Can Market Imperfections Explain the Behavior of Exchange Rates?

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Abstract

The answer is a clear yes. Market imperfections in both the goods and asset markets not only help to explain the short-run behavior of exchange rates but are also the driving force behind it. Indeed they constitute the key element to its understanding. Exactly which market imperfections are responsible for which features of this behavior is, however, disputed. Following one approach based on a model by Devereux and Engel [2002] this thesis will show how each market imperfection contributes to closing the enormous gap between data and theory, that is, between observable characteristics of exchange rate behavior and theoretical concepts like Purchasing Power Parity (PPP) and Uncovered Interest Rate Parity (UIRP).

Keywords: Exchange Rates, Market Imperfections, PPP, UIRP

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1 Introduction

Exchange rates. Since 2002 Europeans have been completely liberated from having to convert money whenever traveling across their continent; from having to keep lira, escudo or drachma stored some place for the next trip only to be unable to find them when the time comes. However, the issue of exchange rates has by no means disappeared. Sudden changes of the euro-US-dollar-rate are now affecting even more countries, governments, firms and consumers than before. With $1,200 billion average daily turnover, the foreign exchange market is a huge market where banks, financial institutions, multinational companies, and central banks buy and sell deposits denominated in foreign currencies. Euro and US dollars make up most trading volume with respective market shares of 38% and 90% .[1]

According to Frankel [1979], the "monetary or asset view" on exchange rates posits that exchange rates are " moving to equilibrate the international demand for stocks of assets"(p.610). However, forces in goods markets also play an important role. Originally the very reason for trade in currencies was the world trade of goods and services across borders, which amounted to $8,025 billion in 2002.[2] Thus payments related to international trade only represent a small fraction of total transactions in foreign currencies. This discrepancy in volumes poses just one of numerous still largely unexplained questions that have arisen from the study of exchange rates and their behavior.

Two of the six major puzzles in international macroeconomics that Obstfeld and Rogoff [2000b] identify are linked to exchange rates, but for every question raised, economists have provided even more possible answers. Most answers suggest deviations from perfectly functioning mechanisms in both goods and asset markets. The main focus of this thesis will be to asses several of these deviations regarding their explanatory power for the behavior of exchange rates. This evaluation has considerable implications. Depending on the underlying forces, effects of exchange rate policies differ to a great extent. A possible but as yet non-existent consensus about the causes of exchange rate behavior will influence the way policy makers act and, in turn, judge the outcome of their actions. In this thesis, we will not go into great detail on policy implications; but rather will focus on the preliminary step of identifying the responsible market imperfections.

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[1]As two currencies are needed for each transaction, percentages add up to 200%, Data for 2001: Bank for International Settlements (BIS). For a detailed description of the foreign exchange market see e.g. Krugman and Obstfeld [1999] (Chap.12-14)

[2]Including intra-EU trade, thus the estimate is even too large. Data: World Trade Organization (WTO)

Before considering any explanation for the behavior of exchange rates, the question: "Can market imperfections explain the behavior of exchange rates?" needs to be clarified. In the following section 2.1 we will do just that by taking a look at data on exchange rates and defining exchange rate behavior. The issue becomes particularly interesting when a presentation of the two basic theoretical concepts behind the determination of exchange rates, Purchasing Power Parity (PPP) and Uncovered Interest Rate Parity (UIRP) in section 2.2 reveals the huge gap between data and theory. It will be the job of section 2.3 to introduce and discuss some economic suggestions for closing this gap.

One particularly appealing approach is based upon a model developed by Devereux and Engel [2002], which will be studied in greater detail in sections 3 - 5. Starting from a stripped down benchmark version, we will literally take this model apart. Each of the market imperfections in the model will be added one at a time in order to discuss each imperfection's particular implications for the behavior of exchange rates. Acknowledging the fact that there is quite some dispute among leading economists about one of these market imperfections, about the question whether prices are sticky in the currency of the producer or in the currency of the consumer, section 6 will briefly consider an alternative combination of market imperfections and evaluate this approach. A critical discussion of the model's ability to account for the behavior of exchange rates follows in section 7. A conclusion of this thesis will wrap things up.

It will be shown that, on the one hand, the model by Devereux and Engel [2002] can fully account for the behavior of exchange rates but that, on the other hand, the model's success relies heavily on the underlying assumption about the way firms set prices for exports (Local Currency Pricing) which, in turn, has strong implications for the influence of exchange rates on other macroeconomic variables. These implications are not supported by any data findings.

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2 Exchange Rate Behavior

2.1 Data
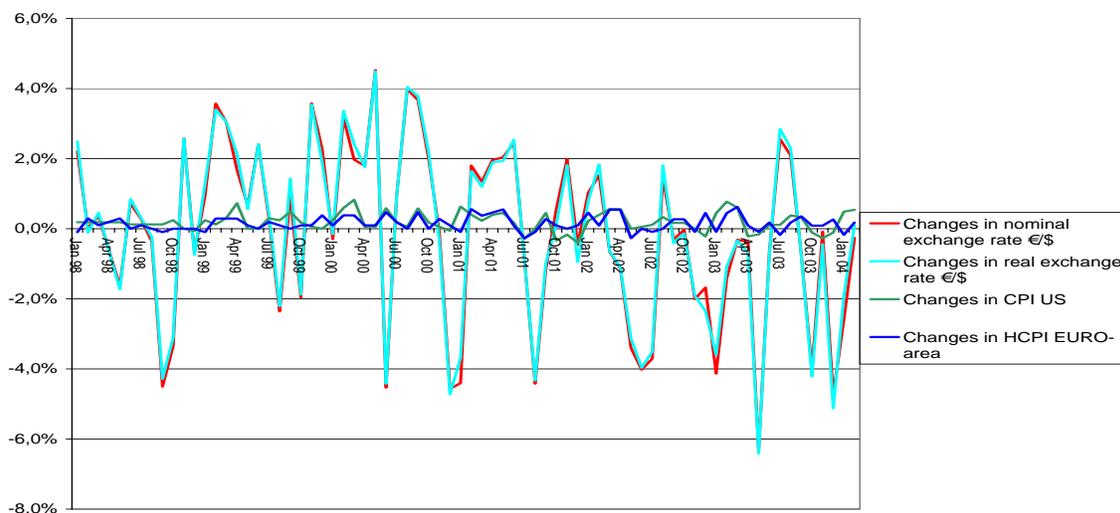
Figure 1: Monthly changes of €/$ rates, CPI, and HCPI
Data: Monthly Bulletin European Central Bank (ECB) and US Department of Labor

Figure 1 provides an impression of short-run behavior of exchange rates. It displays monthly changes of nominal and real exchange rates, euro against US dollar, (€/$), over a period of the last six years. Additionally, movements in prices represented by changes in CPI (Consumer Price Index) for the US and HCPI (Harmonized Consumer Price Index) for the euro area are shown.[3] The first three of the following stylized facts may be observed right away. Short-run behavior of exchange rates displays:

1. High volatility

2. Close correlation of nominal and real rates

3. Low pass-through of volatility to price levels

4. Movements disconnected from other macro variables

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[3] Changes in the real exchange rate have been calculated subtracting changes in HCPI from the sum of changes of nominal exchange rate and CPI. These changes are hence not estimated precisely but for small percentage changes this proceeding is admissible. Up to Dec. 98 exchange ECU/dollar.

These features of €/$ rates are valid for other major currencies of industrialized countries. Currencies of developing countries, on the other hand, are often characterized by a higher volatility of nominal than of real exchange rates as Devereux and Engel [2003] point out. Throughout this thesis we will often refer to these stylized facts. If not indicated otherwise, we will thus be talking about the short-run behavior of exchange rates for industrialized countries with floating exchange rate regimes.

In figure 1, it is the extreme volatility of both rates that sticks out at first glance. Monthly changes of up to 4% are not particularly rare. A closer look reveals the almost perfect co-movement of nominal and real exchange rates, implying a close correlation between the two variables. However, it is not possible to depict any relation between price indices and nominal exchange rates right away, even though for the case of an open economy like the euro-area, one would expect a rise in the exchange rate to trigger some response in the HCPI as this measure includes a certain share of imports. It seems that changes in the exchange rate are not fully passed on to consumers. Campa and Goldberg [2002], who conduct an extensive study on the pass-through of exchange rates to import prices, find the following: "For countries in the OECD we overwhelmingly reject complete pass-through (or PCP) and zero pass-through (or LCP) as a description of aggregate import prices in the short run. Partial pass-through is the best description for import price responsiveness in the first quarter"(p.10).[4]

To constitute the last stylized fact, exchange rate behavior under different exchange rate regimes needs to be compared. Obstfeld [1998] studies the real and nominal lira/Deutschmark rates and finds a tremendous increase in volatility for both rates after 1973 when the system of Bretton-Woods, which pegged all major currencies to the US dollar was abandoned. When both Italy and Germany joined the target zone of the European Monetary System (EMS) in 1979 volatility was again reduced. A more elaborate study is conducted by Baxter and Stockman [1989], who compare the historical behavior of a number of macroeconomic variables in 49 countries. They make two important findings: First, the volatility of both nominal and real exchange rates decline dramatically under fixed regimes, which is confirmed by Obstfeld's observations and thus reinforces the second stylized fact. Second, other economic aggregates do not behave very differently under alternative regimes, making it implausible to attribute the smaller variance of real exchange rates under fixed regimes to a reduced number of real shocks. This suggests a big influence of monetary shocks on the real exchange rate.

However, even nominal fundamentals like money supply and interest rates do not display the kind of volatility that exchange rates do, as Obstfeld and Rogoff [2000b] point out.

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[4]The expressions in parentheses, PCP(Producer Currency Pricing) and LCP(Local Currency Pricing) will be explained in detail in section 2.3

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This has led them to conclude that there exists an 'Exchange Rate Disconnect Puzzle' that they define as "the remarkably weak short-term feedback links between the exchange rate and the rest of the economy"(p.380). But what are these links and what moves exchange rates? Even with an understanding of exchange rate behavior and the sources of the behavior of the goods and asset markets the actual links between the two remain elusive. The next section will shed some light on this, introducing the two basic theoretical concepts of exchange rate determination; Purchasing Power Parity (PPP) for the goods market and Uncovered Interest Rate Parity for the asset market (UIRP).

2.2   Theory

Exchange rates are relative prices of currencies. We define the nominal exchange rate S as units of home currency required for the purchase of one unit of foreign currency, thus the relative price of the foreign currency in terms of home currency $S = \frac{CUR}{CUR^{\star}}$.[5] But what is a currency's value? It should be determined by the goods it pays for now and in the future, its price reflecting its purchasing power within the country and the interest paid on its deposits. Respective forces in goods and asset markets ought to establish this behavior. The exchange rate's task should simply be to relate prices and interest rates of two countries to allow for a comparison of values across borders. The aforementioned discussion constitutes the traditional view on exchange rates.

The Law of One Price (henceforth LOP) states that the nominal exchange rate should account for international price differences. Plainly speaking and assuming away any costs of transportation, if a shirt cost 20€ in the euro area and \$24 in the United States in order to prevent any possible gains from arbitrage, the nominal exchange rate would have to settle at 0.83€/\$. However, many empirical studies have rejected the LOP. Parsley and Wei [2001] comparing prices for 27 traded goods in Japan and the United States report its severe violation. The concept of Purchasing Power Parity (henceforth PPP) is less rigid as it abstains from the idea of all prices being equal when adjusted by the nominal exchange rate. Instead it assumes that currencies can be converted in such a way that one unit of them pays for a representative basket of goods and services in all countries. PPP is formally defined as:

$$S_t = \frac{P_t}{P_t^{\star}} \tag{2.1}$$

P_t represents the overall price level, or for the case of the LOP the price of any given good, at date t.

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[5]Throughout this thesis variables denoted by the symbol $\star$ represent foreign variables. For this section we define the US as the foreign country.

We define the real exchange rate as:

$$V_t = S_t \frac{P_t^\star}{P_t} \qquad (2.2)$$

It is obvious that with PPP holding the real exchange rate always equals unity, or is at least constant as relative PPP states. This version acknowledges that differences in baskets of goods across countries may hinder perfect functioning of absolute PPP. However, all these concepts clearly contradict the stylized facts identified before.

But what about the currency's future value and the returns paid on its deposits? Uncovered Interest Rate Parity (henceforth UIRP), the second basic concept of exchange rate determination, embodies this idea. As a typical example, we take the nominal exchange rate to still be at $0.83€/\$$ and we assume that nominal interest rates for 3-month deposits are at 2% in the euro area and at 1% in the US. Any US-investor could sell 100\$, get 83€ and deposit them in Europe for three months. If the exchange rate were then still at its old level he would make a riskless profit. This opportunity could be exploited by all agents, which would lead to an immediate appreciation and an expected depreciation of the euro as these investors were expected to convert back their returns in 3 months time. A stronger domestic currency would erode the extra interest gained on foreign deposits. UIRP expresses this no arbitrage argument as follows:

$$E_t S_{t+1} = \frac{S_t(1 + i_t)}{1 + i_t^\star} \qquad (2.3)$$

Differences in nominal returns on deposits across countries are accounted for by expectations of changes in the nominal exchange rate. Combining PPP and UIRP, real interest rates across countries have to be equal.

$$r_t = r_t^\star$$

Figure 2: Changes in 3 month ahead €/$ rate and difference in interest rates Data: Monthly Bulletins ECB

Figure 2 displays monthly differences in nominal interest rates paid on 3-month deposits for the euro area and the US and the monthly change in the 3-month ahead nominal €/$ rate for a period of four years.[6] An increase in relative returns on euro deposits seems to be related to a fall in the exchange rate, hence an appreciation of the euro. This is the exact opposite of what the UIRP concept states, however, in a world of uncertainty, expectation and realization will obviously differ. Thus, the fact that UIRP does not hold ex-post cannot lead to its dismissal.

The concept of Covered Interest Rate Parity (henceforth CIRP) takes this last issue into account. It uses the forward rate, a future rate agreed upon today, to approximate for the expected exchange rate. There exists strong empirical evidence that CIRP holds as Krugman and Obstfeld [1999] (Chap.13) point out, but instead of resolving the issue of UIRP this merely changes it. With CIRP holding and UIRP being violated the forward rate cannot be a good approximation of the future expected exchange rate. Hence the relationship $F_t = E_t S_{t+1}$, with F_t denoting the forward rate, cannot be valid. This conclusion has led to numerous econometric studies on the relationship between the expected future exchange rate and the forward rate, testing the following:

$$E_t[s_{t+1} - s_t] = \beta_0 + \beta_1(F_t - S_t) + \epsilon_{t+1}$$

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[6]Assuming this linear version of UIRP: $i_t - i_t^\star = E_t[s_{t+1} - s_t]/s_t$ is only an approximate method that can be applied for the case of small percentage changes

In a survey on some of these studies Engel [1996] concludes that "empirical tests routinely rejected the null hypothesis $[H_0 = (\beta_0 = 0; \beta_1 = 1)]$ that the forward rate is a conditionally unbiased predictor of future spot rates."(p.124) Instead he describes $\beta_1 < 0$ to be a common finding. This implies that one could bet against the market and always win. We assume $F_t > S_t$. It is thus cheaper to buy a forward contract than to purchase euros today. With $\beta_1 < 0$ holding, the spot rate would have appreciated on the expiration date of the forward contract and we could be certain to obtain a riskless profit. Chinn and Meredith [1998] confirm the negative relationship between the forward rate and the future spot rate for the case of short-term assets, as do Flood and Rose [1990] for the case of exchange rates under floating regimes. Though not universal, these findings hold exactly for the type of exchange rates relevant to this thesis.

Even adjusting the forward rate by a risk premium, i.e. accounting for the fact that agents are willing to pay more to be guaranteed a certain future rate, does not explain the negative coefficient, as Frankel and Froot [1990] show. A different study by the same authors [1987] uses survey data on exchange rate expectations instead of the forward rate to approximate the future spot rate, in order to avoid the influence of a risk premium. However, the problem remains because the hypothesis that this data is an unbiased estimator is rejected. The initial suspicion caused by figure 2 has been confirmed. There is a huge gap between the short-run behavior of exchange rates and the concepts of UIRP as well as PPP. Richer economic approaches are needed to provide explanations for theses deviations.

2.3   Closing the Gap between Data and Theory

Searching for answers on the issue of UIRP economists have taken a closer look at the supposedly negative relationship between the forward rate and the future spot rate. Besides risk premia, Engel [1996] in the aforementioned survey identifies another possible reason for the findings: deviations from rational expectations on behalf of investors.[7] The concept of rational expectations, i.e. expectations formed on the basis of all available information, is vital to modern economics. A departure from it constitutes a major change. To incorporate the idea into the representative agent framework without affecting rational choices on other allocations, an additional group of agents, so called noise traders must be introduced. Noise traders receive different or distorted information, form peculiar expectations and thus act in discordance with rational agents.

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[7] He actually mentions two more reasons: Inaccuracies in data samples and estimation procedures and Jensen's inequality terms. Both are likely to be rather small. See e.g. Obstfeld and Rogoff [1996] (Chap.8.7) for an explanation of Jensen's inequality terms.

Many economists have suggested that this approach might be useful for explaining exchange rate behavior. Frenkel and Mussa [1980] point out that, "The evidence from the behavior of spot and forward rates ... indicates that the bulk of exchange-rate changes appear to be due to new information." (p.377). Frankle and Froot [1990] note that, "there is evidence that trading volume, exchange rate volatility and the dispersion of expectations among forecasters are all positively related."(p.183) In the asset pricing framework, the introduction of noise traders has provided a sensible explanation for so-called asset bubbles, pricing schemes inconsistent with fundamental values.[8]

Disconnectedness from fundamental values seems to be a feature shared by exchange rates and asset prices, a phenomenon that Durate and Stockman [2001] call the "common view that exchange rates behave like asset prices."(p.0) Hence, it comes as a natural step to apply a very similar noise traders approach to the foreign exchange rate market. Among others, Jeanne and Rose [2002] do so and find support for their view on the superiority of a fixed exchange rate regime. Their model determines the entry of noise traders endogenously, but lacks any micro-based exchange rate determination. It predicts exactly what has been quoted by Frankle and Froot [1990], finding "increased forecast dispersion, greater UI[R]P deviations, and higher [trading] volume during floating exchange rate regimes"(p.21), when noise traders are more likely to enter. Bacchetta and van Wincoop [2003b] focus on the quantitative impact of noise on exchange rate volatility. Modeling noise as information heterogeneity, they find noise to contribute tremendously to short and medium run behavior of exchange rates. The way Devereux and Engel [2002] incorporate the noise trading approach into their model will be presented in great detail in section 3.3.

Suggestions made so far seem promising. Assuming the existence of noise traders explains deviations from UIRP and might enable us to account for the features of high volatility and the disconnectedness of exchange rates. But what about the close correlation of nominal and real rates and the limited pass-through of movements in exchange rates to consumer prices and the contrast these observations pose to PPP? Answering these questions requires a closer inspection of the source of these prices, the goods market.

We recall that the volatility of prices is far smaller than that of exchange rates and that there is evidence that monetary shocks cause movements in the real exchange rate. However, for those shocks to have any significant effects, a departure from the assumption of a flexible price world is needed.[9]

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[8]For such a model see e.g. DeLong et al. [1990]

[9]As Obstfeld and Rogoff [1996] (Chap.9) point out, under certain assumptions on the utility function -nonseperability- real effects of monetary shocks might also be achieved without sticky prices but these effects are by far weaker.

Obstfeld and Rogoff [1996] (Chap.9, 10) present macro as well as microeconomic evidence on sticky prices and wages, i.e. prices and wages that do not respond instantaneously to real or monetary shocks. With nominal rigidities prevailing, flexible exchange rates have to do all the adjusting and should be more volatile than prices. The first market imperfections in goods markets that might help to account for stylized facts (2.) and (3.) are, hence, sticky prices and wages combined with monopolistic competition, because, as Walsh [1998] points out, "any sort of price rigidity naturally raises the question of who is setting wages and prices, a question the perfectly competitive model begs. Once we need to address the issue of price setting, we must examine models that incorporate some aspects of imperfect competition, such as monopolistic competition." (p.195)

Most recent open macro models include monopolistic competition and some form of nominal rigidities, such as the Obstfeld and Rogoff [1995] model, which incorporates sticky prices. For being the first two country model with complete micro foundations, it is often cited and has provided a basis for many others. However, as prices are sticky in the producer's currency the LOP continues to hold and the real exchange rate in this model remains a constant, causing complete exchange rate pass-through to consumer prices. The authors themselves offer a variant by including non-traded goods. As differences in prices of those goods cannot be arbitraged away, this approach allows for some deviation from the LOP causing movements in the real exchange rate. However, in an extensive study, Engel [1999] finds, "that the relative price of nontraded goods has little import for understanding U.S. real exchange rate movements over the short and medium run." (p.532)

Hence, another approach has been to incorporate so-called border costs accounting for tariffs, transportation costs, commercial barriers and exchange rate risks and leading to deviations in the LOP for traded goods. As non-traded goods are simply too expensive to trade across great distances, the assumption of border costs is merely a generalization allowing for various shades of goods. Obstfeld and Rogoff [2000b] apply the concept of border costs to address their 6 major puzzles in open macroeconomics using so called iceberg-shipping costs, i.e. only a fraction of each good reaches its destination abroad, thus making imports more expensive. However, the authors admit that this assumption is not sufficient to account for the puzzles connected to exchange rates, for which they do not provide a full model.

Burnstein et al. [2003] who also find the dividing line between traded and non-traded goods to be quite arbitrary do provide a full model of a small economy. They claim that any traded good sold in domestic or foreign markets includes large components of services essentially of non-traded goods, reflected in distribution costs, i.e. costs for transport, marketing, sales etc.

Abstaining from nominal rigidities and monopolistic competition and under the assumption that distribution costs make up about one half of consumer prices, their model is able to account for the sharp appreciation of the Argentine real exchange rate in the period shortly after the one-to-one peg of the peso to the US dollar began. For the particular case of such a change in exchange rate regime, the model delivers a good explanation. As periods of exchange-rate based stabilizations are characterized differently in comparison to the exchange rate behavior discussed here, an evaluation of the model's validity to account for the stylized facts identified is limited. However, a high share of local content in products should help to reproduce a close correlation of nominal and real exchange rates as it further restrains the LOP. A reduced pass-through of movements in exchange rates to consumer prices can be expected, limiting the so called expenditure-switching effect of exchange rates, i.e. the way consumers adapt their product choices to swings in the nominal exchange rate.[10]

Deviations from the LOP combined with monopolistic competition have led others to conclude that the expenditure-switching effect might also be limited due to market segmentation on behalf of producing firms. Based on this assumption, Betts and Devereux [2000] develop the concept of Local Currency Pricing (henceforth LCP). In the framework of a two-country model with sticky prices, they allow for a fraction of monopolistic competitive firms to set export prices in the currency of the consumer. This behavior they call LCP. Volatility of exchange rates turns out to be an increasing function of this fraction of firms. LCP is a special form of so called Pricing to Market (henceforth PTM) behavior. PTM assumes that firms with market power can set different prices in separate markets. Depending on the prevailing price elasticities, mark-up factors over marginal costs may differ. Changes in exchange rates are not fully passed on to consumers but lead to adjustments of these factors instead, changing the firm's profits and thus also limiting expenditure-switching on behalf of consumers.

There exists microeconomic evidence on PTM. Verboven [1996], studying the European car market, concludes that domestic firms face lower price elasticities and are thus able to demand a higher mark-up. However, his and other results on PTM in certain industries cannot simply be extended to the case of LCP. Goldberg and Knetter [1997] review some of the studies on PTM and can only offer a nice example for LCP. Consider an internationally sold newspaper. The various prices printed on its cover are not adjusted each time nominal exchange rates change but are sticky in the respective currencies.

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[10]The traditional assumption on the expenditure-switching effect is the so called Marshall-Lerner condition stating that a currency's depreciation will only lead to a surplus of the country's trade balance if the increase in volume of exports is bigger than the loss in its value and if the decrease in volume in imports is large enough to offset the higher prices for imports, i.e. price elasticities of imports and exports have to at least add up to unity.

Evidence for the existence of LCP is hence only anecdotal or as in the following two cases, circumstantial . Studying consumer prices in different cities in the US and Canada Engel and Rogers [1996] find the volatility of these prices within countries to be far smaller than the volatility of these prices across borders, i.e. when adjusted by the nominal exchange rate. As border costs and volatility in wage differentials are not able to fully account for the higher volatility, they propose nominal price stickiness, prices sticky in local currency, as one possible explanation. Parsley and Wei [2001] in the aforementioned study find that changes in the real exchange rate composed of the prices of traded Japanese and US goods are quite similar to changes in the nominal $/¥ rate. As this observation implies that prices are sticky and are not adjusted by firms when the nominal exchange rate changes the authors conclude that many firms fix prices in the currency of the consumer, their invoicing practices thus being as assumed under LCP.

From a microeconomic perspective LCP seems quite reasonable. Firms facing local distribution costs and competition from local firms do care more about prices of their goods in terms of local currencies than in terms of their respective domestic currencies. Applying this intuition to the exporting sector of a semi-small open economy with sticky prices and wages Kollman[2001] is able to achieve high volatility of closely correlated nominal and real exchange rates that is not passed on to consumer prices. However, a high correlation of output and exchange rates arises which contradicts the feature of the disconnectedness of exchange rates. A two country LCP model by Chari et al.[2002] runs in to a similar problem but with relative consumption. Even a departure from their initial assumption of a perfect asset market with a full set of state-contingent bonds does not help to improve results.

Despite some success of the LCP idea, Obstfeld and Rogoff [2000a] make their case against it, arguing that instead Producer Currency Pricing (henceforth PCP), sticky prices in terms of domestic currency, are required to account for the fact "that the expenditure-switching effect of exchange rate changes is alive and well among industrialized economies, and should be a central feature of open economy models."(p.127) They argue that the effect is needed on the macroeconomic level to explain changes in trade flows and countries' competitiveness due to movements in the exchange rates. In support of their view the authors provide empirical correlations between changes in these two measures and changes in the nominal exchange rate. However, models based on the PCP approach estimate the correlation of consumption among countries to be higher than the correlation of output, thus contrasting the findings of various empirical studies like Baxter and Crucini [1995] who show that it is output that is more correlated internationally than consumption. Under the assumption of LCP, on the other hand, this other important macroeconomic issue, the so called consumption-correlation puzzle, is resolved. However, the question of PCP or LCP is not only relevant for explaining observed behavior of macroeconomic variables.

In terms of exchange rate policy, matters are even more controversial as described by Devereux and Engel [2003]. The assertion that flexible rates are preferable to fixed ones when real shocks dominate an economy is only valid under the PCP assumption. If prices are sticky in local currency, as under LCP, the possibility of exchange rates mimicking a flexible price environment does not exist, making fixed rates the desirable option.

Recalling empirical evidence and the quoted statement by Campa and Goldberg [2002] none of the extreme cases of PCP, full pass-through, nor LCP, zero pass-through, is able to explain overall short-run behavior of exchange rates.[11] Some models account for this assertion, by endogenizing a firm's choice on its pricing scheme. Bacchetta and van Wincoop [2003a] introduce an intermediate sector that purchases imports at prices set in foreign currency and that decides whether to pass the exchange rate risk on to consumers or whether to take it on itself and applying LCP. An important distinction is thus made between exchange rate pass-through to import prices and to consumer prices. The authors point out that the former has been found to be more complete. Devereux et al.[2003] are also endogenizing firms's choices on LCP or PCP. In the model by Devereux and Engel [2002] to be presented in the next section, price setting is instead exogenous, but complete LCP only applies to direct sales to foreign consumers. When selling to foreign retailers firms apply PCP.

Before introducing the model, a short note on the second puzzle in open macroeconomics related to exchange rates will be provided. The observation that it takes the real exchange rate about four to ten years to return to a stable path i.e. its half-life being about two to five years, stands in contrast to the important role assigned to monetary shocks in the determination of exchange rates. Its adjustment process should thus be similar to those of prices and wages, which are estimated to be far shorter. This is what Obstfeld and Rogoff [2000b] call the PPP Puzzle. A recent paper by Imbs et al. [2003] finds that the classical method of calculating the real exchange rate by using aggregate CPIs on the nominal exchange rate may be too simple.[12] With certain bias entering the calculations due to the heterogeneity of price adjustment across sectors, the degree of real exchange rate persistence may be overestimated. They conclude that there does not exist a PPP Puzzle after all. Chen and Engel [2004], on the other hand, contradict them. Reassessing the same data set and calculations they find support for the Obstfeld and Rogoff findings.

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[11] "For countries in the OECD we overwhelmingly reject complete pass-through (or PCP) and zero pass-through (or LCP) as a description of aggregate import prices in the short run. Partial pass-through is the best description for import price responsiveness in the first quarter"

[12] This simple method was used in this thesis but for the short-run behavior of the real exchange rate implications are less severe

As the dispute goes on and as the emphasis of this thesis will be laid on the short-run behavior of exchange rates, the issue of real exchange rate persistence will be laid aside, but a valid model of exchange rates would also have to account for their long-run behavior, whatever it may look like.

This section has not been able to close the gap between theory and data nor has it been able to provide a single explanation for the short-run behavior of exchange rates, but it has clearly stated the essential need for market imperfections in both goods and asset markets to account for the observation. The following model and especially the results presented in section 5 will visualize some of the assertions made.

3 The Model

We need a benchmark model in order to adequately asses the effects that market imperfections have on the behavior of exchange rates. We will begin by considering a model of perfect asset markets and flexible prices, introducing market imperfections one at a time and checking in the analysis of the model how each imperfection changes the reaction of the exchange rates to money shocks, and how each contributes to achieving the aforementioned stylized facts of exchange rate behavior: high volatility, close correlation of real and nominal exchange rates, low pass-through and disconnectedness. The final model that contains all the market imperfections introduced, is equal to the model by Devereux and Engel [2002].

The benchmark model can be described as the final model stripped off its market imperfections. However, the model is not truly perfect, because it assumes competitive monopolies in goods markets. Changing to a perfectly competitive set-up would not have any impact on the dynamic features of real or nominal exchange rates. Departing from this assumption, however, reduces the necessary steps leading to the final model, because all other goods market imperfections that will be introduced are based on monopolistic competition. An analysis of the welfare implications of the model would obviously have to consider the perfect competition case as a benchmark. The main focus of this thesis, however, is on the behavior of exchange rates and we will thus abstain from a truly perfect benchmark model. We follow the set-up of the paper by Devereux and Engel. However, the authors assume that money balances enter the utility function logarithmically and they apply a more general utility function only to some parts of their paper. In order to be able to show certain results and to be consistent we keep the more general utility function throughout the entire model. Additionally, we introduce dynamics to all steps and present impulse responses and simulation results for them, to visualize the impact, of each market imperfection on the behavior of exchange rates.

3.1 Benchmark Model

The world is made up of two countries, home and foreign. The entire population indexed by i is normalized to one. There are n identical agents living in the home country and 1-n of them living abroad. There exist just as many firms. The same preference and technology specifications are shared by all. Choices of foreign agents, quantities of foreign origin, and prices and wages denominated in foreign currency are denoted by the symbol ⋆. The following section specifies preferences and technologies for the home country households, firms, and government. As these specifications also hold for the foreign country, the respective equations for foreign country nationals are provided only where needed.

Households   Households maximize their discounted life-time utility by choosing optimal amounts of consumption demanded C, real balances held $\left(\frac{M}{P}\right)$, and labor supplied L. Choices are made by maximizing the following separable utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C, \left(\frac{M}{P}\right), L)$$

where

$$U = \frac{1}{1-\rho}C_s^{1-\rho} + \frac{1}{1-\epsilon}\left(\frac{M_s}{P_s}\right)^{1-\varepsilon} - \frac{\eta}{1+\psi}L_s^{1+\psi}$$

The households' choices are restricted by a budget constraint:

$$P_t C_t + M_t + \sum_{z^{t+1}} H(z^{t+1} \mid z^t)A(z^{t+1}) + \sum_{z^{t+1}} H^{\star}(z^{t+1} \mid z^t)A^{\star}(z^{t+1})S_t + d_t B_{ht} =$$
$$W_t L_t + \Pi_t + M_{t-1} + T_t + A_t + A_t^{\star}S_t + B_{ht-1}$$

$A(z^{t+1})$ and $A^{\star}(z^{t+1})$ denote one-period nominal state contingent bonds issued by the home and foreign country with $H(z^{t+1} \mid z^t)$ and $H^{\star}(z^{t+1} \mid z^t)$ being their respective prices.[13]   A nominal state contingent bond is defined as a bond that pays off one unit of currency if a particular state of the world occurs. In any other case payoff is zero. Complete asset markets allow for households to obtain state contingent bonds, either in the domestic or in the foreign asset market. Agents are able to assure a payoff for any state of the world by purchasing a full set of state contingent bonds, thus being completely insured, and receiving a guaranteed income stream no matter what type of aggregate shock hits their home country. Even though the complete set of state contingent bonds can easily replicate a one-period riskfree bond, it is convenient to model this bond apart as B_{ht} with payoff unity and respective price d_t, in order to derive an equation for the nominal interest rate. In accordance with further notation riskfree bonds maturing at date t are described as B_{ht-1} and z^t represents the state of the world at time t.[14] T_t are government lump-sum transfers and Π_t are profits of home country firms owned by domestic households. S_t is the nominal exchange rate, given as the price of one unit of foreign currency in terms of home country currency. The representative household spends money on consumption, state contingent bonds of either the home country or the foreign country, or it holds some of the money as liquidity.

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[13]Consequently all variables are depended on the state of the world, but to simplify notation this dependence is only modeled explicitly for the relevant case of the state contingent bonds; why this is important will become clear in the analysis of the model in section 4.

[14]This dating convention seems inconsistent but for further implementation into MATLAB using the TOOLKIT program, risk-free bonds must be dated like this, and it did not make sense to define the current state of the world as z^{t-1} for consistency reasons only.

Income is generated through working, the home country firms' profits, last period's left-over liquidity, government transfers, and the payoffs of the claims. Consumption is a CES-index, with ω being the constant elasticity of substitution between domestic goods C_{ht} and imports C_{ft}. The index is defined the following way:

$$C_t = \left(n^{1/\omega} C_{ht}^{1-1/\omega} + (1-n)^{1/\omega} C_{ft}^{1-1/\omega} \right)^{\omega/(\omega-1)}$$

For $\omega = 1$ aggregates of home and foreign country goods are perfectly substitutable. Cole and Obstfeld [1991] show that with $\omega = 1$ holding, perfect risk sharing can be achieved without any trade in bonds, as for this parameter specification the same insurance provided by a full set of state contingent bonds can be replicated by changes in terms of trade. In line with the final model by Devereux and Engel [2002] we will assume $\omega > 1$, thus imperfect competition among domestic goods and imports, and we will keep the assumption of perfect asset markets for the benchmark model instead, in order to be able to highlight the implications of a malfunction of this market later on.

Additionally, there is also monopolistic competition within each country. Goods produced by different firms are imperfect substitutes. Home consumption of domestic and of foreign goods is defined as continua given by the following indices with $\alpha > 1$.

$$C_{ht} = \left[n^{-1/\alpha} \int_0^n C_{ht}(i)^{\alpha-1/\alpha} di \right]^{\alpha/\alpha-1}$$

$$C_{ft} = \left[(1-n)^{-1/\alpha} \int_n^1 C_{ft}(i)^{\alpha-1/\alpha} di \right]^{\alpha/\alpha-1}$$

Consequently the domestic price index, describing the minimum expenditure required for the purchase of one unit of aggregate consumption, is composed of prices for domestic P_{ht} and imported $P_{ft}^{\star}$ goods. The latter are denoted in foreign currency and thus must be adjusted by the nominal exchange rate S_t.

$$P_t = \left[n P_{ht}^{1-\omega} + (1-n) \left(P_{ft}^{\star} S_t \right)^{1-\omega} \right]^{1/1-\omega}$$

$$P_{ht} = \left[\int_0^n P_{ht}(i)^{1-\alpha} di \right]^{1/1-\alpha}$$

$$P_{ft}^{\star} = \left[\int_n^1 P_{ft}^{\star}(i)^{1-\alpha} di \right]^{1/1-\alpha}$$

The last two equations are price indices composed of the individual prices set by home country firms and foreign country firms.

Optimal consumption of domestic and foreign goods and the optimal expenditure for those goods is given by the following equations:

$$C_{ht} = n \left[\frac{P_{ht}}{P_t} \right]^{-\omega} C_t$$

$$C_{ft} = (1 - n) \left[\frac{P_{ft}^{\star} S_t}{P_t} \right]^{-\omega} C_t$$

$$C_{ht}(i) = \frac{1}{n} \left[\frac{P_{ht}(i)}{P_{ht}} \right]^{-\alpha} C_{ht}$$

$$C_{ft}(i) = \frac{1}{1 - n} \left[\frac{P_{ft}^{\star}(i) S_t}{P_{ft}^{\star} S_t} \right]^{-\alpha} C_{ft}$$

$$\int_0^n P_{ht}(i) C_{ht}(i) d(i) = P_{ht} C_{ht}$$

$$\int_n^1 P_{ft}^{\star}(i) S_t C_{ft}(i) d(i) = P_{ft}^{\star} S_t C_{ft}$$

Firms   Firms produce final consumption goods that are sold in the domestic market and exported to the foreign country. Firms demand labor and pay a wage rate, which they take as given. Production is carried out using a linear technology requiring labor as the only input.

$$Y_t(i) = L_t(i)$$

Firms set prices in order to maximize period t profits, given by:

$$\Pi_t(i) = \left(n C_{h,t}(i) + (1 - n) C_{h,t}^{\star}(i) \right) P_{h,t}(i) - W_t L_t(i)$$

There are no transportation costs.

Government   The government issues bonds and prints money, which it pays out to home country agents in form of lump-sum transfers. The exchange rate floats freely without any interventions. Money supply is assumed to be following a stochastic process.

$$E_t \left(\frac{M_t}{M_{t+1}} \right) = \mu$$

where $\mu \leq 1$.

3.2 Imperfections in Goods Markets

Nominal Rigidities   There does not exists any apparent reason for individuals to delay adjustment of prices and wages when facing a real or monetary shock, as the adjustment is required for achieving optimal allocation. Thus, to incorporate nominal rigidities into the rational agent framework the assumption of menu costs, costs of changing prices and wages, is needed. Here these costs are not modeled explicitly, but only assumed implicitly. Prices are set one period in advance and cannot be changed the following period.

Firms's optimal pricing decisions are obtained by maximizing the sum of expected discounted profits.

$$\max_{\{P_{ht}(i)\}} \sum_{t=0}^{\infty} E_{t-1} d_{t-1} \Pi_t(i)$$

E_{t-1} are expectations at date $t-1$ and d_{t-1} is the inverse of the gross nominal interest rate that is used to discount profits. Sales are assumed to be determined by agents' demands.

Local Currency Pricing   Home country firms are able to price-discriminate between domestic and foreign agents. It is assumed that the costs of arbitrage for trade across borders are too high for consumers to bear. The firm's problem is defined as follows:

$$\max_{\{P_{ht}(i), P_{ht}^{\star}(i)\}} \sum_{t=0}^{\infty} E_{t-1} d_{t-1} \Pi_t(i)$$

$$\Pi_t(i) = n C_{ht}(i) P_{ht}(i) + (1-n) C_{ht}^{\star}(i) P_{ht}^{\star}(i) S_t - W_t L_t(i)$$

$P_{ht}^{\star}(i)$ denotes the price in foreign currency charged to foreign consumers by firm i. The problem of foreign firms changes accordingly, thus the price index for the home country is given by:

$$P_t = \left[n P_{ht}^{1-\omega} + (1-n) P_{ft}^{1-\omega} \right]^{1/1-\omega} \tag{3.4}$$

$$P_{ht} = \left[\frac{1}{n} \int_0^n P_{ht}(i)^{1-\alpha} di \right]^{1/1-\alpha}$$

$$P_{ft} = \left[\frac{1}{1-n} \int_n^1 P_{ft}(i)^{1-\alpha} di \right]^{1/1-\alpha}$$

The last two equations are price indices for home country nationals denoted in domestic currency and composed of individual prices set by domestic firms $P_{ht}(i)$ and foreign firms $P_{ft}(i)$.

Optimal consumption of imported goods is as follows:

$$C_{ft} = (1 - n) \left[\frac{P_{ft}}{P_t}\right]^{-\omega} C_t$$

$$C_{ft}(i) = \frac{1}{1 - n} \left[\frac{P_{ft}(i)}{P_{ft}}\right]^{-\alpha} C_{ft}$$

Distributors LCP implies zero pass-through of exchange rate fluctuations to consumer prices. The introduction of distributors accounts for the fact that pass-through to wholesale prices has been found to be higher. When exporting, producing firms can choose whether to sell their products directly to foreign country households at a price set in local currency or whether to sell the same good at a price set in domestic currency, to foreign country nationals who act as distributors. Under the assumption of complete asset markets this framework does not have any impact, nor on the behavior of exchange rates nor on matters of wealth distribution. As long as risk sharing is perfect, ownership of firms is irrelevant.[15] However, later on this strong assumption about the asset markets will be relaxed and hence the distributors' choices will matter.

Distributors are assumed to be in perfect competition. They set prices in advance that they have to stick to because of binding contracts, thus making zero expected profits. They cannot exit the market and might either be left with a profit or a loss, depending on the state of the world. Distributors simply buy and sell imported goods. They do not employ any resources. Unable to market any local products they face zero elasticity of substitution between imports and home country goods. There is hence no expenditure switching effect of exchange rate changes. Distributors are the ones who take on the exchange rate risk. Following Devereux et al. [2003] the problem of a home country national acting as a distributor of foreign goods looks as follows:

$$\max_{\{P_{ft}^D(i)\}} \sum_{t=0}^{\infty} E_{t-1} d_{t-1} \Pi_t^D(i)$$

$$\Pi_t^D(i) = n(P_{ft}^D(i) - Q_{ft}^{\star}(i) S_t) C_{ft}(i)$$

$Q_{ft}^{\star}(i)$ is the retail price set by foreign country producing firm i. The price is denominated in foreign currency.

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[15]Section 4.1.2 provides a proof of this assertion

Producing firms of the home country solve the following problem:

$$\max_{\{P_{ht}(i),P_{ht}^{P\star}(i),Q_{ht}(i)\}} \sum_{t=0}^{\infty} E_{t-1}d_{t-1}\Pi_t^P(i)$$

$$\Pi_t^P(i) = nC_{ht}(i)P_{ht}(i) + (1-n)\left(\theta C_{ht}^\star(i)P_{ht}^{P\star}(i)S_t + (1-\theta)C_{ht}^\star(i)Q_{ht}(i)\right) - W_t L_t(i)$$

θ denotes the share of directly marketed goods. The price for imported goods is defined as an index over prices set by foreign producers P_{ft}^P and home country distributors P_{ft}^D.

$$P_{ft} = \left(\theta^\star \left(P_{ft}^P\right)^{1-\alpha} + (1-\theta^\star)\left(P_{ft}^D\right)^{(1-\alpha)}\right)^{1/(1-\alpha)}$$

$\theta^\star$ denotes the share of directly marketed goods by foreign firms. Total profits received by home country agents are defined as $\Pi_t = \Pi_t^P + \Pi_t^D$.

3.3 Imperfections in Asset Markets

Non Contingent Claims The main implication of perfect asset markets is complete international risk sharing. This type of insurance induces among countries a higher correlation of consumption than of output. As an economy is hit by a country specific shock, the temporarily richer country has to distribute some gains to the temporarily poorer country, leading to smaller differences in changes in consumption than in changes in output. However, empirical studies find the opposite to hold true. Trying to find reasons for this discrepancy has been the focus of numerous studies with suggestions concerning e.g. heterogeneity of agents or the enforceability of risk sharing contracts. Here the reasons for the malfunction of perfect risk sharing are implicitly assumed and trade in state contingent bonds is limited to the domestic level. International asset trading is reduced to non contingent claims. A one period nominal riskfree bond is a typical example of a non contingent claim, a claim whose payoff is not state dependent, but expected to be the same in all states of the world.

Home country households can purchase one period nominal riskfree bonds in either the domestic or the foreign asset market. Their budget constraint is given by:

$$P_tC_t+M_t+\sum_{z^{t+1}} H(z^{t+1} \mid z^t)A(z^{t+1})+d_tB_{ht}+d_t^\star B_{ht}^\star S_t = W_tL_t+\Pi_t+M_{t-1}+T_t+A_t+B_{ht-1}+B_{h,t-1}^\star S_t$$

(3.5)

$B_{ht}^\star$ are foreign bonds held by home country agents. It is assumed that foreign households can only hold foreign bonds, as anything else would be redundant. The budget constraint of a foreign country agent is as follows:

$$P_t^\star C_t^\star + M_t^\star + \sum_{z^{t+1}} H^\star(z^{t+1} \mid z^t)A^\star(z^{t+1}) + d_t^\star B_{ft}^\star = W_t^\star L_t^\star + \Pi_t^\star + M_{t-1}^\star + T_t^\star + A_t^\star + B_{f,t-1}^\star$$

Noise Traders   As an additional group of agents foreign exchange dealers are added to the model to represent noise traders. These dealers are home country nationals who buy foreign bonds and sell them to home country households. International bond trading thus only occurs indirectly. Foreign exchange dealers are in perfect competition. There are no entry barriers to the market. The dealers maximize their discounted future profits by choosing the optimal amount of foreign bonds. Their problem can be written the following way:

$$\max_{\{B_{ht}^\star\}} \sum_{t=0}^{\infty} d_t \Pi_t^{fx}$$

$$\Pi_t^{fx} = E_t^n (H_t S_{t+1} B_{ht}^\star - d_t^\star S_t B_{ht}^\star)$$

Dealers' profits are given by the difference between expected payoff of a foreign bond and the cost of a foreign bond $d_t^\star S_t B_{ht}^\star$. Payoffs are discounted by H_t, the price of a domestic state contingent bond.[16] E_t^n are peculiar expectations of foreign exchange dealers that depart from rational expectations the following way:

$$E_t^n = E_t s_{t+1} + \upsilon_t, E_{t-1}(\upsilon_t) = 0$$
$$Var_t^n(s_{t+1}) = Var_t(s_{t+1})$$
$$Var_{t-1}(\upsilon_t) = \kappa^2 Var_{t-1}(s_t), 0 \leq \kappa < 1$$
$$E_t^n[H_t] = E_t[H_t] \tag{3.6}$$

Small case letters denote log deviations from the steady state value of the respective variable. υ_t can be interpreted as the amount of distorted information causing misconceptions on behalf of the dealers. Conditioned on the information set of date $t-1$ this term is expected to be zero, thus the two period ahead forecast is expected to be unbiased. However, noise traders take possibly noisy information, received at date t, into account when forming their expectations about a change in the exchange rate in t+1, turning the one period ahead forecast into a potentially biased estimator. Noise traders have perfect knowledge of the conditional variance of the change in exchange rates. This knowledge is applied when composing the conditional variance of the noise term, proportional to the former. This assumption implies that foreign exchange dealer know the overall volatility of exchange rates and take it into account. In periods of high exchange rate volatility noise is thus likely to be further away from zero, depending on the choice of κ. Misconception should thus be greater when fluctuations are stronger. This conclusion is in line with the quoted findings of Jeanne and Rose [2002]. Noise traders are aware of their surroundings, e.g. if operating under fixed or floating exchange rate regimes, and they act accordingly.

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[16]The analysis of the model in part 4 will show $d_t = E_t[H_t]$. Foreign exchange dealers' peculiar expectations require a different discount factor than the one introduced for firms, in order not to have an equation containing contradictory expectations

Expectations on the state contingent discount-factor are free of distortion, as this factor can be observed in the domestic asset market as the current price of a nominal state contingent bond. It is further assumed that each period households receive the amount of $S_t B^\star_{ht-1}$ from foreign exchange dealers as returns on their investments of $d^\star_t B^\star_{ht} S_t$. The difference between these two amounts is denoted by:

$$\Pi^f_t = S_t B^\star_{ht-1} - d^\star_t B^\star_{ht} S_t \qquad (3.7)$$

Home country agents receive this amount as an additional source of income. Their budget constraint is given by:

$$P_t C_t + M_t + \sum_{z^{t+1}} H(z^{t+1} \mid z^t) A(z^{t+1}) + d_t B_{ht} = W_t L_t + \Pi_t + \Pi^f_t + M_{t-1} + T_t + A_t + B_{ht-1} \qquad (3.8)$$

The assumption that foreign households do not trade in home country bonds is essential, because if noise traders were to act in both countries their actions would effect the exchange rate in opposite directions and the overall impact would thus be weakened.

4 Analysis of the Model

In the following sections first order conditions and all necessary equations for the definition of the equilibrium are derived. This procedure is applied to the benchmark model, as well as to any changes brought upon by the imperfections introduced. Close attention is being payed to equations that determine the behavior of the exchange rates. After defining the equilibrium conditions, the respective steady states are given and log-linearized equations of the model and its extensions are provided. The log-linearized equations are needed in order to pin down the behavior of the nominal exchange rate to a function of underlying fundamentals.

4.1 Equilibrium

4.1.1 Benchmark Model

When deriving optimizing behavior of agents, we apply the definition of rational expectations as the conditional probability of a state of the world occurring tomorrow given the knowledge of today's state of the world; put formally $E_t = \pi(z^t \mid z^{t-1})$.[17] Keeping this definition in mind, first order conditions of the household describing optimal holdings of domestic and foreign state contingent claims and of domestic riskfree bonds as well as optimal money demand and optimal labor supply can easily be obtained.

$$H(z^t \mid z^{t-1}) = \beta\pi(z^t \mid z^{t-1})\left[\frac{C_t^\rho P_t}{C_{t+1}^\rho P_{t+1}}\right] \tag{4.9}$$

$$H^\star(z^t \mid z^{t-1}) = \beta\pi(z^t \mid z^{t-1})\left[\frac{C_t^\rho P_t S_{t+1}}{C_{t+1}^\rho P_{t+1} S_t}\right]$$

$$d_t = \beta E_t\left[\frac{C_t^\rho P_t}{C_{t+1}^\rho P_{t+1}}\right] \tag{4.10}$$

$$\left(\frac{M_t}{P_t}\right)^\varepsilon = \frac{C_t^\rho}{1 - d_t} \tag{4.11}$$

$$W_t = \eta L_t^\psi P_t C_t^\rho \tag{4.12}$$

The relation between the equilibrium price of a riskfree bond as given in equation (4.10) and the domestic nominal interest rate is the following:

$$i_t = \frac{1 - d_t}{d_t}$$

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[17]See e.g. Cochrane [2001] (Chap. 3) for further details on state contingent claims

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Following Chari et al. [2002] we can derive the following essential risk sharing condition that holds in any economy where international trade in state contingent bonds is unrestraint:[18]

$$S_t \frac{P_t^\star}{P_t} = \Gamma_0 \left[\frac{C_t}{C_t^\star} \right]^\rho \tag{4.13}$$

Γ_o are initial conditions set to unity. The left hand side of the risk sharing condition is equal to the real exchange rate, defined in equation (2.2). With PPP (2.1) holding the real exchange rate remains a constant, thus marginal utilities and even absolute levels of consumption in both countries cannot differ, given the assumption of identical preferences for foreign and home country nationals.

Combining domestic money demand (4.11), its foreign equivalent and the risk sharing condition (4.13) leads to the following equation:

$$S_t = \left(\frac{M_t}{M_t^\star} \right)^\varepsilon \left(\frac{P_t^\star}{P_t} \right)^{\varepsilon-1} \left(\frac{1-d_t}{1-d_t^\star} \right) \tag{4.14}$$

Applying PPP (2.1), the definition of the nominal interest rate, and UIRP (2.3) gives an equation that fully determines the nominal exchange rate of the benchmark model:[19]

$$\left[\frac{S_{t+1}}{S_t} \right] S_t^\varepsilon = \left(\frac{M_t}{M_t^\star} \right)^\varepsilon \frac{i_t}{i_t^\star} \tag{4.15}$$

The exchange rate is influenced by movements in money supplies and nominal interest rates. Firms' pricing decisions have no influence on the determination of the exchange rate. Under the assumption of $\varepsilon = 1$ and thus money balances entering the utility function logarithmically, the nominal interest rate is a constant.[20] In this case the exchange rate is given by the ratio of money supplies $S_t = \frac{M_t}{M_t^\star}$.

A complete benchmark model enables us to thoroughly track changes in the behavior of the exchange rate and in the behavior of other variables, which are brought upon by the imperfections in goods and asset markets. That is why firms' decisions are considered despite their irrelevance for the determination of the exchange rate in the benchmark model. Under the assumption of a symmetric equilibrium all firms of one country are identical and choose the same price $P_{ht}(i) = P_{ht}(k) = P_{ht}$ with $i \neq k$.

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[18]Part A.1 of the appendix shows how the risk sharing condition is derived.
[19]Under the assumption of perfect asset markets UIRP holds with certainty.
[20]Part A.2 of the appendix provides a proof of this assertion

Competitive monopolistic firms know the demand functions of households and can thus take households' decisions on optimal consumption into account when setting profit maximizing prices. With $C_t = C_t^\star$ holding, derivation of the optimal price set by home country firms is straight forward.

$$P_{ht} = \frac{\alpha}{\alpha - 1} W_t$$

Equivalently for the producer of the foreign country :

$$P_{ft}^\star = \frac{\alpha}{\alpha - 1} W_t^\star$$

Competitive monopolistic firms can demand a mark-up factor $\frac{\alpha}{\alpha-1}$ over marginal costs. This factor is determined by the price elasticities of demand which are assumed to be equal in both countries.

In equilibrium goods markets in both countries clear. Using the production function, the equations for optimal consumption and their foreign equivalents, applying PPP (2.1) and $C_t = C_t^\star$, the goods market clearing condition is given by:

$$L_t = \left[\frac{P_{ht}}{P_t} \right]^{-\omega} C_t$$

Money market clearing implies:

$$M_t = M_{t-1} + T_t$$

There is zero net trading in bonds and claims:

$$B_{ht} = B_{ht-1}; A_t = A_{t-1}; A_t^\star = A_{t-1}^\star$$

The benchmark model is fully described by the agents' optimal choices given by equations (4.11), (4.10), and (4.12), the goods market clearing condition, the price index and foreign equivalents for all these equations, as well as the risk sharing condition (4.13), the two pricing equations plus two stochastic processes for the money supplies. Additionally, definitions for the real exchange rate (2.2) and the nominal interest rates for both countries may be included.

4.1.2 Imperfections in Goods Markets

Nominal Rigidities The consumer's problem stays unaltered as PPP continues to hold. Firms, however, face a slightly different problem, and must change their pricing schemes accordingly. Bearing in mind that all prices are predetermined, that they, as well as all variables dated $t - 1$, can be taken out of the expectation as they stay unaffected by any time t shocks, the following prices are derived. For the home country firm:

$$P_{ht} = \frac{\alpha}{\alpha - 1} \frac{E_{t-1}[W_t C_t^{1-\rho}]}{E_{t-1}[C_t^{1-\rho}]} \tag{4.16}$$

and its foreign counterpart:

$$P_{ft}^\star = \frac{\alpha}{\alpha - 1} \frac{E_{t-1}[W_t^\star C_t^{\star 1-\rho}]}{E_{t-1}[C_t^{\star 1-\rho}]} \tag{4.17}$$

Firms charge a mark-up factor over expected marginal costs. These costs are the costs of not being able to adjust prices ex-post. As Devereux and Engel [2003] point out, prices are set such that they equal the ratio of the marginal costs of lowering prices $P_{ht}^{-\alpha-1} C_t W_t$ to the marginal increase in revenues from doing so $P_{ht}^{-\alpha} C_t$, both measures weighted by the discount factor d_{t-1}, that is given by equation (4.10). We take as an example a positive money shock to illustrate this relation. As prices are sticky, this shock causes a fall in real interest rates and thus an increase in consumption. Since sales are demand determined, firms have to produce more to meet demand. Keeping in mind the linearity of production, firms' costs thus rise by $\frac{\partial C_t}{\partial P_{ht}} W_t$ per extra unit produced whereas their revenues increase by $\frac{\partial C_t}{\partial P_{ht}} P_{ht}$ per extra unit sold.[21] The ratio of the two derivatives accounts for the fact that relative costs in terms of revenue matter to firms not their absolute value. As PPP and UIRP continue to hold, the determination of the nominal exchange rate is not altered by these changes. Firm's price setting remains irrelevant for the behavior of exchange rates.

Local Currency Pricing Firms choose two different prices one in domestic currency for home country nationals and one in foreign currency for sales abroad. Prices of goods sold to locals stay as given by equation (4.16). The price charged to foreign country nationals by home country firms is given by:

$$P_{ht}^\star = \frac{\alpha}{\alpha - 1} \frac{E_{t-1}[W_t C_t^\star C_t^{-\rho}]}{E_{t-1}[C_t^\star C_t^{-\rho} S_t]} \tag{4.18}$$

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[21]We use the equations on households' optimal consumption behavior when taking derivatives.

Whereas the price charged by foreign producers to home country nationals can be written as:

$$P_{ft} = \frac{\alpha}{\alpha - 1} \frac{E_{t-1}[W_t^\star C_t C_t^{\star - \rho}]}{E_{t-1}[C_t C_t^{\star - \rho}/S_t]} \qquad (4.19)$$

Prices equal the ratio of costs and benefits of forgone flexible price setting, thus expectations regarding possible changes of the exchange rate enter the pricing equations. As prices are set in the currency of the consumer the marginal revenues of exports $\frac{\partial C_t^\star}{\partial P_{ht}^\star} P_{ht}^\star S_t$, are adjusted by the exchange rate. This causes the deviations from PPP. Movements in the real exchange rate are possible leading to differences in consumption levels across countries, thus to deviations from absolute perfect risk sharing. PPP and absolute perfect risk sharing only hold on expectation. Under the assumption of perfect asset markets the risk sharing condition (4.13) remains valid which allows for simplifications of the pricing equations to: $P_{ht}^\star = \frac{\alpha}{\alpha - 1} \frac{E_{t-1}[W_t C_t^{\star 1-\rho}/S_t]}{E_{t-1}[C_t^{\star 1-\rho}]}$ and $P_{ft} = \frac{\alpha}{\alpha - 1} \frac{E_{t-1}[W_t^\star C_t^{1-\rho} S_t]}{E_{t-1}[C_t^{1-\rho}]}$, as Devereux and Engel [2003] point out. These simplifications, however, do not imply $C_t = C_t^\star$, but as the assumption about perfect asset markets will not continue to hold throughout the whole model, we will not apply these simplifications

The goods market clearing condition is given by:

$$L_t = n \left[\frac{P_{ht}}{P_t}\right]^{-\omega} C_t + (1 - n) \left[\frac{P_{ht}^\star}{P_t^\star}\right]^{-\omega} C_t^\star \qquad (4.20)$$

The nominal exchange rate is determined by equation (4.14). However, without PPP holding, further simplification is not possible. Under LCP price setting has an influence on the behavior of the exchange rate.

Distributors The firms' problem can be described as a two-step Stackelberg game with producers as leaders and distributing firms or retailers as followers. Thus a backward induction approach is applied to solve it. First, the problem of the distributors is solved, then applying the knowledge of those results producers maximize their profits by choosing the optimal wholesale price. It is assumed that distributors know the households' demand function.[22] Keeping this assumption in mind and the facts that expected profits of distributors are zero and that prices are predetermined, the price charged by home distributors to domestic households for imported goods is derived:

$$P_{ft}^D = Q_{ft}^\star \frac{E_{t-1}\left(S_t C_t^{1-\rho}\right)}{E_{t-1}\left(C_t^{1-\rho}\right)}$$

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[22]Even though distributors are not monopolistic competitive, one might assume that they have knowledge of these demand functions through the producing firms.

Equivalently for foreign distributors selling home country goods abroad their retail price is given by:

$$P_{ht}^{D\star} = Q_{ht} \frac{E_{t-1}\left(C_t^{\star 1-\rho}/S_t\right)}{E_{t-1}\left(C_t^{\star 1-\rho}\right)}$$

Given knowledge of these prices home country producers are able to derive optimal wholesale prices:

$$Q_{ht} = \frac{\alpha}{\alpha - 1} \frac{E_{t-1}[W_t C_t^\star C_t^{-\rho}]}{E_{t-1}[C_t^\star C_t^{-\rho}]}$$

Foreign producer charge the following when selling to domestic distributors:

$$Q_{ft}^\star = \frac{\alpha}{\alpha - 1} \frac{E_{t-1}[W_t^\star C_t C_t^{\star -\rho}]}{E_{t-1}[C_t C_t^{\star -\rho}]}$$

Producing firms do not alter their prices for directly marketed products. Following Devereux et al. [2003], we can show that under perfect risk sharing $P_{ht}^{D\star} = P_{ht}^{P\star}$ always holds. When deriving Q_{ht} we simply keep prices dated t inside the expectation and use the risk sharing condition (4.13) to get $Q_{ht} = \frac{\alpha}{\alpha-1} \frac{E_{t-1}[S_t W_t C_t^{\star 1-\rho}]}{E_{t-1}[C_t^{\star 1-\rho}]}$. Plugging this into the equation for the retail price set by foreign distributors, we arrive at equation (4.18) that gives the price set by foreign producers for directly marketed goods. This relation confirms the aforementioned notion that, as long as risk sharing is perfect, distributors do not affect the outcome of the model. However, as the assumption of a full set of state-contingent bonds will be relaxed later on, for further analysis we will stick to the initial retail and wholesale prices derived.

The goods market clearing condition is given by.

$$L_t = n\left[\frac{P_{h,t}}{P_t}\right]^{-\omega} C_t + (1-n)\left(\theta\left[\frac{P_{h,t}^{P\star}}{P_{h,t}^\star}\right]^{-\alpha}\left[\frac{P_{h,t}^\star}{P_t^\star}\right]^{-\omega} + (1-\theta)\left[\frac{P_{h,t}^{D\star}}{P_{h,t}^\star}\right]^{-\alpha}\left[\frac{P_{h,t}^\star}{P_t^\star}\right]\right)C_t^\star$$

Determination of the nominal exchange rate is given by equation (4.14) and without PPP holding there is no simplification possible.

4.1.3   Imperfections in Asset Markets

Non Contingent Claims   The consumer's problem is altered. As agents are unable to purchase foreign state contingent bonds, the risk sharing condition (4.13), does not continue to hold, nor does the equation for the determination of the exchange rate (4.14).

Optimal holdings of foreign one-period nominal riskfree bonds by home country agents are given by:

$$d_t^\star = \beta E_t \left[\frac{C_t^\rho P_t S_{t+1}}{C_{t+1}^\rho P_{t+1} S_t} \right] \tag{4.21}$$

Foreign country nationals can hold the same bonds. Their optimal choices of these bonds is given by:

$$d_t^\star = \beta E_t \left[\frac{C_t^{\star\rho} P_t^\star}{C_{t+1}^{\star\rho} P_{t+1}^\star} \right] \tag{4.22}$$

Combining these two equations with the optimal holdings of domestic bonds (4.10), and keeping in mind the relationship between bond price and nominal interest rate, it is easy to see that UIRP holds. However, without PPP holding real interest rates among countries can differ.

We define the terms of trade as the ratio of export prices to import prices, adjusted by the nominal exchange rate, thus as the relative price of exports in terms of imports in home country currency $TT = \frac{P_{ht}^\star S_t}{P_{ft}}$.[23] We also define the home country current account as $CA = B_{ht}^\star$. If the home country's current account, defined as a its exports minus its imports, is in surplus the foreign country is in debt and hence must use its bonds to pay for its imports. That is why, an increase in net holdings of foreign assets by home country nationals is equal to the home country's current account and can be defined as such.

Under incomplete asset markets wealth effects are not canceled via perfect risk sharing. Money shocks can have permanent effects on a country's relative prosperity. These effects will be reflected in changes in the current account and the country's terms of trade. In order to account for these wealth effects within the framework of the model the agent's budget constraint (3.5) that contains all income aspects is added to the equilibrium definition.

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[23] Note that the usual definition $TT = \frac{P^{ex}}{P^{im} S}$ is reversed under LCP as export prices are set in foreign currency and import prices are set in domestic currency;

Additionally, the sum of all profits received by the representative household is specified as:

$$\Pi_t = nP_{ht}\left(\frac{P_{ht}}{P_t}\right)^{-\omega}C_t + (1-n)\theta S_t P_{ht}^{P\star}\left(\frac{P_{ht}^{P\star}}{P_{ht}^{\star}}\right)^{-\alpha}\left(\frac{P_{ht}^{\star}}{P_t^{\star}}\right)^{-\omega}C_t^{\star}+$$

$$+ (1-n)(1-\theta)Q_{ht}\left(\frac{P_{ht}^{D\star}}{P_{ht}^{\star}}\right)^{-\alpha}\left(\frac{P_{ht}^{\star}}{P_t^{\star}}\right)^{-\omega}C_t^{\star} +$$

$$+ (1-\theta^{\star})(1-n)(P_{ft}^{D} - S_t Q_{ft}^{\star})\left(\frac{P_{ft}^{D}}{P_{ft}}\right)^{-\alpha}\left(\frac{P_{ft}}{P_t}\right)^{-\omega}C_t - W_t L_t \qquad (4.23)$$

The first term denotes home country firms' revenues from sales to domestic residents. The second term gives the earnings from directly marketed domestic goods to foreign households, the third describes those revenues from sales of domestic goods to foreign retailers. The earnings of home country distributors that sell foreign goods to domestic agents are described by the next term. The last term accounts for the costs of production, incurred by the producing firm of the home country. Distributors account for ambiguous wealth effects of exchange rate changes. We take as an example an increase in the nominal exchange rate, hence a depreciation of the home currency. This depreciation will leave producing firms who market their products directly to foreign country agents better off because they receive revenues in foreign currency which has appreciated. Home country distributors, however, are worse off because their earnings denominated in home country currency are reduced whereas their costs, the wholesale prices set in foreign currency by foreign firms, have increased. The exchange rate is determined by a joint working of the UIRP in the asset market and the way firms set prices in the goods markets.

Noise Traders   Foreign exchange dealers make expected zero returns, which implies the following about their choice on the optimal purchase of foreign bonds:

$$d_t^{\star} = E_t^n \frac{H_t S_{t+1}}{S_t} \qquad (4.24)$$

H_t is the state contingent discount factor as given by (4.9). Obviously, UIRP cannot be derived anymore, as expectations are not compatible. None of the basic concepts of exchange rate determination holds. Noise traders' actions and firms' price setting influence the behavior of the exchange rates.

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The full model with noise traders is defined by the household's optimal choices (4.11), (4.12) and (4.10) and their foreign equivalents, the noise traders' optimal demand for foreign bonds (4.24), goods market clearing condition (4.1.2) and price indices for both countries (3.4) with the additional indices for the price of imported goods; furthermore the home country agent's budget constraint (3.8), the definition for total profits received by households (4.23), the return on investment in foreign bonds by home country nationals (3.7) and all pricing equations for goods sold to agents within the country of production, for directly marketed exported goods, and for indirectly marketed exported goods, plus the wholesale prices are needed. For the dynamics of the model the stochastic processes for money supply in the home and foreign country are added.

4.2   Steady State

A symmetric steady state is assumed. Initially all prices are equal and consumption and output across countries are the same. From this assumption it follows directly that the steady state nominal exchange rate is equal to unity. The home country government does not issue bonds. There are no government transfers and net foreign bond holdings are zero, i.e. the current account is balanced.

1. $\bar{S} = 1$

2. $\bar{d} = \bar{d}^\star = \beta$

3. $r = 1/\beta - 1$

4. $\bar{A}_h = \bar{A}_h^\star = \bar{B}_h = \bar{B}_h^\star = 0$

5. $\bar{T} = \bar{T}^\star = 0$

6. $\bar{L} = \bar{C} = \bar{L}^\star = \bar{C}^\star$

7. $\bar{P}_h = \bar{P}_h^\star = \bar{P}_f^\star = \bar{P}_f = \bar{P} = \bar{P}^\star$

8. $\bar{W} = \bar{W}^\star$

9. $\bar{W} = \eta \bar{P} \bar{C}^{\rho + \psi}$

10. $\bar{P} = \frac{\alpha}{\alpha - 1} \bar{W}$

11. $\bar{C} = \left(\frac{1}{\eta} \frac{\alpha - 1}{\alpha} \right)^{\frac{1}{\rho + \psi}}$

12. $\left(\frac{\bar{M}}{\bar{P}} \right)^{\varepsilon} = \frac{\bar{C}^\rho}{1 - \beta} = \left(\frac{\bar{M}^\star}{\bar{P}^\star} \right)^{\varepsilon}$

r denotes the steady state real interest rate, equal for both countries, as in steady state both PPP and UIRP hold.

4.3 Log-Linearization

We define small case letters as log deviations of a variable from its steady state value, thus as: $x_t = ln(1 + X_t)$. These deviations can be interpreted as percentage changes. The focus of this section will be on the equations that define the behavior of the nominal exchange rate. For the benchmark model and the final model complete lists of log-linearized equations are provided in section A.3 of the appendix.

4.3.1 Benchmark Model

The equation that determines the exchange rate in the benchmark model (4.15) is given in its log-linearized form:

$$[s_{t+1} - s_t] + \varepsilon s_t = \varepsilon(m_t - m_t^\star) + i_t - i_t^\star \tag{4.25}$$

UIRP can be written as:

$$i_t - i_t^\star = \frac{1 + r}{r} E_t[s_{t+1} - s_t] \tag{4.26}$$

Money supply is assumed to be following a simple random walk:

$$m_t = m_{t-1} + u_t$$
$$u_t \sim N(0, \sigma_m) \tag{4.27}$$

In addition money shocks are assumed to be uncorrelated across countries.

Combining the above conditions leads to the following equation that fully accounts for the behavior of the exchange rate in the benchmark model:

$$s_t = m_t - m_t^\star \tag{4.28}$$

Movements in the nominal exchange rate are caused by differences in money supplies. Exchange rate volatility is thus determined by the variance of the money supplies. As the real exchange rate remains a constant, the benchmark model is neither able to achieve excess volatility nor a correlation between nominal and real rates. The exchange rate cannot display any disconnectedness from fundamental values.

4.3.2 Imperfections in Goods Markets

Nominal Rigidities As stated in section 4.1.2, the introduction of nominal rigidities into the benchmark model does not influence the way the exchange rate is determined. The behavior of other variables under the assumption of sticky prices will be discussed when results for this case are presented in section 5.2.

Local Currency Pricing Log-linearization of equation (4.14) that determines the exchange rate under LCP and combining this equation with UIRP (4.26) leads to:

$$s_t = \varepsilon(m_t - m_t^\star) + (\varepsilon - 1)(p_t^\star - p_t) + \frac{1}{r}E_t[s_{t+1} - s_t] \tag{4.29}$$

In order to proceed, log-linearized versions of the price indices and of all pricing equations have to be derived.

$$p_t = np_{ht} + (1-n)p_{ft} \tag{4.30}$$

$$p_t^\star = np_{ht}^\star + (1-n)p_{ft}^\star \tag{4.31}$$

and

$$p_{ht} = E_{t-1}w_t \tag{4.32}$$

$$p_{ft}^\star = E_{t-1}w_t^\star \tag{4.33}$$

$$p_{ht}^\star = E_{t-1}[w_t - s_t] \tag{4.34}$$

$$p_{ft} = E_{t-1}[w_t^\star + s_t] \tag{4.35}$$

With UIRP being valid and with PPP holding on expectation, the anticipated movement in the exchange rate should be equal to the change in exchange rate of the benchmark model as given by equation (4.28). We make this guess:

$$E_{t-1}s_t = E_{t-1}[m_t - m_t^\star] \tag{4.36}$$

Additionally, we define:

$$\hat{x}_t = x_t - E_{t-1}x_t$$

$$E_t\hat{x_{t+1}} = E_tx_{t+1} - E_{t-1}x_{t+1}$$

An unexpected change of a variable is given as a deviation from its expected value at date $t - 1$. Applying the pricing equations, after some intermediate steps the change in the exchange rate can be defined as:[24]

$$s_t = \frac{1 + \varepsilon r}{1 + r}(\hat{m}_t - \hat{m_t^\star}) + (m_{t-1} - m_{t-1}^\star) \tag{4.37}$$

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[24]See part A.4 of the appendix for a detailed derivation of these equations

Under LCP an intertemporal aspect is added to the dynamics of the exchange rate. The behavior of the exchange rate depends on this period's as well as on last period's money supplies. The unanticipated change in the exchange rate is determined by the unexpected change in moneys and can thus be defined as:

$$\hat{s}_t = \frac{1 + \varepsilon r}{1 + r}(\hat{m}_t - \hat{m}_t^\star) \tag{4.38}$$

By choosing $\varepsilon > 1$ a reaction of the exchange rate to changes in money supplies out of proportion of the former can be achieved. This way volatility may increase dramatically as impulse responses and simulations for this case in section 5.2 will visualize.

Distributors Log-linearization of the wholesale price charged by home country firms and the consumer price charged by foreign distributors leads to:

$$q_{ht} = E_{t-1} w_t$$

$$p_{ht}^{D\star} = E_{t-1}[q_{ht} - s_t]$$

It is easy to see that $p_{h,t}^{D\star} = p_{h,t}^{P\star}$. Home country firms marketing their products directly to foreign consumers and foreign distributors change their prices equivalently in response to a shock. Regardless of the underlying structure of the asset markets, prices for directly and indirectly marketed products are the same up to a log-linear approximation. Thus from now on, the distinction between the two prices will be dropped. In addition as long as agents are able to obtain a full set of state contingent bonds distributors have no effect on movements in the exchange rate. Hence these movements remain determined by condition (4.37).

4.3.3 Imperfections in Asset Markets

Non Contingent Claims Under the assumption of incomplete asset markets the agent's budget constraint (3.5) had to be added to the equilibrium definition of the model. The log-linearized version of the budget constraint, incorporating the definition of firms' profits (4.23) looks as follows:

$$p_t + c_t + \frac{\beta \bar{d}}{\overline{PC}} b_{ht}^\star = n(-\omega(p_{ht} - p_t) + c_t + p_{ht}) + (1-n)\theta(-\omega(p_{ht}^\star - p_t^\star) + s_t + p_{ht}^\star + c_t^\star) +$$
$$+ (1-n)(1-\theta)(-\omega(p_{ht}^\star - p_t^\star) + q_{h,t} + c_t^\star) + (1-n)(1-\theta^\star)(-\omega(p_{ft} - p_t) + c_t + p_{ft}) -$$
$$- (1-n)(1-\theta^\star)(-\omega(p_{ft} - p_t) + c_t + q_{ft}^\star + s_t) + \frac{\bar{d}}{\overline{PC}} b_{ht-1}^\star \tag{4.39}$$

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Due to the their steady state value of zero, net foreign bond holdings must be defined differently as: $B_{ht}^\star = \bar{B}_h^\star + \frac{B_{ht}^\star - \bar{B}_h^\star}{\bar{d}}\bar{d}$ where $\frac{B_{ht}^\star - \bar{B}_h^\star}{\bar{d}} = b_{ht}^\star$ denotes changes in the home country's net foreign bond holdings. These changes are defined as percentages deviations from the steady state price of the one period riskfree bond. This does not have any particular economic reasoning. In the log-linearized version of the budget constraint, however, the denominator simply cancels and changes in net foreign bond holdings, i.e. changes in the current account, are defined as percentage deviations from steady state nominal consumption. This is a sensitive measure because it takes into account a country's ability to pay back its trade deficits. A country with an enormous absolute trade deficit that consumes a lot will be more likely to balance its debts than a country that consumes little but a large share of imported goods. Thus, by means of the denominator $\bar{P}\bar{C}$ more weight is given to the same absolute change in net foreign bond holdings in case of the latter country.

To derive an equation that determines the nominal exchange rate additional log-linearized equations are needed. Labor supply is given by:

$$w_t = p_t + \rho c_t + \psi l_t \tag{4.40}$$

The goods market clearing condition looks as follows:

$$l_t = n(-\omega(p_{ht} - p_t) + c_t) + (1 - n)(-\omega(p_{ht}^\star - p_t^\star) + c_t^\star) \tag{4.41}$$

Combining these two equations and their foreign equivalents with the budget constraint gives a relation between expectations on the difference in relative consumptions on the left hand side and expected terms of trade and initial net foreign bond holdings on the right hand side.[25]

$$E_{t-1}[c_t - c_t^\star] = (1 - \omega)E_{t-1}[w_t - w_t^\star - s_t] + (1 - \beta)\frac{\bar{d}}{\bar{P}\bar{C}(1 - n)}b_{ht}^\star \tag{4.42}$$

Under the assumption of $\omega > 1$ changes in expected domestic consumption are negatively dependent on movements in expected terms of trade. This condition is due to the fact that an increase in prices of exports by one unit will increase home country firms' revenue by one unit per good sold but at the same time reduces foreign demand for those goods by ω units. On expectation, an appreciation of the home country currency leads to an improvement in its terms of trade and a fall in relative consumption because an anticipated fall in the exchange rate influences firms's price setting. These workings are in line with all PCP models and the empirical observations that Obstfeld and Rogoff [2000a] posit in critic of the LCP approach.

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[25]For the intermediate steps on the entire section consult part A.5 of the appendix

However, recalling the definition of a country's terms of trade for the LCP case, this assertion does not hold true when an unexpected shock occurs. In that case terms of trade worsen as a result of an appreciation and depending on the share of directly marketed goods a fall in the exchange rate can even have a positive influence on relative consumption. Note also that $E_{t-1}[w_t - s_t] = p_{ht}^\star$ is the price for domestic goods sold abroad and $E_{t-1}w_t^\star = p_{ft}^\star$ the price for goods produced by foreign firms sold in the foreign market. A difference in those prices will shift foreign demand towards the cheaper product, hence $p_{ht}^\star > p_{ft}^\star$ will reduce home country firm's profits and thus expected relative consumption. An increase in initial net holdings of foreign bonds is obviously expected to have a positive effect on relative home country consumption because returns on these bonds provide additional income for the home country agent.

The expected change of the nominal exchange rate is given by:

$$E_{t-1}s_t = E_{t-1}[m_t - m_t^\star] - \frac{\rho}{\varepsilon}E_{t-1}[c_t - c_t^\star] \tag{4.43}$$

On expectation, the exchange rate behaves as predicted by standard monetary theory. It is easy to see, that under the assumption of perfect asset markets and PPP holding on expectation, one arrives at the expected exchange rate under LCP pricing as given by equation (4.36). However, without perfect asset markets, expected differences in relative consumption, thus expected terms of trade as well as the changes in initial net foreign bond holdings, i.e. the country's initial trade balance, have an influence on the expectations regarding changes of the exchange rate.

The unexpected change in the exchange rate, on the other hand, can be written as follows:

$$\hat{c}_t - \hat{c}_t^\star + \frac{\beta\bar{d}\bar{b}\hat{}_{ht}^\star}{(1-n)\bar{P}\bar{C}} = \hat{s}_t(\theta - (1 - \theta^\star)) \tag{4.44}$$

An unanticipated rise in the exchange rate, i.e. an unexpected depreciation of the home country currency, has a positive relative wealth effect on home country consumers as long as $\theta > (1 - \theta^\star)$, i.e. as long as the share of directly marketed products by home country firms is larger than the share of foreign products sold by home country retailers. This relation reflects the ambiguous wealth effects that the introduction of distributors to the model has brought along. Whereas producing firms profit from a depreciation distributors are punished. If profits are bigger than loses, agents will use a certain share of their additional wealth to increase current consumption and another part of this wealth to accumulate foreign assets, in order to smooth consumption and to prolong the positive effects of the shock.

This connection between unexpected changes in relative present and future consumption and unanticipated movements in the exchange rate is stated in the following equation:

$$\hat{c}_t - \hat{c}_t^\star + \frac{\sigma}{r}E_t\left[\hat{c_{t+1}} - \hat{c_{t+1}^\star}\right] = \hat{s}_t(\theta - (1 - \theta^\star)) \tag{4.45}$$

where $\sigma = (1 - ((1-\omega)\rho/(1+\psi\omega))) > 0$ is assumed in order to avoid so called immiserizing growth. The concept of immiserizing growth states that a country's growth, if based on an increase in exports, might leave the country worse off if the decrease in prices for its exports is bigger than the increase in volume of products sold. However, as there is no growth in this model the assumption on σ simply assures that an unexpected depreciation of the exchange rate cannot cause a fall in future consumption of home country agents.

Log-linearized versions of optimal holdings of foreign and domestic bonds by home country nationals, optimal bond holdings of foreign country nationals, and of the money demand function are given by:

$$d_t^\star = p_t^\star + \rho c_t^\star - E_t[p_{t+1}^\star + \rho c_{t+1}^\star] \tag{4.46}$$

$$d_t = p_t + \rho c_t - E_t[p_{t+1} + \rho c_{t+1}] \tag{4.47}$$

$$d_t^\star = p_t + \rho c_t - E_t[p_{t+1} + \rho c_{t+1}] + E_t s_{t+1} - s_t \tag{4.48}$$

$$m_t - p_t = \frac{\rho}{\varepsilon}c_t + \frac{1}{r\varepsilon}d_t \tag{4.49}$$

It is easy to verify that even under incomplete asset markets UIRP (4.26) holds. We can express the unanticipated change in the exchange rate as a function of the difference in unexpected changes in money supplies:

$$\hat{s}_t = \frac{(1 + (\sigma/r))\varepsilon(\hat{m}_t - \hat{m}_t^\star)}{((\sigma/r) + \rho(\theta - (1 - \theta^\star)))} \tag{4.50}$$

The ability to generate high volatility of exchange rates under imperfect markets is restraint by the coefficient: $\frac{(1+(\sigma/r))\varepsilon}{((\sigma/r)+\rho(\theta-(1-\theta^\star)))}$ This volatility is highly depended on the shares of directly and indirectly marketed goods. Under the assumption of $\varepsilon > 1$ and by choosing $\theta + \theta^\star = 1$ exchange rate volatility is out of proportion with the volatility of money supplies. While a firm can shield its profits from changes in exchange rates, thus reducing volatility of profits by employing distributors, the existence of distributors on the macroeconomic level implies a higher volatility of exchange rates. This matter will be explored further in section 5, when simulation results for different parameter choices are presented and discussed.

Noise Traders Log-linearizing foreign exchange dealers' optimal choice of foreign bonds (4.24) and applying the definition of the peculiar expectations of noise traders (3.6) yields the following:

$$d_t^\star = p_t + \rho c_t - E_t[p_{t+1} + \rho c_{t+1}] + E_t s_{t+1} + \nu_t - s_t \qquad (4.51)$$

UIRP does not continue to hold. As the following equation demonstrates it is only valid on expectation.

$$\rho E_t[c_{t+1} - c_t] + E_t[p_{t+1} - p_t] = \rho E_t[c_{t+1}^\star - c_t^\star] + E_t[p_{t+1}^\star - p_t^\star] + E_t s_{t+1} - s_t + \nu_t \quad (4.52)$$

The unanticipated change in the exchange rate can be expressed as a function of the difference in unexpected changes to money supplies and the noise term:

$$\hat{s}_t = \frac{(1 + (\sigma/r))\varepsilon(\hat{m}_t - \hat{m}_t^\star) + \frac{\sigma}{r}\nu_t}{((\sigma/r) + \rho(\theta - (1 - \theta^\star)))} \qquad (4.53)$$

The conditional variance of the change exchange rate is hence given by:

$$Var_{t-1}(s_t) = \frac{(1 + (\sigma/r))^2 \varepsilon^2 Var_{t-1}(m_t - m_t^\star)}{\Phi^2 \left(1 - (\kappa\sigma/r\Phi)^2\right)} \qquad (4.54)$$

where $\Phi = \left(\frac{\sigma}{r} + \rho(\theta - (1 - \theta^\star))\right)$.[26] It is clear to see that for the case of $\kappa \to 1$ and $\theta + \theta^\star = 1$ the denominator of equation (4.54) approaches zero, thus the conditional volatility of the exchange rate rises to infinity. Since this increase in volatility does not require equally strong movements in money supplies, the introduction of noise traders pays tribute to the feature of disconnectedness of the exchange rate while at the same time achieving high volatility of exchange rates. Results in part (5) will go into detail on this matter.

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[26]Consult part A.6 of the appendix for intermediate steps and to see why $Var_{t-1}(\hat{s}_t) = Var_{t-1}(s_t)$ and $Var_{t-1}(\hat{m}_t) = Var_{t-1}(m_t)$

Parameter	Value	Description
ρ	4	Inverse of elasticity of intertemporal substitution
ε	4.7	Inverse of elasticity of money demand
ψ	1	Inverse of elasticity of labor supply
ω	8	Elasticity of substitution between home and foreign aggregates
n	0.5	Relative size of home country
r	0.1	(Annual) Steady state real interest rate
σ_m	1%	Variance of domestic money supply
$\sigma_{m^\star}$	1%	Variance of foreign money supply
$\bar{P}\widetilde{C}$	1	Steady state nominal consumption

Table 1: Parameter Specifications

4.4 Calibration

In order to obtain similar results as Devereux and Engel [2002], most of their parameter specifications are adopted. The choice of the first two parameters is particularly crucial because, as Betts and Devereux [2001] and Devereux [1999] point out, their ratio, $\frac{\rho}{\varepsilon}$, is the consumption elasticity of money demand that is essential for the response of the exchange rates to shocks in money supplies. Thus, the parameter values for ρ and ε have a large impact on the behavior of the exchange rates. Betts and Devereux [2001] state that most empirical estimates for the consumption elasticity of money demand are around unity. We use their choice of $\frac{\rho}{\varepsilon} = 0.85$ to determine the value for ε because this parameter value was not specified by Devereux and Engel [2002], as they apply a utility function with money balances entering logarithmically. The choice of ε further affects the second important parameter for the reaction of exchange rates to money shocks; the interest elasticity of money demand, $\frac{\beta}{\varepsilon}$. For this parameter Betts and Devereux [2001] choose a value of 0.12 but they report that a variety of values between 0.02 and 0.25 has been estimated. The value of $\frac{\beta}{\varepsilon} = 0.2$ that is obtained applying the parameter values of table 1 lies between the two extremes of estimation.

The parameter ψ was chosen according to Devereux and Engel [2002], as was the value for ω. The value of the latter is quite high when compared to the estimates of Betts and Devereux [2001] who apply a value of 1.5. However, neither of these values has any significant impact on the quantitative results to be presented in the following section. That is why these values will be adopted. The relative size of the country given by n, has already been specified implicitly, as a symmetric steady state can easily be extended to the relative size of the country, hence n is chosen to be 0.5.

The value of the steady state real interest rate at 10% seems quite high. Engel states that the number is probably even too large to represent an annual value and he explains that their model is not intended to be a business cycle model.[27] Quantitative results of the final model are not significantly altered when the value of r is changed. We will again stick to the assumption of Devereux and Engel [2002]. Variances of money supplies are also chosen according to this paper.

Setting the value of steady state nominal consumption, $\bar{P}\bar{C}$, to unity is a simplification. An explicit calculation would require additional assumptions on the mark-up factor of the firms and the steady state labor supply of households. However, in the case of a symmetric equilibrium steady state nominal consumption is the same in both countries $\bar{P}\bar{C}=\bar{P}^\star\bar{C}^\star$ and there are no trade imbalances. As this value only appears in the budget constraint to measure the relative deviation of the current account, the simplification seems admissible.

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[27]In personal correspondence with Engel from June 2004

5 Results

The following results and figures have been obtained by implementing the benchmark model into the TOOLKIT program for Matlab.[28] The TOOLKIT program was altered, as each imperfection was added one at a time to arrive at the full model.[29] Note that apart from all other variables, nominal interest rates are plotted as absolute deviations from their steady state values, i.e. a 1% decrease in interest rates has to be interpreted as a fall in interest rates by one percentage point.

5.1 Benchmark Model
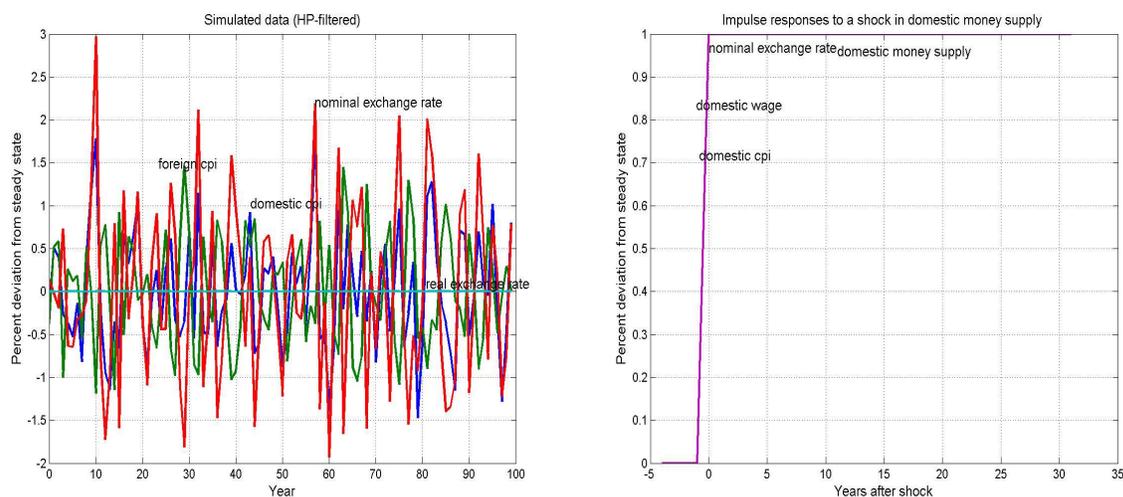
Figure 3: Simulated HP-Filtered Data and Impulse Responses for the Benchmark Model

The analysis of the benchmark model predicted that in a flexible price world where perfect risk sharing is possible and where PPP holds, money shocks are fully absorbed by changes in the nominal exchange rate. Figure 3 visualizes this assertion. Its left graph displays simulated and hp-filtered series for the nominal and real exchange rates and for the price indices (henceforth CPIs) of both countries.

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[28]See part A.9 of the appendix for a short note on the adaptations of the model that were required for implementation into the TOOLKIT program

[29] The TOOLKIT code for the full model can be found in section A.11 of the appendix.

As predicted, the real exchange rate remains a constant and movements of the nominal exchange rate are equal to the difference in changes in CPIs. The benchmark model cannot replicate any stylized fact of exchange rate behavior. A comparison of these simulation results to figure 1 of section 2.1 emphasizes the big difference between the benchmark model and empirical observations of exchange rate behavior. Given that both traditional concepts of exchange rate behavior, PPP and UIRP, hold in the benchmark model this comparison visualizes the huge gap between data and theory.

The right graph of figure 3 shows impulse responses of all variables to a domestic money supply shock. This plot addresses the additional shortcoming of the benchmark model: real variables are not affected by money shocks. In response to a 1% increase in money supply of the home country, domestic nominal wage and domestic CPI rise by the same percentage, as does the nominal exchange rate, causing a depreciation of the home country currency. The domestic nominal interest rate and all variables of the foreign country remain unaltered.

$_{t-1}\sigma^2(s)^*$
2
$*$ Simulation results running 50,000 times, not hp-filtered

Table 2: Conditional Variance of Nominal Exchange Rate in Benchmark Model

As could be expected, the conditional variance of the nominal exchange rate as given in table 2, with a value of 2 is exactly equal to the sum of variances of money supplies and thus completely proportional to the underlying fundamentals.

5.2 Imperfections in Goods Markets
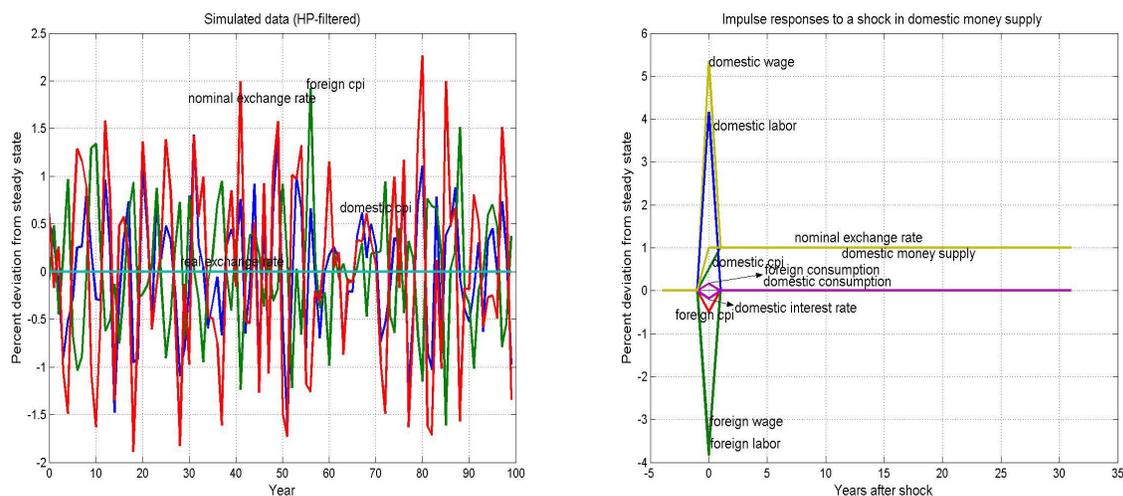
Figure 4: Simulated HP-Filtered Data and Impulse Responses for the Model with Nominal Rigidities

Nominal Rigidities Considering the isolated behavior of exchange rates the introduction of nominal rigidities proved meaningless because the left graph of figure 4 does not display any differences in comparison to the results of the benchmark model.

However, the assumption of sticky prices alters the impulse responses to a domestic money supply shock for several variables. The shock affects nominal and real variables of both home and foreign country. A 1% increase in domestic money supply induces a proportional increase of the nominal exchange rate, thus causing a depreciation of the home currency. This depreciation leads to an immediate rise in import prices and hence home and foreign country agents' expenditure switches towards domestic goods. In order to meet the additional demand, firms have to produce more and thus output rises. Given the linearity of the production function this increase in output is accompanied by an equally large rise in labor demand. This additional demand triggers a rise in domestic wages. Home country agents are left with more money to spend. Due to a fall in the real interest rate the incentive to save is weakened and home country agents consume more. However, given the perfect risk sharing contract with agents of the foreign country, domestic agents have to share their short-term prosperity with foreign households. That way foreign consumption rises despite a fall in foreign wages, due to decreasing foreign labor caused by the reduced demand for foreign goods.

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However, all effects of the money shock are only temporary. After one period prices are free to adjust and thus the domestic CPI fully deviates to its new steady state level 1% above the old one. All other variables, except for the domestic money supply and the nominal exchange rate, return to their former steady state levels. The real exchange rate remains a constant and as such unaffected by the money shock. The conditional variance of the model with nominal rigidities as reported in table 3 remains the same as for the benchmark model.

$_{t-1}\sigma^2(s)^*$
2
$*$ Simulation results running 50,000 times, not hp-filtered

Table 3: Conditional Variance of Nominal Exchange Rate after Introduction of Nominal Rigidities
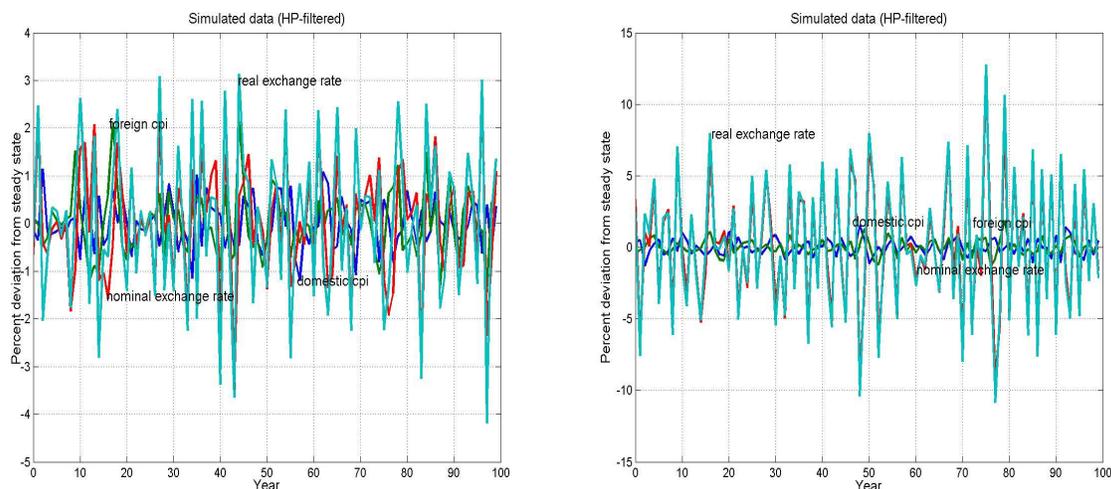
Figure 5: Simulated HP-Filtered Data for the Model with Local Currency Pricing for $\varepsilon = 4.7$ (left graph) and $\varepsilon = 40$ (right graph)

Local Currency Pricing Figure 5 displays two distinct versions of simulated and hp-filtered series for the nominal and real exchange rates and for the CPIs of both countries. These plots have been generated under different specifications for ε, the preference parameter of money demand. Simulations shown in the left graph were obtained under the default assumption as specified in table 1, whereas the simulations in the right graph were generated under the assumption of $\varepsilon = 40$.

Both plots display movements in the real exchange rate. Nominal and real exchange rates are moving quite in line with each other. The introduction of LCP has proven to be a success, as it helps to achieve the close correlation of nominal and real exchange rates. In the left plot of figure 5 volatility of both rates is still very similar to that of both countries' CPIs. Altering parameter specifications as indicated, however, improves this result as the simulations in the right plot show. The plot displays movements of exchange rates that are far more volatile than the changes in the CPIs.

Under the assumption of $\varepsilon = 40$ a 1% increase in domestic money supply causes a tremendous response of real and nominal exchange rates as can be depicted from the impulse responses in the right graph of figure 6. However, the implications for other variables of choosing such a high value for this parameter are quite severe. The shock to the domestic money supply is accompanied by a fall in the domestic nominal interest rate by 4 percentage points.

47
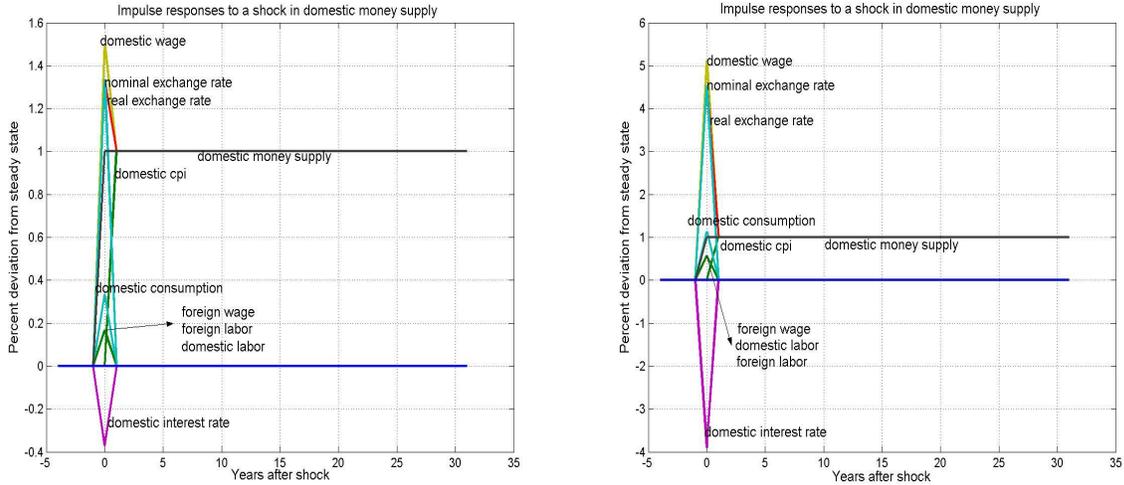
Figure 6: Impulse Responses for the Model with Local Currency Pricing
for $\varepsilon = 4.7$ (left graph) and $\varepsilon = 40$ (right graph)

Even though the simulations for the LCP case under a different parameter specification seemed promising, simply changing a parameter value is the wrong track for achieving the desired features of exchange rate behavior as the impulse responses for the specific case of $\varepsilon = 40$ have shown.

In order to further discuss the effects of a money shock for the model with LCP we will return to the original parameter specifications and turn to the impulse responses plotted in the left graph of figure 6. This graph reveals that under the assumption of LCP reactions of the nominal and real exchange rates to a money shock are characterized by the phenomenon of overshooting.[30] The term overshooting of exchange rates is used to describe a situation in which the initial reaction of the exchange rate to a shock exceeds the exchange rate's long-term response to the shock. In the case considered here, a 1% increase in domestic money supply causes the nominal and real exchange rates to depreciate by slightly more than 1%. After this initial reaction to the shock, both rates appreciate, but whereas the real exchange rate returns to its old steady state level, the nominal exchange rate adjusts to a new steady state level at 1% above the old one. With all prices completely fixed, there is no expenditure switching effect of a depreciation.

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[30]The term was first introduced by Rudiger Dornbusch. For his definition on overshooting see e.g. Dornbusch and Fisher [1993](Chap.20)

Money supply exceeds money demand and in order for the domestic money market to clear nominal interest rates drop 0.4 percentage points. Thus, with prices staying the same, this fall reduces the domestic real interest rate. The opportunity costs of holding money are thus lowered, which discourages agents from saving. That is why, despite their consumption smoothing motive, home country agents consume more. However, their additional consumption does not trigger an increase in foreign consumption because movements in the real exchange rate allow for consumption levels to differ across countries. The increase in domestic consumption is equally met by a rise in domestic and foreign output. This additional output increases labor demand leading to a rise in nominal wages in both countries. This wage increase is bigger in the home country, as domestic firms anticipate higher prices for the following period that will allow for them to pass the additional labor cost on to consumers. The domestic CPI adjusts after one period and it moves, just as the nominal exchange rate does, to its new steady state level.

$_{t-1}\sigma^2(s)^*$
3.58
$*$ Simulation results running 50,000 times, not hp-filtered

Table 4: Conditional Variance of Nominal Exchange Rate after Introduction of LCP

The capability of LCP to induce an overshooting of the real and nominal exchange rates has proven a promising track, when considering the new value of 3.58 for the conditional variance of the nominal exchange rate, that is given in table 4. Note that under the model's specifications for LCP the conditional variance of the nominal exchange rate is equal to the unconditional variance of the real exchange rate. [31]

However, the assumption of a perfect asset market has strong implications for the relation between exchange rates and relative consumption. Though levels of consumption among countries may differ under LCP, as PPP does not continue to hold, the availability of a full set of state contingent bonds implies that differences in consumption be fully reflected in changes in the real exchange rate. However, considering figure 7 that displays differences in changes in real consumption in the euro area and in the US and the change in the real €/$ rate on a quarterly basis over a period of five and a half years, this assertion cannot be sustained. The close correlation between differences in consumption levels and the real exchange rate, as suggested by the risk sharing condition (4.13), is by no means reflected in the data. Volatility of the real exchange rate is higher than volatility of consumption differences.

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[31]Check out equations 13-19 in part A.3.2 of the appendix to see immediately that this is true.

8,0%

6,0%

4,0%

2,0%

0,0%

-2,0%

-4,0%

-6,0%

-8,0%

1998-II 1998-III 1998-IV 1999-I 1999-II 1999-III 1999-IV 2000-I 2000-II 2000-III 2000-IV 2001-I 2001-II 2001-III 2001-IV 2002-I 2002-II 2002-III 2002-IV 2003-I 2003-II 2003-III 2003-IV

Difference in change in real consumption EURO-area-US
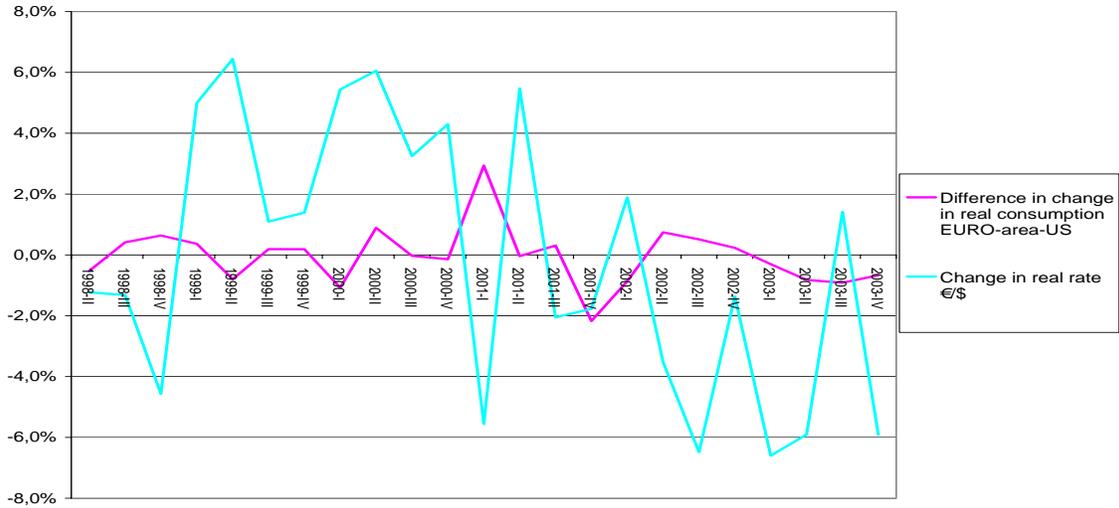
Change in real rate €/$

Figure 7: Changes in the real €/$ rate and differences in changes in real consumption of euro area and US
Data: Monthly Bulletins ECB, Bureau of Economic Analysis

Indeed, figure 7 provides additional support for the stylized fact on the disconnectedness of exchange rate behavior.[32] The close relationship of real exchange rate and consumption differences as suggested by the model under the assumption of perfect asset markets, on the one hand, and empirical observations, on the other hand, pose a big discrepancy. This discrepancy calls for the introduction of imperfect asset markets into the model in order to explain more than just the one stylized fact of high volatility of exchange rates. That is why, though changing the customary order of market imperfections introduced, results for the case of LCP in combination with non contingent claims will be discussed before turning to the case of the model with distributors.

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[32]Again this figure only provides an approximative picture as changes were simply subtracted.

5.3 Imperfections in Asset Markets

The deviation of net foreign bond holdings is defined as its linear deviation from the steady state value of the bond price. Given the choice of parameter values the steady state of net foreign bond holdings takes on the value of -0.182.[33] It is important to keep this value in mind when interpreting the following results.
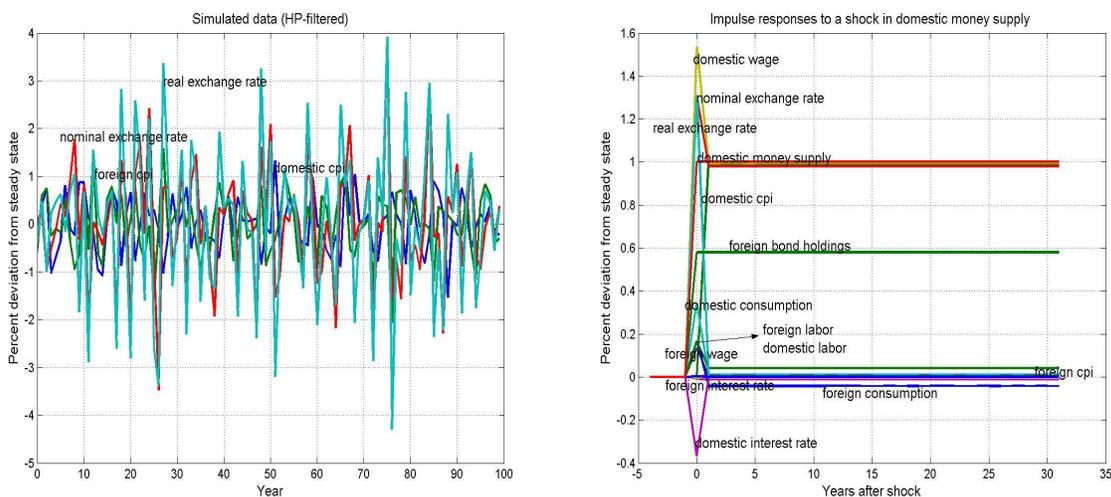
Figure 8: Simulated HP-Filtered Data and Impulse Responses for the Model with Non Contingent Claims

Non Contingent Claims The simulation results for the model with imperfect asset markets, displayed in the left graph of figure 8, resemble the last results for the model assuming LCP and allowing for trade in a full set of state contingent bonds. Nominal and real exchange rates fluctuate jointly and their volatility is only slightly higher than the volatility of CPIs.

At first glance the impulse responses shown in the right graph of figure 8 also seem quite similar to the impulse responses of the last result in figure 6. However, the behavior of home country agents' foreign bond holdings sticks out. The reaction of this variable to a money shock marks the essential difference between the assumption that agents can purchase a full set of state contingent bonds and the assumption of non contingent claims being the only type of claims traded internationally; the latter assumption allows for long-term wealth effects of money shocks.

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[33]Part A.7 of the appendix explains how this value is obtained

Impulse responses of most variables to a domestic money shock are as described before. However, given trade in non contingent claims home country agents can pursue their consumption smoothing motive by purchasing foreign bonds and thus by transferring wealth to the future. Due to the fall in the domestic nominal and real interest rates home country nationals do not turn to the domestic asset market but rather accumulate foreign claims. As prices are fixed, terms of trade improve but without any expenditure switching on behalf of consumers. Thus, the improvement in terms of trade does not induce any preference for domestic products over foreign products. In order to meet the additional demand of home country households output in both countries rises by the same percentage. With foreign bonds in fixed supply, market clearing requires that their increased demand be met by a rise in bond prices, or equivalently a fall in returns on foreign bonds given by the foreign nominal interest rate.

The transfer of wealth enables domestic households to continue consuming slightly above and to work slightly below their old steady state levels even after the period of the shock. However, without perfect risk sharing this long-term prosperity is only possible at the expense of foreign country nationals. They, in turn, have to work more and consume less to pay the returns on their assets to domestic agents. The policy of increasing the domestic money supply in order to induce prosperity in the home country at the costs of the foreign country can be described as the LCP form of the beggar-thy-neighbor policy. This term usually refers to a policy under the PCP assumption implying a wealth increase based on a devaluation of the home country currency, reducing prices for home country products and increasing import prices thus leading to a surplus of the current account and a rise in domestic output at the expense of the foreign country. Under LCP a beggar-thy-neighbor policy, based on a currency's devaluation would have no effect. However, as Betts and Devereux [2000] point out: "In the presence of PTM [LCP], monetary surprises reduce the foreign terms of trade. Foreign residents get less benefit of the world output expansion in consumption, but increase their labour supply to meet the expansion in demand from the other country. This makes them worse off."(p.239)

In addition the change in net foreign bond holdings causes the expected future consumption of home country nationals to differ from the expected consumption of foreign country nationals, as the latter are expected to keep paying the returns on their bonds to domestic agents. This difference in expectation influences the expected change in exchange rate as described by equation (4.43). That is why the long-term response of the nominal exchange rate after the period of the shock is not equal to the long-term reaction of the domestic money supply. The depreciation of the home country currency is thus slightly reduced. These workings are also responsible for the fact that the domestic price level adjusts to a steady state level of slightly less than 1% above its old one.

$_{t-1}\sigma^2(s)^*$
3.43
* Simulation results running 50,000 times, not hp-filtered

Table 5: Conditional Variance of Nominal Exchange Rate after Introduction of Non Contingent Claims

The conditional variance of the nominal exchange rate given in table 5 is slightly lower than the one under the perfect risk sharing assumption as reported in table 4. This reduction in volatility is due to the aforementioned long-term reaction of the exchange rate. This reaction is reduced because part of the money shock is absorbed by expectations regarding future consumption. As seen in the analysis of the model the conditional variance of exchange rates can be described as a function of the shares of directly and indirectly marketed goods. This option is discussed in the following section that presents the results of the model after the introduction of distributors.
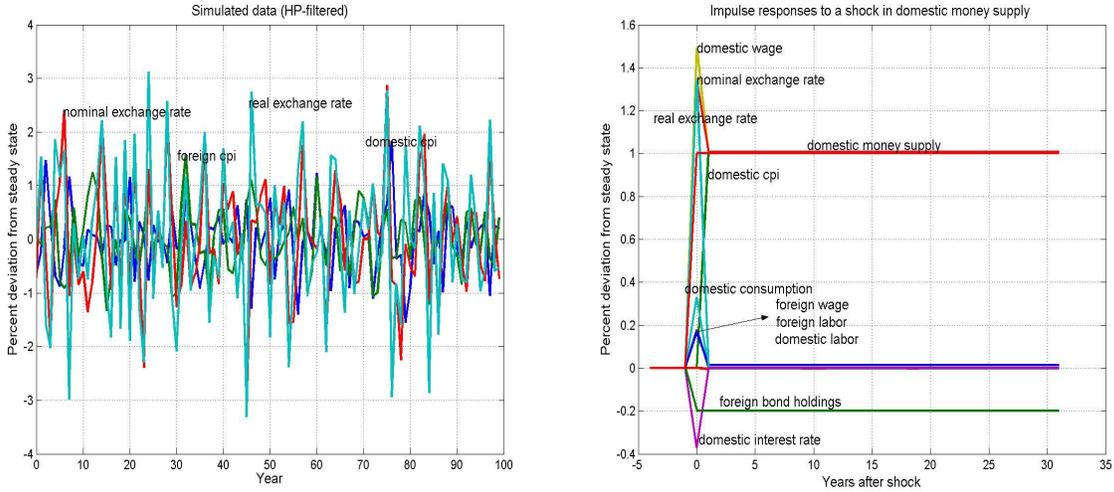
Figure 9: Simulated HP-Filtered Data and Impulse Responses for the Model with Distributors
($\theta = \theta^\star = 0.5$)

Including Distributors   Simulation results and the impulse responses displayed in figure 9 have been generated under the following parameter specifications, $\theta = \theta^\star = 0.5$. These values imply that in each country half of the imported goods are marketed directly by firms of the other country, with prices set in local currency, whereas the other 50% of imports are sold by nationals of the own country acting as distributors who buy goods from foreign producers at a price set in producer's currency. Judging by the simulation results shown in the left graph of figure 9 the introduction of distributors to the model did not alter the volatility of exchange rates nor the changes in CPIs.

However, the impulse responses to a shock in domestic money supply displayed in the right plot have changed. Recalling the steady state value of net foreign bond holdings of -0.182, it becomes clear that all wealth effects of a domestic money shock, discussed in the last section, have been canceled.[34] The depreciation of the home country currency caused by the 1% rise in domestic money supply increases profits of home country producers, as their foreign sales become more valuable, while the depreciation leaves domestic distributors with a loss, as they have to pay more for foreign goods than they receive as revenues from home country households. This is the aforementioned ambiguity of wealth effects caused by the introduction of distributors to the model.

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[34]Note that when running the TOOLKIT code the impulse responses will still display some wealth effects. Due to the fact that the steady state value of $b_{ht}^\star$ was chosen unequal to zero there are initial differences in relative consumption. The plot has been edited to get the point across

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$\theta + \theta^\star$	$_{t-1}\sigma^2(s)^*$	$\rho(s,c)^\dagger$	$\rho(s,\tilde{r}^\ddagger)^\dagger$
2.0	3.43	0.61	-0.59
1.75	3.47	0.61	-0.59
1.50	3.48	0.60	-0.59
1.25	3.57	0.59	-0.59
1.0	3.63	0.59	-0.60

∗ Simulation results running 50,000 times, not hp-filtered

† Moments approximation choosing autocorrelation of money shocks 0.99999, hp-filtered

‡$\tilde{r} = E_t \rho[c_{t+1} - c_t]$

Table 6: Various Simulation Results after Introduction of Distributors

The specific choice of parameter values used to generate the plots implies that the fraction of those who benefit from a depreciation of the home country currency and those who suffer from it is of equal seize in both countries thus any wealth effect is canceled.

Under the assumption of $\theta = \theta^\star = 0.5$ the impulse responses of the model with distributors are equal to the impulse responses of the model with perfect asset markets and LCP that are displayed in the right graph of figure 6. Instead of a full set of state contingent bonds, it is the heterogeneity in distribution in the goods markets that guarantees that risk be perfectly shared. However this risk sharing possibility under imperfect asset markets is highly depended on the parameter specification. Equation (4.44), relating the unexpected change in the nominal exchange rate to unanticipated relative differences in consumption and to changes in the current account shows that for $\theta = \theta^\star = 0.5$ an unexpected change in the nominal exchange rate does not affect either variable. The wealth effects of a change in exchange rates are maximized under the parameter choice, $\theta = \theta^\star = 1$, which implies that goods are only sold directly to consumers of the foreign country. This case has already been discussed in the last section as the model without distributors.

The second row of table 6 displays results for the conditional variance of the nominal exchange rate for both extremes and for intermediate cases of parameter specifications. The choice of $\theta + \theta^\star = 1$, while maximizing the reaction of the exchange rates to money shocks, minimizes the wealth effects of the change in exchange rates. Indeed the increase in exchange rate volatility is not accompanied by a rise in the volatility of other real variables. The third row of table 5 shows that the correlation between nominal exchange rate and domestic consumption is slightly declining, as the volatility of the former increases. The same assertion holds true for the relationship between the nominal exchange rate and the real interest rate, given in the fourth row. The introduction of distributors is a better way to increase the volatility of exchange rates than a change in parameter specifications, because the former pays tribute to the stylized fact of the disconnectedness of exchange rate behavior.

55
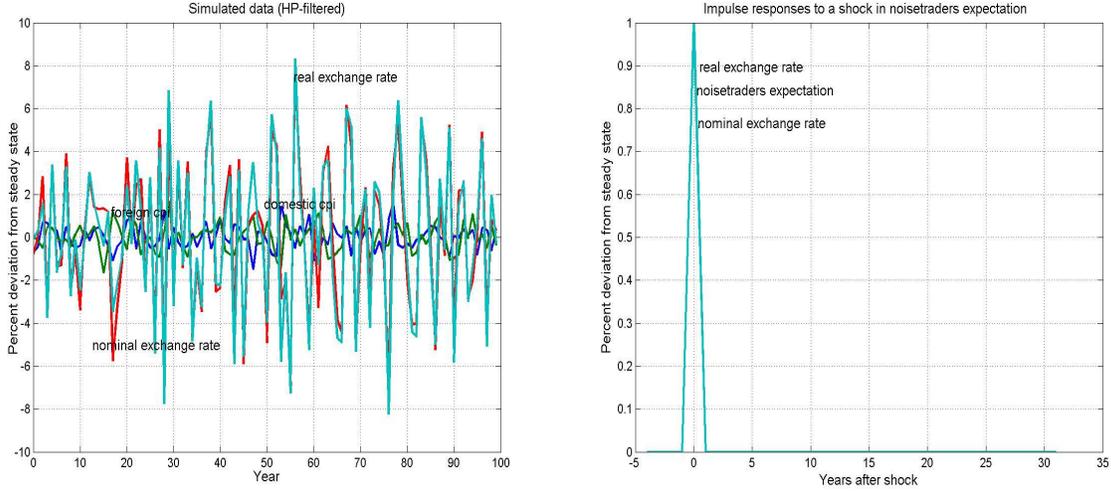
Figure 10: Simulated HP-Filtered Data and Impulse Responses for the Model with Noise Traders $(\theta = \theta^\star = 0.5\kappa = 0.5)$

Noise Traders The simulation results obtained for the model with noise traders under the parameter assumptions, $\theta = \theta^\star = 0.5$ and $\kappa = 0.5$, resemble the real life behavior of exchange rates displayed in figure 1 of section 2. Apart from the time horizons considered the behavior of the simulated exchange rates shown in the left graph of figure 10 is indeed similar to the €/$ rate of the last six years. Nominal and real exchange rates obtained by the simulations display a volatility that is far higher than that of both countries' CPIs. There are changes in exchange rates of up to 8% from one period to the next. The joint movement of both rates shows the high correlation between the nominal and the real exchange rates.

This picture of success is due to the shock to noise trader's expectations. Impulse responses of nominal and real exchange rates to this shock are displayed in the right graph of figure 10. Under the parameter specification, $\theta + \theta^\star = 1$, a shock to noise trader's expectation does not affect any other real or nominal variables. Given any other parameter choice the shock to noise traders' expectations, however, will induce a change in the current account or alter relative consumption as can be depicted from equation (4.44). This equation continues to hold after noise traders are introduced.

κ	$\theta+\theta^\star$	$_{t-1}\sigma^2(s)^*$	$\rho(s,c)^\dagger$	$\rho(s,\tilde{r}^\ddagger)^\dagger$
0.25	2.0	5.86	0.41	-0.38
	1.75	6.05	0.40	-0.38
	1.5	6.27	0.39	-0.38
	1.25	6.45	0.38	-0.37
	1.0	6.69	0.37	-0.37
0.5	2.0	15.13	0.25	-0.19
	1.75	15.90	0.24	-0.19
	1.50	16.54	0.23	-0.19
	1.25	17.93	0.21	-0.20
	1.0	18.93	0.20	-0.20

$*$ Simulation results running 50,000 times, not hp-filtered

$\dagger$ Moments approximation choosing autocorrelation of money shocks 0.99999, hp-filtered

$\ddagger \tilde{r} = E_t \rho [c_{t+1} - c_t]$

Table 7: Various Simulation Results for the Full Model

Combining this equation (4.44) with equation (4.53) of section 4.3 we can write changes in the current account as follows:

$$\frac{\beta \bar{d} \hat{b}^\star_{ht}}{(1-n)\bar{P}\bar{C}} = \frac{(1+(\sigma/r))\varepsilon(\hat{m}_t - \hat{m}^\star_t) + \frac{\sigma}{r}\nu_t}{((\sigma/r) + \rho(\theta - (1-\theta^\star)))} - \hat{c}_t - \hat{c}^\star_t$$

It is easy to see that for the case of $\theta - (1-\theta^\star) \to 1$ the effect of a shock to noise traders' expectations on the current account is minimized and changes in the current account will basically be due to money shocks. Noise traders' expectations thus do have an effect on the behavior of real variables. This effect is, however, negligible as the values given in table 7 show. The increase in the conditional variance of the nominal exchange rate caused by the introduction of noise traders is accompanied by a decline in the correlation of the nominal exchange rate with real variables.

Table 7 displays the conditional variance of the nominal exchange rate and correlations between nominal exchange rate and domestic consumption and between nominal exchange rate and domestic real interest rate for several values of κ and σ.[35] It is fairly easy to further increase exchange rate volatility by choosing a higher value for κ. As can be seen from equation 4.54 the conditional variance of the exchange rate approaches infinity for $\kappa \to 1$ and $\theta + \theta^\star = 1$. The introduction of noise traders to the model helps to achieve a high volatility of exchange rates that is disconnected from movements of other variables.

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[35]Part A.10 of the appendix provides the results obtained under the specifications of the paper by Devereux and Engel [2002]

6 Extension

The introduction of noise traders proved to be a powerful tool for generating the afore-mentioned features of exchange rate behavior like high volatility and disconnectedness. Maybe one should adopt the 'monetary view' on exchange rates stating that forces in the asset markets are solely responsible for the determination of the exchange rates. Rendering to this view implies that all ongoing discussion about price setting behavior of firms and whether LCP or PCP better fits the description of the real world, is unnecessary. However, in the following section it will be briefly shown that a simple adaptation of the noise traders's approach into a PCP framework is not possible.[36]

Producer Currency Pricing   Apart from the way firms set their prices all assumptions of the final model with noise traders remain unaltered. Under the assumption of PCP firms set prices for exports in their own currency. In a very simple version of a PCP model without non-traded goods or border costs this price setting behavior implies that the LOP always holds. Under PCP an unanticipated rise in the nominal exchange rate, a depreciation of the home country currency, has a positive wealth effect on the income of home country nationals. In response to it they either increase their relative consumption or accumulate wealth, as can be depicted from the following equation:

$$\hat{c}_t - \hat{c}_t^{\star} + \frac{\beta \bar{d} \bar{b}_{ht}^{\hat{\star}}}{\bar{P}\bar{C}(1-n)} = (\omega - 1)\hat{s}_t \tag{6.55}$$

However, distributors are not assigned any role in this framework, as it simply does not make any difference whether firms sell their goods directly to foreign country households at a price in the currency of the producer or if those goods are sold for the same price to retailers first. Under PCP, the elasticity of substitution between home and foreign aggregates, ω, becomes a crucial parameter. The wealth effect of an unanticipated rise in the nominal exchange rate is increasing in ω. This relationship is due to the fact that a high value of this parameter implies a strong reaction of consumers to price changes. A depreciation of the home country currency causes an immediate fall in export prices and an increase in prices for imports, thus home and foreign country agents increase their demands for domestic products. The magnitude of this expenditure switching and thus the positive wealth effect for home country agents is governed by ω.

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[36]Part A.8 the appendix shows the intermediate steps for the calculations
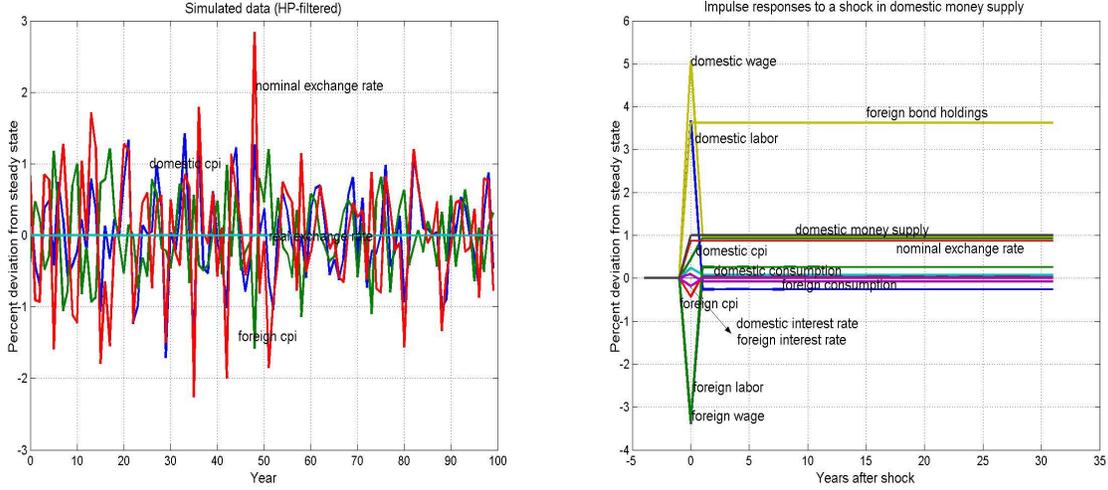
Figure 11: Simulated HP-Filtered Data and Impulse Responses for Full Model with PCP
($\theta = \theta^\star = 0.5\,kappa = 0.5$)

The left graph of figure 11 visualizes this relation. In addition, as UIRP and PPP hold, real interest rates have to be equal across countries. Thus, an increase in domestic money supply also affects the foreign nominal interest rate, causing it to fall in order to account for the drop in foreign CPI. The impulse responses shown in the right graph of figure 11 display the workings of the aforementioned traditional beggar-thy-neighbor policy under PCP. The real exchange rate remains a constant as nothing is causing deviations from the LOP.

The unexpected change in the nominal exchange rate under the assumption of PCP is given by:

$$\hat{s}_t = \frac{(1 + \sigma/r)\varepsilon(\hat{m}_t - \hat{m}_t^\star) + \sigma/r\nu_t}{\rho(\omega - 1) + (1 + \sigma/r)\varepsilon} \tag{6.56}$$

The unanticipated change in the nominal exchange rate is decreasing in ω. As a high value of this parameter implies large changes in expenditure for imports in response to price changes it does not require a big change in the nominal exchange rate to trigger these changes. For the case of $\omega = 1$ and abstaining from the influence of noise traders we arrive at the standard model of perfect risk sharing. The aforementioned findings by Cole and Obstfeld [1991] about the ability of changes in terms of trade to replicate perfect asset markets are confirmed.

κ	$\theta + \theta^\star$	$_{t-1}\sigma^2(s)^*$(PCP)
0.25	2.0	1.59
	1.75	1.59
	1.5	1.58
	1.25	1.58
	1.0	1.57
0.5	2.0	1.70
	1.75	1.68
	1.50	1.70
	1.25	1.69
	1.0	1.70
∗ Simulation results running		
50,000 times, not hp-filtered		

Table 8: Various Simulation Results for the Full Model with PCP

The following equation describes the volatility of the nominal exchange rate as a function of the variance of money supplies for the PCP case:

$$Var_{t-1}(s_t) = \frac{(1 + \sigma/r)^2 \varepsilon^2 Var_{t-1}(m_t - m_t^\star)}{\Upsilon^2 \left(1 - (\sigma\kappa/r\Upsilon)^2\right)} \tag{6.57}$$

where $\Upsilon = \rho(\omega - 1) + ((1 + \sigma/r)\varepsilon)$.

In the original model the introduction of distributors diminishes the wealth effects of changes in the nominal exchange rate but increases the volatility of exchange rates. Under the PCP assumption it is the elasticity of substitution between home and foreign consumption aggregates that, on the one hand, magnifies the discussed wealth effects but, on the other hand, reduces fluctuations in the nominal exchange rate.

Table 8 displays values for the conditional variance of the nominal exchange rate. The simple idea to introduce noise traders into the classical PCP framework did not prove to be of any great success. Even though volatility of exchange rates is slightly increasing in κ, this volatility is by no means comparable to the volatility of exchange rates achieved by the original model. The ability of the model by Devereux and Engel [2002] to account for the short-run behavior of exchange rates is highly dependent on the specific combination of imperfections in the goods and asset markets. The model' success depends in particular on the underlying assumption about the way firms set their prices, on the LCP approach.

7 Discussion

The model by Devereux and Engel is able to account for all the stylized facts of short-run exchange rate behavior. The introduction of noise traders generates the feature of high volatile exchange rates. Prices sticky in the currency of the consumer (LCP) cause movements in the real exchange rate that are very similar to changes in the nominal exchange rate, hence incorporating the close correlation between the two variables. In addition LCP accounts for the fact that the high volatility of exchange rates is not reflected in consumer prices and thus the model pays tribute to the third stylized fact. The introduction of distributors shields other real variables from fluctuations in the exchange rates as this goods market imperfection minimizes the wealth effects of exchange rate volatility. In the framework of this model the two basic concepts of exchange rate determination, UIRP and PPP, still hold on expectation. This assumption accounts for the fact that, on the one hand, these concept are backed by the deeply routed belief that exchange rates should reflect a currency's current and future value but that, on the other hand, their simple mechanisms cannot explain the short-run behavior of exchange rates

In addition to providing an answer to the overall question of this thesis, the model resolves the consumption correlation puzzle. Table 9 displays inter-country correlations of output and consumption under different assumptions about the type of assets traded internationally and about the way firms set their prices. These simulation results clearly show that the puzzle only persists for the PCP case. Under the price setting assumption of LCP correlations of output among countries are always higher than correlations of consumption, regardless of the underlying structure of the financial markets.

International Trade in	Price setting for exports	$\rho(y, y^\star)^\dagger$	$\rho(c, c^\star)^\dagger$
State contingent claims	LCP	1	0.15
	PCP	-1	1
Non contingent claims	LCP	0.94	-0.18
	PCP	-0.99	0.85
† Simulation results hp-filtered, 100 simulations			

Table 9: Inter-country Output and Consumption Correlations under Different Assumptions about International Asset Trade and Price Setting Behavior

As the model is not set up as a business cycle model a direct comparison of the quantitative results to data is not possible. The authors admit this shortcoming of the model but given the current state of research the model's contribution is tremendous because it is the first one that can fully account for the short-run behavior of exchange rates.

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For further improvement of the model Devereux and Engel suggest to include capital accumulation in order to account for the long-run behavior of exchange rates, whatever this turns out to be. They propose to include more detailed microfoundations for the choices of noise traders. The authors state that a more diversified modeling of the price setting behavior of firms would make the model more realistic allowing for some expenditure switching effect of exchange rates.

However, the success of the model to fully account for the behavior of exchange rates as well as the model's additional contribution to the consumption correlation puzzle is heavily dependent on the assumed price setting of firms, on LCP. This assertion was emphasized by the failed attempt to adapt the model to a PCP approach.

The heavy reliance of the model's explanatory power on LCP, on this imperfection of the goods market, is questionable for two reasons.[37] First, there does not exist microeconomic evidence of such a price setting behavior on behalf of producing firms. Second, the macroeconomic implications of this price setting behavior are at least not sustained by data, if not contradictory to empirical findings.

The last point is emphasized by Obstfeld and Rogoff [2000a] who define a country's competitiveness as its multilateral terms of trade, i.e. the ratio of a foreign country index of export prices incorporating prices set for sales to all countries and evaluated with the nominal exchange rate and of an equivalent measure for the home country , $SP^{\star}_{ex}/P_{ex}$. The authors report highly positive correlations between monthly changes in this measure and monthly changes in the respective bilateral nominal exchange rate. They conclude that prices are fixed in the currency of the exporter, (PCP), as under the LCP assumption the export indices would include prices in many different currencies. Given the fact that hence changes in the index could be due to changes in many different bilateral exchange rates the correlation between movements in one of these exchange rates and changes in a country's competitiveness could not be very high. Thus due to LCP the Devereux and Engel model is able to account for the short-run behavior of exchange rates but the implications of this crucial assumption seems to contradict the short-run effects of exchange rate behavior; the effects of exchange rate changes on a country's competitiveness. While the behavior is accounted for by the model, the effects of this behavior are lost along the line of explanation.

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[37]The dependence of the model on a goods market imperfection to account for the behavior of exchange rates is also quite interesting considering the limited volume of transactions in foreign currencies related to world trade of goods

However, this contradiction should not lead to a complete dismissal of the model's success, nor does it imply a lost case for the LCP approach, but it rather calls for a more diverse price setting scheme within the framework of the model as proposed by Devereux and Engel. Even though the introduction of distributors to the model accounts for the difference in pass-through of exchange rates to consumer prices and import prices, zero elasticity of substitution faced by the distributors cancels all possible expenditure switching effects of exchange rate changes. However, altering the model's assumptions about the way prices are set might weaken the achieved feature of disconnectedness of exchange rates as deviations from the zero expenditure switching effect cause real effects of changes in the exchange rate. These effects might be able to reduce the achieved high volatility of exchange rates and negatively affect the disconnectedness predicted by the model. Thus a simple introduction of additional features to the model without endangering its success is not possible. Future research along this line is needed to evaluate if the model's specific combination of market imperfections that can account for the short-run behavior of exchange rates is also able to explain the effects of this behavior on other macroeconomic variables.

The contradiction between the model's predictions and the data claimed by Obstfeld and Rogoff [2000a] also calls for an improvement of empirical studies. Better estimates of the pass-through of exchange rate changes to consumer prices and wholesale prices, e.g. based on more detailed price indices, are needed. Studies on the expenditure switching effect should help to better estimate the real effects of the pass-through of exchange rates. On the microeconomic level empirical studies are needed that provide answers to the question of how firms set their prices when exporting.

8 Conclusion

The answer to the overall question: "Can market imperfections explain the behavior of exchange rates?" has been shown to be a clear yes. Under the assumption of the particular combination of market imperfections in goods and asset markets the model by Devereux and Engel [2002] is able to account for all the stylized facts of short-run exchange rate behavior. At the beginning of this thesis data on exchange rates was presented and these stylized facts were identified. The introduction of the two basic concepts of exchange rate determination, PPP and UIRP, that followed, left a huge gap between data and basic theory. It became clear that market imperfections were needed to account for the behavior of exchange rates. A review on recent research introduced several possible suggestions for closing the gap between data and theory. These suggestions considered imperfections in both the goods and asset market. Then the model by Devereux and Engel [2002] was presented as each of its market imperfections was added one at a time.

Taking this model apart step by step provided a deep understanding of its underlying assumptions and it equipped us with a powerful tool to pin down the workings of each market imperfection. Simulation results and impulse responses for all steps visualized the contribution that each market imperfection has for the behavior of exchange rates. Hence, this thesis enables us to join in on the vivid discussion on exchange rates and possible reasons for their behavior. In addition, the extension of the model, assuming a different price setting behavior by firms, PCP, clearly identified the assumption of LCP as the key to success for the Devereux and Engel model.

This key also poses one of the shortcomings of this model, given the questionable assumption of LCP. However, backed by its success in regard to its explanatory power for the behavior of exchange rates the model provides a solid basis for future research, in particular along the line of including a more diversified price setting scheme. A price setting approach that incorporates the expenditure switching effects of exchange rates is needed in order to better account for the macroeconomic effects of exchange rate behavior. In addition, these macroeconomic effects of exchange rates on trade flows among countries should be the subject to detailed empirical studies. Along these paths a consensus among economists on the way firms set their prices when exporting can be achieved and the question: "Can market imperfections explain the behavior of exchange rates?' can be answered with an unconditional 'yes'.

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A   Appendix

A.1   Deriving the Risk Sharing Condition

It is assumed that agents of the foreign country can only purchase foreign country claims as anything else would be redundant. Their budget constraint is thus given by

$$P_t^\star C_t^\star + M_t^\star + \sum_{z^t} H^\star(z^t \mid z^{t-1})A^\star(z^t) = W_t^\star L_t^\star + \Pi_t^\star + M_{t-1}^\star + T_t^\star + A_{t-1}^\star$$

Easy to see that the first order condition for optimal holdings of these claims will look as follows:

$$H^\star(z^t \mid z^{t-1}) = \beta\pi(z^t \mid z^{t-1})\left[\frac{C_t^{\star\rho}P_t^\star}{C_{t+1}^{\star\rho}P_{t+1}1^\star}\right]$$

Combining it with the home country agent's first order condition for optimal holdings of foreign state contingent claims of the text gives:

$$\beta\pi(z^t \mid z^{t-1})\left[\frac{C_t^{\star\rho}P_t^\star}{C_{t+1}^{\star\rho}P_{t+1}^\star}\right] = \beta\pi(z^t \mid z^{t-1})\left[\frac{C_t^{\rho}P_tS_{t+1}}{C_{t+1}^{\rho}P_{t+1}S_t}\right]$$

As the probabilities that states of the world occur are the same across countries these probabilities can be dropped and one can iterate to arrive at:

$$\frac{C_0^{\star\rho}P_0^\star}{C_t^{\star\rho}P_t^\star} = \frac{C_0^{\rho}P_0S_t}{C_t^{\rho}P_tS_0}$$

Setting initial conditions constant: $\Gamma_0 = \frac{S_0 C_0^{\star\rho}P_0^\star}{C_0^\rho P_0}$ leads to equation (4.13) of the text.

A.2   Interest Rate Determination under Log-Utility

A slightly modified utility function

$$U = \frac{1}{1-\rho}C_s^{1-\rho} + \chi ln\left(\frac{M_s}{P_s}\right) - \frac{\eta}{1+\psi}L_s^{1+\psi}$$

leads to the following money demand:

$$\left(\frac{M_t}{P_t}\right) = \frac{\chi C_t^\rho}{1 - d_t}$$

Forwarding this equation one period and plugging both versions into the money supply equation (3.1) then using the definition of d_t to substitute for C_t and P_t and applying the relationship between bond price and interest rate we see that $E_t(i_{t+1}) = \frac{1}{\mu\beta} - 1$ thus $i_t = \frac{1}{\mu\beta} - 1$. Log-utility of money balances imply a constant nominal interest rate.

A.3 List of log-linearized equations

A.3.1 For the Benchmark Model

1. $m_t - p_t = \frac{\rho}{\varepsilon} c_t + \frac{1}{r\varepsilon} d_t$

2. $m_t^\star - p_t^\star = \frac{\rho}{\varepsilon} c_t^\star + \frac{1}{r\varepsilon} d_t^\star$

3. $w_t = p_t + \rho c_t + \psi l_t$

4. $w_t^\star = p_t^\star + \rho c_t^\star + \psi l_t^\star$

5. $s_t + p_t^\star - p_t = \rho c_t + \rho c_t^\star$

6. $l_t = n(-\omega(p_{ht} - p_t) + c_t) + (1 - n)(-\omega(p_{ht} - s_t - p_t^\star) + c_t^\star)$

7. $l_t^\star = (1 - n)(-\omega(p_{ft}^\star - p_t^\star) + c_t^\star) + n(-\omega(p_{ft}^\star + s_t - p_t) + c_t)$

8. $d_t = p_t + \rho c_t - E_t[p_{t+1} + \rho c_{t+1}]$

9. $d_t^\star = p_t^\star - \rho c_t^\star - E_t[p_{t+1}^\star + \rho c_{t+1}^\star]$

10. $i_t = -(1 + r)d_t$

11. $i_t^\star = -(1 + r)d_t^\star$

12. $v_t = s_t + p_t^\star - p_t$

13. $p_t = n p_{ht} + (1 - n)(p_{ft}^\star + s_t)$

14. $p_t^\star = n(p_{ht} - s_t) + (1 - n)p_{ft}^\star$

15. $p_{ht} = w_t$

16. $p_{ft}^\star = w_t^\star$

17. $m_{t+1} = m_t + u_{t+1}$

18. $m_{t+1}^\star = m_t^\star + u_{t+1}^\star$

Changes of nominal interest rates are defined as absolute deviations from steady state instead of relative deviations.

A.3.2 For the Full Model

1. $m_t - p_t = \frac{\rho}{\varepsilon} c_t + \frac{1}{r\varepsilon} d_t$

2. $m_t^\star - p_t^\star = \frac{\rho}{\varepsilon} c_t^\star + \frac{1}{r\varepsilon} d_t^\star$

3. $w_t = p_t + \rho c_t + \psi l_t$

4. $w_t^\star = p_t^\star + \rho c_t^\star + \psi l_t^\star$

5. $(1 - n\omega)p_t + (1 - n)c_t + \frac{\beta\bar{d}}{PC} b_{ht}^\star = (1 - n\omega - \theta + n\theta)p_{ht} + (1 - n)(\theta - (1 - \theta^\star))s_t +$
   $(1 - n)c_t^\star - (1 - n)(1 - \theta^\star)p_{ft}^\star + (1 - n)(1 - \theta^\star)p_{ft} + (1 - n)(\theta - \omega)p_{ht}^\star + \frac{\bar{d}}{PC} b_{ht-1}^\star$

6. $l_t = n(-\omega(p_{ht} - p_t) + c_t) + (1 - n)(-\omega(p_{ht} - s_t - p_t^\star) + c_t^\star)$

7. $l_t^\star = (1 - n)(-\omega(p_{ft}^\star - p_t^\star) + c_t^\star) + n(-\omega(p_{ft}^\star + s_t - p_t) + c_t)$

8. $d_t = p_t + \rho c_t - E_t[p_{t+1} + \rho c_{t+1}]$

9. $d_t^\star = p_t^\star + \rho c_t^\star - E_t[p_{t+1}^\star + \rho c_{t+1}^\star]$

10. $d_t^\star = p_t + \rho c_t - E_t[p_{t+1} + \rho c_{t+1}] + E_t s_{t+1} + \nu_t - s_t$

11. $i_t = -(1 + r)d_t$

12. $i_t^\star = -(1 + r)d_t^\star$

13. $v_t = s_t + p_t^\star - p_t$

14. $p_t = np_{ht} + (1 - n)p_{ft}$

15. $p_t^\star = np_{ht}^\star + (1 - n)p_{ft}^\star$

16. $p_{ht} = E_{t-1}w_t$

17. $p_{ft}^\star = E_{t-1}w_t^\star$

18. $p_{ht}^\star = E_{t-1}[w_t - s_t]$

19. $p_{ft} = E_{t-1}[w_t^\star + s_t]$

20. $m_{t+1} = m_t + u_{t+1}$

21. $m_{t+1}^\star = m_t^\star + u_{t+1}^\star$

22. $\nu_{t+1} = \epsilon_{t+1}$

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A.4 Exchange Rate Determination under LCP

Combining the two price indices and the four pricing equations and using them in equation (4.29) gives:

$$(1+r)s_t = \varepsilon r(m_t - m_t^\star) - r(\varepsilon - 1)E_{t-1}s_t + E_t s_{t+1}$$

Taking the suggested guess on the behavior of the expected exchange rate (4.36), keeping in mind that money supplies follow the before specified random walk (4.27) it then follows that:

$$E_t s_{t+1} = E_{t-1}s_{t+1} = E_{t-1}[m_{t+1} - m_{t+1}^\star] = m_t - m_t^\star$$

Plugging this result into the above equation and into an expected version of it dated $t-1$ to verify the guess, we arrive at equation (4.37) of the text. We then simply subtract the expected part to get condition (4.38).

A.5 Exchange Rate Determination under Incomplete Asset Markets

Using the log-linearized versions of the labor supply (4.40) and the goods market clearing condition (4.41), solving in both cases for l_t, and using the price indices and pricing equations we arrive at the following intermediate step:

$$\frac{w_t - p_t - \rho c_t}{\psi} = n c_t - (1-n)c_t^\star - \omega(1-n)E_{t-1}[w_t - w_t^\star - s_t]$$

Applying the same procedure to the foreign country equations, taking expectations date $t-1$ and subtracting the foreign country equation from the home country equation we arrive at:

$$E_{t-1}[w_t - w_t^\star - s_t] = \frac{\rho}{(1+\psi\omega)}E_{t-1}[c_t - c_t^\star] \tag{A.58}$$

Now using the log-linearized version of the budget constraint (4.39) and again applying the pricing equations we can derive the following:

$$p_t - p_t^\star + c_t - c_t^\star + \frac{\beta \bar{d}\bar{b}_{ht}^\star}{\bar{P}\bar{C}(1-n)} = (1-\omega)E_{t-1}[w_t - s_t - w_t^\star] + s_t(\theta - (1-\theta^\star)) + (2-\theta-\theta^\star)E_{t-1}s_t + \frac{\bar{d}\bar{b}_{ht-1}^\star}{\bar{P}\bar{C}(1-n)} \tag{A.59}$$

Taking expectations dated $t-1$, employing the price indices and acknowledging that agents do not expect to change their bond holdings, i.e $E_{t-1}b_{ht}^{\star} = b_{ht-1}^{\star}$ we arrive at equation (4.42) of the text. Combining this equation with (A.58) leads to:

$$E_{t-1}[c_t - c_t^{\star}] = \frac{(1-\beta)}{\sigma}\frac{\bar{d}b_{ht}^{\star}}{\bar{P}\bar{C}(1-n)} \tag{A.60}$$

where $\sigma = (1 - ((1-\omega)\rho/(1+\psi\omega)))$.

Now using the log-linearized money demand equation (4.49) and subtracting its foreign equivalent from it, plugging in equations on optimal bond holdings as given by (4.46) and (4.47), taking expectations date $t-1$, using equation (A.60) from above and considering what has been stated about expectations on changes in bond holdings, gives an intermediate step:

$$E_{t-1}s_t = \frac{r\varepsilon}{r\varepsilon+1}E_{t-1}[m_t - m_t^{\star}] - \frac{\rho r}{r\varepsilon+1}E_{t-1}[c_t - c_t^{\star}] + \frac{1}{r\varepsilon+1}E_{t-1}s_{t+1}$$

Forwarding this equation one period, plugging this back into the above equation, keeping in mind that money supplies follow a random walk and applying equation (A.60) again as well as some calculation rules for infinite sums we arrive at equation (4.43) of the text. Now subtract equation (4.42) of the text from (A.59) keeping in mind that prices are predetermined thus $p_t = E_{t-1}p_t$. Using the introduced hat notation to denote the difference between realization and expectation one arrives at condition (4.44) of the text. Plugging in for the change in bond holdings, using the difference between equation (A.60) forwarded one period and its date $t-1$ version, keeping the steady state relationship in mind, that the real interest rate is equal to the inverse of the bond price minus one we can derive equation (4.45) of the text. Then setting the two equations for foreign bond holdings (4.46) and (4.48) equal, again taking expectations dated $t-1$ we get the following:

$$\rho E_t[\hat{c_{t+1}} - \hat{c_{t+1}^{\star}}] = \rho(\hat{c}_t - \hat{c}_t^{\star}) - \hat{s}_t \tag{A.61}$$

Combining this with equation (4.45) of the text we arrive at equation:

$$\hat{s}_t = \frac{(1+(\sigma/r))(\hat{c}_t - \hat{c}_t^{\star})}{((\sigma/r\rho) + \theta - (1-\theta^{\star}))} \tag{A.62}$$

We now want to substitute out for $\hat{c}_t - \hat{c}_t^{\star}$. Taking both linear approximations of money demands for home and foreign country (4.49) and subtracting one from the other we then use the equation for the expected exchange rate (4.43) and the definition for d_t and its foreign equivalent.

We get an intermediate step:

$$(\hat{m}_t - \hat{m}_t^\star) = \frac{\rho}{\varepsilon}(\hat{c}_t - \hat{c}_t^\star) + \frac{1}{r\varepsilon}\left(\rho(c_t - c_t^\star - \rho E_t[c_{t+1} - c_{t+1}^\star]) + \hat{s}_t\right)$$

Then using equation (A.61) to substitute out for $\hat{s}_t$, by means of equation (A.60) and the knowledge that agents do not expect to change their bond holdings we arrive at the following equation:

$$\hat{c}_t - \hat{c}_t^\star = \frac{\varepsilon(\hat{m}_t - \hat{m}_t^\star)}{\rho} \tag{A.63}$$

Finally plugging this result into (A.62) we can derive equation (6.56) of the text.

A.6 Variance of the Exchange Rate in the Full Model

In order to derive an explicit equation for the exchange rate for the case of the model with noise traders we can build on the previous section. Starting from the relationship between unanticipated change in exchange rate, relative consumptions and net foreign bond holdings as given by equation (4.45) of the text then using conditions (4.51) and (4.52) and similar intermediate steps as in part A.5 of the appendix we can derive equation (4.53) of the text.

Keeping in mind the definition of noise traders' variance as being proportional to the conditional variance of the nominal exchange rate, the variance of the latter is easily derived. Use the definition of the conditional variance for $\hat{s}_t$

$$Var_{t-1}(\hat{s}_t) = E_{t-1}[(\hat{s}_t - E_{t-1}(\hat{s}_t))^2]$$

and note that $E_{t-1}\hat{s}_t = 0$. Plugging back the definition for $\hat{s}_t$ we get:

$$Var_{t-1}(\hat{s}_t) = E_{t-1}[(s_t - E_{t-1}s_t)^2] = Var_{t-1}(s_t$$

The same holds true for $Var_{t-1}m_t$. Remember that the difference in variances is equal to the sum of variances minus two times the covariance which is equal to zero in this model.

A.7 Steady State of Bonds

In equation (4.42) of the text the initial current account was defined as:

$$(1 - \beta)\frac{\bar{d}}{\bar{P}\bar{C}(1 - n)}b_{ht}^\star$$

Applying parameter choices as given in section 4.4 on calibration , it is easy to see that in percentages of β, i.e. $\bar{d}$, the value of the steady state current is specified as 0.18182.

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A.8 Exchange Rate Determination under PCP

The same steps as for the case of LCP lead to equation (A.58), given in part A.5 of the appendix on exchange rate determination under non contingent claims. However, loglinerizing the budget constraint and applying PPP to the pricing equations leads to:

$$c_t - c_t^\star + \frac{\beta \bar{d} b_{ht}^\star - \bar{d} b_{ht-1}^\star}{\bar{P}\bar{C}(1-n)} = (1-\omega)(p_{ht} - s_t - p_{ft}^\star)$$

Taking expectations dated $t-1$, noting that prices are sticky and that agents do not expect to change their bond holdings, i.e $E_{t-1}b_{ht}^\star = b_{ht-1}^\star$ one arrives at equation (4.42) of the text. This equation continues to hold even under the different assumption about firms' price setting behavior of PCP. However, taking expectations dated t-1 of the loglinerized budget constraint and subtracting it from the version above, leads to equation (6.55) of the text that only holds for the case of PCP. Combining this equation with the forwarded version of equation (A.60) gives:

$$\hat{c}_t - \hat{c}_t^\star + \sigma/r E_t[\hat{c_{t+1}} - \hat{c_{t+1}^\star}] = -(1-\omega)\hat{s}_t$$

Using UIRP and with PPP holding, one can note, that apart from the noise, the real interest rates will be equal across countries:

$$\rho(\hat{c}_t - \hat{c}_t^\star) = \rho E_t[\hat{c_{t+1}} - \hat{c_{t+1}^\star}] + \nu_t$$

Plugging in for $E_t[\hat{c_{t+1}} - \hat{c_{t+1}^\star}]$ and solving for $\hat{s}_t$ we obtain:

$$\hat{s}_t = \frac{(1+\sigma/r)(\hat{c}_t - \hat{c}_t^\star) + \frac{\sigma}{r\rho}\nu_t}{-(1-\omega)}$$

When subtracting home and foreign money demands PPP holds and s_t does not cancel, thus using equation (4.43) of the text on the expected change in exchange rate leads to equation (6.56) of the text.

A.9 Notes on Implementation into the TOOLKIT Program

In order to implement the assumption of sticky prices, prices set at date t-1, into the TOOLKIT program some adjustments have to be made. As the TOOLKIT program only allows for equations to include expectations date t, additional dummy variables have to be introduced. As an example we take these two pricing equations:

$$p_{ht} = E_{t-1}[w_t]$$

$$p_{ft}^{\star} = E_{t-1}[w_t^{\star}]$$

We define:

$$x_t = E_t[w_{t+1}]$$

We then change the pricing equation above to:

$$p_{ht} = x_{t-1}$$

This way implementation of nominal rigidities into the TOOLKIT program is made possible.

An additional dummy variable 'a' was created because the 'Brute Force' method was applied for implementation. This method requires that all variables be defined as endogenous state variables.[38]

Due to the fact that the autocorrelation between money shocks was set to unity, as money supplies follow simple random walks, the values reported in in tables 2 - 10 could not be obtain as more precise results of frequency domain given by 'moments' in the TOOLKIT program. Thus the values for the conditional variance of the nominal exchange rate were obtained by running simulations $50,000$ times. These values were not hp-filtered in accordance with the results of the Devereux and Engel paper. However, due to the fact that money shocks in the model are assumed to be permanent, this approximative procedure could not be used to obtain more precise results on correlations between nominal exchange rate and consumption and nominal exchange rate and real interest rate. In this case the assumption about the autocorrelation of money shocks was relaxed to 0.99999 and the HP-Filter was applied. That way the results of 'moments' could be obtained.

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[38]See Uhlig [1999] for a more detailed definition of the method.

A.10 Simulation Results under Log Utility

In order to reproduce the results of table 1 of the paper by Devereux and Engel [2002] a utility function with money balances entering logarithmically as specified in part A.2 of the appendix has to be assumed. It is fairly easy to change the TOOLKIT code given in the following section of the appendix by simply setting $\varepsilon = 1$ and changing the first two rows of matrix GG setting $1/r$ to 0. Then the following results that are equal to the ones reported by the authors can be obtained.

κ	$\theta + \theta^\star$	$_{t-1}\sigma^2(s)^*$	$_{t-1}\sigma^2(s)^*$(PCP)	$\rho(s,c)^\dagger$	$\rho(s,\tilde{r}^\ddagger)^\dagger$
0	2.0	1.73	0.72	0.71	-0.71
	1.75	1.82	0.72	0.40	-0.38
	1.5	1.90	0.72	0.39	-0.38
	1.25	2.00	0.72	0.38	-0.37
	1.0	2.11	0.72	0.37	-0.37
0.5	2.0	2.20	0.78	0.25	-0.19
	1.75	2.32	0.78	0.24	-0.19
	1.50	2.46	0.79	0.23	-0.19
	1.25	2.62	0.78	0.21	-0.20
	1.0	2.78	0.8	0.20	-0.20
0.9	1.00	5.32	1.00	0.25	-0.19
	1.75	6.13	1.00	0.24	-0.19
	1.50	7.21	1.00	0.23	-0.19
	1.25	8.81	1.01	0.21	-0.20
	1.0	11.06	1.01	0.20	-0.20

∗ Simulation results running 50,000 times, not hp-filtered

† Moments approximation choosing autocorrelation of money shocks 0.99999, hp-filtered

‡$\tilde{r} = E_t \rho[c_{t+1} - c_t]$

Table 10: Various Simulation Results for the Full Model under Log-Utility

A.11   MATLAB Code for the Full Model

```
% VERSION 2.0, MARCH 1997, COPYRIGHT H. UHLIG.
% EXAMPLO.M:
% Solving the stochastic neoclassical growth model with the "toolkit"

% Copyright: H. Uhlig.  Feel free to copy, modify and use at your own risk.
% However, you are not allowed to sell this software or otherwise impinge
% on its free distribution.

disp('Devereux/Engel: Exchange Rate Pass Through...(Complete
Model)');

disp('Hit any key when ready...'); pause;

% This is the complete model with nominal rigidities, local currency
% pricing, distributors, imperfect asset markets and noise traders.
% The 'brute force' method - declearing all variables endogenous state
% variables - is employed.

% Setting parameters:
rho         = 4 ;   % Parameter of consumption utility
psi         = 1 ;   % Parameter of labor disutility
omega       = 8;    % Elasticity of substitution between domestic and imported goods
n           = 0.5;  % Relative size of home country
zeta        = 1;    % Standard deviation of money shock home.    Units: Percent.
zetastar    = 1;    % Standard deviation of money shock foreign.  Units: Percent.
gamma       = 1;    % Autocorrelation of money shock home
gammastar   = 1;    % Autocorrelation of money shock foreign
gammanoise  = 0;    % Autocorrelation of shock to noise traders' expectation
epsilon     = 4.7;  % Parameter of money balance utility
r           = 0.1;  % Steady state real interest rate

%Additional parameters:
teta        = 0.5;   % Share of directly marketed goods by domestic producers to foreign households
tetastar    = 0.5;   % Share of directly marketed goods by foreign producers to domestic households
kappa       = 0.5;   % Parameter for variance of noise traders' expectations.

% Calculating the steady state:
d_bar       = 1/(1+r); beta       = d_bar; P_bar     = 1; C_bar
= 1;

% Calculating Variance of noise traders' expectation
sigma       = (1-((1-omega)*rho/(1+psi*omega))); phi
=((sigma/r)+rho*(teta-(1-tetastar))); varm       = zeta+zetastar;
vars        =
(((1+(sigma/r))^(2)*epsilon^(2)*(varm/100))/(phi^(2)*(1-((kappa*sigma)/(r*phi))^2)));
zetanoise= kappa^(2) * vars*100; %  Variance of noise traders' expectation. Units: percent

% Declaring the matrices.

 VARNAMES =['domestic cpi          ',
            'foreign cpi           ',
            'domestic consumption  ',
            'foreign consumption   ',
            'domestic wage         ',
            'foreign wage          ',
            'domestic labor        ',
            'foreign labor         ',
            'nominal exchange rate ',
            'real exchange rate    ',
            'domestic interest rate',
            'foreign interest rate ',
            'price home bond       ',
            'price foreign bond    ',
            'foreign bond holdings ',
            'pricehome             ',
            'priceforeign          ',
            'pricehomeforeign      ',
            'priceforeignhome      ',
            'x                     ',
            'xstar                 ',
            'z                     ',
            'rtilde                ',
            'a                     ',
            'domestic money supply ',
            'foreign money supply  ',
            'noisetraders expec.   ',];
```

```
% Translating into coefficient matrices.
% The loglinearized equations are, conveniently ordered:
% 1) 0 = epsilon*m(t)-epsilon*p(t)-rho*c(t)-1/r*d(t)
% 2) 0 = epsilon*mstar(t)-epsilon*pstar(t)-rho*cstar(t)-1/r*dstar(t)
% 3) 0 = w(t)-p(t)-rho*c(t)-phi*l(t)
% 4) 0 = wstar(t)-pstar(t)-rho*cstar(t)-phi*lstar(t)
% 5) 0 = (n*(1-omega)+(1-n))*p(t)-(1-n)*omega*pstar(t)+(1-n)*c(t)-(1-n)*(teta-(1-tetastar))*s(t)-(1-n)*cstar(t)+
%        +beta*d_bar/(P_bar*C_bar)*bstarh(t)-n*(1-omega)-(1-n)*(1-teta)*ph(t)+((1-n)*(1-tetastar))*pfstar(t)-(1-n)*(teta-omega)*phstar(t)-
%        -(1-n)*(1-tetastar))*pf(t))-(d_bar/(P_bar*C_bar))*bstarh(t-1)
% 6) 0 = l(t)-n*(-omega*(ph(t)-p(t))+c(t))-(1-n)*(-omega*(phstar(t)-pstar(t))+cstar(t))
% 7) 0 = lstar(t)-(1-n)*(-omega*(pfstar(t)-pstar(t))+cstar(t))-n*(-omega*(pf(t)-p(t))+c(t))
% 8) 0 = d(t)-p(t)-rho*c(t)+E_t[p(t+1)+rho*c(t+1)]
% 9) 0 = dstar(t)-pstar(t)-rho*cstar(t)+E_t[pstar(t+1)+rho*cstar(t+1)]
%10) 0 = dstar(t)-rho*c(t)+E_t[p(t+1)+rho*c(t+1)]-s(t+1)-noises(t)+s(t)
%11) 0 = p(t)-n*ph(t)-(1-n)*pf(t)
%12) 0 = pstar(t)-n*phstar(t)-(1-n)*pfstar(t)
%13) 0 = v(t)-s(t)-pstar(t)+p(t)
%14) 0 = i(t)+(1+r)*d(t)
%15) 0 = istar(t)+(1+r)*dstar(t)
%16) 0 = ph(t)-x(t-1)
%17) 0 = pfstar(t)-xstar(t-1)
%18) 0 = phstar(t)-x(t-1)+z(t-1)
%19) 0 = pf(t)-xstar(t-1)-z(t-1)
%20) 0 = x(t)-E_t[w(t+1)]
%21) 0 = xstar(t)-E_t[wstar(t+1)]
%22) 0 = z(t)-E_t[s(t+1)]
%23) 0 = rtilde(t)-E_t*rho*[c(t+1)]+rho*c(t) ; definition of domestic real interest rate
%24) m(t+1)=m(t)+error(t+1)
%25) mstar(t+1)=mstar(t)+errorstar(t+1)
%26) noises(t+1)=noises(t)+errornoise(t+1)
%27) 0=a (dummy)
% CHECK: 27 equations, 27 variables.
%
% Endogenous state variables "x(t)":p(t),pstar(t),c(t),cstar(t),w(t),wstar(t),l(t),lstar(t),s(t),v(t),i(t),istar(t),d(t),
%                                   dstar(t),bstarh(t),ph(t), pfstar(t), phstar(t),pf(t),x(t), xstar(t),z(t), rtilde(t)
% Endogenous other variables "y(t)": a
% Exogenous state variables  "z(t)": m(t), mstar(t),noises(t)
% Switch to that notation.  Find matrices for format
% 0 = AA x(t) + BB x(t-1) + CC y(t) + DD z(t)
% 0 = E_t [ FF x(t+1) + GG x(t) + HH x(t-1) + JJ y(t+1) + KK y(t) + LL z(t+1) + MM z(t)]
% z(t+1) = NN z(t) + epsilon(t+1) with E_t [ epsilon(t+1) ] = 0,

% DETERMINISTIC EQUATIONS:
% for x(t):
AA = zeros(1,23);

% for x(t-1):
BB = zeros(1,23);

% for y(t):
CC = [1];

% For z(t):
DD = zeros(1,3);

% EXPECTATIONAL EQUATIONS:
% For x(t+1)
FF = [0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0      % Equ.1)
      0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0      % Equ.2)
      0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0      % Equ.3)
      0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0      % Equ.4)
      0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0      % Equ.5)
      0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0      % Equ.6)
      0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0      % Equ.7)
      1,0,rho,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0    % Equ.8)
      0,1,0,rho,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0    % Equ.9)
      1,0,rho,0,0,0,0,0,-1,0,0,0,0,0,0,0,0,0,0,0,0,0,0   % Equ.10)
      0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0      % Equ.11)
      0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0      % Equ.12)
      0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0      % Equ.13)
      0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0      % Equ.14)
      0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0      % Equ.15)
      0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0      % Equ.16)
      0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0      % Equ.17)
      0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0      % Equ.18)
      0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0      % Equ.19)
      0,0,0,0,-1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0     % Equ.20)
      0,0,0,0,0,-1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0     % Equ.21)
      0,0,0,0,0,0,0,0,-1,0,0,0,0,0,0,0,0,0,0,0,0,0,0     % Equ.22)
      0,0,-rho,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0];% Equ.23)
```

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```matlab
% For x(t)
GG = [-epsilon,0,-rho,0,0,0,0,0,0,0,0,0,-1/r,0,0,0,0,0,0,0,0,0,0                                  % Equ. 1)
      0,-epsilon,0,-rho,0,0,0,0,0,0,0,0,0,-1/r,0,0,0,0,0,0,0,0,0,0                                 % Equ. 2)
      -1,0,-rho,0,1,0,-psi,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0                                       % Equ. 3)
      0,-1,0,-rho,0,1,0,-psi,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0                                       % Equ. 4)
(n*(1-omega)+(1-n)),(-(1-n)*omega),(1-n),-(1-n),0,0,0,0,(-(1-n)*(teta-(1-tetastar))),0,0,0,0,0,beta*d_bar/(P_bar*C_bar),(this
(-n*(1-omega)-(1-n)*(1-teta)),((1-n)*(1-tetastar)),(-(1-n)*(teta-omega)),(-(1-n)*(1-tetastar)),0,0,0,0    % Equ. 5) (is one single equation)
      -n*omega,-(1-n)*omega,-n,-(1-n),0,0,1,0,0,0,0,0,0,0,0,n*omega,0,(1-n)*omega,0,0,0,0,0        % Equ. 6)
      -n*omega,-(1-n)*omega,-n,-(1-n),0,0,0,1,0,0,0,0,0,0,0,0,(1-n)*omega,0,n*omega,0,0,0,0        % Equ. 7)
      -1,0,-rho,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0                                          % Equ. 8)
      0,-1,0,-rho,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0                                          % Equ. 9)
      -1,0,-rho,0,0,0,0,0,1,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0                                          % Equ. 10)
      1,0,0,0,0,0,0,0,0,0,0,0,0,0,-n,0,0,-(1-n),0,0,0,0,0                                          % Equ. 11)
      0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,-(1-n),-n,0,0,0,0,0,0                                          % Equ. 12)
      1,-1,0,0,0,0,0,0,-1,1,0,0,0,0,0,0,0,0,0,0,0,0,0                                              % Equ. 13)
      0,0,0,0,0,0,0,0,0,0,1,0,(1+r),0,0,0,0,0,0,0,0,0,0                                            % Equ. 14)
      0,0,0,0,0,0,0,0,0,0,0,1,0,(1+r),0,0,0,0,0,0,0,0,0,0                                          % Equ. 15)
      0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0                                                % Equ. 16)
      0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0                                                % Equ. 17)
      0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0                                                % Equ. 18)
      0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0                                                % Equ. 19)
      0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0                                                % Equ. 20)
      0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0                                                % Equ. 21)
      0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0                                                % Equ. 22)
      0,0,rho,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1];                                            % Equ. 23)

% For x(t-1)
HH =[0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0                                               % Equ. 1)
     0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0                                               % Equ. 2)
     0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0                                               % Equ. 3)
     0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0                                               % Equ. 4)
     0,0,0,0,0,0,0,0,0,0,0,0,0,-(d_bar/(P_bar*C_bar)),0,0,0,0,0,0,0,0,0,0 % Equ. 5)
     0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0                                               % Equ. 6)
     0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0                                               % Equ. 7)
     0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0                                               % Equ. 8)
     0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0                                               % Equ. 9)
     0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0                                               % Equ. 10)
     0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0                                               % Equ. 11)
     0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0                                               % Equ. 12)
     0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0                                               % Equ. 13)
     0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0                                               % Equ. 14)
     0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0                                               % Equ. 15)
     0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,0,0,0                                              % Equ. 16)
     0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,0,0                                                % Equ. 17)
     0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,0,1,0                                              % Equ. 18)
     0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,-1,0                                               % Equ. 19)
     0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0                                               % Equ. 20)
     0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0                                               % Equ. 21)
     0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0                                               % Equ. 22)
     0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0];                                             % Equ. 23)

% For y(t+1)
JJ = zeros(23,1);

% For y(t)
KK = zeros(23,1);

% For z(t+1)
LL= zeros(23,3);
```

```
% For z(t)
MM =   [epsilon,0,0      % Equ. 1)
        0,epsilon,0      % Equ. 2)
        0,0,0            % Equ. 3)
        0,0,0            % Equ. 4)
        0,0,0            % Equ. 5)
        0,0,0            % Equ. 6)
        0,0,0            % Equ. 7)
        0,0,0            % Equ. 8)
        0,0,0            % Equ. 9)
        0,0,-1           % Equ. 10)
        0,0,0            % Equ. 11)
        0,0,0            % Equ. 12)
        0,0,0            % Equ. 13)
        0,0,0            % Equ. 14)
        0,0,0            % Equ. 15)
        0,0,0            % Equ. 16)
        0,0,0            % Equ. 17)
        0,0,0            % Equ. 18)
        0,0,0            % Equ. 19)
        0,0,0            % Equ. 20)
        0,0,0            % Equ. 21)
        0,0,0            % Equ. 22)
        0,0,0];          % Equ. 23)

% AUTOREGRESSIVE MATRIX FOR z(t)

NN = [gamma,0,0
      0,gammastar,0
      0,0,gammanoise];

Sigma = [ zeta,0,0
          0, zetastar,0
          0,0,zetanoise];

% Setting the options:

[l_equ,m_states] = size(AA); [l_equ,n_endog ] = size(CC);
[l_equ,k_exog  ] = size(DD);

PERIOD    = 1;   % Number of periods per year, i.e. 12 for monthly, 4 for quarterly; 1 for annual
GNP_INDEX = 9;   % Index of nominal exchange rate among the variables selected for HP filter,
IMP_SELECT = [1:15,25,26,27]; %  a vector containing the indices of the variables to be plotted
SIM_SELECT = [1:22];
HP_SELECT  = [1:22]; % Selecting the variables for the HP Filter calcs.
DO_STATE_RESP=0;
DO_SIMUL   = 1; % Calculates Simulations
DO_MOMENTS = 0; % Calculates Moments
DO_NO_ZERO_RESPONSE=1; DO_HP_FILTER=0;
% SIM_LENGTH=50000; % Set for calculation of conditional variance of
% nominal exchange rate given by the standard deviation of the real exchange rate, in order to increase precision. But
% make sure HP-Filter =0

do_it;
```

Erklärung zur Urheberschaft

Hiermit erkläre ich, dass ich die vorliegende Arbeit allein und nur unter Verwendung der aufgeführten Quellen und Hilfsmittel angefertigt habe.

Zoë Kuehn

Berlin, 27. Juli 2004