Implications of Aggregate and Idiosyncratic Shocks for Neoclassical Growth and Wealth Distribution

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To my family and Pinar

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Abstract

The aim of this paper is to examine the effects of aggregate and individual shocks on the dynamics of wealth distribution and macroeconomic variables. For this purpose, the stochastic neoclassical representative agent model is extended to an economy populated by heterogeneous agents with partially uninsurable idiosyncratic risks. Agents differ in their labor productivity and capital endowments whose steady state distributions are given exogenously. Using log-linear approximation around the steady state it is shown that the effects of aggregate shocks depend on the degree of heterogeneity and preferences. Nevertheless, idiosyncratic shocks impose a consistent antagonistic relationship between different types of agents where those experiencing a positive shock are better off leaving the others worse off. Moreover, it is found that agents do not care about the distribution of wealth beyond its mean while making their decisions. In addition, the business cycle properties of the heterogeneous economy are distinguished from those of the corresponding representative agent framework if the persistence of idiosyncratic productivity shocks differs significantly from the persistence of the aggregate technology shock. Given these results, one may conclude that it is time to deviate from the representative agent assumption.

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1 Introduction

Although microfoundations is the key term in modern macroeconomic theory many models are based on the supposition that the economy is populated by a single infinitely living agent who is a proxy for the aggregate economy. It is quite understandable that this framework has affected earlier dynamic models for its strong implications. Essentially, the assumptions of infinite horizon, competitive markets, constant returns to scale production and homogeneous agents indicate that the allocation attained by a decentralized economy will be the same as the allocation achieved by a central planner. As theory evolves, almost all of these assumptions are criticized: Infinite horizon has been replaced with overlapping generations, monopolistic competition has been added via intermediate goods sector and increasing returns to scale has been introduced in innumerable number of studies. However, the homogeneity assumption has remained almost untouched until recently despite the fact that the dynamics of income and wealth distribution is at the center of lively political discussions. This paper touches homogeneity assumption by studying the stochastic dynamics of wealth distribution in an economy inhabited by heterogeneous agents.

Our model is basically an extension of the well-known dynamic stochastic general equilibrium model with an infinitely living representative agent and neoclassical production function. The crucial difference is that we have many agents subject to partially uninsurable idiosyncratic labor productivity shocks affecting their labor earnings. Agents have the same time separable preferences and try to maximize their discounted sum of utility subject to a decentralized budget constraint by using all available information up to the time of decision. The Cobb-Douglas production technology employs capital and labor as production factors. Agents differ in their steady state endowments, i.e. wealth and labor earnings. Labor is supplied inelastically and wealth is the only asset that can be saved to smooth consumption and to insure oneself against exogenous unpredictable events which include idiosyncratic productivity and aggregate technology shocks.

In general, studies that employ heterogenous agents framework focus on simulations and try to approximate the dynamics via limited state-space models[1]. However, the examination of idiosyncratic and aggregate shocks separately has been absent until now. So, our approach is novel for this branch of models because of being the first attempt to assess the contribution of each shock to the dynamics and to interpret the properties of the economy accordingly. Using this approach we will try to answer three main questions:

- What are the effects of aggregate technology and individual productivity shocks on wealth heterogeneity?
- Do wealth and income distributions matter for the dynamics of aggregate capital and correspondingly of other aggregate variables?
- Do the distributional properties of wealth other than its mean matter for the dynamics of individual decisions?

In order to reach our aim, we calculate the impulse responses of individual and aggregate variables to different shocks using the procedure presented in Uhlig (1999) that has proved to be useful in solving dynamic stochastic general equilibrium models in discrete time framework. This procedure involves four main steps that lead to the calculation of impulse responses, simulations and second moments. First of all, one gathers the equations that define the equilibrium of the model including constraints, identities, first order conditions, exogenous processes and other necessary equations. The second step involves finding the (non-stochastic) steady state of these equations. The third step is the log-linearization of the equations around their steady state and finally we employ method of undetermined coefficients to obtain the recursive equilibrium laws of motion assuming that they are linear in log-deviations of state variables. This last step, as well as the calculation of impulse responses, simulations and moments, is automatically done by Toolkit[2].

Moreover, as our objectives concern dynamics of wealth heterogeneity we should also choose a measure of inequality. There are several ways in literature to address wealth inequality among which are

[1]See, for example, Krusell and Smith (1998) and Castañeda, Díaz-Giménez and Ríos-Rull (1998) which will be discussed in the next section in more detail.

[2]©Prof. Harald Uhlig, Ph.D.. The latest information on Toolkit can be found under http://www.wiwi.hu-berlin.de/wpol/html/toolkit.htm .

- *Comparison of percentile distributions* where one percentile is compared to another one or to the aggregation of some other percentiles,

- *Lorenz curve* which is a graphical representation of relative inequality where one can read what fraction of the population owns how much share of total wealth or income,

- *Gini coefficient* which is a proxy for the convexity of Lorenz curve used to quantify inequality, and

- *Standard deviation* which is a well-known statistical quantity that should be considered relative to average wealth rather than in absolute terms.

Although we employ the first three items to present the dimensions of wealth inequality we track its dynamics using the Gini coefficient since it is informative, easy to compare in different model settings and simple to implement once we get the impulse responses from Toolkit. Knowing the approach, we may now discuss some results.

Our first finding is that any wealth distribution is consistent with the non-stochastic steady state of our model. So, the steady state distribution of wealth and thereby of consumption is indeterminate. For this reason, the aggregate and idiosyncratic shocks may and do have persistent effects on wealth distribution and consumption paths of individuals.

Secondly, we find that if agents have the same constant relative risk aversion (CRRA) preferences a positive technology shock promotes wealth equality. Moreover, it is shown that subsequent positive technology shocks can reduce wealth inequality down to a limit where the ratio of individual capital holdings to individual labor productivity is equalized among all agents. In other words, given that agents have the same CRRA utility and that the probability of a positive technology shock is greater than a negative one, the long-run stochastic steady state of the model implies the same capital-labor ratio for all agents.

Thirdly, we observe that the idiosyncratic shocks cause an antagonistic relationship between the agents and that the poor agents are hurt more than the rich ones due to this relationship. Moreover, the distribution of wealth matters for the impulse responses of aggregate capital to idiosyncratic shocks but not to aggregate shocks under CRRA preferences. However, introducing concave absolute risk tolerance to the model changes the latter result such that wealth heterogeneity affects aggregate dynamics in response to aggregate shocks.

Furthermore, we compare the coefficients that have been obtained using the method of undetermined coefficients in order to find out whether the distribution of wealth matters for the aggregate dynamics. This comparison makes it clear that only the mean of the wealth distribution matters for the individuals when they make their decisions. In addition, we see that this property of the model is robust to changes in preferences.

Finally, we introduce a model with persistent idiosyncratic shocks and discuss the effects of persistence and different preferences on the cyclical properties of the aggregate variables. We find that the changes in preference settings does not affect the business cycle characteristics in a significant way. Nevertheless, persistence of idiosyncratic shocks play an important role in the determination of cross-correlations of aggregate variables with aggregate output.

The rest of this paper is organized as follows. In the next section we discuss the literature on the same line of research to get an idea about the current state of knowledge which will help the reader to position our paper on this line. Section 3 presents facts regarding the wealth and earnings distribution in the United States that are relevant to our model. The benchmark model with two types of agents and its implementation are described in Sections 4 and 5 whereas Section 6 provides the corresponding results and their explanation. In Section 7, we present variations of the benchmark model for deeper understanding of model dynamics. Section 8 offers a discussion of the results and alternative methods. Finally, Section 9 concludes with a brief summary and additional remarks.

2 Literature

This section reviews several models of wealth heterogeneity similar to the one discussed in this paper in the sense that they employ infinitely living agents framework. Therefore, these models implicitly assume that each generation is perfectly altruistic toward its offspring. We will start the discussion by focusing on deterministic models and then we move on to the stochastic versions.

In the deterministic models involving heterogeneity earnings are deterministic, households differ in their asset holdings and wealth is indeterminate[3]. Therefore, insurance against unforeseen events is not a motive for the consumers to refrain themselves from spending their income. Moreover, it is clear that if all the agents are alike and face no uncertainty the model collapses to representative agent framework. Using this framework, Chatterjee (1994) shows that the initial distribution of wealth in an economy with infinitely living agents, homothetic preferences and neoclassical production remains constant throughout the equilibrium path regardless of the economy being in the steady state or not.

The typical example of a model economy with stochastic dynamics of idiosyncratic productivity that generates endogenous wealth distribution is introduced by Aiyagari (1994)[4]. In his model, the distribution of labor earnings is exogenous and the inclusion of stochastic earnings dynamics to the model has resulted in precautionary savings against idiosyncratic shocks due to the inexistence of complete insurance markets. Another crucial feature of this model is that there is a lower limit for asset holdings to prevent agents from running a Ponzi scheme. This feature creates distortion at the bottom

[3]In other words, the amount of wealth each agent owns is described via a probability distribution over asset holdings. It may be mentioned, in passing, that the indeterminacy of wealth will also show up in our calculations of steady state and we will see that any wealth distribution is consistent with our non-stochastic steady state.

[4]It may be necessary to mention that Aiyagari (1994) is not the first paper that involves a dynamic stochastic model. For example, Scheinkman and Weiss (1986) employ a simple two-agent framework with idiosyncratic uncertainty. Their paper is similar to Aiyagari (1994) in ignoring aggregate uncertainty. However, it differs from those discussed here since they do not incorporate a neoclassical production function.

such that the savings propensity of very poor agents differ from others[5]. The basic difference of this influential paper from its successors is that it excludes aggregate shocks whereas the constant relative risk aversion utility function, the borrowing constraint and limited discrete state space for the Markov process of labor endowment shocks are widely adapted by later contributors. The main finding of Aiyagari (1994) is that the contribution of idiosyncratic risk, i.e. of the precautionary motive, to aggregate savings is low.

A model which incorporates both aggregate and idiosyncratic uncertainty is generated by Krusell and Smith (1998) which is also an important methodological contribution. The main motivation of their paper is to answer whether the representative agent framework is a reasonable modeling strategy. The answer to this question is positive if aggregate variables in a model with a more realistic setting, in their context one that consists of many agents with idiosyncratic risks, behave like those in the representative agent framework. For this purpose they build a stochastic neoclassical growth model with large number of infinitely living consumers who have CRRA preferences and try to insure themselves against aggregate and idiosyncratic productivity shocks using only one asset, which is capital. They use a novel simulation technique to achieve the comparison of the aggregate properties of this model's stationary stochastic equilibrium with the corresponding representative agent model and conclude that the heterogeneous-agents economy behaves almost identically to its representative-agent counterpart except a slight increase in the equilibrium level of aggregate capital if markets are incomplete[6].

Another stochastic model in this line of research is presented in Castañeda, Díaz-Giménez and Ríos-Rull (1998). They focus on the role of unemployment spells and cyclically moving factor shares in shaping the distribution of income and business cycle dynamics. Their models are similar to the benchmark model in Krusell and Smith (1998) except the fact that they focus primarily on two environments. In one of the environments all agents are ex

[5]See also Krusell and Smith (1998), pp. 880.

[6]In Krusell and Smith (1998), the employment status of agents and the aggregate productivity shocks follow a joint discrete first order Markov process with limited state space. Moreover, idiosyncratic shocks are uncorrelated conditional on the aggregate shock.

ante identical and in the other one they divide the population into five types that vary in their average skill levels and unemployment processes. These two environments, even after the inclusion of cyclically moving factor shares underestimate the wealth concentration observed in the U.S. as in Krusell and Smith (1998). Finally, they propose a model which imposes an exogenous initial wealth distribution and imitates the wealth concentration in the United States better by construction. Nevertheless, they find that the dynamics of this model is not significantly different from their previous model settings where wealth distribution is determined endogenously.

The papers by Krusell and Smith (1998), Castañeda, Díaz-Giménez and Ríos-Rull (1998) and many others utilizing the same approach for the overlapping generations (OLG) model[7] claim that the movement of aggregate variables can almost perfectly be described using only the mean of the capital distribution and the aggregate productivity shock[8]. In other words the dynamics of their model economy are very similar to those of representative agent framework. Moreover, almost all the models studied fail to attain the magnitude of skewness and concentration of wealth distribution represented in the data[9].

3 Facts

There are two main sources on financial inequality which are widely referred in the literature. One of these sources is the Panel Study of Income Dynamics conducted by the Survey Research Center at the University of Michigan and the other one is the Survey of Consumer Finances (SCF) conducted by the

[7]See Storesletten, Telmer and Yaron (2001) for a recent study of the OLG model with idiosyncratic risk. A detailed discussion of the OLG models with heterogeneity can be found in Quadrini and Ríos-Rull (1997).

[8]Krueger and Kubler (2003) employ a different method for the computation of the equilibrium of an OLG model and claim that the models in the other papers are extremely special cases and the distributional properties of capital matter in more general cases.

[9]Two main exceptions are Krusell and Smith (1998) and Castañeda, Díaz-Giménez and Ríos-Rull (2003). Although the equilibrium of the benchmark model in Krusell and Smith (1998) does not show sufficient concentration of wealth they manage to handle this problem via stochastic modeling of the rate of time preference. Castañeda, Díaz-Giménez and Ríos-Rull (2003) employ a model that combines some properties of OLG and infinitely living agents models.

Pop. Percentile %	0-20	20-40	40-60	60-80	80-100	90-95	95-99	99-100
Earnings Share %	7.7	14.2	16.3	20.4	41.5	8.2	12.7	7.5
Wealth Share %	-0.2	1.4	5.3	12.9	80.6	12.3	24.1	31.4

Table 1: Dimensions of Inequality from SCF 1992 data as summarized in Budría et al.(2002). Population percentiles are ordered from the poorest to the richest in terms of wealth.

Pop. Percentile %	0-20	20-40	40-60	60-80	80-100	90-95	95-99	99-100	0-100
Labor %	71.8	83.4	77.4	71.2	48.4	58.1	41.7	31.8	63.0
Capital %	0.5	0.7	2.2	4.8	18.6	15.2	20.7	34.1	10.0
Business %	2.1	2.3	2.7	5.2	18.2	8.9	26.1	29.3	10.3
Transfers %	17.5	7.8	11.8	11.0	7.1	10.2	4.8	2.5	9.4
Others %	8.1	5.8	5.9	7.8	7.7	7.7	6.6	2.4	7.3

Table 2: Income Sources of U.S. households. Population percentiles are ordered from the poorest to the richest in terms of wealth.

National Opinion Research Center at the University of Chicago. We focus on the SCF 1992 data summarized in Budría et al. (2002) since the SCF minimizes top-coding by having a large sample of wealthy households. The relevant numbers are reproduced in Tables 1 and 2 for convenience[10].

When we look at Table 1 and Figure 1 we see that the wealth and earnings are concentrated. First of all, the poorest quintile holds a negative amount of wealth on average, whereas the richest quintile hold 80.6 percent of wealth. The spread in wealth becomes more striking when we see that 31.4 percent of total wealth is possessed by the top percentile meaning that the wealth of the top percentile is almost equal to that of the bottom 90 percent of total population. Moreover, we realize that the distribution of wealth within the top quintile is far from being even[11].

Considering the earnings share that has been obtained by each wealth group we see that the wealthier groups tend to get a higher fraction of earn-

[10] See Table 33 in Budría et al. (2002). Budría et al. (2002) is principally an updated version of Díaz-Giménez, Quadrini and Ríos-Rull (1997).

[11] The Gini coefficient for the wealth distribution within the top quintile is 0.54 according to the numbers in Table 1.

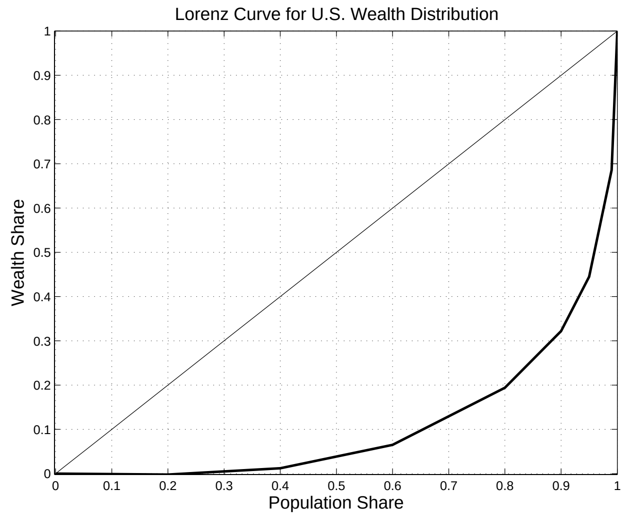

Figure 1: Lorenz curve for the wealth distribution in the United States according to SCF 1992

ings. Given this fact we may infer that the mobility between different groups is pretty limited. Nevertheless, earnings are far more equally distributed than wealth.

Table 2 presents information about the income sources of different wealth groups with its last column informing about the contribution of each income source for an average person in the economy. As expected, the main source of income of the poorest wealth group is labor whereas it also benefits from transfers greatly. The importance of capital and business as a source of income increases as we move to upper wealth groups.

In conclusion, the uneven distribution of wealth within the top quintile and the fact that upper wealth groups get a higher share of labor earnings than the lower groups point to the vital role played by the wealthy households in all aspects of economic inequality. Therefore, we have decided to divide the top quintile into its parts in our model frameworks that involve multiple types of agents. The importance of different sources of income, especially of labor and capital income among others, is another significant aspect that we will consider in the explanation of the results implied by our model.

4 The Model

This section introduces our benchmark neoclassical growth model with two types of infinitely living agents having identical constant relative risk aversion (CRRA) preferences. All agents are endowed with labor and capital which is the only asset that can be stored for consumption in future periods. Using all available information, they try to maximize their discounted sum of utility

$$\max_{\{c_{i,t},k_{i,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_{i,t}^{1-\sigma}}{1-\sigma} \tag{1}$$

subject to a decentralized budget constraint.

$$c_{i,t} + k_{i,t} = R_t k_{i,t-1} + w_t \tilde{l} \theta_{i,t} \tag{2}$$

Here, $0 < \beta < 1$ is the discounting factor, c is consumption, $k_{i,t-1}$ is the capital endowment of an agent of type i and R is the gross rate of return on capital. The wage rate per efficiency unit of labor is denoted by w, whereas θ_i denotes the individual labor productivity of agents of type i and $\tilde{l}$ is an adjustment parameter for labor supply in the economy[12]. The economy uses a Cobb-Douglas technology that takes capital and labor as input for the production of output, y, and capital depreciates at rate δ.

$$\begin{aligned} y_t &= K_t^{\alpha} L_t^{1-\alpha} && (3) \\ K_t &= \sum_i H_i k_{i,t-1} && (4) \\ L_t &= \sum_i H_i \tilde{l} \theta_{i,t} && (5) \end{aligned}$$

K and L are aggregate capital and aggregate labor, which is measured in efficiency units, respectively. H_i is the fraction of households characterized as type i in the whole population and $\sum_i H_i = 1$. The factor prices are determined by their marginal productivities such that we have got the following equations for wage and gross rate of return on capital.

$$\begin{aligned} w_t &= (1-\alpha)(K_t/L_t)^{\alpha} && (6) \\ R_t &= \alpha (K_t/L_t)^{\alpha-1} + (1-\delta) && (7) \end{aligned}$$

[12] In other words, $\tilde{l}\theta_i$ is the efficiency units of labor provided by type i.

Moreover, the labor productivity of each agent follows an exogenous process consisting of permanent aggregate technology shocks, z_t, and temporary type-specific productivity shocks, $u_{i,t}$[13].

$$\log \theta_{i,t} = \log z_t + \log \kappa_i + u_{i,t} \tag{8}$$

$$\log z_t = (1-\rho)\log \bar{z} + \rho \log z_{t-1} + v_t \tag{9}$$

κ_i denotes the steady state labor productivity of type i agents. Finally, it should be clear that the state variables at time t are z_t and the distribution of capital and labor productivity[14].

5 Model Analysis

5.1 Implementation of the benchmark model

In this section, we will present the necessary steps to implement the benchmark model in Toolkit which include collection of the first order conditions and other necessary equations that characterize the equilibrium, calculation of the non-stochastic steady state, log-linearization of the equations to obtain the impulse responses and calibration[15].

The first order conditions of the agents' maximization problem can be summarized in two equations which are the budget constraint already presented in equation (2) and the following Euler equation.

$$c_{i,t}^{-\sigma} = \beta E_t[c_{i,t+1}^{-\sigma} R_{t+1}] \tag{10}$$

Therefore, we may use the equations (2) to (9) along with equation (10) to define the non-stochastic steady state and to analyze the stochastic dynamics

[13]In this context, the idiosyncratic shocks affect all agents belonging to the same type. Therefore, these shocks are perfectly correlated within each group and all agents of the same type are ex ante and ex post identical.

[14]Contrary to other papers there is no exogenously specified borrowing constraint in this paper since we focus on impulse responses which are logarithmic deviations from steady state. This induces an endogenous borrowing constraint for any non-negative initial capital holding and the no-Ponzi-game condition is inherently satisfied.

[15]These steps, as well as Toolkit, are discussed in Uhlig (1999) in detail.

of this model. The steady state versions of these equations, starting with the Euler equation and continuing with the others as they have been ordered before, are summarized as follows[16].

$$
\begin{aligned}
\bar{R} &= 1/\beta && (11)\\
\bar{c}_i &= (\bar{R}-1)\bar{k}_i + \bar{w}\tilde{l}\bar{\theta}_i && (12)\\
\bar{K} &= \sum_i H_i \bar{k}_i && (13)\\
\bar{L} &= \tilde{l}\sum_i H_i \bar{\theta}_i && (14)\\
\bar{w} &= (1-\alpha)(\bar{K}/\bar{L})^{\alpha} && (15)\\
\bar{R} &= \alpha(\bar{K}/\bar{L})^{\alpha-1} + (1-\delta) && (16)\\
\bar{\theta}_i &= \bar{z}\bar{\kappa}_i && (17)\\
\bar{z} &= \bar{z} && (18)
\end{aligned}
$$

It is clear that the equations above, excluding equation (12), are enough to obtain the steady state values of aggregate variables. However, one cannot also get an endogenous steady state distribution of capital holdings from these equations. The reason is that the number of variables exceed the number of equations for the fact that the steady state versions of all individual Euler equations are the same, namely equation (11). This underidentification of the non-stochastic steady state implies that any distribution of capital holdings is consistent with the non-stochastic steady state[17].

The final step of our analysis is the log-linearization of necessary equations characterizing the equilibrium. This step serves for the purpose of making the equations approximately linear in the log-deviations from the steady state. The details of this procedure can be found in Uhlig (1999)[18]. We present the

[16] We skip equation (3) because we work with the decentralized version of the economy and the dynamics of aggregate output is not of particular interest for the explanation of the impulse responses.

[17] Due to the indeterminacy indicated by this property of the model, shocks may and do have permanent effects on wealth distribution as we will see in the next section.

[18] Uhlig (1999) studies the dynamic properties of the models via the method of undetermined coefficients using the calibrated log-linearized equations. Toolkit takes this intermediate step automatically and produces corresponding impulse responses, simulations and moments.

log-linearized equations in the same order as the steady state equations.

$$
\begin{aligned}
E_t[\sigma\hat{c}_{i,t} - \sigma\hat{c}_{i,t+1} + \hat{R}_{t+1}] &= 0 && (19)\\
\bar{c}_i\hat{c}_{i,t} + \bar{k}_i\hat{k}_{i,t} - \bar{R}\bar{k}_i\hat{R}_t - \bar{R}\bar{k}_i\hat{k}_{i,t-1} - \bar{w}\tilde{l}\bar{\theta}_i\hat{w}_t - \bar{w}\tilde{l}\bar{\theta}_i\hat{\theta}_{i,t} &= 0 && (20)\\
\bar{K}\hat{K}_t - \sum_i H_i\bar{k}_i\hat{k}_{i,t-1} &= 0 && (21)\\
\bar{L}\hat{L}_t - \tilde{l}\sum_i H_i\bar{\theta}_i\hat{\theta}_{i,t} &= 0 && (22)\\
\hat{w}_t - \alpha\hat{K}_t + \alpha\hat{L}_t &= 0 && (23)\\
\bar{R}\hat{R}_t - \alpha(\bar{K}/\bar{L})^{\alpha-1}(\alpha-1)\hat{K}_t + \alpha(\bar{K}/\bar{L})^{\alpha-1}(\alpha-1)\hat{L}_t &= 0 && (24)\\
\hat{\theta}_{i,t} - \hat{z}_t &= u_{i,t} && (25)\\
\hat{z}_t &= \rho\hat{z}_{t-1} + v_t && (26)
\end{aligned}
$$

In the above equations the entries with a hat are the log-deviations of the corresponding variables from their steady state, which can be interpreted as the approximate percentage deviation[19]. Having all the equations that describe the dynamics, we now focus on the calibration of the model.

Some of the values used in calibration are pretty standard. For example, for the capital share in production function we take 0.36 and assume a quarterly depreciation rate of 0.025 and 1% real interest rate per quarter as implied in Krusell and Smith (1998). The coefficient of relative risk aversion, σ, is the same for all agents and is equal to 1.5 as in Castañeda, Díaz-Giménez and Ríos-Rull (1998).

For the non-stochastic steady state distribution of labor productivity and capital we benefit from the 1992 Survey of Consumer Finances (SCF) data as documented in Budría et al. (2002) which is summarized in Table 1. Accordingly, we have selected H_1 and H_2 as 0.8 and 0.2, whereas the agents belonging to these groups possess 20% and 80% of aggregate wealth respectively. Moreover, we have employed the labor income as a proxy for labor

[19]The reader may refer to Uhlig (1999) for more information. Moreover, note that we have indexed K_t as an endogenous variable. This was necessary in order to get the correct impulse responses to a change in type-specific capital holdings in Toolkit and for our analysis of multiple types of agents in Sections 7.1 and 7.3. It is validated by cross-checking that this feature does not change the impulse responses to other shocks.

productivity and adjusted its distribution such that the poor group gets 59% and the rich one gets 41% of total labor income. Therefore, capital-labor ratio of poor group is lower than that of rich group such that the main source of income of poor people is labor earnings and that of rich people is capital earnings[20]. In addition, $\bar{z}$ is normalized to 1 and $\tilde{l}$ is chosen such that the aggregate labor supply in steady state, $\bar{L}$, is 1/3. Finally, we choose $\rho = 0.95$ for the autocorrelation of aggregate technology shocks as in Hansen (1985).

5.2 Gini Index

An elaborate way of tracking the dynamics of wealth heterogeneity is using a standard inequality index which is both simple and informative. We make use of Gini coefficient which fulfills these criteria. The Gini coefficient is the ratio of the area between the perfect equality line and the Lorenz curve to the area under the line of perfect equality. Figure 2 illustrates the line of perfect equality and the Lorenz curve that corresponds to the non-stochastic steady state of our benchmark economy. According to the definition above, the Gini coefficient here is given by $A/(A+B)$ or $1-2B$ since $A+B=0.5$. It is important to note that the value of Gini coefficient lies in the range [0, 1]. A zero value indicates perfect equality where everyone holds the same amount of wealth whereas we have perfect inequality, i.e. a single person possesses everything, if the value of Gini coefficient is unity.

We calculate the dynamics of the Gini coefficient for our model as follows. First, we define a_i as the fraction of wealth held by agents of type i and the cumulative sum of a_i, A_i, such that

$$a_{i,t} = \frac{H_i k_{i,t}}{K_{t+1}} \tag{27}$$

$$A_{i,t} = \sum_{j=1}^{i} a_{j,t} \tag{28}$$

hold[21]. Accordingly, the area under the Lorenz curve and the corresponding

[20] The capital to labor ratio for the agents will turn out to be important in the attempt to explain our results and is therefore introduced as a new concept, the *relative endowment*.

[21] The value of K_{t+1} is known at time t for certain since $K_{t+1} = \sum_i H_i k_{i,t}$. Therefore, there are no expectations involved in these equations.

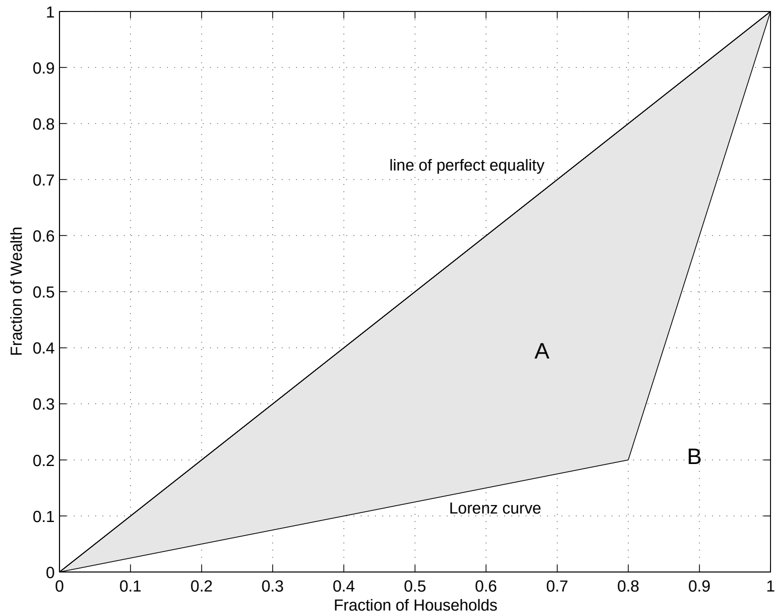

Figure 2: Lorenz curve and line of perfect equality for the benchmark model

Gini coefficient can be calculated as

$$B_t = \sum_i H_i A_{i,t} - 0.5 \sum_i H_i a_{i,t} \tag{29}$$

$$Gini_t = 1 - 2B_t \tag{30}$$

where B_t is the area under the Lorenz curve at time t. The related steady state equations are:

$$\bar{a}_i = \frac{H_i \bar{k}_i}{\bar{K}} \tag{31}$$

$$\bar{A}_i = \sum_{j=1}^{i} \bar{a}_j \tag{32}$$

$$\bar{B} = \sum_i H_i \bar{A}_i - 0.5 \sum_i H_i \bar{a}_i \tag{33}$$

$$G\bar{i}ni = 1 - 2\bar{B} \tag{34}$$

The last step is to log-linearize the equations (27) to (30) to get the following equations that define the dynamics of the Gini coefficient given the dynamics of wealth distribution.

$$\hat{a}_{i,t} = \hat{k}_{i,t} - \hat{K}_{t+1} \tag{35}$$

$$\bar{A}\hat{A}_{i,t} = \sum_{j=1}^{i} \bar{a}_j \hat{a}_{j,t} \tag{36}$$

$$\bar{B}\hat{B}_t = \sum_i H_i \bar{A}_i \hat{A}_{i,t} - 0.5 \sum_i H_i \bar{a}_i \hat{a}_{i,t} \tag{37}$$

$$G\bar{ini}\widehat{Gini}_t = -2\bar{B}\hat{B}_t \tag{38}$$

6 Model Results and Answer

This section will present the impulse responses to different shocks in the benchmark model and the corresponding insights with their explanation. At this stage, it is important to note that all shocks discussed in this paper are one percent deviations of the respective variables from their steady state values. We can summarize the results for the given calibration of the benchmark model as follows:

1. Under CRRA utility function with the same relative risk aversion coefficient for both types of agents, a positive technology shock results in a decrease of wealth heterogeneity.

2. If one considers idiosyncratic shocks, one observes an antagonistic relationship between different types of agents.

3. The impulse responses of aggregate variables, and therefore that of aggregate capital, to aggregate technology shocks does not alter after a change in wealth and labor productivity distribution. However, distributions matter for the dynamics of aggregate variables in response to idiosyncratic labor productivity shocks.

4. The capital-labor ratios of different types of agents are crucial to understand the model dynamics. Given CRRA preferences and higher probability of positive technology shocks with respect to negative ones, the stochastic steady state of the model in the long run is the one that implies the same capital-labor ratio for all agents.

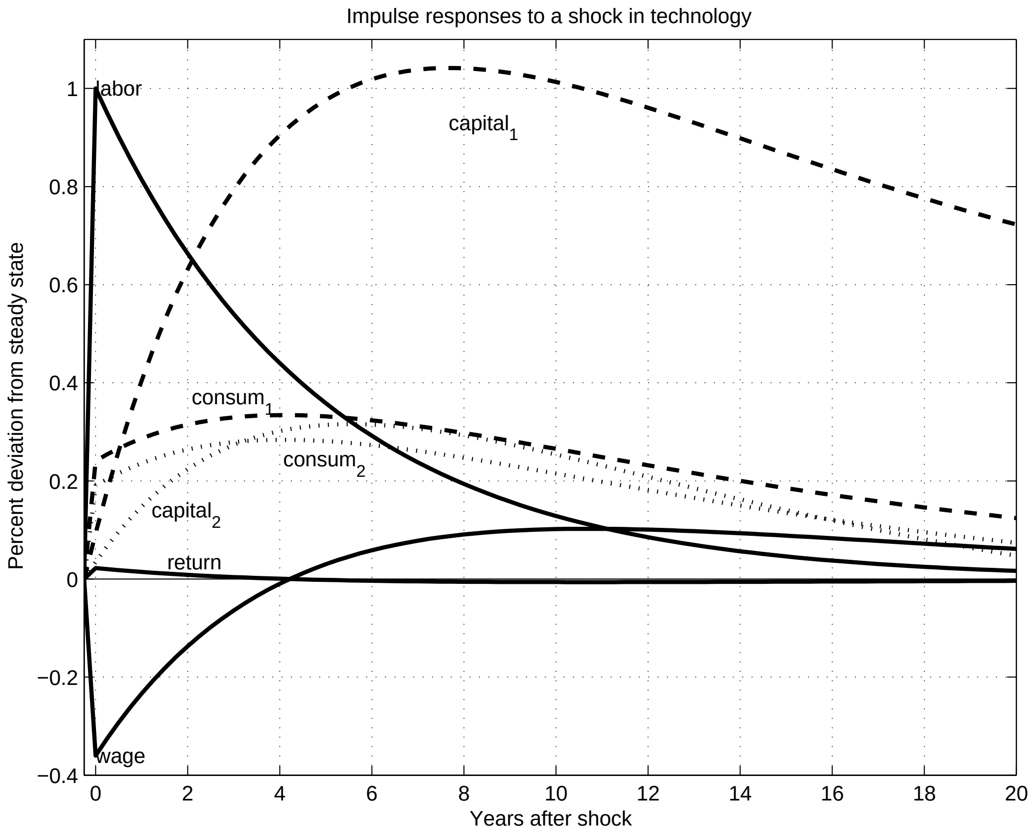

Figure 3: Impulse responses to aggregate technology shock in benchmark model. The impulse responses of labor, $\hat{L}_t$; idiosyncratic productivities, $\hat{\theta}_{1,t}$ and $\hat{\theta}_{2,t}$ and aggregate technology, $\hat{z}_t$ are overlapping.

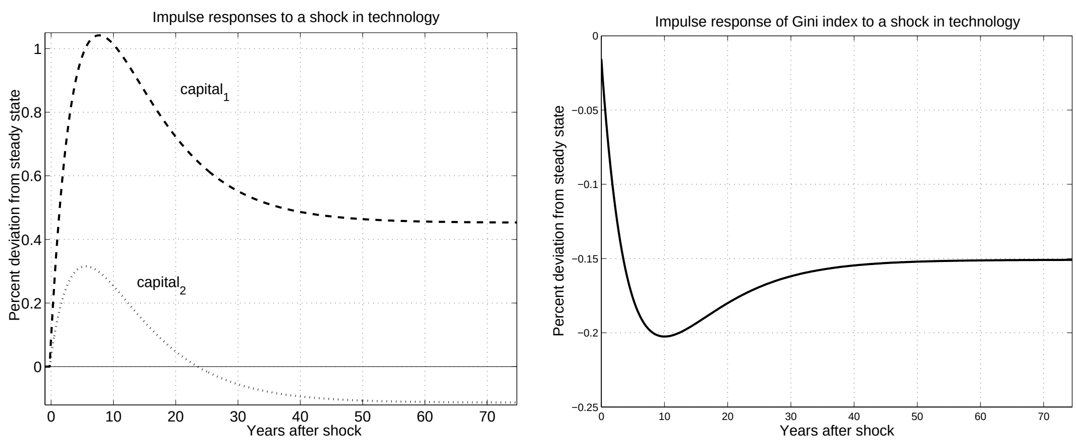

Figure 4: Long run impulse responses of capital holdings and Gini index to aggregate technology shock in benchmark model

6.1 Technology Shocks and Wealth Distribution

Figure 3 shows the evidence for the first result. Here, the impulse response of capital holdings of the first type of agents to an aggregate technology shock is higher than that of agents of second type. Since our aggregate technology is labor augmenting (Harrod-neutral), a positive technology shock that increases labor productivity, and equivalently the aggregate labor supply, causes the wage per efficiency units, w_t, to fall. However, the careful reader will note that the dynamics of the wage per allocated time, $w_t\theta_{i,t}$, is exactly the same as in the case of output augmenting (Hicks-neutral) technology shock by construction. The dynamics of the wage per allocated time can be calculated easily by adding up the impulse responses of w_t and $\theta_{i,t}$.

In addition, Figure 4 presents the impulse responses of capital holdings and the Gini index to the same shock in a longer time horizon. The persistence of the impulse responses of capital holdings and Gini index, whose impulse response is negative, implies that the technology shock has a persistent smoothing effect on wealth distribution. The mathematical reason for this persistence is that there are fewer stable eigenvalues than endogenous state variables which makes the initial non-stochastic steady state of idiosyncratic variables unstable. This reasoning is consistent with our previous finding that the wealth distribution in the steady state is indeterminate. As a consequence, shocks affect the relative wealth of different types which leads to permanent changes in their consumption and saving paths[22].

The technology shock has this smoothing effect on wealth distribution because, due to the temporary nature of labor earnings increase after a technology shock, risk averse agents increase their capital which is the only asset that allows storage of the value and consumption smoothing. Agents whose main source of income is labor earnings, i.e. the poor agents in our benchmark model, enjoy a relatively higher increase in their income[23], such that they can save more compared to other agents as a percentage of their capital holdings. Therefore, the value of impulse responses of type 1 agents are greater than that of type 2 agents.

6.2 Idiosyncratic Shocks and Wealth Distribution

The antagonistic relationship between different types of agents is documented in Figure 5 and Figure 6, which show the impulse responses to type-specific productivity shocks and to a deviation in type-specific capital holdings in benchmark model[24]. One can easily see that a shock which has a positive effect on one type's capital holdings affects the other type's capital holdings negatively and this antagonistic relationship is persistent as shown by the corresponding impulse responses of Gini indices[25]. We provide an intuition for this fact in the following.

The increase in one type's labor productivity drives the wage down. However, the reduction of the wage rate is not enough to offset the positive effect

[22]However, since none of the relevant eigenvalues are greater than one, our model is not explosive such that we can still calculate an equilibrium satisfying the nonlinear equilibrium conditions locally. See Uhlig (1999), p. 46, for more information.

[23]This is obvious if one compares the impulse responses of rate of return on capital, $\hat{R}_t$, and wage per allocated time, $\hat{w}_t + \hat{\theta}_{i,t}$, in Figure 2.

[24]Some impulse responses are omitted from the figures if they are not significantly different from zero, such as *return* in top left panel of Figure 6, or if they tend to ruin the appearance of the corresponding figure, such as *wage* in Figure 5.

[25]Since the aggregate capital turns back to its steady state in the long-run, as it is the case in any neoclassical real business cycle model, the persistence of the impulse response of Gini index indicates that some agents get poorer whereas some others get richer at the end and not that the distribution of wealth changes in a win-win situation. That is why we call it an antagonistic relationship.

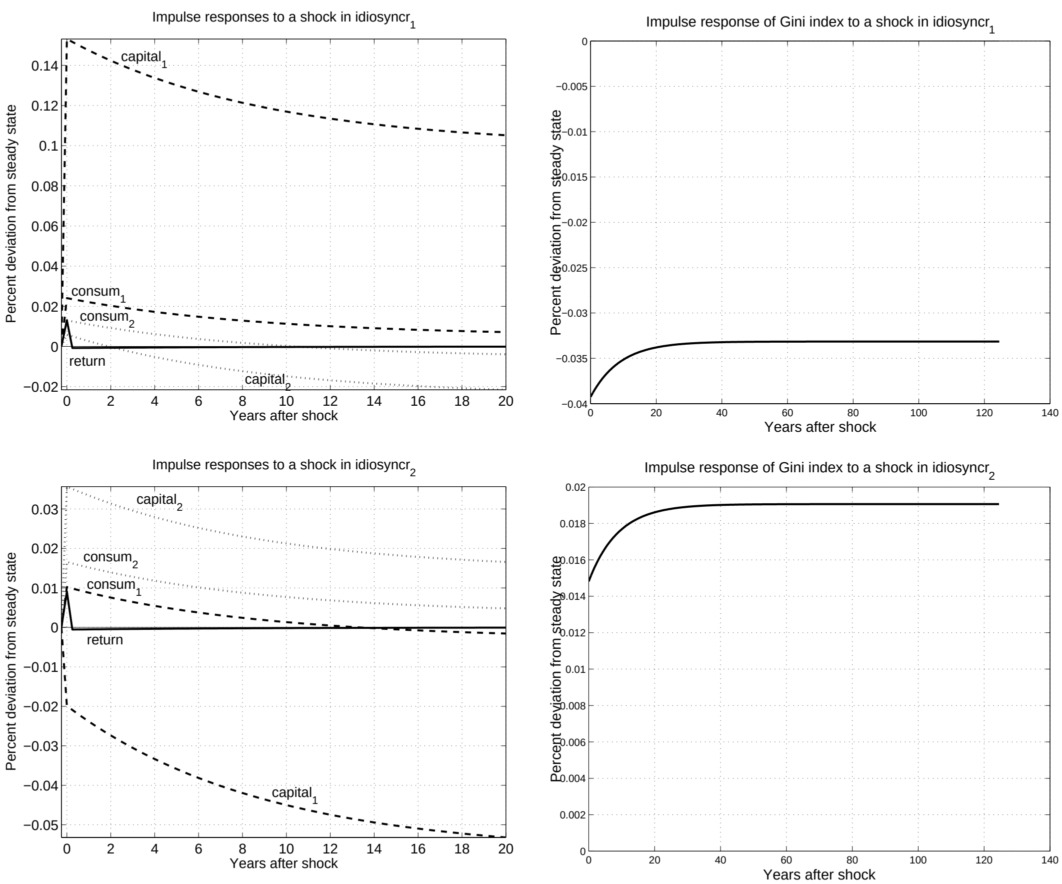

Figure 5: Impulse responses to idiosyncratic productivity shocks in benchmark model

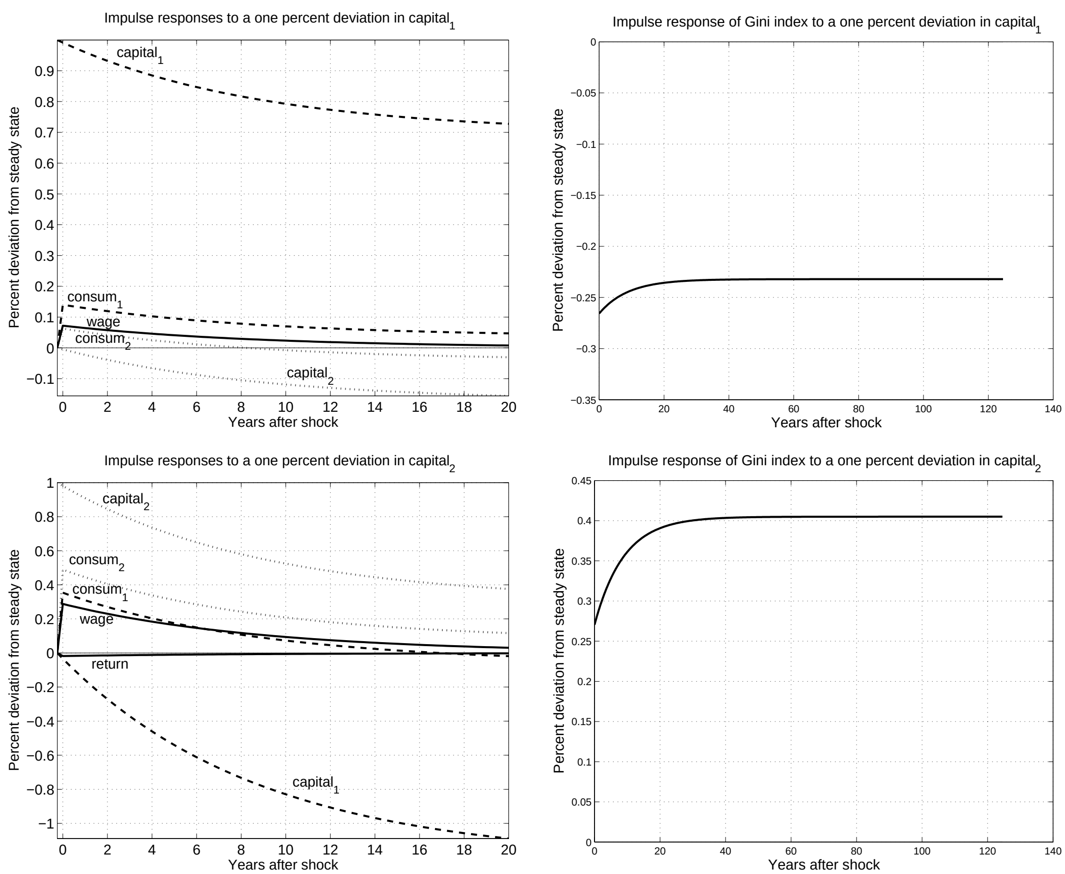

Figure 6: Impulse responses to a 1% deviation in type-specific capital holdings in benchmark model

of the increase in the productivity on the agents that are subject to this shock. Therefore, the benefits from an idiosyncratic increase in productivity is mainly collected by the agents that experience the shock. The positive effect on return on capital, due to the increase in its marginal productivity, does not cover the loss due to the decrease in wage for those who do not experience the same shock. In conclusion, a positive technology shock in a type's productivity benefits this type and harms the other one.

The antagonism in case of capital shocks also stems from the changes in marginal productivities. Any positive change in the amount of capital results in the decrease of the marginal productivity of capital and in the increase of marginal productivity of labor which also effects factor prices, return and wage, in the same way. However, the decrease in return can be offset by the increase in the capital holding of the agent that experiences the shock whereas the agent that is not directly effected by this shock has not this opportunity. In addition to that, the increase in wage is not enough to cover the loss of the latter agent due to the reduction in capital earnings. This results in the change of wealth distribution in favor of the agent that experiences the positive capital shock.

Moreover, Figure 5 and Figure 6 tell us that the effects of this antagonistic relationship on the two types of agents are not symmetric: Poor agents are hurt more than the rich ones in case of shocks that affect their position negatively. The first reason for this asymmetry in case of idiosyncratic productivity shocks is that the rich agents are not affected significantly by a change in their labor income since their main source of income is capital earnings. On the other hand, the poor agents rely mainly on their labor earnings and are therefore greatly affected by a change in their labor income. Another reason is that the amount of capital held by a rich agent is much more higher than the capital held by a poor agent. Therefore, the percentage deviation of a rich agent's capital is not as high as that of a poor agent's capital although the deviations in absolute terms are the same[26].

[26]This becomes more clear if one remembers that the gain of an agent is the loss of another one.

In Figure 6, we see again that the negative effect of this antagonistic relationship on the first type of agents is more severe than that on the second type of agents, i.e. that the negative response of rich agents' capital to a deviation in poor agents' assets is not as strong as the negative response of poor agents' capital to a deviation in rich agents' assets. This has two main reasons. First, a shock to the capital holdings of the first type of agents does not affect the return on capital severely since the capital holdings of first type of agents are much less than that of second type. Thus, the response of rich agents' capital to such a shock is not strong. Exactly the opposite arguments hold for the effect of a shock to second type's capital holdings on first type of agents. Second, the labor earnings, which is the main source of income of poor agents, is positively affected by an increase in the capital holdings of rich agents, whereas the rate of return is negatively affected, obviously due to the changes in marginal productivities. This results in the fact that poor agents prefer to consume more and save less, expecting that their income will remain high in the future due to higher wages. This effect, combined with the decrease in return on capital, gives the poor agents strong incentive to reduce their savings.

6.3 Wealth Distribution and Aggregate Dynamics

Our next result, the independence of the aggregate variables' impulse responses from the distributional properties of wealth and labor productivity in case of an aggregate technology shock, is a very important insight that confirms the results of the studies discussed in previous sections, such as Krusell and Smith (1998). The main reason for this phenomenon is the perfect aggregation of individual quantities which becomes clear if we focus on the log-linearized versions of model equations. The first step to show perfect aggregation includes multiplying both sides of equation (19) with $H_i\bar{c}_i$, summing up the resulting equations over i and dividing both sides of this equation by $\sum_i H_i\bar{c}_i$ to obtain the following aggregate log-linearized Euler equation

$$E_t[\sigma\hat{C}_t - \sigma\hat{C}_{t+1} + \hat{R}_{t+1}] = 0 \tag{39}$$

where $C_t = \sum_i H_i c_{i,t}$ is the aggregate consumption defined in the same way as aggregate capital and labor[27]. The second step is multiplying both sides of equation (20) with H_i, summing up the resulting equations over i and using equations (21) and (22) to obtain the aggregate log-linearized budget constraint.

$$\bar{C}\hat{C}_t + \bar{K}\hat{K}_{t+1} - \bar{R}\bar{K}\hat{R}_t - \bar{R}\bar{K}\hat{K}_t - \bar{w}\bar{L}\hat{w}_t - \bar{w}\bar{L}\hat{L}_t = 0 \tag{40}$$

The equations (39), (40) and (21) to (26), excluding (25), are enough to characterize the stochastic aggregate dynamics of our model in response to an aggregate technology shock[28]. It is important to see that these set of equations are the same as in a model populated by a single representative agent with CRRA utility. Since none of these equations are related with the heterogeneity of agents the distribution of wealth and labor productivity does not matter for the aggregate dynamics. For the same reason, the aggregate dynamics are independent from the distributional properties of the economy also in the case of an exogenous shock to aggregate capital that affects all the individual capital holdings proportionately[29]. A final remark regarding this analysis is that it provides an avenue to a set of models where the distribution of wealth should matter for the aggregate dynamics, which will be discussed in Section 7.2 as a variation of the benchmark model.

Figure 7 is helpful for demonstrating how heterogeneity matters for the dynamics of aggregate capital in response to idiosyncratic productivity shocks. The panels of Figure 7 include plots of the impulse responses of aggregate capital in two different settings. In one of these settings wealth and labor productivity are evenly distributed such that the economy is egalitarian and resembles the representative agent framework whereas the other setting is our benchmark model. Explanation of Figure 7 is straightforward. First of all, it is clear that the impulse responses of aggregate capital in the egalitarian

[27]Equation (39) becomes more clear if one notes that $C_t = \sum_i H_i c_{i,t}$ implies $\bar{C}\hat{C}_t = \sum_i H_i \bar{c}_i \hat{c}_{i,t}$ after log-linearization.

[28]As discussed before, $K_{t+1} = \sum_i H_i k_{i,t}$ and is therefore known at time t. For this reason, there are no expectations involved here.

[29]A 1% increase in both types of capital holdings corresponds to a 1% increase in aggregate capital regardless of the distribution of wealth and labor productivity.

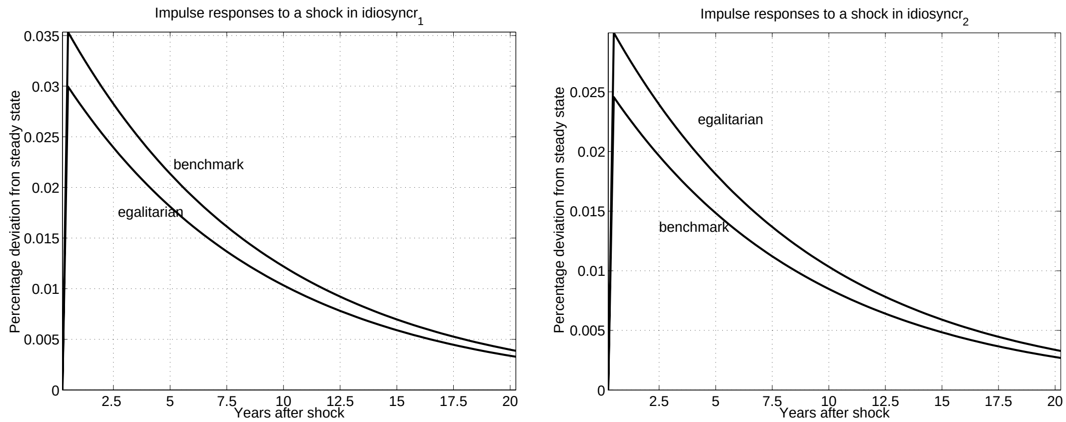

Figure 7: Impulse responses of aggregate capital to idiosyncratic productivity shocks under different distributional settings

society should be the same in case of both idiosyncratic shocks because all agents are identical. So, the difference between the impulse responses of the aggregate capital in our benchmark model is due to the distributional settings where agents of type 1 get a higher fraction of aggregate labor income than the agents of type 2. This issue results in the fact that a (1%) productivity shock affecting only the first type has a greater impact on the economy than a (1%) shock affecting only the second type. Finally, it is noteworthy that if both of the types would experience 1% productivity shocks simultaneously, as if we have a temporary aggregate technology shock, then the impulse responses of aggregate capital in egalitarian and benchmark economies would be the same as a result of our analysis of aggregate dynamics above.

To summarize, we can classify the shocks in two parts. Shocks that affect aggregate variables, such as aggregate technology shocks or exogenous (proportionate) shocks to aggregate capital, cause a balanced expansion of the economy whereas shocks that affect individual variables disproportionately cause an imbalanced expansion. Under CRRA preferences, the aggregate dynamics of the balanced expansion and contraction of is independent from the wealth distribution, whereas distributional properties matter in case of an imbalanced expansion or contraction.

6.4 Importance of the Relative Endowments

In the following, we introduce the concept of *relative endowment*, which is simply the ratio of capital to labor endowment, and show that the implied stochastic steady state of the capital distribution in the long run would be the one that assumes the same relative endowment for all agents under the assumptions that positive technology shocks are more likely than negative ones and that agents have CRRA preferences. For this purpose, we will start with three propositions.

Proposition 6.1 *Given CRRA preferences, the impulse responses of individual capital holdings remain totally the same if the relative endowments of all agents remain the same in two different settings of wealth and income distribution.*

Proof. The only equation which may cause a difference in two different distributional settings is the decentralized budget constrained since the log-linearized Euler equations are the same for all agents. If one divides both sides of the log-linearized budget constraint, equation (20), by the individual labor endowment, $\bar{\theta}_i\tilde{l}$, and calls the relative endowment of agents $\bar{\phi}_i = \bar{k}_i/(\bar{\theta}_i\tilde{l})$ one can easily see that the equation becomes

$$\{(\bar{R}-1)\bar{\phi}_i + \bar{w}\}\hat{c}_{i,t} + \bar{\phi}_i\hat{k}_{i,t} - \bar{R}\bar{\phi}_i\hat{R}_t - \bar{R}\bar{\phi}_i\hat{k}_{i,t-1} - \bar{w}\hat{w}_t - \bar{w}\hat{\theta}_{i,t} = 0 \quad (41)$$

since $\bar{c}_i = (\bar{R}-1)\bar{k}_i + \bar{w}\tilde{l}\bar{\theta}_i$ by equation (12). Therefore, one gets exactly the same set of log-linearized equations for different wealth and labor income distributions, as long as the distribution of the relative endowments, $\bar{\phi}_i$, remain the same. It is obvious that the same set of equations should lead to the same dynamics.

Proposition 6.2 *Both types of agents have the same impulse responses if their relative endowments are the same.*

Proof. The proof is straightforward when one considers equation (41). If both types of agents have the same relative endowment, i.e. $\bar{\phi}_i = \bar{\phi}_j \; \forall i \neq j$, one can easily see that the set of equations for both types of agents, including the budget constraint, are identical. In this case, the agents' responses are also identical. Obviously, this claim holds as long as the agents have the same CRRA preferences such that the Euler equations are the same for all agents[30].

Proposition 6.3 *Positive technology shocks reduce wealth heterogeneity until relative endowments of the agents become equalized. The impulse responses of the agents follow the same path thereafter.*

Proof. It is easy to see this by analytical thinking after combining the result presented in Section 6.1 with the first two propositions that show how relative endowments determine the individual dynamics. However, it may be a good idea to visualize this result. In Figure 8, we see the path of agents' relative endowments after subsequent positive technology shocks[31]. It is clear that the relative endowments of both types converge to the same value which is equal to aggregate capital-labor ratio[32].

The third proposition leads to the immediate result stated before: We can conclude that the stochastic steady state in the long run would be the one that implies the same relative endowment for all agents whereas the non-stochastic steady state may assume any distributional property for wealth and labor productivity. Since all agents have the same relative endowment in this new stochastic equilibrium, the log-deviations of individual variables co-move with each other and with log-deviations of aggregate variables such that the wealth distribution is stable. Obviously, this result is contingent on the conditions that positive technology shocks are more likely than negative ones and that agents have CRRA preferences.

The implications of these results are very important for several reasons. First of all, our results confirm previous studies like Krusell and Smith (1998)

[30]Under different preference settings like constant absolute risk tolerance (CART), the Euler equations will depend on the prosperity of the agents. We will deal with such type of preferences in the next section.

[31]We obtain this figure by updating the steady state wealth distribution in the economy using the long run responses of individual capital holdings after each positive technology shock.

[32]Due to the same equalizing effect, the wealth gap between wealth-rich and wealth-poor agents may increase if the relative endowment (capital-labor ratio) of rich agents is lower than that of poor agents. In that case, the wealth of rich agents increase whereas that of poor agents decrease since the main source of income of rich agents would be labor earnings whereas that of poor ones would be return on capital.

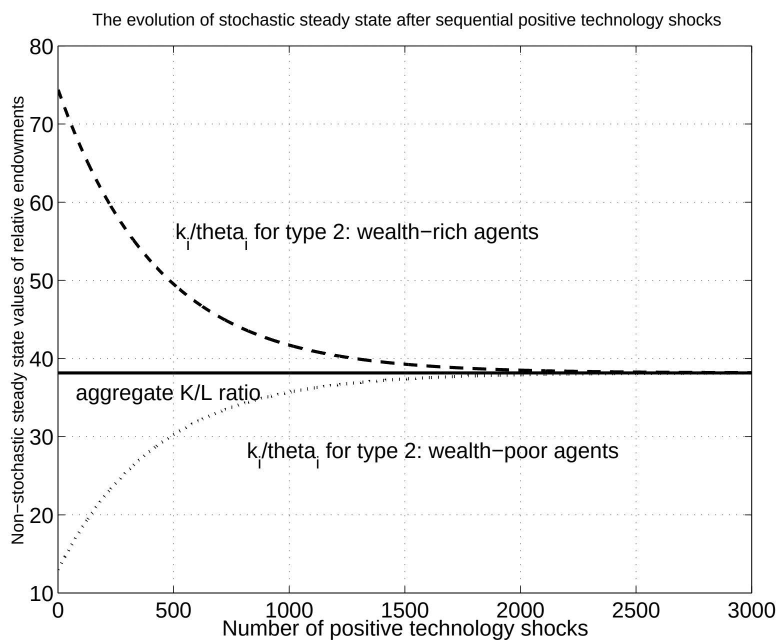

Figure 8: Convergence of relative endowments after positive technology shocks

by showing that the aggregate dynamics in response to aggregate shocks are independent from distributional properties of the model under CRRA preferences. Moreover, we have provided evidence for the antagonistic relationship among different classes of agents and shown that heterogeneity matters for aggregate dynamics in response to idiosyncratic shocks. Finally, our results about the dynamics of wealth heterogeneity in case of a technology shock provide an important intuition why a model with CRRA preferences may fail to generate wealth concentration of the magnitude observed in the data.

7 Variations

Although our benchmark model is well suited to understand the basic properties of the economy, it is necessary to look at further models to answer other questions that are part of our goal. For this purpose, we will introduce three variations of our benchmark model, which include multiple types of agents with CRRA preferences, two-types framework with concave absolute risk tolerance (CART) preferences and many types with CART preferences. Each of these variations will mainly serve for the purpose of answering two questions: Does the wealth distribution matter for the decisions of individuals, i.e. for the impulse responses of individual capital holdings, beyond its mean? Can we think of a model where the distributional properties matter for the aggregate dynamics in response to aggregate shocks? Finally, we will provide cross-correlations of aggregate variables in different settings for deeper understanding of the effects of preferences and persistence of idiosyncratic shocks on the model dynamics.

7.1 Many Types of Agents

The benchmark model provides us with important insights regarding the dynamics of the economy populated by heterogeneous agents. However, one of the main motivating questions remain unanswered: Does wealth heterogeneity matter for the decisions of individuals? This part makes use of the benchmark model with multiple types to deal with this question[33]. However, we should first have some basic knowledge about how our solution method,

[33] Our benchmark model becomes more realistic as we increase the number of types of agents. In the limiting case we would have one type for each single agent in the economy.

the method of undetermined coefficients, work.

Our model economy, like any other neoclassical growth model, consists of three groups of variables among which are endogenous (predetermined) state variables that are in our case the capital holdings of individuals at the beginning of a period; exogenous state variables, which are technology and labor productivity and finally other endogenous variables that include all other variables being part of the model dynamics. The method of undetermined coefficients tries to solve the system of log-linearized equations assuming that the model is describable by recursive laws of motion that are linear in log-deviations of state variables. In this context, we can write the recursive law of motion for the log-deviations of individual capital holdings as

$$\hat{k}_{i,t} = \sum_{j=1}^{n} \tau_{ij}\hat{k}_{j,t-1} + \sum_{j=1}^{n} \varsigma_{ij}\hat{u}_{j,t} + \varsigma_{zi}\hat{z}_t \tag{42}$$

where τ_{ij}, which can be obtained via our solution method, determines how a change in type j's capital holdings affects type i's decision about its assets to be transferred to the next period. The number of types is denoted by n whereas the remaining coefficients denote the contributions of individual labor productivity shocks and aggregate technology shocks. The deterministic part of this equation can also be expressed as

$$\hat{k}_{i,t} = \psi_i\hat{k}_{i,t-1} + \mu_i\sum_{j=1}^{n} \omega_{ij}\hat{k}_{j,t-1} \tag{43}$$

where $\psi_i + \mu_i\omega_{ii} = \tau_{ii}$ and $\mu_i\omega_{ij} = \tau_{ij}$. It should be clear that ω_{ij} should be proportional to the wealth share of type j, $H_j\bar{k}_j/\bar{K}$, if only the mean of the capital distribution matters for the individual decisions[34]. This is what we are going to check using the data from Budría et al.(2002) as tabulated in Table 1 which we have modified slightly[35]. Table 3 presents the values of

[34]Considering equation (21), this proportionality makes the second term on the right-hand side of equation (43) equal to the log-deviation of aggregate capital. Moreover, it is obvious that ω_{ii} is indeterminate and that a single value of $\tau_{ij} = \mu_i\omega_{ij}$ is not informative for our purpose. Therefore, we need more than two types of agents to analyze whether wealth distribution matters for individual decisions.

[35]So, we have eight types of agents in this framework. We have set the wealth share of poorest quintile to 0.001%, instead of -0.2%, to keep the corresponding agents from

	j=1	j=2	j=3	j=4	j=5	j=6	j=7	j=8
i=1 (x10^3)	100.14	-0.0566	-0.0566	-0.0566	-0.0566	-0.0566	-0.0566	-0.0566
i=2	-0.0948	71.4773	-0.0948	-0.0948	-0.0948	-0.0948	-0.0948	-0.0948
i=3	-0.0429	-0.0429	18.8630	-0.0429	-0.0429	-0.0429	-0.0429	-0.0429
i=4	-0.0319	-0.0319	-0.0319	7.7356	-0.0319	-0.0319	-0.0319	-0.0319
i=5	-0.0278	-0.0278	-0.0278	-0.0278	7.8004	-0.0278	-0.0278	-0.0278
i=6	-0.0252	-0.0252	-0.0252	-0.0252	-0.0252	8.1213	-0.0252	-0.0252
i=7	-0.0241	-0.0241	-0.0241	-0.0241	-0.0241	-0.0241	4.1336	-0.0241
i=8	-0.0220	-0.0220	-0.0220	-0.0220	-0.0220	-0.0220	-0.0220	3.1691

Table 3: The ratios $\tau_{ij}/(H_j\bar{k_j}/\bar{K})$ for the benchmark model with multiple types of agents

$\tau_{ij}/(H_j\bar{k_j}/\bar{K})$ since the equality of this ratio for different values of j, given i s. t. $i \neq j$, implies that $\omega_{ij} \propto H_j\bar{k_j}/\bar{K}$ holds for each j, given i.

Table 3 shows the validity of $\omega_{ij} \propto H_j\bar{k_j}/\bar{K}$. So, we can assert that only the first moment of the capital distribution matters for individual decisions. The intuition for this insight is simple. The income of an agent is defined by the movements of aggregate variables that determine the factor prices and by her individual endowment. Therefore, aggregate (mean) capital and one's own capital holdings are the only state variables that have an effect on the non-stochastic dynamics of one's capital holdings.

Finally, Figures 9, 10 and 11 show the impulse responses of Gini index to different types of shocks to confirm that the same dynamics of wealth inequality holds in multi-types framework. Figure 9 illustrates the impulse response of Gini coefficient to an aggregate technology shock which has the similar U-shape as in our benchmark model. Figures 10 and 11 only refer to the shocks that affect the labor productivity and capital holdings of the richest and poorest agents. Although the shapes of the impulse responses in these figures are similar to their counterparts in the benchmark model the magnitudes are different because each type in this framework has less amount

running a Ponzi scheme remembering that our endogenous borrowing constraint depends on the non-negativity of capital holdings in the non-stochastic steady state. Then the sum of the shares are normalized to 1. This change has been kept for all the variations that involve more than two types of agents.

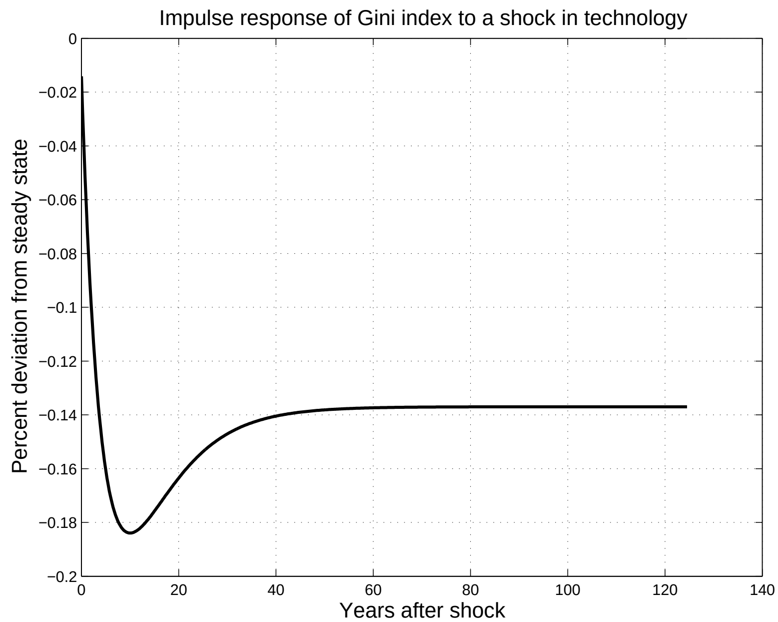

Figure 9: Impulse response of Gini index to an aggregate technology shock

of capital and labor productivity compared to the types in the benchmark model. This limits the effects of corresponding shocks.

7.2 Changing the Preferences

In the previous section, we have seen that the wealth distribution does not matter for the aggregate dynamics of our benchmark model because the individual equations can be added up to obtain perfect aggregation. Since the perfect aggregation of budget constraints is inevitable, one should focus on the Euler equations, thus on preferences, to obtain a model where heterogeneity matters for the economy as a whole. This section introduces concave absolute risk tolerance (CART) for this purpose[36].

The absolute risk tolerance is represented as $ART = -u'(c)/u''(c)$, which is the inverse of absolute risk aversion. A simple form for concave absolute risk tolerance can be given as $ART = c^{\eta}/\sigma$, $0 < \eta < 1$, which implies the

[36]Evidence for concave absolute risk tolerance can be found in Guiso and Paiella (2001) who use data from the Bank of Italy's Survey of Household Income and Wealth.

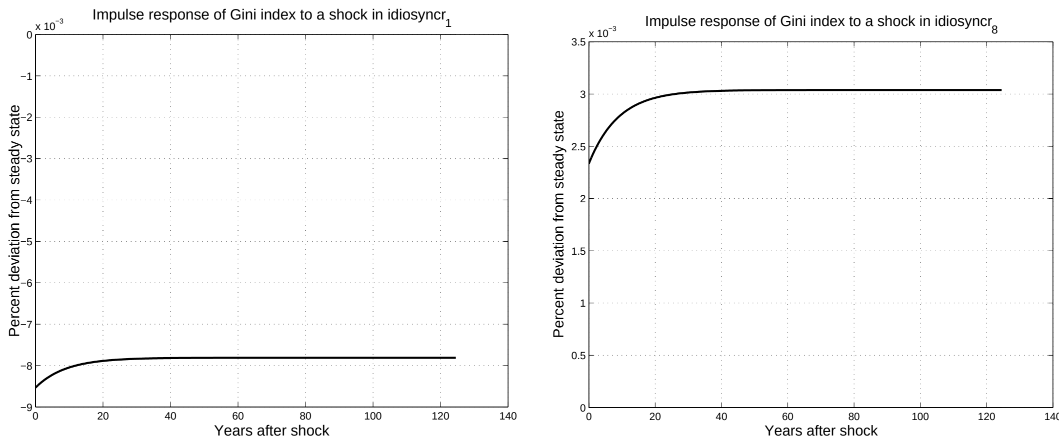

Figure 10: Impulse response of Gini index to a shock in the idiosyncratic productivity of the poorest (left panel) and richest (right panel) agents in benchmark model with many types of agents

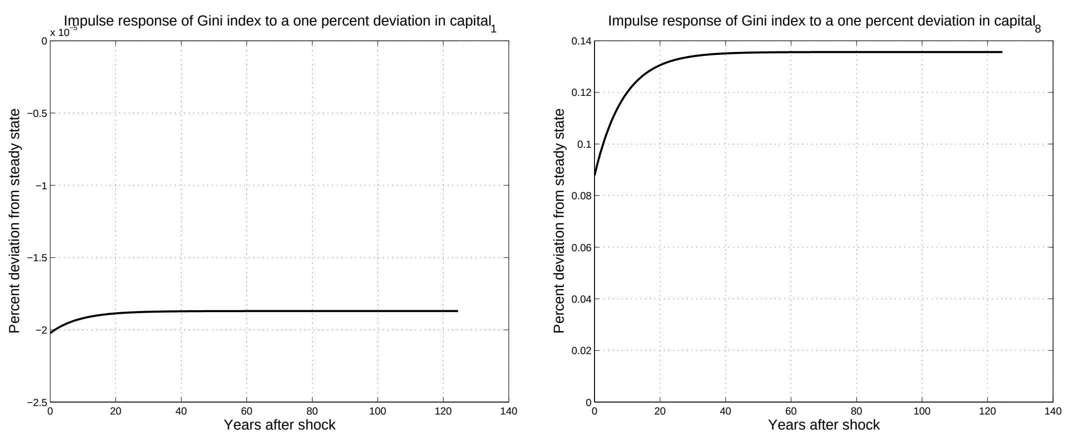

Figure 11: Impulse response of Gini index to a one percent deviation in the capital holdings of the poorest (left panel) and richest (right panel) agents in benchmark model with many types of agents

following form for the utility function adapted from Hatchondo (2003).

$$u(c) = \int_a^c \exp(-\sigma \frac{x^{1-\eta}}{1-\eta})dx \tag{44}$$

The introduction of concave absolute risk tolerance to our model only results in a change of the Euler equation. The new set of log-linearized equations can simply be obtained by replacing σ with $\sigma * \bar{c}^{1-\eta}$ in equation (19) and our new log-linearized Euler equation looks like

$$E_t[\sigma \bar{c}_i^{1-\eta} \hat{c}_{i,t} - \sigma \bar{c}_i^{1-\eta} \hat{c}_{i,t+1} + \hat{R}_{t+1}] = 0. \tag{45}$$

It is obvious that this functional form is quite general since setting $\eta = 0$ results in a constant absolute risk aversion (CARA) utility function whereas setting $\eta = 1$ gives us our original CRRA utility function. The value of η used in the following analysis ranges from 0.1 to 0.9. The values of other parameters are exactly the same as in the benchmark model[37].

Figure 12 compares the impulse responses of aggregate capital in our new economy with that in corresponding egalitarian economy[38] for different values of η. It is obvious that the difference in the dynamics of aggregate capital becomes more visible for higher deviations of η from 0 and 1 although it is still small. The reason is that the aggregation, which is perfect if $\eta = 0$ or $\eta = 1$, becomes the less perfect the more we move away from CARA and CRRA assumptions. To see this more precisely, one should first multiply both sides of equation (45) with $H_i \bar{c}^{\eta}$ and sum the outcome over i to get

$$E_t[\sigma \bar{C} \hat{C}_t - \sigma \bar{C} \hat{C}_{t+1} + \sum_i H_i \bar{c}_i{}^{\eta} \hat{R}_{t+1}] = 0. \tag{46}$$

After dividing both sides of this equation by $\bar{C}^{\eta}$ we end up with the pseudo-Euler equation for aggregate consumption

[37] In case of CARA ($\eta = 0$), one would again see that perfect aggregation holds after multiplying both sides of equation (45) by H_i and summing the resulting equations over i. The outcome will be nothing but the log-linearized Euler equation of a representative agent with CARA preferences regardless of the distribution of wealth and labor productivity. So, as in case of CRRA preferences, the distribution of production factors does not matter for aggregate variables reacting to aggregate shocks.

[38] It is useful to remember that every agent gets the same fraction of wealth and labor earnings in the egalitarian setting such that the impulse response of aggregate capital to an aggregate shock is the same as in the corresponding representative agent framework.

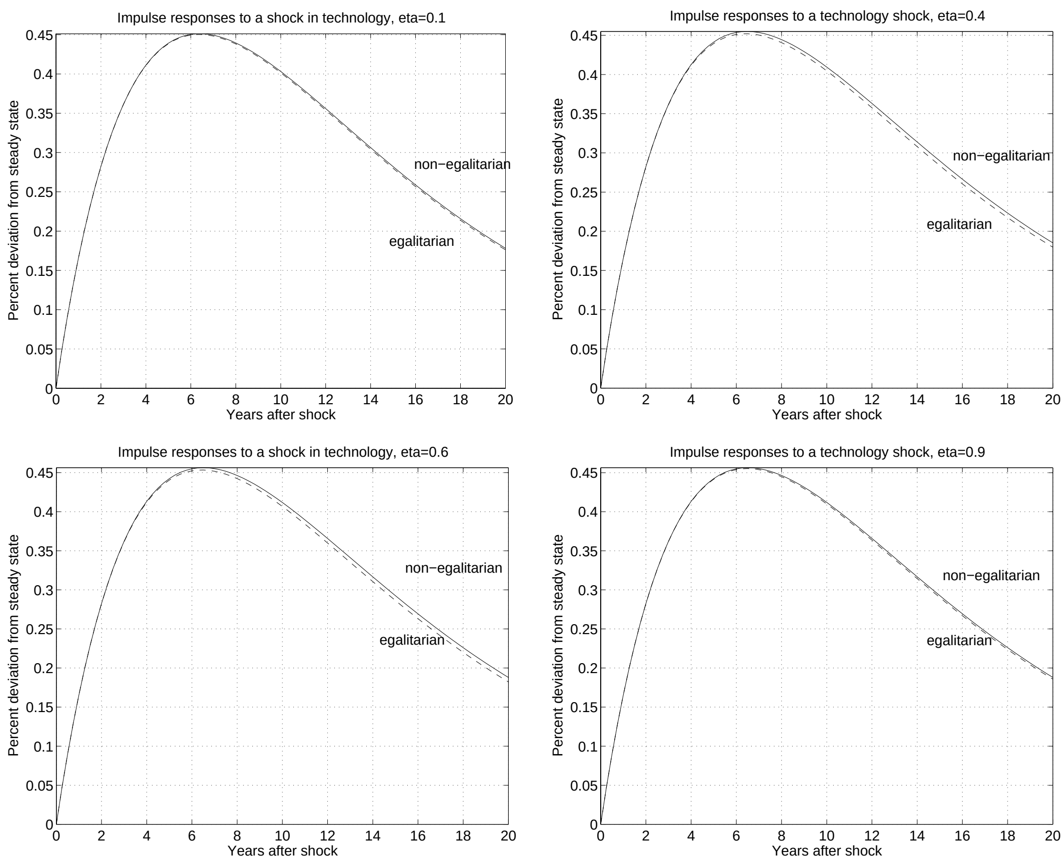

Figure 12: Impulse responses of aggregate capital to aggregate technology shocks under CART preferences, $\eta \in \{0.1, 0.4, 0.6, 0.9\}$

$$E_t[\sigma \bar{C}^{1-\eta}\hat{C}_t - \sigma \bar{C}^{1-\eta}\hat{C}_{t+1} + \frac{\sum_i H_i \bar{c}_i{}^{\eta}}{\bar{C}^{\eta}}\hat{R}_{t+1}] = 0 \tag{47}$$

which is similar to equation (45) except the last term on the left. The value of the multiplier in the last term of equation (47) depends on the distributional properties of steady state consumption and therefore of wealth and labor productivity. This dependence becomes higher if the deviation of η from 0 and 1 gets higher whereas for $\eta = 0$ or $\eta = 1$ or for $\bar{c}_i = \bar{c}_j \ \forall i, j$, this multiplier will become unity[39]. Consequently, *in case of strictly concave absolute risk tolerance, distribution of wealth and labor productivity affects the behavior of aggregate variables in response to macroeconomic shocks and*

[39]Remember that in egalitarian economy $\bar{c}_i = \bar{c}_j = \bar{C}$ and $\sum\limits_i H_i = 1$ as usual.

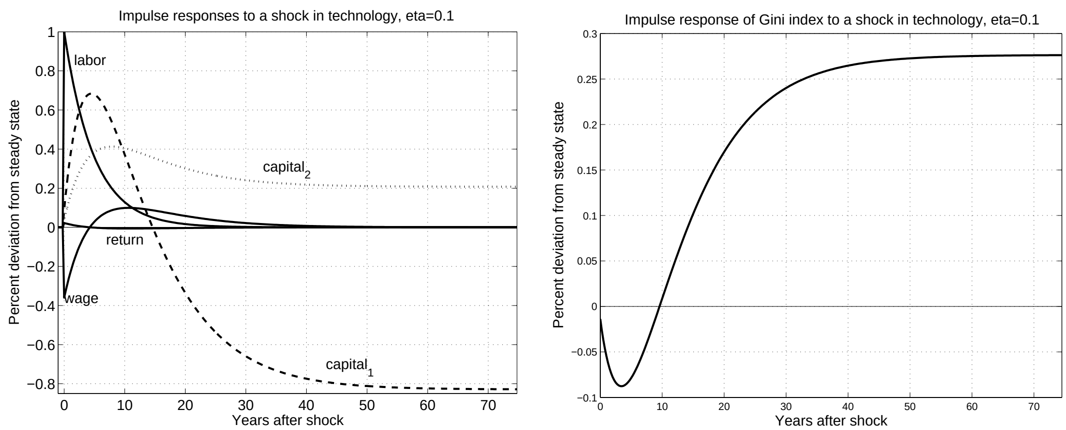

Figure 13: Long-run dynamics of wealth heterogeneity in response to an aggregate technology shock in the benchmark model with CART preferences, $\eta = 0.1$

one cannot simply reduce the aggregate dynamics of the economy to the corresponding representative agent framework.

Figure 12 reveals another interesting phenomenon: The impulse response of aggregate capital in the heterogeneous setting is greater than that in the egalitarian setting. In order to find the intuition behind this insight, we multiply both sides of equation (47) with $A = (\frac{\sum_i H_i \bar{c_i}^\eta}{\bar{C}^\eta})^{-1} \geq 1$ to obtain

$$E_t[\sigma^* \bar{C}^{1-\eta}\hat{C}_t - \sigma^* \bar{C}^{1-\eta}\hat{C}_{t+1} + \hat{R}_{t+1}] = 0 \qquad (48)$$

which is similar to the log-linearized Euler equation of a representative agent having CART utility with $\sigma^* = \sigma A$. If $0 < \eta < 1$ and there is heterogeneity in the model, then $A > 1$, whereas $A = 1$ if the economy is egalitarian[40]. Therefore, the pseudo-representative agent of a heterogeneous economy is more risk-averse than the pseudo-representative agent of an egalitarian economy. So, the aggregate consumption path in heterogeneous economy is smoother, and hence the aggregate savings path lies higher, than those in egalitarian economy.

One may wonder how the dynamics of wealth distribution changes in this new framework in response to aggregate and idiosyncratic productivity

[40] $A < 1$ is not possible since $\bar{c}_i^\eta$ is a concave function of $\bar{c}_i$.

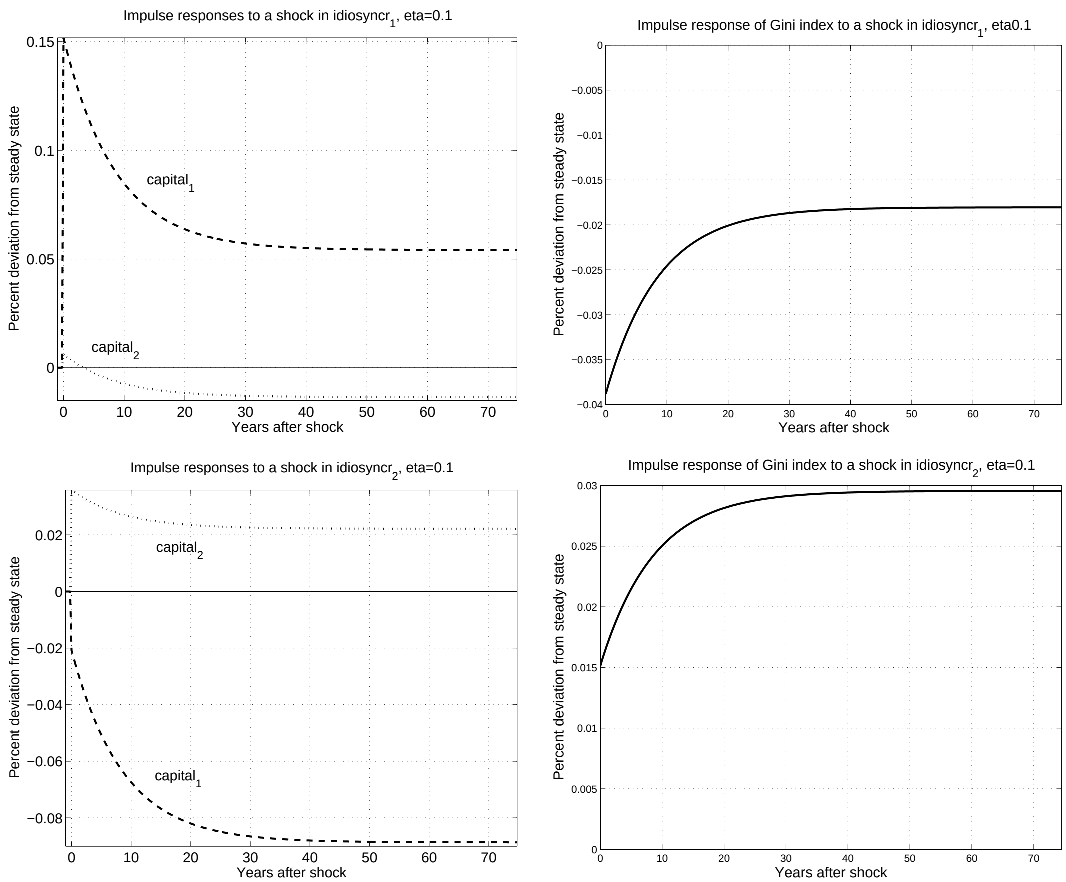

Figure 14: Long-run dynamics of wealth heterogeneity in response to idiosyncratic productivity shocks in the benchmark model with CART preferences, $\eta = 0.1$

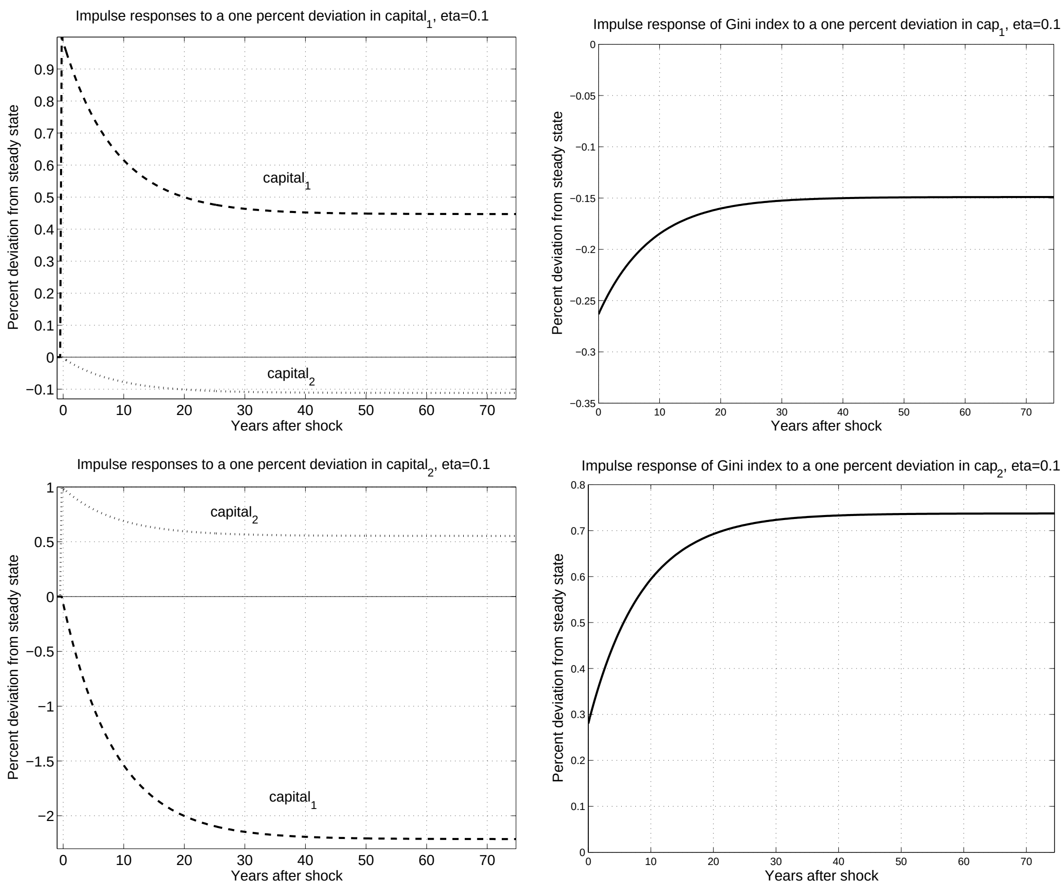

Figure 15: Long-run dynamics of wealth heterogeneity in response to a deviation in individual capital holdings in the benchmark model with CART preferences, $\eta = 0.1$

shocks. Figures 13, 14 and 15 provide the answer to this question. We have set $\eta = 0.1$ since this value is in the range reported in Guiso and Paiella (2001).

The wealth gap between the two types of agents gets smaller at the initial periods after a positive technology shock as indicated by the negative response of Gini coefficient in Figure 13. However, the long-run impulse responses tell us that the wealth distribution changes at the end in favor of rich agents. The reason for this behavior of impulse responses is that we have two competing forces in this framework. One of these forces is the one discussed in Section 6.1, namely that an agent whose main source of income is labor earnings, in our context a poor agent, enjoys higher increase in her income as a percentage of her total income since the rate of return changes only slightly compared to the wage per allocated time[41]. This force tries to change the wealth distribution in favor of poor agents. Second, the risk aversion of rich agents relative to the risk aversion of poor agents has increased due to CART preferences. To see this more clearly, one should remember that the ratio of absolute risk aversion of rich agents to that of poor agents is

$$\frac{ARA_{rich}}{ARA_{poor}} = \left(\frac{\bar{c}_{poor}}{\bar{c}_{rich}}\right)^{\eta} \tag{49}$$

where the ratio $\bar{c}_{poor}/\bar{c}_{rich}$ is less than one and independent of preferences. Therefore, the value of expression (49) increases with decreasing value of η. Knowing that the value of η for a CRRA utility function is greater than the value of η for a strictly CART utility, we can say that the rich agents in this new framework are relatively more risk averse than their analogues in our benchmark model. For this reason, the new preferences of rich agents give them relatively higher incentive to save than before whereas the opposite holds for poor agents. This builds a force that changes the wealth distribution in favor of rich agents. In conclusion, we may say that in our new framework, the first force is more effective than the second in the initial periods whereas the second one overcomes the first one thereafter.

[41] The reader will recall that the wage per allocated time is $w_t\theta_{it}$ whose impulse response can be found by adding the impulse responses of the involving variables.

	j=1	j=2	j=3	j=4	j=5	j=6	j=7	j=8
i=1 (x10^4)	9.9806	-0.0395	-0.0395	-0.0395	-0.0395	-0.0395	-0.0395	-0.0395
i=2	-0.2458	71.3264	-0.2458	-0.2458	-0.2458	-0.2458	-0.2458	-0.2458
i=3	-0.0724	-0.0724	18.8335	-0.0724	-0.0724	-0.0724	-0.0724	-0.0724
i=4	-0.0354	-0.0354	-0.0354	7.7321	-0.0354	-0.0354	-0.0354	-0.0354
i=5	-0.0230	-0.0230	-0.0230	-0.0230	7.8052	-0.0230	-0.0230	-0.0230
i=6	-0.0181	-0.0181	-0.0181	-0.0181	-0.0181	8.1284	-0.0181	-0.0181
i=7	-0.0133	-0.0133	-0.0133	-0.0133	-0.0133	-0.0133	4.1444	-0.0133
i=8	-0.0138	-0.0138	-0.0138	-0.0138	-0.0138	-0.0138	-0.0138	3.1773

Table 4: The ratios $\tau_{ij}/(H_j\bar{k_j}/\bar{K})$ for the CART model with multiple types of agents, $\eta = 0.1$

We see that the second force described above is also in charge in case of idiosyncratic shocks to capital and labor productivity. The shapes of the impulse responses in Figures 14 and 14 are similar to their counterparts in Figures 5 and 6. However, the comparison of the impulse responses of Gini coefficient in both frameworks shows that the impulse responses in CART framework ends up in higher levels than those in CRRA framework. Therefore, the change of the wealth distribution in the long-run is more in favor of rich agents if agents have CART preferences. The driving force behind this phenomenon is again the increase in the risk aversion of rich agents relative to the risk aversion of poor agents which gives the rich agents higher incentive to save compared to the poor agents.

7.3 Many Types of Agents with CART Preferences

We refer again to the multiple types of agents framework to analyze whether the distributional properties of wealth, beyond its first moment, matter for the individual decisions in case of CART preferences. The intuition provided in Section 7.1, which is independent of the functional form of the preferences, points to the idea that the same insight should also be valid here. So, one should expect that the ratios $\tau_{ij}/(H_j\bar{k_j}/\bar{K})$ should be the same for all js, given i s.t. $i \neq j$, indicating that agents' decisions are solely dependent on the first moment of the distribution of capital. Table 4 and 5 confirm this intuition providing evidence that also agents with CART preferences care

	j=1	j=2	j=3	j=4	j=5	j=6	j=7	j=8
i=1 (x10^4)	9.9950	-0.0251	-0.0251	-0.0251	-0.0251	-0.0251	-0.0251	-0.0251
i=2	-0.1960	71.3761	-0.1960	-0.1960	-0.1960	-0.1960	-0.1960	-0.1960
i=3	-0.0639	-0.0639	18.8420	-0.0639	-0.0639	-0.0639	-0.0639	-0.0639
i=4	-0.0353	-0.0353	-0.0353	7.7322	-0.0353	-0.0353	-0.0353	-0.0353
i=5	-0.0251	-0.0251	-0.0251	-0.0251	7.8031	-0.0251	-0.0251	-0.0251
i=6	-0.0204	-0.0204	-0.0204	-0.0204	-0.0204	8.1261	-0.0204	-0.0204
i=7	-0.0158	-0.0158	-0.0158	-0.0158	-0.0158	-0.0158	4.1419	-0.0158
i=8	-0.0149	-0.0149	-0.0149	-0.0149	-0.0149	-0.0149	-0.0149	3.1762

Table 5: The ratios $\tau_{ij}/(H_j\bar{k}_j/\bar{K})$ for the CART model with multiple types of agents, $\eta = 0.4$

only about the mean of the wealth distribution[42].

7.4 Business Cycle Implications

Until now, we have assumed that the idiosyncratic shocks are of temporary nature since introducing persistence would not change our main insights in the previous sections. Nevertheless, one can anticipate that persistence of idiosyncratic shocks will turn out to be important for cyclical features of aggregate variables. Therefore, this section aims to provide information about how different preference settings and persistence of idiosyncratic shocks affect business cycle properties of macroeconomic variables such that we may obtain a better understanding of the effects of introducing heterogeneity to the model. For this reason, we have extended our economy such that it is inhabited by 30 agents divided into five wealth groups. Each wealth group corresponds to a different quintile of the economy as summarized in Table 1 and includes equal number of agents. Moreover, agents belonging to the same group have the same endowments and the idiosyncratic productivity process of each agent is independent. In that sense, this modification is more realistic compared to the analysis conducted in previous sections. Finally, the aggregate output and consumption are now officially introduced to the model framework. These changes are brought to life via following equations

[42]The result implied by Tables 4 and 5 is robust for other values of η and that is why we only provide two tables.

$$\hat{y}_t - \alpha\hat{K}_t - (1-\alpha)\hat{L}_t = 0 \tag{50}$$

$$\bar{C}\hat{C}_t - \sum_i H_i\bar{c}_i\hat{c}_{i,t} = 0 \tag{51}$$

$$u_{i,t} = \tilde{\rho}u_{i,t-1} + \epsilon_{i,t} \tag{52}$$

where $\tilde{\rho}$ is the persistence parameter for idiosyncratic shocks and y and C are the aggregate output and consumption respectively[43]. The standard deviation of idiosyncratic shocks are selected in such a way that the unconditional variance of the mean of idiosyncratic productivity processes is equal to the unconditional variance of aggregate technology. The details regarding the calculation of standard deviations can be found in Section B of Appendix.

Tables 6 to 10 here and 12 to 26 in Appendix C provide the cross-correlations of aggregate variables with aggregate output for different levels of concavity of risk tolerance and for different persistence levels of idiosyncratic shocks[44]. Moreover, the corresponding moments of aggregate variables in representative agent framework are provided in Tables 11 and 27 to 29. All moments are obtained via Toolkit using the frequency domain based calculations after filtering the series with Hodrick and Prescott (HP) filter[45]. We have selected the value of the smoothing parameter for HP-filter as 1600 since we assume that each period is a quarter.

If one compares the tables for different preferences for a given value of $\tilde{\rho}$ one observes that the difference between cross-correlations is less than 0.01 in absolute value. Therefore, we provide here only the tables for the CRRA preferences ($\eta = 1$) and the rest in Appendix C. This fact also leads us to the conclusion that preferences do not matter significantly for the effects of heterogeneity on the business cycle properties of HP-filtered macroeconomic

[43]The reader will remember that $y_t = K_t^\alpha L_t^{1-\alpha}$ and $C_t = \sum_i H_i c_{i,t}$. The latter equation implies $\bar{C} = \sum_i H_i\bar{c}_i$ for its steady state.

[44]Since the equilibrium of aggregate variables is stable, this is a legitimate approach. Tables presenting the moments of aggregate variables are slightly modified versions of Toolkit's output. This modification has been discussed in Section A of Appendix.

[45]Details regarding the HP-filter can be found in Hodrick and Prescott (1997). For information about the frequency domain representation and Fourier transform of the HP filter, see King and Rebelo (1993).

series and one should rather focus on persistence of idiosyncratic shocks.

The most obvious effect of introducing persistence of idiosyncratic shocks is the increase in the cross-correlations of aggregate variables. Knowing that the main source of variability in our model is aggregate and idiosyncratic productivity shocks, this is not a counterintuitive result. If autocorrelation of the idiosyncratic shocks is lower than that of the aggregate shock, there will be a lower dependence of total productivity, which is composed of both type of shocks, on its past values such that the cross-correlations are lower than their counterparts in the representative agent framework. As $\tilde{\rho}$ increases the dependence of total productivity on its past values gets higher which directly effects the autocorrelation of output in a positive way since productivity is the main driving force in the economy. This also increases the degree of co-movements of aggregate capital and consumption with aggregate output since agents' behavior tend to be more persistent with increasing persistence of own and total productivity.

Comparing Tables 10 and 11 reveals the information that the values of the cross-correlations in the representative agent framework is exactly the same as in the heterogeneous agents framework if the autocorrelation of the idiosyncratic shocks matches that of the aggregate technology shock. The reason is that the autocorrelation of total productivity converges to ρ as the value of $\tilde{\rho}$ gets closer to the value of ρ. To see this more clearly one should first remember that the aggregate labor in efficiency units is our proxy for total productivity[46]. So, combining equations (25), (26), (52) and $\rho = \tilde{\rho}$ leads to

$$\begin{aligned}\hat{\theta}_{i,t} &= \rho\hat{z}_{t-1} + \rho u_{i,t-1} + v_t + \epsilon_{i,t} \\ &= \rho\hat{\theta}_{i,t-1} + m_{i,t} \qquad (53)\end{aligned}$$

where $m_{i,t} = v_t + \epsilon_{i,t}$. Using equation (53) in equation (22) one may obtain the following expression for aggregate labor

$$\bar{L}\hat{L}_t = \tilde{l}\sum_i H_i\bar{\theta}_i(\rho\hat{\theta}_{i,t-1} + m_{i,t})$$

[46] This also explains why aggregate labor's cross-correlations with output are similar to the autocorrelation of output if one considers the variation in total productivity as the main source of variability in output.

		Cross Correlations with output, corr(X_{t+j}, y_t) (HP-filtered series)								
	σ_x %	$t-4$	$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$	$t+4$
L	2.65	-0.03	-0.01	0.01	0.04	1.00	0.00	-0.02	-0.04	-0.05
K	0.25	-0.24	-0.24	-0.24	-0.22	0.40	0.38	0.35	0.31	0.27
y	1.68	-0.04	-0.02	0.00	0.03	1.00	0.03	0.00	-0.02	-0.04
C	0.24	-0.09	-0.06	0.00	0.07	0.49	0.40	0.31	0.24	0.17

Table 6: Frequency domain based calculation of moments, $\eta = 1, \tilde{\rho} = 0$

		Cross Correlations with output, corr(X_{t+j}, y_t) (HP-filtered series)								
	σ_x %	$t-4$	$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$	$t+4$
L	2.58	-0.06	-0.03	0.06	0.29	1.00	0.24	0.00	-0.08	-0.10
K	0.29	-0.32	-0.33	-0.30	-0.15	0.37	0.48	0.46	0.41	0.34
y	1.63	-0.08	-0.05	0.04	0.27	1.00	0.27	0.04	-0.05	-0.08
C	0.25	-0.16	-0.13	-0.06	0.11	0.54	0.48	0.39	0.30	0.22

Table 7: Frequency domain based calculation of moments, $\eta = 1, \tilde{\rho} = 0.3$

which may be manipulated to get

$$\begin{aligned} \bar{L}\hat{L}_t &= \rho\tilde{l}\sum_i H_i\bar{\theta}_i\hat{\theta}_{i,t-1} + \tilde{l}\sum_i H_i\bar{\theta}_i m_{i,t} \\ &= \rho\bar{L}\hat{L}_{t-1} + \tilde{l}\sum_i H_i\bar{\theta}_i m_{i,t}. \end{aligned} \tag{54}$$

Dividing both sides of equation (54) leads us to a total productivity process whose autocorrelation is the same as the aggregate technology and which differs from equation (26) only in the standard deviation of its innovation. In conclusion, if someone is just interested in the cross-correlations as a proxy for the goodness of fit of the model to the data and if the data indicates that the autocorrelation of aggregate shocks is close to that of idiosyncratic shocks, one can be happy with the representative agent framework[47].

[47]The differences in the standard deviations is just a matter of calibration. For σ_v in representative agent framework, we choose 2*0.7 percent to make up for the loss of idiosyncratic variability whereas it is 0.7 if there are heterogeneous agents. Actually, the variability does not matter since the cross-correlations remain essentially the same in this framework regardless of the value of σ_v.

		Cross Correlations with output, corr(X_{t+j}, y_t) (HP-filtered series)								
	σ_x %	$t-4$	$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$	$t+4$
L	2.41	-0.04	0.06	0.24	0.53	1.00	0.48	0.17	-0.02	-0.12
K	0.33	-0.42	-0.39	-0.28	-0.06	0.35	0.53	0.58	0.55	0.49
y	1.53	-0.08	0.03	0.21	0.51	1.00	0.51	0.21	0.03	-0.08
C	0.28	-0.25	-0.17	-0.02	0.23	0.64	0.59	0.50	0.40	0.30

Table 8: Frequency domain based calculation of moments, $\eta = 1, \tilde{\rho} = 0.6$

		Cross Correlations with output, corr(X_{t+j}, y_t) (HP-filtered series)								
	σ_x %	$t-4$	$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$	$t+4$
L	1.71	0.14	0.30	0.49	0.72	1.00	0.67	0.41	0.19	0.03
K	0.29	-0.43	-0.33	-0.18	0.04	0.33	0.52	0.63	0.67	0.66
y	1.09	0.09	0.25	0.46	0.70	1.00	0.70	0.46	0.25	0.09
C	0.32	-0.11	0.06	0.28	0.56	0.90	0.76	0.61	0.47	0.35

Table 9: Frequency domain based calculation of moments, $\eta = 1, \tilde{\rho} = 0.9$

		Cross Correlations with output, corr(X_{t+j}, y_t) (HP-filtered series)								
	σ_x %	$t-4$	$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$	$t+4$
L	1.39	0.16	0.32	0.51	0.73	1.00	0.69	0.43	0.22	0.06
K	0.23	-0.42	-0.32	-0.16	0.05	0.33	0.51	0.62	0.66	0.66
y	0.89	0.12	0.28	0.48	0.72	1.00	0.72	0.48	0.28	0.12
C	0.32	-0.02	0.15	0.36	0.63	0.96	0.77	0.60	0.44	0.30

Table 10: Frequency domain based calculation of moments, $\eta = 1, \tilde{\rho} = 0.95$

		Cross Correlations with output, corr(X_{t+j}, y_t) (HP-filtered series)								
	σ_x %	$t-4$	$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$	$t+4$
L	1.82	0.16	0.32	0.51	0.73	1.00	0.69	0.43	0.22	0.06
K	0.30	-0.42	-0.32	-0.16	0.05	0.33	0.51	0.62	0.66	0.66
y	1.17	0.12	0.28	0.48	0.72	1.00	0.72	0.48	0.28	0.12
C	0.42	-0.02	0.15	0.36	0.63	0.96	0.77	0.60	0.44	0.30

Table 11: Frequency domain based calculation of moments, $\eta = 1$, representative agent framework

8 Discussion

Throughout the analysis in this paper we have seen that the effects of shocks on aggregate variables and wealth heterogeneity are dependent on the preferences whereas persistence of uninsurable idiosyncratic shocks play an important role for the cyclical properties of aggregate variables. We have found that we can perfectly aggregate individual log-linearized equations if agents have CRRA preferences whereas aggregation is imperfect if we have CART preferences. In case of perfect aggregation, heterogeneity of agents does not matter for the dynamics of aggregate variables in response to aggregate technology shocks. However, we have shown that heterogeneity results in increased aggregate savings in response to aggregate shocks if aggregation is imperfect. Nevertheless, this increase is quite small and we do not observe its effects on the business cycle properties of aggregate variables. Finally, we have obtained the result that it is not possible to differentiate the moments of aggregate variables in heterogeneous agent framework from those in representative agent framework if the autocorrelation of idiosyncratic shocks equals to the autocorrelation of aggregate technology shock. All of these findings have interesting implications for economic theory and policy.

The theoretical importance of our analysis is related with the adequacy of representative agent framework for the analysis of the behavior of aggregate variables. There are two options to justify the representative agent assumption. The first one is having an environment in which there are complete markets such that agents can perfectly insure themselves against idiosyncratic risks since this will result in the collapse of our heterogeneous economy to the corresponding representative agent framework. However, we do not have perfect insurance markets in real economies. The second option would be showing that the aggregate variables in a heterogeneous economy behave in a similar way to their counterparts in the representative agent economy. Our results based on impulse responses and cross-correlations indicate that representative agent framework is not adequate especially if there is sufficient heterogeneity and concave risk tolerance making the aggregation imperfect or if there is a significant difference between the persistence of idiosyncratic and aggregate shocks.

Moreover, the dynamics of wealth heterogeneity in response to different shocks gives us important clues about policy implications of our model although our economy does not directly include a government. First of all, we may say that any policy affecting the labor income positively will primarily benefit the poorer groups and smooth the wealth distribution since labor earnings is the main source of income of poor households. Besides, an unemployment insurance would be an important device to avoid significant fluctuations in the income of poor agents due to the strong effects of the antagonistic relationship between different groups of people.

In addition, we have seen that the impulse response of the aggregate capital in the heterogeneous economy is higher than that in the corresponding homogeneous economy if we have an environment where heterogeneity affects the behavior of aggregate variables in response to aggregate shocks. The policy implications of this phenomenon are two-sided. One may claim that greater inequality leads to higher aggregate savings and therefore higher output in good times, i.e. when we have positive aggregate shocks. So, a government which places heavy emphasis on the macroeconomic aggregates may favor inequality during booms. However, there is also the other side of the medallion since the amount of savings in heterogeneous framework is not Pareto-optimal, i.e. the impulse responses are inefficiently high. Therefore, a government may improve welfare by eliminating heterogeneity such that perfect aggregation holds and the decentralized economy can imitate the Pareto-optimal allocation. In conclusion, one should be very careful in interpreting the results for not being the devil's advocate.

Last but not least, it is necessary to discuss the advantages and disadvantages of our method. The main disadvantage of our method is that it is a linear approximation to the model dynamics meaning that our results are reliable locally and for small deviations from steady state. However, our method does not require statistical procedures or inference in order to reach the necessary conclusions such that our results are not subject to estimation errors. Moreover, we do not limit us to particular values of a discrete state-space matrix like in Aiyagari (1994), Krusell and Smith (1998) and others following the same approach since our exogenous processes are well defined and the value domain for state variables is continuous. Finally and most importantly, our approach allows us to focus on the dynamics after each kind of shock separately which provides much better information about what is happening behind the picture[48].

9 Conclusion

Being the first one of its kind, this paper studies a simple dynamic stochastic model of heterogeneity and analyzes the effects of exogenous aggregate and individual shocks separately. Using this model and the solution approach detailed in Uhlig (1999) we have tried to find out the effects of aggregate and idiosyncratic shocks on wealth heterogeneity and whether wealth and income distribution matters for macroeconomic dynamics and for the decisions of individuals. We do not claim that this model can perfectly capture the dynamics of a real economy which is the cost of using a simple model framework. Nevertheless, we believe that our results are pretty instructive and this is how the simplicity pays off.

Papers following the approach of Krusell and Smith (1998) define an equilibrium as the case where the state vector is convergent such that wealth distribution is well defined in the steady state. However, our stationary economy is the one in which aggregate variables are fixed but idiosyncratic variables may move between different states over time. This property of our model is due to the indeterminacy of the non-stochastic steady state of wealth distribution[49].

The indeterminacy of wealth distribution results in persistent effects of shocks on wealth distribution. We have seen that idiosyncratic shocks affect

[48]Two alternative methods are the Parameterized Expectations Approach of Marcet and Lorenzoni (1999) and the limited state space approach by Krusell and Smith (1998). The first approach allows calculation of impulse responses but it employs a statistical procedure to approximate a nonlinear recursive law of motion. The second one uses statistical inference as a proof of approximate aggregation of individual variables. The method employed in this paper helps us to calculate impulse responses and to show exactly under which conditions perfect aggregation holds and which environments violate the conditions necessary for perfect aggregation without making use of any statistical procedure.

[49]Such stationary economies are also present in several other papers in the literature, such as Loury (1981), Banerjee and Newman (1991, 1993) and Hopenhayn (1992).

the wealth distribution in favor of the agents that experience the positive shock. Nevertheless, whether aggregate shocks lead to a smoother or more asymmetric distribution is heavily dependent on the choice of preferences which determine the relative effects of two competing forces on the dynamics of wealth heterogeneity. One of these forces changes the wealth distribution in favor of poor agents since aggregate technology shocks have a positive effect especially on labor earnings which constitute the main endowment of poor agents. The second force is due to the preferences that determine the risk aversion of rich agents relative to poor agents. If the ratio of risk aversion of rich agents to that of poor agents is sufficiently high, this second force, which gives the rich agents higher incentive to save compared to the poor agents, may overcome the first force such that one observes an increase in wealth heterogeneity in the long-run after an aggregate shock.

In addition, we have shown that wealth heterogeneity does not matter for the decisions of the individuals and for the dynamics of aggregate variables responding to aggregate shocks if households have CRRA utilities. Introducing CART utility function does not change the first result whereas the impulse response of aggregate capital changes if one moves away from CRRA and CARA assumptions. Moreover, we have compared the aggregate business cycle traits of various models that differ in their preference settings and persistence of idiosyncratic shocks. We have shown that the preferences play almost no role in the determination of cyclical properties of aggregate variables. However, the difference between the persistence levels of aggregate and idiosyncratic shocks is important to explain the deviation of cyclical properties of a heterogeneous economy from those belonging to the representative agent framework.

The potential extensions of our model are only limited to the consumption and saving motives of individuals. The most immediate extension would be introducing labor decisions of agents since occupational choices of individuals are likely to depend on their wealth. This would result in greater imperfection in the economy if one considers upper or lower bounds for the labor supply of each agent. Another possible extension is the consideration of the life cycle motive which can be introduced via the overlapping generations framework. Moreover, we have seen in Table 2 that transfers play an important role as an income source of poor households. Therefore, one can introduce government and transfers to the model to study the reactions of the economy in case of a change in fiscal policy that affects taxes and transfers. All of these extensions are candidates for future research proposals.

References

[1] Aiyagari, S. Rao. "Uninsured Idiosyncratic Risk and Aggregate Saving." *Quarterly Journal of Economics* 109 (August 1994): 659-84.

[2] Banerjee, Abhijit V., and Newman, Andrew F. "Risk-bearing and the Theory of Income Distribution." *Review of Economic Studies* 58 (April 1991): 211-35.

[3] Banerjee, Abhijit V., and Newman, Andrew F. "Occupational Choice and the Process of Development." *Journal of Political Economy* 101 (April 1993): 274-98.

[4] Budría, Santiago; Díaz-Giménez, Javier; Quadrini, Vincenzo; and Ríos-Rull, José-Víctor. "New Facts on the Distributions of Earnings, Income and Wealth in the U.S." Manuscript. Philadelphia: University of Pennsylvania, 2002.

[5] Castañeda, Ana; Díaz-Giménez, Javier; and Ríos-Rull, José-Víctor. "Exploring the Income Distribution Business Cycle Dynamics." *Journal of Monetary Economics* 42 (June 1998): 93-130.

[6] Castañeda, Ana; Díaz-Giménez, Javier; and Ríos-Rull, José-Víctor. "Accounting for the U.S. Earnings and Wealth Inequality." *Journal of Political Economy* 111 (April 1993): 818-857.

[7] Chatterjee, Satyajit. "Transitional Dynamics and the Distribution of Wealth in a Neoclassical Growth Model." *Journal of Public Economics* 54 (May 1994): 97-119.

[8] Díaz-Giménez, Javier; Quadrini, Vincenzo; and Ríos-Rull, José-Víctor. "Dimensions of Inequality: Facts on the U.S. Distributions of Earnings, Income, and Wealth." *Federal Reserve Bank of Minneapolis Quarterly Review* 21 (Spring 1997): 3-21.

[9] Guiso, L. and Paiella, M. "Risk Aversion, Wealth and Background Risk." *CEPR Discussion Paper* no. 2728. London, March 2001.

[10] Hansen, Gary D. "Indivisible Labor and The Business Cycle." *Journal of Monetary Economics* 16 (November 1985): 309-327.

[11] Hatchondo, Juan Carlos. "Asset Pricing with Nonlinear Risk Tolerance." Manuscript. New York: University of Rochester, 2003.

[12] Hodrick, Robert J., and Prescott, Edward C. "Postwar U.S. Business Cycles: An Empirical Investigation" *Journal of Money, Credit and Banking* 29 (February 1997): 1-16.

[13] Hopenhayn, Hugo A. "Entry, Exit, and Firm Dynamics in Long Run Equilibrium." *Econometrica* 60 (September 1992): 1127-50.

[14] King, Robert G., and Rebelo, Sergio. "Low frequency filtering and real business cycles." *Journal of Economic Dynamics and Control* 17 (January-March 1993): 207-31.

[15] Krueger, Dirk, and Kubler, Felix. "Computing equilibrium in OLG models with stochastic production." *Journal of Economic Dynamics and Control* 28 (April 2004): 1411-1436.

[16] Krusell, Per, and Smith, Anthony A. "Income and Wealth Heterogeneity in the Macroeconomy." *Journal of Political Economy* 106 (October 1998): 867-96.

[17] Loury, Glenn C. "Intergenerational Transfers and the Distribution of Earnings." *Econometrica* 49 (July 1981): 843-67.

[18] Marcet, Albert, and Lorenzoni, Guido. "Parameterized Expectations Approach; Some Practical Issues." in *Computational Methods for the Study of Dynamic Economies*, edited by Ramon Marimon, and Andrew Scott. Oxford: Oxford University Press, 1999.

[19] Quadrini, Vincenzo; and Ríos-Rull, José-Víctor. "Understanding the U.S. Distribution of Wealth." *Federal Reserve Bank of Minneapolis Quarterly Review* 21 (Spring 1997): 22-36.

[20] Scheinkman, Jose A., and Weiss, Laurence. "Borrowing Constraints and Aggregate Economic Activity." *Econometrica* 54 (January 1986): 23-46.

[21] Storesletten, Kjetil; Telmer, Chris; and Yaron, Amir. "Asset Pricing with Idiosyncratic Risk and Overlapping Generations." *CEPR Discussion Paper* no. 3065. London, November 2001.

[22] Uhlig, Harald : "A Toolkit for Analysing Nonlinear Dynamic Stochastic Models Easily." in *Computational Methods for the Study of Dynamic Economies*, edited by Ramon Marimon, and Andrew Scott, 30-61. Oxford: Oxford University Press, 1999.

A Further Discussion

One will note that there is a difference between the moments provided by Toolkit and the tables in Section 7.4: The contemporaneous correlation of aggregate capital with output in Toolkit's results is quite low and sometimes even negative in an unexpected manner. To understand the reason for this strange result we have set the number and types of agents equal to one and the value of the variance of idiosyncratic shocks close to zero (10^{-30}). This has allowed us to compare our results with that of exampl0.m provided in Toolkit. (The relevant file, comparison.m, is provided in the accompanying compact disc.) The comparison between the outputs of these two files and the comparison of the correlation coefficients for individual capital with that of the aggregate capital, which should be the same since the number of types is restricted to one, has shown that we should shift the correlation coefficients for aggregate capital one period back in order to obtain consistent results. Once this is done, we obtain the same cross-correlations as in the output of exampl0.m[50]. This is why there is difference between the output of Toolkit and the tables provided here.

The reason for this issue is that aggregate capital has been defined as an endogenous variable rather than a state variable[51]. Since we have shifted the cross correlations of aggregate capital with output one period back one should consider K in the relevant tables as the capital to be carried over to the next period, rather than the capital employed in current production. We use this notation to avoid confusion in case when someone else uses Toolkit's approach in a different framework where aggregate capital is state variable and wants to compare her results with ours. Nevertheless, this issue does not result in any significant change of our main insights in Section 7.4.

[50]The comparison is based on the frequency-domain method based calculation of moments. The only difference between the outputs of the two programs, comparison.m and exampl0.m, is the values of the standard deviations since we have a labor augmenting technology whereas exampl0.m employs an output augmenting technology with the same variance of innovations.

[51]This changes the usual dating convention in Toolkit. So, there is no magic involved here.

B Calculations

This section explains the determination of σ_ϵ for equation (52) such that the variability of the unconditional mean of idiosyncratic productivity processes is the same as the unconditional variability of the aggregate technology process. First of all, since the aggregate technology shock, $\hat{z}_t$, follows an AR(1) process we calculate its unconditional standard deviation as $\sigma_z = \sigma_v / \sqrt{(1-\rho^2)}$ where σ_v is the standard deviation of the innovations in equation (26). Similarly, we can write the unconditional standard deviation of idiosyncratic productivity shocks as $\sigma_u = \sigma_\epsilon / \sqrt{(1-\tilde{\rho}^2)}$ using equation (52). Finally, we know that the standard deviation of the average of n independent random variables can be given as $\sigma/\sqrt{n}$, σ being the standard deviation of each random variable.

Using this information we determine σ_u as follows: We first set the value of σ_v equal to 0.7 percent following Hansen (1985) and obtain σ_z accordingly. Thereafter, we find the unconditional standard deviation of idiosyncratic productivity shocks via $\sigma_u = \sigma_z \sqrt{n}$ where n is the number of agents in the economy. The last step is finding the standard deviation of idiosyncratic innovations using $\sigma_\epsilon = \sigma_u \sqrt{(1-\tilde{\rho}^2)}$. And we are done!

C Tables

		Cross Correlations with output, corr(X_{t+j}, y_t) (HP-filtered series)								
	σ_x %	$t-4$	$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$	$t+4$
L	2.65	-0.03	-0.01	0.01	0.04	1.00	0.00	-0.02	-0.04	-0.05
K	0.25	-0.24	-0.24	-0.23	-0.22	0.40	0.38	0.35	0.31	0.27
y	1.68	-0.04	-0.02	0.00	0.03	1.00	0.03	0.00	-0.02	-0.04
C	0.24	-0.10	-0.06	-0.01	0.06	0.49	0.40	0.32	0.24	0.17

Table 12: Frequency domain based calculation of moments, $\eta = 0.1, \tilde{\rho} = 0$

		Cross Correlations with output, corr(X_{t+j}, y_t) (HP-filtered series)								
	σ_x %	$t-4$	$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$	$t+4$
L	2.58	-0.06	-0.03	0.06	0.29	1.00	0.24	0.00	-0.08	-0.10
K	0.29	-0.32	-0.33	-0.29	-0.15	0.37	0.48	0.46	0.41	0.34
y	1.63	-0.08	-0.05	0.04	0.27	1.00	0.27	0.04	-0.05	-0.08
C	0.25	-0.17	-0.13	-0.06	0.10	0.54	0.48	0.39	0.30	0.22

Table 13: Frequency domain based calculation of moments, $\eta = 0.1, \tilde{\rho} = 0.3$

		Cross Correlations with output, corr(X_{t+j}, y_t) (HP-filtered series)								
	σ_x %	$t-4$	$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$	$t+4$
L	2.41	-0.04	0.06	0.24	0.53	1.00	0.48	0.17	-0.02	-0.12
K	0.33	-0.42	-0.39	-0.28	-0.06	0.35	0.53	0.58	0.55	0.49
y	1.53	-0.08	0.03	0.21	0.51	1.00	0.51	0.21	0.03	-0.08
C	0.28	-0.25	-0.17	-0.03	0.22	0.64	0.59	0.50	0.40	0.31

Table 14: Frequency domain based calculation of moments, $\eta = 0.1, \tilde{\rho} = 0.6$

	σ_x %	Cross Correlations with output, corr(X_{t+j}, y_t) (HP-filtered series)								
		$t-4$	$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$	$t+4$
L	1.71	0.14	0.30	0.49	0.72	1.00	0.67	0.41	0.19	0.03
K	0.29	-0.43	-0.33	-0.18	0.04	0.33	0.52	0.63	0.67	0.66
y	1.09	0.09	0.25	0.46	0.70	1.00	0.70	0.46	0.25	0.09
C	0.32	-0.11	0.05	0.27	0.55	0.90	0.76	0.62	0.48	0.35

Table 15: Frequency domain based calculation of moments, $\eta = 0.1, \tilde{\rho} = 0.9$

	σ_x %	Cross Correlations with output, corr(X_{t+j}, y_t) (HP-filtered series)								
		$t-4$	$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$	$t+4$
L	2.65	-0.03	-0.01	0.01	0.04	1.00	0.00	-0.02	-0.04	-0.05
K	0.25	-0.24	-0.24	-0.24	-0.22	0.40	0.38	0.35	0.31	0.27
y	1.68	-0.04	-0.02	0.00	0.03	1.00	0.03	0.00	-0.02	-0.04
C	0.24	-0.09	-0.06	0.00	0.07	0.49	0.40	0.31	0.24	0.17

Table 16: Frequency domain based calculation of moments, $\eta = 0.4, \tilde{\rho} = 0.0$

	σ_x %	Cross Correlations with output, corr(X_{t+j}, y_t) (HP-filtered series)								
		$t-4$	$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$	$t+4$
L	2.58	-0.06	-0.03	0.06	0.29	1.00	0.24	0.00	-0.08	-0.10
K	0.29	-0.32	-0.33	-0.30	-0.15	0.37	0.48	0.46	0.41	0.34
y	1.63	-0.08	-0.05	0.04	0.27	1.00	0.27	0.04	-0.05	-0.08
C	0.25	-0.16	-0.13	-0.06	0.11	0.54	0.48	0.39	0.30	0.22

Table 17: Frequency domain based calculation of moments, $\eta = 0.4, \tilde{\rho} = 0.3$

	σ_x %	Cross Correlations with output, corr(X_{t+j}, y_t) (HP-filtered series)								
		$t-4$	$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$	$t+4$
L	2.41	-0.04	0.06	0.24	0.53	1.00	0.48	0.17	-0.02	-0.12
K	0.33	-0.42	-0.39	-0.28	-0.06	0.35	0.53	0.58	0.55	0.49
y	1.53	-0.08	0.03	0.21	0.51	1.00	0.51	0.21	0.03	-0.08
C	0.28	-0.25	-0.17	-0.02	0.23	0.64	0.59	0.50	0.40	0.30

Table 18: Frequency domain based calculation of moments, $\eta = 0.4, \tilde{\rho} = 0.6$

		Cross Correlations with output, corr(X_{t+j}, y_t) (HP-filtered series)								
	σ_x %	$t-4$	$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$	$t+4$
L	1.71	0.14	0.30	0.49	0.72	1.00	0.67	0.41	0.19	0.03
K	0.29	-0.43	-0.33	-0.18	0.04	0.33	0.52	0.63	0.67	0.66
y	1.09	0.09	0.25	0.46	0.70	1.00	0.70	0.46	0.25	0.09
C	0.32	-0.11	0.06	0.28	0.56	0.90	0.76	0.61	0.47	0.35

Table 19: Frequency domain based calculation of moments, $\eta = 0.4, \tilde{\rho} = 0.9$

		Cross Correlations with output, corr(X_{t+j}, y_t) (HP-filtered series)								
	σ_x %	$t-4$	$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$	$t+4$
L	2.65	-0.03	-0.01	0.01	0.04	1.00	0.00	-0.02	-0.04	-0.05
K	0.25	-0.24	-0.24	-0.24	-0.22	0.40	0.38	0.35	0.31	0.27
y	1.68	-0.04	-0.02	0.00	0.03	1.00	0.03	0.00	-0.02	-0.04
C	0.24	-0.09	-0.05	0.00	0.07	0.49	0.40	0.31	0.23	0.17

Table 20: Frequency domain based calculation of moments, $\eta = 0.7, \tilde{\rho} = 0.0$

		Cross Correlations with output, corr(X_{t+j}, y_t) (HP-filtered series)								
	σ_x %	$t-4$	$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$	$t+4$
L	2.58	-0.06	-0.03	0.06	0.29	1.00	0.24	0.00	-0.08	-0.10
K	0.29	-0.32	-0.33	-0.30	-0.15	0.37	0.48	0.46	0.41	0.34
y	1.63	-0.08	-0.05	0.04	0.27	1.00	0.27	0.04	-0.05	-0.08
C	0.25	-0.16	-0.13	-0.05	0.11	0.54	0.48	0.39	0.30	0.21

Table 21: Frequency domain based calculation of moments, $\eta = 0.7, \tilde{\rho} = 0.3$

		Cross Correlations with output, corr(X_{t+j}, y_t) (HP-filtered series)								
	σ_x %	$t-4$	$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$	$t+4$
L	2.41	-0.04	0.06	0.24	0.53	1.00	0.48	0.17	-0.02	-0.12
K	0.33	-0.42	-0.39	-0.28	-0.06	0.35	0.53	0.58	0.55	0.49
y	1.53	-0.08	0.03	0.21	0.51	1.00	0.51	0.21	0.03	-0.08
C	0.28	-0.24	-0.17	-0.02	0.23	0.64	0.58	0.50	0.40	0.30

Table 22: Frequency domain based calculation of moments, $\eta = 0.7, \tilde{\rho} = 0.6$

		Cross Correlations with output, corr(X_{t+j}, y_t) (HP-filtered series)								
	σ_x %	$t-4$	$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$	$t+4$
L	1.71	0.14	0.30	0.49	0.72	1.00	0.67	0.41	0.19	0.03
K	0.29	-0.43	-0.33	-0.18	0.04	0.33	0.52	0.63	0.67	0.66
y	1.09	0.09	0.25	0.46	0.70	1.00	0.70	0.46	0.25	0.09
C	0.32	-0.10	0.06	0.28	0.56	0.90	0.76	0.61	0.47	0.34

Table 23: Frequency domain based calculation of moments, $\eta = 0.7, \tilde{\rho} = 0.9$

		Cross Correlations with output, corr(X_{t+j}, y_t) (HP-filtered series)								
	σ_x %	$t-4$	$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$	$t+4$
L	1.39	0.16	0.32	0.51	0.73	1.00	0.69	0.43	0.22	0.06
K	0.23	-0.42	-0.31	-0.16	0.05	0.33	0.51	0.62	0.67	0.66
y	0.89	0.12	0.28	0.48	0.72	1.00	0.72	0.48	0.28	0.12
C	0.32	-0.03	0.14	0.36	0.63	0.95	0.77	0.60	0.45	0.31

Table 24: Frequency domain based calculation of moments, $\eta = 0.1, \tilde{\rho} = 0.95$

		Cross Correlations with output, corr(X_{t+j}, y_t) (HP-filtered series)								
	σ_x %	$t-4$	$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$	$t+4$
L	1.39	0.16	0.32	0.51	0.73	1.00	0.69	0.43	0.22	0.06
K	0.23	-0.42	-0.32	-0.16	0.05	0.33	0.51	0.62	0.66	0.66
y	0.89	0.12	0.28	0.48	0.72	1.00	0.72	0.48	0.28	0.12
C	0.32	-0.02	0.15	0.36	0.63	0.96	0.77	0.60	0.44	0.30

Table 25: Frequency domain based calculation of moments, $\eta = 0.4, \tilde{\rho} = 0.95$

		Cross Correlations with output, corr(X_{t+j}, y_t) (HP-filtered series)								
	σ_x %	$t-4$	$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$	$t+4$
L	1.39	0.16	0.32	0.51	0.73	1.00	0.69	0.43	0.22	0.06
K	0.23	-0.42	-0.32	-0.17	0.05	0.33	0.51	0.62	0.66	0.66
y	0.89	0.12	0.28	0.48	0.72	1.00	0.72	0.48	0.28	0.12
C	0.32	-0.02	0.15	0.36	0.63	0.96	0.77	0.60	0.44	0.30

Table 26: Frequency domain based calculation of moments, $\eta = 0.7, \tilde{\rho} = 0.95$

		Cross Correlations with output, corr(X_{t+j}, y_t) (HP-filtered series)								
	σ_x %	$t-4$	$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$	$t+4$
L	1.82	0.16	0.32	0.51	0.73	1.00	0.69	0.43	0.22	0.06
K	0.30	-0.41	-0.31	-0.16	0.05	0.33	0.52	0.62	0.67	0.66
y	1.17	0.12	0.28	0.48	0.72	1.00	0.72	0.48	0.28	0.12
C	0.42	-0.03	0.14	0.36	0.63	0.95	0.77	0.61	0.45	0.31

Table 27: Frequency domain based calculation of moments, $\eta = 0.1$, representative agent framework

		Cross Correlations with output, corr(X_{t+j}, y_t) (HP-filtered series)								
	σ_x %	$t-4$	$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$	$t+4$
L	1.82	0.16	0.32	0.51	0.73	1.00	0.69	0.43	0.22	0.06
K	0.30	-0.42	-0.31	-0.16	0.05	0.33	0.51	0.62	0.67	0.66
y	1.17	0.12	0.28	0.48	0.72	1.00	0.72	0.48	0.28	0.12
C	0.42	-0.03	0.14	0.36	0.63	0.96	0.77	0.60	0.45	0.31

Table 28: Frequency domain based calculation of moments, $\eta = 0.4$, representative agent framework

		Cross Correlations with output, corr(X_{t+j}, y_t) (HP-filtered series)								
	σ_x %	$t-4$	$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$	$t+4$
L	1.82	0.16	0.32	0.51	0.73	1.00	0.69	0.43	0.22	0.06
K	0.30	-0.42	-0.32	-0.16	0.05	0.33	0.51	0.62	0.67	0.66
y	1.17	0.12	0.28	0.48	0.72	1.00	0.72	0.48	0.28	0.12
C	0.42	-0.03	0.14	0.36	0.63	0.96	0.77	0.60	0.44	0.31

Table 29: Frequency domain based calculation of moments, $\eta = 0.7$, representative agent framework

D MATLAB® Codes

D.1 Benchmark Model with Two Types of Agents

```
% VERSION 2.0, MARCH 1997, COPYRIGHT H. UHLIG.
% Code by ALI K OZDAGLI for the M.A. thesis, July 2004
% "Implications of Aggregate and Idiosyncratic Shocks
%   for Neoclassical Growth and Wealth Distribution"
% DENEME_RAMSEY_BASIC_REAL.M: 2 agents with CRRA preferences
% Wealth and labor productivity distribution is exogenously given

% This is the generic version of the code that can be easily extended to
% include many agents which is limited by the speed of your processor.

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% However, you are not allowed to sell this software or otherwise impinge
% on its free distribution.

disp('DENEME_RAMSEY_BASIC_REAL:: Implications of aggregate and idiosyncratic shocks on neoclassical growth');
disp('and wealth distribution; 2 types of agents');

disp('Hit any key when ready...');
pause;


% Setting parameters:
number=2; %number of types/groups

 H_i = [0.8 0.2]'; % fraction of households in each group

% H_i =[0.5 0.5]'; % homogenous agents

a_i=zeros(number,1);   % contains share of wealth possessed by each group

 a = [0.2 0.8];

% a =[0.5 0.5];  % homogenous agents

a_i = (a/sum(a))';

 chi = [0.59 0.41];  % total earnings(productivity) in each group as a % of aggregate economy

% chi = [0.5 0.5]; % homogenous agents

chi_i=(chi/sum(chi))';

Z_bar     = 1;    % Normalization, aggregate technology shock
alpha       = .36;  % Capital share
delta     = .025; % Depreciation rate for capital
R_bar     = 1.01; % One percent real interest per quarter

gamma_i = ones(number,1)*1.5;  % Relative risk aversion coefficient (sigma in paper)
```

```
rho        = 0.95;  % autocorrelation of technology shock
sigma_tech = .712; % Standard deviation of aggregate technology shock.  Units: Percent.
sigma_eps  = 0.5*ones(1,number); % Standard deviation of idiosyncratic shock.  Units: Percent.
L_tilda = 1/(3*Z_bar); % labor supply parameter, chosen such that L_bar is 1/3

% Calculating the steady state:

betta   = 1.0/R_bar;  % Discount factor beta

KL_bar = ((alpha)/(R_bar - 1 + delta))^(1.0/(1 - alpha));  % aggregate capital labor ratio
W_bar = (1-alpha)*(KL_bar)^alpha;  % wage per efficiency unit

eps_i = Z_bar*chi_i./H_i;  % labor productivity of an "average person" in group i (theta in paper)

L_bar = L_tilda*sum(eps_i.*H_i);  % aggregate (mean) labor supply
K_bar = KL_bar*L_bar;  % aggregate (mean) capital

k_i = (a_i*K_bar)./H_i; % capital holdings of an "average person" in group i

c_i   = (R_bar-1)*k_i + W_bar*L_tilda*eps_i; % consumption of an "average person" in group i

% Declaring the matrices.


VARNAMES = ['capital_1  ',
            'capital_2  ',
            'consum_1   ',
            'consum_2   ',
            'return     ',
            'wage       ',
            'labor      ',
            'labor_1    ',
            'labor_2    ',
            'capitaltot ',
            'technology ',
            'idiosyncr_1',
            'idiosyncr_2',

        ];

% Translating into coefficient matrices.
% The loglinearized equations are given in Section 5 of the paper:
% The variables are ordered as (each variable with index 'i' is individual variable)
% Endogenous state variables "x(t)": k_i(t)
% Endogenous other variables "y(t)": c_i(t),  R(t), w(t), L(t), eps_i(t), K(t)
% Exogenous state variables  "z(t)": z(t), u_i(t)
% Switch to that notation.  Find matrices for format
% 0 = AA x(t) + BB x(t-1) + CC y(t) + DD z(t)
% 0 = E_t [ FF x(t+1) + GG x(t) + HH x(t-1) + JJ y(t+1) + KK y(t) + LL z(t+1) + MM z(t)]
% z(t+1) = NN z(t) + epsilon(t+1) with E_t [ epsilon(t+1) ] = 0,

% DETERMINISTIC EQUATIONS with the corresponding equation number in the paper:

BIG=alpha*(K_bar/L_bar)^(alpha-1);  % parameter prededined to simplify the eqn. for R(t)
```

```
% for x(t): k_i(t), 1-number
AA=zeros(2*number+4,number);  %Initializing the matrix

for i=1:number
    AA(i,i)=k_i(i);  % eqn. 20
end

% for x(t-1): k_i(t-1), 1-number
BB=zeros(2*number+4,number);  %Initializing the matrix

for i=1:number;
    BB(number+1, i)=-H_i(i)*k_i(i);   % eqn. 21
end

for i=1:number
    BB(i,i)=-R_bar*k_i(i);   % eqn. 20
end



% For  y(t): c_i(t), 1toN;  R(t), N+1; w(t), N+2; L(t), N+3; eps_i(t), N+4to2N+3;
%            K(t), 2N+4

CC=zeros(2*number+4,2*number+4);   %Initializing the matrix

for i=1:number                      % eqn. 20
    CC(i,i) = c_i(i);
    CC(i,number+1)= -R_bar*k_i(i);
    CC(i,number+2)= -W_bar*L_tilda*eps_i(i);
end

CC(number+2,number+3) = L_bar;          %eqn 22
CC(number+3,number+2) = 1;              % eqn 23
CC(number+3,number+3) = alpha;          % eqn 23
CC(number+4,number+1) = R_bar;          % eqn 23
CC(number+4,number+3) = BIG*(alpha-1);  % eqn 23

for i=number+4:2*number+3
    CC(i-(number+3),i) = -W_bar*L_tilda*eps_i(i-(number+3));    %eqn 20
    CC(number+2,i)= -H_i(i-(number+3))*L_tilda*eps_i(i-(number+3));    %eqn 22
end

CC(number+5:2*number+4,number+4:2*number+3)=eye(number);        % eqn 25

%K_t carried here!
CC(number+3,2*number+4)=-alpha;             % eqn 23
CC(number+4,2*number+4)=-BIG*(alpha-1);     % eqn 24

CC(number+1,2*number+4)=K_bar;              % eqn 21
%end of carriage

% For z(t): z(t), 1; u_i(t), 2toN+1

DD=zeros(2*number+4,number+1);          %Initializing the matrix
```

```
DD(number+5:2*number+4,1)=-ones(number,1);          % eqn 25
DD(number+5:2*number+4,2:number+1)=-eye(number);    % eqn 25


% EXPECTATIONAL EQUATIONS:

% For x(t+1)
FF = zeros(number,number);

% For x(t)
GG = zeros(number,number);

% For x(t-1)
HH = zeros(number,number);

% For y(t+1): c_i(t+1), 1toN;  R(t+1), N+1; w(t+1), N+2; L(t+1), N+3; eps_i(t+1), N+4to2N+3;
%            K(t+1), 2N+4
% All for equation 19.
JJ = zeros(number,2*number+4);   %Initializing the matrix


for i=1:number
    JJ(i,i) = -gamma_i(i);
end

JJ(1:number,number+1) = ones(number,1);

% For y(t): c_i(t), 1toN;  R(t), N+1; w(t), N+2; L(t), N+3; eps_i(t), N+4to2N+3;
%            K(t), 2N+4
% All for equation 19.
KK = zeros(number,2*number+4);      %Initializing the matrix

for i=1:number
    KK(i,i) = gamma_i(i);
end



% For z(t+1)
LL = zeros(number,number+1);

% For z(t)
MM = zeros(number,number+1);

% AUTOREGRESSIVE MATRIX FOR z(t)

NN = zeros(number+1,number+1);      %Initializing the matrix

NN(1,1) = rho;                  % The idiosyncratic shocks are not persistent

Sigma_vector = [ sigma_tech^2, sigma_eps.^2];
Sigma = diag(Sigma_vector);

% Setting the options:
```

```
[l_equ,m_states] = size(AA);
[l_equ,n_endog ] = size(CC);
[l_equ,k_exog  ] = size(DD);

PERIOD     = 4;  % number of periods per year, i.e. 12 for monthly, 4 for quarterly
GNP_INDEX  = 4; % Index of output among the variables selected for HP filter
IMP_SELECT = 1:(m_states+n_endog+k_exog);   %  a vector containing the indices of the variables to be plotted
HP_SELECT  = 1:(m_states+n_endog+k_exog); % Selecting the variables for the HP Filter calcs.
DO_SIMUL   = 0; % Calculates Simulations
DO_MOMENTS = 0; % Calculates Moments
DISPLAY_IMMEDIATELY = 1;
DISPLAY_AT_THE_END = 0;
DO_QZ=1;
HORIZON=300;
IMP_SELECT = [1:2 ];

DO_STATE_RESP = 0;
SELECT_STATES = 2;

DO_SHOCK_RESP = 1;
SELECT_SHOCKS = 1;

SIM_SELECT=1:4;
%DO_HP_FILTER= 0;
SIM_LENGTH  = 2000;
SIM_MAX = 2000;
% Starting the calculations:

do_it;
```

D.2 Impulse Response of Gini Index

```
% GINI.M:
% This program calculates and plots the impulse response of the Gini index
% WARNING: Run the code that produces the impulse response of capital first
% Input: k_i, a_i, H_i, K_bar, impulse responses of k_i and K
% Output: Lorenz curve, Impulse response of Gini index
% See section 5 of the paper for details

A_i = cumsum(a_i);

area_B = H_i'*A_i - H_i'*a_i/2;

Gini_bar = 1 - 2* area_B;

disp('Press any key to see the Lorenz curve');  pause;

plot([[0; cumsum(H_i)], [0; A_i]], [0; A_i])  % plots the Lorenz curve and the line of perfect equality
axis tight
grid on
xlabel('Population Share','FontSize',14)
ylabel('Wealth Share','FontSize',14)
title('Lorenz Curve for Wealth Distribution','FontSize',14)
```

```
%--------------------------------------------------------------------------

gini_resp = zeros(1,HORIZON-1);   disp('Press any key to see impluse response of Gini index');
pause;

for time=1:HORIZON-1

aa_i = Response(1:number,time) - Response(3*number+4,time+1);  % \hat a_it

AA_i = cumsum(a_i.*aa_i);                              % \barA_i \hatA_it

BB_hat = H_i'*AA_i - H_i'*(a_i.*aa_i)/2;               % \barB \hatB_t

gini_resp(time) = - 2 * BB_hat / Gini_bar;          %impulse resp. of gini index

end

%plot([0:1/PERIOD:(HORIZON-2)/PERIOD], gini_resp)

plot([0:1/PERIOD:(HORIZON-2)/PERIOD], [gini_resp; zeros(1,HORIZON-1)])

xlim([0 (HORIZON-2)/PERIOD])

xlabel('Years after shock','FontSize',14)
ylabel('Percent deviation from steady state','FontSize',14)
if DO_STATE_RESP == 1;
title('Impulse response of Gini index to a one percent deviation in','FontSize',14)
else
title('Impulse response of Gini index to a shock in','FontSize',14)
end

grid on
```

D.3 Convergence of Relative Endowments

```
% LOOP_RAMSEY_BASIC_REAL.M:
% Produces the plot for the convergence of relative endowments
% in case of subsequent technology shocks

tot_iter = 3000;   % Upper limit number of iterations
number=2;  % number of types of agents
a_i=zeros(number,1);   % share of wealth possessed by each group
 a = [0.2 0.8];

a_i = (a/sum(a))';

getKL=zeros(number,tot_iter);       % This matrix will contain the new values of the
                                    % share of wealth possessed by each wealth group
                                    % after an aggregate technology shock

simulation_ramsey_basic_real;      % This is simply our benchmark model except that
                                   % the values of a_i is taken from the output of this program
iter=0;
ask=abs(k_i(1)/(eps_i(1)*L_tilda)-KL_bar);   % Precision of convergence
```

```
while ask>10^-4
    iter=iter+1;
    a_i=k_i.*(1+Response(1:number,HORIZON)/100).*H_i/K_bar;
    a_i=(a_i/sum(a_i));
    getKL(1:number,iter)=a_i;
    simulation_ramsey_basic_real;

    if iter==tot_iter
        break
    end

    ask=abs(k_i(1)/(eps_i(1)*L_tilda)-KL_bar);
end

plot([getKL(1,1:iter)'*K_bar./(chi_i(1)*Z_bar*L_tilda), ...
... getKL(2,1:iter)'*K_bar./(chi_i(2)*Z_bar*L_tilda), KL_bar*ones(1,iter)'])
```

D.4 The Benchmark Model with CART preferences

The code is essentially the one in Section D.1 with the addition of the following code to make the necessary changes:

```
eta = 1;  % concavity parameter of absolute risk tolerance set equal to 1
% for CRRA and to 0 for CARA preferences

...

gamma_i=gamma_i.*(c_i.^(1-eta));
```

D.5 Multiple Types of Agents

We only change the number of the agents and the corresponding entries for the distribution of wealth and labor earnings.

```
number=8

H_i = [0.2 0.2 0.2 0.2 0.1 0.05 0.04 0.01]'; % fraction of households in each group

a = [0.001 1.4 5.3 12.9 12.8 12.3 24.1 31.4];
% share of capital of each agent
% The poorest agents capital holdings are chosen to be
% non-negative for budget constraint considerations,
% the actual value should be -0.2

chi = [7.7 14.2 16.3 20.4 13.1 8.2 12.7 7.5];
% total earnings(productivity) in each group as a % of economy
```

D.6 Many Agents and Persistent Idiosyncratic Shocks

We change the number of the agents and the corresponding entries for the distribution of wealth and labor earnings. Moreover, we add a parameter for the autocorrelation of idiosyncratic shocks and change the NN matrix accordingly.

```
number = 30

H_i = 1/number*ones(number,1) ; % fraction of households in each group

a = [0.001*ones(1,number/5) 1.4*ones(1,number/5) 5.3*ones(1,number/5) 12.9*ones(1,number/5)...
...80.6*ones(1,number/5)]/(number/5);   % share of capital of each agent

chi = [7.7*ones(1,number/5) 14.2*ones(1,number/5) 16.3*ones(1,number/5) 20.4*ones(1,number/5)...
  ... 41.5*ones(1,number/5)]/(number/5);
 % total earnings(productivity) in each group as a % of economy


rho_idio=0.3;           % autocorrelation of idiosyncratic shocks

...

NN(1,1) = rho;
for i=2:number+1
    NN(i,i) = rho_idio;
end
```

Declaration of Authorship

I hereby confirm that I have authored this master thesis independently and without use of others than the indicated resources. All passages, which are literally or in general matter taken out of publications or other resources are marked as such.

Ali Kivilcim Özdagli

Berlin, 27th July 2004