Technology Shocks and Labor: An Analysis Using Medium-Run Identification

Diplomarbeit

zur Erlangung des Grades
einer Diplom-Volkswirtin

an der Wirtschaftswissenschaftlichen Fakultät
der Humboldt-Universität zu Berlin

vorgelegt von

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March 18, 2004

Abstract

The question how technology shocks affect aggregated hours worked is widely discussed in the literature and delivers a touchstone for Real-Business-Cycle theory. Initiated by Galí (1999), the impact and dynamic propagation of single shocks on macroeconomic variables are of interest. In this context, the response of labor to technology shocks needs special treatment, since it has not uniquely been determined so far. Usually, "long-run identificiation" is applied to disentangle the disturbances in the macroeconomic system, meaning that technology shocks are restricted to be the only influence of productivity in the long run.

While there exists a large body of research already, this thesis considers two recent approaches that shed some new light on the debate. Uhlig (2003a) introduces medium-run identification as an alternative to long-run identification. He states that there may exist more sources in the variation of productivity than technology only and allows for labor hoarding in his model. Fisher (2002) assumes investment-specific technology as a complement to the neutral technology analyzed by Galí. He identifies the investment-specific shock as the only influence on the investment price, while both kinds of technology are the only driving forces of labor productivity in the long run.

In this thesis, I first derive an identification method that decomposes the forecast revision variance using Cholesky as proposed by Uhlig (2003a). I then solve the model introduced by Fisher as well as an extension with labor hoarding. Next, I derive identifying restrictions for the shocks from the solution of the model using forecast revision variances and test them on artificial data simulated from the model specification. Finally, I identify technology shocks in real data.

My results can be summarized as follows: The theoretical impulse responses from the model solution are qualitatively matched by Fisher's empirical ones. The response of labor after an innovation in investment-specific technology is understated in the model. The identification strategy proposed by Fisher is valid for the original model and the labor hoarding model, if there are no other permanent shocks influencing productivity. I show that medium-run identification could be used instead and derive an alternative identification strategy that identifies neutral technology as the only influence on productivity and both technology shocks as the only driving force of labor in the medium run. I show that this strategy works equivalently well.

In real data, my Fisher identification delivers responses of productivity and labor that fall after an innovation in investment-specific technology, which strongly contradicts Fisher's own findings. This is probably due to a different data measure and should be handled with care; however, it sheds light on the specification's sensitivity to the investment price series employed and indicates the desirability of an alternative specification. I find that the alternative specification in fact identifies the same shocks as Fisher. However, the response of labor is negative on impact and in the long run after an innovation in neutral technology. While this contradicts RBC theory, one may argue that, due to the strong rise of labor after investment-specific technology, which is quite robust across specifications, technology as a combined measure of neutral and investment-specific components induces hours to rise. This defends technology-driven business cycle theory.

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1 Introduction

One of the recently most disputed topics in macroeconomics concerns the effects that technology shocks have on the labor market. While technological progress should lead to a rise in productivity and output in the long run, its effect on labor is not that obvious. On the one hand, there may be Schumpeterian creative destruction, since outdated and unproductive jobs are destroyed. Hence, technical progress may be labor-saving. On the other hand, existing jobs may use the new technology and become more productive, resulting in labor-augmenting technical progress. Meanwhile, of course, new jobs may be created due to the new advancements. Clearly, hours worked decrease if technical upgrading can not exceed creative destruction or if labor market frictions, that is sluggishness on the labor market, slows the creation of new jobs. If this is not the case or, in analogy, technical upgrading is sufficiently high, hours may rise due to an innovation in technology.[1]

Standard Real-Business-Cycle (RBC) models use technology shocks in order to explain aggregate macroeconomic fluctuations. For this class of models, labor rises after a positive shock in technology. In general, the RBC-theory claims empirical success based on the unconditional second moments of the time series generated by these models. More precisely, the movements of the model variables induced by the shocks that hit the system and their propagation over time match the ones of the aggregated variables observed in reality in terms of correlations and covariances. This concerns not only the co-movement of the real variables with output but also the co-movement of the variables with each other. Even though "non-technology" shocks may be introduced into the models in order to further improve the empirical performance[2], the role technology shocks play in these model as the key driving force of aggregate fluctuations as well as their dynamic effects in the model remain unchanged.

Since the contribution of Galí (1999), RBC models are subject to a renewed empirical check that disentangles the effects of technology shocks from other disturbances to the model. For this, Galí investigates the correlations of the macro variables conditional on single shocks using structural vector autoregression. By applying a procedure introduced by Blanchard and Quah (1989) as well as by Shapiro and Watson (1988), he imposes identifying assumptions that restrict technology shocks to be the only influence on labor productivity in the long run. This method will be called long-run identification in the following and is discussed in detail in section 2. Two results of Galí's examination have fueled a debate on the empirical foundation of Real-Business-Cycle theory:

1. Positive technology shocks lead to a decline in hours worked.
2. Technology shocks can only explain a small fraction of the hours worked.

Hence, technology shocks are labor-saving and do not serve to explain procycli-

[1] See Michelacci and Lopez-Salido (2003).

[2] This could be done for example through the introduction of government spending shocks as done by Christiano and Eichenbaum (1992) in order to account for the Dunlop–Tarshis observation. This observation finds the empirical correlation of labor productivity and labor to be close to zero.

cal hours worked. Quite drastically, this implies that the "technology-driven real business cycle hypothesis is dead"[3].

Concerning the fall of hours worked after a positive shock in technology, the literature has parted into two branches, supporting Galí or defending RBC respectively. On the one hand, Galí's findings have been backed by the application of his technique to long run data as shown by Francis and Ramey (2003) or by testing Galí's identifying restrictions for overidentification (Francis and Ramey (2002)). In addition, several authors observe a negative correlation between hours and technology shocks by measuring technology via alternative methods, such as the use of data on patents and R&D by Shea (1998) or of Hall-style regressions as done by Basu, Fernald and Kimball (1998).

On the other hand, however, there also exists a body of research concluding that hours worked rise after a positive shock in technology. Using the same identification strategy as Galí, the difference is due to assuming hours worked to be stationary rather than integrated of order one. Although both assumptions may be justified using classical tests, Christiano, Eichenbaum and Vigfusson (2003a) argue that these standard tests may not always be valid and, hence, hours worked may be subject to overdifferencing. Using posterior odds ratios, the authors test the plausibility of both specifications and conclude that the level specification of hours is to be preferred over the difference specification. Their results are supported by Christiano, Eichenbaum and Vigfusson (2003b) who use direct measures of technology or by allowing for a large variety of shocks and variables as is done by Altig, Christiano, Eichenbaum and Linde (2002). To put in a nutshell, assuming stationarity of hours worked is crucial for the fall or rise of labor after a shock in technology and positions the researcher in one of these two groups of scholars.

In addition to the direction of labor movement after the shock, Galí finds that technology shocks play only a minor role in the explanation of the cyclical movement in hours worked. Hence, technology shocks are not able to explain procyclical hours as found in the data. Regardless of the time series properties of labor, this result is supported by almost all authors contributing to the debate. There exist certain attempts to account for this failure, such as the approach by Wen (2002)[4] or Fisher (2002). The latter author allows for two different types of technology shocks: changes in neutral technology affect the production of consumption and investment goods homogeneously, while investment-specific technology affects investment goods only. Fisher shows that investment-specific technology is responsible for most of the variation in hours worked, while neutral technology is relatively unimportant. Assuming stationarity of hours, he reports a rise of labor after innovations in both technology shocks. In order to identify both technology shocks, Fisher remains in the structural vector autoregression framework used by Galí, but introduces alternative identifying assumptions. That is, he assumes investment-specific technology to be the sole influence of the investment price in the long run and he restricts the long-

[3]Francis and Ramey (2002), p. 1.

[4]Wen (2002) proposes to turn away from technology shocks altogether and explain all fluctuations using demand shocks only.

run influence on productivity to be exclusively driven by both kinds of technology shocks. This will hereafter be called the Fisher identification.

Another contribution to this debate is the work by Uhlig (2003a). Within the framework of identifying shocks as proposed by Galí (1999), Uhlig re-investigates the theoretical foundations of the identifying restrictions given by a particular model by using forecast revision variances. In his model he allows for labor hoarding and additional sources of variation in labor productivity. According to his results, there exist more possibilities for an identification strategy than merely the long run. He shows that for his model, medium-run identification works best in identifying technology shocks. In order to apply medium-run identification, Uhlig states that using the Cholesky decomposition of the forecast revision variance is equivalent to the method of Blanchard and Quah (1989) in the long run, but can also be applied for shorter horizons. I will derive and further discuss this procedure in Section 2. Assuming stationary hours, Uhlig's results stand in line with Christiano, Eichenbaum and Vigfusson (2003a) as stated above.

In this work, I apply Uhlig's method in the framework of the Fisher model. I find this approach especially suitable, since it tries to address both of Galí's findings and allows for more than one shock influencing productivity by introducing investment-specific technology. I also consider an extension to the Fisher model by allowing for labor hoarding in line with Uhlig's (2003a) approach. I will solve both versions of the model and compare the resulting impulse responses to Fisher's empirical findings. Investigating the possibilities of short-, medium- and long-run identification, I recheck the theoretical foundation of Fisher's identifying assumptions using forecast revision variances derived from the model. Then, I apply my resulting identification procedure to simulated data from the model in order to further base my identification strategy. As a consequence, I investigate Fisher's results in two ways. First, I check whether his empirically applied identifying assumptions are theoretically backed by his model. Second, I examine whether the empirical results do in fact support the model proposed. Last, I apply my identification strategy to real data and carry out my own empirical analysis.

Several results can be reported. First, Fisher's estimated impulse responses match the ones obtained from all versions of the model. However, the model understates the importance of the investment-specific shock on labor. Second, the investigation of the model-based forecast revision variance decomposition delivers that the Fisher identification is appropriate in the original Fisher model as well as in the labor hoarding model with additional shocks. These shocks may not have a permanent effect on productivity for this identification to hold. Moreover, the Fisher identification does not only hold in the long run, but also in the short- and medium-run horizon. In addition, I develop an alternative strategy for the identification of the two technology shocks. This procedure restricts neutral technology to be the only influence on labor productivity, while both technology shocks only explain hours worked. I show that this identification strategy should be valid in short- to medium-run horizons.

Last, I apply both identification schemes to real data. Here, the Fisher identification delivers unsatisfactory results that are probably due to the measure of the investment price included in the specification. I conclude that this specification is very sensible to the price data employed. However, my alternative identification delivers productivity results that are similar to the obtained by Fisher. This means that this procedure in fact identifies the shocks as the Fisher identification, but without the need of investment price data. Furthermore, medium-run identification can be applied to identify the two technology shocks. The response of labor after a shock in investment-specific technology increases strongly, while labor drops on impact after an innovation in neutral technology, then increases above zero and converges to a slightly negative value in the long run. This implies labor-saving neutral technology and contradicts RBC. When considering technology to consist of both neutral and investment-specific components, however, one can conclude that labor rises in response to positive technology shocks. This defends technology-driven business cycles.

The remainder of this work is organized as follows. Part two explains the empirical strategy of identification in all horizons based on Blanchard and Quah (1989) and extended by Uhlig (2003a). Part three introduces the Fisher model and its solution and provides impulse responses; it furthermore contains an extended version of this model allowing for labor hoarding. Part four derives the identifying assumptions based on both versions of the model using variance decompositions and applies them to simulated data. Part five contains the identification of technology shocks in real data. Part six concludes.

2 Structural VARs and Identification

2.1 How to Determine Technology Shocks?

Real-Business-Cycle models deal with the effects of technology and non-technology shocks to aggregate macroeconomic variables. As Galí (1999) has criticized, these models have for a long time been checked for their empirical performance based on the overall cyclical behavior of time series that are simulated by the models. The impact and propagation of the single shocks in the model, however, have not always been subject to empirical testing. Most importantly, the influence of technology shocks, the main driving force in RBC models, on macroeconomic variables is of interest in Galí's work. Since it is impossible to observe shocks to the economic system directly, one needs to employ some method that disentangles the effects of single shocks on real variables from other disturbances. More precisely, one needs to identify the shocks of interest using appropriate assumptions from economic theory. This means that economic theory helps us to get the dynamic behavior of the economic system into order and to be able to investigate the effects of a few shocks exclusively.[5]

Several strategies to identify technology shocks can be found in the literature. However, there are some common features that should be shared by all of these measures. First, the shocks that hit the model variables over time should be exogenous to these variables. This may cause problems when using direct measures of technology as Evans (1992) shows[6]. Second, all shocks that disturb the system should be uncorrelated in order to be analyzed separately. Intuitively, one therefore considers each shock to contain some specific information on economic reality that is disjunct from the information other shocks may carry.[7] The method employed by Galí, Christiano, Eichenbaum and Vigfusson (2003a), Fisher (2002), Uhlig (2003a) and many others is the identification of technology shocks using structural vector autoregression and goes back to the work of Blanchard and Quah (1989) and Shapiro and Watson (1988). This procedure fulfills orthogonality of the shocks by assumption. When using vector autoregression, one estimates the relationships between macroeconomic variables over time. Since the shocks are nothing else than the residuals from this estimation, they can also be considered exogeneous. However, these assumptions are not sufficient for complete identification of the shocks and need to be supplemented by additional restrictions stemming from economic theory. In the framework of Galí, these restrictions concern the effect of the shocks of interest on macroeconomic variables over time. For this, Galí and others use assumptions on the long-run impact of technology shocks, whereas Uhlig shows that medium-run restrictions should be taken into account as well. I will present the identification based on structural vector autoregression for all possible identification horizons in the following.

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[5] See Breitung, Brügemann and Lütkepohl (2003), and Breitung (1998).

[6] However, Christiano, Eichenbaum and Vigfusson (2003b) and Shea (1998) check carefully for exogeneity of their direct technology measures.

[7] See Breitung (1998) for a good description of economic motivation for identification.

2.2 Structural (Identified) VARs

As stated above, the estimation of structural vector autoregressive processes (SVARs) or, equivalently, the identification of vector autoregressive processes (VARs) focusses on the determination of the errors of a system rather than of the autoregressive coefficients as in "standard" VAR estimation. While these standard vector autoregressions, which are often referred to as the "reduced form" of a VAR, have some difficulty to be connected to some particular economic interpretation, structural representations of these models try to relate the estimated parameters to the "deep" parameters of a model, e.g. technology.[8]

One can achieve the structural representation of a VAR using the following procedure.[9] Consider an estimated reduced form of a VAR of order p,

$$X_t = D_1 X_{t-1} + D_2 X_{t-2} + ... + D_p X_{t-p} + v_t,$$

where $X_t = (X_{it}, ..., X_{nt})'$ is an $n \times 1$ vector of n endogenous model variables at date t, $D_1, ..., D_p$ are $n \times n$ coefficient matrices and v_t is the $n \times 1$ reduced-form error vector with $Var(v) = \Omega$. Due to its recursive nature, one can transform this process into the Wold moving-average representation:

$$\begin{aligned} X_t &= D_1 \left[D_1 X_{t-2} + ... + D_p X_{t-p-1} + v_{t-1} \right] \\ &\quad + D_2 X_{t-2} + ... + D_p X_{t-p} + v_t \\ &= v_t + D_1 v_{t-1} + (D_1^2 + D_2) X_{t-2} + (D_1 D_2 + D_3) X_{t-3} + ... \\ &= \Phi_0 v_t + \Phi_1 v_{t-1} + ... \\ &= \sum_{s=0}^{\infty} \Phi_s v_{t-s}, \end{aligned}$$

where $\Phi_0 = I_n$, $\Phi_s = \sum_{j=1}^{s} \Phi_{s-j} D_j$ and D_j is an $n \times n$ zero matrix for $j > p$. Note "that the coefficients of this representation may be interpreted as reflecting the responses to impulses hitting the system"[10]. We now want to relate this reduced form to the structural form in which the residuals are assumed to be orthogonal. This means that their variance-covariance matrix (VCV) is diagonal. Blanchard and Quah (1989) additionally assume the VCV matrix to be the identity matrix. For this purpose, consider a decomposition of the VCV matrix of the estimated reduced form residuals such that

$$\Omega = AA',$$

where A is of dimension $n \times n$. This matrix can then be used to transform the estimated residuals into the structural form residuals, since

$$e_t = A^{-1} v_t, \; E[e_t e_t'] = I.$$

[8] See Breitung, Brüggemann and Lütkepohl (2003), Sarte (1997).

[9] The following procedure is drawn from Blanchard and Quah (1989) as well as Breitung, Brüggemann and Lütkepohl (2003). VAR representations, moving average representations and impulse responses are in line with Lütkepohl (1993), Hamilton (1994) and Breitung and Lütkepohl (1997).

[10] Breitung, Brüggemann und Lütkepohl (2003), p. 136

The structural form of the process can then be expressed as

$$\begin{aligned} X_t &= \Phi_0 A A^{-1} v_t + \Phi_1 A A^{-1} v_{t-1} + \Phi_2 A A^{-1} v_{t-2} + ... \\ &= \Phi_0 A e_t + \Phi_1 A e_{t-1} + \Phi_2 A e_{t-2} + ... \\ &= \sum_{s=0}^{\infty} \Phi_s A e_{t-s}. \end{aligned}$$

Now, the coefficients of the structural representation may be interpreted as the responses to impulses from the structural disturbances. However, the matrix A has to be fully specified in order to achieve this structural form of the model; that is, in a system with n model variables, all n^2 components of A have to be determined. Considering that the covariance matrix Ω is symmetric, the relationship $\Omega = AA'$ delivers $n(n+1)/2$ relationships between the unknown elements of A and the known elements of Ω[11]. Hence, we need at least $n^2 - n(n+1)/2 = n(n-1)/2$ additional restrictions to be able to solve for A. Stemming from economic theory, different possibilities of delivering the required additional restrictions will be discussed in the following sections.

2.3 Decomposition of Forecast Revision Variance

First, I introduce the decomposition of the forecast revision for vector autoregressive processes, since it is of key importance for the identification procedure outlined in later sections.[12] The forecast variance decomposition separates the variation in an endogenous variable into the contribution of each shock contained in the VAR. Thus, it provides information on the relative importance of each random innovation in affecting the variables in the process. For this, assume the following form of a VAR of order p,

$$X_t = D_1 X_{t-1} + ... + D_p X_{t-p} + A\epsilon_t,$$

where ϵ_t are the structural residuals with $E\left[\epsilon_t \epsilon_t'\right] = I$. Here, the VAR has been estimated using levels of the endogenous variables. It is assumed that the form of the matrix A has been determined already. One can then express the k-step ahead forecast revision by

$$\begin{aligned} \epsilon_{t,k} &= E_t\left[X_{t+k}\right] - E_{t-1}\left[X_{t+k}\right] \\ &= D_1 E_t[X_{t+k-1}] + ... + D_p E_t[X_{t+k-p}] \\ &\quad - D_1 E_{t-1}[X_{t+k-1}] - ... - D_p E_{t-1}[X_{t+k-p}] \\ &= \Phi_k A \epsilon_t, \end{aligned}$$

where the Φ_i are determined as in the previous section. Intuitively, the forecast revision measures the mistake that remains when we revise our k-step ahead forecast today compared to yesterday. The corresponding variance-covariance matrix is then equal to

$$\Sigma_k = \Phi_k A A' \Phi_k'.$$

[11]Since $n + (n-1) + ... + 1 = \sum_{i=1}^{n} i = \frac{1}{2} n(n+1)$.

[12]This has been proposed by Uhlig (2003a), p. 6. We will use an equivalent notation to his.

Note that in the case of a first-order structural VAR, $\Phi_k = B^k$, where B^k is the coefficient matrix of the first lag in the specification.[13]

If, in turn, the model variables are first-difference stationary and the VAR is accordingly estimated in differences, the forecast revision for the *levels* of these variables can be derived as follows: Consider

$$\widetilde{X}_t = D_1\widetilde{X}_{t-1} + D_2\widetilde{X}_{t-2} + ... + D_p\widetilde{X}_{t-p} + Ae_t,$$

where $E\left[e_t'e_t\right] = I$ and

$$\widetilde{X}_t = \Delta X_t = X_t - X_{t-1}.$$

We can now express the relationship between some future level X_{t+k} and the future difference variables as

$$\begin{aligned} X_{t+k} &= X_{t-1} + X_t - X_{t-1} + X_{t+1} - X_t \\ &\quad +... + X_{t+k} - X_{t+k-1} \\ &= X_{t-1} + \widetilde{X}_t + ... + \widetilde{X}_{t+k} \\ &= X_{t-1} + \sum_{i=0}^{k} \widetilde{X}_{t+i}. \end{aligned}$$

The k-step ahead forecast revision for the level of some future variable can accordingly be written as

$$\begin{aligned} e_{t,k} &= E_t\left[X_{t+k}\right] - E_{t-1}\left[X_{t+k}\right] \\ &= E_t\left[X_{t-1} + \sum_{i=0}^{k}\widetilde{X}_{t+i}\right] - E_{t-1}\left[X_{t-1} + \sum_{i=0}^{k}\widetilde{X}_{t+i}\right] \\ &= E_t\left[\sum_{i=0}^{k}\widetilde{X}_{t+i}\right] - E_{t-1}\left[\sum_{i=0}^{k}\widetilde{X}_{t+i}\right] \\ &= \sum_{i=0}^{k} E_t[\widetilde{X}_{t+i}] - \sum_{i=0}^{k} E_{t-1}[\widetilde{X}_{t+i}] \\ &= \Phi_k Ae_t + \Phi_{k-1}Ae_t + ... + \Phi_0 Ae_t \\ &= (\sum_{i=0}^{k}\Phi_i)Ae_t. \end{aligned}$$

The respective VCV matrix is then equal to

$$\Sigma_k' = (\sum_{i=0}^{k}\Phi_i)AA'(\sum_{i=0}^{k}\Phi_i)'.$$

Both of these variance-covariance matrices can now be decomposed into the contributions of each shock $j = 1, ..., k$.[14] That is

$$\Sigma_k = \sum_{j=1}^{k}\left(\Sigma_{k,j}\right), \text{ where } \Sigma_{k,j} = \Phi_k AE_{jj}A'(\Phi_k')$$

[13] Uhlig (2003a) considers only the lag order one in his specification. I will, however, use more lags in some of my estimations later.

[14] This is in line with Uhlig (2003a) as well as Lütkepohl (1993) and Breitung, Brüggemann and Lütkepohl (2003).

in the case of the level VAR. Here, E_{jj} is the zero matrix, with only the j-th element on the diagonal replaced by 1. This can be used to determine the share of shock j in the total variance of variable i, k periods ahead:

$$\phi_{i,j,k} = \frac{(\Sigma_{k,j})_{ii}}{(\Sigma_k)_{ii}}.$$ [15]

This is equivalently true for the forecast revision variance for differenced variables.

2.4 Long-Run Identification

This section outlines long-run identification as applied by Galí (1999), Christiano, Eichenbaum and Vigfusson (2003a), Fisher (2002) as well as Uhlig (2003a) and various others. According to Blanchard and Quah (1989), the additional restrictions needed to determine the matrix A stem from the long-run influence of the shocks of the model on the variables of the model.

Using a classical econometric approach, the time series properties of the variables in the VAR are of interest and, as will be shown, crucial for the exact specification of the restrictions on the long-run influence of shocks on the model variables. In this context, it has to be noted that these restrictions induce one or a few shocks of the model to be the sole influence on the *level* of one of the variables in the long run, even if the VAR may be estimated in differences. We will outline the possible long-run restrictions for different time series properties in the following two cases.

Case 1: All variables in the VAR are stationary

The model can regularly be estimated in levels, i.e.

$$X_t = D_1 X_{t-1} + D_2 X_{t-2} + ...D_p X_{t-p} + v_t.$$

As noted above, the impulse responses are derived from the coefficient matrices of the shocks in the moving-average representation. The responses of the model variables in period k after shocks in period zero can then be expressed as

$$X_{t+k} = \Phi_k v_t.$$

More precisely, the impulse responses of the model variables to one particular shock are given by the respective column of the response matrix as shown here. After orthogonalizing the residuals and considering a long-run (infinite) horizon impulse response, one can write

$$C \equiv \lim_{i \to \infty} \Phi_i A.$$

Setting $n(n-1)/2$ elements (i,j) of this matrix C equal to zero does now imply that the respective shock j has no influence on the level of variable i in the long run. Consequently, these conditions deliver the additional restrictions needed in order to calculate all elements of the matrix A. The level specification is employed by Uhlig (2003a) who uses Bayesian inference instead of the classical approach.

Case 2: All/some variables in the VAR have a unit root and there is no cointegration[16]

Estimate a stationary model

$$\widetilde{X}_t = D_1\widetilde{X}_{t-1} + D_2\widetilde{X}_{t-2} + ... + D_p\widetilde{X}_{t-p} + v_t,$$

using $\widetilde{X}_t = \Delta X_t$. The impulse response for this first-differenced variables can then be expressed as

$$\widetilde{X}_{t+k} = \Phi_k v_t.$$

Considering the respective level variables yields

$$\begin{aligned} X_{t+k} &= X_{t-1} + \sum_{i=0}^{k} \widetilde{X}_{t+i} \\ &= X_{t-1} + \sum_{i=0}^{k} \Phi_i v_t, \end{aligned}$$

where the first part of the sum is known and hence the second part represents the impulse response function to the shocks of the system. Again, after orthogonalizing and considering the infinite horizon, one can express the long-run impulse responses as

$$C \equiv \sum_{i=0}^{\infty} \Phi_i A.$$

The matrix C is now subject to imposing long-run restrictions just as in the level variable case.

If one estimated a mixed system of integrated variables in first differences and stationary variables in levels, the impulse responses of the levels of the single variables in the VAR are calculated according to cases one and two as well. It will be shown later that not all of the zero-restrictions imposed on the matrix C need an economic interpretation, but that the focus of this procedure is on the identification of one or two shocks only. Typically, one restricts one or two shocks to be the only influence on a particular variable. It has to be made sure that the impulse responses of the model variables due to the shocks of interest are uniquely determined. That is, they need to be determined regardless of the additional restrictions imposed on the matrix C in order to achieve a structural form of the process. Therefore, when imposing long-run restrictions, the time series properties of the variables that are subject to the restrictions determine the nature of the matrix C as shown above. Restricting the long run influence of labor productivity to be caused by technology shocks only, this variable is employed in first differences in the specification of Blanchard and Quah (1989) as well as Galí (1999) and most of the other authors contributing to this debate.

Note that for notational convenience, there are no constants or other deterministic terms in the specification of the VAR. The reason for this is that deterministic

[16] The consideration of cointegration is left to different research.

terms are not affected by impulses on the system. When estimating, one may adjust the model for deterministic terms; for example, a constant is usually included in the specification. These are however, not considered when calculating the impulse response functions.[17]

2.5 Medium-Run and Short-Run Identification

Medium-run identification as proposed by Uhlig (2003a) differs from long-run identification only in the considered horizon $0 < k < \infty$. Thus, the derivations of the last section also hold for medium-run identification through just replacing ∞ with k. The matrix C in the case of unit roots is then

$$C \equiv \sum_{i=0}^{k} \Phi_i A$$

and, equivalently, when dealing with stationary variables, one uses

$$C = \Phi_k A$$

in order to impose restrictions. As Uhlig points out, medium-run identification is not concerned with the development of an alternative method of identification, but rather with the identification of technology shocks in a shorter horizon than the very long run. Thus, the economic interpretation of technology shocks that motivated long-run identification is altered to allow these shocks to be the only influence of labor productivity in an intermediate horizon.

In addition, there exists the possibility of short-run identification. Imposing short-run restrictions concerns the influence of the shocks of the system on certain variables on impact. As can easily be seen, the impulse response to the contemporaneous shock has to be considered in this framework. Thus, identification imposes zero restrictions on the components of the matrix A itself. This is in fact equivalent to the identification in the longer horizons where k is simply set to zero. Hence, the procedures described above hold in this case as well. Note, though, that the investigation of time series properties is not important for achieving the restriction matrix C in this case, since C is equal to A in all of the cases described above. Obviously, the differences between medium- and short-run identification only depend upon the definition of a short or medium horizon. Clearly, setting $k = 2$ would also be a very short horizon. In this context, however, short run restriction is bound to $k = 0$ only, while, according to Uhlig (2003a), a medium-run horizon considers the range of three to ten years which is more or less equivalent to business cycle frequency.

2.6 Decomposing the Forecast Revision Variance with Cholesky

For the specification of the matrix A, I follow the method suggested by Uhlig (2003a). Uhlig states that imposing restrictions according to Blanchard and Quah (1989)

[17] See Breitung, Brüggemann and Lütkepohl (2003), p. 134.

is equivalent to decomposing the forecast revision variance for the identification horizon. In this section, I will show that this is indeed the case.

First, consider the case of unit roots in the variables, that is $C \equiv \sum_{i=0}^{k} \Phi_i A$, where $k = \{0, k, \infty\}$ for he short, medium or long run. When multiplying C with its transpose,

$$CC' = (\sum_{i=0}^{k} \Phi_i) AA' (\sum_{i=0}^{k} \Phi_i)',$$

one can see that this is equivalent to the k-step ahead forecast revision variance Σ_k' as derived in Section 2.3. After estimating the reduced form model, one may therefore calculate Σ_k' and apply the Cholesky decomposition to this variance in order to obtain some lower triangular $n \times n$ matrix Q such that

$$QQ' \equiv \Sigma_k' = (\sum_{i=0}^{k} \Phi_i) \Omega (\sum_{i=0}^{k} \Phi_i)'.$$

Note thate the forecast revision variance needs to be a square, positive definite matrix for the Cholesky decomposition[18]. As can easily be seen,

$$Q = (\sum_{i=0}^{k} \Phi_i) A$$

has to hold, and the matrix A can then be obtained through

$$A = (\sum_{i=0}^{k} \Phi_i)^{-1} Q.$$

Thus, matrix Q is equivalent to the restriction matrix C which is described in the previous section. In this matrix, the zero-restrictions are ordered in such a way that C is lower triangular. The lower triangular shape in fact implies that there exist $n(n-1)/2$ zero restrictions in a $n \times n$ system. However, for this specification to be reasonable, the vector of endogenous variables in the VAR must be conveniently ordered. It has to be noted that this may not always be feasible in such a way that all zero restrictions reflect a sensible economic interpretation. This problem will be addressed in the next section.

Alternatively, the model may be estimated in levels, where $C = \Phi_k A$. In analogy to what has been derived above,

$$CC' = \Phi_k AA' \Phi_k'$$

is equivalent to the k-step ahead forecast revision variance Σ_k. Consequently, applying the Cholesky decomposition to this variance delivers the following solution for the matrix A:

$$Q^* Q^{*\prime} \equiv \Sigma_k = \Phi_k \Omega \Phi_k'$$

[18]Lütkepohl (1993), p. 461.

$$\iff Q^* = \Phi_k A$$
$$\iff A = (\Phi_k)^{-1} Q^*.$$

Again, the Cholesky decomposition is equivalent to arranging the zero-restrictions in C in a lower triangular shape.

Note that there exist different approaches for applying long-run restrictions. Using Cholesky as described above is quite close to the procedure outlined by Blanchard and Quah (1989) that is followed by Galí (1999). Blanchard and Quah decompose the variance-covariance matrix of the reduced from residuals, and not the forecast revision variance, into a lower triangular matrix using Cholesky. The matrix that corresponds to the matrix A in my specification is then equal to the orthonormal transformation of this lower triangular factor that satisfies the long-run restrictions imposed.[19] In contrast, another large body of the literature uses the instrumental-variable approach introduced by Shapiro and Watson (1988)[20]. In this procedure, each equation of the VAR is estimated separately, while possible interdependencies between the shocks and variables are accounted for by appropriate instruments. Long-run restrictions are applied by using double-differences of the variables that are not affected by the particular shock in the long run. The shocks are then equivalent to the residuals of the single equations. A yet different approach by Breitung, Brüggemann and Lütkepohl (2003) numerically minimizes a nonlinear problem consisting of all restrictions imposed on the system[21].

All methods stated above have been developed in order to apply long-run restrictions to a VAR. When taking medium-run identification into account, however, decomposing the forecast revision variance with Cholesky works especially well, since this procedure is easily applicable to any identifying horizon.

2.7 Uniqueness of A

Recall that fully specifying the $n \times n$ matrix A is nothing else than solving a system of n^2 equations. Even though enough equations describing the components of A have been found and, hence, this matrix should be just identified, it has to be noted that these equations contain nonlinear relationships between the variables. Namely, the components of A can be determined uniquely in their absolute values, but some of them appear in squares in the system of equations and hence potentially produce two possible solutions. As a consequence, the matrix A yielded by the Cholesky decomposition method represents only one possible solution to the system of equations. Nevertheless, considering more information, that is economic intuition, on the effect of certain shocks on variables in the system, I will show that Cholesky delivers appropriate impulse responses for the shocks of interest.

Let me elaborate this in greater detail. Note that any matrix $\widetilde{Q}$ that is a trans-

[19] See Blanchard and Quah (1989), p. 657.

[20] Among these are Francis and Ramey (2003), Christiano, Eichenbaum and Vigfusson (2003a) and Fisher (2003).

[21] Breitung, Brüggemann and Lütkepohl (2003), p. 139.

formation of the lower triangular Cholesky factor Q (or Q^* equivalently) such that

$$\widetilde{Q} = QT,$$
$$\text{with } T = diag(i_j) \text{ and } i_j \in \{1, -1\},\ j = 1, ..., n,$$

satisfies the restrictions imposed on the system[22] as shown above as well. For an $n \times n$ matrix Q, in a system of n variables, there then exist 2^n possible solutions of Q and hence of A as well. In order to diminish this quantity of solutions, one has to remind oneself that the interest of this investigation lies on the effect of one, at most two, shocks only. Hence, these are the shocks that give a reasonable economic meaning to some of the zero restrictions. To summarize, the aim of the specification described in the previous section is not necessarily to identify all possible shocks that hit the system, but rather to determine restrictive influence of technology or some other shock solely. For this, one needs to assure that the shocks of interest and their respective impulse responses are determined uniquely by the method applied.

Deciding about the sign of the first element (or the first two elements) on the diagonal of the matrix T uniquely determines the first of Q (or first and second row, respectively). Knowing that $A = (\sum_{i=0}^{k} \Phi_i)^{-1} Q$, one can easily see that the first (first and second) column of A are then uniquely specified as well. Obviously, this is true for the use of the level specification, too. As a consequence, for all remaining possible solutions of A, the impulse responses of the model variables due to the first (the first and second) shock are fully determined.

How to decide about the sign of the first elements in T? Consider two possible identification strategies. Identification according to Galí restricts the appearance of a shock in neutral technology to be the only influence on labor productivity. In turn, Fisher (2002) proposes that both neutral and investment specific technology may drive labor productivity in a certain time horizon, while investment-specific technology shocks are bound to be the only driving force of the price in investment. In the first case, ordering labor productivity first among the variables in the VAR and using the Cholesky procedure determines the first shock in the system to be the technology shock. In the second case, ordering the investment price first, followed by labor productivity, defines the first shock to be the shock in investment-specific technology and the second one to be the neutral technology shock. In both cases, the response of the first variable in the VAR, that is labor productivity or the investment-price respectively, due to the first shock is positive in the identifying horizon if the upper-left element of Q, denoted as q_{11}, is positive. Considering the algorithm of the Cholesky decomposition, this approach indeed delivers a positive q_{11}. In addition, all elements on the diagonal of Q will also be positive.[23] Therefore,

[22]That is, $AA' = \Omega$ plus the additional zero restrictions.

[23]The algorithm for the Cholesky decomposition is as follows:

$$\begin{aligned} q_{ii} &= \sqrt{(\sigma_{ii} - \sum_{k=1}^{i-1} q_{ik}^2)}, \text{ for } i = 1, ..., n; \\ q_{ji} &= (\sigma_{ji} - \sum_{k=1}^{i-1} q_{jk} q_{ik})/q_{ii}, \text{ for } j = i+1, ..., n, \end{aligned}$$

Cholesky not only restricts the influence of some shocks to be the only influence on certain model variables, but also determines the sign of some of the responses to these shocks to be positive. In the case of the Fisher identification one therefore identifies a negative shock in investment-specific technology, since positive investment-specific technology shocks are supposed to have a negative impact on the price of investment. In order to avoid this, one might use the inverted investment price rather than the regular series in the specification. Another possibility of dealing with this feature of the algorithm of the Cholesky decomposition is to transform the matrix Q using T as described above.[24] In any case, some economic intuition or assumption on the direction of the variable responses must be applied in order to uniquely define the relevant columns in A.

Mostly, the remaining shocks and their respective impulse responses are not in the focus of the investigation and will be considered as the "residual" disturbances independent from the shocks identified. Since potentially there do not exist any reasonable restrictions of the Blanchard–Quah type in order to identify the remaining shocks, other methods have to be employed in order investigate their dynamic impacts on the system. Altig, Christiano, Eichenbaum and Linde (2002), for example, apply an instrumental-variable estimation for the relevant shock equations only and then identify the rest of the shocks in the system following Uhlig (2003b). Here, by a principal component analysis, those disturbances are identified that the are most important sources of fluctuations. Within this work, however, the remaining shocks will be neglected.

I tested my version of the Cholesky decomposition method as derived above using the original data of Francis and Ramey (2003)[25]. The results of this examination are practically identical to the ones found by Francis and Ramey and can be viewed in Appendix C.

2.8 A Note on the Theoretical Foundation

How to decide about the application of short-, medium- or long-run identification? Since the identifying restrictions derived above serve as minimal assumptions for the determination of technology shocks, they cover a large class of models. In particular, long-run restrictions address the low frequency component of the shock. In order to be influential in the long run, the respective shocks need to be permanent, that is, they have a unit root. Hence, when considering the effects of technology, long-run identification implies the investigation of neoclassical growth models.[26] Fisher

where σ_{ii} denote the elements of the covariance matrix decomposed. This matrix needs to be symmetric and positive definite for the expression under the square root to be positive as well (Source: http://mathworld.wolfram.com).

[24] When transforming, note that all responses for the respective shock change their sign. In the application of the identification procedure, this transformation will rarely be used. One reason is that only one shock is identified mostly. The Fisher identification as well does not need to be transformed if the inverted investment price is used. In general, however, there may be cases, where a transformation is appropriate.

[25] This data was provided to me by Harald Uhlig.

[26] Uhlig (2003a) notes that these assumptions do not work in the framework of many other models such as endogenous growth models in which all shocks may affect labor productivity in the long

(2002) states that even though most RBC models consider persistent, but transitory technology shocks and even though both in reality and theory transitory components of technology cannot be neglected, the focus on permanent technology shocks can be seen as "presenting a lower bound on the contribution to business cycles of technology shocks more generally conceived"[27].

In contrast, Uhlig (2003a) has tested a model with transitory technology shocks. Moreover, he allows for further possible sources of long-run influence on labor productivity than only technology. For this, Uhlig has found that medium-run identification is to be preferred over long-run identification. Summing up, Uhlig shows that augmenting the standard models by labor hoarding in order to improve their empirical performance, may invalidate long-run restrictions as the best identification procedure. Hence, it is worth investigating the models of interest thoroughly before deriving the identifying restrictions for the technology shocks. Doing this, the possibilities of short- as well as medium- and long-run identification should be taken into account and their performance be compared.

One may argue that models with transitory technology shocks should be estimated with medium-run identification, while shocks in growth models should be identified in the long run. Unfortunately, it is not that easy. Altig, Christiano, Eichenbaum and Linde (2002) mention that "a possibility is that technology shocks have two components. One component has a long run impact on productivity, and the other one has only a transitory impact."[28] Therefore, long-run identification concerns the first component, while medium-run identification concerns the second one. Which strategy will be more effective in identifying the technology is, however, not known in advance. Hence, a model-based derivation of the identifying restrictions is appropriate.

2.9 A Note on VAR Estimation

Regardless of the technique employed, a large part of the debate on estimating the models described above concerns the time series properties of the variables employed, first of all labor. Galí (1999) and Francis and Ramey (2003), among others, use standard Augmented Dickey Fuller (ADF) tests to decide about unit roots in the variables. They conclude that hours should be specified in differences or deviations from a trend in the model. This results in hours responding negatively to a positive shock in technology. In contrast, Christiano, Eichenbaum and Vigfusson (2003a) mention that this should be handled with care. Since "standard tests cannot reject the null hypothesis that per capita hours worked are difference stationary, but standard tests can also not reject the null hypothesis that hours worked are stationary", they claim that "univariate tests are simply not very informative under these circumstances"[29]. Christiano, Eichenbaum and Vigfusson show that both the level and the growth rate (i.e. difference) specification of hours worked encompasses the

run.

[27]Fisher (2003), pp. 3–4.

[28]Altig, Christiano, Eichenbaum and Linde (2002), p. 34.

[29]Christiano, Eichenbaum and Vigfusson (2003a), p. 2.

respective alternative model. However, the level specification is better suited to explain both possibilities. That is, applying the level specification to a first-difference stationary process may cause problems in the asymptotics but does not result in a specification error, whereas differencing hours even though the true data generating process is stationary does result in a specification error.

In addition, Christiano, Eichenbaum and Vigfusson test detrended versions of hours worked as proposed by Galí and Francis and Ramey using similar techniques of relative plausibility of different specification.[30] They find that the level specification is to be preferred over the detrended specification as well. Moreover, the authors propose the use of per-capita hours worked rather than total hours worked, since this measure is closer to the one implied by RBC models. They reason that, both empirically and theoretically, the measure of hours worked should be bounded which is not easy to reconcile with a unit root in hours.

Galí states that while the "identification strategy hinges critically on the presence of a unit root in productivity, it can accommodate both I(0) and I(1) hours"[31]. Hence, long-run restrictions as applied by most of the researchers contributing to this debate are bound to a unit root in the variables under restriction. According to Uhlig (2003a) and as shown above, this does not necessarily have to be the case, however. While the use of first-differenced variables implies the investigation of the accumulated impulse responses over the horizons of interest, the level approach uses the limit of the impulse responses over a particular horizon. Clearly, the level approach imposes a somewhat "weaker" criterion on the long- or medium-run effects of certain shocks, since it concerns the impulse response of one period only. However, due to the autoregressive structure of the system, imposing restrictions on the influence of the shocks in one period will also affect the influence of these shocks in the periods shortly before and after the identifying horizon.

According to this, identification is also possible in systems that are estimated in levels. Sticking to a classical estimation approach, one needs to assure that the estimators of the coefficients of the VAR, or – more precisely – of the forecast variance, are consistent and the restrictions in the medium or long term are imposed on appropriate matrices. If the variables in the VAR are stationary, the estimators of the coefficients are consistent and asymptotically normally distributed[32]. Lütkepohl (2003)[33] states that this "asymptotic distribution ... is also obtained for nonstationary systems with integrated variables. ... In that case it is important to note, however, that the covariance matrix ... is singular, whereas it is nonsingular in the usual I(0) case. In other words, if there are integrated or cointegrated variables, some estimated coefficients or linear combinations of coefficients converge with a faster rate than" in the stationary case. "Therefore, the usual t-, χ^2- and F-tests for inference regarding the VAR parameters may not be valid in this case. ... It is

[30] In particular, they employ posterior odds ratios.

[31] Gali (1999), p. 257. I(0) stands for integrated of order zero, that is stationary, whereas I(1) denotes integrated of order one, that is first-difference stationary.

[32] See Lütkepohl (1993) or Hamilton (1994).

[33] Lütkepohl (2003), p. 80.

perhaps worth noting, however, that even in VAR models with I(1) variables, there are also many cases where no inference problems occur." It has been shown that "if all variables are I(1) or I(0) and if a null hypothesis is considered which does not restrict elements of each of the coefficients of the lagged values the usual tests have their standard asymptotic properties. For example, if the VAR order $p \geq 2$, the t-ratios have their usual asymptotic standard normal distributions."[34]

More important than significance tests is the performance of the impulse responses, however. If all variables are stationary, the coefficients of the impulse responses ϕ_i, that are nonlinear functions of the VAR coefficients, converge to zero. Hence the effect of an impulse on the variables of the VAR, which may be in differences, is transitory. In this case, the impulse response coefficients $\Phi_{ij,h}$ are asymptotically normally distributed according to

$$\sqrt{T}(\widehat{\Phi}_{ij,h} - \Phi_{ij,h}) \xrightarrow{d} N(0, \sigma^2_{ij,h}),$$

where i, j denote the variables and shocks, respectively, h is the lag and $\sigma^2_{ij,h} = \frac{\partial \Phi_{ij,h}}{\partial \alpha'} \Sigma_{\widehat{\alpha}} \frac{\partial \Phi_{ij,h}}{\partial \alpha}$. Here, α denotes the vector of VAR coefficients.[35] Since finding an expression for the asymptotic variance of the coefficients of the impulse responses is complicated, bootstrap methods are widely used in order to calculate confidence intervals for these impulse responses.[36]

If some variables in the VAR are I(1), the ϕ_i do no longer converge to zero. Since the forecast error is still unbiased in the unstable case according to Lütkepohl (1993)[37], the tools for generating impulse responses and forecast variance decompositions all remain valid. Furthemore, the forecast variance is unbounded and the forecast uncertainty may be very large. However, the covariance matrix of the VAR coefficients $\Sigma_{\widehat{\alpha}}$ may be singular, as noted above, and the limiting normal distribution may not hold any more.

An alternative approach to deal with possibly non-stationary variables in a level estimation is the use of Bayesian inference. This has, for instance, been applied by Uhlig (2003a) and Smets and Wouters (2003). Here, estimation is possible "without first having to take a stand on whether the data is integrated or trend stationary"[38]. Even though Bayesian inference delivers a more sophisticated way of dealing with a level estimation, I will stick to the classical approach for two reasons. First, long-run restrictions as proposed by Galí use a classical approach, and equivalent identification strategies formulated in levels of the variables are then straightforward extensions of this specification. Second, the qualitative results generated by a classical approach remain to be valid in terms of the impulse responses. This is sufficient to determine the direction of labor after a technology shock.

[34]This has been shown by Toda and Yamamoto (1995) and Dolado and Lütkepohl (1996).

[35]See Breitung, Brüggemann and Lütkepohl (2003) as well as Lütkepohl (1993).

[36]For this, see Kilian (1998) and Sims and Zha (1999). My bootstrap procedure is presented in Appendix F.

[37]Lütkepohl (1993), Chapter 11.

[38]Uhlig (1994), p. 645.

3 The Fisher Model

3.1 The Original Fisher Model ("Version 1")

In the following section, I will investigate the model proposed by Jonas Fisher (2002) that delivers the foundation for the later identification of technology shocks. Fisher has added a new viewpoint to the debate on the influence of technology shocks on labor and labor productivity: He emphasizes that, regardless of the sign of the reaction of labor due to a positive innovation in technology, all existing work has shown that technology shocks account for a small fraction of business cycle variation only. In line with Greenwood, Hercowitz and Krusell (1997), Fisher states that "investment-specific, not neutral change is the major source of economic growth"[39]. Here, neutral technological change is assumed to affect investment and consumption goods homogeneously, whereas investment-specific technological progress influences investment goods only. Thus, the dynamics stemming from the rise of new and more efficient capital goods should be disentangled from traditional neutral technological progress, as considered by Galí (1999) and others. This additional source of technology shocks to business cycle movements of macroeconomic variables has not been taken into consideration so far. Evidence for this "additional" productivity in investment goods has been found in US data by Greenwood, Hercowitz and Krusell who state that, in all data frequencies, the real price of equipment has fallen while the stock of equipment has risen considerably over time. Thus, significant technological change in the production of new equipment must have caused this to become less expensive[40].

For his identification strategy, Fisher assumes investment-specific technology to be the only influence on the investment price in the long run, while both neutral and investment-specific technology are considered to be the only variables that have a long-run impact on labor productivity. Fisher finds that investment-specific technological change accounts for 48% in business cycle variation of hours worked, while the importance of neutral technology is rather small (6%).[41]

The theoretical basis of Fisher's analysis consists of the following social-planner problem:

$$\max_{\{C_t,H_t\}_t^\infty} E\sum_{t=0}^{\infty}\beta^t U(C_t,H_t)$$

subject to, for all t,

$$C_t + X_t \leq A_t K_t^\alpha H_t^{1-\alpha},$$

$$K_{t+1} \leq (1-\delta)K_t + V_t X_t,\ K_0 \text{ given},$$

[39] Fisher (2003), p. 1.

[40] Greenwood, Hercowitz and Krusell (1997), p. 342–344.

[41] Note that a very similar empirical approach to Fisher is applied by Michelacci and Lopez-Salido (2003). However, these authors test a labor market search model. This shows that the identifying assumptions employed by Fisher may be useful in various theoretical models.

and

$$
\begin{aligned}
A_t &= \exp(\gamma + C_a(L)\varepsilon_{at})A_{t-1}, \\
V_t &= \exp(\nu + C_v(L)\varepsilon_{vt})V_{t-1}, \\
[\varepsilon_{at}, \varepsilon_{vt}]' &\sim N(0, D), \; D \text{ diagonal.}
\end{aligned}
$$

Here, the representative agent maximizes the infinite sum of expected utility U, where β is the discount factor and C_t, H_t, X_t and K_t stand for consumption, hours worked, investment and capital, respectively. The first constraint represents the budget constraint implying that, in each period, expenditures should not exceed production. The second one measures the accumulation of capital over time. A_t measures the level of neutral technology and V_t the level of investment-specific technology. $C_a(L)$ and $C_v(L)$ are square summable polynomials in the lag operator; α, δ, γ and ν are parameters with $\alpha \in (0,1), \delta \in (0,1), \gamma, \nu \geq 0$.

Neutral technology affects the production function in this economy and, therefore, influences all consumption and investment goods homogeneously. In turn, investment-specific technology augments investment in the capital accumulation equation and can thus be related to the price of investment. In addition, both of these technology shocks are permanent, in contrast to the specification of many RBC models where only persistent but transitory shocks are considered. This means that the neutral technology shocks are of the kind of shocks considered by Galí. According to Fisher, the unit root in the investment-specific technology is in line with data on asset prices exhibiting unit roots.

In the subsequent sections, I will first amend missing specifications to the model in order to be able to solve and implement it into the Toolkit presented in Uhlig (1999)[42]. Then, some special characteristics of the model as well as the necessary steps for implementation into the Toolkit will be discussed. Finally, plots and impulse responses for the model will be provided. In a second part, I will augment the model by allowing for labor hoarding which introduces some kind of sluggishness to the specification. This second version of the model will be solved and impulse responses will be evaluated.

It has to be noted that Fisher does not provide an analytical solution of his model at all, but rather uses it as a rough idea for his econometric specification. Therefore, the resulting impulse responses cannot be compared to an equivalent derived by Fisher, but only to the results of his empirical investigation. This way, I will check whether Fisher's empirical results actually support the model he proposes.

Uhlig (2003a) proposes to use the solution of the model, which consists of the recursive law of motion, to calculate the forecast revision variance decomposition of the model shocks on the variables of the model. Following this proposition, I will use the information gained by these calculations to determine a feasible strategy for the identification of the shocks.

[42]For the solution, I need to express the model by a system of log-linearized equations. These are then solved by the "Toolkit routines", programmed in MATLAB, that are available on Harald Uhlig's website.

3.1.1 Specification

Having added some specifications to the general version of the Fisher model as stated in the previous section, we face the following social-planner problem:

$$\max_{\{c_t,n_t\}_t^\infty} E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\left[c_t^{1-\theta}(1-n_t)^\theta\right]^{1-\eta} - 1}{1-\eta} \right)$$

subject to, for all t,

$$c_t + x_t = a_t k_{t-1}^\alpha n_t^{1-\alpha}$$

$$k_t = (1-\delta)k_{t-1} + v_t x_t,\ k_0 \text{ given},$$

and

$$\begin{aligned} a_t &= \exp(\gamma + \varepsilon_{at})a_{t-1},\ \gamma \geq 0, \\ v_t &= \exp(\nu + \varepsilon_{vt})v_{t-1},\ \nu \geq 0, \\ [\varepsilon_{at}, \varepsilon_{vt}]' &\sim N(0, D),\ D \text{ diagonal}. \end{aligned}$$

As can be seen, the utility function is assumed to take a quite general form that allows for adjustment of relative risk aversion $\eta > 0$ or the leisure share $0 < \theta < 1$. Note that I assume that labor input is bounded by time endowment $(= 1)$. While this is widely used in RBC models, it is also supported by Fisher who employes per-capita hours in his specification. In addition, I chose the simplest lag polynomial possible:

$$C_a(L) = C_v(L) = \alpha_1 + \alpha_2 L + \alpha_3 L^2 + ...,\ \text{where } \alpha_1 = 1, \alpha_j = 0 \text{ for } j > 1.$$

In contrast to the notation of Fisher, I use lower-case letters. In addition, I rename H_t by n_t and use the non-satiation assumption to obtain equality in the constraints. Furthermore, I introduce an index shift $k_{t+1} \equiv k_t$.

The derivation of the first-order conditions for this model via Lagrange is shown in Appendix A.1. After substituting out the Lagrange multiplier λ_t, one obtains the following system of relevant model equations:

$$\begin{aligned} (1-\alpha)\frac{y_t}{c_t} &= \frac{\theta}{1-\theta}\frac{n_t}{1-n_t}, \\ y_t &= c_t + \frac{1}{v_t}k_t - (1-\delta)\frac{1}{v_t}k_{t-1}, \\ y_t &= a_t k_{t-1}^\alpha n_t^{1-\alpha}, \\ R_t &= \frac{1}{v_t}(1-\delta) + \alpha\frac{y_t}{k_{t-1}}, \\ \frac{1}{v_t} &= \beta E_t \left[\left(\frac{c_t}{c_{t+1}}\right)^{\eta(1-\theta)+\theta} \left(\frac{1-n_{t+1}}{1-n_t}\right)^{-\eta\theta+\theta} R_{t+1} \right], \\ a_t &= \exp(\gamma + \varepsilon_{at})a_{t-1}, \\ \nu_t &= \exp(\gamma + \varepsilon_{\nu t})\nu_{t-1}. \end{aligned}$$

3.1.2 Characteristics of the Model Variables

Since the Fisher model is a growth model, it cannot be implemented into the Toolkit directly. Considering

$$\ln a_t = \gamma + \varepsilon_{at} + \ln a_{t-1},$$

one can see that any change in a_{t-1} affects a_t with the factor 1. For investment-specific technology, the equivalent is true. Hence, we face permanent shocks to technology and any new shock will drive the system to a new steady state. Additionally, the AR(1)-representations of the shocks have an intercept, i.e., a deterministic trend. This means that in steady state, when the shock is equal to zero, neutral technology grows with γ. Equivalently, investment-specific technology then grows with the rate ν. Consequently, there exists a trend in the model that completely changes the steady-state character of most of the model variables. More precisely, some endogenous variables are not constant, but grow in steady state. As a consequence, Fisher mentions that a transformation of certain endogenous variables is necessary in order to render them stationary along a balanced growth path.[43] I will quickly check this proposed transformation for a better understanding of the model dynamics.

The first question to answer is which variables will grow in steady state and which will not? Since neutral technology affects both consumption and investment goods, it is reasonable for consumption and capital to grow and, consequently, output will grow as well. In contrast, labor enters the utility function with a negative sign and is bounded by time endowment; hence, it should not grow and can already be considered stationary.

Next, the rates of growth of the variables have to be determined. As stated, consumption as well as output and investment grow in steady state. According to

$$y_t = c_t + x_t,$$

they should all grow with the same rate z. Considering

$$k_t = (1-\delta)k_{t-1} + v_t x_t,$$

let $g_t = k_t - (1-\delta)k_{t-1}$ denote gross investment in t. If the capital stock in the current and previous period grow with the rate κ, gross investment should also grow with this rate. One can easily show that

$$\ln g_t = \ln(v_t x_t)$$

$$\Longleftrightarrow \ln g_t - \ln g_{t-1} = \ln v_t - \ln v_{t-1} + \ln x_t - \ln x_{t-1}.$$

If x_t grows with z and v_t grows with ν (according to the evolution of investment-specific technology), capital should grow with $\kappa = \nu + z$. We now use the production function to determine the growth rate of output as

$$z = \gamma + \alpha(\nu + z),$$

[43] See Fisher (2003), p. 4.

which can be rearranged to yield

$$z = \frac{\gamma + \alpha\nu}{1 - \alpha}.$$

This then represents the growth rate of output, consumption, investment and labor productivity (since labor is stationary). The growth rate of capital is accordingly derived as

$$\kappa = \frac{\gamma + \nu}{1 - \alpha}.$$

These growth rates are equivalent to the results that Fisher presents in his paper.

3.1.3 Transformation of the Model Variables and Log-linearization

Knowing the rates at which the respective variables grow in steady state, one may represent the steady-state paths by

$$\overline{y}_t = y_0 e^{(\frac{\gamma+\alpha\nu}{1-\alpha})t}, \; t = 1, 2, ...,$$

in the case of output for example, where y_0 represents the starting value. However, in order to be able to use the Toolkit for solving this model, one needs to find variables with a constant steady state. For this, I define new, "detrended" variables that are simply the ratios of model variables and some other variables that grow with the same rate. Hence, the resulting ratios will in fact show a constant steady state.[44] I will call these new variables "ratio"-variables in the following. An appropriate variable for detrending is already proposed by Fisher:

$$z_t = a_t^{\frac{1}{1-\alpha}} v_t^{\frac{\alpha}{1-\alpha}},$$

which, as is easy to see, grows with the rate z.

We define the ratio-variables by

$$\begin{aligned} y_t^{ratio} &\equiv \frac{y_t}{z_t}, \\ c_t^{ratio} &\equiv \frac{c_t}{z_t}, \\ k_t^{ratio} &\equiv \frac{k_t}{v_t z_t}. \end{aligned}$$

In addition, the return R_t grows with the rate $-\nu$ in steady state, since $\frac{1}{v_t}$ and $\frac{y_t}{k_{t-1}}$ both grow with $-\nu$. Thus, define the new variable for the return as

$$R_t^{ratio} \equiv v_t R_t.$$

In order to express the model equations in these new variables, the model has to

[44] A similar approach is also presented by Uhlig (2003c).

be transformed accordingly to yield the following system of equations

$$\begin{aligned}
(1-\alpha)\frac{y_t^{ratio}}{c_t^{ratio}} &= \frac{\theta}{1-\theta}\frac{n_t}{1-n_t}, \\
y_t^{ratio} &= c_t^{ratio} + k_t^{ratio} - (1-\delta)k_{t-1}^{ratio}e^{-\frac{1}{1-\alpha}(\gamma+\varepsilon_{at}+\nu+\varepsilon_{vt})}, \\
y_t^{ratio} &= n_t^{1-\alpha}(k_{t-1}^{ratio})^{\alpha}e^{-\frac{\alpha}{1-\alpha}(\gamma+\varepsilon_{at}+\nu+\varepsilon_{vt})}, \\
R_t^{ratio} &= (1-\delta) + \alpha\frac{y_t^{ratio}}{k_{t-1}^{ratio}}e^{\frac{1}{1-\alpha}(\gamma+\varepsilon_{at}+\nu+\varepsilon_{vt})}, \\
1 &= \beta E_t[\left(\frac{c_t^{ratio}}{c_{t+1}^{ratio}}\right)^{\eta(1-\theta)+\theta}\left(\frac{1-n_{t+1}}{1-n_t}\right)^{-\eta\theta+\theta}R_{t+1}^{ratio} \\
&\quad \cdot e^{-\frac{1}{1-\alpha}[\phi_a(\gamma+\varepsilon_{at+1})+\phi_v(\nu+\varepsilon_{vt+1})]}],
\end{aligned}$$

where $\phi_a \equiv \eta(1-\theta)+\theta$ and $\phi_v \equiv 1-\alpha+\alpha[\eta(1-\theta)+\theta]$. For this, I used

$$\begin{aligned}
\frac{a_t}{a_{t-1}} &= \exp(\gamma+\varepsilon_{at}),\ \frac{v_t}{v_{t-1}} = \exp(\nu+\varepsilon_{vt}), \\
\frac{z_t v_t}{z_{t-1}v_{t-1}} &= (\frac{a_t}{a_{t-1}})^{\frac{1}{1-\alpha}}(\frac{v_t}{v_{t-1}})^{\frac{\alpha}{1-\alpha}}\frac{v_t}{v_{t-1}} \\
&= \exp(\frac{1}{1-\alpha}(\gamma+\varepsilon_{at}+\nu+\varepsilon_{vt})).
\end{aligned}$$

Next, one has to express the linear relationship between the deviation of the model variables from their steady state values, that is, one has to log-linearize the model equations. Through the transformation above, I got rid of the variables measuring the evolution of the shock over time (a_t and v_t) and, hence, am now only considering the technology shocks directly, namely $\varepsilon_{at}, \varepsilon_{vt}$. This is reasonable, because the ratio-variables are considered here and not the movement of the actual model variables over time. In the case of neutral technology, a_t grows with γ, so any value of ε_{at} already expresses the deviation from steady state (in levels!), since ε_{at} itself is equal to zero in steady state. The deviations of the other model variables are the standard percentage deviations from steady state. The set of log-linearized equations can be viewed in Appendix A.2.

This system of equations can now be implemented into the Toolkit. Plots of the respective impulse responses are provided in Appendix A.3. How can these impulse responses of the ratio-variables be interpreted? Here, it is important to look at the base from which simulations are plotted, since impulse responses show the percentage deviation of the model variables from this base. Here, this base is represented by the growth path of the respective variable. Being the ratio between a growing variable and a different variable that grows with the same rate, the ratio-variables have a constant steady state. If now a shock occurs in this system, we observe that technology jumps to a new level, while consumption, output and capital react to this shock more gradually and adjust more slowly to their respective new steady states. Thus, any steady-state deviation of a ratio-variable shows that the respective real variable has not reached its new steady state yet. It has to be noted, that labor, which is stationary and therefore not represented by a ratio-variable, already shows the actual impulse response of labor due to both shocks.

3.1.4 Deviation of the Real Variables

The behavior of the real variables rather than that of the ratios is of interest when investigating the results of the model. To derive these, use

$$\begin{aligned}\widehat{y}_t^{ratio} &= \ln(\frac{y_t^{ratio}}{y^{ratio}}) \\ &= \ln(\frac{y_t}{z_t}/\frac{\overline{y}_t}{\overline{z}_t}),\end{aligned}$$

where $\overline{y}_t, \overline{z}_t$ are the balanced growth paths of output and z_t[45]. Rearranging yields

$$\widehat{y}_t^{ratio} = \ln(\frac{y_t}{\overline{y}_t}) + \ln(\frac{\overline{z}_t}{z_t})$$

$$\Rightarrow \widehat{y}_t = \widehat{y}_t^{ratio} + \widehat{z}_t.$$

The same is true for consumption, since the definition of the ratio is equivalent to the one for output. Hence,

$$\widehat{c}_t = \widehat{c}_t^{ratio} + \widehat{z}_t.$$

Analogously, the deviation from steady state for capital can be expressed by

$$\widehat{k}_t = \widehat{k}_t^{ratio} + \widehat{z}_t + \widehat{v}_t,$$

and for the return one gets

$$\widehat{R}_t = \widehat{R}_t^{ratio} - \widehat{v}_t.$$

Thus, we can represent the deviation of the real variables as the sum of the respective ratio-variables, which we have determined already, plus the deviation of the technology shock variables. To be able to express these deviations in real variables in the model solution, the equations stated above have to be added to the system of loglinearized equation derived in the previous section.

Next, we must further investigate the deviations of neutral and investment-specific technology from steady state. We know that a_t grows with γ in steady state. If $\widehat{a}_t$ is to govern the deviation of neutral technology from its base steady state in every period, this means that the stochastic part of the neutral technology hitting the system has to be added. This is expressed by the following equations

$$\widehat{a}_t = \widehat{a}_{t-1} + \varepsilon_{at},$$

$$\widehat{v}_t = \widehat{v}_{t-1} + \varepsilon_{vt},$$

$$\widehat{z}_t = \frac{1}{1-\alpha}\widehat{a}_t + \frac{\alpha}{1-\alpha}\widehat{v}_t,$$

which I also add to the system of loglinearized equations of the model.

[45] As shown above, these can be represented by $\overline{y}_t = y_0 e^{(\frac{\gamma+\alpha\nu}{1-\alpha})t}$ and $\overline{z}_t = z_0 e^{(\frac{\gamma+\alpha\nu}{1-\alpha})t}$, where $t = 1, 2, ...$ Hence, their ratio is constant.

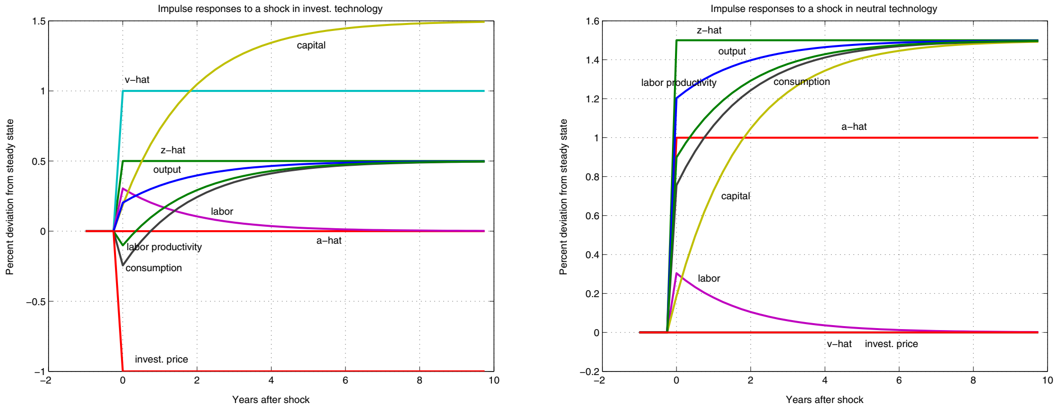

Figure 1: Impulse responses for the original Fisher model

Finally, the movement of labor productivity and the investment price due to innovations in technology should also be expressed in the solution. To make the behavior of these variables visible in the impulse responses, specify

$$w_t = \frac{y_t}{n_t},$$
$$p_t = \frac{1}{v_t},$$

and add another two loglinearized equations to the system:

$$0 = \widehat{y}_t - \widehat{w}_t - \widehat{n}_t,$$
$$0 = \widehat{p}_t + \widehat{v}_t.$$

3.1.5 Impulse Responses for Version 1 of the Model

The system of loglinearized equations as derived in the previous section can be implemented and solved using the Toolkit by Uhlig (1999). As calibrated parameters, I choose widely used standard values: $\delta = 0.025$, $\beta = 0.99$, $\alpha = \frac{1}{3}$, $\theta = \frac{2}{3}$, $\eta = 1.1$, $\nu = \gamma = 0.02$, $\sigma_a = \sigma_\nu = 0.1$. The impulse responses to both kinds of technology shocks are represented in Figure 1.[46]

Before interpreting the results, let us consider what these impulse responses actually represent. In steady state, all variables grow with a certain rate. This growth path is now equivalent to the baseline in the impulse responses. If a shock occurs, the shock variables immediately jump to their new levels as can be seen in the plots. Quite intuitively, this new level lies parallel and above the old one. Equivalently, the real variables react to these shocks and also converge to their respective new steady states. Since the responses of the variables are measured with respect to the baseline, that is, the old growth paths, they do not simply diverge, but converge to a new, parallel level instead.

[46]The Toolkit code for this version of the model can be viewed on the CD enclosed. A similar code for the extended version of the model is shown in Appendix B.3.

Consider the response due to an innovation in neutral technology. "z-hat" measures the deviation of the "growth"-variable z_t. Consequently, all variables but labor converge to $\widehat{z}_t$, since they all grow with the same rate, that is z, in the new steady state[47]. For an innovation in investment-specific technology, one can see that capital moves up strongly, because both the variable $\widehat{z}_t$ and the deviation of investment-specific technology increase $\widehat{v}_t$ ("v-hat"). The price of investment falls, since it is moved by $\widehat{v}_t$ only, while all other endogenous variables converge to the level of $\widehat{z}_t$ as before. Due to the higher weight of neutral technology in $\widehat{z}_t$, the effect of the neutral shock on all endogenous variables except for capital and labor is higher than the effect of an innovation in investment-specific technology. In the case of labor, one can see that this variable rises after both shocks in technology and that both of these shocks have an identical impact.

The impulse responses due to an innovation in neutral technology look quite standard, since all endogenous variables rise as expected. Clearly, labor productivity rises less than output on impact, due to increasing labor. However, the impulse responses to a shock in investment-specific technology need a little more explanation. One can see that labor productivity falls on impact, then overshoots the zero line and responds positively in the long run. The reason for this is the weak reaction of output compared to the one of labor. This can be explained by the notion that the agents react to the shock in investment-specific technology with increasing investment, since capital is now more productive. Hence, consumptions falls on impact. Since investment needs time to have an impact on the capital stock and since they want to compensate the loss in consumption, the agents reduce their leisure and work more directly after the shock. Thus, immediately after the shock productivity is driven by hours. Since labor subsequently converges back to its old steady state and output rises due to the strong increase of capital in the following periods, productivity rises as well.

One may now alternate some parameter values in order to investigate the effect of such changes to the model dynamics. When allowing for a relatively high coefficient of risk aversion ($\eta = 2$), the negative impact of labor productivity in response to a shock in investment-specific technology is not that strong any more, since the increase of labor is weaker. If agents are risk averse, they want to smooth consumption. Hence, they do not easily accept a drop in consumption after an innovation in technology. As a consequence, they do not invest as much and do not need to increase labor as much as risk-neutral agents.

As a different alternative, one could reduce the share of leisure in the utility function to $\theta = \frac{1}{3}$, for example. This also results in a weaker response of labor compared to the specification used before. Since labor is weighted more highly, certain utility levels can now be achieved with a lower input of labor. In general, changing parameter values does only lead to small quantitative, but no substantial differences regarding the impulse responses.

[47]Even capital grows with the same rate, since there is no innovation in investment-specific technology.

How do these impulse responses relate to the ones Fisher derives empirically[48]? The empirical responses of productivity and output after an innovation in neutral technology as well as the responses of the investment price and productivity after a shock in investment-specific technology match the ones derived from the model. Hence, the responses of the variables that are restricted in the identification are well determined by Fisher. In the case of labor, Fisher finds that an investment-specific technological innovation drives hours up a lot more than neutral technology. This is in line with his result that this kind of technological change should account for most of the business cycle variation in hours worked. In addition, hours worked shortly drop below zero after the impact of a neutral technology shock. Both of these features are clearly not reflected in the impulse responses shown in Figure 1 and one can conclude that the Fisher model understates the importance of investment-specific technology for labor. In addition, Fisher's empirical response of output due investment-specific technological advancements does not match the theoretical response either. In Fisher's result, output rises strongly after an innovation in investment-specific technology and then decreases to a comparably low (even though still positive) new steady state value.[49] In Figure 1, in turn, output does not react strongly on impact, but quite gradually rises to its new steady state.

3.2 Labor Hoarding Model ("Version 2")

3.2.1 Specification

Technology shocks are probably not the only force that drives labor productivity. It can be shown, for example, that productivity remains procyclical even in periods where empirical fluctuations are demand-driven[50]. In addition, productivity is strongly correlated with output, while only weakly correlated with employment.[51] These empirical features, which Wen (2002) calls the "productivity puzzle", can, however, not be met by standard technology-driven RBC models.[52] In the following, I therefore augment the model by introducing sources of variation in labor productivity different from technology. Christiano and Eichenbaum (1992) propose to model this via additional shocks in government spending in order to account for the Dunlop–Tarshis observation. I, in turn, will follow Uhlig (2003a) as well as Wen (2002) and extend the Fisher model by allowing for labor hoarding, since labor hoarding aims at explaining procyclical productivity as well.

Extending the model by additional shocks may alter the dynamics significantly and, as a consequence, the identification strategy for the shocks of interest as well. Fisher states that "it is straightforward to extend the simple growth model in other ways to incorporate a large variety of alternative propagation mechanisms, including

[48]For plots of these, see Fisher (2002), p. 18.

[49]In fact, from merely looking at the graph provided by Fisher, it is not that obvious whether output does not even converge back to its old steady state. Due to the permanence of the shock, this should not be the case, however.

[50]Wen (2002), p. 3.

[51]This is the Dunlop–Tarshis observation. See Christiano and Eichenbaum (1992) for details.

[52]See Wen (2002), pp. 2-4.

models with money or sticky prices"[53], that is, some form of sluggishness as in Galí (1999). Thus, Fisher's identifying restrictions for the technology shocks may change when augmenting the model with additional shocks that permanently influence labor productivity. Because these restrictions are based on a long-term horizon, they will remain valid, however, as long as additional shocks have transitory effects on labor productivity only.

Labor hoarding accounts for the fact that measured labor input does not necessarily equal actual labor input and thus may cause observed labor productivity to differ from actual labor productivity. This can be modelled in various different ways. Burnside, Eichenbaum and Rebelo (1993) use "fixed costs" in terms of hours that cannot be seen as "effective" work and simply corresponds to lost time when working. Agents may be employed or unemployed and this specification and productivity is driven by technology. In contrast, Wen (2002) additionally models employment-adjustment costs and uses demand shocks only to explain procyclical productivity. In turn, Uhlig (2003a) proposes a different modelling technique for labor hoarding. I will follow his approach described in the following.

Contract hours, that is measured hours, can be represented by

$$n_{ct} = n_t + l_{nt},$$

where n_t is the actual work input, meaning actual hours worked, and l_{nt} represents leisure spent at the working place. Uhlig calls this mismeasurement "social attitude towards the working place", accounting for the fact that time at work is increasingly spent for social and leisure activities such as using the internet, lunch-time meetings or communication with the colleagues to name only a few. According to this, contract labor and labor productivity are mismeasurements of the actual variables. However, it is assumed that wages still reflect the return to the actual labor input. In addition, contract hours can be described by an exogenous process, where the difference between contract hours and some multiple of actual hours, measured by ϕ, closes with a certain speed:

$$n_{ct} = \rho(n_{c(t-1)} - \phi n_{t-1}) + (1-\rho)\phi n_t + \eta_t,$$

given that $\eta_t = \psi_\eta \eta_{t-1} + \epsilon_{\eta t}$ and $\epsilon_{\eta t} \sim N(0, \sigma_\eta^2)$. The value of ρ and the persistence parameter of the contract shock, ψ_η, determine the short- or long-run impact of these shifts in attitude toward the working place on contract hours. Choosing $\psi_\eta = 1$ therefore introduces another permanent source of variation of labor productivity in addition to the two types of technology shocks.

Note that the newly introduced variable for contract labor is not subject to the optimal choice by representative agent, but rather determined in addition to the endogenous variables; as Uhlig states, "there is no economic force pinning down contract hours"[54]. Therefore, contract labor is not considered in the utility maximization, but added to the system of model equations afterwards.

[53] Fisher (2003), p. 5.
[54] Uhlig (2003a), p. 4.

In addition, the time endowment μ_t is stochastic and used according to

$$\mu_t - n_t = l_{nt} + l_{at},$$

where l_{at} is home leisure. Agents, however, care only about total leisure. The time endowment follows an exogenous process and is subject to so-called preference shocks

$$\mu_t = (1-\psi_\mu)\overline{\mu} + \psi_\mu \mu_{t-1} + \epsilon_{\mu t},$$

where $\epsilon_{\mu t} \sim N(0, \sigma_\mu^2)$. I assume that the steady state time endowment $\overline{\mu} = 1$, i.e., it equals the time endowment of Version 1 of the model.

The social-planner problem to be solved is very close to the one in version 1 of the model and can be viewed in Appendix B.1. Here, the according first-order conditions and the system of equations that completely describes the new model are also provided. Next, the model has to be transformed in order to express the system in "ratio"-variables as described in Version 1 of the model. Since labor is bounded, leisure and contract labor are as well. Hence, the definitions of the ratio variables are equivalent to the ones used in Version 1 above. The transformed system of equations is then:

$$\begin{aligned}
(1-\alpha)\frac{y_t^{ratio}}{c_t^{ratio}} &= \frac{\theta}{1-\theta}\frac{n_t}{\mu_t - n_t}, \\
y_t^{ratio} &= n_t^{1-\alpha}(k_{t-1}^{ratio})^{\alpha} e^{-\frac{\alpha}{1-\alpha}(\gamma+\varepsilon_{at}+\nu+\varepsilon_{vt})}, \\
y_t^{ratio} &= c_t^{ratio} + k_t^{ratio} - (1-\delta)k_{t-1}^{ratio} e^{-\frac{1}{1-\alpha}(\gamma+\varepsilon_{at}+\nu+\varepsilon_{vt})}, \\
R_t^{ratio} &= (1-\delta) + \alpha\frac{y_t^{ratio}}{k_{t-1}^{ratio}}\exp(\frac{1}{1-\alpha}(\gamma+\varepsilon_{at}+\nu+\varepsilon_{vt})), \\
1 &= \beta E_t[\left(\frac{c_t^{ratio}}{c_{t+1}^{ratio}}\right)^{\eta(1-\theta)+\theta}\left(\frac{\mu_{t+1}-n_{t+1}}{\mu_t - n_t}\right)^{-\eta\theta+\theta} R_{t+1}^{ratio} \\
&\quad \cdot \exp(-\frac{1}{1-\alpha}\{\phi_a(\gamma+\varepsilon_{at+1}) + \phi_\nu(\nu+\varepsilon_{vt+1})\})],
\end{aligned}$$

where the shock equations are omitted and ϕ_a and ϕ_ν are defined as in Version 1 of the model. The equations for labor productivity and the investment price from Version 1 of the model are added to this system. Also, the development of contract labor productivity and contract labor is to considered in the solution and, hence, the system of model equations is completed by the following two equations

$$\begin{aligned}
n_{ct} &= \rho(n_{c(t-1)} - \phi n_{t-1}) + (1-\rho)\phi n_t + \eta_t, \\
w_{ct} &= \frac{y_t}{n_{ct}}.
\end{aligned}$$

After calculating the steady-state values of the endogenous variables, the parameters of the model are calibrated using $\delta = 0.025$, $\beta = 0.99$, $\alpha = \frac{1}{3}$, $\theta = \frac{2}{3}$, $\eta = 1.1$, $\rho = 0.8$, $\overline{\mu} = 1$, $\phi = 1$, $\nu = \gamma = 0.02$, $\sigma_a = \sigma_\nu = 0.1$, $\psi_\eta = 1$, $\psi_\mu = 0.8$, $\sigma_\eta = 0.1$, $\sigma_\mu = 0.3$.[55] One can now solve the model by implementing the system of

[55]The new parameters concerning the preference and contract shocks are in line with Uhlig (2003a). The growth rates of neutral and investment technology have been chosen to be in line with real empirical growth.

log-linearized equations into the Toolkit. See Appendix B.2 for the system of equations for the detrended ratio variables. Again, we want to express the deviations of the real variables rather than those of the ratios. For this reason, we add the equations that express the relationship between the ratios and the real variables as derived for Version 1 of the model to the system (that is, four equations for the real variables of capital, consumption, output and the return). Equivalently, we also augment the model by equations for labor productivity and the investment price as shown in Version 1. The log-linearized equations for contract labor productivity and contract labor are added to the system as well:

$$\begin{aligned} 0 &= -\overline{n_c}\widehat{n}_{ct} + \rho\overline{n_c}\widehat{n}_{ct-1} - \rho\phi\overline{n}\widehat{n}_{t-1} + (1-\rho)\phi\overline{n}\widehat{n}_t + \widehat{\eta}_t, \\ 0 &= \widehat{y}_t - \widehat{w}_{ct} - \widehat{n}_{ct}. \end{aligned}$$

Finally, include the log-linearized shock equations:

$$\begin{aligned} \widehat{\eta}_t &= \psi_\eta \widehat{\eta}_{t-1} + \sigma_\eta, \\ \widehat{\mu}_t &= \psi_\mu \widehat{\mu}_{t-1} + \sigma_\mu, \\ \varepsilon_{at} &= \sigma_a, \\ \varepsilon_{\nu t} &= \sigma_\nu. \end{aligned}$$

3.2.2 Impulse Responses for Version 2 of the Model

The impulse responses for this version of the model are plotted in Figure 2. The respective Toolkit program can be viewed in Appendix B, Section 3. Note that only the responses of labor, contract labor, the respective measures of productivity, output and the investment price are plotted for better graph clarity. Also note that a contract shock only influences contract labor and contract labor productivity, but permanently in this specification. Not surprisingly, the responses to a shock in neutral technology are very similar to the ones of Version 1 of the model. Contract labor and contract labor productivity behave differently than actual labor input and productivity. Contract labor rises in a humpshape, and thus more slowly than actual labor, after both technology shocks and reacts less strongly than the actual measure. Contract productivity, in turn, rises with output and thus more strongly than actual productivity, slows down to grow after the strong impact due to the subsequent rise of contract labor and then converges to its new steady state more gradually than actual labor productivity. Hence, contract labor productivity may better explain the procyclical behavior of productivity than actual labor productivity.

The movement of the contract measures after an innovation in investment-specific technology differs from the responses to neutral technology only in the magnitude of the response. This is due the smaller impact of investment-specific technology on $\widehat{z}_t$.[56] The rest of the variables behaves as in Version 1 of the model.

Last, investigate the responses due to an impulse in preference, that is, time endowment. Here, one can see that contract labor productivity co-moves strongly

[56] This is discussed in Version 1 of the model.

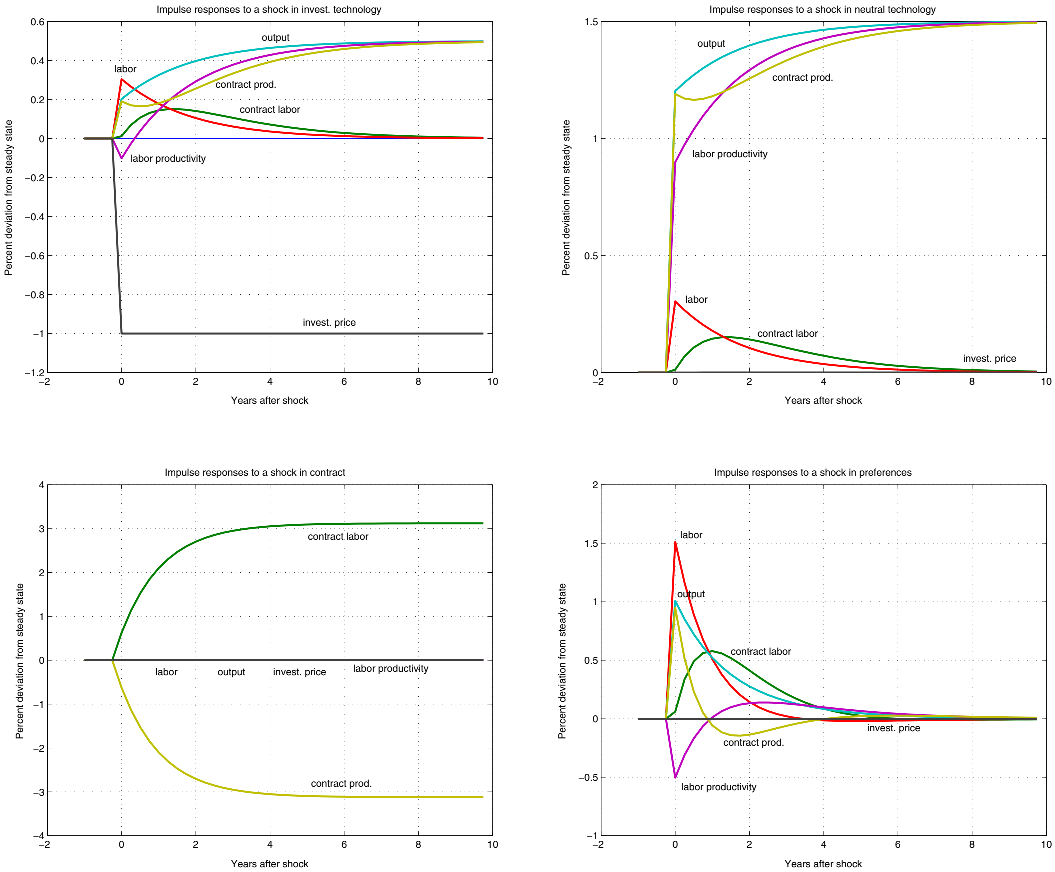

Figure 2:
Impulse responses for the labor hoarding model with permanent contract shocks

with output, especially shortly after the shock. This, again, is due to the relatively slow and humpshaped response of contract labor due to the shock. In turn, this shock induces actual labor to rise more strongly than output and hence to cause a decline in actual labor productivity. Thus, as for the other shocks, contract productivity may explain procyclical productivity better than the actual variable.

Recall that so far I have assumed contract shocks to be persistent. This was motivated by adding another source of long-run productivity movements to the model. This means that shifts in the social attitude towards the working place as described above are permanent. In turn, one might prefer to model transitory effects of the contract shocks only. Economically, this means that stochastic shifts in the social attitude towards the working place do not last. Intuitively, one might compare these attitude shifts to “trends” dying out. For example, while a few years ago internet surfing during the work hours was widely spread, there exist certain limits for this today, such as the restricted access to the internet or pnealties for exessive private e-mail communication. Therefore choosing $\psi_\eta = 0$ means that I still allow contract labor and labor to differ; thus there is still sluggishness in the model. Contract shocks do still have a large impact on contract labor, but in a very

Table 1: Comparison of Correlations

	empirical[57]	Version 1	Version 2 ($\psi_\eta = 1$)	Version 2 ($\psi_\eta = 0$)
$\text{corr}(y_t, n_t)$	0.86	0.65	0.37	0.47
$\text{corr}(y_t, \frac{y_t}{n_t})$	0.54	0.99	0.68	0.78
$\text{corr}(n_t, \frac{y_t}{n_t})$	0.04	0.91	–0.43	–0.19

short-run horizon only. The equivalent is true for contract productivity. Clearly, the impulse responses for all shocks but social attitude towards the working place are quite similar to before.

3.2.3 Empirical Performance

Comparing the impulse responses predicted by the labor hoarding model to the ones Fisher finds empirically, the problems described for the previous version of the model remain equally valid, since not much has changed fot the technology shocks. Admittedly, the humpshape response of contract labor to both technology shocks is closer to the response of Fisher than the one of actual labor. Nevertheless, the strong response of labor after an investment shock as found by Fisher is still not reflected in the model. As before, the response of output for the same shock does not match the result of Fisher. While the empirical response of labor productivity is closer to the one of contract productivity than to actual productivity for an innovation in neutral technology, the opposite is true for an investment-specific shock. Here, actual productivity falls initially after the shocks which corresponds to Fisher's findings. For both versions of the model, it can be stated in general that the response to neutral technology is more closely matched by the empirical investigation than the response to investment-specific technology.

In order to shed light on the choice of the model specification, I investigate the unconditional correlations of the different versions and compare them to empirically found correlations. Let us see whether extending the model has provided some answers to the productivity puzzle as stated above, that is productivity moves procyclically while the correlation with labor is low. In Table 1, the empirical correlations of US postwar data are compared to the equivalent measures from both versions of the model. For the labor hoarding model, I give results for permanent as well as transitory results.

First, it has to be noted that the correlations from the different versions of the model are bound to the particular calibration choice as stated in the previous sections. Hence, the correlations are not optimized according to the values of the model parameters. As one can see, the correlation of labor productivity with output is best matched for the original model. Regarding the productivity puzzle, the labor haording specification with transitory contract shocks works best in explaining the empirical correlations. However, it fails to explain the high correlation of labor with output. To conclude, labor hoarding helps in explaining the productivity puzzle. Transitory contract shocks are to be preferred over permanent contract shocks.

4 Deriving the Identifying Restrictions

4.1 Procedure

Since Fisher (2002), Galí (1999) and various other authors impose restrictions on the influence of certain shocks on labor productivity, one may wonder whether the models under consideration support these restrictions theoretically. As explained in Section 2, the forecast revision variance provides information on the importance of the shocks on certain variables over time. Hence, this is the relevant measure for deriving the identifying restrictions.

When solving the model with the Toolkit, one obtains a recursive law of motion for the model dynamics which is equivalent to an *identified* VAR of order one, since the shocks are orthogonal by construction. From this, one can calculate the forecast revision variance.[58] As shown in Section 2, this variance can then be decomposed into the contributions of each shock for all possible horizons. One therefore obtains a measure of the actual influence of technology shocks in the model over time and can derive a theoretically based identification structure. More precisely, I use the variance decomposition not only for an investigation of the influence of certain shocks on the model variables, but also for a comparison of this influence in different horizons. This means that, in contrast to the approach of Galí and others, I will take the possibility of short-run or intermediate identification horizon into account.[59]

Adopting Uhlig's (2003a) strategy, I will try to test different identification strategies using simulated data from the model in order to decide about the final identification when taking the model to real data. For this, I generate extensive simulated time series for labor, labor productivity and the investment price using the Toolkit[60]. One has to note that due to the permanent shocks in neutral and investment-specific technology, the simulated series of labor productivity and the investment price my differ largely between different simulations of the model. That is, depending on the amount of early negative or positive technology shocks, the series may taper off in any direction, growing strongly or decreasing by a lot, but they may as well evolve as random walks around zero. However, the estimation results should not be affected insofar their ability to identify the shocks correctly is concerned.

For identification, I will decompose the forecast revision variance using Cholesky as derived in Section 2.5. When doing this, the variables that are restricted to be solely influenced by the technology shocks have to be ordered first in the VAR. In order to test the procedure in general as well as to rule out possible programming mistakes, I apply the method to the original data of Francis and Ramey (2003)[61]. As can be seen in Appendix C, my results match the ones of the two authors almost

[58]Note that in Section 2 the structural residuals were assumed to be orthogonal with an identity VCV matrix, while here this matrix is merely diagonal. However, one may well calculate the forecast revision variance using a formula slightly different from that derived in Section 2. This can be viewed in the MATLAB-file "fisher_VAR_decomp.m" provided in Appendix D.

[59]The possibility of medium-run identification has been introduced by Uhlig (2003a).

[60]This can be viewed in the Toolkit program for the respective models, see Appendix B. The simulated series contain 10 000 observations.

[61]This data was provided to me by Harald Uhlig.

perfectly.

4.2 The Original Fisher Model (Version 1)

4.2.1 Forecast Revision Variance Decomposition

Investigating the variance decomposition of the original Fisher model serves as a check on the identification strategy proposed by Fisher.[62] As stated already, this strategy consists of two components. First, investment-specific technology is assumed to be the only influence on the investment price in the long run. As is obvious from the model, investment-specific technology is the only shock that moves the price at all. Hence, this shock explains 100% of the forecast revision of the investment price in all horizons, meaning that the identification of this shock is theoretically perfect and that it is not bound to any particular identification horizon. As a second identification step, both neutral and investment-specific technology are restricted to be the only driving force of labor productivity movements in the long run.

The decompositions of the forecast revision variance for labor productivity and labor are shown in Figure 3.[63] When investigating the productivity plot, one must note the relatively minor influence of the investment-specific shock on this variable. Clearly, both technology shocks influence labor productivity in the long-run horizon as proposed by Fisher, but the importance of neutral technology for this variable by far exceeds the one of investment-specific technology. That is, neutral technology accounts for at least 90% of the variance in all horizons and could therefore be justified as the only influence of labor productivity. This restriction is then equal to the one Galí (1999), Francis and Ramey (2003), Christiano, Eichenbaum and Vigfusson (2003a) and various other authors have used before.

One may conclude that both technology shocks should be identified separately.[64] While this could as well be done on grounds of the variance decomposition, it should not have significant effects on the results. The reason is that investment-specific technology is perfectly identified through the investment price. Furthermore, since single identification of neutral technology identifies this shock as being the only influence of productivity in the long run, the imminent importance of this shock on productivity even in the presence of investment-specific technology assures that the results can be compared to Galí and others. Fisher compares the responses for single identification of the neutral shock to the ones from the combined identification strategy and achieves quite similar results for both. This finding is supported by Michelacci and Lopez-Salido (2003) as well. It has to be noted, though, that there is no definite, unique model-based identification horizon. More precisely, both shocks may be identified at any horizon and medium-run identification is as reasonable

[62] Recall that Fisher does not provide a theoretical examination of his model in his paper. Note that I approximate the long run by 20 years as proposed in Uhlig (2003a).

[63] Note that I use quarters as periods. The respective MATLAB-files can be viewed in Appendix D.

[64] This is also proposed by Michelacci and Lopez-Salido (2003).

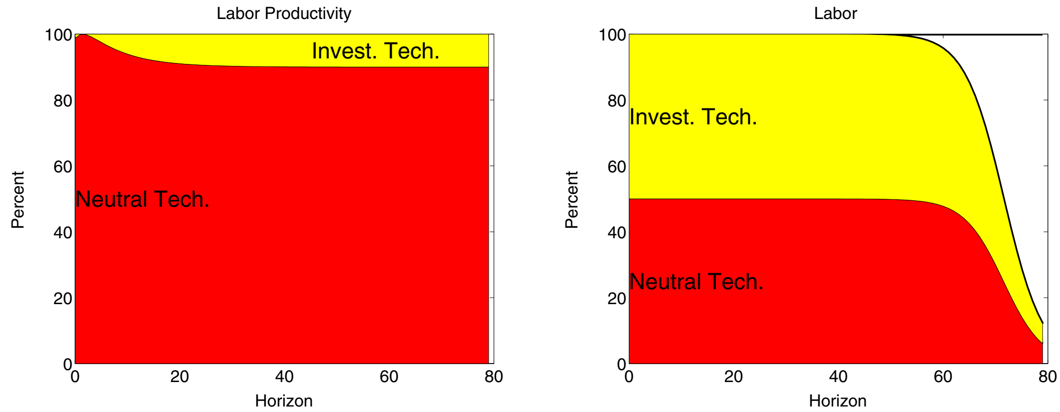

Figure 3:
Forecast revision variance decomposition for the original Fisher model

as long-run identification. Thus, medium- or short-run identification serves as an additional option for investigating the dynamic effects of technology shocks.

Since the influence of the shocks on the movement of labor is of interest as well, let us consider the variance decomposition of labor. As was already seen in the impulse responses for the original model in Figure 1, both technology shocks have an equal influence on the movement of labor. That is, both technology shocks induce labor to increase after a positive innovation, before it converges back to zero in the very long run. While this is not surprising, note, however, that Fisher finds investment-specific technology to be a lot more influential in the variance of labor than neutral technology. Clearly, this finding is not reflected in the model.

Let us take a second look at the variance decomposition plots. Having Fisher's identification scheme in mind and recalling that we are interested in the movement of labor rather than that of the investment price or productivity, why not take labor as the benchmark for identifying technology shocks? I therefore introduce an alternative identification strategy for technology shocks that is theoretically based by the model as follows: First, assume neutral technology to be the only source driving labor productivity in the short, medium or long run. Second, I define neutral and investment-specific technology to be the only influence of labor (in the short, medium or long run). While this approach is suggested by the variance decomposition of this particular model, I am aware that it may not easily be defended on intuitive grounds. Clearly, there should exist more driving forces of labor than technology shocks only, especially in the short and medium run. However, since I tie my identification strategy to the predictions derived from some particular models in which, as standard in RBC, all fluctuations are driven by technology, I will give this alternative identification scheme a try.

4.2.2 Identifying Simulated Data

In this section, I investigate identification of simulated data from the "original" Fisher model. In order to replicate Fisher's method I use first differences for the simulated time series of productivity and the investment price. This is feasible, because due to the unit roots in the technology shocks, these variables quite obviously also have unit roots. I put the investment price in the first position in my VAR specification, followed by productivity and labor. For this, I invert the price series, since I want to examine the responses to a positive shock in investment-specific technology. Since the investment price is assumed to be affected negatively by a positive innovation in investment-specific technology, the use of the inverted series is appropriate.[65] Note that I incorporate constants, but no trends in my specification. Since the series are generated using the recursive law of motion which is a VAR(1), I include one lag in the specification. The error bands are computed using bootstrap techniques.[66]

The results of the Fisher identification plotted versus the theoretical impulse responses from the model are shown in Figure 4 for the long run (the identifying horizon is approximated by 20 years).[67] To my great delight, almost all of the estimated impulse responses perfectly match the theoretical ones. Even if the estimated responses deviate from the theoretical ones, as the investment price response after an innovation in neutral technology for example, the theoretical responses are always within the confidence intervals. Hence, there exists no significant deviation. The impulse response plot for the medium-run (4 years) and short-run (zero horizon) identification can be viewed in Appendix E.2. Even if some of these plots are closer, some farther away from the theoretical impulse responses than in the long-run identification, there is still no significant deviation of the estimated plots from the theoretical ones. Therefore, the identification strategy is able to identify the shocks that drive the system. Here, any horizon may be picked as an identification horizon offering medium- or short-run identification as an option. This is in line with the results from the variance decomposition.[68] Note that the response of the investment price after an innovation in neutral technology differs largely across identification horizons, even though it is not significantly different from zero for all of these. Clearly, this is due to the restrictions imposed on the system, since the response of the prices crosses the zero line at the identifying horizon.

Next, consider the second identification strategy outlined in the previous section which will hereafter be called the "alternative" identification scheme. Since we now restrict productivity as well as labor, one needs to specify both of these variables

[65] As noted before, the upper-left element of the identification matrix is always positive when applying Cholesky.

[66] For details on this, see Appendix F.

[67] The MATLAB-code for the identification of this version of the model including all identification strategies is provided on the enclosed CD. A very similar code for the application to real data can be found in Appendix E.

[68] Note also that single identification of neutral technology, too, delivers estimated impulse responses that match the theoretical ones. Since this is not of crucial interest here, the results are not provided.

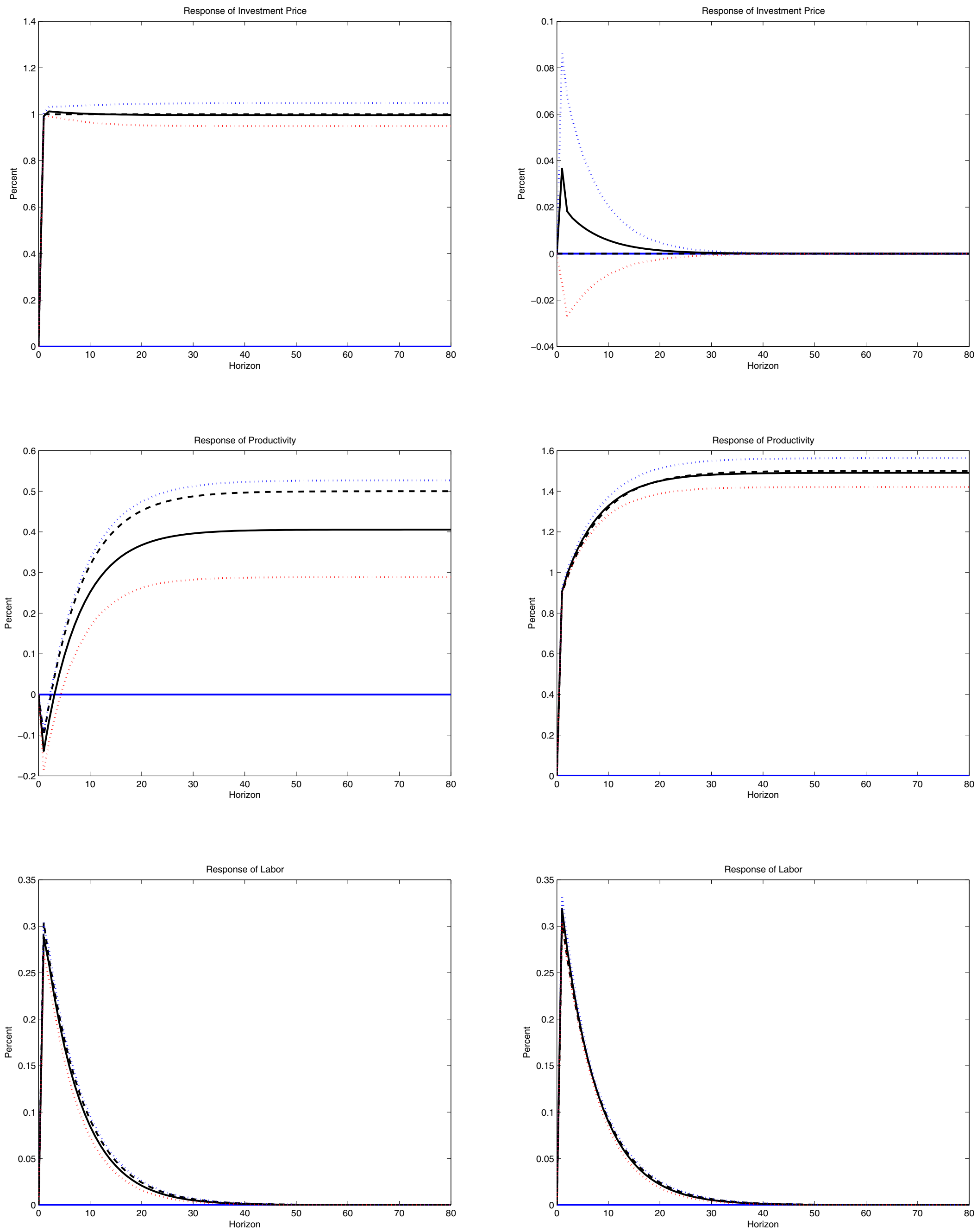

Figure 4:
Impulse responses for the long-run Fisher identification
of simulated data from the original Fisher model.
Left column: responses to innovation in investment-specific technology.
Right column: responses to innovation in neutral technology.

either in levels or in differences in order to apply the identification procedure. As stated above, productivity is first-difference stationary while labor is stationary in levels. To avoid overdifferencing labor, I decide to estimate the entire VAR in levels. Regarding the later application of this procedure to real data, where the productivity and price series are not as obviously integrated as in the simulated data, one may even avoid the overdifferencing of the other variables in the level specification. As elaborated in Section 2.8, this may cause some problems with the asymptotics of the impulse responses. However, one is still be able to say something about the reaction of the variables due to shocks. Again, I use a constant and a lag order of one in the specification.

The responses of labor after both types of technology shocks are shown in Figure 5 for the short, medium and long run. The corresponding responses of productivity are provided in Figure 6. The investment-price is not of crucial importance in this investigation, especially not the reaction of the price after a neutral technology shock, since this reaction is not restricted in any appropriate way. The responses of the price after the two shocks can be viewed in Appendix E.2. There are several things to note here. First, the alternative identification procedure matches the theoretical impulse responses of labor and the responses of productivity after a shock in neutral technology in all horizons. Clearly, these are the variables that are subject to the restrictions imposed. While the response of the investment price after an innovation in investment-specific technology is also considerably close to the theoretical impulse response over time, the response of labor after this type of shock is best matched in the short run. Here, medium-run identification does also work quite well, while the response of labor for the long-run identification is not significantly different from zero. Note that short run identification works best for the response of labor productivity after an innovation in investment-specific technology. This is again due to the fact that the identifying horizon determines where the response crosses the zero line. Hence, $k = 2$ would deliver the best result for this particular response.

One can conclude that both identification strategies work pretty well when applied to simulated data. The results for the Fisher identification are, however, slightly better than the ones of the alternative specification. The reason is that, first, the identification of neutral technology in the alternative identification is not as perfect as the identification of investment-specific technology in the Fisher identification. In addition, I restrict the influence of the investment-specific shock on productivity to be zero in the respective identification horizon. When this identification horizon is relatively short, productivity can still rise in the long run as implied by the model. Hence, the application of medium- or short-run identification is to be preferred over long-run identification for the alternative identification procedure.

4.3 Labor Hoarding Model (Version 2)

4.3.1 Forecast Revision Variance Decomposition

As shown, Fisher's identification scheme is theoretically backed by his model. The question remains, however, whether it is also supported by a more realistic modeling,

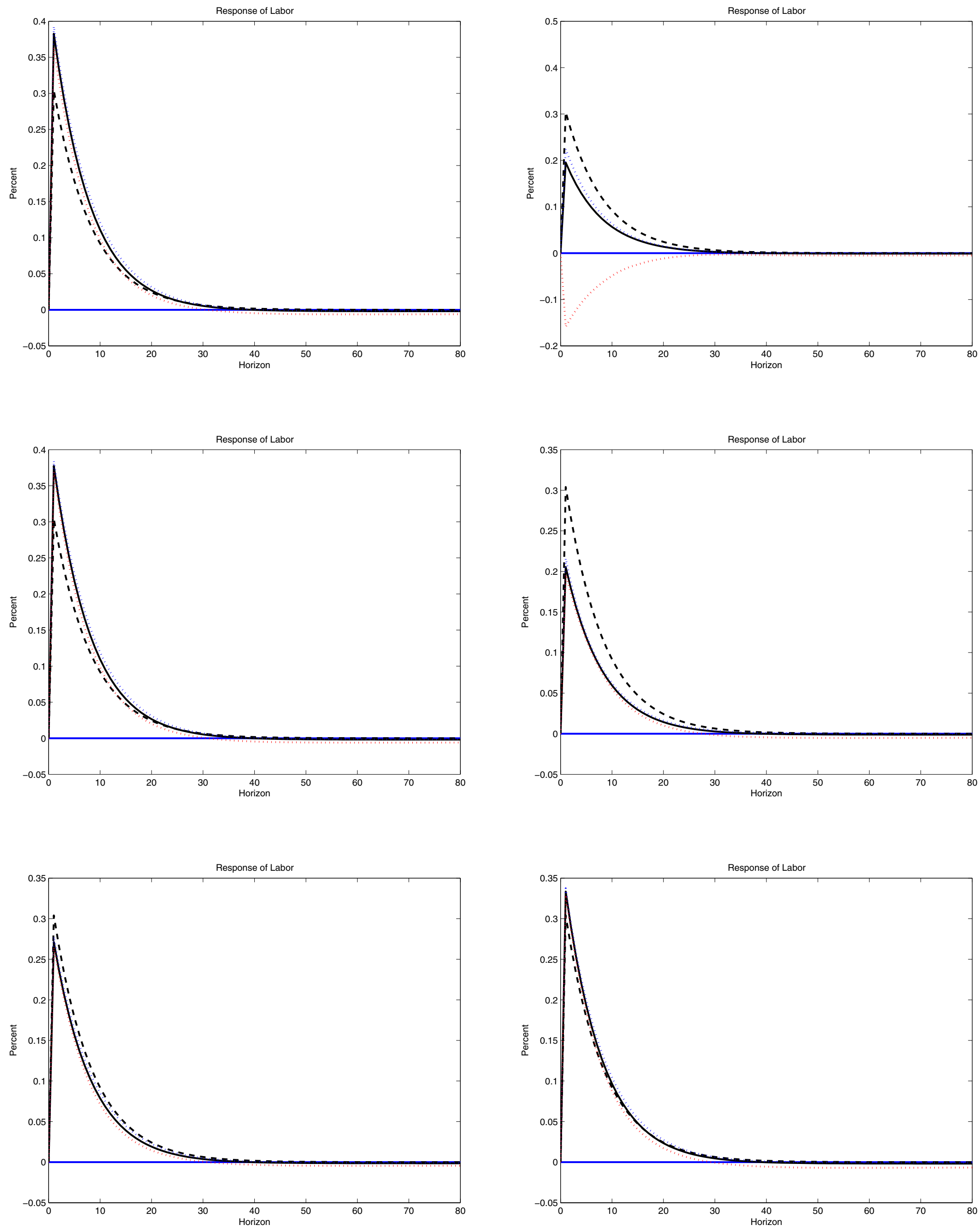

Figure 5:
Impulse responses of labor for short-, medium- and long-run
using the alternative identification
of simulated data from the original Fisher model.
Top panels: long run; middle panels: medium run; bottom panels: short run.

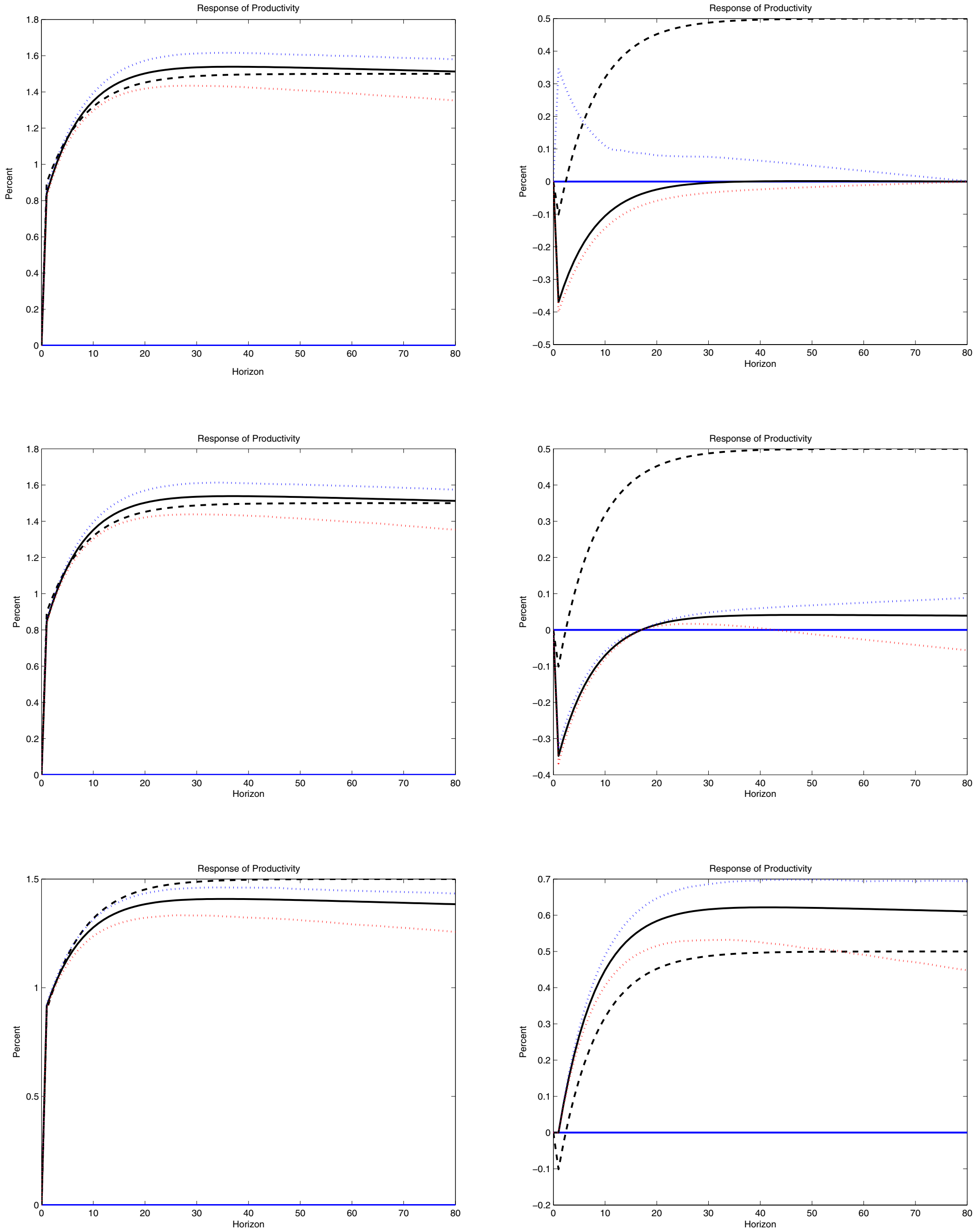

Figure 6:
Impulse responses of productivity for short-, medium- and long-run
using the alternative identification
of simulated data from the original Fisher model.
Top panels: long run; middle panels: medium run; bottom panels: short run.

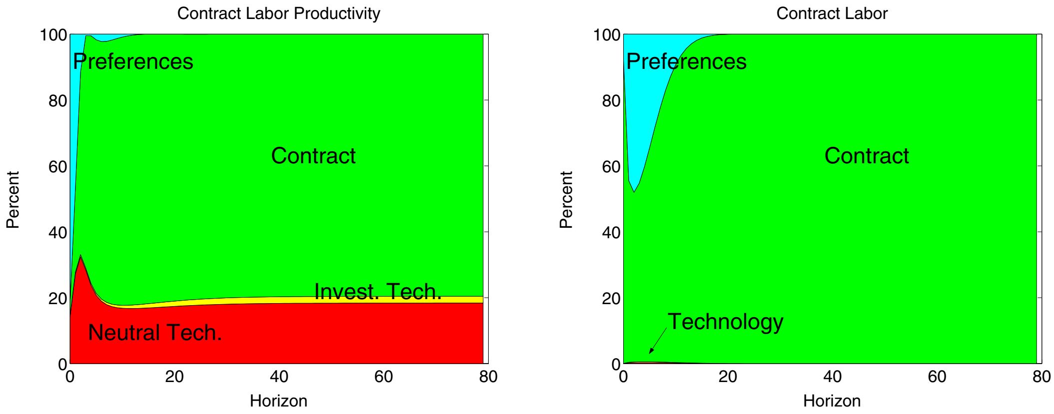

Figure 7:
Forecast revision variance decomposition
for the labor hoarding model with permanent contract shocks

that is, when allowing for labor hoarding. Here, the movements of contract labor productivity and contract labor are considered to be a mismeasurement of labor productivity and labor. Regarding the impulse responses, it has already been shown that these two new variables indeed move differently to labor and labor productivity in the previous specification. However, they are still considerably close to Fisher's empirical findings and may thus serve as a true alternative theoretical foundation to his empirical investigation.

In this framework, the model contains disturbances additional to the two technology shocks that may affect productivity and labor and therefore influence the identification scheme. In equivalence to Section 3.2, I will consider the possibility of persistent as well as transitory contract shocks.

First, assume the presence of persistent contract shocks and consider the forecast revision variance decompositions of contract labor and productivity that are shown in Figure 7.[69] One can see that more than only the technology shocks drive labor productivity in all horizons. More precisely, the permanent contract shock plays a role in the movement of this variable as well. Hence, the identification of the technology shocks as the only influence of labor productivity in the long run is no longer perfect and there exists no horizon in which the technology shocks alone explain labor productivity. However, since investment-specific technology is perfectly identified by its influence on the price, and since neutral technology explains more than 80% of productivity variance for almost all horizons (starting after the first year) and in addition both technology shocks together explain more than 90% of the variance in the long run, Fisher's identification scheme is under attack in this specification, but can still be considered to remain valid.

Regarding the variance decomposition for contract labor, one must note the negligible role of the two technology shocks in the movement of this variable. Obvi-

[69]For the calculation of the variance decomposition, see Appendix D.

ously, contract labor is moved by preference shocks in the short run, while contract shocks drive labor in the medium to long run. In contrast, in his empirical analysis Fisher finds that investment-specific shocks contribute to the variance of labor by a lot more than neutral technology shocks. Therefore, the model underestimates the importance of both components of technology with respect to labor and, again, Fisher's empirical findings regarding labor do not support the results of the model.

As stated before, this thesis focuses on the impact of technology on labor rather than other macroeconomic variables. Therefore, the idea of tying the identification strategy to labor is intriguing and favors the alternative identification scheme presented for Version 1 of the model. However, this alternative strategy may not be derived on grounds of the labor hoarding model with permanent contract shocks as shown above. In order to save this approach, one may assume that contract shock are not persistent. Recall that, in Section 3.2, a comparison of persistent and transitory contract shocks in terms of overall correlation of the variables delivers that the choice of transitory shocks may be even more appropriate than permanent shocks. More interesting, however, are the forecast revision variance decompositions of labor and labor productivity for this version of the model which are shown in Figure 8. Concerning the Fisher identification, the variance decomposition for productivity is quite close to the one obtained from the original Fisher model. Hence, this identification procedure may be valid for this version of the labor hoarding model as well. When regarding the variance decomposition of labor, one sees that both types of technology shock now do play a role in the movement. Even though most of the labor variance is explained by preference shocks, the two technology shocks explain 50% each of labor variance in the intermediate horizon. Therefore, this result gives us a very precise alternative identification scheme, since neutral and investment-specific technology are the sole driving forces of labor in the intermediate horizon (25 quarters).

In addition, the labor hoarding model with transitory contract shocks is better supported by the findings of Fisher (2002) than the version with permanent shocks, since there exists at least some contribution of investment-specific shocks for the variance of labor. In general, this contribution is still understated by the model, however.

4.3.2 Identifying Simulated Data

Equivalent to the original Fisher model, one may test the identifying procedures on simulated data from the model. In line with the variance decomposition in the previous section, the Fisher identification should be applicable to simulated data regardless of the persistence of the contract shock, while the alternative identification should work for simulated data with transitory contract shocks in the intermediate horizon. However, there arise many problems from the identification of simulated data.

First, consider the impulse responses for the long-run (20 years) Fisher identification for the model specification with permanent contract shocks as shown in Figure

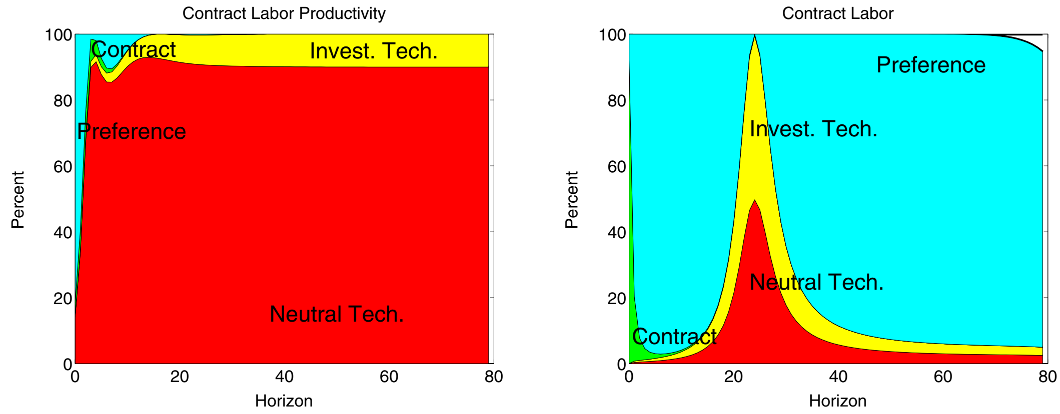

Figure 8:
Forecast revision variance decomposition
for the labor hoarding model with transitory contract shocks

9. While the identification of the investment price is perfect, all other responses only poorly match the theoretical ones. Comparing the responses for neutral technology with the theoretical responses of all shocks as shown in Figure 2, one may even conclude that the procedure identifies a contract shock rather than a neutral technology shock. One can show that these responses are also obtained when applying different identifying horizons.[70]

A possible explanation for this very unsatisfactory result is delivered by the long-run influence of the contract shock to productivity. Recall that Fisher already mentions that his procedure is not valid for models with additional permanent disturbances to productivity. Regarding the variance decompositions, his method should hold, however, in the case of transitory contract shocks. The responses for the long-run Fisher identification for this simulated data set are shown in Figure 10. Unfortunately, the responses are also not very close the theoretical ones in this case. While the investment price is, again, perfectly identified after an investment-specific shock, the other responses due to this shock differ largely from theory. However, the theoretical impulse responses are at least more or less within the large estimated confidence bands. This is also the case for the reaction of the investment price to an innovation in neutral technology. In turn, the responses of productivity and labor to this shock differ significantly from theory. While productivity at least moves in the "correct" direction, labor declines in response to a positive technology shock and therefore strongly contradicts the movement implied by the model. Again, it can be shown that the choice of short- or medium-run identification does not improve these results.

Not surprisingly, the alternative identification does not deliver satisfactory results either when applied to simulated data from the model with transitory contract shocks. The responses for the medium-run identification for this specification can

[70]This is, however, not shown here.

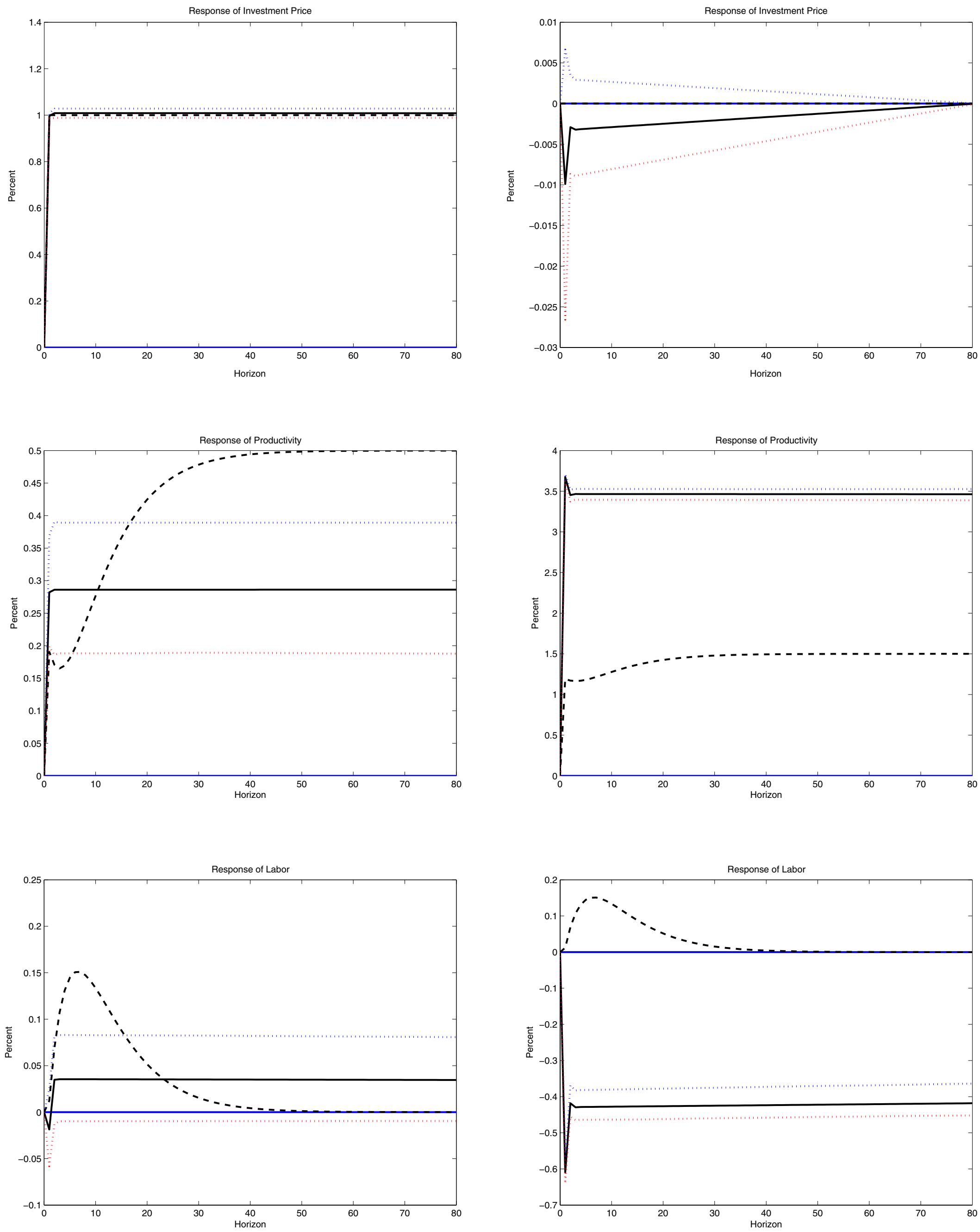

Figure 9:
Impulse responses for the long-run Fisher identification
of simulated data from the labor hoarding model with permanent contract shocks.
Left column: responses to innovation in investment-specific technology.
Right column: responses to innovation in neutral technology.

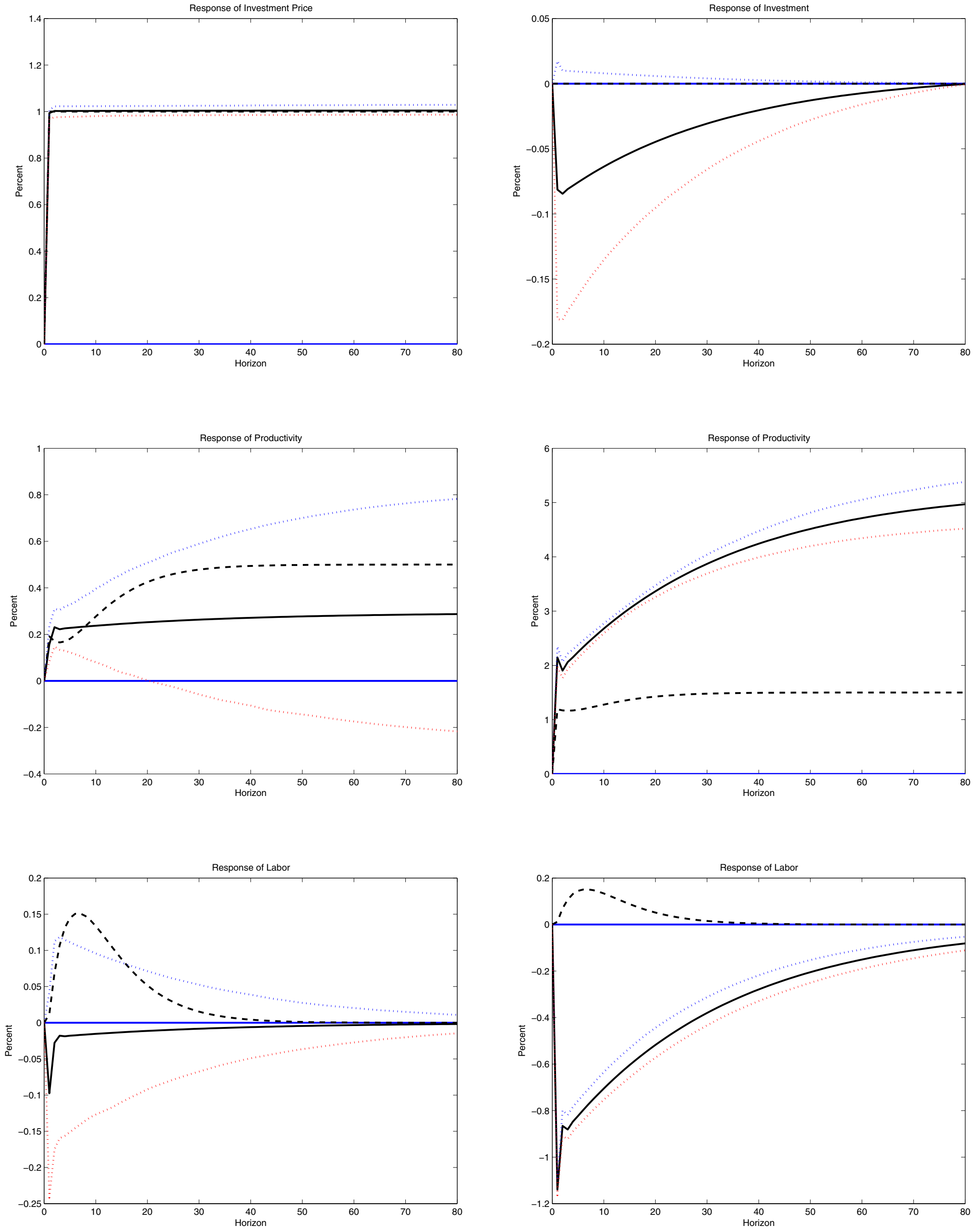

Figure 10:
Impulse responses for the long-run Fisher identification
of simulated data from the labor hoarding model with transitory contract shocks.
Left column: responses to innovation in investment-specific technology.
Right column: responses to innovation in neutral technology.

be viewed in Appendix E.2. Note that the responses of productivity and labor after a neutral technology shock are similar to the ones obtained with the Fisher identification.

4.4 Discussion

Part 4 concerns the theoretical foundation and derivation of restrictions that identify neutral and investment-specific technology in the data. Here, the investigation of the forecast variance decompositions delivers two possible strategies for the identification of technology shocks. As proposed by Uhlig (2003a), these strategies may now be tested on artificial data generated from the model. This works very well for the data generated from Version 1 of the Fisher model and thus supports the validity of these identification procedures. However, the results of the identification of simulated data for the labor hoarding version of the model are pretty bad. What are possible reasons for this?

As stated above, the identification program and procedure have been validated using data from Francis and Ramey (2003). In addition, I increased the sample length of the simulated series up to 100 000 without achieving any improvement. Therefore, this problem may only stem from the specification of the model and the introduction of additional shocks. While I am not able to pin down the source of the problem, I will only list some suspicion on this. Note that the investment price response to investment-specific technology is perfectly identified for the Fisher identification even in the labor hoarding model with permanent contract shocks. Here, the time series for this variable is generated by shocks in investment-specific technology only, as is obvious from the model. Therefore, the Fisher identification is able to detect this shock in the estimation. Even though the responses of productivity and labor are quite different from the theoretical ones, they are still more or less within the confidence bands of the estimation. These are, however, very wide implying that there is a lot of uncertainty in the estimation.

More severe are the problems to identify the responses after a shock in neutral technology. Most importantly, why does labor decline strongly following a shock in neutral technology? As noted above, the alternative identification strategy delivers results similar to the Fisher identification, and the response of productivity for both of these strategies is overstated compared to theory. Hence, everything besides investment-specific technology that explains labor productivity is contained in the response to a neutral shock. Since permanent contract shocks permanently drive down productivity and, in addition, this response is a lot stronger than the movement of productivity after the other shocks, it is quite obvious that the Fisher identification cannot work in the presence of permanent contract shocks. Regarding the forecast variance for transitory contract shocks, it should work, in turn. However, note that even in the transitory case, contract and preference shocks move the productivity by a lot more than the two technology shocks. This, in turn, is not reflected in the forecast variance decomposition. Assume that a lot of these shocks additional to technology occur and recall that contract shocks induce a contradicting response of productivity and labor. This is also true for the effect of preference shocks in the medium run. If the effect of these shocks is then added to the "true" responses of productivity and labor due to a neutral technology shock, productivity would increase more strongly and labor could fall. However, the identification procedure should be able to disentangle these effects, which is why these results remain puzzling.

Even though the identification of simulated data from the labor hoarding model is unsatisfactory, I use my encouraging results from the analysis of the variance decomposition and the identification of simulated data from the original version to take the identification strategies to real data.

5 Empirical Investigation

5.1 Describing the Data

In line with Fisher (2002) and many other authors, I use US labor data from the Bureau of Labor Statistics (BLS). More precisely, I take the BLS nonfarm business measures of "hours of all persons" (total hours worked) and "output per hour of all persons" (productivity)[71]. Per-capita hours are calculated by dividing total hours by an appropriate labor force measure. For this, I use the civilian labor force (age 16 years and over) that is also provided by the BLS. According to Fisher, the real price of investment is measured by the ratio of an investment index and a consumption deflator. I derive US data for these deflators from the National Income and Product Accounts (NIPA) that are provided on the website of the Bureau of Economic Analysis (BEA). My consumption deflator is the index for personal consumption expenditures of the BEA. For the investment price, I use indexes for private fixed investment in structures as well as in equipment and software. All series are quarterly, seasonally adjusted and range from the first quarter of 1948 until the fourth quarter of 2003. Plots of the data can be viewed in Appendix E.2.

However, this measure of the real investment price may cause problems. As Fisher states, "since NIPA investment deflators are known to be poorly measured, finding suitable investment deflators is the main challenge for constructing real investment prices"[72]. This is mainly due to the NIPA measures not taking into account quality change in durable goods. As a consequence, several, rather complicated adjustment procedures are proposed in order to deal with this bias in the data. Fisher uses both an approach by Cummins and Violante (2002), who introduce an equipment deflator, as well as his own, nonetheless similar measure of a total investment deflator. He shows that the adjusted equipment deflator deviates strongly from the NIPA measure, that is it shows a much stronger decline in the price of equipment than the official series.

As already mentioned in Section 3.1, the quantity of investment has risen considerably in postwar data. A strong decline in the price over the same period then implies that investment-specific technology plays an important role for growth. Note that while the fall in the total investment measure is not as strong as in the equipment measure, this series corresponds more closely to investment as used in RBC models[73]. Even though there is still a difference between Fisher's adjusted measure and the official NIPA data, this gap is no longer as large as for the equipment price and can not be considered as "dramatic" as stated by Fisher. To conclude, the NIPA measure for the total investment price is closer to an adjusted measure than the one for the equipment price and thus serves as the relevant one in my specification.

When testing the time series properties of the data employed using the Augmented Dickey-Fuller test, the null hypothesis of first-difference stationarity cannot be rejected for all three series. Next, I check these results using the KPSS test

[71] These are also taken by Fisher (2003).
[72] Fisher (2003), p. 9.
[73] Fisher (2003), p. 10.

which tests the null hypothesis of stationarity. This null hypothesis can in turn not be rejected for the investment price (5% and 1% significance) and for hours worked (all significance values), but is rejected in the case of labor productivity. In line with Christiano, Eichenbaum and Vigfusson (2003a), I conclude that classical tests may not work, especially in the case of labor, and that there is a danger of overdifferencing the variables in the estimation. In my own, alternative identification procedure, estimating all variables in levels should thus not be a problem. In the case of the Fisher estimation, I will include differences of price and productivity in order to be able to compare my results to his.[74]

5.2 The Fisher Identification

The results for the Fisher identification (in the long run, 20 years) are shown in Figure 11.[75] As one can see, there are some obvious differences to the findings of Fisher. While the response of the investment price after an innovation in investment-specific technology is quite similar to Fisher's, the responses of productivity and labor are not. Here, labor productivity falls on impact as in Fisher's results, but then stays persistently below zero. In contrast to the response of labor estimated by Fisher, the one found in this investigation is negative. Note that error bands are wide and that the responses of labor and productivity after a shock in investment-specific technology do not differ significantly from zero. In the case of a neutral technology shock, the investment price rises before converging back to zero, while in Fisher's results, this measure falls. However, the response of productivity and labor for this shock are quite close to Fisher's, even though labor rises more strongly in my specification. Note that these last two responses are also achieved when identifying neutral technology only.[76] While the neutral technology shock is therefore measured in line with Fisher, the problems mainly arise in the case of investment-specific technology. It can be shown that including a productivity measure that is detrended using the consumption deflator, as is also proposed by Fisher, or employing the equipment rather than the total investment price does not significantly change the results. Hence, the differences are probably due to the data employed for the investment price which differ from Fisher's data input, as already mentioned in the previous section.

Let us consider the impulse responses resulting from the Fisher identification scheme when applied to short-run identification. These are shown in Figure 12. One can see that the results are much closer to the ones found by Fisher than for long-run identification. While this looks puzzling at first sight, explanations can be found nonetheless. Fisher states that the correct real investment price measure is especially important in long-run identification. Here, the decline of the investment price over time reflects the assumption that investment-specific shocks are important

[74] The results of the stationarity tests were calculated in EViews and can be viewed in "real_data.wf1" on the enclosed CD.

[75] Note that in all estimations, I include a constant in my specification and choose the lag length according to the appropriate information criteria (Akaike, Schwartz, Hennan–Quinn) that are provided in EViews. The respective file on the enclosed CD is called "real_data.wf1".

[76] That is, neutral technology being the only influence on labor productivity in the long run.

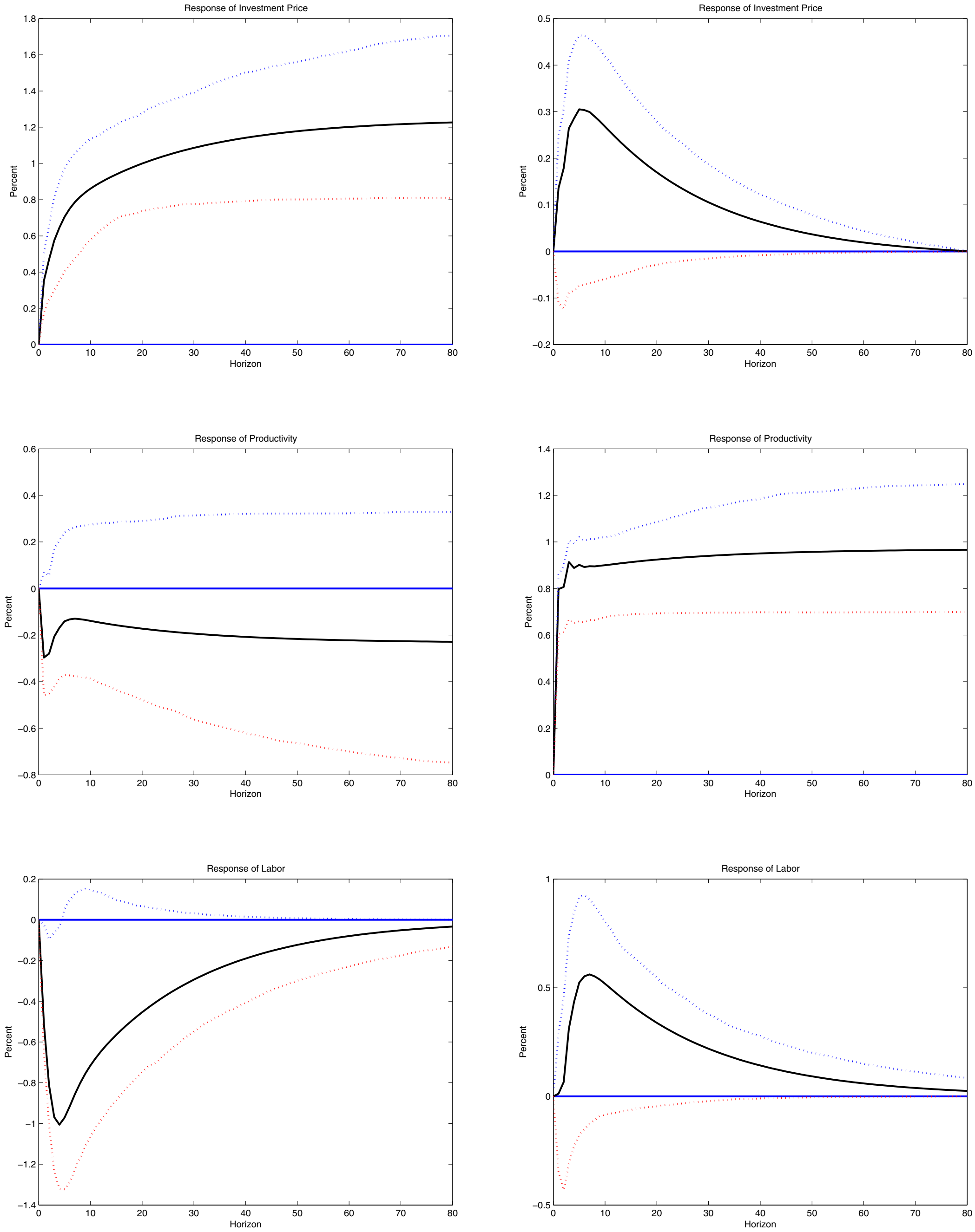

Figure 11:
Impulse responses for the long-run Fisher identification of real data.
Left column: responses to innovation in investment-specific technology.
Right column: responses to innovation in neutral technology.

in the long run, that is, they lead to a significant price fall after a number of periods. In short-run identification, the contemporaneous effect of the shock on the variable is of interest. Based on the results from the model, an investment-specific shock should move the price both in long and short run. Since the results of this identification are quite close to Fisher's, especially compared to the long-run identification, one may conclude that the difference between my price measure and the adjusted price used by Fisher is not of great importance when considering the direct impact of investment-specific shocks. The question remains whether this shock then still reflects technological progress in investment as described by Fisher and Greenwood, Hercowitz and Krusell (1997), since here the long run decline of the price is crucial. Therefore, these results should be handled with care.

5.3 The Alternative Identification

Obviously, the Fisher identification is very sensible to the investment price data employed. To avoid possible problems arising from the choice of this variable, I recommend my alternative identification strategy as described in Section 4. Here, as by Galí (1999), Francis and Ramey (2003) and various other authors, the neutral technology shock is identified as being the only source of fluctuations in labor productivity. Investment-specific technology together with neutral technology is then the main driving force of hours. Therefore, investment-specific technology is determined independently from the influence on the price. While this is clearly supported by the original Fisher model for any identifying horizon, as shown above, it is legitimate to ask whether this restriction can hold in systems with more disturbances than shocks in technology only. Introducing transitory contract and preference shocks clearly reduces the influence of technology shocks on labor. The forecast revision variance shows, however, that technology shocks do play an important role in the intermediate horizon (roughly six years) even in these models.

Note that I will compare results of this identification procedure to my Fisher identification and the responses obtained by Fisher in his empirical investigation. Under the hypothesis that this alternative identification strategy delivers the same shocks as the Fisher identification strategy, the responses of the variables that are restricted in the Fisher identification, that is productivity for both shocks and the investment price for the investment-specific shock, should be close to each other. Due to the somewhat unreliable investment price data and the use of quite similar labor data to Fisher's data, I may take the identification of Fisher himself as a good benchmark.

Let us investigate the results of the alternative specification for the medium run (25 quarters) in Figure 13. Here, the investment price is not included in the specification. While the response of productivity is close to the one obtained with the Fisher identification as shown above and Fisher's own results, labor behaves differently after an innovation in neutral technology. First, it drops below zero after the impact of the shock and then rises strongly above zero. After a certain number of periods, however, it decreases and crosses the zero line again, converging to a small,

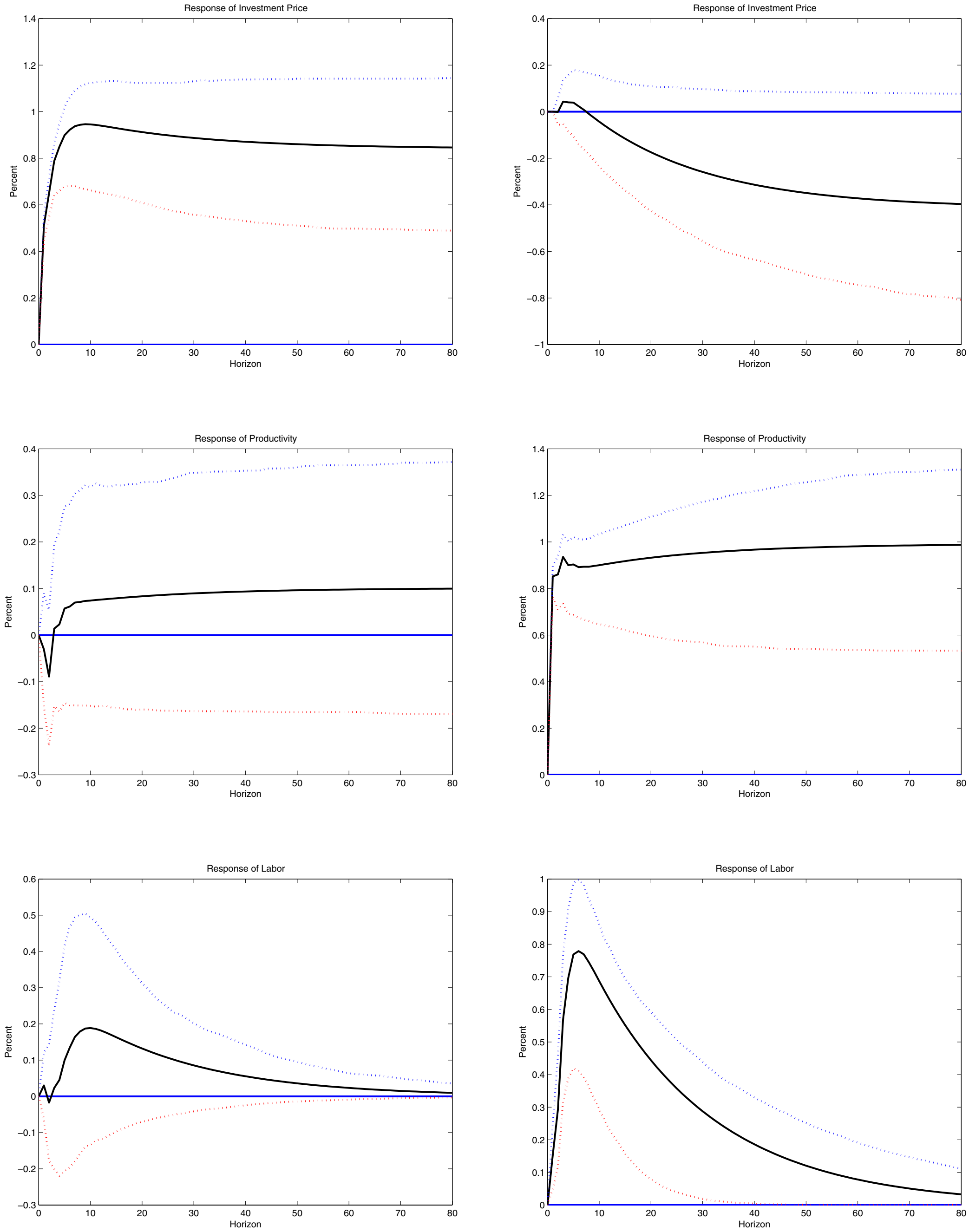

Figure 12:
Impulse responses for the short-run Fisher identification of real data.
Left column: responses to innovation in investment-specific technology.
Right column: responses to innovation in neutral technology.

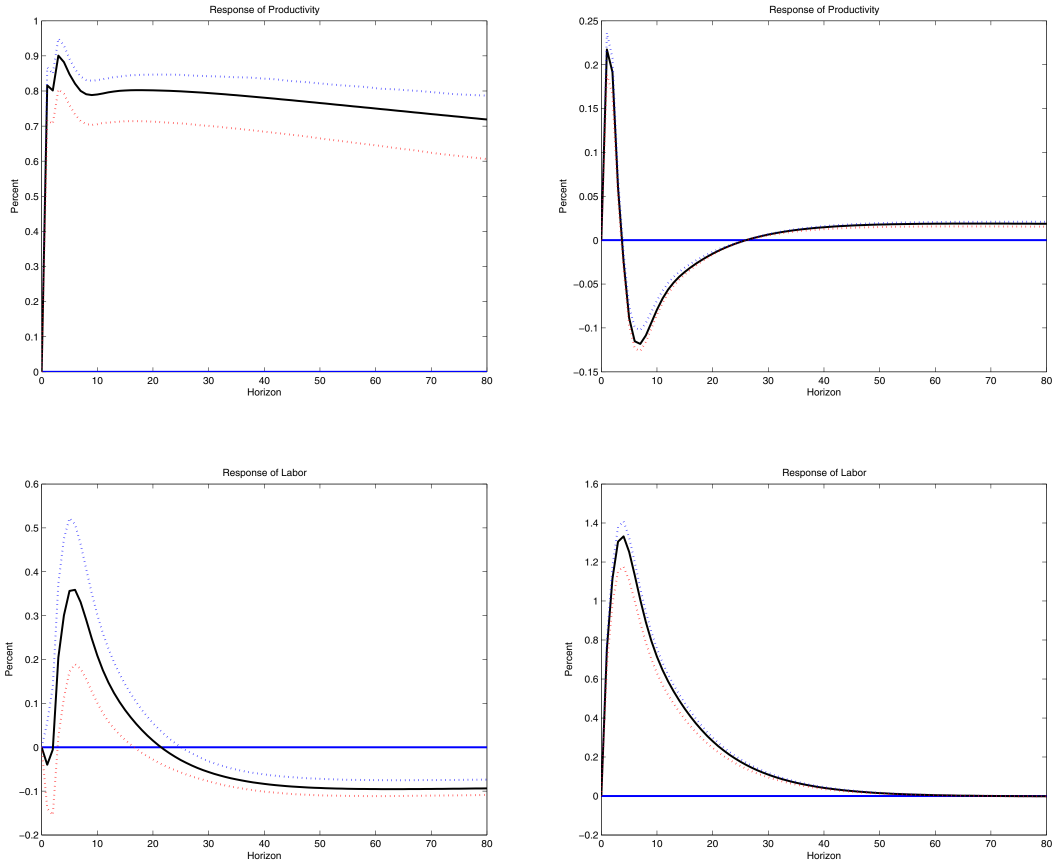

Figure 13:
Impulse responses for the medium-run alternative identification
of real data, no price included.
Left column: responses to innovation in neutral technology.
Right column: responses to innovation in investment-specific technology.

but negative level. While the initial fall and subsequent rise is similar to the results of Fisher, the negative long-run response is not. In the case of investment-specific technology, the responses for labor productivity and labor are very close to the ones Fisher finds; in particular, they are a lot closer than the results of my own Fisher identification with different price data. Since the responses of the variables that are restricted in the Fisher identification – productivity for both technology shocks – are very close to the ones of Fisher, one may conclude that I in fact have identified shocks in neutral and investment-specific technology.

Comparing different identification horizons for this alternative strategy, the medium-run horizon delivers the closest results to Fisher. Graphs for short- and long-run identification are shown in Appendix E.2. For the long run, labor increases more strongly after a neutral technology shock, while after an investment-specific technology shock productivity does not show a positive long-run response. A similar, but even stronger behavior in these directions can be seen for short-run identification.

Next, I include my measure of the investment price in the specification. The results for the medium-run identification are shown in Figure 14. It can be noted that the investment price rises after a shock in neutral technology. Since there is no restriction on the influence of this shock on the price, this should neither be too surprising nor too important for the investigation. The response for the investment price after an innovation in investment-specific technology, however, is very close to the one of my Fisher identification as described above. Quite nicely, this delivers one more hint that the alternative identification procedure in fact identifies the investment-specific technology shock and not some other shock influencing labor such as a shock in preferences.[77]

As before, the responses of productivity to both shocks and the behavior of labor after an investment shock are close to Fisher's results. Note, however, that the initial and the long-run decline of labor after a neutral shock is stronger than in the specification without the investment price. Note also that the comparison with different identification periods does not achieve results that are closer to Fisher than the medium-run responses. This can be seen in Appendix E.2. Thus, even if the price is poorly measured, the results are quite robust for including or not including it in the specification.

5.4 Discussion

The main focus of the identification of shocks is the reaction of labor. As has been shown, labor rises after a shock in investment-specific technology for both identification strategies described above. Fisher as well as Michelacci and Lopez-Salido (2003) support this result in their investigations. In the case of an innovation in neutral technology, the response of labor is not robustly determined across approaches used. While Michelacci and Lopez-Salido find that hours worked fall on impact after a shock in neutral technology and then rise gradually towards zero afterwards, Fisher states that after a small initial drop, hours worked rise before converging back to zero in the long run. This response however is not significantly different from zero when taking confidence bands into account[78].

My alternative specification, in turn, follows Fisher's response in the early periods after the shock and is even significantly positive. However, it is also significantly negative in the long run and implies that neutral technology is labor-saving. This means that the shock in neutral technology creates a large number of jobs in the medium run, when the new technology is applied already, but in the long run more jobs are destroyed by technological progress than are newly created. The jobs that remain are, however, a lot more productive than before the shock, as the response of productivity shows. This result is not in line with standard RBC theory.

Obviously, a unique determination of the response of labor after an innovation in neutral technology is extremely difficult. As already shown in the discussion against

[77]Note that the response of labor and productivity after a shock in preferences is somewhat similar to the one for an innovation in investment-specific technology.

[78]See Fisher, p. 20.

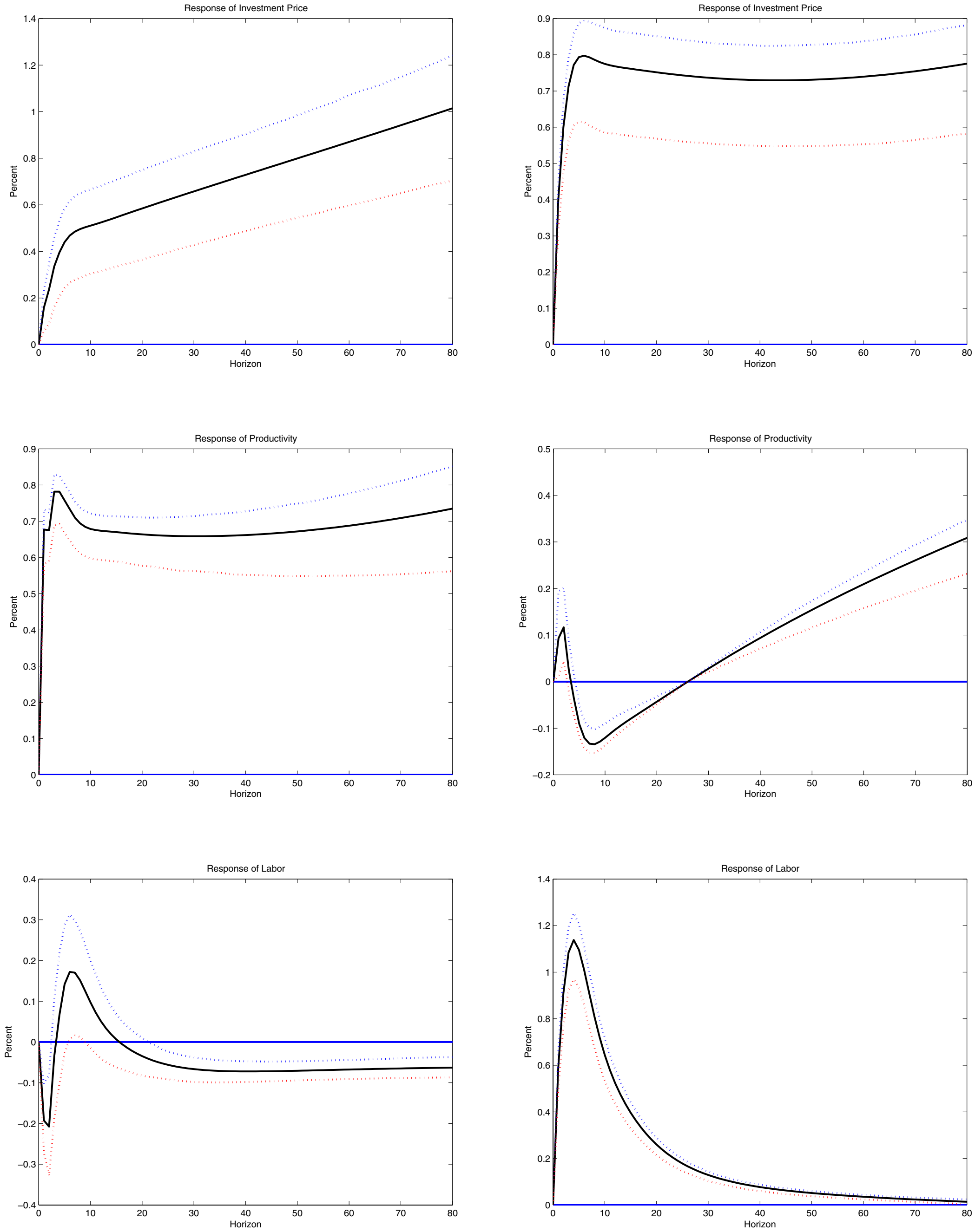

Figure 14:
Impulse responses for the medium-run alternative identification
of real data, price included.
Left column: responses to innovation in neutral technology.
Right column: responses to innovation in investment-specific technology.

and in favor of the Galí findings, this response is not robust against the differencing of hours. The introduction of investment-specific technology complementary to neutral technology does not help to deal with this problem, since contradictory results are also achieved here. However, this approach does shed some light on the dynamic behavior of labor induced by technology as a combined measure of neutral and investment-specific components. Most importantly, Fisher has found a second type of technology shock that is able to explain procyclical labor. Even though I have not further investigated the importance of technology in the business cycle frequency of hours, the impulse responses of labor already show that investment-specific technology drives labor up a lot more than neutral technology may drive labor down. Moreover, this result is quite robust across different specifications. Therefore, assuming that both technology shocks occur in reality and given the result that the influence of investment-specific technology is a lot stronger than the one of neutral technology, labor should still move up in response to technological innovations. This defends the theory of technology-driven business cycles.

Regarding the match of the empirical results to the model implications, one may state that the impulse responses are quite close to each other, at least for the variables investigated in the identification. The influence of the investment-specific technology shock on labor is, however, understated in the model compared to the empirical results.

For further support of these results, more variables could be included in the VAR specification in order to account for possible omitted variables bias. Here, the sensitivity of the results to the choice of variables is to be considered as well. Moreover, one may test for a trend in the variables included in the specification or for structural breaks in the data set. In addition, a further investigation of the identified shocks such as the forecast error variance decomposition or the decomposition of historical time series into the shock components may be appropriate in order to decide whether investment-specific technology shocks are in fact able to explain business cycles in hours.

6 Conclusion

Real-Business-Cycle theory explains macroeconomic fluctuations using shocks in technology. While this widely applied approach has long been tested for its empirical validity on grounds of unconditional correlations between macroeconomic variables, a considerably large debate regarding the correlations of these variables conditional on certain disturbances has been initiated by the work of Galí (1999). Here, the impact and dynamic propagation of single shocks of the model on the variables of the model are of interest rather than the behavior of these variables due to many disturbances. One therefore needs to apply restrictions that identify the effects of technology shocks on the system in order to test the impulse responses implied by RBC models empirically. For this, Galí and various other authors use so-called "long-run identification" based on Blanchard and Quah (1989). This method uses structural vector autoregressions in combination with restrictions on the long-run influence of technology shocks on the model variables and has been discussed in detail in Section 2.

There already exists a considerably large body of research in which technology shocks are identified as the only source of long-run movements in labor productivity, and their dynamic effects are investigated. Here, the effect of technology on labor is crucial in the debate, since its impact on this variable depends on the specification of hours worked in the estimation. If included in levels, hours worked rise after a shock in technology, whereas they fall on impact if specified in first differences. However, regardless of the measure of hours worked included in the specification, most of the research finds that technology shocks do only account for a small fraction of the business cycle fluctuations of hours and are thus not appropriate to explain strongly procyclical hours.

This thesis has considered some recent approaches that have been added to the debate in order to shed light on some of these problems. Uhlig (2003a) states that more sources in the variation of labor productivity may be present in reality. He proposes a model allowing for labor hoarding in order to account for some of these sources. In addition, he employs a procedure of model-based derivation of the restrictions for identifying technology shocks. Here, it is crucial that identification may not be restricted to the long-run effects of technology shocks, but can possibly work better with medium- or short-run effects of these shocks. For this, Uhlig states that the Blanchard–Quah procedure applied by Galí is equivalent to the Cholesky decomposition of the infinite forecast revision variance. This implies that the forecast revision variance derived from the recursive law of motion of a particular theoretical model can well be used in order to determine the identifying restrictions. In addition, this alternative approach can easily be applied in short and intermediate identification horizons as well. While this has been stated but not shown by Uhlig, a large part of this thesis (Section 2) is contributed to the derivation and discussion of this identification method.

Fisher (2002) introduces the identification of investment-specific technology as a complement to neutral technology (the latter being used by Galí) and finds that this technology component is in fact able to explain business cycle fluctuations in hours. He identifies the investment-specific shock as the only influence on the investment price in the long-run, while both neutral and investment-specific technology together are the only driving forces of labor productivity in the long run. Since Fisher has not solved and investigated the theoretical model on which his identifying assumptions are built, Section 3 fills this gap. Here, I also introduce an extension of Fisher's model by allowing for labor hoarding following Uhlig (2003a) and find that the degree of persistence of contract shocks is important for the overall correlations of the model. Here, the labor hoarding model with transitory shocks is to be preferred over the one with permanent shocks. As an additional result, I showed that the theoretical impulse responses from the model solution are matched by the empirical results of Fisher. However, the response of labor after an innovation in investment-specific technology is understated in the model. This response, however, is crucial for the importance of technology as a combined measure of neutral and investment-specific components and should therefore definitely be reflected in the model.

Using Uhlig (2003a), I then derived identifying restrictions for the two technology shocks in both versions of the model based on forecast revision variances. I tried to test these identifying restrictions on simulated data from the model. While this works well for the original Fisher model, there remain some unsolved problems for the labor hoarding model. However, there are several results that arise from this investigation. First, the Fisher identification strategy can be supported for all versions of the model but the presence of permanent contract shocks in the labor hoarding model. Additional to this, I derived an alternative identification strategy that bases the identification of the technology shocks on labor productivity and labor only. This has the advantage that both technology shocks can be determined without taking the price of investment into account for which it is not easy to obtain good data. Applying both procedures to simulated data from the original Fisher model, I found that the Fisher identification may be valid not only in the long-run horizons, but also in short- and medium-run horizons. In turn, the alternative identification strategy works best in the short- to medium-run horizons.

It has to be kept in mind that not all classes of RBC models are embraced by these two identification schemes. First, I consider growth models, that is, the shocks in technology are permanent rather than transitory. Second, I restrict my identification strategy to models where only these technology shocks have permanent effect on productivity. Clearly, this is for example not the case for endogenous growth models, where all disturbances of the system influence the model variables.

As a last part, I took both identification procedures to real data. The Fisher identification delivers responses of productivity and labor that fall after an innovation in investment-specific technology. This strongly contradicts Fisher's own findings. Since I use different data for the investment-price that may be poorly measured, I conclude that these results have to be handled with care and show that the specification is very sensible to the investment-price series employed. This, in turn, favors my alternative specification. Applying this to the data, the responses of productivity after both shocks and labor after the investment-specific shock are close to the investigation done by Fisher. Hence, the procedure identifies the same shocks as Fisher's. However, the response of labor is negative not only on impact but also in the long run after an innovation in neutral technology. While this contradicts RBC theory, one may argue that, due to the strong rise of labor after investment-specific technology shock, a result which is quite robust across specifications, technology understood as a *combined* measure of neutral and investment-specific components induces hours to rise and therefore defends technology-driven business cycle theory.

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A Version 1 of the Fisher Model

A.1 First-Order Conditions

Maximizing

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left[(\frac{\left[c_t^{1-\theta}(1-n_t)^{\theta}\right]^{1-\eta} - 1}{1-\eta}) - \lambda_t (c_t + \frac{1}{v_t} k_t - (1-\delta)\frac{1}{v_t} k_{t-1} - a_t k_{t-1}^{\alpha} n_t^{1-\alpha}) \right]$$

yields as the derivative with respect to consumption

$$\lambda_t = \left[c_t^{1-\theta}(1-n_t)^{\theta}\right]^{-\eta} (1-\theta) \left(\frac{1-n_t}{c_t}\right)^{\theta},$$

with respect to labor

$$\lambda_t (1-\alpha) \frac{y_t}{n_t} = \left[c_t^{1-\theta}(1-n_t)^{\theta}\right]^{-\eta} \theta \left(\frac{c_t}{1-n_t}\right)^{1-\theta},$$

capital

$$\lambda_t \frac{1}{v_t} = \beta E_t \left[\lambda_{t+1} R_{t+1}\right]$$

and lambda

$$0 = c_t + \frac{1}{v_t} k_t - (1-\delta)\frac{1}{v_t} k_{t-1} - y_t,$$

where

$$y_t = a_t k_{t-1}^{\alpha} n_t^{1-\alpha}$$

and

$$R_{t+1} \equiv \frac{1}{v_{t+1}}(1-\delta) + \alpha \frac{y_{t+1}}{k_t}.$$

A.2 Log-linearized Equations for the Ratio-Variables

The system of log-linearized equations for the ratio-variables can be represented as follows:

$$\begin{aligned}
0 &= \widehat{y}_t^{ratio} - \widehat{c}_t^{ratio} - \frac{1}{1-\overline{n}} \widehat{n}_t \\
0 &= -\overline{y}^{ratio} \widehat{y}_t^{ratio} + \overline{c}^{ratio} \widehat{c}_t^{ratio} + \overline{k}^{ratio} \widehat{k}_t^{ratio} - (1-\delta) \overline{k}^{ratio} \mu (\widehat{k}_{t-1}^{ratio} - \frac{1}{1-\alpha}(\varepsilon_{at} + \varepsilon_{vt})) \\
0 &= -\widehat{y}_t^{ratio} + (1-\alpha)\widehat{n}_t + \alpha \widehat{k}_{t-1}^{ratio} - (\frac{\alpha}{1-\alpha})(\varepsilon_{at} + \varepsilon_{vt}) \\
0 &= -\overline{R}\widehat{R}_t^{ratio} + \alpha \frac{\overline{y}}{\overline{k}} \frac{1}{\mu} (\widehat{y}_t^{ratio} - \widehat{k}_{t-1}^{ratio} + \frac{1}{1-\alpha}(\varepsilon_{at} + \varepsilon_{vt})) \\
0 &= E_t[\frac{1-\overline{n}}{\theta - \theta\eta}[\phi_a(\widehat{c}_t^{ratio} - \widehat{c}_{t+1}^{ratio}) - \frac{1}{1-\alpha}(\phi_a \varepsilon_{at+1} + \phi_v \varepsilon_{vt+1}) + \widehat{R}_{t+1}^{ratio}] + \overline{n}\widehat{n}_t - \overline{n}\widehat{n}_{t+1}] \\
\varepsilon_{at} &= \sigma_a \\
\varepsilon_{vt} &= \sigma_v,
\end{aligned}$$

where and $\mu \equiv e^{-\frac{1}{1-\alpha}(\gamma+\nu)}$.

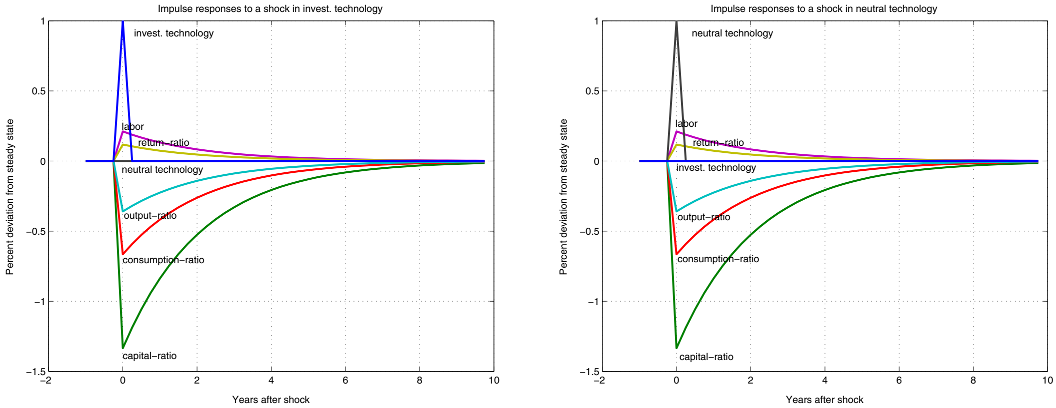

Figure 15:
Impulse responses of the ratio-variables of the original Fisher model

A.3 Impulse Responses to Ratio-Variables

The impulse responses for the ratio-variables are shown in Figure 15. As can be seen from the log-linearized equations above, both technology shocks have an identical impact on the system. Consequently, the impulse responses for both shocks are identical as well which is sensible when looking at the system of equations. The Toolkit program corresponding to these plots, can be found on the enclosed CD.

B Version 2 of the Model

B.1 Social Planner Problem and First-Order Conditions

The social planner problem can be represented as follows:

$$\max E_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{\left[c_t^{1-\theta} (\mu_t - n_t)^{\theta} \right]^{1-\eta} - 1}{1-\eta} \right]$$

s.t.

$$\begin{aligned} y_t &= c_t + \frac{1}{v_t} k_t - (1-\delta) \frac{1}{v_t} k_{t-1} \\ y_t &= a_t k_{t-1}^{\alpha} n_t^{1-\alpha} \end{aligned}$$

further contract labor develops according to

$$n_{ct} = \rho(n_{ct-1} - \phi n_{t-1}) + (1-\rho)\phi n_t + \eta_t,$$

but since "there is no economic force pinning down contract hours", this equation is not relevant for the maximization problem. More precisely, contract labor is no control variable of the agent, since it is determined by an exogenous process. Nevertheless, the relationship between contract labor, actual labor and work time leisure is important for the agent, since both labor and work time leisure have an impact on the agent's utility. In the following, I therefore substitute our labor in the utility function and the constraints and maximize with respect to work time leisure. To complete the model, the shock equations are represented by

$$\begin{aligned}
\eta_t &= \psi_\eta \eta_{t-1} + \epsilon_{\eta t} \\
\mu_t &= (1-\psi_\mu)\overline{\mu} + \psi_\mu \mu_{t-1} + \epsilon_{\mu t}. \\
a_t &= \exp(\gamma + \varepsilon_{at}) a_{t-1} \\
v_t &= \exp(\nu + \varepsilon_{vt}) v_{t-1}.
\end{aligned}$$

After substituting labor, one has to maximize with respect to work time leisure and hence have solve the following maximization problem:

$$L = E_0[\sum_{t=0}^{\infty} \beta^t (\frac{\left[c_t^{1-\theta}(\mu_t - n_t)^\theta\right]^{1-\eta} - 1}{1-\eta}) - \lambda_t (c_t + \frac{1}{v_t} k_t - (1-\delta)\frac{1}{v_t} k_{t-1} - a_t k_{t-1}^\alpha {n_t}^{1-\alpha})]$$

yields for derivative with respect to consumption

$$\lambda_t = \left[c_t^{1-\theta}(\mu_t - n_t)^\theta\right]^{-\eta} (1-\theta) \left(\frac{\mu_t - n_t}{c_t}\right)^\theta,$$

with respect to labor

$$\left[c_t^{1-\theta}(\mu_t - n_t)^\theta\right]^{-\eta} \theta \left(\frac{c_t}{\mu_t - n_t}\right)^{1-\theta} = \lambda_t (1-\alpha) \frac{y_t}{n_t},$$

capital

$$\lambda_t \frac{1}{v_t} = \beta E_t \left[\lambda_{t+1} R_{t+1}\right]$$

and lambda

$$0 = c_t + \frac{1}{v_t} k_t - (1-\delta) \frac{1}{v_t} k_{t-1} - y_t,$$

where

$$y_t = a_t k_{t-1}^\alpha {n_t}^{1-\alpha},$$

and

$$R_{t+1} \equiv \frac{1}{v_{t+1}}(1-\delta) + \alpha \frac{y_{t+1}}{k_t}.$$

After substituting λ_t, we write the system of equations that now completely describes the new model in the following convenient way:

$$\begin{aligned}
(1-\alpha)\frac{y_t}{c_t} &= \frac{\theta}{1-\theta}\frac{n_t}{\mu_t - n_t} \\
y_t &= a_t k_{t-1}^{\alpha} n_t^{1-\alpha} \\
y_t &= c_t + \frac{1}{v_t}k_t - (1-\delta)\frac{1}{v_t}k_{t-1} \\
R_t &= (1-\delta)\frac{1}{v_t} + \alpha\frac{y_t}{k_{t-1}} \\
\frac{1}{v_t} &= \beta E_t\left[\left(\frac{c_t}{c_{t+1}}\right)^{\eta(1-\theta)+\theta}\left(\frac{\mu_{t+1}-n_{t+1}}{\mu_t - n_t}\right)^{-\eta\theta+\theta} R_{t+1}\right] \\
\eta_t &= \psi_\eta \eta_{t-1} + \epsilon_{\eta t} \\
\mu_t &= (1-\psi_\mu)\overline{\mu} + \psi_\mu \mu_{t-1} + \epsilon_{\mu t} \\
a_t &= \exp(\gamma + \varepsilon_{at})a_{t-1} \\
v_t &= \exp(\nu + \varepsilon_{vt})v_{t-1}.
\end{aligned}$$

B.2 Log-linearized Equations for the Ratio-Variables

The following system of equations for the ratio-variables is to be implemented into the Toolkit:

$$\begin{aligned}
0 &= \widehat{y}_t^{ratio} - \widehat{c}_t^{ratio} + \frac{\overline{\mu}}{\overline{\mu}-\overline{n}}\widehat{\mu}_t - \frac{\overline{\mu}}{\overline{\mu}-\overline{n}}\widehat{n}_t \\
0 &= -\widehat{y}_t^{ratio} + (1-\alpha)\widehat{n}_t + \alpha\widehat{k}_{t-1}^{ratio} - (\frac{\alpha}{1-\alpha})(\varepsilon_{at}+\varepsilon_{vt}) \\
0 &= -\overline{y}^{ratio}\widehat{y}_t^{ratio} + \overline{c}^{ratio}\widehat{c}_t^{ratio} + \overline{k}^{ratio}\widehat{k}_t^{ratio} - (1-\delta)\overline{k}^{ratio}\xi(\widehat{k}_{t-1}^{ratio} - \frac{1}{1-\alpha}(\varepsilon_{at}+\varepsilon_{vt})) \\
0 &= -\overline{R}\widehat{R}_t^{ratio} + \alpha\frac{\overline{y}}{\overline{k}}\frac{1}{\xi}(\widehat{y}_t^{ratio} - \widehat{k}_{t-1}^{ratio} + \frac{1}{1-\alpha}(\varepsilon_{at}+\varepsilon_{vt})) \\
0 &= E_t[\frac{\overline{\mu}-\overline{n}}{\theta-\theta\eta}\{\phi_a(\widehat{c}_t^{ratio} - \widehat{c}_{t+1}^{ratio}) + \widehat{R}_{t+1}^{ratio} - \frac{1}{1-\alpha}(\phi_a\varepsilon_{at+1} + \phi_v\varepsilon_{vt+1})\}... \\
&\quad ... + \overline{\mu}\widehat{\mu}_{t+1} - \overline{\mu}\widehat{\mu}_t - \overline{n}\widehat{n}_{t+1} + \overline{n}\widehat{n}_t],
\end{aligned}$$

where $\xi \equiv e^{-\frac{1}{1-\alpha}(\gamma+\nu)}$.

B.3 Toolkit Program for impulse responses of Real Variables

```
% Fisher Model, Version 1, Real Variables
% using H. Uhlig, "A toolkit for solving nonlinear dynamic
% stochastic models easily".

clear;

disp('Fisher Model, Version 4, "Real" Variables');
disp('With contract and preference shocks"');
disp('Jonas D.M. Fisher, "Technology Shocks Matter,"');
```

```
disp('Federal Reserve Bank Chicago, 2002');

disp('Hit any key when ready...');
pause;

% Setting parameters:

delta       = 0.025;   % Depreciation rate for capital
betta       = 0.99;    % discount factor
alpha       = 1/3;     % capital share in production
teta        = 2/3;     % share of utility spend for leisure
eta         = 1.1;     % relative risk aversion
mu_bar      = 1;       % mean time endowment
rho         = 0.8;     % persistence of eta(t) in equation of soc. att.
                       % tow. working place
phi         = 1;       % can be >= 1; adjustment betw. contract
                       % and actual hours
nu          = 0.02;    % mean growth rate of inv.-specific technology
gamma       = 0.02;    % mean growth rate of neutral technology
psi_eta     = 0.99999; % parameter for contract shock, change to zero
                       % for non-permanent shocks
psi_mu      = 0.8;     % parameter for preference shocks
sigma_epsa  = 0.1;     % Standard deviation neutral technology shock.
                       % Units: Percent.
sigma_epsv  = 0.1;     % Standard deviation inv.-specific technology shock.
                       %  Units: Percent.
sigma_eta   = 0.1;     % Standard deviation contract shock.  Units: Percent.
sigma_mu    = 0.3;     % Standard deviation preference shock.  Units: Percent.

% Calculating the steady state:

zet     = exp((-1/(1-alpha))*(gamma + nu));
phi_a   = eta*(1-teta)+teta;
phi_v   = 1-alpha+alpha*phi_a;
R_bar   = (1/betta)*(exp((1/(1-alpha))*(phi_a*gamma+phi_v*nu)));
lambda  = R_bar-1+delta;
omega   = (1-alpha)*((alpha/lambda)^(alpha/(1-alpha)))*((1-teta)/teta);

N_bar   = omega/(((alpha/lambda)^(alpha/(1-alpha))) + omega -
          (((alpha*(zet^(alpha-1)))/lambda)^(1/(1-alpha)))*(1-zet*(1-delta)));
K_bar   = N_bar*(((alpha*(zet^(alpha-1)))/lambda)^(1/(1-alpha)));
Y_bar   = N_bar*((alpha/lambda)^(alpha/(1-alpha)));
C_bar   = Y_bar - K_bar + (1-delta)*K_bar*zet;
```

```
Nc_bar  = (phi/(1-rho))*N_bar;


% Declaring the matrices.

VARNAMES = ['capital-ratio      ',
            'contract labor     ',
            'labor              ',
            'z-hat              ',
            'a-hat              ',
            'v-hat              ',
            'consumption-ratio  ',
            'output-ratio       ',
            'return-ratio       ',
            'capital            ',
            'consumption        ',
            'output             ',
            'return             ',
            'labor productivity ',
            'contract prod.     ',
            'invest. price      ',
            'contract           ',
            'preferences        ',
            'neutral technology ',
            'invest. technology '];

% Translating into coefficient matrices.
% The equations are, conveniently ordered:
% 1)  0 = y(t)_ratio - c(t)_ratio + (mu_bar/(mu_bar-N_bar)) * mu(t)
%         - (mu_bar/(mu_bar+N_bar)) * n(t)
% 2)  0 = -Y_bar * y(t)_ratio + C_bar * c(t)_ratio + K_bar * k(t)_ratio
%         - (1-delta)*K_bar*zet * k(t-1)_ratio +
%           (1-delta)*K_bar*zet*(1/(1-alpha)) * epsv(t)
%            + (1-delta)*K_bar*zet*(1/(1-alpha)) * epsa(t)
% 3)  0 = - y(t)_ratio + (1-alpha) * n(t) + alpha * k(t-1)_ratio
%         - (alpha/(1-alpha)) * epsa(t) - (alpha/(1-alpha)) * epsv(t)
% 4)  0 = -Nc_bar * nc(t) + rho*Nc_bar * nc(t-1) + rho*phi*N_bar * n(t-1) +
%         (1-rho)*phi*N_bar * n(t) + eta(t)
% 5)  0 = -R_bar * R(t)_ratio + (alpha/zet)*(Y_bar/K_bar) * y(t)_ratio
%         -(alpha/zet)*(Y_bar/K_bar) * k(t-1)_ratio
%         + (alpha/zet)*(Y_bar/K_bar)*(1/(1-alpha)) * epsa(t)
%         + (alpha/zet)*(Y_bar/K_bar)*(1/(1-alpha)) * epsv(t)
% 6)  0 = y(t) - w(t) - n(t)
```

```
% 7)  0 = y(t) - wc(t) - nc(t)
% 8)  0 = p(t) + v-hat(t)
% 9)  0 = y(t) - y(t)_ratio - z-hat(t)
% 10) 0 = c(t) - c(t)_ratio - z-hat(t)
% 11) 0 = k(t) - k(t)_ratio - z-hat(t) - v-hat(t)
% 12) 0 = R(t) - R(t)_ratio + v-hat(t)
% 13) 0 = z-hat(t) - (1/(1-alpha)) * a-hat(t) + -(alpha/(1-alpha)) * v-hat(t)
% 14) 0 = a-hat(t) - a-hat(t-1) - epsa(t)
% 15) 0 = v-hat(t) - v-hat(t-1) - epsv(t)
% 16) 0 = E_t [ ((Ln_bar-La_bar)/(teta-teta*eta))*phi_a * c(t)_ratio
%         -((Ln_bar-La_bar)/(teta-teta*eta))*phi_a * c(t+1)_ratio
%         -((Ln_bar-La_bar)/(teta-teta*eta))*(phi_a/(1-alpha)) * epsa(t+1)
%         -((Ln_bar-La_bar)/(teta-teta*eta))*(phi_v/(1-alpha)) * epsv(t+1)
%         + ((Ln_bar-La_bar)/(teta-teta*eta)) * R(t+1)_ratio -Ln_bar * ln(t)
%         -La_bar * la(t) + Ln_bar * ln(t+1) + La_bar * la(t+1) ]
% 17) eta(t+1) = psi_eta * eta(t) + sigma_eta
% 18) mu(t+1)  = psi_mu * mu(t) + sigma_mu
% 19) epsa(t) = sigma_epsa^2
% 20) epsv(t) = sigma_epsv^2
% CHECK: 20 equations, 20 variables.
%
% Endogenous state variables "x(t)": k(t)_ratio, nc(t), n(t), z-hat(t),
%                              a-hat(t), v-hat(t)
% Endogenous other variables "y(t)": c(t)_ratio, y(t)_ratio, R(t)_ratio,
%                              k(t), c(t), y(t), R(t), w(t), wc(t), p(t)
% Exogenous state variables  "z(t)": eta(t), mu(t), epsa(t), epsv(t).
% Switch to that notation.  Find matrices for format
% 0 = AA x(t) + BB x(t-1) + CC y(t) + DD z(t)
% 0 = E_t [ FF x(t+1) + GG x(t) + HH x(t-1) + JJ y(t+1) + KK y(t)
%     + LL z(t+1) + MM z(t)]
% z(t+1) = NN z(t) + epsilon(t+1) with E_t [ epsilon(t+1) ] = 0,

% Order k(t)_ratio  nc(t)   n(t)  z-hat(t)  a-hat(t)  v-hat(t)
AA = [  0,  0,   -mu_bar/(mu_bar-N_bar),  0, 0, 0,
  K_bar,  0,  0,  0,  0,  0,
  0,  0,  1-alpha,  0,  0,  0,
  0,  -Nc_bar,   (1-rho)*phi*N_bar, 0,  0,  0,
  0,  0,  0,  0,  0,  0,
  0,  0,  -1,   0,  0,  0,
  0,  -1,   0,  0,  0,  0,
  0,  0,  0,  0,  0,  1,
  0,  0,  0,  -1,   0,  0,
  0,  0,  0,  -1,   0,  0,
```

```
  -1,   0,  0,  -1,   0,  -1,
  0,  0,  0,  0,  0,  1,
  0,  0,  0,  1,  -(1/(1-alpha)), -(alpha/(1-alpha)),
  0,  0,  0,  0,  1,  0,
  0,  0,  0,  0,  0,  1 ];

% Order k(t-1)_ratio  nc(t-1)   n(t-1)  z-hat(t-1)  a-hat(t-1)  v-hat(t-1)
BB = [ 0,   0,  0,  0, 0, 0,
 -(1-delta)*K_bar*zet,  0,  0,  0,  0,  0,
 alpha,   0,  0,  0,  0,  0,
 0,   rho*Nc_bar,  rho*phi*N_bar, 0,  0,  0,
  -(alpha/zet)*(Y_bar/K_bar), 0,  0,  0,  0,  0,
 0,   0,  0,  0,  0,  0,
 0,   0,  0,  0,  0,  0,
 0,   0,  0,  0,  0,  0,
 0,   0,  0,  0,  0,  0,
 0,   0,  0,  0,  0,  0,
 0,   0,  0,  0,  0,  0,
 0,   0,  0,  0,  0,  0,
 0,   0,  0,  0,  0,  0,
 0,   0,  0,  0,  -1,   0,
 0,   0,  0,  0,  0,  -1 ];

%Order: c(t)_ratio  y(t)_ratio  R(t)_ratio  k(t)  c(t)  y(t)  R(t)  w(t)
% wc(t) p(t)
CC = [  -1, 1,  0,  0, 0, 0, 0, 0, 0, 0,
  C_bar,  -Y_bar,   0,  0,  0,  0,  0,  0,  0,  0,
  0,  -1,   0,  0,  0,  0,  0,  0,  0,  0,
  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,
  0,  (alpha/zet)*(Y_bar/K_bar),  -R_bar, 0,  0,  0,  0,  0,  0,  0,
  0,  0,  0,  0,  0,  1,  0,  -1, 0,  0,
  0,  0,  0,  0,  0,  1,  0,  0,  -1, 0,
  0,  0,  0,  0,  0,  0,  0,  0,  0,  1,
  0,  -1,   0,  0,  0,  1,  0,  0,  0,  0,
  -1, 0,  0,  0,  1,  0,  0,  0,  0,  0,
  0,  0,  0,  1,  0,  0,  0,  0,  0,  0,
  0,  0,  -1, 0,  0,  0,  1,  0,  0,  0,
  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,
  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,
  0,  0,  0,  0,  0,  0,  0,  0,  0,  0 ];

%Order: eta(t)  mu(t) epsa(t)    epsv(t)
DD = [  0, mu_bar/(mu_bar-N_bar), 0, 0,
```

```
  0,  0,  (1-delta)*K_bar*zet*(1/(1-alpha)), (1-delta)*K_bar*zet*(1/(1-alpha)),
  0,  0,  -(alpha/(1-alpha)),  -(alpha/(1-alpha)),
  1,  0,  0,   0,
  0,  0,  (alpha/zet)*(Y_bar/K_bar)*(1/(1-alpha)), ...
  ...(alpha/zet)*(Y_bar/K_bar)*(1/(1-alpha)),
  0,  0,  0,   0,
  0,  0,  0,   0,
  0,  0,  0,   0,
  0,  0,  0,   0,
  0,  0,  0,   0,
  0,  0,  0,   0,
  0,  0,  0,   0,
  0,  0,  0,   0,
  0,  0,  -1,  0,
  0,  0,  0,  -1 ];


FF = [ 0,   0,  -N_bar,  0,  0,  0 ];

GG = [ 0,   0,  N_bar,  0,  0,  0 ];

HH = [ 0,   0,  0,  0,  0,  0 ];

JJ = [ -((mu_bar-N_bar)/(teta-teta*eta))*phi_a,  0, ...
        ((mu_bar-N_bar)/(teta-teta*eta)),  0,  0,  0,  0,  0,  0,  0 ];

KK = [ ((mu_bar-N_bar)/(teta-teta*eta))*phi_a,  0,  0, ...
        0,  0, 0, 0,  0,  0,  0 ];

LL =
[0,mu_bar,-((mu_bar-N_bar)/(teta-teta*eta))*(phi_a/(1-alpha)),...
      -((mu_bar-N_bar)/(teta-teta*eta))*(phi_v/(1-alpha)) ];

MM = [ 0,   -mu_bar,  0,   0 ];

NN = [ psi_eta, 0,      0,   0,
       0,       psi_mu, 0,   0,
       0,       0,      0,   0,
       0,       0,      0,   0 ];

Sigma = [ sigma_eta^2   0,           0,                 0,
          0,            sigma_mu^2, 0,                 0,
          0,            0,           sigma_epsa^2,     0,
```

```
          0,             0,          0,              sigma_epsv^2 ];

% Setting the options:

[l_equ,m_states] = size(AA);
[l_equ,n_endog ] = size(CC);
[l_equ,k_exog  ] = size(DD);


PERIOD     = 4;  % number of periods per year, i.e. 12 for monthly,
                 % 4 for quarterly
GNP_INDEX  = 2;%12; % Index of output among the variables
                 % selected for HP filter
IMP_SELECT = [2,3,12,(14:16)];
   %  a vector containing the indices of the variables to be plotted
DO_PLOTS=1;
HORIZON = 80;
DO_MOMENTS = 1; % Calculates moments based on frequency-domain methods
HP_SELECT  = 1:(m_states+n_endog+k_exog);
% Selecting the variables for the HP Filter calcs.
DO_STATE_RESP=0;
% keine abweichung von endogenous state vom steady state
DO_SIMUL   = 1; % Calculates simulations
SIM_LENGTH = 500;%10000;
DO_HP_FILTER=1;%0;
SIM_MODE=1;
SIM_GIVEN_EPS = 0;
SIM_RANDOM_START = 1;
SIM_DISCARD = 200;
SIM_SELECT =  1:(m_states+n_endog+k_exog);
DO_QZ = 1;
%to make calculation of moments possible, QQ has to be invertable


% Starting the calculations:

do_it;

% saving simulated series and theoretical impulse responses
% this part of the program is based on "tax_labhoard_simul.m" provided to
% me by Harald Uhlig
plot_simul=0;
SAVE_IMPRESP = 0;
```

```
if plot_simul,
    sel = [2,15,16];
    %selects contract labor, contract labor productivity and the
    % investment price
    %similar to 3-variable data set by Fisher.

    series = sim_xyz(sel,:);

    fid = fopen('fisher_nocontr_data_simlong.m','w');
    % fid = fopen('fisher_hoard_data_simlong.m','w'); if psi_eta == 0

    fprintf(fid,'%% Simulation, %6.0f periods in length for the variables\n',...
                SIM_LENGTH);
    fprintf(fid,'%% labor, labor productivity and investment price\n');
    fprintf(fid,' \n');
    fprintf(fid,'series = [ ...\n');
    fprintf(fid,'    %10.5f,  %10.5f,  %10.5f\n',series(:,1:SIM_LENGTH-1));
    fprintf(fid,'    %10.5f,  %10.5f,  %10.5f];\n',series(:,SIM_LENGTH));
    fclose(fid);

    if psi_eta == 0,
        fisher_nocontr_data_simlong;
    else
        fisher_hoard_data_simlong;
    end;

    xline = 1:10000;
    hndl=plot(xline,series(:,1),'k-',xline,series(:,2),'k:',xline,...
         series(:,3),'k*');
end;

if SAVE_IMPRESP,
    ndx_labor = 2;
    ndx_labprod = 15;
    ndx_price = 16;
    help = m_states+n_endog+k_exog;
    sel_contract = [ndx_labor,ndx_labprod,ndx_price];
    sel_preference = [help+ndx_labor,help+ndx_labprod,help+ndx_price];
    sel_neutral = [help*2+ndx_labor,help*2+ndx_labprod,help*2+ndx_price];
    sel_invest = [help*3+ndx_labor,help*3+ndx_labprod,help*3+ndx_price];
    ir_theory_contract = [   0: (HORIZON-1);
                            Resp_mat(sel_contract,:) ];
```

```
    ir_theory_preference = [   0: (HORIZON-1);
                            Resp_mat(sel_preference,:) ];
    ir_theory_neutral = [   0: (HORIZON-1);
                            Resp_mat(sel_neutral,:) ];
    ir_theory_invest = [    0: (HORIZON-1);
                            Resp_mat(sel_invest,:) ];



    fid = fopen('fisher_nocontr_ir_theory.m','w');
    % fid = fopen('fisher_hoard_ir_theory.m','w'); if psi_eta == 0

    fprintf(fid,'%% Theory-impulse response of\n');
    fprintf(fid,'%% labor, labor productivity and investment price to ...
             neutral and investment_specific technology shock.\n');
    fprintf(fid,' \n');
    fprintf(fid,'ir_theory_contract = [ ...\n');
    fprintf(fid,'    %10.5f,  %10.5f,  %10.5f,  %10.5f\n',...
                ir_theory_contract(:,1:HORIZON-1));
    fprintf(fid,'    %10.5f,  %10.5f,  %10.5f,  %10.5f];\n',...
                ir_theory_contract(:,HORIZON));
    fprintf(fid,' \n');
    fprintf(fid,'ir_theory_preference = [ ...\n');
    fprintf(fid,'    %10.5f,  %10.5f,  %10.5f,  %10.5f\n',...
                ir_theory_preference(:,1:HORIZON-1));
    fprintf(fid,'    %10.5f,  %10.5f,  %10.5f,  %10.5f];\n',...
                ir_theory_preference(:,HORIZON));
    fprintf(fid,' \n');
    fprintf(fid,'ir_theory_neutral = [ ...\n');
    fprintf(fid,'    %10.5f,  %10.5f,  %10.5f,  %10.5f\n',...
                ir_theory_neutral(:,1:HORIZON-1));
    fprintf(fid,'    %10.5f,  %10.5f,  %10.5f,  %10.5f];\n',...
                ir_theory_neutral(:,HORIZON));
    fprintf(fid,' \n');
    fprintf(fid,'ir_theory_invest = [ ...\n');
    fprintf(fid,'    %10.5f,  %10.5f,  %10.5f,  %10.5f\n',...
                ir_theory_invest(:,1:HORIZON-1));
    fprintf(fid,'    %10.5f,  %10.5f,  %10.5f,  %10.5f];\n',...
                ir_theory_invest(:,HORIZON));
    fclose(fid);
end;
```

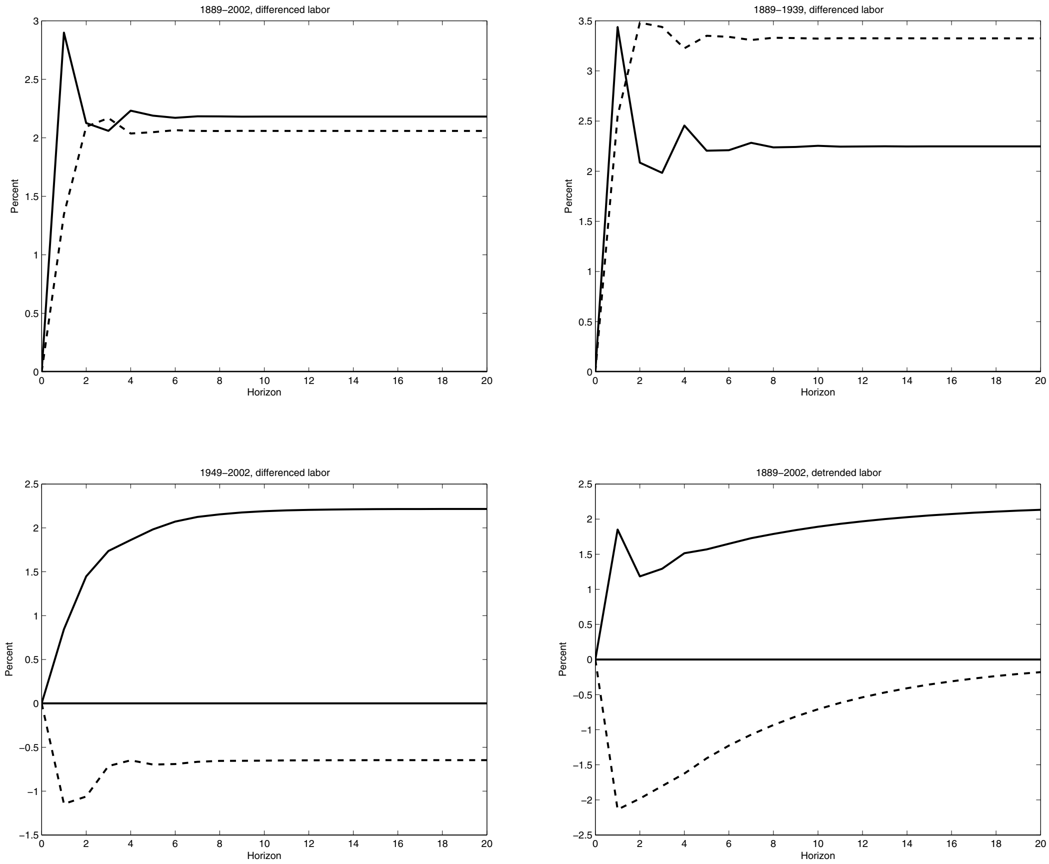

Figure 16:
Applying the Cholesky method to the Francis and Ramey data.
Solid line: labor productivity; dashed line: labor.

C Francis and Ramey

In order to rule out programming mistakes, I applied the Cholesky method to the original data set of Francis and Ramey (2003) that was provided to me by Harald Uhlig. Figure 16 contains plots for the specification with differenced labor for the long, prewar and postwar sample and the specification with detrended labor for the long sample. The solid line denotes the response for labor productivity, while the dashed line represents labor. These plots are considerably close the results of Francis and Ramey as shown in their paper, meaning that the values of the impulse response coefficients deviate by little. The respective MATLAB-program is available on the enclosed CD. Note that this program uses the same procedure as outlined in Appendix E.

D Forecast Variance Decomposition for Models

In Section 4, the forecast revision variance decomposition is investigated for all versions of the model. Here, I show the MATLAB-program for Version 1 of the model. The programs for the other versions can be viewed on the enclosed CD, but are very similar to this file.

```
% for Fisher_orig_real
% Generating plots of variances for the Fisher model
% is based on Uhlig's file "tax_labhoard_plots.m" which generates some plots
% for ex_tax_labhoard.m example.
% fisher_orig_real has to be executed before
disp('Calculating and Plotting the Forecast Variances of the Fisher Model');
disp('Hit any key when ready...');
pause;
ndx_labprod = 13; % the index in the total list of variables,
                  % containing labor productivity
ndx_labor = 7; % the index in the total list of variables, containing labor
ndx_set = [ndx_labprod, ndx_labor];
k_ndx_ntec = 1;
% index among exog. variables of neutral technology shock
k_ndx_itec = 2;
% index among exog. variables of invest.-specific technology shock
fisher_VAR_decomp;

for ndx_ndx =1: length(ndx_set),
 ndx_pick = ndx_set(ndx_ndx);
 frac_ntec = 100*squeeze(frac_rev_tab(ndx_pick,k_ndx_ntec,:));
 frac_itec = 100*squeeze(frac_rev_tab(ndx_pick,k_ndx_itec,:));

 xline = 0:(HORIZON_VAR_decomp-1);
 reverse = HORIZON_VAR_decomp : -1 : 1;
 mn_ndx = 2;
 mx_ndx = floor(HORIZON_VAR_decomp*.7);
 [m,i]=max(frac_ntec(mn_ndx:mx_ndx));
 x_ntec = (i-1); y_ntec = m/2;
 [m,i]=max(frac_itec(mn_ndx:mx_ndx));
 x_itec = (i-1); y_itec = m/2 + frac_ntec(i);

 hndl=plot(xline,frac_ntec,'k-',xline,frac_ntec+frac_itec,'k-',xline,...
          xline*0+100,'k-');
 hold on;
 fill([0,xline,max(xline)],[0;frac_ntec;0],'r');
 fill([xline,xline(reverse)],[frac_ntec+frac_itec;frac_ntec(reverse)],'y');
```

```
 hold off;
 if ndx_pick == ndx_labprod,
 title(['Labor Productivity']);
 else
 title(['Labor ']);
 end;
 xlabel('Horizon');
 ylabel('Percent');
 text(x_ntec,y_ntec,'Neutral Tech.');
 text(x_itec,y_itec,'Invest. Tech.');
 enlarge;
 disp('Inspect Figure');
 disp('Hit any key when ready...');
 pause;
end;
```

The actual variance decomposition is calculated in fisher_VAR_decomp:

```
% using Uhlig's "VAR_decomp.m" which does a variance-decomposition
% for the k-step ahead prediction errors
% from the recursive law of motion (RLLN).
% It needs or computes an RLLN or VAR of the form
% x_tilde(t) = PP_tilde x(t-1) + QQ_tilde eps(t)
% with Sigma, a matrix assumed diagonal,
% as the variance-covariance matrix of eps(t)
% This routine can be used "stand-alone",
% if PP_tilde and QQ_tilde are given, or in
% connection with the "Toolkit", where  x_tilde is
% the vector [x(t)', y(t)', z(t)' ]
% and where PP_tilde and QQ_tilde are computed from the "toolkit"-RLLN.
% It is "general purpose" and not tied to a particular example.

DO_CALC_PP_TILDE = 1;
HORIZON_VAR_decomp = 80;
VAR_decomp_TINY = .00000000001;  % to avoid divisions by zero

if DO_CALC_PP_TILDE,
    % consolidate
    % x(t) = PP x(t-1) + QQ z(t)
    % y(t) = RR x(t-1) + SS z(t)
    % z(t) = NN z(t-1) + eps(t)
    % to
    % x_tilde(t) = PP_tilde x(t-1) + QQ_tilde eps(t)
    % where x_tilde(t)' = [ x(t)', y(t)', z(t)']
    % and PP_tilde, QQ_tilde contain the appropriate matrices, see below.
```

```
    if ~exist('m_states'),
        [n_endog,m_states]=size(RR);
        k_exog=min(size(NN));
        % note: NN is a square matrix, and for the Fisher model NN is a zero
        % matrix!
    end;
    PP_tilde = [ PP,    zeros(m_states,n_endog), QQ*NN,
                 RR,    zeros(n_endog, n_endog), SS*NN,
                 zeros(k_exog,m_states+n_endog),    NN ];
    QQ_tilde = [ QQ
                 SS
                 eye(k_exog) ];
end;
[n_tilde,k_tilde]=size(QQ_tilde);

% Initializations:

% Variance of k-step ahead forecast revision error, i.e. of
% eps(t,k) = E_{t-1}[x(t+k)]-E_t[x(t+k)]
% This is the relevant object for Blanchard-Quah-type decompositions.
var_rev_tab = zeros(n_tilde,k_tilde,HORIZON_VAR_decomp);
sum_rev_tab = zeros(n_tilde,HORIZON_VAR_decomp);
frac_rev_tab = var_rev_tab;

% Calculations:

QQ_multistep = QQ_tilde;
QQ_square_sum = 0*QQ_tilde;
Sigma_tilde = diag(diag(Sigma)); % to make sure of diagonality
for hor_j = 1 : HORIZON_VAR_decomp,
    var_rev_tab(:,:,hor_j)=QQ_multistep.^2*Sigma_tilde;
    for n_j = 1 : n_tilde,
        sum_rev_tab(n_j,hor_j) = sum(var_rev_tab(n_j,:,hor_j));
        frac_rev_tab(n_j,:,hor_j)=var_rev_tab(n_j,:,hor_j)/...
                             (sum_rev_tab(n_j,hor_j)+VAR_decomp_TINY);
    end;
    QQ_multistep = PP_tilde*QQ_multistep;
end;
```

E Identification

E.1 Code

Here, I provide the MATLAB-code for the identification of real data. However, the procedure is equivalently used in the identification of the simulated data and can be viewed on the enclosed CD.

```
% impulse responses due to long-, short- and medium-run identification
% using the Cholesky decomposition of the forecast revision variance
% using real data from the Bureau of Labor Statistics and Bureau of
% Economic Analysis

clear;
data;
sample = 224; % number of observations
HORIZON_impresp = 80; % horizon for impulse responses
HORIZON_ident = 80; % identifying horizon

% options:
SPEC_VERSION = 5; % chose identification strategy
do_ci = 1; % plots confidence intervals

switch SPEC_VERSION,
    case 1 % Fisher identification
        % setting some parameters
        lag_length = 2; % determined through appropriate criteria in EViews
        use_diff = 1; % if differenced variables are used
        num_vars = 3; % number of variables in the VAR
        num_vars_diff = 2; % number of differenced variables in the VAR
        num_vars_lev = 1; % number of level variables in the VAR
        ndx_price = 1; % index of price, position in hte VAR specification
        ndx_prod = 2; % index of productivity
        ndx_labor = 3; % indes of labor
        shocks_ident = 2; % number of shocks to be identified
        level_ident = 0;
        % if identifying restrictions relate to level variables
        do_trans = 0; % no transformation necessary for real data
                      % but used in fisher_orig_ident
        % data choice
        DATA = zeros(sample,num_vars);
        DATA(:,ndx_price) = -log(series(:,1)); % inverted price
        DATA(:,ndx_prod) = log(series(:,3));
        DATA(:,ndx_labor) = log(series(:,5));
```

```
case 2 % Fisher identification, neutral shock only
    % setting some parameters
    lag_length= 2;
    use_diff = 1;
    num_vars = 2;
    num_vars_diff = 1;
    num_vars_lev = 1;
    ndx_price = 0;
    ndx_prod = 1;
    ndx_labor = 2;
    shocks_ident = 1;
    level_ident = 0;
    do_trans = 0;
    % data choice
    DATA = zeros(sample,num_vars);
    DATA(:,ndx_prod) = log(series(:,3));
    DATA(:,ndx_labor) = log(series(:,5));

case 3 % Fisher identification, using EQUIPMENT price
    % setting some parameters
    lag_length= 2;
    use_diff = 1;
    num_vars = 3;
    num_vars_diff = 2;
    num_vars_lev = 1;
    ndx_price = 1;
    ndx_prod = 2;
    ndx_labor = 3;
    shocks_ident = 2;
    level_ident = 0;
    do_trans = 0;
    % data choice
    DATA = zeros(sample,num_vars);
    DATA(:,ndx_price) = -log(series(:,2));
    DATA(:,ndx_prod) = log(series(:,3));
    DATA(:,ndx_labor) = log(series(:,5));

case 4 % Fisher identification, using DEFLATED productivity
    % setting some parameters
    lag_length= 2;
    use_diff = 1;
    num_vars = 3;
    num_vars_diff = 2;
```

```
        num_vars_lev = 1;
        ndx_price = 1;
        ndx_prod = 2;
        ndx_labor = 3;
        shocks_ident = 2;
        level_ident = 0;
        do_trans = 0;
        % data choice
        DATA = zeros(sample,num_vars);
        DATA(:,ndx_price) = -log(series(:,2));
        DATA(:,ndx_prod) = log(series(:,6));
        DATA(:,ndx_labor) = log(series(:,5));

    case 5 % alternative identification, using levels
        % setting some parameters
        ndx_price = 3;
        lag_length = 3;
        use_diff = 0;
        if ndx_price == 0,
            num_vars = 2;
            num_vars_lev = 2;
        else
            num_vars = 3;
            num_vars_lev = 3;
        end;
        num_vars_diff = 0;
        ndx_prod = 1;
        ndx_labor = 2;
        shocks_ident = 2;
        level_ident = 1;
        do_trans = 0;
        % data choice
        DATA = zeros(sample,num_vars);
        if ndx_price ~= 0;
            DATA(:,ndx_price) = -log(series(:,2));
        end;
        DATA(:,ndx_prod) = log(series(:,3));
        DATA(:,ndx_labor) = log(series(:,5));

    otherwise
        break;
end;
```

```
% defining the appropriate series from raw data
datamass;

% the series are now called "Ynew" for zero lag and "Ynlagone", "Ynlagtwo",
% "Ynlagthree" for the respective lags of the series

switch lag_length,
    case 1
        ZZ = [  Ynlagone
                ones(1,sample)];
    case 2
        ZZ = [  Ynlagone
                Ynlagtwo
                ones(1,sample)];
    case 3
            ZZ = [ Ynlagone
                   Ynlagtwo
                   Ynlagthree
                   ones(1,sample)];
    otherwise
        break;
end;

% estimation
Btildenew = Ynew*(ZZ')*(inv(ZZ*(ZZ')));
UU = Ynew - Btildenew*ZZ;
Omega = (1/sample)*(UU*(UU'));
Btilde = zeros(num_vars,num_vars,HORIZON_impresp);
Btilde(:,:,1) = Btildenew(:,(1:num_vars));
if lag_length >=2,
    Btilde(:,:,2) = Btildenew(:,((num_vars+1):(2*num_vars)));
end;
if lag_length == 3,
    Btilde(:,:,3) = Btildenew(:,(2*num_vars+1):(3*num_vars));
end;

% calculation of matrix A
Phi = zeros(num_vars,num_vars,HORIZON_ident+1);
Phi(:,:,1) = eye(num_vars,num_vars);
% caution, zero subscript are not allowed,
% thus, use Phi(:,:,1) instead of Phi(:,:,0)!!!
% and shift the matrices in the MA-representation accordingly
```

```
% calculation of CCtilde which is subject to short-, medium- or
% long-run restrictions;
CCtilde = eye(num_vars,num_vars);
for s = 2 : HORIZON_ident+1,
    Phitemp = zeros(num_vars,num_vars);
    for j= 1:(s-1),
        Phitemp = Phitemp + Phi(:,:,(s-j))*Btilde(:,:,j);
    end;
    Phi(:,:,s) = Phitemp;
    CCtilde = CCtilde + Phi(:,:,s);
end;

if level_ident,
    CCtilde = Phi(:,:,HORIZON_ident+1);
end;

% we are now looking for some matrix A, such that AA'=Omega and such that
% CCtilde*A is lower triangular
Sigma=CCtilde*Omega*CCtilde';
Qtrans=chol(Sigma);
Q=Qtrans';
Atilde = inv(CCtilde)*Q;

% confidence intervals
if do_ci,
    do_real = 1;
    bootstrap;
end;

% calculating and plotting the impulse responses
% the shock is assumed to occur in period 1

 Time_axis = (0 : HORIZON_impresp);

 for shock_counter = 1 : shocks_ident,
     Response = zeros(num_vars,HORIZON_impresp);
     shock_vector = zeros(num_vars,1);
     shock_vector(shock_counter) = 1;
     Phi = zeros(num_vars,num_vars,HORIZON_impresp);
     Phi(:,:,1) = eye(num_vars,num_vars);
         for s = 2 : HORIZON_impresp,
             Phitemp = zeros(num_vars,num_vars);
             for j= 1:(s-1),
```

```
                Phitemp = Phitemp + Phi(:,:,(s-j))*Btilde(:,:,j);
            end;
            Phi(:,:,s) = Phitemp;
        end;

     CCneutilde = zeros(num_vars,num_vars);
     for time_counter = 1 : HORIZON_impresp,
         CCneutilde = CCneutilde + Phi(:,:,time_counter);
         Resphelp = CCneutilde*Atilde*shock_vector;
         for i = 1:num_vars_diff,
             Response(i,time_counter) = Resphelp(i);
         end;
     end;
     for time_counter = 1 : HORIZON_impresp,
         Resphelp = Phi(:,:,time_counter)*Atilde*shock_vector;
         for k = 1:num_vars_lev,
              Response(num_vars_diff+k,time_counter) = Resphelp(num_vars_diff+k);
         end;
     end;

    % in order to plot the percentage deviation
    Response = Response*100;

    if ndx_price ~= 0,
        if do_ci,
            hndl = plot(Time_axis,0*Time_axis,Time_axis,..
                        [0,Response(ndx_price,:)],'k-',...
                        Time_axis,[0,CI_upper(ndx_price,:,shock_counter)],'b:',...
                        Time_axis,[0,CI_lower(ndx_price,:,shock_counter)],'r:');
        else
            hndl = plot(Time_axis,0*Time_axis,Time_axis,...
                        [0,Response(ndx_price,:)],'k-');
        end;
            title(['Response of Investment Price']);
            xlabel('Horizon');
            ylabel('Percent');
            set(hndl,'LineWidth',2);
            disp('Inspect Figure');
            disp('Hit any key when ready...');
            pause;
    end;

    if do_ci,
```

```
        hndl = plot(Time_axis,0*Time_axis,Time_axis,...
                    [0,Response(ndx_prod,:)],'k-',...
                    Time_axis,[0,CI_upper(ndx_prod,:,shock_counter)],'b:',...
                    Time_axis,[0,CI_lower(ndx_prod,:,shock_counter)],'r:');
    else
        hndl = plot(Time_axis,0*Time_axis,Time_axis,[0,Response(ndx_prod,:)],'k-');
    end;
    title(['Response of Productivity']);
    disp('Inspect Figure');
    xlabel('Horizon');
    ylabel('Percent');
    set(hndl,'LineWidth',2);
    disp('Hit any key when ready...');
    pause;

    if do_ci,
      hndl = plot(Time_axis,0*Time_axis,Time_axis,...
                    [0,Response(ndx_labor,:)],'k-',...
                    Time_axis,[0,CI_upper(ndx_labor,:,shock_counter)],'b:',...
                    Time_axis,[0,CI_lower(ndx_labor,:,shock_counter)],'r:');
    else
        hndl = plot(Time_axis,0*Time_axis,Time_axis,[0,Response(ndx_labor,:)],'k-');
    end;
    title(['Response of Labor']);
    xlabel('Horizon');
    ylabel('Percent');
    set(hndl,'LineWidth',2);
    disp('Inspect Figure');
    disp('Hit any key when ready...');
    pause;

end; % closes shock loop
```

The identification file uses the program "datamass.m" which calculated differences and adjusts the time series according to the lag-length of the specification:

```
% defining the appropriate series for identification
% need to set certain options in real data

% generating lags and differences
YY = DATA';
Ylagone = DATA';
Ylagtwo = DATA';
Ylagthree = DATA';
```

```
Ydiff = YY(:,(2:sample))' - YY(:,(1:sample-1))';
Ydiff = Ydiff';
Ydlagone = Ydiff;
Ydlagtwo = Ydiff;
Ydlagthree = Ydiff;

switch lag_length,
    case 1
        if use_diff,
            Ydiff(:,1) = [];
            Ydlagone(:,sample-1) = [];
            YY(:,(1:2)) = [];
            Ylagone(:,sample) = [];
            Ylagone(:,1) = [];
            sample = sample-2;
        else
            YY(:,1) = [];
            Ylagone(:,sample) = [];
            sample = sample-1;
        end;
    case 2
        if use_diff,
            Ydiff(:,(1:2)) = [];
            Ydlagone(:,sample-1) = [];
            Ydlagone(:,1) = [];
            Ydlagtwo(:,((sample-2):(sample-1))) = [];
            YY(:,(1:3)) = [];
            Ylagone(:,sample) = [];
            Ylagone(:,(1:2)) = [];
            Ylagtwo(:,((sample-1):sample)) = [];
            Ylagtwo(:,1) = [];
            sample = sample-3;
        else
            YY(:,(1:2)) = [];
            Ylagone(:,sample) = [];
            Ylagone(:,1) = [];
            Ylagtwo(:,((sample-1):sample)) = [];
            sample = sample-2;
        end;
    case 3
        if use_diff,
            Ydiff(:,(1:3)) = [];
            Ydlagone(:,sample-1) = [];
```

```
            Ydlagone(:,(1:2)) = [];
            Ydlagtwo(:,((sample-2):(sample-1))) = [];
            Ydlagtwo(:,1) = [];
            Ydlagthree(:,((sample-3):(sample-1))) = [];
            YY(:,(1:4)) = [];
            Ylagone(:,sample) = [];
            Ylagone(:,(1:3)) = [];
            Ylagtwo(:,((sample-1):sample)) = [];
            Ylagtwo(:,(1:2)) = [];
            Ylagthree(:,((sample-2):sample)) = [];
            Ylagthree(:,1) = [];
            sample = sample-4;
        else
            YY(:,(1:3)) = [];
            Ylagone(:,sample) = [];
            Ylagone(:,(1:2)) = [];
            Ylagtwo(:,((sample-1):sample)) = [];
            Ylagtwo(:,1) = [];
            Ylagthree(:,((sample-2):sample)) = [];
            sample = sample-3;
        end;
    otherwise
end;

% final VAR specification
Ynew = zeros(num_vars,sample);
Ynlagone = zeros(num_vars,sample);
Ynlagtwo = zeros(num_vars,sample);
Ynlagthree = zeros(num_vars,sample);

if num_vars_diff == 1 | num_vars_diff == 2,
    Ynew(ndx_prod,:) = Ydiff(ndx_prod,:);
    Ynlagone(ndx_prod,:) = Ydlagone(ndx_prod,:);
    if lag_length >= 2,
       Ynlagtwo(ndx_prod,:) = Ydlagtwo(ndx_prod,:);
    end;
    if lag_length == 3,
        Ynlagthree(ndx_prod,:) = Ydlagthree(ndx_prod,:);
    end;
    if ndx_price ~= 0,
        Ynew(ndx_price,:) = Ydiff(ndx_price,:);
        Ynlagone(ndx_price,:) = Ydlagone(ndx_price,:);
        if lag_length >= 2,
```

```
            Ynlagtwo(ndx_price,:) = Ydlagtwo(ndx_price,:);
        end;
        if lag_length == 3,
            Ynlagthree(ndx_price,:) = Ydlagthree(ndx_price,:);
        end;
    end;
    Ynew(ndx_labor,:) = YY(ndx_labor,:);
    Ynlagone(ndx_labor,:) = Ylagone(ndx_labor,:);
    if lag_length >= 2,
        Ynlagtwo(ndx_labor,:) = Ylagtwo(ndx_labor,:);
    end;
    if lag_length == 3,
        Ynlagthree(ndx_labor,:) = Ylagthree(ndx_labor,:);
    end;
else
    if num_vars_diff == 3,
        Ynew = Ydiff;
        Ynlagone = Ydlagone;
        if lag_length >= 2,
            Ynlagtwo = Ydlagtwo;
        end;
        if lag_length == 3,
            Ynlagthree = Ydlagthree;
        end;
    else
        Ynew = YY;
        Ynlagone = Ylagone;
        if lag_length >= 2,
            Ynlagtwo = Ylagtwo;
        end;
        if lag_length == 3,
            Ynlagthree = Ylagthree;
        end;
    end;
end;
```

E.2 Some additional plots

Here, additional plots for different identification procedures that were mentioned in the text are provided.

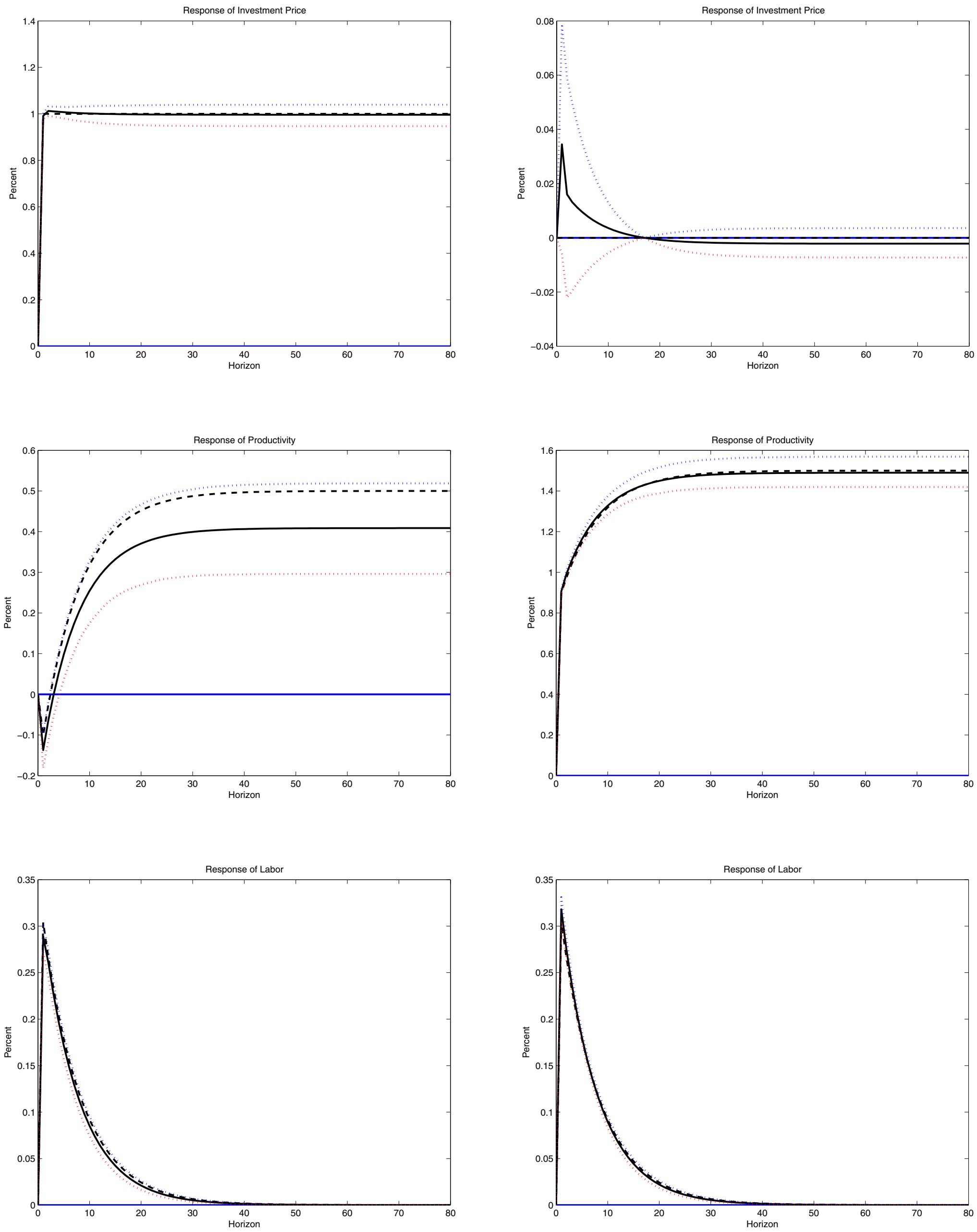

Figure 17:
Impulse responses for the medium-run Fisher identification (k=16) of simulated data from the original Fisher model.
Left column: responses to innovation in investment-specific technology.
Right column: responses to innovation in neutral technology.

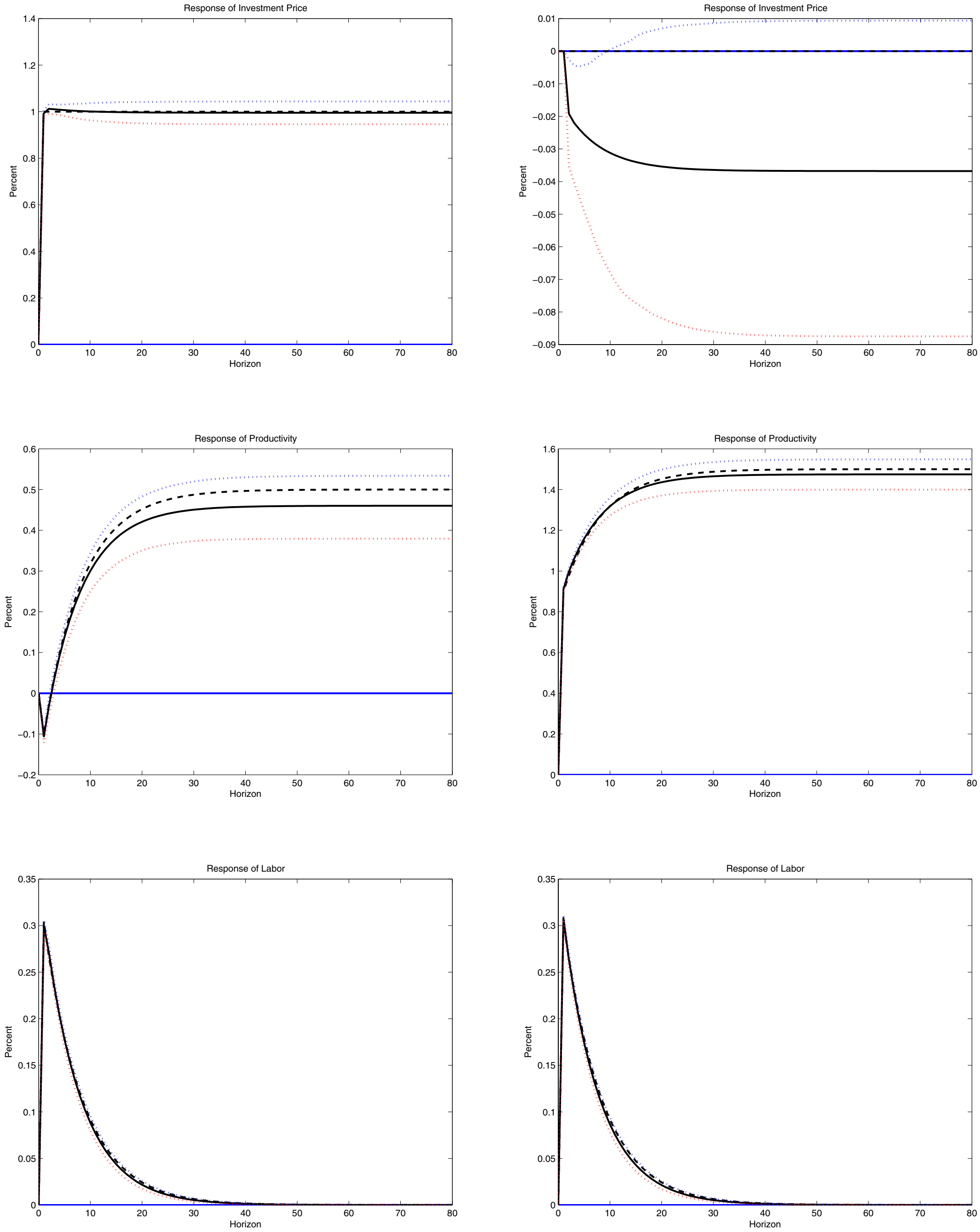

Figure 18:
Impulse responses for the short-run Fisher identification (k=0)
of simulated data from the original Fisher model.
Left column: responses to innovation in investment-specific technology.
Right column: responses to innovation in neutral technology.

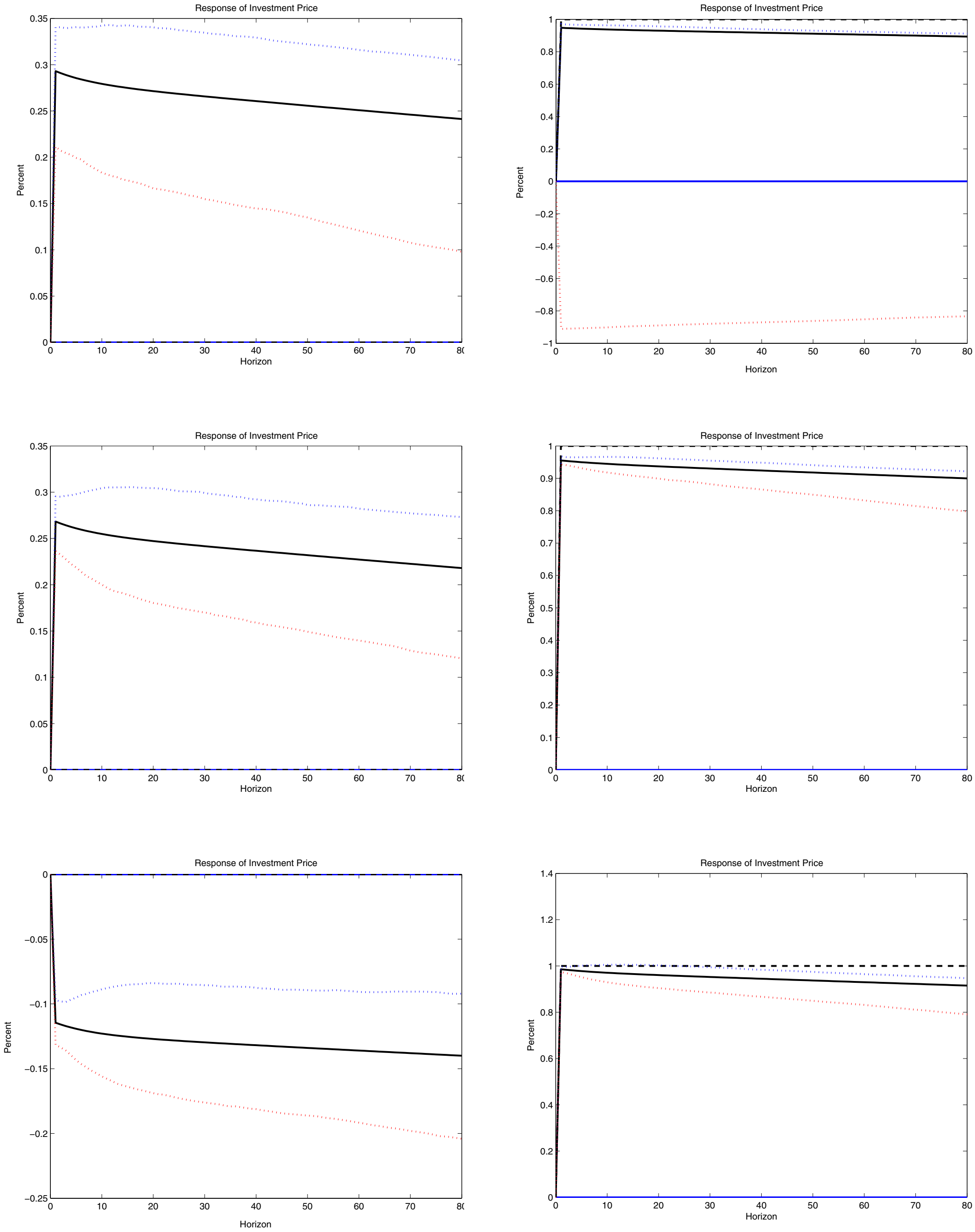

Figure 19:
Impulse responses of the investment price for short-, medium- and long-run using the alternative identification of simulated data from the original Fisher model.
Top panels: long run; middle panels: medium run; bottom panels: short run.

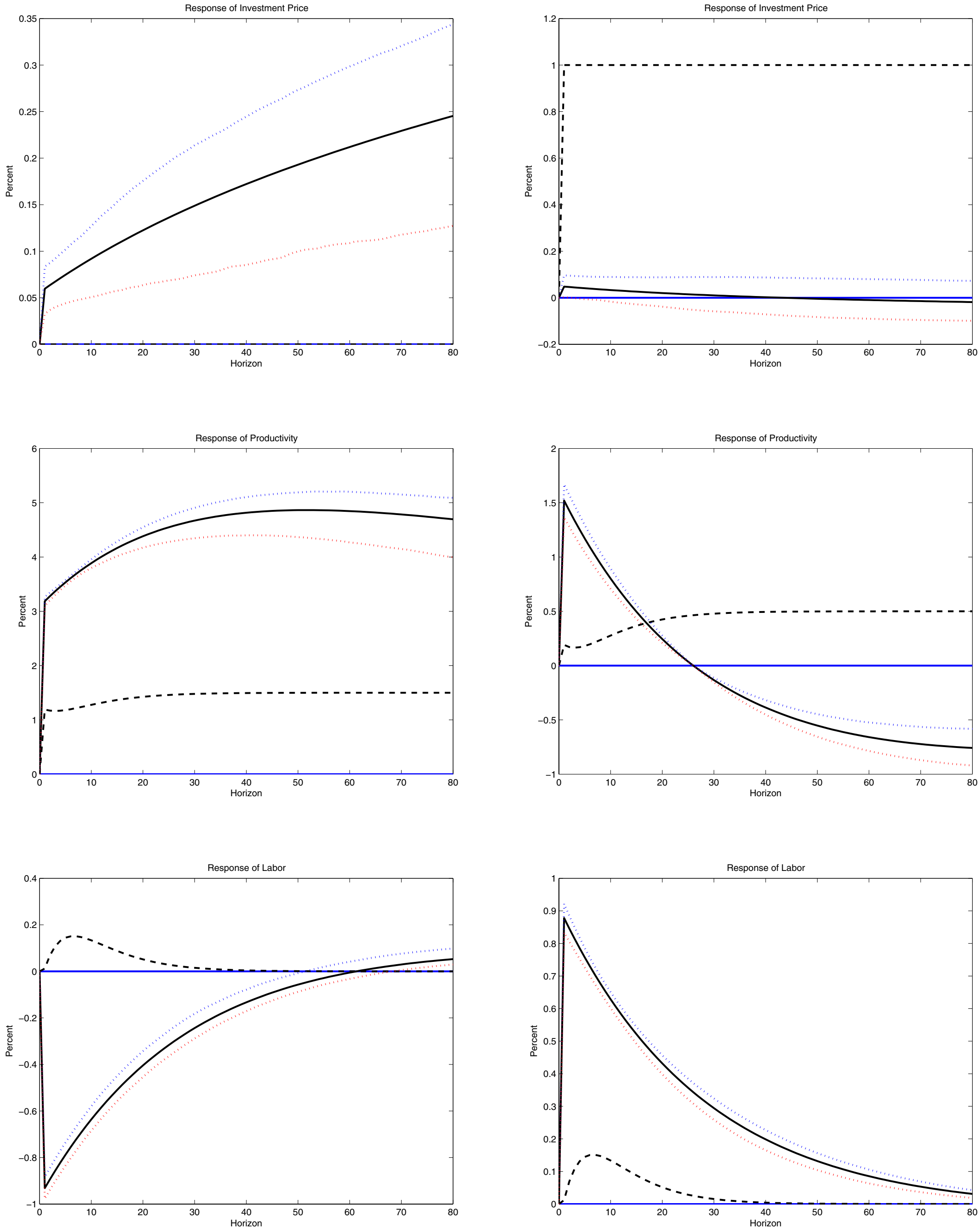

Figure 20:
Impulse responses for the medium-run alternative identification (k=25)
of simulated data from the labor hoarding model with transitory contract shocks .
Left column: responses to innovation in neutral technology.
Right column: responses to innovation in investment-specific technology.

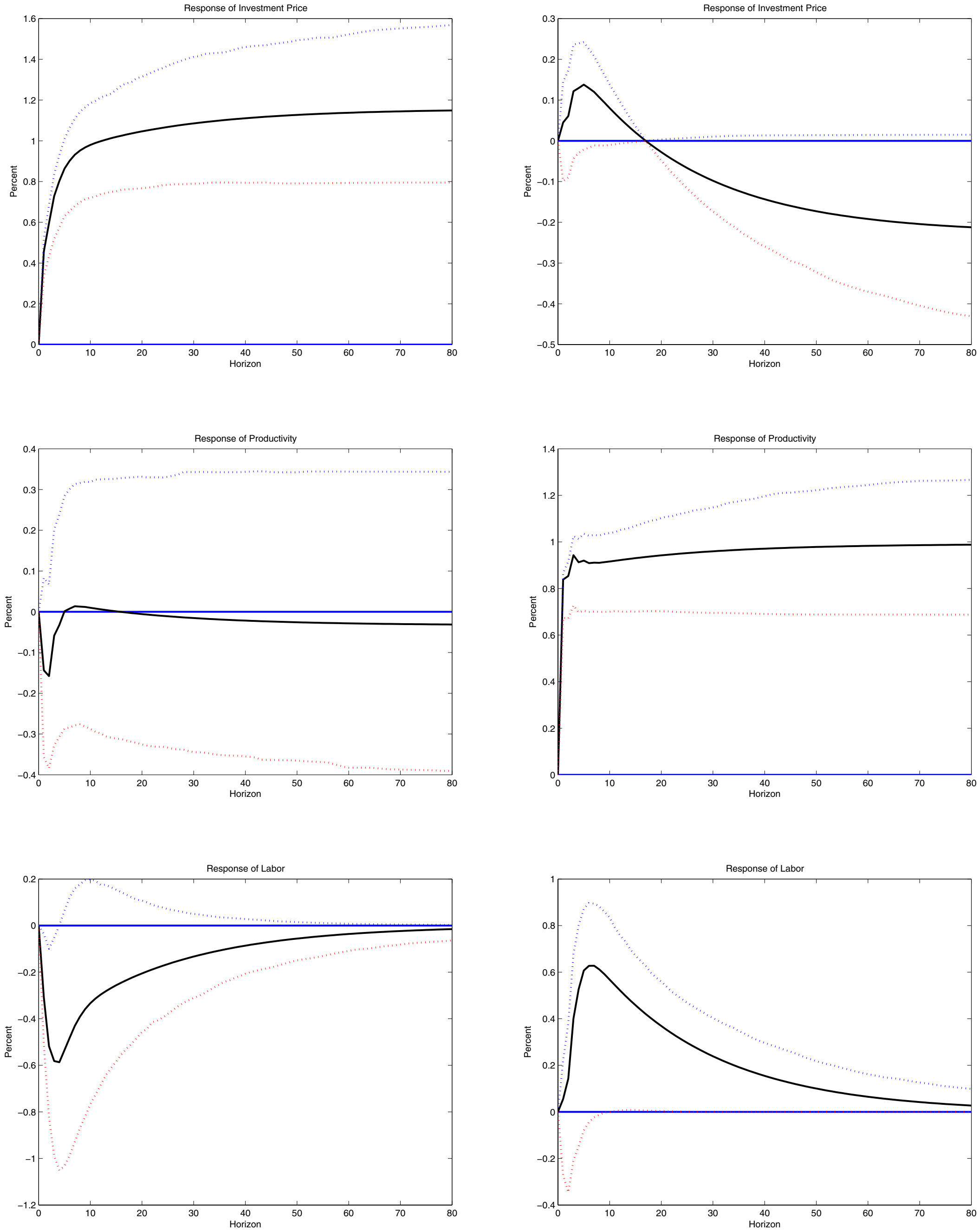

Figure 21:
Impulse responses for the medium-run Fisher identification
of real data, (k=16).
Left column: responses to innovation in investment-specific technology.
Right column: responses to innovation in neutral technology.

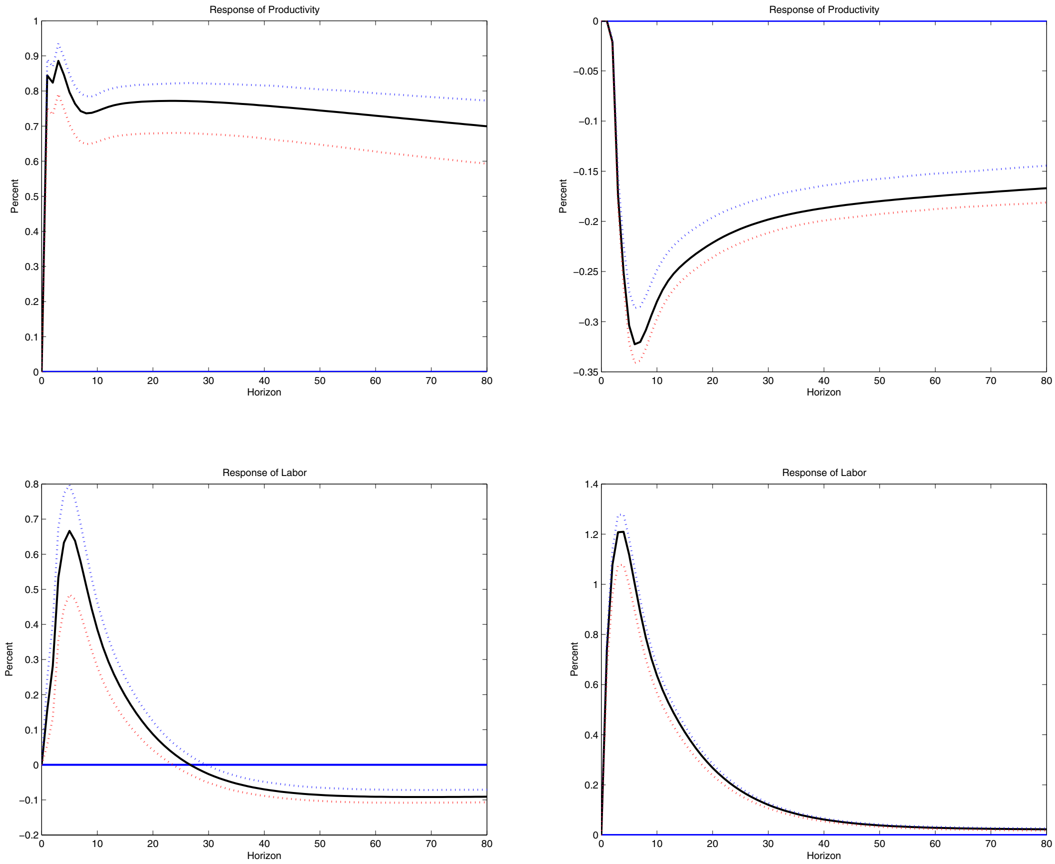

Figure 22:
Impulse responses for the short-run alternative identification
real data without price, (k=0).
Left column: responses to innovation in neutral technology.
Right column: responses to innovation in investment-specific technology.

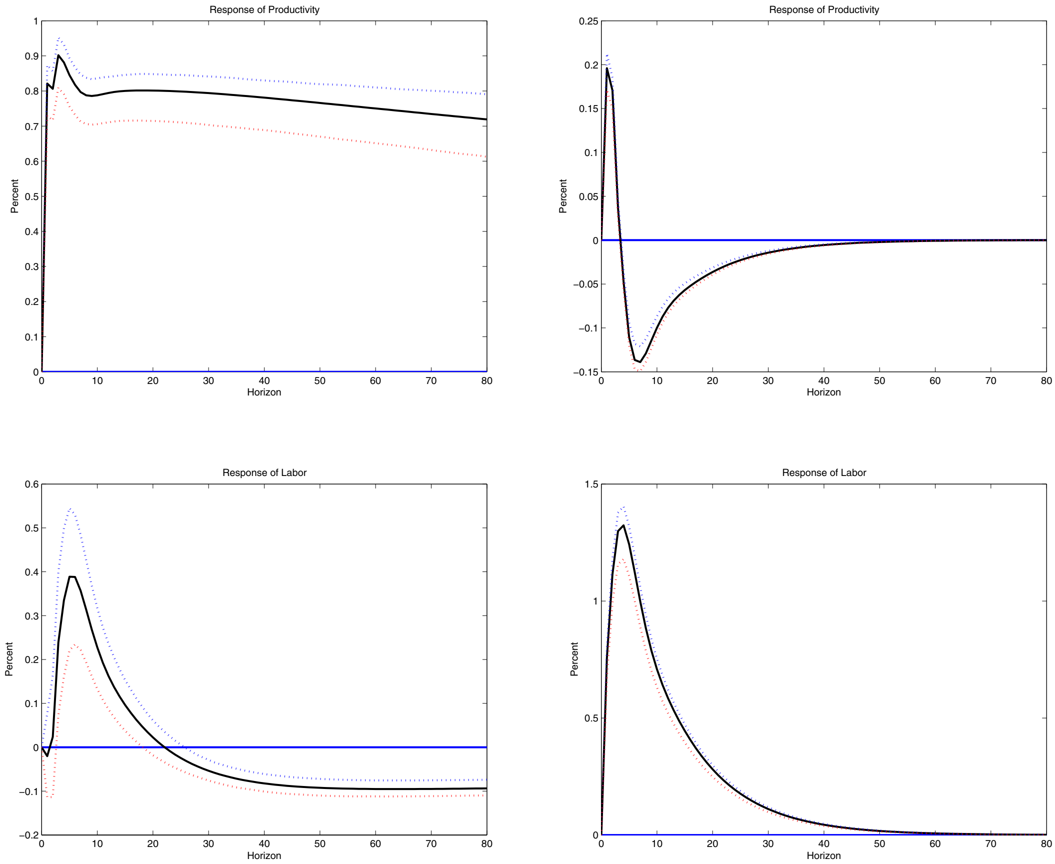

Figure 23:
Impulse responses for the long-run alternative identification
real data without price, (k=80).
Left column: responses to innovation in neutral technology.
Right column: responses to innovation in investment-specific technology.

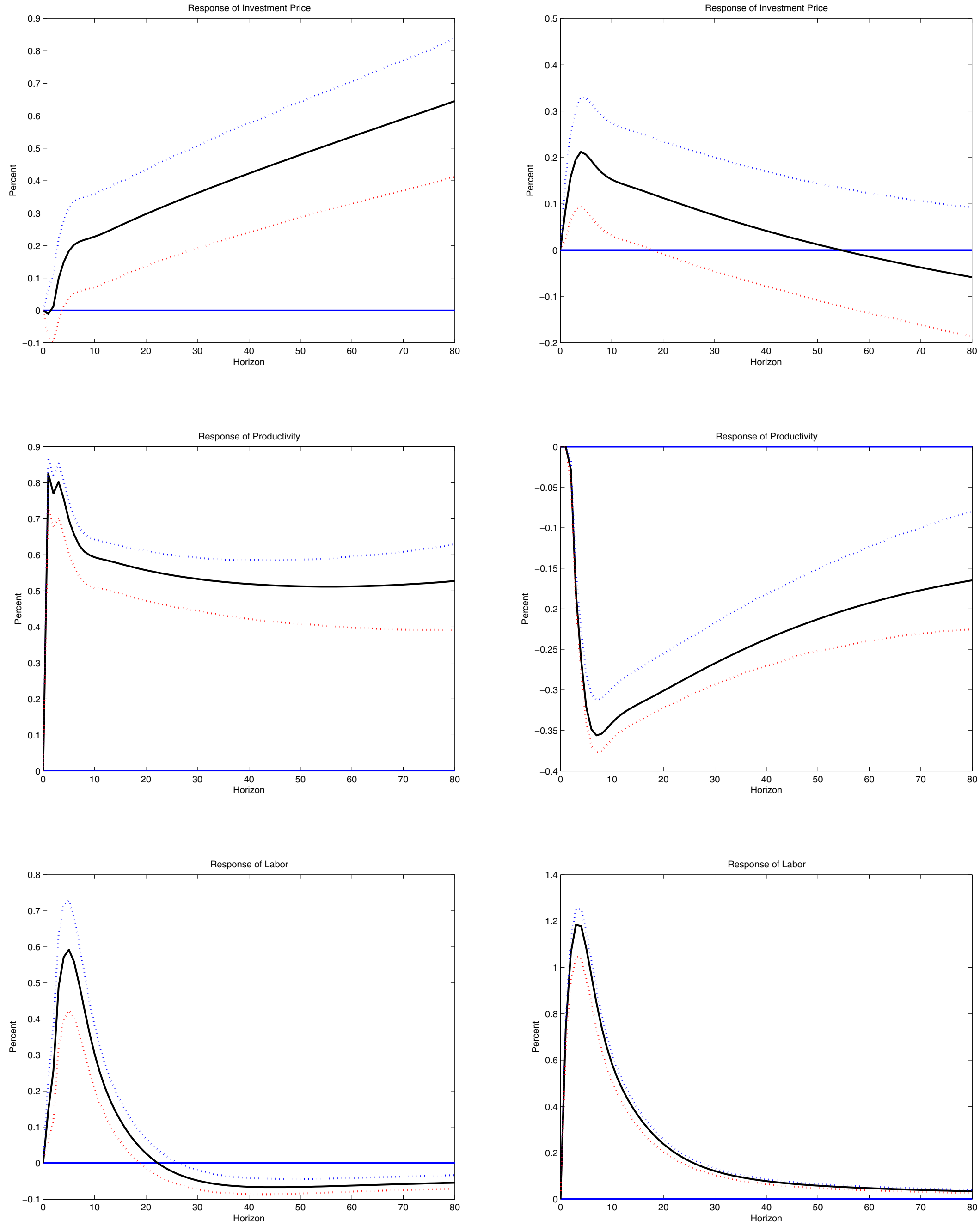

Figure 24:
Impulse responses for the short-run alternative identification
real data with price, (k=0).
Left column: responses to innovation in neutral technology.
Right column: responses to innovation in investment-specific technology.

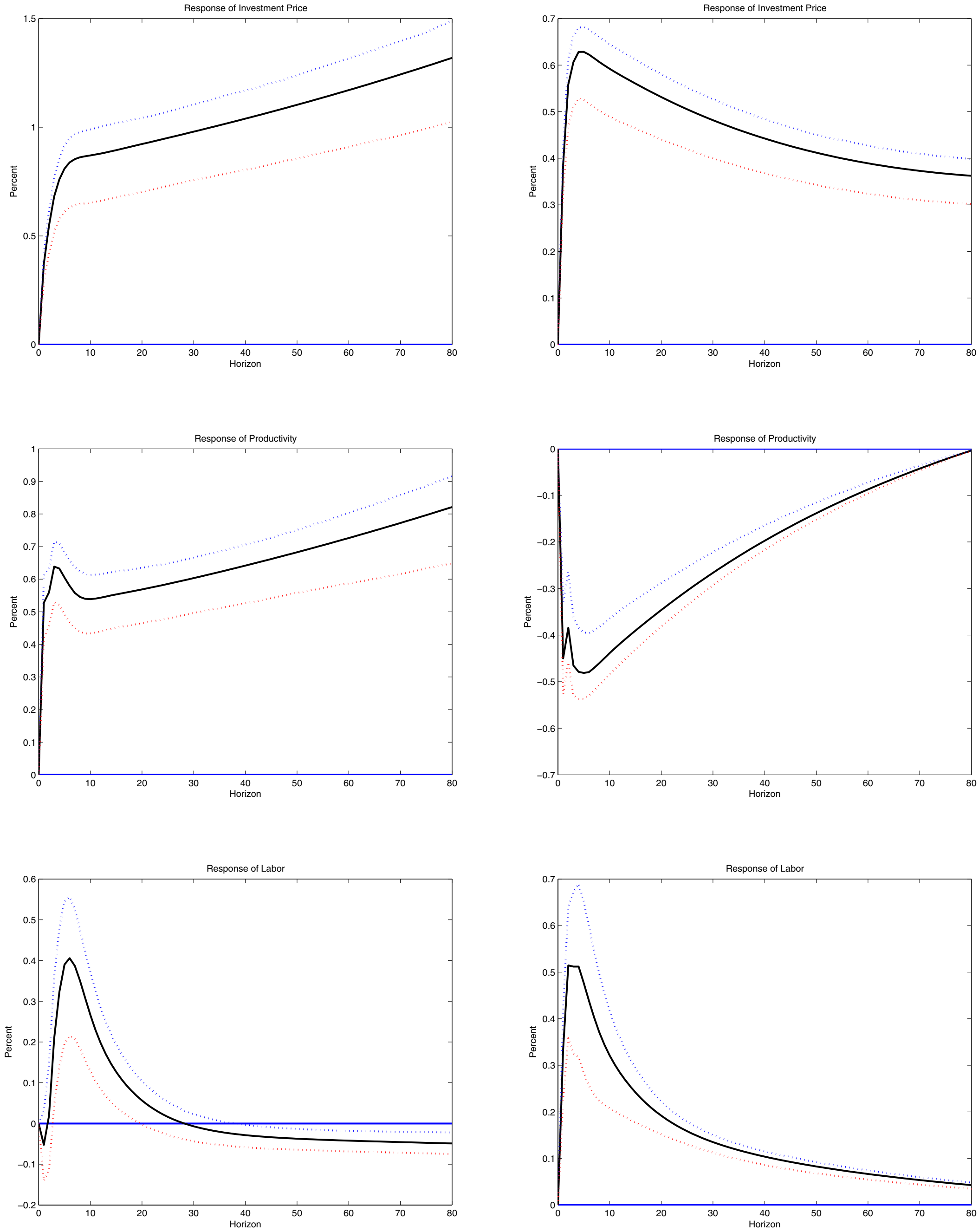

Figure 25:
Impulse responses for the long-run alternative identification
real data with price, (k=80).
Left column: responses to innovation in neutral technology.
Right column: responses to innovation in investment-specific technology.

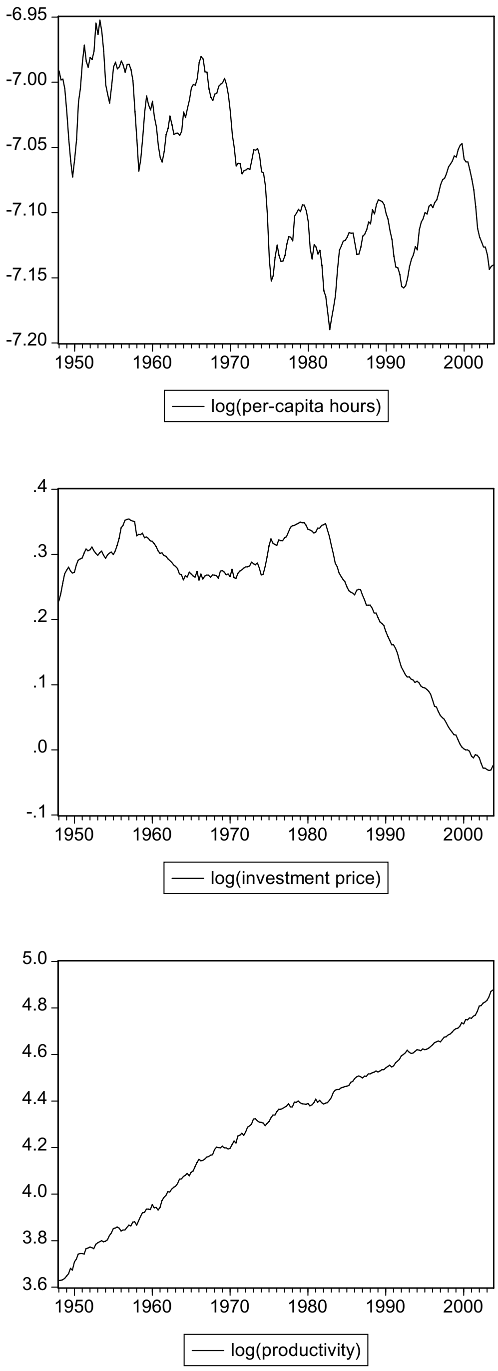

Figure 26:
Data for the specification
Per-capita hours, total investment price and productivity (in logs).
Sources: BLS, BEA.

F Bootstraps

Confidence intervals for impulse responses are often computed using bootstraps, since the determination of the variance of the asymptotic distribution is quite difficult. Moreover, Kilian (1998) shows that, in small samples, they even achieve better results than asymptotic theory. Even though Sims and Zha (1999) state that Bayesian methods for error bands are to be preferred over bootstraps, I will stick to the overall classical approach in this work. I programmed the confidence intervals for the impulse responses following a procedure outlined in Breitung, Brüggemann and Lütkepohl (2003), pp. 141-142. This procedure works as follows: First, the model is estimated. The resulting residuals are then centered with respect to their mean. Second, I draw with replacement from these residuals and generate new time series given pre-sample values from the data. With these, the model is estimated again and the respective impulse responses are determined. I repeat these steps a large number of times an obtain the quantiles of the impulse response "distribution". These are then my confidence intervals.

```
% bootstrapping confidence intervals
% to be used by "fisher_orig_indent", "fisher_hoard_ident"
% and "real_data"

% create centered residuals
Umean = mean(UU,2); Ucentered = UU; for i = 1:num_vars,
    Ucentered(i,:) = UU(i,:) - Umean(i);
end;

% start bootstrap loop
REPEAT = 1000;
Boot = zeros(num_vars,HORIZON_impresp,shocks_ident,REPEAT);
% collects all impulse responses in all loops
for repeat = 1:REPEAT,

% random draw from centered residuals
NDX = randint(sample,1,[1,sample]); Uboot =
zeros(num_vars,sample); for j = 1:sample,
    ndx = NDX(j);
    Uboot(:,j) = Ucentered(:,ndx);
end;

% generating new time series
% generates differences or levels automatically as in specification
Newdata = zeros(num_vars,sample); Newdata(:,1) = Btildenew*ZZ(:,1)
+ Uboot(:,1); if lag_length == 2,
    Hellp = [ Newdata(:,1)
```

```
              Ynlagone(:,1)
              ones(1,1) ];
    Newdata(:,2) = Btildenew*Hellp + Uboot(:,2);
end; if lag_length == 3,
        Heelp = [ Newdata(:,1)
                  Ynlagone(:,1)
                  Ynlagtwo(:,1)
                  ones(1,1) ];
        Hellp = [ Newdata(:,2)
                  Newdata(:,1)
                  Ynlagone(:,1)
                  ones(1,1) ];

    Newdata(:,2) = Btildenew*Heelp + Uboot(:,2);
    Newdata(:,3) = Btildenew*Hellp + Uboot(:,3);
end; for i = (lag_length+1):sample,
    Helpp = [];
    for j = 1:lag_length,
        Helpp = [ Helpp
                  Newdata(:,i-j)];
    end;
    Helpp = [ Helpp
              ones(1,1) ];
    Newdata(:,i) = Btildenew*Helpp + Uboot(:,i);
end;

switch lag_length,
    case 1
        Lagone = Newdata(:,1:sample-1);
        Newdata(:,1) = [];
        ZZboot = [ Lagone
                   ones(1,sample-1)];
    case 2
        Lagone = Newdata(:,2:(sample-1));
        Lagtwo = Newdata(:,1:(sample-2));
        Newdata(:,(1:2)) = [];
        ZZboot = [ Lagone
                   Lagtwo
                   ones(1,sample-2)];
    case 3
        Lagone = Newdata(:,3:(sample-1));
        Lagtwo = Newdata(:,2:(sample-2));
        Lagthree = Newdata(:,1:(sample-3));
```

```
        Newdata(:,(1:3)) = [];
            ZZboot = [ Lagone
                       Lagtwo
                       Lagthree
                       ones(1,sample-3) ];
    otherwise
        break;
end;

% new estimation
Bootnew = Newdata*(ZZboot')*(inv(ZZboot*ZZboot')); UUboot =
Newdata - Bootnew*ZZboot; Omegaboot =
(1/sample)*(UUboot*(UUboot')); Bboot =
zeros(num_vars,num_vars,HORIZON_impresp); Bboot(:,:,1) =
Bootnew(:,(1:num_vars)); if lag_length >=2,
    Bboot(:,:,2) = Bootnew(:,((num_vars+1):(2*num_vars)));
end; if lag_length == 3,
    Bboot(:,:,3) = Bootnew(:,(2*num_vars+1):(3*num_vars));
end;

% calculation of matrix A
Phiboot = zeros(num_vars,num_vars,HORIZON_ident+1); Phiboot(:,:,1)
= eye(num_vars,num_vars);

CCtildeboot = eye(num_vars,num_vars); for s = 2 : HORIZON_ident+1,
    Phitemp = zeros(num_vars,num_vars);
    for j= 1:(s-1),
        Phitemp = Phitemp + Phiboot(:,:,(s-j))*Bboot(:,:,j);
    end;
    Phiboot(:,:,s) = Phitemp;
    CCtildeboot = CCtildeboot + Phiboot(:,:,s);
end;

if level_ident,
    CCtildeboot = Phiboot(:,:,HORIZON_ident+1);
end;

Sigmaboot=CCtildeboot*Omegaboot*CCtildeboot';
Qtrans=chol(Sigmaboot); Qboot=Qtrans'; if do_trans,
    Transboot = [   1,  0,  0,
                    0,  -1, 0,
                    0,  0,  1 ];
    Qboot = Qboot*Trans;
```

```
end; Atildeboot = inv(CCtildeboot)*Qboot;

% impulse responses
Bootresponse = zeros(num_vars,HORIZON_impresp,shocks_ident);
% contains all responses to all shocks for single loop
for shock_counter = 1 : shocks_ident,
    Responseboot = zeros(num_vars_diff+num_vars_lev,HORIZON_impresp);
    shock_vector = zeros(num_vars,1);
    shock_vector(shock_counter) = 1;
    Phiboot = zeros(num_vars,num_vars,HORIZON_impresp);
    Phiboot(:,:,1) = eye(num_vars,num_vars);
    for s = 2 : HORIZON_impresp,
        Phitempboot = zeros(num_vars,num_vars);
        for j= 1:(s-1),
            Phitempboot = Phitempboot + Phiboot(:,:,(s-j))*Bboot(:,:,j);
        end;
        Phiboot(:,:,s) = Phitempboot;
    end;
    for i = 1: num_vars_diff,
        CCtildeboot = zeros(1,num_vars);
        for time_counter = 1 : HORIZON_impresp,
            CCtildeboot = CCtildeboot + Phiboot(i,:,time_counter);
            Resphelpboot = CCtildeboot*Atildeboot*shock_vector;
            Responseboot(i,time_counter) = Resphelpboot;
        end;
    end;
    for k = 1:num_vars_lev,
        for time_counter = 1 : HORIZON_impresp,
            Resphelpboot = Phiboot(num_vars_diff+ k,:,time_counter)*...
                               Atildeboot*shock_vector;
            Responseboot(num_vars_diff+k,time_counter) = Resphelpboot;
        end;
    end;
    Bootresponse(:,:,shock_counter) = Responseboot;
end; % closes loop for different shocks

Boot(:,:,:,repeat) = Bootresponse;
end; % closes "bootstrap" loop

% getting the quantiles

CI_upper = zeros(num_vars,HORIZON_impresp,shocks_ident); CI_lower
= zeros(num_vars,HORIZON_impresp,shocks_ident);
```

```
for j = 1:shocks_ident;
    for i = 1:HORIZON_impresp,
        if ndx_price ~= 0,
            NoSortprice = reshape(Boot(ndx_price,i,j,:),REPEAT,1);
            Sortprice = sort(NoSortprice);
            CI_upper(ndx_price,i,j) = Sortprice(950);
            CI_lower(ndx_price,i,j) = Sortprice(50);
        end;
        NoSortprod = reshape(Boot(ndx_prod,i,j,:),REPEAT,1);
        NoSortlab = reshape(Boot(ndx_labor,i,j,:),REPEAT,1);
        Sortprod = sort(NoSortprod);
        Sortlab = sort(NoSortlab);
        CI_upper(ndx_prod,i,j) = Sortprod(950);
        CI_lower(ndx_prod,i,j) = Sortprod(50);
        CI_upper(ndx_labor,i,j) = Sortlab(950);
        CI_lower(ndx_labor,i,j) = Sortlab(50);
    end;
end;

if do_real,
    CI_upper = 100*CI_upper;
    CI_lower = 100*CI_lower;
else
    CI_upper = 10*CI_upper;
    CI_lower = 10*CI_lower;
end;
```

Erklärung zur Urheberschaft

Hiermit erkläre ich, dass ich die vorliegende Arbeit allein und nur unter Verwendung der aufgeführten Quellen und Hilfsmittel angefertigt habe.

Almut Balleer

Berlin, den 18. März 2004