0.01
B1S5	1.34	0.19	0.39	0.07	0.03	0.05	0.19	-0.01
B2S1	1.75	0.12	-0.22	-0.35	-0.13	-0.12	0.14	0.00
B2S2	2.05	0.05	0.26	-0.22	-0.07	-0.01	0.01	0.01
B2S3	-0.31	0.46	-0.37	0.16	-0.03	-0.06	0.47	0.00
B2S4	-1.34	0.10	-0.60	-0.18	-0.35	-0.24	0.10	0.01
B2S5	-0.33	0.02	0.23	-0.14	0.03	0.05	0.01	0.00
B3S1	0.85	0.09	-0.30	0.39	-0.04	0.11	0.08	0.00
B3S2	-0.26	-0.17	-0.30	-0.14	-0.03	0.03	-0.14	0.00
B3S3	0.31	-0.07	0.03	-0.04	-0.01	-0.02	-0.06	0.00
B3S4	-0.98	0.05	-0.33	0.06	-0.11	-0.08	0.06	0.00
B3S5	0.07	0.07	0.18	0.08	0.13	0.08	0.09	0.00
B4S1	0.34	0.10	-0.02	0.11	0.24	0.10	0.09	0.00
B4S2	-0.56	-0.42	0.01	-0.38	-0.10	0.00	-0.44	0.00
B4S3	-0.76	-0.17	0.06	-0.04	0.05	0.01	-0.16	0.00
B4S4	-0.76	0.03	-0.22	0.10	0.04	0.06	0.02	0.00
B4S5	-0.07	-0.03	0.17	0.00	0.12	0.01	-0.04	0.01
B5S1	-2.08	-0.17	-0.18	-0.16	-0.04	-0.06	-0.20	0.00
B5S2	-0.14	-0.43	-0.07	-0.03	-0.08	0.12	-0.41	0.00
B5S3	-2.18	-0.32	0.17	-0.07	-0.03	-0.02	-0.32	0.00
B5S4	-1.61	-0.16	0.31	-0.02	0.00	-0.03	-0.17	0.00
B5S5	-1.94	-0.06	0.40	0.03	0.09	0.00	-0.04	0.00
$\sqrt{\sum \hat{\alpha}_i^2}$	6.98	1.10	1.31	0.93	0.59	0.40	1.09	0.02
χ^2 statistics	296.82	7.33	10.00	8.44	5.47	2.06	6.85	0.00
5% Critical Value	36.42	36.42	33.92	35.17	33.92	31.41	35.17	31.41

Note: Average Pricing Errors are in percent. For model specification please refer to Chapter 5. Sorting information please see Chapter 4.

Table B.4 Average Pricing Errors with Industrial Data

Model	1	2	3	4	5	6	7	8
B1S1	0.12	0.02	0.27	-0.06	-0.09	0.02	0.02	0.00
B1S2	1.58	-0.10	-0.39	0.12	0.13	0.10	-0.11	0.00
B1S3	0.29	0.01	-0.04	0.49	0.71	0.26	0.01	0.00
B1S4	-1.28	-0.30	0.16	-0.11	-0.01	-0.09	-0.30	0.00
B1S5	-0.24	-0.03	0.20	-0.15	-0.26	-0.05	-0.03	0.00
B2S1	0.07	0.18	-0.36	0.19	0.02	0.02	0.18	0.01
B2S2	0.38	0.15	-0.29	0.04	-0.17	-0.01	0.16	-0.01
B2S3	-0.36	-0.26	-0.09	-0.14	-0.09	-0.12	-0.26	-0.01
B2S4	-1.47	0.04	-0.15	0.13	-0.40	0.02	0.04	0.00
B2S5	-0.28	-0.05	-0.06	-0.02	-0.38	0.08	-0.05	0.00
B3S1	-0.40	0.10	-0.16	-0.21	-0.10	-0.25	0.10	0.00
B3S2	0.12	0.15	0.38	-0.14	0.08	-0.05	0.15	0.00
B3S3	-0.46	-0.08	-0.28	0.01	0.14	-0.04	-0.08	0.00
B3S4	0.25	0.24	0.02	0.21	0.30	0.06	0.23	0.00
B3S5	-0.62	0.06	-0.16	0.15	-0.25	0.02	0.06	0.00
B4S1	1.38	0.21	0.45	0.02	0.21	0.20	0.20	-0.01
B4S2	-0.58	-0.46	0.08	-0.33	0.03	-0.31	-0.45	0.00
B4S3	0.76	-0.44	0.03	-0.37	-0.01	-0.15	-0.44	0.00
B4S4	-1.13	0.03	-0.37	0.40	0.26	0.35	0.04	0.00
B4S5	0.41	-0.94	0.73	-0.56	0.24	0.14	-0.94	0.00
B5S1	3.88	0.73	0.06	0.47	-0.05	-0.05	0.74	0.01
B5S2	0.67	0.23	0.29	-0.02	0.23	0.07	0.23	0.00
B5S3	-2.00	0.41	-0.23	0.08	0.07	-0.01	0.41	-0.01
B5S4	-0.31	0.16	-0.12	0.05	0.11	-0.02	0.16	0.00
B5S5	-0.78	-0.07	0.00	-0.27	-0.74	-0.19	-0.07	0.01
$\sqrt{\sum \hat{\alpha}_i^2}$	5.66	1.56	1.36	1.23	1.38	0.72	1.56	0.03
χ^2 statistics	140.02	0.75	11.46	14.21	6.68	0.88	1.15	0.00
5% Critical Value	36.42	36.42	33.92	35.17	33.92	31.41	35.17	31.41

Note: Average Pricing Errors are in percent. For model specification please refer to Chapter 5. Sorting information please see Chapter 4.

Declaration of Authorship

I hereby confirm that I have authored this master thesis independently and without use of others than the indicated resources. All passages, which are literally or in general matter taken out of publications or other resources, are marked as such.

Qi Sun

Berlin and St. Andrews, December 18, 2003