HABIT FORMATION WITH NONSEPARABILITY BETWEEN CONSUMPTION AND LEISURE

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Abstract

Habit formation has had some success in replicating some of the historical financial data. Furthermore, habit formation brings difficulties to the simultanous study of both financial markets and business cycles. This thesis reviews the performance of habit formation and presents an external habit with nonseparability between consumption and leisure. The asset pricing implications obtained, as well as the ability to match main business cycle facts, will be presented and compared with other preferences.

This thesis contains an analysis of historicsl data from the past thirty-eight years, wherein the co-movement between leisure, consumption, and the stock market were measured. This analysis shows some key problems of consumption-based asset pricing literature, e.g., an intensified volatility puzzle, as well as changes over the last decades that have an impact on the asset pricing literature.

The counterfactuality of nonseparability between consumption and leisure for asset pricing will be shown, as well as the poor performance of habit formation as a whole. Moreover, the advantages of nonseparablility in matching business cycle facts will be shown. The model provided overcomes the smoothness of consumption as well as the non-volatility of leisure, whereby these results can be obtained by introducing capital adjustment costs. Finally, the difficulties with this and the possible counterfactuality of leisure responses will be discussed.

Contents

1 Introduction

The thematical context and motivation of this thesis, to present the asset pricing implications as well as the ability of habit formation with nonseparability between consumption and leisure to replicate business cycle facts, go hand in hand. After the introduction of the asset pricing literature, e.g., by Lucas (1978), the research has been interested in replicating financial market data with economic models. This interest has brought forth some puzzles, which are still unsolved today. More then that, it looks as if the attempt to solve one of them individually would be followed by a hand full of new puzzles.

This phenomenon can be illustrated by the three main puzzles of the asset pricing literature. Mehra and Prescott (1985) formulated the *equity premium puzzle*, which was followed by Weil (1989), who introduced the *risk-free rate puzzle*. Finally, Campbell (1996) closed with the *volatility puzzle*, where he mentioned the smoothness of consumption, whereby the volatility of consumption is one of the key facts of the consumption-based asset pricing literature. There can still be no doubt today, two decades after Mehra and Prescott (1985), that there still exists a high equity premium alongside small and unvolitile risk-free rate; but no satisfying model, which is able to replicate this facts without rejecting other stylized facts of economic research. Many have claimed, but as of yet no one conclusively has, to have solve these puzzles.

This research cannot only be measured by its unability to solve the prominent puzzles, as the desire to solve them, brought some interesting discussions about market imperfections as well some new kinds of preferences. These results brings the question about the existing economic research, as well they brings the knowledge, that the desire to transform human proceed in relative easy mathmatical models looks limited.

The cornerstone of this thesis is habit formation, which this is one of the new preference structures that has been developed in recent years. The idea of this paper is to review habit formation, which was introduced by Sundaresan (1989) and Constantinides (1990) fifteen years ago, and to solidify it as a main direction in the asset pricing literature, which can be observed in the rampantly growing literature from recent years. Next to this specific preference structure, this review goes hand in hand with a review of the whole consumption-based asset pricing literature (more comprehensive reviews van be found, e.g., in Campbell, 2002; Cochrane, 2001; Mehra and Prescott, 2003). Habit formation has had some success in explaining key facts of financial markets, these postive aspects will be highlighted alongside the "tricks" used and the costs and problems by using this utility specification to resolve the stylized facts. Furthermore, this paper will not attempt to present a possible parameter set that can solve the puzzles, like is all too often done in the literature, rather it will show the "absurdity" of the parameters implied so that these puzzles be solved. All together this thesis can be understood as a respectful criticism of habit formation as part of the consumption-based asset pricing research.

In contrast, to many papers in the asset pricing literature, the motivation should not only be to recreate the financial data, but also to confirm to the results of business cycle research. The problem that the asset pricing literature has had in fitting financial data without rejecting bussiness cycle facts was mentioned, e.g., by Lettau and Uhlig (2000), and it reflects one of the main problems this thesis will try to cover. There exist many habit models in the literature, that look to combine financial implications with business cycle implications (e.g., Boldrin et al., 2001; Jermann, 1998; Lettau and Uhlig, 2000). The different key intuitions behind this models - for example, e.g., Boldrin et al. (2001) concentrated on financial aspects, whereas Lettau and Uhlig (2000) were more interested in the volatility of consumption - illustrates the problem in successfully combining the two.

Past research has shown that the extension of prefernces with leisure helps to match main business cycle facts (see, e.g., Kydland and Prescott, 1982; Hansen, 1985), so that many habit models try to cover the importance of leisure for business cycle theory. Because of the counterfactual influence of leisure if it is part of the Lucas Asset Pricing Formula (Lettau, 2003), most research uses separable preferences. Another consequence of use of separable leisure is that these utility functions, introduced by Hansen (1985), make the agent less able to smooth consumption.

The use of separable preferences between leisure and consumption is a legitimitate adjustment, but the neglected habit in leisure its not. The habit literature gives a good intuitive explanation for the existence of habit formation - that today's decision is also affected by past decisions - but it cannot explain why it is important to respect past consumption and, simutanously, neglect past leisure. Of course, Lettau and Uhlig (2000) included a leisure habit and show that it is not very inducive of better results: in this kind of preferences, a leisure habit does not increase the volatility of consumption, but decreases the volatility of leisure. The result emanates for the restrictive influence of leisure on the utility function. These results are not very comforting, but however to neglect leisure and/or leisure habit is no solution. A removal of the leisure habit only to present better results is not satisfying. Moreover it is an indication of the poor performance of the entire habit formation. This thesis postulates a leisure habit and, in contrast to most of the literature, a nonseparable preference structure between consumption and leisure. It will be shown that nonseparable leisure extensions in habit models can yield non-negative solutions.

To illustrate how habit formation works, the paper is broken down into two main parts. Firstly, the asset pricing implications with different utility functions will be discussed, whereby, next to the standard external difference habit and the leisure-extended version thereof, the standard power utility function used by Mehra and Prescott (1985) and power utility with nonseparability between consumption and leisure, comparable to the one used in Eichenbaum et al. (1988), will be compared. This analysis provides some asset pricing implications and illustrates the problems that consumption-based asset pricing has in fitting the stylized facts of financial markets. The solutions obtained from these calculation will be critical discussed and the counterfactuality of a leisure extension for consumption-based asset pricing will be corroporated. More than that the poor performance of habit formation can be corroporated.

Secondly, a simple RBC model will be presented, which will be used to form an idea as to how well habit formation can replicate main business cycle facts. For this analysis, some adjustments from previous models in the literature were used to fit the data as well as possible. The comparison with other preferences, mentioned above, shows that, for business cycle analysis an extension with leisure can be helpful. For example, it will be shown that leisure helps to overcome the smoothness of consumption, which is a main problem of habit formation, and, especially, of the seperable leisure habit model by Lettau and Uhlig (2000)- due to its unsolveability. Furthermore, the second advantage of the nonseparability used in this thesis is the ability to generate more volatile leisure. But these solutions are not a victory in theirself, but rater they look like a "pyrrhic-victory", because they can only be obtained by concentrating on business cycles, especially, on consumption volatility, and by simultanously neglecting financial market facts. Furthermore, the problem of less volatile leisure, as mentioned by Lettau and Uhlig (2000), still exists.

Finally, this paper concludes by comparing the results of asset pricing implications with those of the business cycle implications. Furthermore, it illustrates rather the problem with consumption-based asset pricing and less the problem with a leisure extension, especially nonseparability between consumption and leisure. Of course, the counterfactuality of nonseparable leisure can be corroporated, but the counterfactuality between fitting financial data and business cycle facts look be heavier. What at first glance may look like another indication for the bad performance of habit formation can alternativly interpreted as a success, because it shows that habit formation is able to fit main business cycle facts, and generates simultanously better results by fitting financial facts, in contrast to standard preferences. Of course, this "success" is marginal, but it is nonetheless a step in the right direction: the motivation, as previously mentioned, was not to match the one whlie neglecting the other, but rather to show that matching both as well as possible.

This thesis is divided into six sections, the first of which is the nearly completed introduction. A short literature review examining both the developments in and the interesting directions of the asset pricing literature will preface the analysis. Section three presents and discusses some stylized facts of business cycles and statistics of financial markets as a background for comparision with the results obtained thereafter in this thesis. Section four begins the introduction of the model, first by reviewing the basic asset pricing implications, then by introducing the specific implications resulting from a utility function with nonseparability between consumption and leisure, and finally by comparing and discussing the usefulness of the results. Section five presents an RBC model able to fit many of the main business cycle facts, it then continous with a discussion of the model and its computed results, troughout which, section five describes the way followed and the adjustments necessary. Section six juxtaposes the results from the two mainparts of this thesis and then concludes.

2 Literature

The Real Business Cycle Theory has had great success in recounting most of the empirical business-cycle data (e.g. Kydland and Prescott, 1982), and as such, much of today's economical intuition comes from these neoclassical growth models (Mehra and Prescott, 2003). The foundations were laid down by, e.g., Lucas (1978) when he implemented a representative agent model in which per capita consumption and the consumption by a typical agent were perfectly correlated (a similiar discussion is given by Kocherlakota, 1996). This illustrates one of the key facts in RBC theory, where the difference between today's consumption and the expectation of tomorrow's represents the willingness of the individual to substitute between these. The main result of Lucas (1978), the Lucas Asset Pricing Formula, shows that in these kind of models every risk of an asset should be able to be represented by the covariance between per capita consumption and the asset return (see Kocherlakota, 1996, pp. 42-43).

A look at financial data shows the weakness of these models to replicate key empirical elements, e.g, stock returns. This problem was mentioned by Mehra and Prescott (1985) when they showed that only an implausibly large risk averseness of the representive agent would fit the historical differences between stock returns and the return on a relative riskless asset: they called this the equity premium puzzle.

This was extended by Weil (1989), who found a second puzzle in the data: the risk-free rate puzzle. This second anomaly is given by the fact that if the risk premium is high enough to require a high level of risk aversion , the individual, in standard preference models, would dislike consumption growth; all of which is consistent with a high risk-free rate. Unfortunately, the real return of a relatively riskless asset has been empirically observered to be extremely small (see table 1) - implying that the individual does indeed like consumption growth (see Kocherlakota, 1996)! Thenceforth, the literature looks to solve these two puzzles. A look into recent research shows that probable solutions might be those that argue that possible market imperfections could solve the puzzles, or, on the other hand, that alternative preferences could be a point of departure to resolve these anomalies. This distinction can be found in the textbooks of Campbell et al. (1997) and Cochrane (2001), as well as in the comprehensive articles of Kocherlakota (1996), Cochrane (1997), Campbell (2000, 2002), Mehra and Prescott (2003), and Constantinides (2002); because of the exellence of these reviews, I will often lean on these authors to review the main directions.

A frequently used assumption in economic models is that investors form rational expectations. A deviation from this direction in the asset pricing literature looks at the consequences of assuming irrational beliefs (see, e.g., Cecchetti et al. (1998) and Hansen et al. (1999)). Campbell (2002) discusses these models and determined that the limitations of this direction are given by the fact that its models only function in partial equilibrium but do not consider general equilibrium issues. A similiar methodology is followed by Rietz (1988), where the investor believes in a small probability of a catastrophic event that consequently drives asset prices, such a situation is called the "peso problem" (see Campbell et al., 1997, p. 310). Rietz (1988) postulated that if the investor concern himself with a catastrophe that has not yet, and may not ever, occur, a high equity premium would go hand in hand with a small risk-free rate (see a comprehensive discussion about this argumentation in Mehra and Prescott, 2003). The paper of Campbell (2000) criticizes the argumentation of Rietz (1988) with the rebutal that it would require - next to the probability of the catastrophe - that stockholders be more serious in trading assets than short-term bondholders are.

Another alternative is that there exists stock market segmentation. This means that in contrast to the standard model by Mehra and Prescott (1985), which assumes one representive agent, it is assumed to be possible to distinguish between consumption of stockholders and nonstockholders. Mankiw and Zeldes (1991) have shown that consumption by stockholders is more volatile and more highly correlated with stock market data than consumption by nonstockholders. A similiar discussion about market segmentation can be found in most of the asset pricing review literature - so, in Kocherlakota (1996), Campbell (2002) and Constantinides (2002). As a result it is to be concluded that there is consensus that market segmentation alone cannot solve the anomalies. Next to a different consumption decision, there are several papers, e.g., Heaton and Lucas (1997, 2000), that distinguish between different investor behaviors - like portfolio choice or saving behaviors.

The next necessary direction which should be discussed is the theory of idiosyncratic income shocks (a main study is provided by Constantinides and Duffie, 1996). To make it possible to simulate the stock market data, these models require some specifications of the shock process; these can be summerized as follows: the shock must be uninsurable, persistent, heteroskedastic, and have counter-cyclical conditional variance (see Constantinides, 2002). The problem of this model is that it needs a implausibly high risk aversion as in other models (see the discussion in Cochrane, 1997; Mehra and Prescott, 2003). Furthermore, the empirical evidence of idiosyncratic uncertainty was tested by Lettau (2001), and he observed that only extreme cases reached a data-equivalent.

If there is a sizeable fraction of agents with borrowing constraints - as Heaton and Lucas (1996) found out in their numerical work - then the risk-free rate may be substantially lower than when there are no constraints. But Heaton and Lucas (1996) have furthermore shown that such constraints have no important effects on the risk premium (see Kocherlakota, 1996, p. 64). Another point of depature, with respect to borrowing constraints, is given by Constantinides et al. (2002); the authors implemented heterogeneity in a overlapping three-generation model. The result from this framework is that the youngest generation had the strongest incentives to hold equity stocks (a comprehensive explenation is, next to the authors, given by Mehra and Prescott, 2003). It is then possible to generate a high equity premium when these agents have borrowing constraints on their ability to buy equity stocks.

In a perfect market, the agent can sign a contract to insure herself against all possible risks in her consumption stream. Like in Kocherlakota (1996), the existence of such a complete insurance market will be neglected. So, e.g., Alvarez and Jermann (2000) found out that it is possible to obtain a solution by assuming that risk sharing is limited. They argued that this assumption would drive the agent to be more impatient and risk tolerant, and, because for that, the risk-free rate decreases. Furthermore, the authors pointed out that asset prices depend on the individual risk-affinity of an agent. The necessary heterogeneity, as in the models above, will be characterized by: "…agents whose income process is similiar to the aggregate are irrelevant for asset prices." (Alvarez and Jermann, 2000, p. 791).

Finally the section dealing with incomplete markets should be completed by a short discussion of transaction costs. The perfect market assumption that there are no necessary costs when trading assets seems to be unrealistic. Buying or holding assets goes hand in hand with many kinds of transaction costs like taxes, bank fees, and information costs. The possible relationship of such transaction costs were studied, e.g., by Aiyagari and Gertler (1991) and Heaton and Lucas (1996). They found out that transaction costs could possibly explain the equity premium; but in order for it to do so, there must exist a high difference in these costs across equity and bond markets. Further, Kocherlakota (1996) saw only a little evidence to support this proposition. Another point of view was maintained in the study by McGratten and Prescott (2001). The authors found out that the period between 1960 and 2000 was economically and politically stable, but that the unanticipated change in capital taxation could have increased the capital gain on corporate equity (see Mehra and Prescott, 2003).

The completeness of asset markets is one of the key frameworks in finance research. As it would be undesireable to have to reject it, another direction of the literature looks, alternatively, to solve the two puzzles by modifying preferences (Kocherlakota, 1996). The research in solving the two puzzles by modifying preferences can be grouped into two directions. Mehra (2003) distinguishes between the modification of the conventional time-and-state-seperable utility function and the use of habit formation.

Because of the problems of the standard utility function used by Mehra and Prescott (1985), Epstein and Zin (1989, 1991) and Weil (1989) introduced a new kind of preference class. These preferences are called “Generalized Expected Utility” (GEU). An important advantage of these preferences is that it is posssible to use a different parameterization for the coefficent of risk aversion and the elasticity of intertemporal substitution (see also Cochrane, 2001; Campbell et al., 1997; Campbell, 2002; Mehra and Prescott, 2003), which helps to remove one of the main problems with standard power utility, where the elasticity of intertemporal substitution and risk aversion are rigidly linked to each other (Mehra and Prescott, 2003). Another advantage is that a high risk aversion does not go hand in hand with a dislike of consumption growth, as with power utility preferences (see discussion above). This type of preferences can help solve the equity premium puzzle and, especially, the risk-free rate puzzle. Furthermore, the literature describes the assumptions that are needed to resolve the puzzles as critical. For example, Mehra and Prescott (2003) argue that finding the right parameters to calibrate the model is crucial. Part of the preferences is a wealth function, which includes human capital as well as financial data - which data should be used? Epstein and Zin (1989, 1991) approximated this by using the data of the “market portfolio” for calibration (Mehra and Prescott, 2003). A main result was that the equity premium can be explained not only by risk aversion, but also by a higher variance in wealth than in consumption. The latter is rejected by Campbell (2002) with the argumentation that wealth and consumption are linked to each other, so that different volatilities would bring us a new puzzle. Another difficulty is the high elasticity of intertemporal substitution, which would allow the risk-free rate puzzle to be solved. Campbell (2002) describes the problem with the empirical evidence of a low elasticity of intertemporal substitution in consumption. Finally, Tallarini (2000) has shown that preference parameter settings, which are chosen to fit the financial data, increase the welfare costs of business cycles.

From now on, the discussion of the literature is concentrated on the second alternative, often used to modifiy preferences - habit formation. The growing importance of habit formation in the asset pricing literature goes back to the studies by Sundaresan (1989) and Constantinides (1990). The key intuition behind habit formation is that consumers’ consumption is not only a decision reached purly in the present, but is also influenced by past consumption, i.e. their preferences are habitually formed.

There are different modeling issues, which will be summerized in the next sentences by refering to most of the important work and some new directions that these preferences have brought forth during the last fifteen years. Firstly, two main directions as to how the habit stock is modeled with aggregate consumption should be distinguished: ratio models and difference models. Ratio models imply that the agent's utility is a power function of the ratio of today's consumption to the habit stock, this kind of habit formation modeling can be found in Abel (1990, 1999) and Collard et al. (2003). In contrast, most of the literature uses a power utility function, whereby the agent maximizes her utility over the difference between consumption and the habit stock. This was first used by Sundaresan (1989) and Constantinides (1990) [1]. The choice of the model is not trivial - so Chen and Ludvigson (2003) argue that ratio models have difficulty accounting for the predictability of excess stock returns. By contrast, difference models can generate time variation - because relative risk aversion varies countercyclically unlike ratio models, with which the relative risk aversion is constant (see Chen and Ludvigson, 2003). Another motivation for using difference models is the argumentation by Mehra and Prescott (2003): difference models can generate a high risk premium, but ratio models generate similiar risk premiums as standard preferences (see Campbell et al., 1997, pp. 327-329). Finally, a new model intuition was found by Budria-Rodriguez (2002), where the author implemented a multiplicative habit; because this kind of habit is hitherto uncommon in the literature, it will not be discussed in any detail.

A second point that differentiates the literature is the question of who is actually influencing the habit stock, whereby the two different kinds are called "internal" and "external". "Internal" habit formation assumes that the agent's consumption decision today depends on her own past consumption. Such habit formation was introduced by Sundaresan (1989) and Constantinides (1990), as well as in the work of Boldrin et al. (1995, 1997, 2001), Allais (2003), Collard et al. (2003), Chapman (2002), and Heaton (1995). In contrast to this assumption is the use of an "external" habit, first used by Abel (1990) and can be found as well as in Gali (1994), Campbell and Cochrane (1999, 2000), Wachter (2002), and Menzly et al. (2002). The formation of an "external" habit is often called "catching up with the Joneses", which was mentioned in the work of Abel (1990). This phrase "…reflects the assumption that consumers care about the lagged value of aggregate consumption" (Abel, 1990). The choice between these alternatives to define the habit is important. Next to the different affects on the Lucas Asset Pricing Formula (see, e.g., Campbell et al., 1997), both have different implications on the optimal tax and welfare policies (see Ljungqvist and Uhlig, 2000). Chen and Ludvigson (2003) tested both variants and obtained the solution that an internal habit formation should be prefered over an external specification. Cochrane (2001, ch. 21) argues that the use of internal or external habit formation is only a technical convenience and that, in model by Campbell and Cochrane (1999), it does not make a difference. This was supported by Chen and Ludvigson (2003) with the argumentation that if the habit is a nonlinear function, the asset pricing implications must be the same.

[1] There are more paper that use this kind of implementation, e.g., Boldrin et al. (1995, 1997, 2001), Campbell and Cochrane (1999, 2000), Heaton (1995), Jermann (1998), Wachter (2002), and Menzly et al. (2002).

This argumentation brings us to the third issue in modeling habit formation: is the habit a linear or nonlinear function? The most recent paper that deals with a nonlinear habit function is Campbell and Cochrane (1999). This nonlinearity was implemented further by, e.g., Campbell and Cochrane (2000), Wachter (2002), and Menzly et al. (2002). In contrast, the models of Sundaresan (1989), Constantinides (1990), Boldrin et al. (1995, 1997, 2001), Heaton (1995), Jermann (1998), and Allais (2003) concentrated on a linear function of the habit. For the importance of the nonlinearity of the habit, Campbell and Cochrane (1999) argued, and subsequently implemented in their model, that the assumption of nonlinearity is imperative that the model conform to the Sharpe Ratio and the predictability of excess asset returns. A simliar resolution - that nonlinearity is favorable - was given by Chen and Ludvigson (2003), whereby they used a null hypothesis test to distinguish between the functional forms as well as to distinguish between the aforementioned internal and external habit formations.

Contrastingly, the nonlinear habit model, for example, from Campbell and Cochrane (1999, 2000), has greater difficulties in handling risk aversion. Thus, the linear habits by Constantinides (1990) and Boldrin et al. (1995, 1997, 2001) were able to generate a higher risk premium with lower risk aversion, but failed with respect to predictability. The argument by Chen and Ludvigson (2003) in favor of an internal habit and nonlinearity, confronts us with the question of why the usage of an internal habit is preferable if the nonlinearity begets the same results irrespective of the type of habit - internal or external. Because the authors tested only the asset pricing implications and did not implement any policy, Cochrane (2001) concludes that every kind of habit formation hitherto implemented in the asset pricing literature has difficulties in replicating all necessary empirics.

> "No current model generates the equity premium with a low and relatively constant interest rate, low risk aversion, and the right pattern predictability - high prices forecast low returns, not high returns, and consumption is roughly a random walk."
> (Cochrane, 2001, p. 473)

Finally, although leisure is one of the variables most frequently added to the utility function (Cochrane, 1997), and had great success in explaining most of the main business cycle facts (see, e.g., Kydland and Prescott, 1982; Hansen, 1985), leisure is generally omitted in most of the asset pricing literature, not because its theoretical importance is questionable, but because the results that its inclusion cause are undesireable. So, for example, Eichenbaum et al. (1988) have used a standard power utility function with a nonseparable leisure variable, similar to the work by Kydland and Prescott (1982). The authors demonstrated the influence of leisure - by combining consumption and leisure in this way - on the stochastic discount factor. The inability to generate a satisfying risk premium is discussed, as well by Lettau (2003) and Lettau and Uhlig (2000), who argued for a negative correlation between consumption and leisure - which brings even more counterfactual asset pricing implications. On the other hand, the implementation of leisure by Jagannathan and Wang (1996) and Campbell (1996) has shown that adding labor-income growth can be useful in explaining the cross-section of average stock returns (see the discussion in Cochrane, 1997).

The surfacing of counterfactual asset pricing implications when modeling consumption and leisure nonseperably has led the literature to more frequently use seperable utility functions. Whereas Lettau (2003) used a standard model in the form of Hansen (1985), papers like Boldrin et al. (1995, 1997, 2001) avoid the nonseparability assumption by utilizing the characteristics of the power utility, namely, that it is logarithmic if the power parameter approaches one. Furthermore, the use of leisure in habit formation, as, e.g., by Boldrin et al. (1995, 1997, 2001), is most conspicuously marked by the fact that leisure is not part of habit stock; the intuition behind which is unsatisfying: the consumer concerns herself with past consumption decisions but completely ignores past leisure decisions - if one analyzes the past, one should analyze the entire past. An exception is the paper by Lettau and Uhlig (2000), where the authors implemented a model, separable between consumption and leisure, containing two different habit stocks: one with respect to consumption and the other with respect to leisure. Lettau and Uhlig (2000) presented the results that consumption is extremely smooth and unresponsive to shocks, as well as not strongly affected by the leisure habit.

It is to be concluded that the consumption-based asset pricing literature has problems conforming to all stylized financial facts (Cochrane, 2001) whilst simultanously explaining stock market facts and business cycle facts (Lettau and Uhlig, 2000).

3 Historical Data and Stylized Facts

Because the motivation is to examine utility functions with nonseparability between consumption and leisure, its follows that leisure influence the Euler equation. Furthermore, leisure will be part of the asset pricing implications, which forces me, to examine the relationship between leisure and the stock market as well. The comprehensive stock market examinations by, e.g., Campbell (2002) and Shiller (2000) do not contain this relationship, so it was necessary to examine the important data series by myself. The first part of this section contains the explanation of the series, some remarks, and the results of the examination. In the second part, the data will be prepared for the endogenous solution.

3.1 Statistics of Consumption, Leisure, and Asset Returns

The data series used are post-war annual US time series [2]. Many empirical analyses of stock prices work with annual data sets, for example Mehra and Prescott (1985), which is a focus of this paper. Mehra and Prescott (1985) used a larger data set [3], beginning in 1889. Forced by limited access to some time series, this thesis uses data from 1964 to 2002. This time span of data should allow for some sort of idea as to how or if asset returns are affected by leisure growth. A discussion of stock returns, over time, is always followed by the question of which return average should be used: the geometric or arithmetic one?[4] The literature is full of arguments, pro and contra the arithmetic

[2] The study of Campbell (2002), for example takes a look at international data, for exapmle.

[3] Shiller (2000) provided comprehensive quarterly and annual data sets, including consumption and asset returns. An updated version is available on the author's homepage.

[4] Next to every finance handbook, a discussion can be found in Mehra and Prescott (2003)

average. The arithmetic average was choosen by assuming that the stock returns are uncorrelated over time (e.g. Mehra and Prescott, 1985).

All together, this thesis is based on seven basic time series for the period from 1965 to 2002. The first five series look like the series by Mehra and Prescott (1985), the last two were inspired by Hansen (1985). A short describtion of each time series follows below:

1. *Series SP*: Year-end annual Standard and Poor's Composite Price Index corrected by the consumption deflator series.

2. *Series D*: Annual real dividends for the Standard and Poor's series, provided by using series SP and the annual dividend yield of the S&P 500 Index.

3. *Series C*: Per capita real consumption [5] of non-durables and services using data from the Bureau of Economic Analysis (BEA) and the Federel Reserve Bank of St. Louis.

4. *Series CD*: Consumption deflator series, obtained by using the series of nominal and real consumption of non-durables and services by the BEA.

5. *Series TB*: Nominal yield on a 3-month Treasury Bill.

6. *Series LW*: Average leisure-hours per week in a year for an employee in non-agriculture sectors, by using the series of average weekly total private hours worked - by the Bureau of Labor Statistics (BLS).

7. *Series LT*: Average leisure-hours per week of a US citizen, by using series LW, total employment, and total population data series from the Federal Reserve Bank of St. Louis.

The series SP and D were used to generate the annual real return of the S&P 500 Composite Index, where the return can be expressed as:

$$R^e_{t+1} = \frac{SP_{t+1} + D_{t+1}}{SP_t}.$$

To get the real return on the treasury bill, I used the series TB and CD, so that follows:

$$R^f_{t+1} = TB_{t+1} - \frac{CD_{t+1} - CD_t}{CD_t}.$$

The annual real returns on a stock and a treasury bill are plotted in figure 1. The real annual growth of consumption and dividends is nothing more than the annual change in the time series and is plotted in figure 2. This figure illustrates very well one of the problems of consumption-basd asset pricing: the observeable smoothness of consumption growth, called the volatility puzzle by Campbell (1996). Figure 3 shows the annual growth rates of the two kinds of leisure - they look similiar but are indeed different.

From an aggregate point of view, the leisure per worker is continously increasing, but the average leisure of a citizen is continously decreasing. That means that the individual employee is

[5] All real datas in this paper are in year 2000 dollars

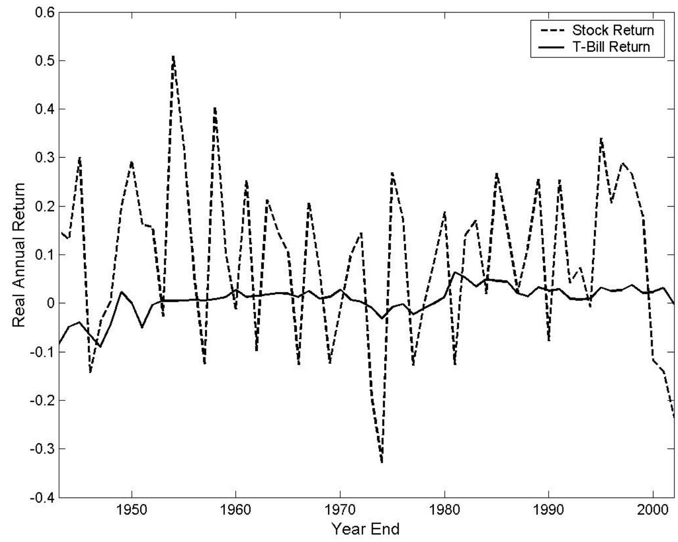


Figure 1: REAL ANNUAL RETURNS OF THE S&P 500 AND 3-MONTH TREASURY BILLS FROM 1943-2002

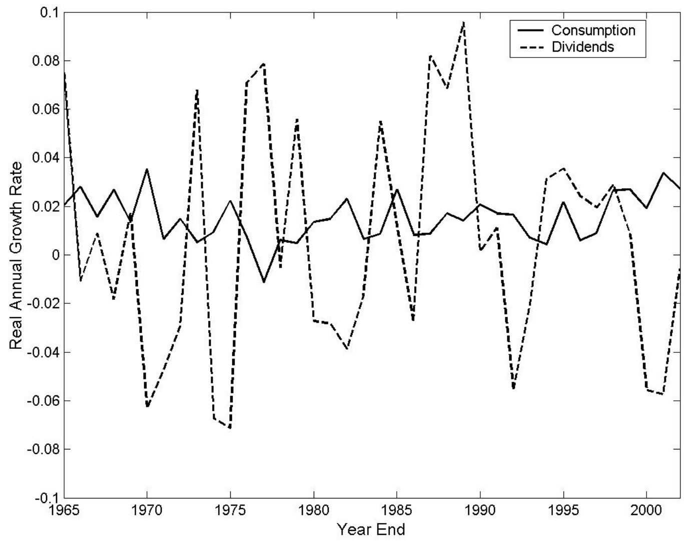


Figure 2: ANNUAL GROWTH RATES OF REAL CONSUMPTION AND REAL DIVIDENDS FROM 1965 - 2002

working less but society is working more, which could be explained by an increasing participation rate (e.g. it is known that the participation of women was strongly increasing during this time). It is clear that using employment statistics are problematical, because, e.g., homework is not part of official statistics. Hansen (1985) discussed this problem too and used non-establishment statistics, like those from the Current Population Survey; the study of Kydland and Prescott (1991) compares such data series with official data. It is clear that the leisure statistics used here are a weak point of

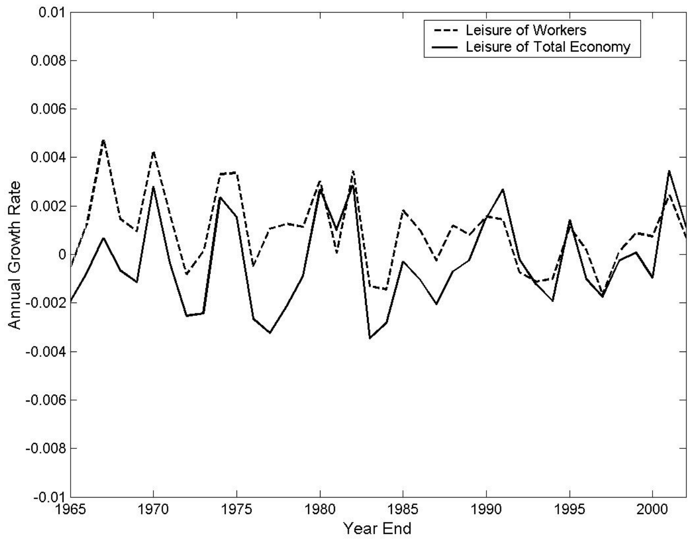


Figure 3: ANNUAL GROWTH RATES OF AVERAGE LEISURE HOURS PER WEEK OF WORKER AND WHOLE CITIZENS BETWEEN 1965 AND 2002

this paper. However, the observations brings us the question of possible differentiated individual decision making, a similiar observation was made by Mankiw and Zeldes (1991), were the authors found that the consumption growth of stockholders is more volatile than that of nonstockholders. The observation in this thesis does not distinguish between stockholders and nonstockholders, but it looks as if there were a heterogeneity between the importance of leisure for different citizens. If we observe the entire period, than it can be seen that leisure did not change very much over time, but consumption increased continously. This phenomenon is mentioned, e.g., by McGratten and Rogerson (1998), where the authors examined the entire post-war period. The same observation was made by Prescott (1986); the author observed that virtually no significant trend in leisure exists, but real wages continously increased. This can be concluded from the fact that it looks as if the income- and substitution effects were approximately equal during the last fifty years. Nevertheless, the importance of leisure for individuals, through heterogeneity exists, has increased, which could go hand in hand with changing risk- and consumption decisions. This result increases the worth of the time span used, because the dramatic change in the last fifty years should be examined seperately; any observation of a different period, e.g., of the last one hundred years, would not replicate this fact.

Finally, the equity premium was derived as the difference between real stock returns and the real returns on a treasury bills. The equity premium varied dramatically over time, some annual premiums are even negative as figure 4 illustrates. It is quite clear that the variation of the risk premium depends on the time horizon over which it is being measured. With respect to Mehra and Prescott (2003), this was illustrated by plotting the risk premium over a 20-year period (figure 5).

In respect to the changes in the risk premium over time, different periods for calculations will be used, wherein the horizons differ in length. The choice of the different samples was motivated by

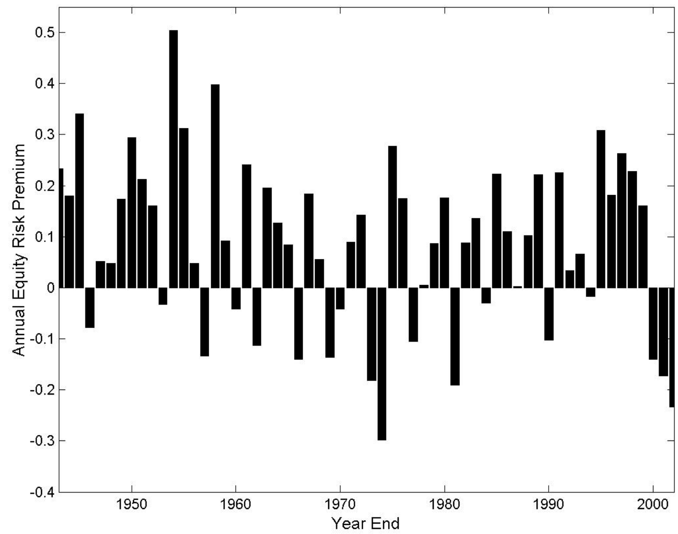


Figure 4: ANNUAL RISK PREMIUM FROM 1943-2002

the different risk premiums, Sharpe Ratios, and, moreover, by the decision to show the differences in the co-movements of leisure growth, consumption growth, and asset returns. Mehra and Prescott (1985) obtain the result that the risk premium depends only on the relative risk aversion of the individual and the covariance between consumption growth and stock returns. Vice versa it should hold that the covariance between consumption growth and stock returns increase when the risk premium increases (assuming constant relative risk aversion). It is clear at this point in the paper that this can only be a first examination. As an introduction, it helps only to show what the

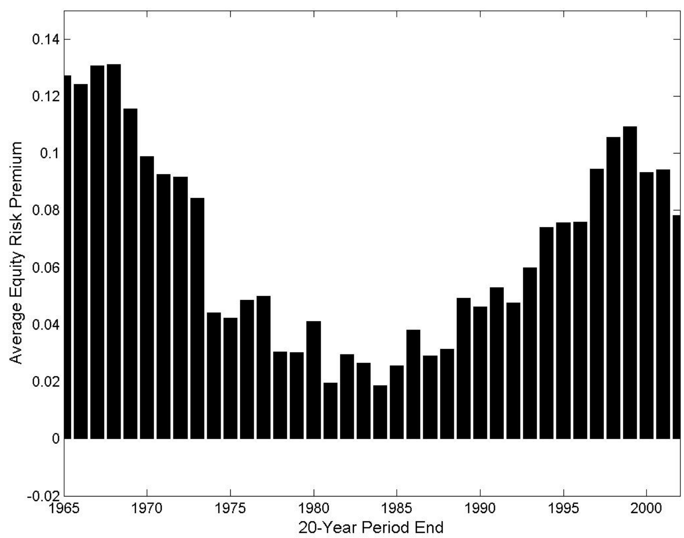


Figure 5: AVERAGE RISK PREMIUM OVER 20-YEAR PERIOD FROM 1965-2002

background motivation is. A more deeply discussion of the Mehra-Prescott Model and extensions will follow later in this paper.

The chosen periods and the necessary statistics are given in table 1. Firstly, it can be observed the equity premium still exists. Of course, it varies over the chosen periods but it cannot be dismissed. Another key result from this data is that a positive relationship between the risk premium and the covariance of consumption growth and stock returns cannot be observed. Furthermore, the risk premium increases continously, but the covariance of consumption growth and stock returns fluctuates. For example, in the period 1965-2002, there was a risk premium of 4.82%, between 1975 and 2002 the risk premium increased to 7.42%, but the covariance between consumption growth and stock returns, in the latter period, decrease. In the short-term period from 1985 to 2002, the risk premium climbed to 8.10%, the covariance increased with respect to the mid-term period, but is lower than in the long-term period. Furthermore, the extremely small standard deviation observed strengthens the claim of the volatility puzzle that consumption growth be very smooth.

The argumentation by Mehra and Prescott (1985) that dividends and stock return be perfectly correlated can also not resolved with these data sets. However, a look at the relationship between the risk premium an the covariance of consumption growth and dividend growth shows that would not beget the use of it better results. It should be possible to show that there is a positive relationship, but the covariance of consumption growth and dividend growth is decreasing while simutanously the risk premium is increasing.

A answer could be that different risk aversions exist in every period. That relative risk aversion alone cannot solve the puzzle is stated by many authors, for example Mehra and Prescott (1985, 1988) and Mehra (2003) themselves as well as in Cochrane (2001) and Campbell et al. (1997). The questions, known from the previous sections, as to whether other variables or market imperfections, like taxes, are involved or whether individuals have changed their fundamental preferences, come back. Campbell (2000, p. 1522) argues that individuals may be compensating lower returns with higher dividend yields. A look at the data shows that this cannot be confirmed for the observed period. A higher volatility of consumption cannot be the solution either. The only logical relationship is the expected negative correlation between the different kinds of leisure growth and stock returns, but this is smaller than assumed and, for the time span between 1975 and 2002, is unexpectedly positive.

A look at on correlations in table 1 reveals some interesting facts. First, the correlation of leisure growth of a citizen in the last period with dividend growth is starkly positive, but, in the mid- and long-run, it was starkly negative. Treasury bills are always positivly correlated with the leisure growth of a citizen, which goes hand in hand with a lower mean of treasury bill returns. The reason may be a fallen risk aversion or a higher stochastic discount factor. The correlation of total leisure with stock returns played, in the mid- and long-run, no important role.

The necessary stylized facts can be cocluded as. The average real return being high and volatile with a standard deviation of about sixteen percent. The riskless interest rate is very small, approximately two pecent, and involatile with a standard deviation of not more then 2.5%. The volatility

Table 1: U.S. ECONOMY SAMPLE STATISTICS - ANNUAL DATA FROM 1965 - 2002

Variable	Mean	Standard Deviation (%)	Correlation with Consumption Growth	Correlation with Dividend Growth	Correlation with Total Leisure Growth	Correlation with Leisure per Worker Growth
Period: 1965-2002						
Consumption Growth	1.0223	1.20	1.0000	0.3295	-0.7142	-0.3521
Total Leisure Growth	0.9997	0.19	-0.7142	-0.5322	1.0000	0.7295
Leisure per Worker Growth	1.0009	0.16	-0.3521	-0.4110	0.7295	1.0000
Dividend Growth	1.0054	4.74	0.3295	1.0000	-0.5322	-0.4110
Stock Return	1.0660	16.55	0.0538	0.1875	-0.0197	-0.0536
Treasury Bill Return	1.0178	2.06	-0.0463	-0.0115	0.1978	-0.0728
Risk Premium	0.0482	15.87	0.0621	0.1975	0.2721	-0.0464
Period: 1975-2002						
Consumption Growth	1.0203	1.08	1.0000	0.3139	-0.7462	-0.3722
Total Leisure Growth	0.9997	0.19	-0.7462	-0.4903	1.0000	0.7196
Leisure per Worker Growth	1.0007	0.14	-0.3722	-0.3255	0.7196	1.0000
Dividend Growth	1.0097	4.67	0.3139	1.0000	-0.4903	-0.3255
Stock Return	1.0955	15.61	0.0074	0.1688	0.0467	0.0627
Treasury Bill Return	1.0213	2.07	-0.1338	-0.1326	0.2572	-0.0534
Risk Premium	0.0742	15.31	0.0256	0.1901	0.0129	0.0712
Period: 1985-2002						
Consumption Growth	1.0200	0.94	1.0000	0.1671	-0.5618	-0.0942
Total Leisure Growth	0.9999	0.16	-0.5618	-0.3048	1.0000	0.7235
Leisure per Worker Growth	1.0005	0.11	-0.0942	-0.0942	0.7235	1.0000
Dividend Growth	1.0110	4.44	0.1671	1.0000	0.3048	-0.0942
Stock Return	1.1050	16.96	0.0294	0.4624	-0.1621	-0.0974
Treasury Bill Return	1.0240	1.28	0.1623	0.0946	0.1491	0.4659
Risk Premium	0.0810	16.24	0.0180	0.4752	-0.1809	-0.1384

Consumption growth is the change in real consumption per capita of nondurable goods and services. Leisure per worker growth is the change in leisure, where leisure is given by the difference of total hours per week and the average working hours per week of employees. Total leisure growth is the change in average leisure hours per week per capita. The stock return is real return of the S&P 500 index. Dividend growth is the change in real dividends, given by the dividend yield of the S&P 500 index. Treasury bill return is the real annual return of a three month treasury bill. All data are annual.

of dividend growth is unexpectedly smooth with a standard deviation of less five percent, as is the correlation of dividend growth to stock returns, which is, in the long-run and mid-run, small at 0.2, but increases dramatically to 0.4626 in the short-run. Lastly, the real consumption growth of non-durables and services is extremely smooth, with a maximum value of 1.2% in the long run, and, furthermore, decreases with the length of the observations. Next to the observed smoothness, if we compare these results with those of longer-horizon studies (e.g., Mehra and Prescott, 1985), it can be observed that the volatility of real consumption growth is dramatically smaller; for example the variance is a tenth of the value measure by Mehra and Prescott (1985).

Concluding this subsection, it should be kept in mind that it looks as if consumption growth alone can not solve the problem. The data supports the volatility puzzle, as well as the existence of the equity premium. The correlations and means of consumption growth and leisure growth have no indentical directions and the fluctuations are too random to be able to obtain anything conclusive. The discussion above has shown that the consumption/leisure decisions have changed and it looks, especially, it looks as if leisure had won importance.

3.2 De-Trending the Data

As mentioned previously, this thesis trys to examine the ability of a habit model to replicate main business cycle facts. For this it was necessary to de-trend the data series, because the study should be irrespective of growth. This "de-trending" procedure follows Hodrick and Prescott (1997), for this, the time series were logged. Because the observations are of a whole economy, the used series now are not per capita series as above. The series - leisure hours of an employee - has changed to leisure hours for all employees. Hours-worked is the known series of average working hours times the number employed. To get a better overview of how the model works, I extended the time series. Real output, private fixed investment, and employment were also observed. Productivity was measured as real output divided by total hours worked. The choice of series was influenced, e.g., by Hodrick and Prescott (1997), Kydland and Prescott (1982), and Hansen (1985). A difference to much of the recent research is the choice of real consumption; all consumption data in this paper is simply real consumption of nondurables and services.

Like in most of the literature, quarterly data was used, in this thesis from 1964:II to 2002:IV. From this choice it follwes that the parameters for the Hodrick-Prescott filter, were set to 155 for the length of observations and λ was set to 1600 - because of quarterly observations (Hodrick and Prescott, 1997, p.4). This thesis will not discuss the Hodrick-Prescott filter in detail, so interested readers should refer to the comprehensive literature (e.g., Kydland and Prescott, 1982, 1991; Hodrick and Prescott, 1997; Prescott, 1986; Hansen, 1985).

Table 2 shows the results of HP-filtering. Comparing these results with those of Hodrick and Prescott (1997), Kydland and Prescott (1982), and Hansen (1985)[6], shows that the results line up with each other well. The working-hours-influenced time series (like leisure, hours worked, and productivity) are an exception; as written above, it looks as if the use of different time series should

[6]That can only be an approximation, because the authors have looked at other periods, but it helps to evaluate the results.

Table 2: SAMPLE STANDARD DEVIATIONS AND CORRELATIONS WITH REAL OUTPUT AND REAL CONSUMPTION - U.S. ECONOMY 1964: II - 2002: IV

Variable	Standard Deviation (%)	Correlation with Real Output	Correlation with Real Consumption
Output	1.59	1.0000	0.8332
Consumption	0.80	0.8332	1.0000
Private Fixed Investment	5.12	0.9033	0.7986
Employment	0.99	0.8245	0.6665
Hours Worked	1.27	0.8885	0.7158
Leisure Hours	0.90	0.7492	0.6060
Dividends	2.93	0.4990	0.3834
Productivity	0.74	0.6217	0.5600
Risk-Free Rate $*$	0.99	-	-

Leisure hours are measured over all employees, productivity is given by the ratio of output with hours worked.
$*$ The Risk-free rate is real and annual, all other data is real and quarterly. The quarterly data series were HP-filtered, with a length of 155 and paramater $\lambda = 1600$, the riskless interest rate was HP-filtered by using a length of 38 and $\lambda = 6.25$.

have a strong influence on the results. This difference was dicussed by Kydland and Prescott (1991). This is a weak point to be sure, but Kydland and Prescott (1991) have shown that non-establishment data series do not work better. This discussion will be completed when the US data and the simulated data are be compared.

Table 3: SAMPLE CROSS-CORRELATIONS WITH REAL OUTPUT - U.S. ECONOMY 1964: II - 2002: IV

Variable	Time-Lag / Time-Lead							
	-1	0	1	2	3	4	5	6
Output	0.8675	1.0000	0.8675	0.6844	0.4621	0.2417	0.0214	-0.1553
Consumption	0.8502	0.8332	0.7096	0.5408	0.3441	0.1347	-0.0382	-0.1990
Private Fixed Investment	0.8355	0.9033	0.8206	0.6605	0.4514	0.2136	-0.0225	-0.2379
Employment	0.6610	0.8254	0.8757	0.8259	0.7074	0.5337	0.3428	0.1373
Hours Worked	0.7487	0.8885	0.8713	0.7444	0.5787	0.3674	0.1655	-0.0332
Leisure Hours	0.5752	0.7492	0.8426	0.8464	0.7638	0.6222	0.4466	0.2449
Dividends	0.4235	0.4990	0.5551	0.5628	0.5395	0.4342	0.3061	0.1748
Productivity	0.5783	0.6217	0.3649	0.1888	-0.0096	-0.1191	-0.2459	-0.2813

Leisure hours are measured over all employees, productivity is given by the ratio of output with hours worked. All datas are real, quarterly and HP-filtered, with a length of 155.

Next to that, the results show that total leisure of employees is positivly correlated with real output and real consumption, approximately, as strong as hours worked with real output and real consumption. The strong correlation of total hours worked, which can be observed, is driven by higher employment rate rather than by a higher average number of working hours per week (Prescott, 1986). Likewise, by McGratten and Rogerson (1998) it will be argued that aggregate-leisure is more substitutable than individual leisure, if we distinguish between citizens as aggregate group and employess as individuals, the observed results support this. Two other main problems of business cycle theory and, especially, of habit formation are the volatility of real consumption and the volatility of the risk-free rate. This will win importance when the data is compared, but should be mentioned here. As dicussed, habit formation further smooths the consumption path of the individual; standard habit models are unable to solve this anomaly (Lettau and Uhlig, 2000)[7]. Another problem is the volatility of the riskless interest rate, many models have problems matching this value. Both of these problems stand in the middle of the analyses, so they are especially worthy of note.

Finally, table (3) presents the cross-correlations between the observed data and output.

4 Asset Pricing Implications with Different Utility Functions

4.1 The Basic Asset Pricing Theory

At the beginning, it is necessary to figure out which decisions lead an investor to buy an asset. This analysis encompasses how much an investor wants to consume, to save, and/or, especially, what kind of assets she wants to hold. Most of the financial textbooks contain the following steps, for example Cochrane (2001) or Campbell et al. (1997). Next to that, Campbell (2000) gives a comprehensive review of the basic asset pricing implications.

To capture most investment situations, consider a typical decision of valuing a stream of uncertain cash-flows. The individual wants to know the price p_t of a investment, which generates a payoff, x_{t+1}, in period $t+1$. If the investor buys, e.g., a stock, the payoff is distinguishable between the price of the stock in the next period, p_{t+1}, and the dividends, d_{t+1}. The variable x_{t+1} is random, so the investor has only an expectation in time t of the outcome in $t+1$. It follows that the value of an investment depends on the utility function of the investor, for a start, this can be defined by:

$$U\left(c_t, c_{t+1}\right) = u\left(c_t\right) + \beta E_t\left[u\left(c_{t+1}\right)\right].$$

Where the period utility function is increasing to reflect a desire of more consumption and is concave to capture the declining marginal utility of a additional unit of consumption. The parameter β reflects the subjective discount factor - the impatience of the investor. For the maximization problem of the representive investor, assume that she chooses an amount, ζ, of an asset, which is freely tradeable, the following expression is binding,

$$\begin{aligned} \max_{\zeta} \quad & u\left(c_t\right) + \beta E_t\left[u\left(c_{t+1}\right)\right] \\ \textit{s.t.} \quad & c_t = e_t - p_t \zeta, \\ & c_{t+1} = e_{t+1} + x_{t+1}\zeta. \end{aligned} \tag{1}$$

[7] Of course, there exist habit model extensions that solve this problem, but it is a general problem of standard habit formation, as mentioned above, this is one of the main problems this thesis trys to cover.

The variable e denotes the income of the individual, which she can consume without any investment. The first order condition for this problem brings,

$$\frac{\partial U\left(c_t, c_{t+1}\right)}{\partial \zeta} = -p_t u'\left(c_t\right) + E_t\left[\beta u'\left(c_{t+1}\right) x_{t+1}\right] = 0.$$

The standard marginal condition for an optimal amount of an asset is the point, whereat the investor is indifferent between buying or selling the asset. That means that the expected marginal utility benefit of an asset is equal to its marginal utility cost. The marginal utility cost is the loss in utility for the investor if she buys an additional infinitesmally small unit of an asset. The expected marginal utility benefit is the discounted increase in utility the investor expects from the payoff of the investment, which she gets from having bought an amount of the asset in time t and selling the amount at time $t+1$. Rewriting the equation above brings us the central asset pricing formula,

$$1 = E_t\left[m_{t+1} R^i_{t+1}\right], \tag{2}$$

which is also known as the Lucas Asset Pricing Formula. The variable $m_{t+1} = \delta u'\left(c_{t+1}\right)/u'\left(c_t\right)$ is the stochastic discount factor, or pricing kernel. Furthermore R^i_{t+1} describes the gross return of the asset, with $R^i_{t+1} = x_{t+1}/p_t$.

Because the return on a risk-free asset $\left(R^f_t\right)$ is known ahead of time, equation (2) reduces to $1 = E_t\left[m_{t+1}\right] R^f_t$. So that these expression shows that the risk-free rate is related only to the pricing kernel,

$$R^f_t = \frac{1}{E_t\left[m_{t+1}\right]}. \tag{3}$$

Using covariance decomposition[8], equation (2) can be rewritten as,

$$1 = E_t\left[m_{t+1}\right] E_t\left[R^i_{t+1}\right] - \text{cov}\left(m_{t+1}, R^i_{t+1}\right),$$

so that for any asset, the following condition must be satisfied:

$$E_t\left[R^i_{t+1}\right] = \frac{1 - \text{cov}\left(m_{t+1}, R^i_{t+1}\right)}{E_t\left[m_{t+1}\right]}. \tag{4}$$

The intuition of this equation is that expected returns are proportional to the covariance of the princing kernel with the return, such that the covariance term is the risk adjustment of a asset. This will be more intuitive by assuming that there is an asset whose covariance is zero, often called the zero-beta asset, in which case the equation reduces to the known expression of the risk-free rate above. The implied risk premium of an asset over the risk free rate is nothing more then the difference between the returns,

$$E_t\left[R^i_{t+1}\right] - R^f_t = -R^f_t \cdot \text{cov}\left(m_{t+1}, R^i_{t+1}\right). \tag{5}$$

[8] The definition of the covariance implies that $\text{cov}\left(y, z\right) = E\left[yz\right] - E\left[y\right] E\left[z\right]$ holds.

The intuition behind this result is given by Campbell et al. (1997) as follows: an asset whose covariance with m_{t+1} is small, tends to have low returns when the investor's marginal utility of consumption is high. So, the asset is risky in that it fails to deliver wealth when wealth is most valuable to the investor. Therefore, she demands a larger risk premium to hold the asset.

To simplify the previous equation, the literature often follows Hansen and Singleton (1983, 1982) and assumes that the joint conditional distribution of asset returns and the stochastic discount factor is lognormal and homoskedastic (Cochrane, 2001; Campbell et al., 1997). The underlying assumptions are unrealistic, but like Campbell (2002) argued, they do make it easier to discuss the main forces that determine asset prices. If the logarithm of a variable z is IID lognormally distributed, with

$$\ln z \sim N\left(\mu_z, \sigma_z^2\right),$$

it follows that for the expectation of the variable z, the following condition[9] must hold:

$$E\left(z^a\right) = E\left[\exp\left(a \ln z\right)\right] = \exp\left(a\mu_z + \frac{1}{2}\left(a^2\sigma_z^2\right)\right).$$

Where the term $\sigma_z^2 = E_t\left[\left(\log z - E_t\left[\log z\right]\right)^2\right]$ can - with the addition of condional homoskedasticity - be rewritten as $\sigma_z^2 = \mathrm{var}_t\left[\log z - E_t\left[\log z\right]\right]$. As in Campbell et al. (1997), with joint conditional lognormality and homoskedasticity of asset returns and the stochastic discount factor, after taking logs of equation (2) we obtain:

$$0 = \mu_{R^i} + \mu_m + \frac{1}{2}\left(\sigma_{R^i}^2 + \sigma_m^2 + 2\sigma_{R^i m}\right), \tag{6}$$

where μ_R and μ_m are the means of continously compounted asset returns and the continously compounted pricing kernel, and σ_R and σ_m are the unconditional standard deviations of $\log R$ and $\log m$. Furthermore, σ_{Rm} denotes the covariance between $\log R$ and $\log m$ [10]. With the knowledge that the variance of the risk-free rate is zero, and thus the covariance as well, it can be obtained,

$$\hat{R}_t^f = \mu_R = -\mu_m - 1/2\sigma_m^2, \tag{7}$$

what is nothing more than the logarithmic version of equation (3). The hat denotation indicates that it is the logarithmic risk-free rate. For the log equity premium we obtain,

$$E_t\left[\hat{R}_{t+1}^i - \hat{R}_t^f\right] + \frac{1}{2} \cdot \sigma_{R^i}^2 = -\sigma_{R^i m}, \tag{8}$$

The variance term of the left-hand side is a Jensen's Inequality adjustment (see Campbell, 2002), which is what is left over when one takes the expectation of the logged returns instead of the logged expectation of the returns. Equation (8) can be rewritten to eliminate this term; by taking

[9] Appendix A.1 shows more properties of the lognormal distribution.

[10] Where the covariance can be evaluate alternatively by $\sigma_{Rm} = \rho\sigma_R\sigma_m$, where ρ is the correlation-coefficent between of the logarithms of asset returns and the pricing kernel.

logs out of the parentheses, the new risk premium is given by,

$$\log E_t \left(\frac{R_{t+1}^i}{R_t^f} \right) = -\sigma_{R^i m}. \tag{9}$$

Finally, another interesting characterization of a securities will be discussed. The ratio between the risk premium - mean excess return - and the standard deviation of an asset. This is called the Sharpe Ratio. The equation for the Sharpe Ratio can be expressed as follows:

$$\frac{E\left[R^i\right] - R^f}{\sigma_{R^i}}.$$

The intuition behind this ratio is more interesting than the mean return alone (Cochrane, 2001). To deriving this formula, remember the previous equation, cov $(y, z) = E[yz] - E[y]E[z]$, which can be rewritten as,

$$E\left[R^i\right] - R^f = -\rho_{R^i,m} \frac{\sigma_m}{E[m]} \sigma_{R^i}.$$

Because of the restriction on the correlation coefficent, that it cannot be greater than one in absolute value, the following condition holds:

$$\left| \frac{E\left[R^i\right] - R^f}{\sigma_{R^i}} \right| \leq \frac{\sigma_m}{E[m]} \sigma_{R^i}. \tag{10}$$

Equation (10) describes the intuition of the Sharpe Ratio, namely, that it is limited by the volatility of the discount factor (m). Like in Cochrane (2001), the interpretation behind this is that a more risky asset or a higher risk aversion is followed by a steeper maximal risk-return trade-off. Furthermore, a more gerneral intuition for asset pricing is that every asset, which is priced by the discount factor, must be bounded by this equation (Campbell et al., 1997). A comprehensive discussion on the usefulness of the Sharpe Ratio can be found, e.g., in Lettau and Uhlig (1997).

4.2 Asset Pricing with Power Utility

Considering the restrictions on the utility function (see previous subsection), the aim of this subsection will be to discuss one of the most frequently-used models in the asset pricing literature: the power utility function. This kind of utility function has a long tradition in the Asset Pricing literature, for example in the classic papers of Lucas (1978) and Mehra and Prescott (1985). Furthermore, it builds the background for many extensions - discussed in the previous section, 2 - that tackle the three known puzzles. This section will shortly present the properties of this kind of utilty function. Lastly, the asset pricing implications will be described, which deliver the main intuition behind the risk-free rate- and equity premium puzzles.

Let there be a representative agent who maximizes a time-separable utility function of the following form:

$$U(c_t) = \frac{c_t^{1-\gamma} - 1}{1-\gamma}, \tag{11}$$

where c_t denotes aggregate consumption. This utility function is a constant relative risk aversion utility function (CRRA), as relative risk aversion is equal to the parameter γ. This parameter often has different names in the literature; next to the power parameter, it is often called the concavity or curvature parameter. For the simple case of power utility, the power parameter is often named the relative risk aversion coefficient or the elasticity of intertemporal substitution parameter. The second name comes from another special property, namely that the reciprocal of γ, equals the elasticity of intertemporal substitution. It looks confusing, but it is necessary to discuss these different notations, because with more difficult power-utility-models, e.g., habit formation, these two properties no longer hold. To satisfy the requirement of the utility function that it reflect asset pricing implications- positive and concave - the parameter γ must be bigger than zero. For the case that γ approches one, equation (11) approches the logarithmic form $U\left(c_t\right) = \log\left(c_t\right)$. This specification is often used in the asset pricing literature to avoid nonseparabilities between different variables, which would force the Lucas Asset Pricing Formula to be dependent upon more than just consumption (e.g., by Boldrin et al., 1995, 1997, 2001).

This utility function has two more important properties, which are often used to justify its use. Campbell (2002) argues that the property of scale-invariance with constant return distribution satisfies the fact that in the last two centuries welfare and consumption increased without the risk-free rate and the risk premium having demonstrated any trends; as well as the related property that different investors with the same power utility function can be aggregated into a single representative investor.

Henceforth, the asset pricing implications of this utility function will be discussed, whereby the classic work of Mehra and Prescott (1985), which used this kind of utility function, will be referenced. After finding the marginal utility, $U'\left(c_t\right) = c_t^{-\gamma}$, it is possible to solve for the stochastic discount factor,

$$m_{t+1} = \beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}. \tag{12}$$

This condition changes the Euler equation, discussed above, to:

$$1 = E_t\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} \cdot R_{t+1}^i\right]. \tag{13}$$

By following the assumptions and steps mentioned above, this equation can be solved as was first done by Hansen and Singleton (1983). The equation (6) can be replaced by the formula:

$$0 = -\log\beta + \mu_{R^i} - \gamma\mu_c + \frac{1}{2}\left(\sigma_{R^i}^2 + \gamma^2\sigma_c^2 - 2\gamma\sigma_{R^i c}\right), \tag{14}$$

where μ_c denotes $E_t\left[\Delta\log c_t\right]$. Likewise as mentioned above, σ_c denotes the unconditional variance of log-consumption growth, and $\sigma_{R^i c}$ is the unconditional covariance bewteen the log-return of an asset (i) and log-consumption growth. Furthermore, the equation for the risk-free rate (7) can now be demonstrated to equal,

$$\log R_t^f = -\log \beta + \gamma \mu_c - \frac{1}{2}\gamma^2 \sigma_c^2. \tag{15}$$

It is easy to see that the risk-free rate is linearly dependent on expected log-consumption growth with slope γ. The literature, e.g., Campbell et al. (1997), interpretes the negative effect of the second term on the right hand side as precautionary savings. Hence, the solution for the risk premium can be expressed by the condition,

$$\log E_t\left[R_{t+1}^i\right] - \log R_{t+1}^f = \gamma \sigma_{R^i c}, \tag{16}$$

whereby this condition is used to obtain an expression for the risk premium absent of Jensen's Inequality adjustment; furthermore, it implicity defines, together with equation (15), the expected return of a risky asset i. Finally, to conclude the asset pricing implications of a standard power utility model, the Sharpe ratio (equation 10) can be approximately expressed by,

$$\left|\frac{E\left[R^i\right] - R^f}{\sigma_{R^i}}\right| \leq \frac{\sigma_m}{E\left[m\right]}\sigma_{R^i} \approx \gamma \rho_{R^i c}\sigma_c. \tag{17}$$

Before interpreting this expression, these solutions should be compared with those in the paper by Mehra and Prescott (1985).

The authors have assumed that the price of an asset is homogenous of degree one in its dividends, $p_t = w \cdot d_t$, which implies that the return of an asset is perfectly correlated with its dividends. This assumption made it possible to solve the equations (15-17) as dependent on expected dividend growth rather than on expected asset returns. Because the stylized facts found in section (3) cannot justify this assumption, this paper works with the conditions given in this section. However, because of the importance of the work by Mehra and Prescott (1985), it should only be mentioned here - the explicit methods of the authors can be found in the appendix of this thesis.

Lastly, the basic conditions found in this section will be discussed. Recall, as shown by section (3), that the volatility of consumption growth being extremely small leads to the known volatility puzzle mentioned by Campbell (1996). The intuition behind the equity premium puzzle can be easily underdstood by equation (16). To resolve the premium with the data from the previous section, would necessitate that investors be extremely risk averse. The necessary parameter settings will be discussed more deeply later in this paper, but, for more illustration, consider that, irrespective of the time span, only a γ of more than 200 can fit the data. Mehra and Prescott (1985) argued that a maximum value of ten is plausible, other authors (e.g., Hansen and Singleton, 1983; Prescott, 1986) even claimed that γ should not stray far from one. Without any discussion of the exact value of γ, it easy to see the absurdity of the discovered value, this is called the *equity premium puzzle*.

A look at equation (15) illustrates the next puzzle. As one can see, the unconditional mean of the risk-free rate depends on three components. A solution of the equity premium would imply a high γ, but this would be followed by a high risk-free rate, however the stylized facts showed a rate of approximately two percent. Moreover, such a high γ is followed by a discount factor, β, higher than one, this implication of negative time preference is called the *risk-free rate puzzle* by Weil (1989). Of course, the last term on the right hand side of equation (15) shows that a high γ would bring a high precautionary motive, which could reduce the risk-free rate and possibly fit the data without changing β. An comprehensive discussion is given by, e.g., Hansen and Jagannathan (1991), wherein the authors analyze the volatility of the stochastic discount factor. This example shows that the selection of the parameters to fit the data depends on the moments (see table 5) used in these equations; as the actual values to be used will be discussed later, this should be understood as a mere introduction that the puzzles be identified.

4.3 Habit Formation with Nonseparability Between Consumption and Leisure

After finding the asset pricing implications with a standard power utility model, this subsection concentrates on the implications given by habit formation. As previously discussed, there are many different possible ways to generate habit formation. The focus here is on the difference model, as given by, e.g., Constantinides (1990), wherewith, at the beginning, the habit was modeled unspecifically to derive a general solution for the Lucas Asset Pricing Formula. After this, the habit will be specified as an external and linear function of past consumption and leisure. The externality and the restriction of past decisions to the last period were used to reduce the complexity of the analysis, because the motivation, as mentioned above, was to show the influence of nonseparabilty between consumption and leisure, as well as to get an idea as to how well habit formation is able to fit financial data. As shown by Chen and Ludvigson (2003) and Cochrane (2001), there are differences in the ability of the several formations to fit the data, but the use of a difference model is the most necessary assumption. Nonlinear versus linear habit stock, as well as internal versus external habit formation are necessary assumptions, but every one has pros and cons, so for a introduction to the motivation, the specifications used should be good enough.

The following analysis is preformed with respect to the paper by Chen and Ludvigson (2003), wherein the authors comprehensively described most of the steps, as well as to those by Ferson and Constantinides (1991) and Otrok et al. (2001). As discussed, an extension with leisure, which is nonseparable with consumption, has not yet been analyzed in the literature, so the steps by the quoted authors are only similiar to the following, but can, with the following, be reproduced by setting the consumption share equal to one [11].

Consider a infinitly-lived representative investor, who maximizes her expected utility with respect to consumption and leisure,

$$E\left[\sum_{t=0}^{\infty}\beta^{t}U\left(c_{t},l_{t}\right)\right],$$

[11] A comprehensive analysis of this preference structure, as well as the analysis of habit formation without leisure, e.g., by Chen and Ludvigson (2003), is given in appendix A

where the utility function has the following form,

$$U\left(c_t, l_t\right) = \frac{\left(c_t^{\chi} \cdot l_t^{1-\chi} - x_t\right)^{1-\gamma} - 1}{1-\gamma}, \tag{18}$$

γ is the known power parameter, with the extensively discussed requirement that it be bigger than zero. The parameter χ determines the consumption share of the investor; as mentioned above, setting this equal to one reduces the preference stucture to a standard difference habit formation. Furthermore, the habit stock, x_t, is a unspecified function, which depends on past consumption and leisure,

$$x_t = f\left(c_{t-1}l_{t-1}, c_{t-2}l_{t-2}, \ldots, c_{t-L}l_{t-L}\right).$$

Taking the first derivative of equation (18), to find the marginal utility in consumption, brings us,

$$\begin{aligned} MU_t \quad = \quad & \left(c_t^{\chi} l_t^{1-\chi} - x_t\right)^{-\gamma} \chi \left(\frac{l_t}{c_t}\right)^{1-\chi} \cdot E_t \left[1 - \sum_{j=0}^{L} \beta^j \left(\frac{c_{t+j}^{\chi} l_{t+j}^{1-\chi} - x_{t+j}}{c_{t+1}^{\chi} l_{t+1}^{1-\chi} - x_{t+1}}\right)^{-\gamma}\right. \\ & \left. \cdot \chi \left(\frac{l_{t+j}}{c_{t+j}}\right)^{1-\chi} \frac{\vartheta x_{t+j}}{\vartheta c_{t+1}}\right]. \end{aligned} \tag{19}$$

Substituting this into the condition for the stochastic discount factor, which can then be expressed by:

$$\begin{aligned} m_{t+1} = \beta \frac{MU_{t+1}}{MU_t} \quad = \quad & \beta \left(\frac{c_{t+1}^{\chi} l_{t+1}^{1-\chi} - x_{t+1}}{c_t^{\chi} l_t^{1-\chi} - x_t}\right)^{-\gamma} \left(\frac{l_{t+1}}{l_t}\right)^{1-\chi} \left(\frac{c_{t+1}}{c_t}\right)^{\chi-1} \cdot \\ & \cdot \frac{E_{t+1}\left[1 - \sum_{j=0}^{L} \beta^j \left(\frac{c_{t+j+1}^{\chi} l_{t+j+1}^{1-\chi} - x_{t+j}}{c_{t+1}^{\chi} l_{t+1}^{1-\chi} - x_{t+1}}\right)^{-\gamma} \cdot \chi \left(\frac{l_{t+j+1}}{c_{t+j+1}}\right)^{1-\chi} \frac{\vartheta x_{t+j+1}}{\vartheta c_{t+1}}\right]}{E_t\left[1 - \sum_{j=0}^{L} \beta^j \left(\frac{c_{t+j}^{\chi} l_{t+j}^{1-\chi} - x_{t+j}}{c_{t+1}^{\chi} l_{t+1}^{1-\chi} - x_{t+1}}\right)^{-\gamma} \chi \left(\frac{l_{t+j}}{c_{t+j}}\right)^{1-\chi} \frac{\vartheta x_{t+j}}{\vartheta c_{t+1}}\right]}. \end{aligned} \tag{20}$$

After a little bit of algebra, with respect to that used used by Chen and Ludvigson (2003), the Lucas Asset Pricing Formula can be found to take the following specification:

$$1 = E_t \left[\beta \left(\frac{c_{t+1}^{\chi} l_{t+1}^{1-\chi} - x_{t+1}}{c_t^{\chi} l_t^{1-\chi} - x_t}\right)^{-\gamma} \left(\frac{l_{t+1}}{l_t}\right)^{1-\chi} \left(\frac{c_{t+1}}{c_t}\right)^{\chi-1} \cdot R_{t+1}^i \cdot \tilde{F}_{i,t+1}\right], \tag{21}$$

with

$$\begin{aligned} \tilde{F}_{i,t+1} \quad \equiv \quad & 1 - \sum_{j=0}^{L} \beta^j \left(\frac{c_{t+j+1}^{\chi} l_{t+j+1}^{1-\chi} - x_{t+j}}{c_{t+1}^{\chi} l_{t+1}^{1-\chi} - x_{t+1}}\right)^{-\gamma} \chi \left(\frac{l_{t+j+1}}{c_{t+j+1}}\right)^{1-\chi} \frac{\vartheta x_{t+j+1}}{\vartheta c_{t+1}} + \\ & + \sum_{j=0}^{L} \beta^{j-1} \left(\frac{c_{t+j}^{\chi} l_{t+j}^{1-\chi} - x_{t+j}}{c_t^{\chi} l_t^{1-\chi} - x_t}\right)^{-\gamma} \chi \left(\frac{l_{t+j}}{c_{t+j}}\right)^{1-\chi} \frac{\vartheta x_{t+j}}{\vartheta c_{t+1}} \cdot \frac{1}{R_{e,t+1}} \left(\frac{l_{t+1}}{l_t}\right)^{\chi-1} \left(\frac{c_{t+1}}{c_t}\right)^{1-\chi}. \end{aligned}$$

Consider, that up to this point, that equation (21) holds for any kind of difference habit formation

- for external and internal habits as well as for a linear or nonlinear habit stock.

In order to specify the model, it is necessary to decide between external and internal habit formation as well as to specify the habit stock. As mentioned previously, the aim of this thesis is to examine an external habit. Because of this externality, the function $\tilde{F}_{i,t+1}$ reduces to one, likewise done by Chen and Ludvigson (2003) and Campbell et al. (1997). The intuition behind this simplification can be found, e.g., in Cochrane (2001), and is summed up by the fact that the investor's habit is determined by everyone else's consumption and leisure, such that it is permissable to ignore terms when current consumption or leisure decisions affect expected future habits. The second specification to choose was the habit stock; to simplify the analysis it was assumed that it depends only on the last period, so that it can be rewritten as follows:

$$x_t = \theta \cdot c_{t-1}^{\chi} \cdot l_{t-1}^{1-\chi}, \tag{22}$$

where θ is the habit persistence or subsistence level, with the requirement that $0 \leq \theta < 1$. Both assumptions help to reduce the Lucas Asset Pricing Equation to the following form:

$$1 = E_t \left[\beta \left(\frac{c_{t+1}^{\chi} l_{t+1}^{1-\chi} - \theta c_t^{\chi} l_t^{1-\chi}}{c_t^{\chi} l_t^{1-\chi} - \theta c_{t-1}^{\chi} l_{t-1}^{1-\chi}} \right)^{-\gamma} \left(\frac{l_{t+1}}{l_t} \right)^{1-\chi} \left(\frac{c_{t+1}}{c_t} \right)^{\chi-1} \cdot R_{e,t+1} \right]. \tag{23}$$

After finding the asset pricing implications, and to make the results easier to compare with solutions in the literature, the previous equation was rewritten in terms of growth rates. The steps used are analogous to those used by Otrok et al. (2001). Finally, the stochastic discount factor can be written as follows:

$$m_{t+1} = \beta \cdot \left[\frac{\left(\frac{ct+1}{c_t} \right)^{\chi} \left(\frac{l_{t+1}}{l_t} \right)^{1-\chi} \cdot \left[1 - \theta \left(\frac{ct}{c_{t+1}} \right)^{\chi} \left(\frac{l_t}{l_{t+1}} \right)^{1-\chi} \right]}{\left[1 - \theta \left(\frac{ct-1}{c_t} \right)^{\chi} \left(\frac{l_{t-1}}{l_t} \right)^{1-\chi} \right]} \right]^{-\gamma} \cdot \left(\frac{c_{t+1}}{c_t} \right)^{\chi-1} \left(\frac{l_{t+1}}{l_t} \right)^{1-\chi}.$$

By substituting the growth rate of consumption and of leisure with g_c and g_l respectively, the pricing kernel can be represented by:

$$m_{t+1} = \beta g_{c,t+1}^{a} g_{l,t+1}^{b} \left(1 - \theta g_{c,t+1}^{-\chi} g_{l,t+1}^{\chi-1} \right)^{-\gamma} \left(1 - \theta g_{c,t}^{-\chi} g_{l,t}^{\chi-1} \right)^{\gamma}, \tag{24}$$

where a and b are simplifications used to allow a better overview:

$$a = \chi - \gamma\chi - 1 \qquad b = (1-\chi)(1-\gamma).$$

Because of the specific form of equation (24), log-linearizing the Lucas Asset Pricing Formula is not as easy here it previously was. The most commonly used way to linearize a function is by means of a Taylor expansion. The analyses have shown a first-order Taylor expansion to be sufficient, as the second moments did not fall out - which would be the requirement for a second- or higher-order Taylor expansion. As expansion points, the steady states of log-consumption- and log-leisure growth were chosen, whereby these could reproduced by the discrete means of consumption- and leisure growth, the following equation illustrates the algorithm:

$$\log f\left(g_{c,t}, g_{l,t}\right)\big|_{\bar{g}_c,\bar{g}_l} \approx f\left(\bar{g}_c, \bar{g}_l\right) + \frac{\partial f\left(\bar{g}_c, \bar{g}_l\right)}{\partial \bar{g}_c}\left(\log g_{c,t} - \bar{g}_c\right) + \frac{\partial f\left(\bar{g}_c, \bar{g}_l\right)}{\partial \bar{g}_l}\left(\log g_{l,t} - \bar{g}_l\right)$$

This method of log-linearization brings us the following form for the Lucas Asset Pricing Formula:

$$\begin{aligned} 0 &= \log\beta + E_t\left[\left(a - \frac{\gamma\chi\theta\frac{\bar{g}_l^{\chi-1}}{\bar{g}_c^{\chi}}}{1-\theta\frac{\bar{g}_l^{\chi-1}}{\bar{g}_c^{\chi}}}\right)\hat{g}_{c,t+1} + \left(b + \frac{\gamma\left(\chi-1\right)\theta\frac{\bar{g}_l^{\chi-1}}{\bar{g}_c^{\chi}}}{1-\theta\frac{\bar{g}_l^{\chi-1}}{\bar{g}_c^{\chi}}}\right)\hat{g}_{l,t+1}\right] + \\ & \left(\frac{\gamma\chi\theta\frac{\bar{g}_l^{\chi-1}}{\bar{g}_c^{\chi}}}{1-\theta\frac{\bar{g}_l^{\chi-1}}{\bar{g}_c^{\chi}}}\right)\hat{g}_{c,t} + \left(-\frac{\gamma\left(\chi-1\right)\theta\frac{\bar{g}_l^{\chi-1}}{\bar{g}_c^{\chi}}}{1-\theta\frac{\bar{g}_l^{\chi-1}}{\bar{g}_c^{\chi}}}\right)\hat{g}_{l,t} + E_t\left[\hat{R}_{t+1}^i\right]. \end{aligned} \tag{25}$$

The hat-denoted variables are log growth-rates, but the bar denoted variables are discrete growth-rates. Using simplifing expressions, like $\zeta = \gamma/\left(1-\theta\bar{g}_l^{\chi-1}\bar{g}_c^{-\chi}\right)$; the two following equations,

$$\psi_c = \zeta\chi - \chi + 1 \quad \text{and} \quad \psi_l = \left(1-\chi\right)\left(1-\zeta\right);$$

as well as the known assumptions of conditional lognormal ditribution and homoskedasticity, let us rewrite the equation as:

$$\begin{aligned} 0 &= \log\beta - \psi_c\mu_{c,t+1} + \psi_l\mu_{l,t+1} + \mu_{R^i,t+1} + \left(\rho\chi - \gamma\chi\right)\hat{g}_{c,t} + \left(\gamma\chi + \rho - \gamma - \rho\chi\right)\hat{g}_{l,t} + \\ & + \frac{1}{2}\left(\sigma_{R^i}^2 + \psi_c^2\sigma_c^2 + \psi_l^2\sigma_l^2 - 2\psi_c\psi_l\sigma_{cl} + 2\psi_c\sigma_{cr_e} - 2\psi_l\sigma_{lr_e}\right). \end{aligned} \tag{26}$$

After having obtained this equation, it is easy to solve for the necessary asset price implications. The solution is given in the next equation, wherein it was assumed that the known and expected log growth-rates are equivalent, which helps to reduce the equation above and solve for the risk-free rate as mentioned in the previous subsections:

$$\log R_{t+1}^f = -\log\beta - a\mu_c - b\mu_l - \frac{1}{2}\left(\psi_c^2\sigma_c^2 + \psi_l^2\sigma_l^2 - 2\psi_c\psi_l\sigma_{cl}\right). \tag{27}$$

The risk premium can also be easily found by using the known expression to neglect the Jensen's Inequality adjustment:

$$\log E_t\left[R_{t+1}^i\right] - \log R_{t+1}^f = \psi_c\sigma_{cr_e} - \psi_l\sigma_{lr_e}.$$

The first look at the risk premium shows that it now depends, not only on the unconditional covariance between consumption growth and asset returns, but also on the unconditional covariance between leisure growth and asset returns. Because the ψ_c is positive and ψ_l is negative - this is binding because of the requirements for θ, γ, and χ - it is easy to see that the nonseparability between consumption and leisure brings us, for a negative correlation between leisure and asset returns, a counterfactual effect for the equity premium, which supports the statements by, e.g., Lettau and Uhlig (2000) and Lettau (2003).

Before interpreting the solution more deeply, relative risk aversion should be examined. Because of the changed preference structure, the property of the standard power utility, that γ is equal to the relative risk aversion, does not hold anymore. In contrast to the previous model, utility now depends not only on consumption but also on leisure. That this property be satisfied, the relative risk aversion based on leisure should be solved for alongside consumption-based relative risk-aversion, the latter being the usual focus in the literature. The distinction between the two is made clear by means of the parameter χ; should χ decrease, relative risk aversion based on consumption will likewise decrease, while that based on leisure will increase.

The relative risk aversion based on a variable x is defined by the following condition:

$$rra_x = x \cdot \frac{\partial^2 U_x(x)}{\partial x^2} \cdot \frac{\partial x}{\partial U_x(x)}.$$

For the given utility of a investor, for the relative risk aversion based on consumption can be solved after finding the first and second order conditions with respect to consumption, which would bring the following relative risk aversion based on consumption:

$$rra_c = \frac{\gamma\chi}{1-\theta\frac{c_{-1}^{\chi}l_{-1}^{1-\chi}}{c^{\chi}l^{1-\chi}}} - \chi + 1 = \left[\frac{\gamma\chi}{1-\theta\bar{g}_c^{-\chi}\bar{g}_l^{\chi-1}}\right] - \chi + 1 = \psi_c. \tag{28}$$

A similiar way is followed by Mehra and Prescott (2003): the authors look at a difference habit model without leisure, in which, in order to express the rra_c in terms of growth rates, it is necessary to assume a fixed subsistence level. The solution shows that the rra_c is equal to the parameter ψ_c, which presents the evidence that the risk premium depends , as before, on the rra_c. On the other hand, this shows, that, for more complicated power utility models, the rigid connection between γ and rra_c is broken. Furthermore, this solution shows very vividly how habit formation works to generate a higher risk premium without changing the power parameter. If all other variables are constant, then the limit of rra_c for $\theta \rightarrow 1$ is equal to infinity.

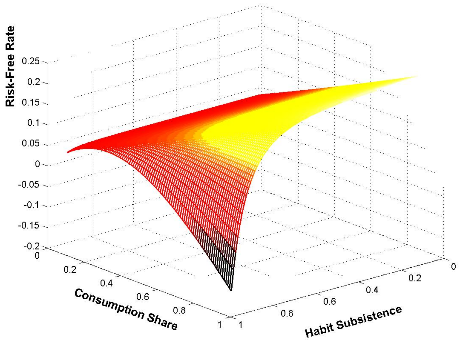
The previous equations shows that the chosen consumption share influences the rra_c, so that for $\chi < 1$, the relative risk aversion based on leisure can be solved for by similiar steps as above, which bring the condition:

$$rra_l = \left[\frac{\gamma\,(1-\chi)}{1-\theta\bar{g}_c^{-\chi}\cdot\bar{g}_l^{\chi-1}}\right] - \chi = 1 - \psi_l. \tag{29}$$

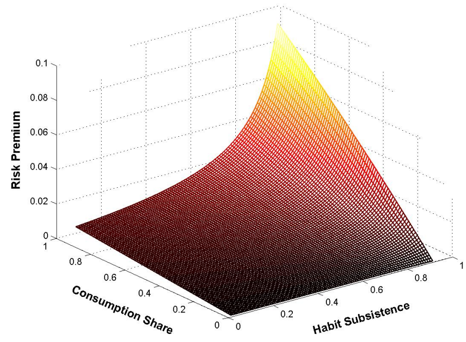
Because the Lucas Asset Pricing Formula depends only on the marginal utility of an additional unit of consumption, of course, the rra_l is not totaly equal to ψ_l. But as an approximation it should be mentioned that the risk premium will be driven by rra_c as well as by rra_l.

Figure 6 represents how a habit model works [12], more than that, it shows the “trick” this kind of preference uses, to generate a high risk premium by holding γ constant. The illustration 6(b) shows that every value of an equity premium can be generated by having the habit subsistence level

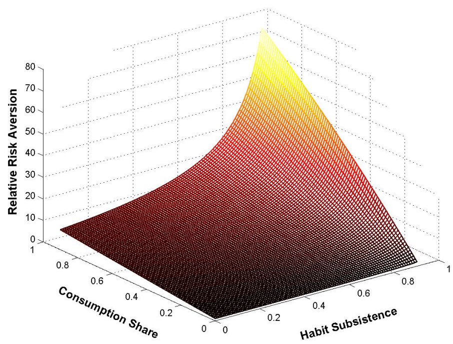
[12] Consider that this figure should be a demonstration, whereby the calculations uses data from table (5) in the time span from 1965 to 2002, the other parameters were choosen in respect to Mehra and Prescott (1985), but are interchangeable.



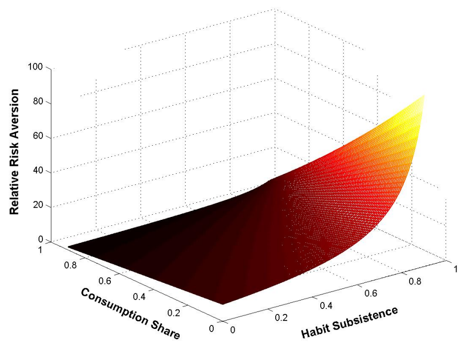
(a) Risk-Free Rate



(b) Risk Premium



(c) Consumption Based Risk Aversion



(d) Leisure Based Risk Aversion

Figure 6: DIFFERENT CONSEQUENCES BY COMBINING LEISURE AND CONSUMPTION IN A HABIT

The habit subsistence level is the parameter θ, have restricted between $0 \leq \theta < 0.9$; χ contains data betweem 0.01 and 0.91. This was done to have a better overview, but should be enough to see how a habit model works. Furthermore, the parameters $\lambda = 10$ and $\beta = 0.99$ are fixed.

approach one. The consequences on the relative risk aversions are observeable in the subfigures 6(c) and 6(d), which demonstrate that habit formation has the same weakness in producing high risk premiums as the standard model discussed previously discussed. Furthermore, subfigure 6(a) shows the ability to reduce the risk-free rate, if θ increases. More then that, this ability shows the limitation when setting the habit subsistence level parameter, because of the ability of parameter θ to dramatically increase the risk premium brings, on the other side, a dramatically decreased risk-free rate. The discussion will continue in the next subsection, so finally, this subsection will conclude by presenting the asset pricing implications for different utility functions (Table 4).

This overview contains four utility functions next to the two discussed in this section, two more to illustrate the consequences of nonseparabilty between consumption and leisure better will be presented. At first, the standard utility model was extended by nonseparable leisure, this utility function is often used in the business cycle literature (e.g., Kydland and Prescott, 1982), wherein it had great success in explaining main business cycle facts. The usage of this preference structure in the asset pricing literature is very limited, a discussion of the asset pricing implication of this function was investigated by Eichenbaum et al. (1988). Secondly, the third utility which will be presented in the table, is an external habit model, which can be generated out of the discussed habit formation if the consumption share is equal to one. So this table compares the consequences of nonseparability between consumption and leisure for two often used preferences in the asset pricing literature [13].

4.4 Presenting and Comparing the Results

Before starting the discussion, it should be recalled that the assumption of lognormal distribution implies that the data from table (1) needs to be modified. This was done, with the results being presented in table (5). The difference to the previous data table is that here the growth rates as well the different growth rates are obtained by continously compounding.

With these data sets, it is now possible, dependent on the adjustment of the parameters, to solve for the asset pricing implications with different utility functions, mentioned in table (4), with different time spans. Beginning with a standard power utility function, used especially by Mehra and Prescott (1985), it should be noted that the equity premium, as well the risk-free rate, can be exactly replicated. Because the equity permium depends, for a given covariance between consumption growth and asset returns, only on the power parameter, it can be obtained that a value of $\gamma = 220$ will solve the equity premium in the time span from 1965 till 2002. Furthermore, for the time span 1975-2002, the power parameter must be increased to 2273 and, lastly, for 1985-2002 it would be necessary that γ equal 1252. Of course, this show us that it is mathematically easy to resolve the equity risk premium. But what is the economic intuition behind this values?

Consider that, in this standard power utility function, the power parameter is equal to the relative risk aversion. It follows, firstly, that in every period, the investors must be extremely risk averse. Moreover, the investors' relative risk aversion must change extremely from period to period. A benchmark of relative risk averseness is given by Mehra and Prescott (1985), where the authors argued that a maximum value of ten would be acceptable [14]. This high risk averseness goes ahead, in this specific preference structure, with an extremely small elasticity of intertemporal substitution because this elasticity is the reciprocal of γ. A small elasticity of intertemporal substitution yields the intuiton that the investor is less willing to substitute consumption between the periods. In our case, the individual is not less willing: she is totally unwilling to substitute. It was mentioned that γ increases the precautionary motive of the individual. It is easy to see that this precautionary motive is extremely high for the given data set.

After finding the "right" parameter γ, it should be possible to solve for the riskless interest rate. The solution for the long-run is that a $\beta = 4.7$ would be necessary. A discount factor of greater than one implies that the individual has negative time preferences, which means that she

[13] The analysis for both models is presented in appendix A.

[14] This benchmark was used, as other authors postulate smaller values.

Table 4: OVERVIEW OF SOME ASSET PRICING IMPLICATIONS WITH DIFFERENT UTILITY MODELS

Variable	Standard Power Utility	Power Utility With Nonseparable Leisure	External Habit Formation	External Habit Formation With Nonseparable Leisure
Risk-Free Rate	$-\log\beta+\gamma\mu_c$ $-\frac{1}{2}\gamma^2\sigma_c^2$	$-\log\beta-a\mu_c-b\mu_l$ $-\frac{1}{2}\left(a^2\sigma_c^2+b^2\sigma_l^2+2ab\sigma_{cl}\right)$ with: $a=\chi-\gamma\chi-1$ $b=(1-\gamma)(1-\chi)$	$-\log\beta+\gamma\mu_c$ $-\frac{1}{2}\rho^2\sigma_c^2$ with: $\rho=\gamma/\left(1-\theta\bar{g}_c^{-1}\right)$	$-\log\beta-a\mu_c-b\mu_l$ - $\frac{1}{2}\left(\psi_c^2\sigma_c^2+\psi_l^2\sigma_l^2-2\psi_c\psi_l\sigma_{cl}\right)$ with: $a=\chi-\gamma\chi-1$ $b=(1-\gamma)(1-\chi)$ $\psi_c=\zeta\chi-\chi+1$ $\psi_l=(1-\chi)(1-\zeta)$ $\zeta=\gamma/\left(1-\theta\bar{g}_l^{\chi-1}\bar{g}_c^{-\chi}\right)$
Equity Risk Premium	$\gamma\sigma_{cr_e}$	$-a\sigma_{cr_e}-b\sigma_{lr_e}$ with: $a=\chi-\gamma\chi-1$ $b=(1-\gamma)(1-\chi)$	$\rho\sigma_{cr_e}$ with: $\rho=\gamma/\left(1-\theta\bar{g}_c^{-1}\right)$	$\psi_c\sigma_{cr_e}-\psi_l\sigma_{lr_e}$ with: $\psi_c=\zeta\chi-\chi+1$ $\psi_l=(1-\chi)(1-\zeta)$ $\zeta=\gamma/\left(1-\theta\bar{g}_l^{\chi-1}\bar{g}_c^{-\chi}\right)$
Consumption Based Relative Risk Aversion (rra_c)	γ	$\gamma\chi-\chi+1$	$\gamma/\left(1-\theta\bar{g}_c^{-1}\right)$	$\left[\frac{\gamma\chi}{1-\theta\bar{g}_c^{-\chi}\bar{g}_l^{\chi-1}}\right]-\chi+1$
Leisure Based Relative Risk Aversion (rra_l)	-	$\gamma+\chi-\gamma\chi$	-	$\left[\frac{\gamma(1-\chi)}{1-\theta\bar{g}_c^{-\chi}\cdot\bar{g}_l^{\chi-1}}\right]-\chi$

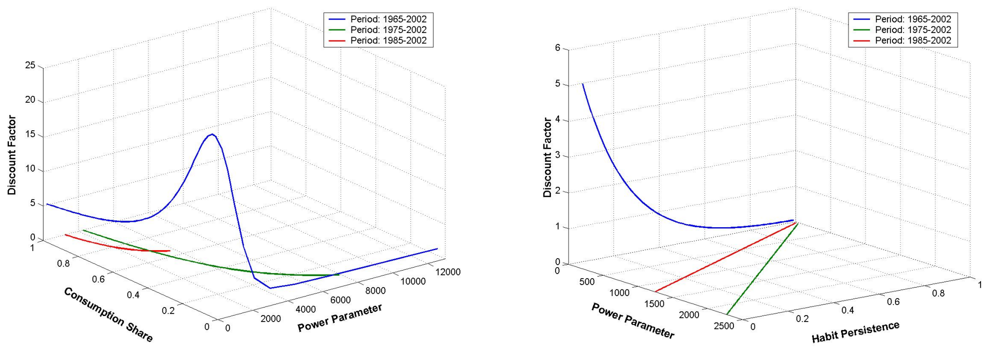
Table 5: NECESSARY LOGARITHM DATA- U.S. ECONOMY 1965 - 2002

Variable	Mean	Variance	Covariance with Consumption	Covariance with Leisure
Period: 1965-2002				
Consumption Growth	2.20	0.0135	0.0135	-0.0015
Total Leisure Growth	-0.03	0.0003	-0.0015	0.0003
Stock Return	5.13	2.6348	0.0164	-0.0015
Treasury Bill	1.74	0.0399		
Risk Premium	3.38	2.4113		
Sharpe Ratio	0.2085			
Period: 1975-2002				
Consumption Growth	2.00	0.0109	0.0109	-0.0015
Total Leisure Growth	-0.03	0.0004	-0.0015	0.0004
Stock Return	8.08	2.1447	0.0026	0.0005
Treasury Bill	2.09	0.0396		
Risk Premium	5.99	2.0592		
Sharpe Ratio	0.4091			
Period: 1985-2002				
Consumption Growth	1.98	0.0081	0.0081	-0.0008
Total Leisure Growth	-0.01	0.0002	-0.0008	0.0002
Stock Return	8.79	2.4788	0.0051	-0.0047
Treasury Bill	2.36	0.0185		
Risk Premium	6.43	2.2729		
Sharpe Ratio	0.4081			

For data sources, see table 1; the difference to the previous data is that the data is continous now. All of the data is real and in percentages.

has no desire to borrow from the future. The solution for the long-run period, let us conclude, is that the individual is extremely risk averse, is unwilling to substitute intertemporally, and has no desire to borrow from the future. This solution alone illustrates every prominent puzzle of the asset pricing literature, but a look at the other periods shows more drastic problems. Because of the dramatically increased relative risk aversion just mentioned, in otrder to solve for the riskless interest rate it would be necessary that the individual have an infinitely high desire to borrow from the future. This follows from the fact that the precautionary motive was dramatically increased. These solutions make no sense: an individual who is totally unwilling to substitute intertemporally would seem to not have a high desire to borrow.

After this, the asset pricing implications with more complicated preferences should now be discussed. At first, sub-figure (7(a)) shows that for a preference, like the one used by Eichenbaum et al. (1988), where consumption is nonseparable for leisure, there exists multiple points that can resolve the equity premium as well as the riskless interest rate. As mentiond previously,



(a) Standard Power Utility with Nonseparability Between Consumption and Leisure

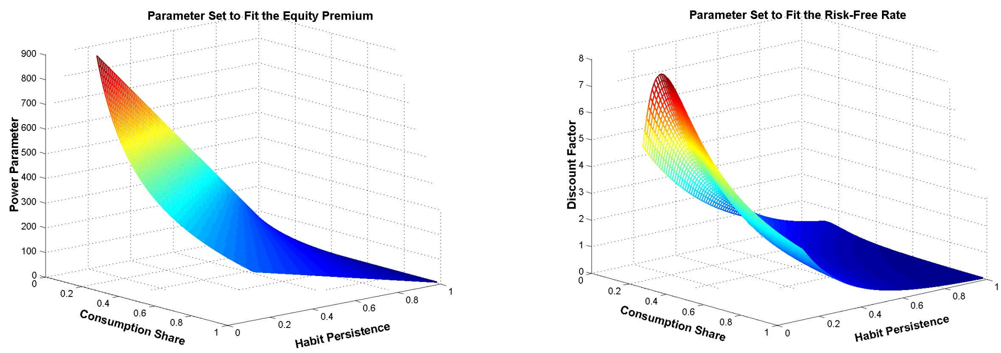
(b) External, Difference Habit Formation

Figure 7: POSSIBLE PARAMETER COMBINATIONS FOR DIFFERENT PREFERENCES TO RESOLVE FINANCAL MARKET FACTS

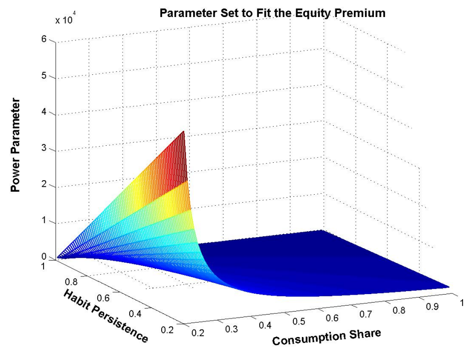
nonseparable leisure has a counterfactual influence on the risk premium, so that, for this, we can expect higher values for γ if the consumption share χ decreases. This will be shown by sub-figure (7(a)), where higher values for γ as well as for the discount factor were obtained. Recall the discussion above: it is easy to see that such a preference structure does not bring better performance. More than that, in the period between 1975 and 2002, it was not possible to find positive parameter settings for $\chi \leq 0.5$. In, the absence of a discussion of economic intuition, the sub-figure illustrates that it seems that, the analysis only provide more or less logical solutions for the period starting in 1965.

Sub-figure 7(b) shows the possible parameter settings with standard external habit formation. It can be seen that the values necessary for the parameters γ and β can be reduced by increasing the habit persistence. This "trick" of habit formation was previously mentioned, because it is possible to match the data without without having to neglect a parameter. A solution with respect to holding the, e.g., discount factor constant, would reduce the problem of negative time preferences mentioned above. But it is to be kept in mind that this advantage is only possible by increasing the influence of other disadvantages. As shown similiarly in figure 6, the relative risk aversion increases if the habit persistence also increases. Of course, the mentioned algorithm for solving for the discount rate brings plausible obeservations only for the time span from 1965 till 2002, because, for the other time spans, the observed volatility of consumption is definitely too small.

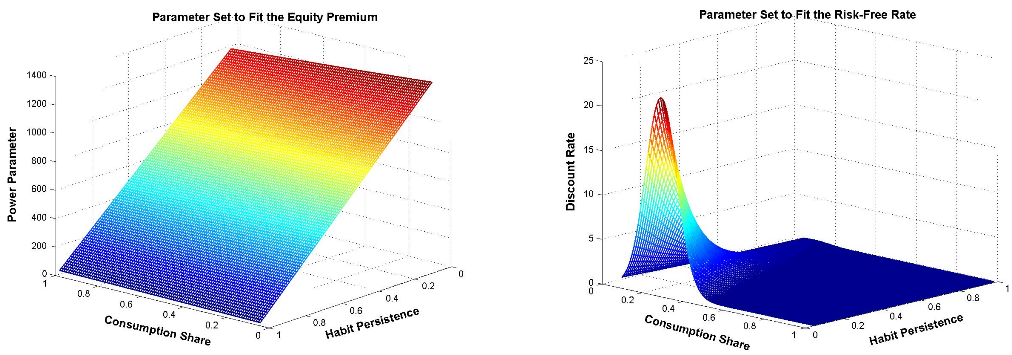
Lastly, will be discussed the ability of external habit formation with nonseparabilty between consumption and leisure to fit the data will be discussed. Figure 8 gives an illustration for every examined time span. Because in this model is possible to change four different parameters, the left figure of every time span shows the necessary settings to resolve the equity premium, whereas the right figure shows the implicit discount factor these setting would require. As mentioned above, it was not possible to resolve the riskless interest rate in the time span starting from 1975 be-



(a) Period: 1965 -2002



(b) Period: 1975-2002



(c) Period: 1985-2002

Figure 8: POSSIBLE PARAMETER COMBINATIONS FOR DIFFERENT PREFERENCES TO RESOLVE FINANCAL MARKET FACTS

The limitation of $0.13 \leq \chi \leq 1$ was chosen to obtain better results. The left figure of every time span replicates the necessary parameters to solve the equity premium, with which it was possible to solve for the required discount rate necessary to reslove the riskless interest rate.

cause of the extremely small volatility of consumption, which would require a infinitely small β. The other results do not look better, but, like above they are able to show the general procedure. The procedure will be described with respect to sub-figure 8(a). Like in the nonseparable model without habit formation, it can be seen that a smaller consumption share requires a higher power parameter - the habit formation makes it possible to counteract this effect by increasing the habit persistence. This counterfactuality is helpful for resolving the riskless interest rate as well. The advantage of this preference structure is that another parameter exists, to assist in resolving the puzzles. Advantagously it can be seen that a consumption share smaller than one reduces the relative risk averseness based on consumption. The problem of speaking about advantages with respect to habit models is, as mentioned above, that every success is based on a "trick", which increases the problems with the other parameters. The following analogy may help in understanding this: a bucket of water with five holes cannot be fixed by plugging just two of the holes.

Up to this point, the discussion has not compared its solution with those in the literature, but as quoted in section 2, Cochrane (2001) argued similiarly that no model can solve every problem following from the use of consumption-based asset pricing. For example Campbell and Cochrane (1999), closed two other "holes of the bucket" by using a fixed riskless interest rate and random-walk consumption. Finally, it is to be concluded that habit formation shows how preferences can be modified to match some facts of financial markets, but one should always keep an eye open for the fact that these modifications are usually followed by another new and/or increasingly present problem. As shown, is it possible to resolve the model with financial data if some parameters or characteristics are fixed. The fixing of characteristics, the production of new puzzles, and the intensification of the known problems unsatisfying. Futhermore, the analyzes of the last thirty-eight years have shown that consumption growth is extremely non-volatile: this smoothness is increased with shorter observations, this seems to be a key indicator of the poor performance of consumption-based asset pricing. The completed extension with nonseparable leisure demonstrates no great success for asset pricing implications, the contrary effect of leisure and asset returns is smaller than expected but exists nevertheless. The only justification for the use of such a nonseparable habit utility function, is the fact that one more variable helps in plugging the "bucket".

5 An RBC Model with Habit Formation

In this section, the interest is now in how well a model with habit formation generates business cycles. The endogenous solution is based on a simple RBC model. In the first part of this section, the economy that was built, the market decentralization used throughout this thesis, and some specifications will be described. Later I present some necessary steps to solve the planner's welfare maximization problem with the method used by Uhlig (1999). Furthermore, in the third part, the calibration used to implement the model will be discussed. Lastly, the solutions will be presented and compared by extensions - made by myself - with given solutions of habit theory.

5.1 The Model

HOUSEHOLDS

The households of the economy are infinitely-lived and identical. The representative agent maximizes expected discounted utility,

$$max \quad E_t \sum_{t=0}^{\infty} \beta^t U(c_t, n_t), \tag{30}$$

where E_t is the expectation operator, conditional on information available at time t. The parameter β represent the household's discount factor, with $0 < \beta < 1$. The agents consumes, every period t, the consumption good c_t and places an amount of labor n_t at the economy's disposal. The utility function has the following form,

$$U(c_t, n_t) = \frac{\left(c_t^{\chi}(1-n_t)^{1-\chi} - X_t\right)^{1-\gamma} - 1}{1-\gamma}, \tag{31}$$

where $1 - n_t$ is nothing more than leisure, so that the agent choose that level of labor, which generates her the highest utility in leisure with respect to possible consumption. X_t is the habit stock, given by the next equation:

$$X_t = \theta \cdot \sum_{s=1}^{\infty} \left(c_{t-s}^{\chi}(1-n_{t-s})^{1-\chi}\right). \tag{32}$$

The power parameter or concavity parameter is given by $\gamma > 0$. As discussed above, for a simpler version with time-separability over consumption ($\theta = 0$ and $\chi = 1$), the parameter γ represents the relative risk aversion. As shown in the previous section, this is not the case in this model, however, γ is still part of the relative risk aversion but differs substantially from the inverse of the elasticity of consumption. The parameter θ, with $0 \leq \theta < 1$, represents the fraction of the aggregated sum of lagged consumption and leisure; it generates a habit subsistence level of today's consumption and leisure. The parameter χ denotes the level the agent wants to consume.

When the agent maximizes her expected lifetime utility, the following budget constraint needs to be satisfied every period:

$$w_t n_t + d_t k_{t-1} + = I_t + c_t. \tag{33}$$

The parameter w_t is the wage the agent recieves per unit of labor (n_t), and d_t denotes the dividends per unit of invested capital in the previous period. The left hand side of equation (33) can be called the income per period of the individual, resulting from the chosen amount of labor and capital investment. The right hand side are the expenditures of the agent, where she can decide between consumption, c_t, and investment, I_t. The amount of money the agent invests depends on the capital adjustment costs, ϕ_t, in this period; because a unit of capital, which brings dividends in the next period, has different prices like a stock, that depend on the expected returns from this unit of capital, which will be discussed further down the road in this thesis.

FIRM

In this economy, there is a representative, infinitely-lived firm, which has access to the following, Cobb-Douglas production function:

$$y_t = \bar{\kappa} e^{z_t} k_{t-1}^{\alpha} n_t^{1-\alpha}. \tag{34}$$

Production in time t depends on the demand for labor in period t and the existing capital stock at the end of the previous period. The parameter α, $0 < \alpha \leq 1$, indicates the capital share used to produce a unit of output. The term $\bar{\kappa}\exp(z_t)$ is total factor productivity, which can be observed by the firm before making decisions about capital and labor demand in time t. It is assumed that the technology shock follows a stochastic process: in this AR(1) process, z_t follows the law of motion given by,

$$z_{t+1} = \rho z_t + \varepsilon_{t+1}^{z}, \tag{35}$$

whith $\varepsilon \sim i.i.dN(0, \sigma^2)$ for all $t > 0$ and $0 < \rho < 1$. TFP is implied to reflect the fact, that not every change in post-war U.S. GDP can be accounted for by changes in labor and capital (Hansen, 1985). This is known as the Solow residual, which will be interpreted as in the literature, e.g. by Kydland and Prescott (1982), as a technological shock.

Given the production function, the firm has to maximize its profits:

$$max \quad \pi_t = y_t - w_t n_t - d_t k_{t-1}, \tag{36}$$

with respect to n_t and k_{t-1}, which are the demanded input factors by the firm. The wages w_t and the dividends d_t are given by the market. The firm's capital stock follows the intertemporal accumulation equation with adjustment costs,

$$k_t = (1-\delta)\, k_{t-1} + \phi_t k_t, \tag{37}$$

where δ represents the depreciation rate of the capital stock. The function ϕ_t is the previously known capital-adjustment-cost function, which is similiar to those of Boldrin et al. (2001, 1997, 1995) and Jermann (1998). This is a positive and concave function of the following form,

$$\phi_t = \frac{a_1}{1-\frac{1}{\zeta}} \left(\frac{I_t}{k_t}\right)^{1-\frac{1}{\zeta}} + a_2, \tag{38}$$

The parameter ζ is the elasticity of investment I_t with respect to Tobin's q, which for $\zeta = \infty$, evaporates and function reduces such that the standard capital accumulation formula is given as,

$$k_t = (1-\delta)\, k_{t-1} + I_t.$$

This interpretation is similiar to the discussion in Jermann (1998), that such a function captures the idea that a fast change in capital stock is costlier than changing the stock slowly. The result the author gave is that this specification allows the shadow price of the installed capital to deviate from the price of a new unit of capital. For the definition of the variables a_1 and a_2, this thesis follows Jermann (1998), whereby the variables are chosen to yield the same steady state as without adjustment costs.

MARKET CLEARING CONDITION

Finally, the equilibrium must be described. To satisfy this simple model, the agent has to consume or invest everything the economy produces. This fact is captured by the following condition,

$$c_t + I_t = y_t. \tag{39}$$

Next to that, the labor market has to clear - this means that the demand for labor by the firm must be satisfied by the agent's supply. Second, the financial market must be in equilibrium, this requires that the agent buys every outstanding unit of capital until the demand for capital by the firm is zero. This argument will be supported by the adjustment cost function, because the price of a capital share depends on the demand for it. Furthermore, if perfect markets are required, perfect competition is required too. The consequence is that the firm's profit in the equilibrium has to be zero.

5.2 Solving The Nonlinear Stochastic Growth Model

This subsection will be provide an overview of the necessary steps in analyzing the described nonlinear stochastic growth model. The literature has studied different solution methods for solving such models - a comparison is given by Taylor and Uhlig (1990). This thesis forgoes a discussion of the different methods and leans on the procedure provided by Uhlig (1999).

The advantage of the chosen method is, next to the prescribed "cookbook recipe" to make the solving of nonlinear models easy (Uhlig, 1999), the provided toolkit. This computional framework made it possible to analyze - relativly easily - the solution via impulse responses and compare second-order properties of the model with the observed moments of HP-filtered U.S. data (see section 3.2). The necessary input algorithm is provided in appendix B.

The first step of the general procedure was to find the necessary equations which describe the equilibrium. Part of the equilibrium are the shock process (35) and the described constraints as before, so consider the production function (34), the capital accumulation process (37), the adjustment cost equation (38), and the market clearing condition given by equation (39). This equation will be extended by the necessary first order conditions of the representive investor and the firm. As known, the firm maximizes its profit by choosing an amount of capital and labor; this brings the following binding conditions for dividends,

$$d_t = \frac{\alpha \cdot y_t}{k_{t-1}} \tag{40}$$

and wages,

$$w_t = (1-\alpha)\frac{y_t}{n_t}. \tag{41}$$

The other equations needed to complete the equilibrium can be derived from the first order conditions of the representative investor. The derivatives with respect to consumption and leisure are given by:

$$\begin{aligned} c_t \quad : \quad & \beta^t \left[\left(c_t^{\chi} l_t^{1-\chi} - \theta c_{t-1}^{\chi} l_{t-1}^{1-\chi} \right)^{-\gamma} \chi \left(\frac{l_t}{c_t} \right)^{1-\chi} - \lambda_t \right] \\ & + \beta^{t+1} \left[- \left(c_{t+1}^{\chi} l_{t+1}^{1-\chi} - \theta c_t^{\chi} l_t^{1-\chi} \right)^{-\gamma} \chi\theta \left(\frac{l_t}{c_t} \right)^{1-\chi} \right] = 0 \\ l_t \quad : \quad & \beta^t \left[\left(c_t^{\chi} l_t^{1-\chi} - \theta c_{t-1}^{\chi} l_{t-1}^{1-\chi} \right)^{-\gamma} (1-\chi) \left(\frac{l_t}{c_t} \right)^{-\chi} - w_t\lambda_t \right] \\ & + \beta^{t+1} \left[\left(c_{t+1}^{\chi} l_{t+1}^{1-\chi} - \theta c_t^{\chi} l_t^{1-\chi} \right)^{-\gamma} \theta (1-\chi) \left(\frac{l_t}{c_t} \right)^{-\chi} \right] = 0. \end{aligned}$$

Consider the discussion of external habit formation in the previous section; it is possible to simplify the necessary first order condition by neglecting every term with expected consumption and leisure decisions. With this, the equations can be rewritten as follows,

$$c_t \quad : \quad S_t^{-\gamma} \chi \left(\frac{l_t}{c_t} \right)^{1-\chi} - \lambda_t = 0 \tag{42}$$

$$l_t \quad : \quad S_t^{-\gamma} (1-\chi) \left(\frac{l_t}{c_t} \right)^{-\chi} - w_t\lambda_t = 0, \tag{43}$$

where S_t simplifies the equations and measures the surplus between the decided consumption and leisure amounts in time t and the habit stock in t, which can expressed by the formula:

$$S_t = c_t^{\chi} l_t^{1-\chi} - \theta c_{t-1}^{\chi} l_{t-1}^{1-\chi}. \tag{44}$$

Finally, the derivative with respect to capital brings us the Euler equation,

$$1 = E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} R_{t+1}^f \right], \tag{45}$$

with

$$R_t^f = d_t \left[\frac{\left(a_1 / \left(1 - \frac{1}{\zeta} \right) \right)}{\phi_{t-1} - a_2} \right]^{\zeta/(\zeta-1)}. \tag{46}$$

After finding the equlibrium, the next step was to solve for the steady state. This can be done by using the parameter settings from table (6), as well as the assumption that $\bar{n} = 1/3$, that value is cited from in the literature, and by finding values for a_1 and a_2, which satisfy the necessary assumption dicussed above.

Lastly, before implementing the algorithm in the toolkit, it is necessary to linearize the equations which determine the equlibrium. The procedure of log-linearization is concerned with the principle of a Taylor expansion around the steady-state, the problems with which, as well as an introduction to using this approximation can be found in Uhlig (1999). So, that this paper forgoes an introduction to these methods as well as the log-linearized equations. This was done, because of the simplicity of the chosen model, whhich makes it possible to find most of the equations, as well as the log-linearized solutions, in the literature; for example, Uhlig (1999) provided this similiarities.

Furthermore, the distinction between different kinds of variables must be mentioned. Because past capital, consumption, leisure, hours worked, and the adjusment costs are known in time t, they have to be considered endogenous state variables, this was an unavoidable consequence of the habit formation process, and is the only important divergence from the standard equilibrium, comprehensivly discussed by Uhlig (1999).

5.3 Parameterizing the Model

For the calibration of the model, the literature was, in most instances, followed. Some adjustments were necessary to reflect the results from the previous section and to examine wether these are reasonable. The chosen parameter settings are documented in table (6).

Table 6: BENCHMARK PARAMETER VALUES

Preferences	Discount factor, $\beta = 0.99$
	Concavity parameter, $\gamma = 2.0$
	Consumption share, $\chi = 1/3$
	Habit subsistence level, $\theta = 0.6$
Technology	Capital share, $\alpha = 0.36$
	Depreciation rate, $\delta = 0.025$
	Elasticity of investment, $\zeta = 0.23$
Shock Process	Shock persistence, $\rho = 0.95$
	Shock variance, $\sigma_z^2 = 0.0074^2$

The relative size of consumption and leisure to each other in the utility function will be set, as in most of the literature, extremely low. Kydland and Prescott (1982) chose a consumption share of one-third; they argued that the household's allocation of time to nonmarket activities is twice as large as the allocation to market activities. A even lower value for the consumption share is given by Eichenbaum et al. (1988), where the authors argued for a value near 0.16. For the benchmark model, the argumentation by Kydland and Prescott (1982) was followed, and a value of one-third was used. The exogenous solution has shown that a maximization of the risk premium by equivalently minimizing the risk-free rate and the relative risk aversion goes hand in hand with a higher consumption share. However, this a benchmark value and will be more deeply discussed with experiments later in this thesis.

For the habit subsistence level, the results from the previous section deliver a high θ to reflect the equity premium; the setting is similiar to those in the literature. For example, Constantinides (1990) use a $\theta = 0.8$ and Boldrin et al. (2001, 1997, 1995) used subsistence levels of 0.6 and 0.9. The benchmark model will start with a $\theta = 0.6$, this value, as well as the consumption share, are the crux of the following experiments. For the discount rate (β), the standard value of 0.99 for quarterly data was used, as by Christiano and Eichenbaum (1992). Prescott (1986) discused the concavity parameter γ, and found out that γ is near 1. Taking the limit of $\gamma \to 1$ would change the power-utility function to a logarithm function. So the chosen parameter should be unequal to but not far from 1; a the value of 2 is used.

For the depreciation rate, a quarterly value $\delta = 0.025$ was selected, and the capital share parameter in the Cobb-Douglas production function was set equal to $\alpha = 0.36$; these values are used in most of the RBC literature [15]. This model did not assume a quarterly growth trend ($\bar{\kappa} \exp(\bar{z}) = 1$), which is in contrast to Jermann (1998) and Boldrin et al. (2001), who did and assumed growth trends of 0.5% or 0.4% respectively. Consequently, the equation for the adjustment cost is simplified [16]. This simplification satisfies the condition for a_1 and a_2 as given by Jermann (1998), that the same steady state be yielded. For choice of the elasticity of investment, I followed the authors and set the parameter $\zeta = 0.23$.

For the calibration of the Solow residual, the main literaturen was followed, a discusion is given by, e.g., Prescott (1986). As persistence level $\rho = 0.95$ was chosen. To find the standard deviation of technology, I used the productivity data (Table 2), in which the standard deviation was 0.0074. This estimate might over- or understate the variance of a technology shock, but it is within of the range used in most of the literature.

5.4 Comparing the Results

This subsection will try to present and discuss those business cycle facts that a habit with nonseparabilty between consumption and leisure is able to replicate. To assess the results, the analysis compares the results with those of the known, utility functions often used in this thesis. Recall that, for different preferences, it was necessary to modify some of the details of the economy, previously described.

To analyze the effect of these different utility functions on macroeconomic facts, the same positive one percent shock in technology will be introduced. The simulated moments will be presented in table (7), whereby it is to be observed that the analysis is distinguishable in two main ways. Firstly, the habit formation models were simulated with and without capital adjustemnt costs and, secondly, different consumption shares were used to get some sort of idea as to how

[15] For more details see, e.g., Kydland and Prescott (1982), Prescott (1986), Christiano and Eichenbaum (1992), and Hansen (1985)

[16] In the paper of Boldrin et al. (2001), the parameters a_1 and a_2 depend on the exponent of the mean growth rate - but here, it follows from using a zero growth rate are simplified such that the fit the models's, and, as such, differ from those used by Jermann (1998) and Boldrin et al. (2001).

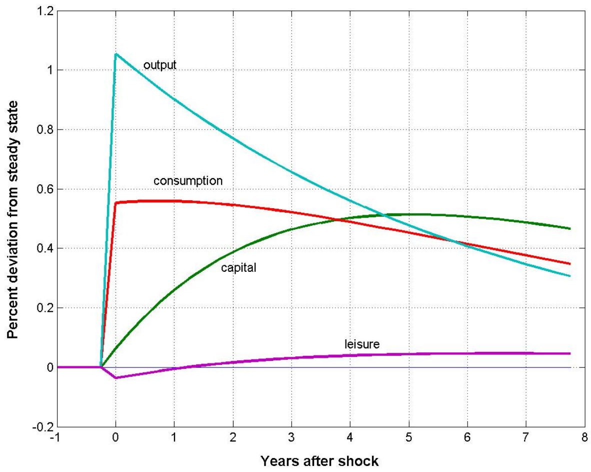


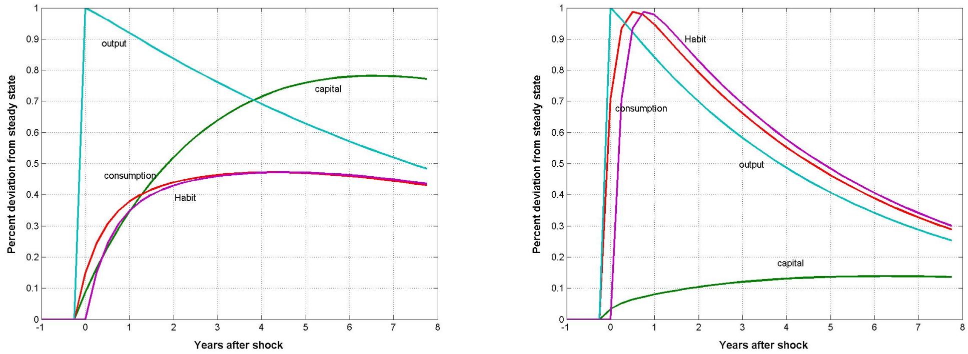
Figure 9: IMPULSE RESPONSES TO A ONE PERCENT TECHNOLOGY SHOCK, WITH STANDARD PREFERENCES

Consider that these figures represent the impulse responses to a one percent technology shock, by using the benchmark parameter values. The used preference contains nonseparability between consumption and leisure, but without habit formation and adjustment costs, so it is comparable to the preference used by Eichenbaum et al. (1988).

strong this consumption share affects the results. Furthermore, the table presented solutions for different parameter values of the consumption share; these were chosen with respect to the recent literature. A $\chi = 1/3$ with respect to Kydland and Prescott (1982) as well as $\chi = .16$ with respect the results of Eichenbaum et al. (1988) will be used.

Figure 9 shows the impulse responses to a one percent shock in technology for a economy without habit formation as well as without adjustment costs. Table (7) refer to the moments for this economy in the fourth coloum with the heading; standard power utility with nonseparable leisure. It can be seen in figure 9 that the individual reacts in the moment of the shock by supplying more labor to participate in the technological gain from the shock; in the following periods, the higher payoff from her increased investment makes it possible for her to reduce her labor supply. Consumption is very persistent, but the Hodrick-Prescott data shows that consumption is still too smooth, unaffected by different consumption shares. This first look shows, next to the smoothness of consumtion, a second problem mentioned prefiously: the non-volatility of the risk-free rate. The intuition is that because individuals are willing to work more, there is no pressure to increase the interest rate.

Figure 10 shows the impulse responses to the same shock as before. The sub-figures try to illustrate how capital adjustment costs involve the individual's reaction. Beginning with figure 10(a), it can be seen that such a habit increases the smoothness of consumption; because the individual is forced to smooth her consumption path over time, she will invest more money in the capital stock. This makes her able to stretch out the increased level of consumption over time - with respect to the steady state value - but a lower level in comparision with the model without habit formation, this shows how an individual can be forced to invest more rather then to consume



(a) Habit Formation Without Capital Adjustment Costs

(b) Habit Formation With Capital Adjustment Costs

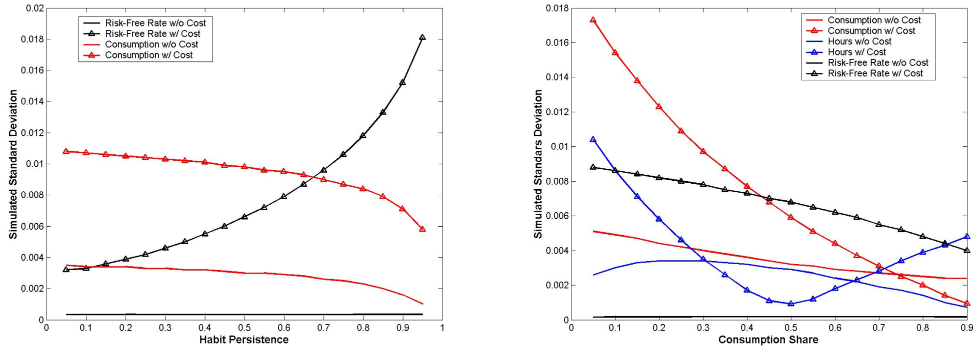
Figure 10: IMPULSE RESPONSES TO A ONE PERCENT TECHNOLOGY SHOCK, FOR AN EXTERNAL HABIT MODEL

Consider that these figures represent the impulse responses to a one percent technology shock, by using the benchmark parameter values.

more. If, now, capital adjustment costs are introduced, see figure 10(b), an investment, in the moment of the shock, is relativly expensive because of the increased demand for capital by the firm has increased the price of capital - this shows that the individual is forced now to consume more than she would like. Another effect, which can be seen in the data of table (7), is that the volatility of the riskless interest rate increases - this comes from the reduced willigness of the individual to invest, so the interest rate has to increase to make an investment more attractive. This forced a difference to the first model, because the labor input is fixed and the input factors cannot be substitute for each other.

Sub-figure 11(a) illustrates the effect of introducing capital adjustment costs into the economy on the simulated volatility of consumption and the riskless interest rate. More then that, it shows the influence of different habit persitence levels on these moments. Capital adjustment costs increase the volatility of consumption, but a contrary influence occurs for high habit persistence. As mentioned above, a higher habit persistence increases the desire of the individual to smooth consumption. On the other hand, the higher persistence level increases the volatility for a riskless investment, the intuition behind which is that such a high value of θ forces the individual to invest her money, because she does not want to consume more, The sub-figure 11(a) shows that the chosen benchmark value of $\theta = 0.6$ reflects the business cycle facts trying to be covered: to increase the volatility of the riskless rate while not increasing the smoothness of consumption.

The discussion of the nonseparable habit formation between consumption and leisure starts with sub-figure 12(a), which shows the impulses to a one percent technology shock in the absence of capital adjustment costs. Similiar to the habit formation results only insofar as consumption: the individual does smooth consumption, but not as strongly as before, because she can adjust her habit by decreasing leisure. The result is a higher volatility in consumption than with habit formation

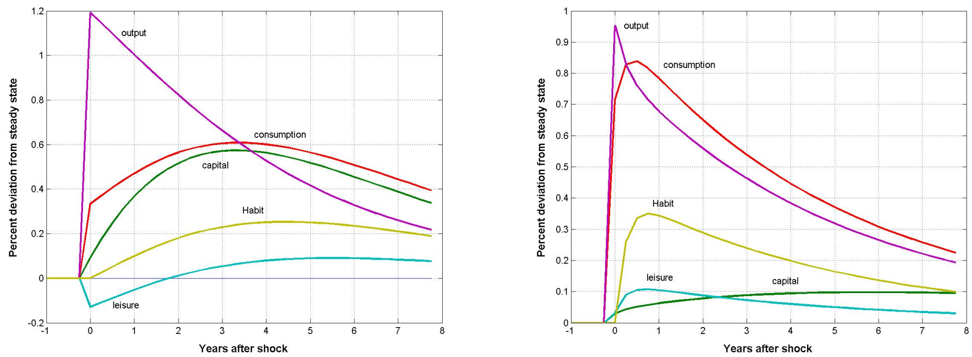


(a) Influences of Different Habit Persistence Levels

(b) Influences of Different Consumption Shares

Figure 11: Influences on Simulated Standard Deviations of Introducing Capital Adjustment Costs

The figure describes the influences of capital adjusment costs in external habit models on simulated standard deviations as well as the possible influence by choosing different consumption shares and habit persitence levels.



(a) Without Capital Adjustment Costs

(b) With Capital Adjustment Costs

Figure 12: Impulse Responses to a one percent Technology Shock, for an External Habit Model with Nonseparability Between Consumption and Leisure

Consider that these figures represent the impulse responses to a One Percent technology shock, by using the benchmark parameter values.

only in consumption without adjustment costs. Introducing captital adjustment costs, increases the volatilities above as before, but it depend strongly on the consumption share (see subfigure 11(b)). If the consumption share is smaller than 0.5, then the individual wants to consume more and wants more leisure: she wants utilize the technology shock for both. This phenomenon can be described by the fact that capital adjustment costs increase the price of capital, and thus, the desire to invest in this capital reduces. The firm is forced to pay higher wages because it cannot accumulate enough capital; it follows that together with the existing habit in leisure an decreasing substitution effect can be observed, which is supported by the decreasing volatility of leisure in table 7. The volatility of the riskless interest rate increases if the consumption share decreases, but this effect cannot be replicated by any leisure decision because of leisure's different directions of possible movement. For the case that $\chi > 0.5$, there exists the "normal" and expected case that a shock in technology would be followed by more hours worked. That can be described by the fact that the individual can now increase consumption to alevel that is very persistent over time - so that the volatility decreases, as can to be observed in figure 11(b). A similiar discussion is given in Boldrin et al. (2001), the model used by the authors differs in many points, but they argued that the increase in leisure if a technology shock occur depends on the capital adjustment costs. The parameter value $\chi = 0.5$ refers to the point, at which the income- and substitution effects are equal - consequently, where the volatility of leisure is smallest. Left from this point the income effect dominates the substitution effect; right from this, the substitution effect dominates the income effect. Another problem, if we observe the Hodrick-Prescott data, is the decreased volatility of output. For this, the argumentation of Boldrin et al. (2001) can be also cited, but it can easily be obtained, by recalling the unwillingness of the individual to work if $\chi < .5$. For this, the firm cannot produce that output, which it could produce in the absence of capital adjustment costs. Another effect which results from the introduction of capital adjustment costs to the model with nonseparability is is the increased ability to generate a relatively high volatility in consumption. This seems to be an advantage over the model discussed by Lettau and Uhlig (2000), because their leisure habit decreased the volatility whereas the nonseparable case used in this thesis increases the volatility of leisure.

Concluding this disccussion, it can be observed that capital adjustemnt costs can help to overcome the smoothness of consumption in habit formation as similiarly argued by Jermann (1998). Furthermore, introducing adjustement costs is helpful in replicating the volatility of the riskless interest rate. The ability of habit formation with nonseparability between consumption and leisure to match further business cycle facts must be distinguished from the previous results that, in any case, should not be dismissed. A combination of leisure and capital adjustment cost would appear to be difficult, because the adjustment costs can be followed by a dominating income effect that would go hand in hand with a contrary effect on the volatility of output. The nonseparability between consumption and leisure brings the advantage, in contrast to nonseparability as in Boldrin et al. (2001), that this domination is reversible by increasing the consumption share. Such a high consumption share would stand in contrast to recent literature (Kydland and Prescott, 1982; Eichenbaum et al., 1988), as well as to the dicussion in section 3, where an incresasing importance

Table 7: HISTORICAL AND SIMULATED BUSINESS CYCLE FACTS

Variable	U.S. Data	Standard Power Utility	Standard Power Utility With Nonseparable Leisure		Standard External Habit		Standard External Habit With Nonseparable Leisure			
					without cost	with cost	without cost		with cost	
			$\chi = 1/3$	$\chi = 0.16$			$\chi = 1/3$	$\chi = 0.16$	$\chi = 1/3$	$\chi = 0.16$
σ_y	1.59	0.96	1.02	1.04	0.97	0.97	1.17	1.15	0.85	0.60
σ_c	0.80	0.35	0.54	0.54	0.29	0.95	0.38	0.46	0.83	1.26
σ_H	1.27	-	0.10	0.14	-	-	0.34	0.33	0.23	0.65
σ_L	0.90	-	0.04	0.02	-	-	0.14	0.05	0.10	0.28
σ_D.	2.93	0.98	0.41	0.42	0.98	0.95	0.50	0.30	0.30	0.21
$corr(y, c)$	0.8332	0.9719	0.9881	0.9840	0.7751	0.9636	0.8287	0.8955	0.9652	0.9104
$corr(y, H)$	0.8885	-	0.8063	0.8083	-	-	0.8945	0.8452	-0.7626	-0.8263
$corr(y, L)$	0.7492	-	-0.8063	-0.8083	-	-	-0.8945	-0.8452	0.7626	0.8263
$corr(y, D)$	0.4990	0.9683	0.8599	0.8562	0.9636	0.9983	0.7651	0.8106	0.9889	0.9784
$corr(\Delta y)$	0.8700	0.7179	0.7153	0.7144	0.7215	0.7171	0.7230	0.7085	0.6560	0.5683
σ_c/σ_y	0.5031	0.3646	0.5294	0.5192	0.2990	0.9794	0.3248	0.4000	0.9765	2.1000
σ_H/σ_y	0.7987	-	0.0980	0.1346	-	-	0.2906	0.2870	0.2706	1.0833
σ_L/σ_y	0.5660	-	0.0392	0.0192	-	-	0.1197	0.0435	0.1176	0.4667
σ_D/σ_y	1.8428	1.0208	0.4020	0.4038	1.0103	0.9794	0.4274	0.2609	0.3529	0.3500
σ_{R^f}	0.99	0.03	0.01	0.01	0.03	0.79	0.02	0.02	0.71	0.78

of leisure was argued. The result should not be that such a preference model with nonseparability between consumption and leisure together with capital adjustment costs be rejected, but that it show that a distinction between individuals could be helpful. Boldrin et al. (2001) were able to generate better results by intoducing a two sector model as well as by favoring a time-to-plan model over the inclusion of capital adjustment costs.

6 Conclusion

This thesis studied how well habit formation with nonseparability between consumption and leisure is able to replicate the historical data of the last thirty-eight years, with respect to the financial market as well as to the business cycle. Whereby the study compared the solutions with two main groups of preferences. Firstly, the standard power utility model, which introduced the asset pricing literature (Lucas, 1978), which, with modifications had great success in replicating the main business cycle facts. Secondly, a difference habit utility function, which had some success in replicating financial data and hich has recently been frequently used in the literature. The motivation to use nonseparability between consumption and leisure was driven, next to the poor justification used in the literature to avoid doing so, by the ability of such a preference structure to replicate many business cycle facts.

The analyses of the asset pricing implications have shown that a nonseparability between consumption and leisure lends the two diametrically oppossed influences. This can be concluded with the support of the literature, which states that such a type of preference does not bring results that are any better. However, it was shown that every examined utility function is unable the replicate financial market data without rejecting monumental cornerstones of economic research. For example, the analysis of the historical data brought forth that the volatility puzzle still exists and has been, in recent years, further intensified, so that every examined model of consumption-based asset pricing has had some trouble. Furthermore, the thesis has discussed the absurdity of the necessary parameters, which are often proclaimed in the literature, these models would require in order that they match the financial market data. The analysis of the asset pricing implications was concluded by supporting the argumentation by Cochrane (2001) that no model can replicate every fact of financial markets. More over, the analysis of the historical data, the background of this thesis, as well as the discussed problems in the models let us conclude that it would seem as if a solution of the prominent puzzles of asset pricing literature were far away. The observations have shown that the examined models are unable to provide satisfying solutions with respect to historical data from the last thirty-eight years. As shown, it should be not much longer data sets should not be used, because the consumption decisions of the last decades have dramatically changed, but the stylized facts of financial markets, e.g., the equity premium exist just as high as in longer data sets - this should not be neglected.

The analysis of business cycle facts, brought forth some interesting solutions. It was shown that habit formation with nonseparability between consumption and leisure had some success in replicating these. So, the RBC model used was able to overcome the smoothness of consumption, that habit formation implies. Furthermore, a leisure habit like the one used in this thesis is able to increase the the volatility of hours worked. This is an advantage over the model used by Lettau and Uhlig (2000), wherein the separable leisure habit reduces this volatility whilst being unable to increase the volatility of consumption. Another positive result is the ability to increase the volatility of the riskless interest rate. Of course, thess positive solutions are obtainable because of the introduction of capital adjustment costs. The disadvantage of this introduction is the countercyclical effect of leisure, which brings a dominating income effect and also reduces the volatility of output. These problems were also mentioned by Boldrin et al. (2001), but in contrast to the work of these authors, the model in this thesis can overcome these problems by increasing the consumption share. Furthermore, in contrast to Boldrin et al. (2001), this model does not over-estimate the volatility of the riskless interest rate.

Although it can be concluded that the arguments against habit formation with nonseparability between consumtion and leisure, with respect to asset pricing, can be supported, but this nonseparability had some success in explaining business cycles. Above and beyond that, irrespective of specific results, it can be postulated that habit formation model, which are extended by leisure, should also contain a leisure habit to make the argumentation behind habit formation quite clear. It is not easy to weigh out the disadvantages for asset pricing implications with they advantages for business cycle implications in this preference structure. However, no benchmark model exists that could replicate the empirics from financial market very well and as such, the worst that could be said about this model is that it merely performs a little worse than all the other models used for comparison in this thesis.

All of this leads us to the conclusion already drawn in the literature: it is the fundamentally difficult to study financial markets and business cycles simultanously (Lettau and Uhlig, 2000). This can be supported, because it seems as if the counterfactuality between business cycle research and asset pricing research would be more heavily weighted than the countefactuality of nonseparability between consumption and leisure.

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A Necessary Properties and Derivations for Solving Asset Pricing Implications

This appendix gives a more detailed analysis of the asset pricing implications, described in section (4.3). Next to some key facts of the lognormal distribution, it contains the analysis of the Mehra and Prescott (1985) model and an extension thereof. The third part looks at habit formation as a standard difference model; whereas the final part (A.4) juxtaposes this with a model that uses nonseparability between consumption and leisure.

A.1 Properties of the Lognormal Distribution

By following, e.g., Campbell et al. (1997), when assuming that continuously compounded single period returns are IID normal distributed (which would implies that single period gross simple returns are distributed as IID lognormal variates), it then holds for the variable z that:

$$\ln z \sim N\left(\mu_z, \sigma_z^2\right), \text{then} \quad a \ln z \sim N\left(a\mu_z, a^2\sigma_z^2\right).$$

For an expected values of z, the following condition holds:

$$E\left(z^a\right) = E\left[\exp\left(a \ln z\right)\right] = \exp\left(a\mu_z + \frac{1}{2}\left(a^2\sigma_z^2\right)\right).$$

By combining two different variables the previous equation can be extended to

$$a \ln z + b \ln x \sim N\left(a\mu_z + b\mu_x + \frac{1}{2}\left(a^2\sigma_z^2 + b^2\sigma_x^2 + 2ab\,\rho\sigma_x\sigma_z\right)\right),$$

where ρ denotes the correlation parameter, or as an expression for the covariance, the next equation usually holds:

$$\text{cov}\left(z, x\right) = \sigma_{zx} = \rho\,\sigma_z\sigma_x.$$

$$E\left(z^a\, x^b\right) = \exp\left(a\mu_z + b\mu_x + \frac{1}{2}\left(a^2\sigma_z^2 + b^2\sigma_x^2 + 2ab\,\sigma_{zx}\right)\right)$$

more thoroughly:

$$E\left(\prod_{i=1}^{n} z_i\right) = \exp\left(\sum_{i=1}^{n}\mu_i + \frac{1}{2}\left(\sum_{i=1}^{n}\sigma_i^2 + \sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}\right)\right)$$

The variance of a lognormally ditributed variable x is given by the following condition:

$$\begin{aligned} \text{var}\left(x\right) &= E\left(x^2\right) - \left[E\left(x\right)\right]^2 \\ &= \exp\left(2\mu_x + 2\sigma_x^2\right) - \exp\left(2\mu_x + \sigma_x^2\right) \\ &= \exp\left(2\mu_x + \sigma_x^2\right)\left[\exp\left(\sigma_x^2 - 1\right)\right] \\ &= \left[E\left(x\right)\right]^2\left[\exp\left(\sigma_x^2 - 1\right)\right] \end{aligned}$$

Solving for the variance of $\ln x$, this expression, which depends on the variance and mean gross return of a simple discrete variable, is,

$$\exp\left(\sigma_x^2\right) = 1 + \frac{\text{var}\,(x)}{[E\,(x)]^2};$$

hence,

$$\sigma_x^2 = \ln\left\{1 + \frac{\text{var}\,(x)}{[E\,(x)]^2}\right\}.$$

The same can be done for the mean of the lognormal variable, the solution is given in the next two equations,

$$\ln\, E\,(x) = \mu_x + \frac{1}{2}\sigma_x^2$$

therefore,

$$\mu_x = \ln\, E\,(x) - \frac{1}{2}\sigma_x^2.$$

A.2 The Mehra-Prescott Model and Extension with Leisure

This section shows the analysis of the Mehra and Prescott (1985) model [17].

$$E\left[\sum_{t=0}^{\infty}\beta^t U\,(c)\right] \qquad \text{with} \quad 0 < \beta < 1$$

$$U\,(c) = \frac{c^{1-\gamma} - 1}{1-\gamma}$$

Where

$$g_{c,t+1} = \frac{c_{t+1}}{c_t}$$

$$g_{d,t+1} = \frac{d_{t+1}}{d_t}$$

are the growth rates of consumption and dividends. The Lucas Asset Pricing Formula is given by the equation,

$$p_t = \beta\, E_t\left[(p_{t+1} + d_{t+1})\,\frac{U'\,(c_{t+1})}{U'\,(c_t)}\right],$$

where $p_{t+1} + d_{t+1}$ is the payoff of the asset in the next period. Inserting the first derivative of $U\,(c)$,

$$U'\,(c) = MU = c^{-\gamma},$$

the Lucas Asset Pricing Formula can be written as:

$$p_t = \beta\, E_t\left[x_{t+1}\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}\right] \text{, with} \quad x_{t+1} = p_{t+1} + d_{t+1}.$$

[17] The analysis explicitly follows the methods used, for example, by Mehra (2003)

By using the definition of the consumption growth rate we get:

$$p_t = \beta\, E_t \left[x_{t+1} \left(g_{c,t+1} \right)^{-\gamma} \right].$$

To set the asset price dependent on the dividend growth, Mehra and Prescott (1985) assume that p_t is homogenous of degree 1 in d_t, fromwhich it follows:

$$p_t = w d_t,$$

$$w = \beta\, E_t \left[(w+1) \frac{d_{t+1}}{d_t} \cdot g_{c,t+1}^{-\gamma} \right].$$

Using the dividend growth rate expression and solving for w brings us the following equation:

$$w = \frac{\beta\, E_t \left[g_{d,t+1} \cdot g_{c,t+1}^{-\gamma} \right]}{1 - \beta\, E_t \left[g_{d,t+1} \cdot g_{c,t+1}^{-\gamma} \right]}$$

We know that:

$$E_t \left[R_{e,t+1} \right] = E_t \left[\frac{p_{t+1} + d_{t+1}}{p_t} \right],$$

which is the same as,

$$E_t \left[R_{e,t+1} \right] = E_t \left[\frac{w d_{t+1} + d_{t+1}}{w d_t} \right] = \left[\frac{w+1}{w} \right] \cdot E_t \left[g_{d,t+1} \right]$$

It can be shown that,

$$\frac{w+1}{w} = \frac{1}{\beta\, E_t \left[g_{d,t+1} \cdot g_{c,t+1}^{-\gamma} \right]}.$$

So that, by remembering the condition for the risk-free rate and the return on equity, the following equations hold,

$$E_t \left[R_{f,t+1} \right] = \frac{1}{\beta\, E_t \left[g_{c,t+1}^{-\gamma} \right]}$$

$$E_t \left[R_{e,t+1} \right] = \frac{E_t \left[g_{d,t+1} \right]}{\beta\, E_t \left[g_{d,t+1} \cdot g_{c,t+1}^{-\gamma} \right]}.$$

By assuming conditional lognormality and homoskedasticity of asset reurns and consumption, and by using the properties of lognormal distribution written in A.1, it is possible to show that

$$E_t \left[R_{f,t+1} \right] = \frac{1}{\beta \, \exp \left(-\gamma \mu_c + \frac{1}{2} \gamma^2 \sigma_c^2 \right)}$$

and

$$E_t \left[R_{e,t+1} \right] = \frac{\exp \left(\mu_d + \frac{1}{2} \sigma_d^2 \right)}{\beta \, \exp \left(\mu_d - \gamma \mu_c + \frac{1}{2} \left(\sigma_d^2 + \gamma^2 \sigma_c^2 - 2\gamma \sigma_{cd} \right) \right)}.$$

After taking logs the expressions are reformulated as:

$$\log E_t\left[R_{f,t+1}\right] = -\log\beta + \gamma\mu_c - \frac{1}{2}\gamma^2\sigma_c^2$$

and

$$\log E_t\left[R_{e,t+1}\right] = -\log\beta + \gamma\mu_c - \frac{1}{2}\left(\gamma^2\sigma_c^2 - 2\gamma\sigma_{cd}\right).$$

By not assuming degree-one homogenity of p_t in d_t, the solution follows in respect to asset returns (not to dividends), as in Cochrane (2001) or Campbell et al. (1997). The use of the Mehra and Prescott (1985) assumption would imply that dividends are perfectly correlated with stock returns. Because the data do not support that, the asset pricing implications should be expressed in the following form:

$$\log E_t\left[R_{e,t+1}\right] = -\log\beta + \gamma\mu_c - \frac{1}{2}\left(\gamma^2\sigma_c^2 - 2\gamma\sigma_{cr_e}\right)$$

for the return on equity, and especially for the risk premium the following condition:

$$\log E_t\left[R_{e,t+1}\right] - \log E_t\left[R_{f,t+1}\right] = \gamma\sigma_{cr_e}.$$

The absolute risk aversion (*ara*) is given by $-U''(c)/U'(c)$, that implies for the Mehra-Prescott model:

$$\mathit{ara} = -\frac{U''(c)}{U'(c)} = \frac{\gamma}{c}$$

The relative risk aversion will be explained by:

$$\mathit{rra} = -\frac{c \cdot U''(c)}{U'(c)} = \gamma$$

A Leisure Extension of the Mehra-Prescott Model

From this point on, I am leaving Mehra and Prescott (1985) and extending the model with a leisure expression. The new utility function can be written as a power function of the product $c^{\chi}l^{1-\chi}$, where χ is the consumption-share parameter.

$$U(c,l) = \frac{\left(c^{\chi}l^{1-\chi}\right)^{1-\gamma} - 1}{1-\gamma}$$

The following steps are the same, which was explained for the standard model explicitly. The derivatives for c_t and c_{t+1}, which are given below, look familiar to the known derivatives from the beginning of the subsection.

$$U_{c_t}(c_t, l_t) = \left[c_t^{\chi} \cdot l_t^{1-\chi}\right]^{-\gamma} \cdot \chi \left(\frac{l_t}{c_t}\right)^{1-\chi}$$

$$U_{c_{t+1}}(c_{t+1}, l_{t+1}) = \left[c_{t+1}^{\chi} \cdot l_{t+1}^{1-\chi}\right]^{-\gamma} \cdot \chi \left(\frac{l_{t+1}}{c_{t+1}}\right)^{1-\chi}$$

It is easily to see that the standard model of Mehra and Prescott (1985) is a special case ($\chi = 1$) of the leisure combined model. The stochastic discount factor m_{t+1} is now given by:

$$m_{t+1} = \beta \left[\left(\frac{c_{t+1}}{c_t}\right)^{\chi} \cdot \left(\frac{l_{t+1}}{l_t}\right)^{1-\chi}\right]^{-\gamma} \cdot \left(\frac{l_{t+1}}{l_t}\right)^{1-\chi} \cdot \left(\frac{c_{t+1}}{c_t}\right)^{\chi-1}$$

Now inserting

$$g_{c,t+1} = \frac{c_{t+1}}{c_t} \quad \text{and} \quad g_{l,t+1} = \frac{l_{t+1}}{l_t},$$

brings us the following stochhastic discount factor:

$$m_{t+1} = \beta \left[g_{c,t+1}^{(\chi-\gamma\chi-1)} \cdot g_{l,t+1}^{(1-\chi-\gamma+\gamma\chi)}\right].$$

For a better overview I substitute

$$a = \chi - \gamma\chi - 1 \quad \text{and} \quad b = 1 - \chi - \gamma + \gamma\chi.$$

The rest of the calculations is analogous to the standard model discussed above. The return on a risk-free asset is given by,

$$E_t\left[R_{f,t+1}\right] = \frac{1}{\beta\, E_t\left[g_{c,t+1}^{a} \cdot g_{l,t+1}^{b}\right]}$$

using expectations of a lognormal distribution and taking logs, you get

$$\log E_t\left[R_{f,t+1}\right] = -\log\beta - a\mu_c - b\mu_l - \frac{1}{2}\left(a^2\sigma_c^2 + b^2\sigma_l^2 + 2ab\sigma_{cl}\right).$$

For the asset return, it follows under risk correction:

$$\log E_t\left[R_{e,t+1}\right] = \log E_t\left[E_{f,t+1}\right] + \frac{1}{2}\left(2a\sigma_{cr_e} + 2b\sigma_{lr_e}\right)$$

$$\log E_t\left[R_{e,t+1}\right] - \log E_t\left[R_{f,t+1}\right] = -a\sigma_{cr_e} - b\sigma_{lr_e}.$$

Plugging in the expressions for a and b, the risk premium can be expressed by,

$$\log E_t\left[R_{e,t+1}\right] - \log E_t\left[R_{f,t+1}\right] = -\gamma\chi\sigma_{cr_e} + (1-\chi)\left(\sigma_{cr_e} + (\gamma-1)\,\sigma_{lr_e}\right).$$

COSUMPTION- AND LEISURE-BASED RISK AVERSION

As above, the relative risk aversion is one of the key problems in the asset pricing literature. As well as the consumption based risk aversion, it is necessary to look to the leisure based risk aversion. The calculation of the absolute- and relative consumption based risk aversion seems to be a little bit more tricky than in the standard model. After finding $U''(c)$,

$$\frac{\partial^2 U(c,l)}{\partial c^2} = \chi \cdot l^{-\gamma(1-\chi)+1-\chi} \cdot c^{\chi-2-\chi\gamma} \cdot (-\gamma\chi + \chi - 1)$$

we can find the *ara* by using the known first derivative of U, with respect to c, and we get the following conditions:

$$ara_c = -\frac{\chi \cdot l^{-\gamma(1-\chi)+1-\chi} \cdot c^{\chi-2-\chi\gamma} \cdot (-\gamma\chi + \chi - 1)}{\chi \cdot l^{-\gamma(1-\chi)+1-\chi} \cdot c^{\chi-1-\gamma\chi}}$$

$$ara_c = \frac{\gamma\chi - \chi + 1}{c}$$

$$rra_c = \gamma\chi - \chi + 1.$$

For the leisure based risk aversion, the formulas looks similiar,

$$ara_l = -\frac{U_l''(c,l)}{U_l'(c,l)} \qquad \text{and} \qquad rra_l = -l \cdot \frac{U_l''(c,l)}{U_l'(c,l)}$$

The first and second derivative can be expressed as follows:

$$\frac{\partial U(c,l)}{\partial l} = (1-\chi) \cdot \left(c^{\chi} l^{1-\chi}\right)^{-\gamma} \left(\frac{c}{l}\right)^{\chi}$$

$$\frac{\partial^2 U(c,l)}{\partial l^2} = (1-\chi) \cdot c^{\chi-\gamma\chi} \cdot l^{\gamma\chi-\gamma-\chi-1} \cdot (\gamma\chi - \gamma - \chi)\,.$$

With these expressions, it is possible to solve for the absolute- and realtive risk aversion based on leisure:

$$ara_l = \frac{\gamma + \chi - \gamma\chi}{l} \qquad \text{and} \quad rra_l = \gamma + \chi - \gamma\chi = 1 - b.$$

A.3 Internal and External Habit Formation Without Leisure Extension

Following Chen and Ludvigson (2003) for solving habit formation:

$$E\left[\sum_{t=0}^{\infty} \beta^t U\left(c_t, x_t\right)\right] \qquad \text{with} \quad 0 < \beta < 1$$

$$U\left(c_t, x_t\right) = \frac{\left(c_t - x_t\right)^{1-\gamma} - 1}{1-\gamma}$$

$$x_t = f\left(c_{t-1}, c_{t-2}, \ldots, c_{t-L}\right)$$

The marginal utility functions for c_t and c_{t+1} are given by the following conditions:

$$MU_t = (c_t - x_t)^{-\gamma} - E_t\left[\sum_{j=0}^{L}\beta^j\left(c_{t+j} - x_{t+j}\right)^{-\gamma}\frac{\vartheta x_{t+j}}{\vartheta c_t}\right]$$

$$MU_{t+1} = (c_{t+1} - x_{t+1})^{-\gamma} - E_{t+1}\left[\sum_{j=0}^{L}\beta^j\left(c_{t+j+1} - x_{t+j+1}\right)^{-\gamma}\frac{\vartheta x_{t+j+1}}{\vartheta c_{t+1}}\right]$$

Remembering the the equation for for the stochastic discount factor, brings the solution:

$$m_{t+1} = \beta\frac{MU_{t+1}}{MU_t}$$

$$m_{t+1} = \beta\left(\frac{c_{t+1} - x_{t+1}}{c_t - x_t}\right)^{-\gamma}\cdot\frac{E_{t+1}\left[1 - \sum_{j=0}^{L}\beta^j\left(\frac{c_{t+j+1}-x_{t+j+1}}{c_{t+1}-x_{t+1}}\right)^{-\gamma}\frac{\vartheta x_{t+j+1}}{\vartheta c_{t+1}}\right]}{E_t\left[1 - \sum_{j=0}^{L}\beta^j\left(\frac{c_{t+j}-x_{t+j}}{c_t-x_t}\right)^{-\gamma}\frac{\vartheta x_{t+j}}{\vartheta c_t}\right]}$$

So, the Lucas Asset Pricing Formula can be written as:

$$E_t\left\{\frac{E_{t+1}\left[\beta\left(\frac{c_{t+1}-x_{t+1}}{c_t-x_t}\right)^{-\gamma}\cdot\left[1 - \sum_{j=0}^{L}\beta^j\left(\frac{c_{t+j+1}-x_{t+j+1}}{c_{t+1}-x_{t+1}}\right)^{-\gamma}\frac{\vartheta x_{t+j+1}}{\vartheta c_{t+1}}\right]\right]\cdot R_{e,t+1}}{E_t\left[1 - \sum_{j=0}^{L}\beta^j\left(\frac{c_{t+j}-x_{t+j}}{c_t-x_t}\right)^{-\gamma}\frac{\vartheta x_{t+j}}{\vartheta c_t}\right]}\right\} = 1,$$

after some transformations, the formula above can be rewritten with the contitional moment restriction as:

$$E_t\left[\beta\left(\frac{c_{t+1} - x_{t+1}}{c_t - x_t}\right)^{-\gamma}\cdot R_{e,t+1}\cdot\tilde{F}_{i,t+1}\right],$$

with

$$\tilde{F}_{i,t+1} \equiv 1 - \sum_{j=0}^{L}\beta^j\left(\frac{c_{t+j+1} - x_{t+j+1}}{c_{t+1} - x_{t+1}}\right)^{-\gamma}\frac{\vartheta x_{t+j+1}}{\vartheta c_{t+1}} + \sum_{j=0}^{L}\beta^{j-1}\left(\frac{c_{t+j} - x_{t+j}}{c_{t+1} - x_{t+1}}\right)^{-\gamma}\frac{\vartheta x_{t+j}}{\vartheta c_t}\cdot\frac{1}{R_{e,t+1}}.$$

For the special case

$$x_t = \theta\cdot c_{t-1} \qquad \text{with} \quad 0 < a \leq 1.$$

it is easy to show that the expression above can simplify to the following equation, which is equivalent to the expression in Ferson and Constantinides (1991);

$$E_t\left[\beta\left(\frac{c_{t+1} - \theta c_t}{c_t - \theta c_{t-1}}\right)^{-\gamma}\cdot R_{e,t+1}\cdot\tilde{F}_{i,t+1}\right]$$

with

$$\tilde{F}_{i,t+1} \equiv 1 - \theta\beta\left(\frac{c_{t+2} - \theta c_{t+1}}{c_{t+1} - \theta c_t}\right)^{-\gamma} + \frac{\theta}{R_{e,t+1}}.$$

Asset Pricing Implications for a External Habit

For the external habit, the condition $\tilde{F}_{i,t+1} = 1$ holds, which simplifies the contitional moment restriction to

$$E_t\left[\beta\left(\frac{c_{t+1}-\theta c_t}{c_t-\theta c_{t-1}}\right)^{-\gamma}\cdot R_{e,t+1}\right].$$

The stochastic discount factor can be rewritten in terms of growth rates, as in Kocherlakota (1996) or Otrok et al. (2001):

$$m_{t+1}=\beta\left(\frac{\frac{c_{t+1}}{c_t}-\frac{\theta c_t}{c_t}}{1-\frac{\theta c_{t-1}}{c_t}}\right)^{-\gamma}=\beta\left(\frac{\frac{c_{t+1}}{c_t}\left(1-\frac{\theta c_t}{c_{t+1}}\right)}{1-\frac{\theta c_{t-1}}{c_t}}\right)^{-\gamma}.$$

This expression is the same as the following - only the notation has changed:

$$m_{t+1}=\beta\cdot g_{c,t+1}^{-\gamma}\left(1-\theta\cdot g_{c,t+1}^{-1}\right)^{-\gamma}\cdot\left(1-\theta\cdot g_{c,t}^{-1}\right)^{\gamma}.$$

Taking logs, it follows:

$$\log\beta-E_t\left[\gamma\hat{g}_{c,t+1}\right]-E_t\left[\gamma\log\left(1-\theta\exp\left(-\hat{g}_{c,t+1}\right)\right)\right]+\gamma\log\left(1-\theta\exp\left(-\hat{g}_{c,t}\right)\right),$$

where $\hat{g}_c$ denotes log-consumption growth. Taking first-order taylor approximation, the previous expression is solved by,

$$\log\beta-E_t\left[\left(\frac{\gamma}{1-\theta\frac{1}{\bar{g}_c}}\right)\hat{g}_{c,t+1}\right]+\left[\frac{\gamma\theta\frac{1}{\bar{g}_c}}{1-\theta\frac{1}{\bar{g}_c}}\right]\hat{g}_{c,t}.$$

The variable $\bar{g}_c$ is the average consumption growth, using knowledge about lognormality distribution and the risk-free rate, I can solve for the risk-free rate as follows,

$$\log R_t^f=-\log\beta+\rho E_t\left[\hat{g}_{c,t+1}\right]-(\rho-\gamma)\,\hat{g}_{c,t}-\frac{1}{2}\rho^2\sigma_c^2,$$

with $\rho=\gamma/\left(1-\theta\bar{g}_c^{-1}\right)$. Assuming that the expected log-consumption growth is constant over time, the equation can be simplified to,

$$\log R_t^f=-\log\beta+\gamma\mu_{g_c}-\frac{1}{2}\rho^2\sigma_c^2.$$

After finding the log-risk-free rate, it is now easy to solve for the other necessary asset pricing implications.

$$\log E_t\left[R_{t+1}^e\right]=-\log\beta+\gamma\mu_{g_c}-\frac{1}{2}\left(\rho^2\sigma_c^2-2\rho\sigma_{cr_e}\right).$$

$$\log E_t\left[RP_{t+1}\right]=\rho\sigma_{cr_e}$$

COSUMPTION-BASED RISK AVERSION

For the next necessary step, finding the relative risk aversion, I followed Mehra and Prescott (2003). Next to the known conditions for the absolute and relative risk aversion, which bring

$$rra = \frac{\gamma}{1-\frac{x}{c}},$$

I assume a fixed subsistence level. This assumption helps to solve the relative risk aversion dependent on average consumption growth, from which it follows that

$$rra = \frac{\gamma}{1-\theta\frac{1}{\bar{g}_c}}$$

holds - this is nothing more than the parameter ρ, known from previous equations.

A.4 Internal and External Habit Formation With Leisure Extension

To solve a model with habit formation and nonseparabilty between consumption and leisure, the necessary steps are exactly the same as in the case without leisure. The following equations look difficult but bring the same result as above for $\chi = 1$

$$E\left[\sum_{t=0}^{\infty}\beta^t U\left(c_t, l_t, x_t\right)\right] \qquad \text{with} \quad 0<\beta<1$$

$$U\left(c_t, l_t, x_t\right) = \frac{\left(c_t^{\chi}\cdot l_t^{1-\chi}-x_t\right)^{1-\gamma}-1}{1-\gamma}$$

$$x_t = f\left(c_{t-1}l_{t-1}, c_{t-2}l_{t-2}, \ldots, c_{t-L}l_{t-L}\right)$$

The marginal utility in consumption can be found as,

$$\begin{aligned} MU_t \quad = \quad & \left(c_t^{\chi}l_t^{1-\chi}-x_t\right)^{-\gamma}\chi\left(\frac{l_t}{c_t}\right)^{1-\chi}\cdot E_t\left[1-\sum_{j=0}^{L}\beta^j\left(\frac{c_{t+j}^{\chi}l_{t+j}^{1-\chi}-x_{t+j}}{c_{t+1}^{\chi}l_{t+1}^{1-\chi}-x_{t+1}}\right)^{-\gamma}\right. \\ & \left.\cdot\chi\left(\frac{l_{t+j}}{c_{t+j}}\right)^{1-\chi}\frac{\vartheta x_{t+j}}{\vartheta c_{t+1}}\right] \end{aligned}$$

and

$$\begin{aligned} MU_{t+1} \quad = \quad & \left(c_{t+1}^{\chi}l_{t+1}^{1-\chi}-x_{t+1}\right)^{-\gamma}\chi\left(\frac{l_{t+1}}{c_{t+1}}\right)^{1-\chi}\cdot E_{t+1}\left[1-\sum_{j=0}^{L}\beta^j\left(\frac{c_{t+j+1}^{\chi}l_{t+j+1}^{1-\chi}-x_{t+j}}{c_{t+1}^{\chi}l_{t+1}^{1-\chi}-x_{t+1}}\right)^{-\gamma}\right. \\ & \left.\cdot\chi\left(\frac{l_{t+j+1}}{c_{t+j+1}}\right)^{1-\chi}\frac{\vartheta x_{t+j+1}}{\vartheta c_{t+1}}\right]. \end{aligned}$$

The stochastic discount factor can now be expressed by:

$$m_{t+1} = \beta \frac{MU_{t+1}}{MU_t} = \beta \left(\frac{c_{t+1}^{\chi} l_{t+1}^{1-\chi} - x_{t+1}}{c_t^{\chi} l_t^{1-\chi} - x_t} \right)^{-\gamma} \left(\frac{l_{t+1}}{l_t} \right)^{1-\chi} \left(\frac{c_{t+1}}{c_t} \right)^{\chi-1} \cdot \\ \cdot \frac{E_{t+1}\left[1 - \sum_{j=0}^{L} \beta^j \left(\frac{c_{t+j+1}^{\chi} l_{t+j+1}^{1-\chi} - x_{t+j}}{c_{t+1}^{\chi} l_{t+1}^{1-\chi} - x_{t+1}} \right)^{-\gamma} \cdot \chi \left(\frac{l_{t+j+1}}{c_{t+j+1}} \right)^{1-\chi} \frac{\vartheta x_{t+j+1}}{\vartheta c_{t+1}} \right]}{E_t\left[1 - \sum_{j=0}^{L} \beta^j \left(\frac{c_{t+j}^{\chi} l_{t+j}^{1-\chi} - x_{t+j}}{c_{t+1}^{\chi} l_{t+1}^{1-\chi} - x_{t+1}} \right)^{-\gamma} \chi \left(\frac{l_{t+j}}{c_{t+j}} \right)^{1-\chi} \frac{\vartheta x_{t+j}}{\vartheta c_{t+1}} \right]}.$$

The Lucas Asset Pricing Equation now takes the following form:

$$1 = E_t \left\{ \beta \left(\frac{c_{t+1}^{\chi} l_{t+1}^{1-\chi} - x_{t+1}}{c_t^{\chi} l_t^{1-\chi} - x_t} \right)^{-\gamma} \left(\frac{l_{t+1}}{l_t} \right)^{1-\chi} \left(\frac{c_{t+1}}{c_t} \right)^{\chi-1} \cdot R_{e,t+1} \\ \frac{E_{t+1}\left[1 - \sum_{j=0}^{L} \beta^j \left(\frac{c_{t+j+1}^{\chi} l_{t+j+1}^{1-\chi} - x_{t+j}}{c_{t+1}^{\chi} l_{t+1}^{1-\chi} - x_{t+1}} \right)^{-\gamma} \chi \left(\frac{l_{t+j+1}}{c_{t+j+1}} \right)^{1-\chi} \frac{\vartheta x_{t+j+1}}{\vartheta c_{t+1}} \right]}{E_t\left[1 - \sum_{j=0}^{L} \beta^j \left(\frac{c_{t+j}^{\chi} l_{t+j}^{1-\chi} - x_{t+j}}{c_{t+1}^{\chi} l_{t+1}^{1-\chi} - x_{t+1}} \right)^{-\gamma} \chi \left(\frac{l_{t+j}}{c_{t+j}} \right)^{1-\chi} \frac{\vartheta x_{t+j}}{\vartheta c_{t+1}} \right]} \right\}.$$

This expresssion can be rewritten, as in the subsection before as:

$$1 = E_t \left[\beta \left(\frac{c_{t+1}^{\chi} l_{t+1}^{1-\chi} - x_{t+1}}{c_t^{\chi} l_t^{1-\chi} - x_t} \right)^{-\gamma} \left(\frac{l_{t+1}}{l_t} \right)^{1-\chi} \left(\frac{c_{t+1}}{c_t} \right)^{\chi-1} \cdot R_{e,t+1} \cdot \tilde{F}_{i,t+1} \right],$$

with

$$\tilde{F}_{i,t+1} \equiv 1 - \sum_{j=0}^{L} \beta^j \left(\frac{c_{t+j+1}^{\chi} l_{t+j+1}^{1-\chi} - x_{t+j}}{c_{t+1}^{\chi} l_{t+1}^{1-\chi} - x_{t+1}} \right)^{-\gamma} \chi \left(\frac{l_{t+j+1}}{c_{t+j+1}} \right)^{1-\chi} \frac{\vartheta x_{t+j+1}}{\vartheta c_{t+1}} + \\ + \sum_{j=0}^{L} \beta^{j-1} \left(\frac{c_{t+j}^{\chi} l_{t+j}^{1-\chi} - x_{t+j}}{c_t^{\chi} l_t^{1-\chi} - x_t} \right)^{-\gamma} \chi \left(\frac{l_{t+j}}{c_{t+j}} \right)^{1-\chi} \frac{\vartheta x_{t+j}}{\vartheta c_{t+1}} \cdot \frac{1}{R_{e,t+1}} \left(\frac{l_{t+1}}{l_t} \right)^{\chi-1} \left(\frac{c_{t+1}}{c_t} \right)^{1-\chi}.$$

Asset Pricing Implications for a External Habit with Nonseparability Between Consumption and Leisure

As before, the assumption for an external habit, $\tilde{F}_{i,t+1} = 1$, holds. More than that, I assume a simple habit condition of the following form:

$$x_t = \theta \cdot c_{t-1}^{\chi} \cdot l_{t-1}^{1-\chi} \qquad \text{with} \quad 0 < \theta \leq 1,$$

so that the Lucas Aset Pricing Equation reduces to

$$1 = E_t \left[\beta \left(\frac{c_{t+1}^{\chi} l_{t+1}^{1-\chi} - \theta c_t^{\chi} l_t^{1-\chi}}{c_t^{\chi} l_t^{1-\chi} - \theta c_{t-1}^{\chi} l_{t-1}^{1-\chi}} \right)^{-\gamma} \left(\frac{l_{t+1}}{l_t} \right)^{1-\chi} \left(\frac{c_{t+1}}{c_t} \right)^{\chi-1} \cdot R_{e,t+1} \right].$$

The stochastic discount factor can be written with growth rates again;

$$m_{t+1} = \beta \cdot \left[\frac{\left(\frac{ct+1}{c_t}\right)^{\chi} \left(\frac{l_{t+1}}{l_t}\right)^{1-\chi} \cdot \left[1 - \theta \left(\frac{ct}{c_{t+1}}\right)^{\chi} \left(\frac{l_t}{l_{t+1}}\right)^{1-\chi}\right]}{\left[1 - \theta \left(\frac{ct-1}{c_t}\right)^{\chi} \left(\frac{l_{t-1}}{l_t}\right)^{1-\chi}\right]} \right]^{-\gamma} \cdot \left(\frac{c_{t+1}}{c_t} \right)^{\chi-1} \left(\frac{l_{t+1}}{l_t} \right)^{1-\chi}$$

or,

$$m_{t+1} = \beta g_{c,t+1}^{a} g_{l,t+1}^{b} \left(1 - \theta g_{c,t+1}^{-\chi} g_{l,t+1}^{\chi-1}\right)^{-\gamma} \left(1 - \theta g_{c,t}^{-\chi} g_{l,t}^{\chi-1}\right)^{\gamma},$$

where a and b are the known simplifications

$$a = \chi - \gamma\chi - 1 \qquad b = (1-\chi)(1-\gamma).$$

After taking logs, this equation looks like,

$$\begin{aligned} \hat{m}_{t+1} \quad = \quad & \log\beta + E_t\left[a\hat{g}_{c,t+1}\right] + E_t\left[b\hat{g}_{l,t+1}\right] + E_t\left[-\gamma\log\left(1 - \theta\exp\left((\chi-1)\,\hat{g}_{l,t+1} - \chi\hat{g}_{c,t+1}\right)\right)\right] + \\ & \left[\gamma\log\left(1 - \theta\exp\left((\chi-1)\,\hat{g}_{l,t} - \chi\hat{g}_{c,t}\right)\right)\right]. \end{aligned}$$

First-order Taylor expansion brings us the following expression:

$$\begin{aligned} & \log\beta + E_t\left[\left(a - \frac{\gamma\chi\theta\frac{\bar{g}_l^{\chi-1}}{\bar{g}_c^{\chi}}}{1-\theta\frac{\bar{g}_l^{\chi-1}}{\bar{g}_c^{\chi}}}\right)\hat{g}_{c,t+1} + \left(b + \frac{\gamma(\chi-1)\,\theta\frac{\bar{g}_l^{\chi-1}}{\bar{g}_c^{\chi}}}{1-\theta\frac{\bar{g}_l^{\chi-1}}{\bar{g}_c^{\chi}}}\right)\hat{g}_{l,t+1}\right] + \\ & \left(\frac{\gamma\chi\theta\frac{\bar{g}_l^{\chi-1}}{\bar{g}_c^{\chi}}}{1-\theta\frac{\bar{g}_l^{\chi-1}}{\bar{g}_c^{\chi}}}\right)\hat{g}_{c,t} + \left(-\frac{\gamma(\chi-1)\,\theta\frac{\bar{g}_l^{\chi-1}}{\bar{g}_c^{\chi}}}{1-\theta\frac{\bar{g}_l^{\chi-1}}{\bar{g}_c^{\chi}}}\right)\hat{g}_{l,t}. \end{aligned}$$

The hat-denoted variables are average log growth rates, but the bar denoted variables are average growth rates. Using a similiar expression as in the habit-model without leisure extension, like $\zeta = \gamma / \left(1 - \theta\bar{g}_l^{\chi-1}\bar{g}_c^{-\chi}\right)$ and the two following new equations,

$$\psi_c = \rho\chi - \chi + 1 \quad \text{and} \quad \psi_l = (1-\chi)(1-\rho),$$

the stochastic discount factor can be expressed by

$$\begin{aligned} \log E_t\left[\hat{m}_{t+1}\right] \quad = \quad & \log\beta - \psi_c E[\hat{g}_{c,t+1}] + \psi_l E[\hat{g}_{l,t+1}] + (\rho\chi - \gamma\chi)\,\hat{g}_{c,t} + (\gamma\chi + \rho - \gamma - \rho\chi)\,\hat{g}_{l,t} + \\ & + \frac{1}{2}\left(\psi_c^2\sigma_c^2 + \psi_l^2\sigma_l^2 - 2\psi_c\psi_l\sigma_{cl}\right) \end{aligned}$$

After finding this equation, it is easily to solve for the necessary asset price implications. For this

I assumed the known and expected log growth rates to be equivalent:

$$\log E_t\left[R^f_{t+1}\right] = -\log\beta - a\mu_c - b\mu_l - \frac{1}{2}\left(\psi_c^2\sigma_c^2 + \psi_l^2\sigma_l^2 - 2\psi_c\psi_l\sigma_{cl}\right)$$

$$\log E_t\left[R^e_{t+1}\right] = -\log\beta - a\mu_c - b\mu_l - \frac{1}{2}\left(\psi_c^2\sigma_c^2 + \psi_l^2\sigma_l^2 - 2\psi_c\psi_l\sigma_{cl} - 2\psi_c\sigma_{cr_e} + 2\psi_l\sigma_{lr_e}\right)$$

$$\log E_t\left[RP_{t+1}\right] = \psi_c\sigma_{cr_e} - 2\psi_l\sigma_{lr_e}.$$

COSUMPTION- AND LEISURE-BASED RISK AVERSION

Solving for the relative risk aversion, first recall the utility function,

$$U(c,l) = \frac{\left(c^\chi l^{1-\chi} - x\right)^{1-\gamma} - 1}{1-\gamma}$$

which, after having found the first and second order conditions, with respect to consumption, bring the absolute risk aversion:

$$ara_c = \frac{\gamma\chi c^{\chi-1} l^{1-\chi}}{c^\chi l^{1-\chi} - x} - \frac{\chi - 1}{c}.$$

The relative risk aversion under the assumption of a constant subsistance level, can be written as,

$$rra_c = \frac{\gamma\chi}{1 - \theta\frac{c_{-1}^\chi l_{-1}^{1-\chi}}{c^\chi l^{1-\chi}}} - \chi + 1 = \left[\frac{\gamma\chi}{1 - \theta\bar{g}_c^{-\chi}\bar{g}_l^{\chi-1}}\right] - \chi + 1 = \psi_c.$$

The same steps are to done again to solve for the relative risk aversion based on leisure, which bring the condition:

$$rra_l = \left[\frac{\gamma(1-\chi)}{1 - \theta\bar{g}_c^{-\chi}\cdot\bar{g}_l^{\chi-1}}\right] - \chi = 1 - \psi_l.$$

B MATLAB Input of the Stochastic Growth Model

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% Diplomarbeit - Martin Kliem
%
% Habit Formation with Nonseperability Between Consumption and Leisure
%
% by using H. Uhlig's Toolkit
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


disp('Model:   Diplomarbeit Martin Kliem - Asset Pricing Implication,');
disp('         Stochastic Growth Model with External Habit Formation');
disp('         and Nonseparability Between Consumption and Leisure');
disp('         and Adjustment Costs                              ');

disp('Hit any key when ready...');
pause;

% Setting parameters:

kappa     = 1;
alpha     = .36;
delta     = .025;
beta      = 0.99;
chi       = .15;
theta     = .6;
gamma     = 2;
rho       = 0.95;
sigma_z   = 0.0074;
zeta      = 0.23;
n_bar     =1/3;

% Defining the adjustment cost parameters:

a_1       = 0.000000325991497
a         =(a_1)/(1-1/zeta)
a_2       =(-delta)/(zeta-1)

% Calculating the steady state:

r_bar=1/beta
phi_bar=delta
ky=alpha/r_bar*(a/(phi_bar-a_2))^(zeta/(zeta-1))
d_bar=alpha*ky^(-1)
cy=(1-alpha)*chi/(1-chi)*(1-n_bar)/n_bar
y_bar=kappa^(1/(1-alpha))*ky^(alpha/(1-alpha))*n_bar
l_bar=1-n_bar
k_bar=ky*y_bar
c_bar=cy*y_bar
w_bar=(1-alpha)*y_bar/n_bar
x_bar=theta*c_bar^chi*l_bar^(1-chi)
s_bar=(1-theta)*c_bar^chi*l_bar^(1-chi)
I_bar=k_bar*(((phi_bar-a_2)/a)^(zeta/(zeta-1)))
```

```
% helpful functions:

help_1=a*(I_bar/k_bar)^(1-1/zeta)*(1-1/zeta)
help_2=(phi_bar/(phi_bar-a_2))*(zeta/(zeta-1))
help_3=c_bar^chi*l_bar^(1-chi)


%Matrices

VARNAMES=[  'capital        ',
            'consumption    ',
            'leisure        ',
            'labor          ',
            'adjustment cost',
            'output         ',
            'investment     ',
            'risk-free rate ',
            'Habit          ',
            'Surplus        ',
            'dividend       ',
            'wage           ',
            'lambda         ',
            'technology     ',];

 % Setting the options:

   %capital       consumption          leisure        labor       adjustment

 AA=[0            (1-chi)             -(1-chi)         0            0
     0               chi                -chi           0            0
     0                0                  l_bar        n_bar         0
     0                0                   0            0            0
     0                0                   0           -1            0
    -help_1           0                   0            0           -phi_bar
     0                0                   0            0            0
     0                0                   0         (1-alpha)       0
   (phi_bar-1)*k_bar  0                   0            0         phi_bar*k_bar
     0              c_bar                 0            0            0
     0                0                   0            0            0
     0            chi*help_3         (1-chi)*help_3    0            0];


 BB=[0               0                    0            0            0
     0               0                    0            0            0
     0               0                    0            0            0
     -1              0                    0            0            0
     0               0                    0            0            0
     0               0                    0            0            0
     -1              0                    0            0          -help_2
     alpha           0                    0            0            0
    (1-delta)*k_bar  0                    0            0            0
     0               0                    0            0            0
     0              chi                (1-chi)         0            0
     0             -chi*help_3      -(1-chi)*help_3    0            0];
```

```
     % output   investment   risk free    Habit    Surplus  dividend   wage   lambda
CC=[   0           0          0            0       gamma    0          0        1
       0           0          0            0      -gamma    0         -1       -1
       0           0          0            0        0       0          0        0
       alpha       0          0            0        0      -1          0        0
      (1-alpha)    0          0            0        0       0         -1        0
       0        help_1        0            0        0       0          0        0
       alpha       0         -1            0        0       0          0        0
      -1           0          0            0        0       0          0        0
       0           0          0            0        0       0          0        0
      -y_bar    I_bar         0            0        0       0          0        0
       0           0          0           -1        0       0          0        0
       0           0          0            0      -s_bar    0          0        0];


DD=[0
    0
    0
    0
    0
    0
    0
    1
    0
    0
    0
    0];

FF=[0 0  0   0 0];

GG=[0 0  0   0 0];

HH=[0 0  0   0 0];

JJ=[   0          0         1            0         0      0           0     1  ];

KK=[   0          0         0            0         0      0           0    -1  ];

LL=[0
];

MM=[0
];

NN=[rho];

Sigma=[sigma_z^2];


[l_equ,m_states] = size(AA);
[l_equ,n_endog ] = size(CC);
[l_equ,k_exog  ] = size(DD);


PERIOD     = 4;  % number of periods per year
GNP_INDEX  = 6; % Index of output among the variables selected for HP filter
IMP_SELECT = [1,2,5,6,7,8,9,10];
```

```
DO_SIMUL   = 1; % Calculates simulations
SIM_LENGTH = 155;
SIM_MODE = 2;
SIM_N_SERIES = 100;
DO_MOMENTS = 1; % Calculates moments based on frequency-domain methods
HP_SELECT  = 1:(m_states+n_endog+k_exog);
HORIZON=32;
SELECT_SHOCKS=1:k_exog;
DO_STATE_RESP=0;
IMP_SINGLE=0;



% Starting the calculations:

do_it;
```

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Eidesstattliche Erklärung

Ich erkläre hiermit von Eides statt, dass ich die vorliegende Arbeit selbständig und ohne Benutzung anderer als der angegebenen Hilfsmittel angefertigt habe; die aus fremden Quellen direkt oder indirekt übernommenen Gedanken sind als solche kenntlich gemacht.
Die Arbeit wurde bisher in gleicher oder ähnlicher Form keiner anderen Prüfungsbehörde vorgelegt und auch noch nicht veröffentlicht.

Berlin, den 19. Februar 2004

Martin Kliem