An Analysis of the Dynamics of the Vintage Capital

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Abstract

Why does the vintage capital explain well the economic inequality? The technology shock will lead to new unemployment. This paper wants to give a numerical analysis of the vintage model to reveal this fact. In the vintage capital model the economy can not replace all its old capital at each date, instead they renew their capital step by step, and different technologies exist at the same time. If different technologies reside in production tools and the plant and its labor can use just one technology at a time, a variety of productive tools will be in use at the same time, since each worker uses different vintage production tools therefore labor's productivities will differ. This vintage capital incurred inequality can be used to explain today huge inequality in per capita outputs among countries. Income disparity is not a consequence of different initial conditions, but the result of different investment choices made by each economy. This paper attempts to explore the route how do aggregate shocks affect aggregate employment by changing the fraction of plants that choose to adjust. The neutral technological shocks increase job reallocation and reduce aggregate employment. The increasing adjustment hazard influences the job reallocation by affecting the marginal product of capital for all vintages, therefore the capital flows from the updated sector 0 to old sector 1. Then labor forces in the old sector with old technology are endowed with more capital, which boosts their relative wages, the labor forces in old sector will increase. Aided with the endogenous adjustment ratios, the previous vintage model gets a new explanation. Empirical study results demonstrate the strongly positive correlation between GNP and private fixed investment of the United State data during the period from 1974 quarter 1 to 2004 quarter 2. There is an investment-driven business cycle in the United State case. In line with this fact, economic policy should pique growth through technology and capital intensity two channels. The policy maker should also impose strength on boosting national savings as well as translating savings to productive investments and enhance the level of education.

Keywords: Vintage Capital, Investment Adjustment, Technological Change, Unemployment, Labor Market.

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1 Introduction

Is technological progress good or bad for workers? This question is interesting to the average person in the street as well as to professional economists. The information technology revolution induces advent of the Internet economy which has spurred our curiosity further about this matter. We can guess that some workers with outmoded skills will be losers, but that other workers will be made more productive by having better tools at their disposal. What will be the main storyline in the next century's history books? Will the final analysis be that technology replaced flesh-and-blood workers, and result in generally higher unemployment and lower real wages? Or will it be that humans become more valuable as they could use their time more effectively?

Economists mostly believe that changes in the growth rate of productivity have profound influences on the quality of workers' economic lives. Cadiou, Dees and Laffargue [2000] uses the French data to reveal the job creation and destruction in France. Previous efforts have been made by Jovanovic [1998], by introducing the vintage capital model to reveal the aggregate employment fluctuation on the technology shock.

Michelacci and Lopez-Salido [2005] uses Structural VAR models to reveal that technological progress can lead to the destruction of technologically obsolete jobs and cause unemployment. They find that neutral technology shocks increase job destruction and job reallocation and reduce aggregate employment. The neutral technological progress prompts waves of Schumpeterian creative destruction.

The previous empirical literature uncovers one key stylized fact that: It is an important route that aggregate shocks affect aggregate employment by changing the fraction of plants that choose to adjust. Investigation of investment dynamics in Thomas [2002] answers the question of whether the individual economic agents' lumpy decision matters in the aggregate. The answer is sometimes. Thomas and Khan [2003] use a similar approach to study (S,s) inventory accumulation and shows that these adjustment probabilities are functions of the difference between plants' actual and target employment, therefore concludes that it is an important route that aggregate shocks affect aggregate employment by changing the fraction of plants that choose to adjust.

Standard partial adjustment model relates current employment number to target or desired employment. But partial adjustment model is inconsistent with the behaviour of individual plants. Plants exhibit nonlinear responses to shocks. It is difficult to determine equilibrium when the aggregate state involves a distribution of production units. State dependent adjustment frameworks extending to general equilibrium have been limited. Generalized (S,s) model were first studied by Caballero and Engel [1999] to explain the observed lumpiness of plant level investment demand. It allows us to examine the influence of deep parameters on the adjustment process. With a large number of plants, the model is similar to the traditional partial adjustment model in that it yields a smooth market labor demand. Caballero [1993] construct a general framework for studying aggregate employment changes that can incorporate a variety of assumptions about how adjustment hazards are related to aggregate conditions.

A generalized partial adjustment model in which individual production units adjust in a discrete and occasional manner is studied by Thomas [2002], yet there is smooth adjustment at the aggregate level. In the generalized model the adjustment rate is an endogenous function of the state of the economy. While the traditional partial adjustment model uses time -invariant aggregate partial adjustment. Impulse response establishes that it retains the basic features of gradual partial adjustment. By undertaking (S,s) analysis within a general equilibrium framework, the influence of aggregate shocks on equilibrium adjustment pattern may be systematically studied.

This paper attempts to explore the route how do aggregate shocks affect aggregate employment by changing the fraction of plants that choose to adjust. Based on the analysis of the vintage model in Thomas [2002], the conclusion shows that the technology shock will lead the labor productivity increase and total labor force number decrease correspondingly. This paper analyzes the high persistence that characterizes the dynamics of firms' neutral technology and the frequency of firms' capital adjustment. The neutral technological shocks increase job reallocation and reduce aggregate employment. The increasing adjustment hazard influences the job reallocation by affecting the marginal product of capital for all vintages, which are increasing function over vintages, for example, the R_{1t+2} is larger thanR_{0t+1}. Full capital mobility induces a general equilibrium feedback that enlarges inequality: factor-price equalization requires capital to flow from the sector 0 to sector 1. Therefore labor force in the sector 1 with old technology are endowed with more capital, which boosts their relative wages, the labor forces in sector 1 will increase. Aided with the endogenous adjustment ratios, the previous vintage model like Jovanovic [1998], which indicates a worker who is with the best machine will acquire more skill therefore earn higher wages, gets a new explanation.

Frictional labor market and the decentralized production structure assumptions necessitate the matching model in Pissarides [2000], research on the relation between embodied productivity growth and unemployment in Aghion and Howitt [1994] and Mortensen and Pissarides [1998]. They pioneered the research on the relation between embodied productivity growth and unemployment in a frictional labor market. In their models new capital adjustment is frictionless, as a result, vacancies all consist of the newest capital. Moreover Hornstein, Krusell and Violante [2002] models the existence of vacancy heterogeneity induced by vintage capital under these structures. An upgrading option is valueable only if it is very costly for firms to meet workers, therefore their result indicates that the quantitative differences between unprading and replacement is negligible.

The paper proceeds as follows: Section 2 reviews the previous literature on the vintage capital. Section 3 explains the theoretical background of the model. Section 4 introduces how the model is set? Section 5 executes the Toolkit initiated by Uhlig [1999] to analyze the dynamic stochastic model easily. Section 6 explains the impulse response functions, the second moment and calibrates the model using the standard parameter choices; furthermore empirically studies the US historical data to detect whether there is a investment driven business cycle. Section 7 extends the frictionless model to include frictional labor market structure and specifies the origin of the technological shock. Section 8 discusses the difference among models and policy implication. Section 9 concludes.

2 Literature

This section reviews several previous models of vintage capital relevant to the one discussed in this paper. Cadiou, Dees and Laffargue [2000] presents a vintage capital model assuming putty clay investment and perfect foresight. The traditional drawback of a putty-clay production function is the presence of variables with long leads and long lags. As putty-clay technology involves some stickiness in the production process, these works can investigate properly the sluggish adjustment of production factors to shocks. This framework clearly studies movements in job creation and job destruction related to economic obsolescence, replacement of productive capacity and expectations over the lifetime of the new units of production. On the contrary to recent works developing model in continuous time, their model is in discrete time. They identify the echo effect characterizing vintage capital models and the related dynamics of job creation and job destruction. This model is also proved useful to explain the medium-term movements in the distribution of income between production factors that putty-putty models lack. A key feature of the putty-putty specification, Especially it illustrates quite well the change in the wage share in value-added in France during the last three decades.

A key feature of the putty-putty specification is that all the vintages of capital have the same capital intensity, which is the same case in the model discussed in this paper initiated with adjustment ratio. On the contrary, they expect the current technology to be only available to the newly created units of production. Whereas this is also in the similar case of this paper. This is precisely what the putty-clay specification does, where current economic conditions affect the capital intensity of the new production units (their technological choice) and the number of these units created (investment in the economy). The other production units maintain their original technology they were endowed at their creation. Current economic conditions affect their profitability and nonprofitable production units face the scrapping. Therefore the aggregate capital-labor ratio changes gradually with the fluctuation of investment and the scrapping of old obsolete production units. Putty-clay investment provides mid-term dynamics in the distribution of income. This specification has some other advantages: the irreversibility of investment is embedded in the model and firing costs can easily be introduced, which gives a convincing foundation to the stickiness of employment.

Whereas the putty-clay technology encounters a serious drawback: its implementation in a macroeconomic model is difficult for two reasons. First, the model has a long memory since it keeps track in working order of all the vintages of capital created in the past. Hence the model has "variables with long lags". Secondly, the investors' planning horizon extends far into the future. The decision concerning the creation of new production units involves forward variables that cover the expected lifetime of these units. The model has then “variables with long leads”.

The vintage capital model in Jovanovic [1998] explains the occurrence of income inequality under four similar assumptions: new technology is embodied in production tools; quantities of capital and labor are matched in fixed proportions; capital quality and labor skill are complements and assignment is frictionless. If machine quality and skill are complements, a worker who is with the best machine will acquire more skill therefore earn higher wages, which is different from the case in the paper, therefore inequality will persist. In his model the efficiency of each vintage of capital is endogenous and it varies when the economy is not on its balanced growth path. He explains incomes inequality but is lack of the studies of inequality in consumption and wealth, one would need to carry out the full dynamics, presumably from an initial condition under which all agents start out equal

Recent discussions on growth theory emphasize the ability of vintage capital models to explain growth facts. An AK-type (Y = AK)endogenous growth model with vintage capital is studied by Boucekkine, Licandro, Puch and Rio. [2005], which is different from previous production function with the property of diminishing returns to capital. This AK model is absence of diminishing returns to capital, which is the key difference between this endogenous growth model and the one discussed in this paper. The inclusion of vintage capital leads to oscillatory dynamics governed by replacement echoes, which influence the intercept of the balanced growth path. The convergence is non-monotonic due to the existence of replacement echoes. As a consequence, investment rates do not move in lock step with growth rates. This coincides with the conclusion of this paper. To characterize the complete resolution of the model they develop analytical and numerical methods that should be of interest for the general resolution of endogenous growth models with vintage capital. These features, which are in sharp contrast to those from the standard AK model, can contribute to explaining the short-run deviations observed between investment and growth rates time series.

Their findings also indicate that there is much to be learned from the explicit modelling of variable depreciation rates. An extension of this line of research is to include an endogenous decision for the scrapping time. Whereas in the model discussed in this paper, the extension is achieved by an endogenous state dependent investment adjustment ratio. A is a constant measuring the amount of output produced for each unit of capital. One extra unit of capital produces A extra units of output, regardless of how much capital there is. The $\frac{\Delta K}{K} = sA - \delta$, if it is positive, the income grows forever, even without the assumption of exogenous technology progress. Therefore a simple change in the production function can alter the predictions about economic growth. This is so since their numerical methods can be used to deal with time dependent and state dependent leads and lags.

In the model discussed in this paper savings leads to growth temporarily but diminishing returns to capital eventually force the economy to approach a steady state in which growth depends only on exogenous technological progress. On the contrary, in the endogenous model, saving and investment can lead to persistent growth. Moreover, all models discussed so far are unable to explain the the relation between vintage capital embodied productivity growth and unemployment in a frictional labor market.

3 Explain the Model

It is as one key stylized fact uncovered in the previous empirical literature that an important route through which aggregate shocks affect aggregate employment is by changing the fraction of plants that choose to adjust. In the model the adjustment rate is an endogenous function of the state of the economy. The generalized model is not observationally equivalent to traditional partial adjustment model with time -invariant aggregate partial adjustment.

The model is based on a generalized (S,s) framework ((S,s) policy is individual deterministic policy with economy state-dependent investment adjustment ratio α_{jt}. An adjustment hazard, following econometric literature on discrete choice, is defined as the probability that an individual production unit makes a discrete change in a particular date.

Different from the traditional (S,s) model, this uses the stochastic adjustment costs with a probabilistic adjustment thresholds that can capture the rising hazards to simultaneously yield lumpy plant-level investment and smooth aggregates.

To model the capital difference in the process of technology change, a vintage capital framework where capitals and labors are costly to create units of capital different ages, corresponding to technologies with different productivity levels is introduced. A unit measure of production units is differentiated by their stocks of capital with different vintage J. Production unit last acquires new capital j periods in the past with the subscript j. The number j of firm types means the vintage, which is endogenously determined and varies with factors such as the average inflation rate and elasticity of production demand.

Depending on the current economy state and its adjustment cost, each plant chooses to invest or not.

The judgement is based on the function:
flow profits of plants = output - wage payments - investment - adjustment cost. If it is beneficial to invest, a current cost payment ξw_t occurs for the production in date t+1, where w_t means the real wage at date t. All plants share the same production technology and same distribution of adjustment costs.

The cross section distribution of the establishments over capital levels is summarized by the distribution of plants over vintage groups. Each vintage group contains a marginal plant whose investment cost is just worthwhile for it to make adjustment. All plants with

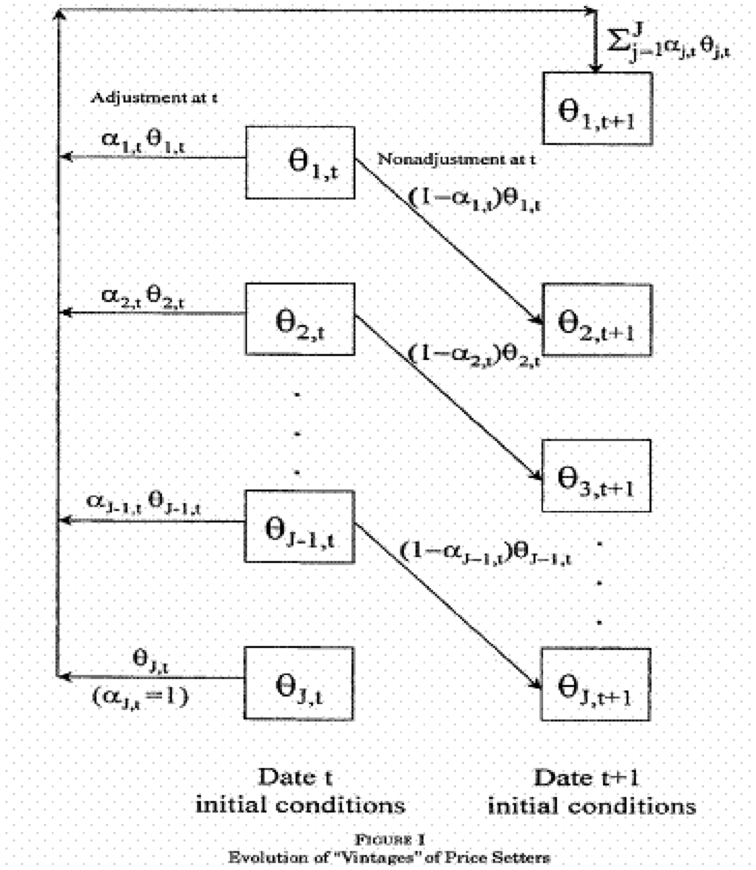

Figure 1: The fraction of plants over vintages

the same or less than this critical investment cost draw have the willingness to invest. Therefore a adjustment ratio α_{jt} in each vintage group can be derived as a function of the adjustment, a known cumulative distribution $G(\xi)$.

If it is not beneficial to invest, the plant's capital stock at date t+1 remains the same as after production in date t. The flow profits of the producers are returned in lump-sum fashion to households. The figure 1 [1] illustrates the distribution of investment plants evolving over time, which depends on the state of the economy.

The cross section distribution of the establishments over capital levels is summarized by the distribution of plants across vintage groups. Each vintage group contains a marginal plant whose investment cost is just worthwhile for it to make adjustment. All plants with the same or less than this critical investment cost draw have the willingness to invest. Therefore a adjustment ratio α_{jt} in each vintage group can be derived as a function of the adjustment, a known cumulative distribution $G(\xi)$.

[1]Quoted from Thomas [2002]

4 The Model

The competitive equilibrium is determined by solving a sequence of variables: $\{C_t, n_{j,t}, N_t, i_{jt}, \alpha_{jt}, \Theta_{j,t+1}, k_{j,t}\}_{t=0}^{\infty}$, where $n_{jt}, i_{jt}, \alpha_{jt}, \theta_{j,t+1}, k_{j,t}$ are variables with vintage j=0. . . ∞.

Competitive equilibrium allocations are determined by solving of a social planning problem indicated by Bellman equation, as in the form of function 1.

Bellman equation

$$V(K_t, \Theta_t, A_t) = \max_{C_t, n_t, N_t, i_t, \alpha_t, \Theta_{t+1}, k_{0,t+1}} [u(C_t, 1 - N_t) + \beta E_t V(K_{t+1}, \Theta_{t+1}, A_{t+1})] \quad (1)$$

subject to the budget constraint:

$$C_t + k_{0t+1} = \bar{y_0} + R_{j,t+j+1} k_{jt} \quad (2)$$

C_t is the consumption, k_{0t+1} is the next period adjusted capital endowment of all the plants choosing to adjust in this period, $R_{j,t+j+1}$ is the gross rate of return on the capital, k_{jt} are the capital endowment in this period of all the plants. The cross section distribution of the establishments across groups is summarized by two vectors indexed by vintage. $K_t = \{k_{jt}\}$, is the vector of capital level across vintage group. The expectation E_t is used as the information of date t+1 is not known at date t. $\Theta_t = \{\theta_{jt}\}$, θ_{jt} is the fraction of plants currently owning each vintage j capital level. The investment adjustment ratio α_{jt} means in period t a fraction of vintage j firms decides to adjust. $0 < \beta < 1$ is the discount factor parameter.

$$\begin{aligned}
k_{0,t+1} &= (1 - \delta) k_{jt} + i_{jt} && (3)\\
y_{jt} &= A_t k_{jt}^{\gamma} n_{jt}^{\nu} && (4)\\
i_{jt} &= y_{jt} - C_t && (5)\\
u(C_t, 1 - N_t) &= log C_t + \zeta(1 - N_t) && (6)\\
A_t &= X_t z_t && (7)\\
z_t &= z_{t-1}^{\rho} e^{\epsilon_t} && (8)\\
\epsilon_t &\sim N(0, \sigma_{\epsilon}^2) && (9)
\end{aligned}$$

Each plant's production applies to Cobb-Douglas production function characterized by diminishing returns with respect to variable inputs in production. Producers use labor and capital as variable inputs. i_{jt} is the investment. The final good can be either consumed or invested. Output, consumption and investment are divided by z_t, and the capital stock is divided by z_{t-1}. Household owns the portfolio of plants and supplies labor. Its endowment, following Hansen (1985) of indivisible labor, one unit of time per period is split to leisure L_t and market activities N_t. Household values consumption and leisure in each period with momentary utility $u(c_t, 1-N_t)$ in form of function 6. Current consumption is financed in forms of wages and profits from plants. X_t is the trend component and evolves with growth rate Θ_A. The z_t is a neutral aggregate technology shock (different from the investment-specific technology shock, q shock) which follows a mean zero AR(1) process in logs. δ is the parameter defining the depreciation rate of the capital. γ and ν are the parameters defining the exponential of capital and labor in production function, following the diminishing returns assumption they should be summed less than 1. ζ is the parameter determining to which extent the labor produces a negative utility. ρ is the autocorrelation of technology shock.

Other constraints

$$k_{j+1,t+1} = (1-\delta)k_{jt} \tag{10}$$

If the plant choose not to invest at the end of period t, its capital stock at t+1 remains the same as pervious period except the depreciation.

$$C_t \leq \sum_{j=0}^{\infty} \theta_{jt} y_{jt} - \sum_{j=0}^{\infty} \theta_{jt} \alpha_{jt} i_{jt} \tag{11}$$

$$\sum_{j=0}^{\infty} \theta_{jt} n_{jt} + \sum_{j=0}^{\infty} \theta_{jt} \Xi(\alpha_{jt}) \leq N_t \tag{12}$$

Function 11 specifies that household consumption can not exceed aggregate production net of plants' adjusting investments. Equality is used to determine the equilibrium.

Function 12 constrains that the household's work hours can not be less than weighted sum of employment in production and adjustment activities across groups. Equality is used to determine the equilibrium.

$$\theta_{0,t+1} = \sum_{j=0}^{\infty} \alpha_{jt} \theta_{jt} \tag{13}$$

$$\theta_{j,t+1} = \theta_{j-1,t}(1-\alpha_{j-1,t}) \tag{14}$$

The adjustment rate α_j responses smoothly to the aggregate state of economy. Inflation erodes a plant's relative value, the noninvesting plants see their price erode more quickly with higher inflation, hence they choose to maintain a given price for fewer periods when inflation is high. The higher is the inflation rate, the greater are the benefits to invest for any j, therefore the higher is α_j. The higher inflation endogenously generates a smaller value for the number of vintage J. The figure 1 illustrates the distribution of investment plants evolving over time, which depends on the state of the economy. In period t, a fraction α_{jt} of vintage j plants decides to invest, and at the same time, a fraction $1-\alpha_{jt}$ of plants decide not to invest. As an assumption, all vintage J plants choose to invest, therefore $\alpha_{Jt} = 1$. The total fraction of investment plants are equation 13. The fraction of non-investment plants is expressed as 14.

$$\begin{aligned}
\Xi(\alpha_{jt}) &= \int_0^{G^{-1}(\alpha_{jt})} xdG(x) && (15)\\
G(\xi) &= \alpha_{jt} && (16)\\
G^{-1}(\alpha_{jt}) &= B\alpha_{jt} && (17)\\
G(\xi) &= \frac{\xi}{B} && (18)\\
\Xi(\alpha_{jt}) &= \frac{B}{2}\alpha_{jt}^2 && (19)\\
\alpha_{jt} &= G(\frac{v_{0t} - v_{j+1,t} - i_{jt}}{w_t}) && (20)
\end{aligned}$$

They may adjust labor usage frictionlessly, and a fixed labor cost occurs in adjusting capital stock. Any individual production unit faces a random cost ξ. The discrete adjustment has the randomized fixed costs ξ, which is independently and identically distributed across time with a known cumulative distribution $G(\xi)$ and p.d.f $g(\xi)$. $G(\xi)$ increases with inflation. This cost is drawn from a time-invariant distribution over interval [0,B], where B is finite upper bound. As the assumption $G(0) = 0 < G(\xi) < 1 = G(B)$, ξ is a linear function of $\alpha_{j,t}$, which means the higher adjustment ratio is, the higher adjustment cost the investment plant will have.

5 Analyze the Model

5.1 First Order Conditions

In this section, I will implement this model in Toolkit which include find first order necessary conditions and other necessary constraints that characterizing the equilibrium, calculate the steady state, log-linearize the equations around the steady states, solve for the recursive law of motion, therefore obtain the impulse responses, HP-filtered moments and calibration will be covered in section 6.

The model can be solved by using the techniques of dynamic programming. The first order conditions can be obtained from the lagrangian.

Set lagrangian

$$L(.) = \max_{C_t, n_t, k_{0,t+1}} \{u(C_t, 1 - N_t) + \beta V(K_{t+1})\} - \lambda_t[C_t + k_{t+1} - \bar{y_0} - Rk_t] \quad (21)$$

First order conditions

$$\frac{\partial L}{\partial \lambda_t} : 0 = C_t + k_{t+1} - \bar{y_0} - Rk_t \quad (22)$$

$$
\begin{aligned}
\frac{\partial L}{\partial C_t} : 0 &= \lambda_t - \frac{1}{C_t} && (23)\\
\frac{\partial L}{\partial (1-N_t)} : 0 &= -\zeta + w_t \lambda_t && (24)\\
\frac{\partial L}{\partial K_t} : 0 &= -\lambda_t + E_t[\beta \lambda_{t+1} R_{0t+1} + \beta^2 \lambda_{t+2}(1-\delta)(1-\alpha_{0t+1})R_{1t+2} + .. && (25)\\
&\quad \beta^3 \lambda_{t+3}(1-\delta)^2(1-\alpha_{0t+1})(1-\alpha_{1t+2})R_{2t+3} + ..\\
&\quad \beta^4 \lambda_{t+4}(1-\delta)^3(1-\alpha_{0t+1})(1-\alpha_{1t+2})(1-\alpha_{2t+3})R_{3t+4} + ..\\
&\quad \beta^5 \lambda_{t+5}(1-\delta)^4(1-\alpha_{0t+1})(1-\alpha_{1t+2})(1-\alpha_{2t+3})(1-\alpha_{3t+4})R_{4t+5} + ..\\
&\quad \beta^6 \lambda_{t+6}(1-\delta)^5(1-\alpha_{0t+1})(1-\alpha_{1t+2})(1-\alpha_{2t+3})(1-\alpha_{3t+4})(1-\alpha_{4t+5})R_{5t+6}]\\
R_{j=0..5,t+j+1} &= \frac{\gamma y_{j=0..5,j=t+j+1}}{k_{j=0..5,t+j+1}} + (1-\delta)\alpha_{j=0..5,t+j+1} && (26)\\
\frac{\partial L}{\partial K_{t+1}} : 0 &= -\lambda_t v_{j=0..4,t} + E_t[\beta \lambda_{t+1} g_{j=0..4,t+1}] && (27)\\
g_{j=0..4,t+1} &= \pi_{j=0..4,t+1} + \alpha_{j=0..4,t+1} v_{0,t+1} + (1-\alpha_{j=0..4,t+1}) v_{j=1..5,t+1} - w_{t+1}\frac{B}{2}\alpha^2_{j=0..4,t+1} && (28)\\
\pi_{j=0..4,t+1} &= y_{j=0..4,t+1} - w_{t+1} n_{j=0..4,t+1} - \alpha_{j=0..4,t+1} i_{j=0..4,t+1} && (29)\\
0 &= v_{j=0..4,t} - w_t B \alpha_{j=0..4,t} - i_{i=0..4,t} - v_{j=1..5,t} && (30)
\end{aligned}
$$

The first order conditions of the plants' maximization problem can be summarized in two equations which are the budget constraint as in equation 2 and the following Euler equation 25. The equation 25 is the Lucas asset pricing equation. The product of $\varphi_{j,t+1+j} = \Pi_{i=0}^{j}(1-\alpha_{i,t+1+i})$ is the probability of nonadjustment from t to t+1+j and it is not time-varying. The steady states can be solved by dropping the time indices of the first order conditions and the constraints.

The investment decision of the plants should take three factors into consideration:

Its real value if it invest v_{0t}, gross of the adjustment cost. It can be obtained by setting equations with j=0.

Its value if it does not invest.

Its currently realized fixed adjustment costs.

v_{0t} as in the expression in equation 27 is the real value of the plant that last set its price j period ago. It is expressed in the way of dynamic program to get the optimal price satisfying the Euler equation. $\frac{\lambda_{t+1}}{\lambda_t}$ is the marginal utility of consumption $D_1 u(C_t, 1-N_t)$ future to current. It is used as the appropriate discount factor for future real profits. Combined with β, it can get the present value of next period's expected value, i.e. $\frac{1}{1+r} = \frac{\beta C_t}{C_{t+1}}$.

5.2 Steady States Values

The steady states of the model can be obtained by dropping the time subscripts and stochastic shocks of the equations listed above. The variables with bars are steady state values. By studying the vintage characteristics of all variables in the model, I can find the way to solve the steady states values. There are only four variables without vintage, w_t, C_t, N_t, z_t, representing real wage, consumption, employment and exogenous technology shock respectively. The remaining variables are all with vintage subscript, namely called vintage variables. The key to solve steady states value of each vintage variable is finding of the relationship between each two contiguous vintages by write-off the non-vintage variables. Starting with Euler equation, all the steady states values can be expressed as the functions of the state dependent adjustment ratios and real wage rate, with the assumption $(1-\delta)^{\frac{1-\gamma-\nu}{1-\nu}} = a$.

$$
\begin{aligned}
\bar{R}_{01} &= \frac{1}{\beta} + \frac{\beta(1-\delta)^2\bar{\alpha}_0(1-\bar{\alpha}_0)}{a} + \frac{\beta^2(1-\delta)^3\bar{\alpha}_0(1-\bar{\alpha}_0)(1-\bar{\alpha}_1)}{a^2}.. \\
&.. + \frac{\beta^3(1-\delta)^4\bar{\alpha}_0(1-\bar{\alpha}_0)(1-\bar{\alpha}_1)(1-\bar{\alpha}_2)}{a^3}.. \\
&.. + \frac{\beta^4(1-\delta)^5\bar{\alpha}_0(1-\bar{\alpha}_0)(1-\bar{\alpha}_1)(1-\bar{\alpha}_2)(1-\bar{\alpha}_3)}{a^4}.. \\
&.. + \frac{\beta^5(1-\delta)^6\bar{\alpha}_0(1-\bar{\alpha}_0)(1-\bar{\alpha}_1)(1-\bar{\alpha}_2)(1-\bar{\alpha}_3)(1-\bar{\alpha}_4)}{a^5}.. \\
&.. - \beta(1-\delta)^2(1-\bar{\alpha}_0)\bar{\alpha}_1 - \beta^2(1-\delta)^3(1-\bar{\alpha}_0)(1-\bar{\alpha}_1)\bar{\alpha}_2.. \\
&.. - \beta^3(1-\delta)^4(1-\bar{\alpha}_0)(1-\bar{\alpha}_1)(1-\bar{\alpha}_2)\bar{\alpha}_3.. \\
&.. - \beta^4(1-\delta)^5(1-\bar{\alpha}_0)(1-\bar{\alpha}_1)(1-\bar{\alpha}_2)(1-\bar{\alpha}_3)\bar{\alpha}_4.. \\
&.. - \beta^5(1-\delta)^6(1-\bar{\alpha}_0)(1-\bar{\alpha}_1)(1-\bar{\alpha}_2)(1-\bar{\alpha}_3)(1-\bar{\alpha}_4).. \\
\bar{R}_{02} &= 1 + \frac{\beta(1-\delta)(1-\bar{\alpha}_0)}{a} + \frac{\beta^2(1-\delta)^2(1-\bar{\alpha}_0)(1-\bar{\alpha}_1)}{a^2}.. \\
&.. + \frac{\beta^3(1-\delta)^3(1-\bar{\alpha}_0)(1-\bar{\alpha}_1)(1-\bar{\alpha}_2)}{a^3}.. \\
&.. + \frac{\beta^4(1-\delta)^4(1-\bar{\alpha}_0)(1-\bar{\alpha}_1)(1-\bar{\alpha}_2)(1-\bar{\alpha}_3)}{a^4}.. \\
&.. + \frac{\beta^5(1-\delta)^5(1-\bar{\alpha}_0)(1-\bar{\alpha}_1)(1-\bar{\alpha}_2)(1-\bar{\alpha}_3)(1-\bar{\alpha}_4)}{a^5} \\
\bar{R}_0 &= \frac{\bar{R}_{01}}{\bar{R}_{02}} \qquad (31) \\
\bar{R}_{j=1..5} &= \frac{\bar{R}_0 - (1-\delta)\bar{\alpha}_0}{a^{j=1..5}} + (1-\delta)\bar{\alpha}_{j=1..5} \qquad (32) \\
\frac{\bar{y}_{j=0..5}}{\bar{k}_{j=0..5}} &= \frac{\bar{R}_{j=0..5} - (1-\delta)\bar{\alpha}_{j=0..5}}{\gamma} \qquad (33)
\end{aligned}
$$

$$\bar{C} = \frac{\bar{w}}{\zeta} \tag{34}$$

$$k_{01} = 1-(1-\delta)(1+(1-\delta)+(1-\delta)^2+(1-\delta)^3+(1-\delta)^4+(1-\delta)^5)-..$$
$$..[\frac{\bar{y}_0}{\bar{k}_0}+(1-\delta)\frac{\bar{y}_1}{\bar{k}_1}+(1-\delta)^2\frac{\bar{y}_2}{\bar{k}_2}+(1-\delta)^3\frac{\bar{y}_3}{\bar{k}_3}+(1-\delta)^4\frac{\bar{y}_4}{\bar{k}_4}+(1-\delta)^5\frac{\bar{y}_5}{\bar{k}_5}]$$

$$\bar{k}_0 = \frac{-6\bar{C}}{k_{01}} \tag{35}$$

$$\bar{y}_{j=0..5} = \bar{k}_{j=0..5}\frac{\bar{y}_{j=0..5}}{\bar{k}_{j=0..5}} \tag{36}$$

$$\bar{i}_{j=0..5} = \bar{y}_{j=0..5}-\bar{C} \tag{37}$$

$$\frac{\bar{y}_{j=0..5}}{\bar{n}_{j=0..5}} = \frac{\bar{w}}{\nu} \tag{38}$$

$$\bar{n}_{j=0..5} = \frac{\bar{y}_{j=0..5}}{(\frac{\bar{y}_{j=0..5}}{\bar{n}_{j=0..5}})} \tag{39}$$

$$\bar{\theta}_{01} = 1+(1-\bar{\alpha}_0)+(1-\bar{\alpha}_0)(1-\bar{\alpha}_1)(1-\bar{\alpha}_2)..$$
$$..(1-\bar{\alpha}_0)(1-\bar{\alpha}_1)(1-\bar{\alpha}_2)(1-\bar{\alpha}_3)+(1-\bar{\alpha}_0)(1-\bar{\alpha}_1)(1-\bar{\alpha}_2)(1-\bar{\alpha}_3)(1-\bar{\alpha}_4)$$

$$\bar{\theta}_0 = \frac{1}{\bar{\theta}_{01}} \tag{40}$$

$$\bar{\theta}_1 = (1-\bar{\alpha}_0)\bar{\theta}_0 \tag{41}$$

$$\bar{\theta}_2 = (1-\bar{\alpha}_1)(1-\bar{\alpha}_0)\bar{\theta}_0 \tag{42}$$

$$\bar{\theta}_3 = (1-\bar{\alpha}_2)(1-\bar{\alpha}_1)(1-\bar{\alpha}_0)\bar{\theta}_0 \tag{43}$$

$$\bar{\theta}_4 = (1-\bar{\alpha}_3)(1-\bar{\alpha}_2)(1-\bar{\alpha}_1)(1-\bar{\alpha}_0)\bar{\theta}_0 \tag{44}$$

$$\bar{\theta}_5 = (1-\bar{\alpha}_4)(1-\bar{\alpha}_3)(1-\bar{\alpha}_2)(1-\bar{\alpha}_1)(1-\bar{\alpha}_0)\bar{\theta}_0 \tag{45}$$

$$\bar{N} = \bar{\theta}_0\bar{n}_0+\bar{\theta}_1\bar{n}_1+\bar{\theta}_2\bar{n}_2+\bar{\theta}_3\bar{n}_3+\bar{\theta}_4\bar{n}_4+\bar{\theta}_5\bar{n}_5..$$
$$..+\frac{B}{2}\bar{\theta}_0\bar{\alpha}_0^2+\frac{B}{2}\bar{\theta}_1\bar{\alpha}_1^2+\frac{B}{2}\bar{\theta}_2\bar{\alpha}_2^2+\frac{B}{2}\bar{\theta}_3\bar{\alpha}_3^2+\frac{B}{2}\bar{\theta}_4\bar{\alpha}_4^2+\frac{B}{2}\bar{\theta}_5 \tag{46}$$

$$\bar{v}_1 = \frac{2B\bar{w}\bar{\alpha}_0-B\beta\bar{w}\bar{\alpha}_0^2+2\bar{i}_0-2\beta\bar{y}_0^2+2\beta\bar{w}\bar{n}_0}{2\beta-2} \tag{47}$$

$$\bar{v}_0 = \frac{2\beta(\bar{y}_0-\bar{w}\bar{n}_0-\bar{\alpha}_0\bar{i}_0)+2\beta(1-\bar{\alpha}_0)\bar{v}_1-B\beta\bar{w}\bar{\alpha}_0^2}{2-2\beta\bar{\alpha}_0} \tag{48}$$

$$\bar{v}_{j=2..5} = \bar{v}_0-\bar{i}_{j=1..5}-\bar{w}B\bar{\alpha}_{j=1..5} \tag{49}$$

$$\bar{g}_{j=0..4} = \bar{y}_{j=0..4}-\bar{w}\bar{n}_{j=0..4}-\bar{\alpha}_{j=0..4}\bar{i}_{j=0..4}+\bar{\alpha}_{j=0..4}\bar{v}_0+(1-\bar{\alpha}_{j=0..4})\bar{v}_{j=1..5}-\frac{B}{2}\bar{w}\bar{\alpha}_{j=0..4}^2 \tag{50}$$

$$\bar{z} = 1 \tag{51}$$

5.3 Log-linearized First Order Conditions

The final step of the analysis is the log-linearization of the necessary equations defining the equilibrium. This step replaces the dynamic nonlinear equations by dynamic linear equations. This step can be fulfilled by expressing of the linear formation in percent deviation from the steady state. The expressions are in the same order as the steady state equations.

$$\begin{aligned}
0 &= E_t[\hat{C}_t + \beta\bar{R}_0(-\hat{C}_{t+1} + \hat{R}_{0,t+1}) + \beta^2\bar{R}_1(1-\delta)(1-\bar{\alpha}_0)(-\hat{C}_{t+2} + \hat{R}_{1,t+2}) + .. \\
&\quad ..\beta^2\bar{R}_1(1-\delta)(1-\bar{\alpha}_0)(-\hat{C}_{t+2} + \hat{R}_{1,t+2}) + .. \\
&\quad ..\beta^3\bar{R}_2(1-\delta)^2(1-\bar{\alpha}_0)(1-\bar{\alpha}_1)(-\hat{C}_{t+3} + \hat{R}_{2,t+3}) + .. \\
&\quad ..\beta^4\bar{R}_3(1-\delta)^3(1-\bar{\alpha}_0)(1-\bar{\alpha}_1)(1-\bar{\alpha}_2)(-\hat{C}_{t+4} + \hat{R}_{3,t+4}) + .. \\
&\quad ..\beta^5\bar{R}_4(1-\delta)^4(1-\bar{\alpha}_0)(1-\bar{\alpha}_1)(1-\bar{\alpha}_2)(1-\bar{\alpha}_3)(-\hat{C}_{t+5} + \hat{R}_{4,t+5}) + .. \\
&\quad ..\beta^6\bar{R}_5(1-\delta)^5(1-\bar{\alpha}_0)(1-\bar{\alpha}_1)(1-\bar{\alpha}_2)(1-\bar{\alpha}_3)(1-\bar{\alpha}_4)(-\hat{C}_{t+6} + \hat{R}_{5,t+6})] \qquad (52) \\
0 &= -\bar{R}_{j=0..4}\hat{R}_{j=0..4,t+j+1} + \gamma\frac{\bar{y}_{j=0..4}}{\bar{k}_{j=0..4}}(\hat{y}_{j=0..4,t+j+1} - \hat{k}_{j=0..4,t+j+1}) + (1-\delta)\hat{\alpha}_{j=0..4,t+j+1} \qquad (53) \\
0 &= -\bar{R}_5\hat{R}_{5,t+6} + \frac{\gamma\bar{y}_5}{\bar{k}_5}(\hat{y}_{5,t+6} - \hat{k}_{5,t+6}) \qquad (54) \\
0 &= -\hat{y}_{j=0..5,t} + \hat{z}_t + \gamma\hat{k}_{j=0..5,t} + \nu\hat{n}_{j=0..5,t} \qquad (55) \\
0 &= (1-\delta)\bar{k}_0\hat{k}_{0t} + (1-\delta)\bar{k}_1\hat{k}_{1t} + (1-\delta)\bar{k}_2\hat{k}_{2t} + (1-\delta)\bar{k}_3\hat{k}_{3t} + (1-\delta)\bar{k}_4\hat{k}_{4t}.. \\
&\quad .. + (1-\delta)\bar{k}_5\hat{k}_{5t} + \bar{i}_0\hat{i}_{0t} + \bar{i}_1\hat{i}_{1t} + \bar{i}_2\hat{i}_{2t} + \bar{i}_3\hat{i}_{3t} + \bar{i}_4\hat{i}_{4t} + \bar{i}_5\hat{i}_{5t} - \bar{k}_0\hat{k}_{0t+1} \qquad (56) \\
0 &= \hat{w}_t - \hat{C}_t \qquad (57) \\
0 &= \bar{y}_{j=0..5}\hat{y}_{j=0..5,t} - \bar{C}\hat{C}_t - \bar{i}_{j=0..5}\hat{i}_{j=0..5,t} \qquad (58) \\
0 &= (1-\delta)^{j=1..5}\bar{k}_0\hat{k}_{0t} - \bar{k}_{j=1..5}\hat{k}_{j=1..5,t+1} \qquad (59) \\
0 &= \frac{1}{1-\nu}\hat{z}_t + \frac{\gamma}{1-\nu}\hat{k}_{j=0..5,t} - \hat{n}_{j=0..5,t} - \frac{1}{1-\nu}\hat{w}_t \qquad (60) \\
0 &= \bar{\theta}_0\hat{\alpha}_{0t} + \bar{\theta}_1\hat{\alpha}_{1t} + \bar{\theta}_2\hat{\alpha}_{2t} + \bar{\theta}_3\hat{\alpha}_{3t} + \bar{\theta}_4\hat{\alpha}_{4t}.. \\
&\quad .. + \bar{\alpha}_0\hat{\theta}_{0t} + \bar{\alpha}_1\hat{\theta}_{1t} + \bar{\alpha}_2\hat{\theta}_{2t} + \bar{\alpha}_3\hat{\theta}_{3t} + \bar{\alpha}_4\hat{\theta}_{4t} + \hat{\theta}_{5t} - \hat{\theta}_{0t+1} \qquad (61) \\
0 &= (1-\bar{\alpha}_{j=1..3})\hat{\theta}_{j=1..3t} - \bar{\theta}_{j=1..3}\hat{\alpha}_{j=1..3t} - \hat{\theta}_{j=2..4t+1} \qquad (62) \\
0 &= \hat{\theta}_{0t} + \hat{\theta}_{1t} + \hat{\theta}_{2t} + \hat{\theta}_{3t} + \hat{\theta}_{4t} + \hat{\theta}_{5t} \qquad (63) \\
0 &= -\bar{N}\hat{N}_t + (\bar{n}_0 + \frac{B}{2}\bar{\alpha}_0^2)\hat{\theta}_{0t} + (\bar{n}_1 + \frac{B}{2}\bar{\alpha}_1^2)\hat{\theta}_{1t} + (\bar{n}_2 + \frac{B}{2}\bar{\alpha}_2^2)\hat{\theta}_{2t} + (\bar{n}_3 + \frac{B}{2}\bar{\alpha}_3^2)\hat{\theta}_{3t}.. \\
&\quad .. + (\bar{n}_4 + \frac{B}{2}\bar{\alpha}_4^2)\hat{\theta}_{4t}(\bar{n}_5 + \frac{B}{2})\hat{\theta}_{5t} + \bar{n}_0\bar{\theta}_0\hat{n}_{0t} + \bar{n}_1\bar{\theta}_1\hat{n}_{1t} + \bar{n}_2\bar{\theta}_2\hat{n}_{2t} + \bar{n}_3\bar{\theta}_3\hat{n}_{3t} + \bar{n}_4\bar{\theta}_4\hat{n}_{4t}.. \\
&\quad .. + \bar{n}_5\bar{\theta}_5\hat{n}_{5t} + B\bar{\alpha}_0\bar{\theta}_0\hat{\alpha}_{0t} + B\bar{\alpha}_1\bar{\theta}_1\hat{\alpha}_{1t} + B\bar{\alpha}_2\bar{\theta}_2\hat{\alpha}_{2t} + B\bar{\alpha}_3\bar{\theta}_3\hat{\alpha}_{3t} + B\bar{\alpha}_4\bar{\theta}_4\hat{\alpha}_{4t} \qquad (64) \\
0 &= \bar{y}_0\bar{\theta}_0\hat{y}_{0t} + \bar{y}_1\bar{\theta}_1\hat{y}_{1t} + \bar{y}_2\bar{\theta}_2\hat{y}_{2t} + \bar{y}_3\bar{\theta}_3\hat{y}_{3t} + \bar{y}_4\bar{\theta}_4\hat{y}_{4t} + \bar{y}_5\bar{\theta}_5\hat{y}_{5t} + (\bar{y}_0 - \bar{i}_0\bar{\alpha}_0)\hat{\theta}_{0t}.. \\
&\quad .. + (\bar{y}_1 - \bar{i}_1\bar{\alpha}_1)\hat{\theta}_{1t} + (\bar{y}_2 - \bar{i}_2\bar{\alpha}_2)\hat{\theta}_{2t} + (\bar{y}_3 - \bar{i}_3\bar{\alpha}_3)\hat{\theta}_{3t} + (\bar{y}_4 - \bar{i}_4\bar{\alpha}_4)\hat{\theta}_{4t}.. \\
&\quad .. + (\bar{y}_5 - \bar{i}_5)\hat{\theta}_{5t} - \bar{i}_0\bar{\theta}_0\hat{\alpha}_{0t} - \bar{i}_1\bar{\theta}_1\hat{\alpha}_{1t} - \bar{i}_2\bar{\theta}_2\hat{\alpha}_{2t} - \bar{i}_3\bar{\theta}_3\hat{\alpha}_{3t} - \bar{i}_4\bar{\theta}_4\hat{\alpha}_{4t}.. \\
&\quad .. - \bar{i}_0\bar{\alpha}_0\bar{\theta}_0\hat{i}_{0t} - \bar{i}_1\bar{\alpha}_1\bar{\theta}_1\hat{i}_{1t} - \bar{i}_2\bar{\alpha}_2\bar{\theta}_2\hat{i}_{2t} - \bar{i}_3\bar{\alpha}_3\bar{\theta}_3\hat{i}_{3t} - \bar{i}_4\bar{\alpha}_4\bar{\theta}_4\hat{i}_{4t} - \bar{i}_5\bar{\theta}_5\hat{i}_{5t} - \bar{C}\hat{C}_t \qquad (65) \\
0 &= -B\bar{w}\bar{\alpha}_{j=0..4}\hat{w}_t - B\bar{w}\hat{\alpha}_{j=0..4t} - \bar{i}_{j=0..4}\hat{i}_{j=0..4t} - \bar{v}_{j=1..5}\hat{v}_{j=1..5t} + \bar{v}_0\hat{v}_{0t} \qquad (66) \\
0 &= E_t[\hat{C}_t - \hat{C}_{t+1} - \hat{v}_{j=0..4t} + \hat{g}_{j=0..4,t+1}] \qquad (67) \\
0 &= -\bar{g}_{j=0..4}\hat{g}_{j=0..4,t+1} + \bar{v}_0\bar{\alpha}_{j=0..4}\hat{v}_{0,t+1} + \bar{v}_{j=1..5}(1-\bar{\alpha}_{j=0..4})\hat{v}_{j=1..5,t+1}..
\end{aligned}$$

$$
\begin{aligned}
&.. - \bar{i}_{j=0..4}\bar{\alpha}_{j=0..4}\hat{\bar{i}}_{j=0..4,t+1} + \bar{y}_{j=0..4}\hat{y}_{j=0..4,t+1} - (\bar{w}\bar{n}_{j=0..4} + B\bar{w}\bar{\alpha}^2_{j=0..4})\hat{w}_{t+1}.. \\
&.. - \bar{w}\bar{n}_{j=0..4}\hat{n}_{j=0..4,t+1} - (\bar{i}_{j=0..4} + \bar{v}_{j=1..5} - \bar{v}_0 + B\bar{w}\bar{\alpha}_{j=0..4})\hat{\alpha}_{j=0..4,t+1} \quad (68) \\
0 \quad = \quad & -\hat{z}_t + \rho\hat{z}_{t-1} + \epsilon_t \quad (69)
\end{aligned}
$$

In the above equations the variables with hat are the log-deviations of the corresponding variables from their steady states, which can be regarded as the approximate percentage deviation. The calibration of the model can be achieved based on the log-linearized equations.

Some of the values in the calibration are standard and others are based on guess. For example, the adjustment ratio and wage are based on the reasonable assumption.

6 Model Result and Answer

6.1 Technology Shock and Plant Distribution

It is worth mentioning that all shocks I will discuss in this paper are one percent deviations of the respective variables from their steady state values. The positive technology shock have the positive effect on wage and all of the vintage 1 variables except capital 1 and value 1. On the contrary, it has a negative effect on all of the vintage 0 variables, capital 1 and employment.

The impulse response functions(of all important variables except the value 0 and value 1 since they response too significantly) to a technology shock were shown in the figure 2. Technology shock has fraternized effect of distinguishing the value and capital among vintages, whereas distinguishes antagonistic output, investment and population density over vintages. Why does the technology shock has a opposite influence on both types of labor, population density, output and investment but a negative influence on both capital stocks? Why does the technology shock have a positive effect on most of the vintage 1 variables but negatively affects vintage 0 variables? I will give an explanation intuitively as the follows:

All vintage 0 variables are hurt from the technology shock, which reflect on the impulse response functions negatively. All vintage 1 variables benefit from the technology shock except capital stock 1, which reflect positively on the impulse response functions. This is straightforward, because technology shock will benefit more on the outdated economy than the up-to-date one. New investment do not attain their full potential as soon as they are introduced, but rather their productivity can stay temporarily below the productivity of older capital that was introduced some time age with outdated technology. This feature is attributed to learning effects. When a new vintage arrives, the most skilled worker abandons his machine and switches to the best one. Thereafter the second best worker gets the machine just abandoned by the most skilled one, and so on. This process continues until the lowest skilled worker scraps his machine. This process is

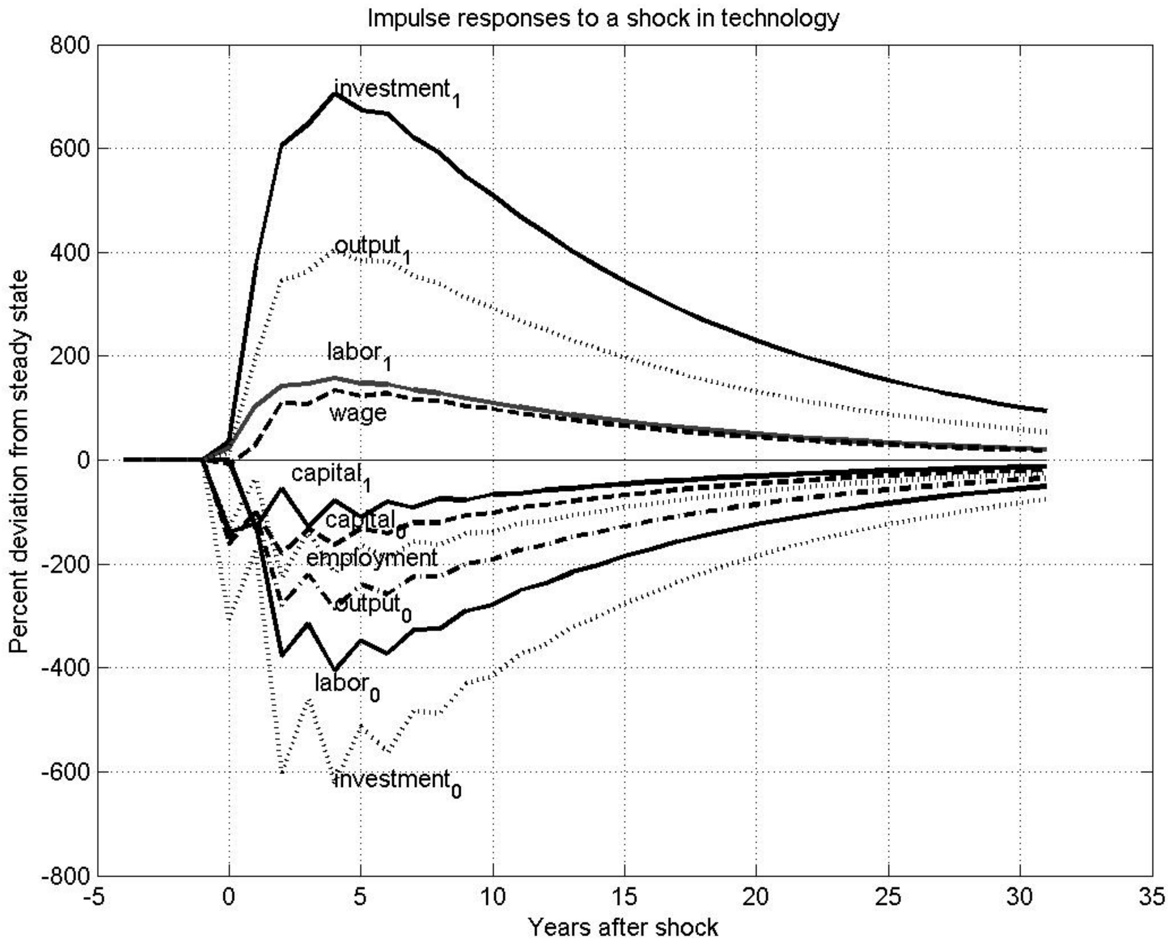

Figure 2: Long-term impulse response of all variables to aggregate technology shock J=1

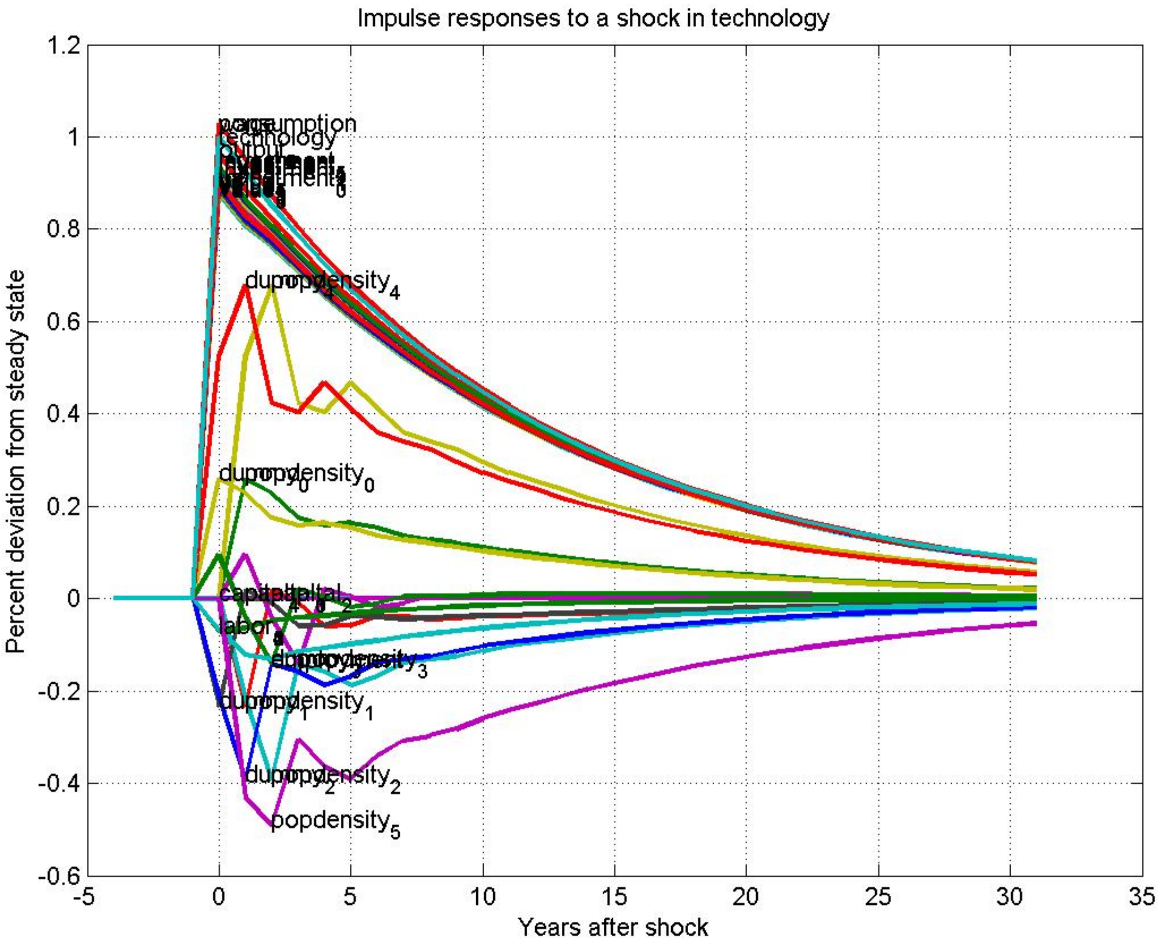

Figure 3: Long-term impulse response of all variables to aggregate technology shock J=5

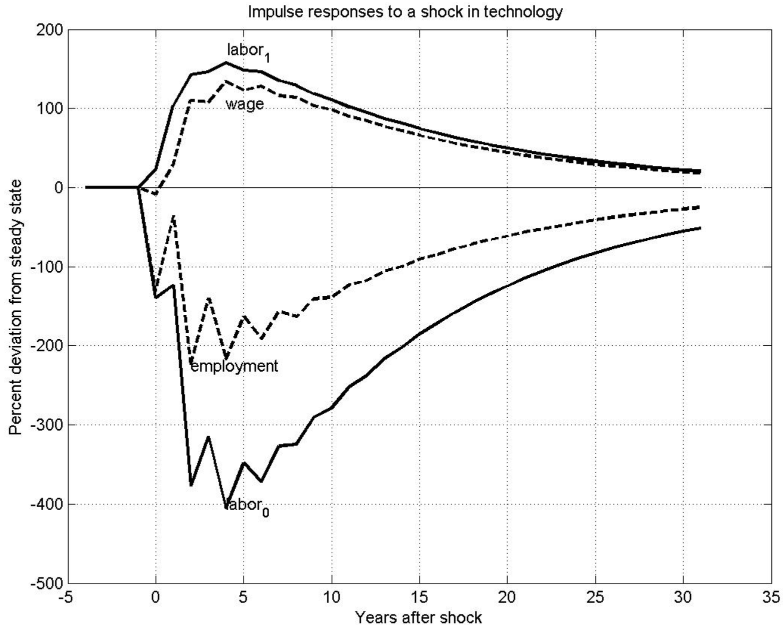

Figure 4: Long-term impulse response of labor, wage and employment to aggregate technology shock J=1

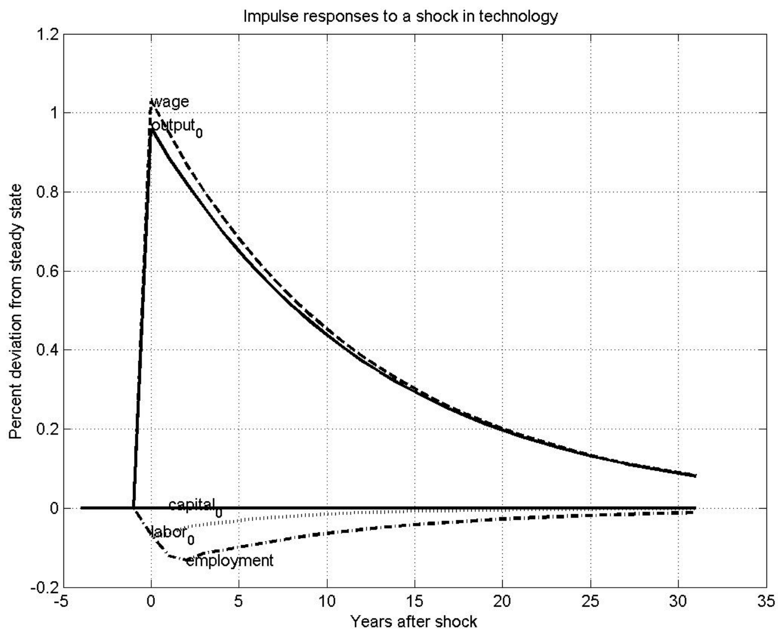

Figure 5: Long-term impulse response of capital, output, labor, wage and employment to aggregate technology shock J=5

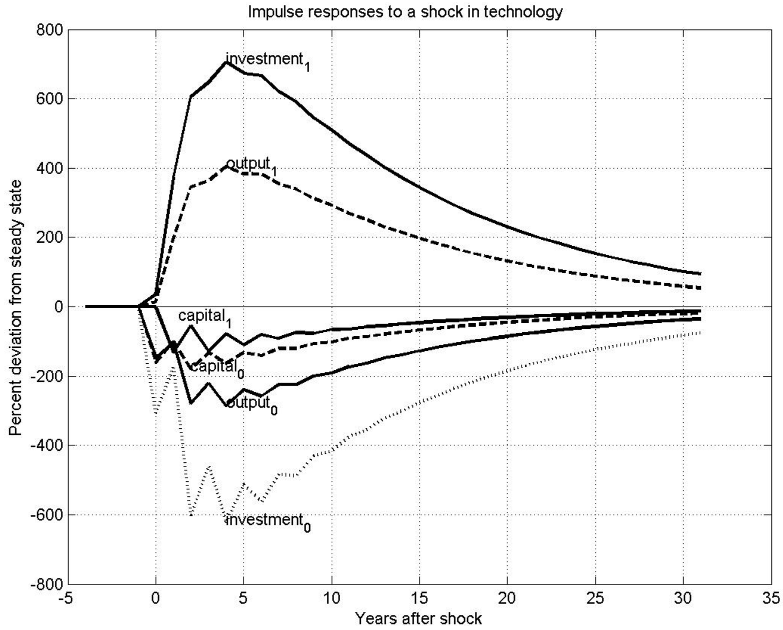

Figure 6: Long-term impulse response of capital holdings, investments and outputs to aggregate technology shock J=1

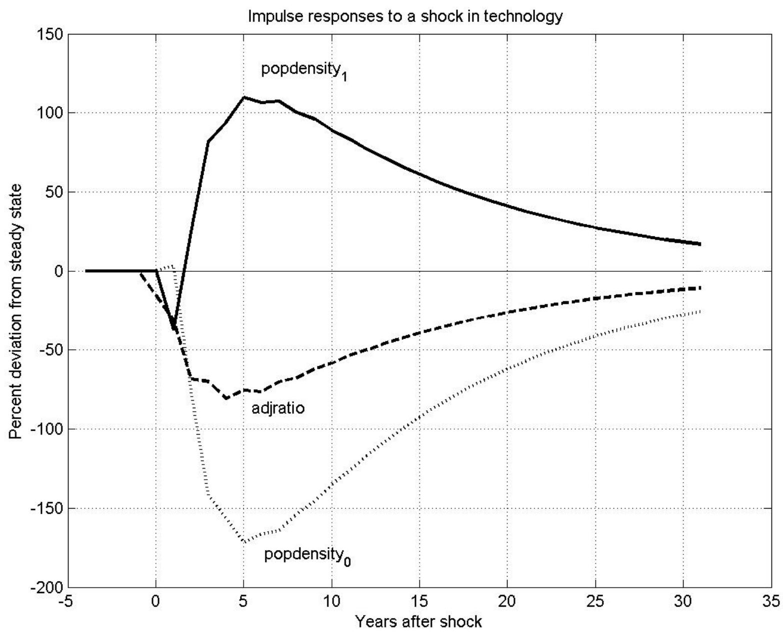

Figure 7: Long-term impulse response of adjustment ratio and population densities to aggregate technology shock J=1

called learning-by-doing(LBD).

Why is the capital stock 1 an exception? Both capital stocks response to the technology shock negatively. Since the technology shock is labor augmenting i.e. the Harrod-neutral technical process, not capital augmenting i.e. the Solow-neutral technical progress, therefore a positive shock increases labor productivity of both old and current vintages, consequently decrease the aggregate labor demand.

The technology shock will lead the labor productivity increase and total labor force number decrease correspondingly. Therefore the wage of per efficiency of labor unit will increase. This wage increase is due to the positive effect of the increase in the productivity of labor, therefore the labor force will decrease anyway. In absolute value, labor 0 responses negatively more than labor 1. Due to the inhomogeneous production labor force, I treat the men of different vintages, distinguished by age and training. Men of current vintage 0, e.g. those with currently training are more productive than those of previous vintage 1, therefore more out of supply and expensive on the labor market. Obviously in absolute value, the technology progress will influence more the current vintage labor force than the old vintage labor force.

The reason can be revealed by organizational learning. Organizational learning is reflected in an increase in the productivity of labor at the plant level: in an steady-state equilibrium where labor is mobile, productivity is equalized across plants, and wage inequality will disappear. In absence of learning effects, the anticipation of a future technical shock embodied in new capital investment can result in a transitional phase shown as a slowdown of economic activity. During the waiting period between the announcement and the actual availability of the new technology, the existing firm chooses to wait for new investment and the new plants prefer to delay entering. Therefore, their output falls temporarily until their full acceptance of the new technology, the fulfillment of the process learning-by-doing.

Numerically the per capita output of vintage 1 sector is the same as of the vintage 0 sector. From the equation 38 listed above, the per capita outputs are same over all vintages, i.e. $\frac{y_j}{n_j} = \frac{w}{\nu}$ at steady states for all j. The same relationship of $\frac{y_j}{k_j}$ holds for all j at steady states. It is the same for the consumption, because $\frac{w}{C} = \zeta$ for all t. What makes the difference? The remaining variables with vintage subscript determine the changes.

The adjustment ratio of vintage 0 plant decreases and so does the population density of plant 0. Therefore the total number of the plants with vintage 0 choosing to add their investment decreases, the opposite case suits for the vintage 1 plant. In this way, the vintage 0 investment decreases and the vintage 1 investment increases. The reason can be revealed by studying the changes in the marginal product of capital for all the vintages, which are different over vintages, for example, the $R_{0t+1} = \frac{\gamma y_{0t+1}}{k_{0t+1}} + (1-\delta)\alpha_{0t+1}$, is smaller than $R_{1t+2} = \frac{\gamma y_{1t+2}}{k_{1t+2}} + (1-\delta)$. Because the increasing adjustment hazard in the inflation economy and along the balanced growth path, the adjustment ratio $\alpha_{0t+1}$is less than α_{1t+1} which is equal to one and at steady state, $\frac{y_0}{k_0} = \frac{y_1}{k_1}$. Full capital mobility induces a general equilibrium feedback that enlarges inequality: factor-price equalization requires capital to flow from the sector 0 to sector 1. Therefore labor force in the sector 1 with old technology are endowed with more capital, which boosts their relative wages, the labor forces in sector 1 will increase. Since the inflation makes the old sector capital cheaper.

The investment increase leads output to increase $y_j = i_j + C$ in the same way for the same vintage. The change of the output is significantly different vintage 1 from vintage 0, which is also caused by the changes of the capital stocks to the technology shock are all negative and but in absolute value are small compared with the changes of labor. The two production factors have different responses to the technology shock, which work oppositely in determining the outputs to technology shock. The vintage 1 output is positive, because the marginal product of labor is more positive therefore offset the negative marginal product of capital, overall the output shows a positive change of vintage 1. Similarly, the vintage 0 output is negative, because the labor marginal productivity is not positive enough to offset the negative marginal product of capital.

If the plants with new capital investment injection are embodied with the most advanced technology, they are more productive than the old ones. As the consequence the society can not provide each labor with a new machine all of the time and inequality will result. If different technologies reside in production tools and the plant and its labor can use just one technology at a time, a variety of productive tools will be in use at the same time, since each worker uses different vintage production tools therefore labor's productivities will differ. Because not everyone can be given the latest vintage productive tools all of the time. If machine quality and skill are complements, a worker who is with the best machine will acquire more skill therefore earn higher wages, which is different from the case in the paper, therefore inequality will persist.

The vintage capital model has a natural nonconvexity, when machines are indivisible. New machines are better than old ones. Under the assumption of complements between new technologies and skills, the new machines will be used by the most skilled workers. Therefore, the inequality will increase. Due to the nonconvexity, small differences in skills will be transferred to larger differences in productivities. This is contrary to the convex, where the improvement of existing machines by small increment takes place instead of producing some much better machines.

It is shown as a summary of the observation of output responses to all capital deviations under J=1 (figure 6.4)that to both shocks of capital 0 and capital 1 deviation influence both outputs first response negatively. Output 0 oscillates around stead state. Output 1 oscillates below the stead state. This means capital shock does not lead to the output increase.

The cases for J = 5 capital responses to all capital shocks are shown similarly as in figure 9. All investments response negatively to the capital shock.

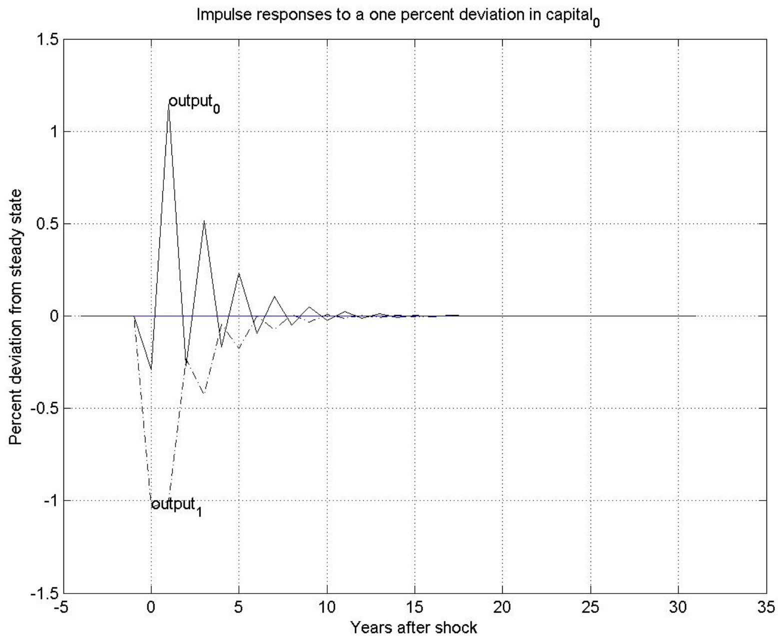

Figure 8: Long-term impulse response of outputs to capital 0 to 1 deviation J=1

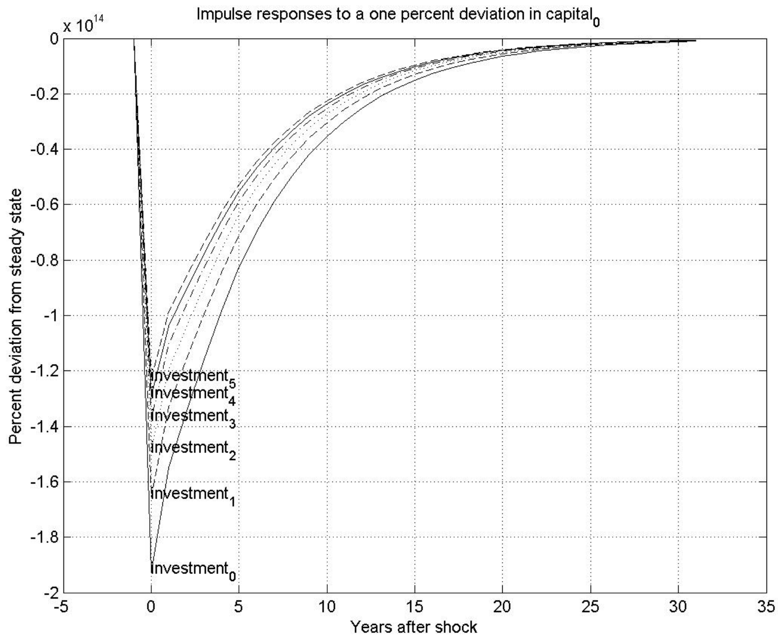

Figure 9: Long-term impulse response of capitals to capital 0 to 5 deviation J=5

		Autocorrelation Table J=1 (HP-filtered series) corr(v(t+j),GNP(t))						
	$\sigma\%$	t-3	t-2	t-1	t	t+1	t+2	t+3
popdensity0	0	-0.24	-0.19	-0.26	0.22	0.27	0.37	0.37
capital0	0	-0.41	0.72	-0.47	0.87	-0.52	0.24	-0.40
capital1	0	0.19	-0.40	0.73	-0.55	0.87	-0.63	0.30
value0	2.79	-0.27	0.56	-0.03	0.51	-0.05	-0.24	-0.22
output0	0	-0.48	0.57	-0.52	1.00	-0.52	0.57	-0.48
output1	0	0.22	0.19	-0.19	-0.35	-0.30	-0.39	0.28
labor0	0	-0.47	0.42	-0.47	0.96	-0.43	0.68	-0.48
labor1	0	0.20	0.11	-0.33	-0.32	-0.37	-0.20	0.33
wage	0	0.27	0.13	0.11	-0.48	-0.01	-0.66	0.28
adjratio	0	-0.41	0.12	-0.16	0.73	0.01	0.57	-0.33
investment0	0	-0.47	0.55	-0.57	0.99	-0.57	0.63	-0.48
investment1	0	0.20	0.20	-0.26	-0.30	-0.37	-0.31	0.26
consumption	0	0.22	-0.22	0.67	-0.56	0.71	-0.75	0.25
employment	0	-0.44	0.58	-0.63	0.97	-0.68	0.64	-0.48
popdensity1	0	0.20	0.08	0.42	-0.18	-0.08	-0.32	-0.44
value1	1.73	-0.27	0.56	-0.03	0.51	-0.05	-0.24	-0.22
technology	0	0.25	-0.60	0.09	-0.47	0.13	0.27	0.19

Table 1: Autocorrelation table J=1 (HP-filtered series),corr(v(t+j),GNP(t))

6.2 Business Cycle Simulation

In this section I examine the quantitative properties of the equilibrium fluctuation by numerical simulations. I study whether the different vintage sectors would add up in the model to the observed aggregate fluctuations and generate the business cycle patterns.

My aim is to reproduce the second moment structure of the business cycles. I particular, I want to explain the mechanism for the positive autocorrelation of the business cycle variables and the positive correlation between production and demand components. I remove the trend by Hodrick-Prescott filter with smoothing parameter $\lambda = 6.25$, since I assume each period is one year.

Table 1 reports the standard deviations and the cross-correlations with GNP for the HP-filtered series in the model. It can be used to study the fitness of the model to the real business cycle data. As it is shown, output with vintage 0 is totally correlated with the GNP in the real business cycle data, and most of other vintage 0 variables are positively correlated with the GNP. On the contrary, most of other vintage 1 variables are negatively correlated with the GNP. This makes sense, the up-to-date variables are most reliable to report the business cycle.

Simulations show that the right magnitude of fluctuations is obtained when the vintage capital with up-to-date technology. It shows that the correlation structure of the production and demand components match the empirical business cycle patterns.

Similarly the autocorrelation table for vintage = 5 is as the following[2]:

Business cycle simulation examines quantitative properties of the equilibrium fluctuation by numerical simulations. I ask whether the different vintage capital with different productivities oscillations would add up in our model to the observed aggregate fluctuations and generate the business cycle patterns. The answer is true when the vintage capital with the latest productivity technology. I explain the mechanism for the positive correlation of the business cycle variables and the positive correlation between production factor components.

Thomas [2002] adapts the neoclassical business cycle model allowing for the lumpy capital adjustments with individual plants. It confines the aggregate productivity shocks as the sole exogenous source of the fluctuation and concludes that the importance of market-clearing adjustment behavior does not play a substantial role in aggregate dynamics. The inclusion of lumpy plant-level investment does not significantly alter the equilibrium predictions of the traditional neoclassical equilibrium business cycle model. The general equilibrium effects dampens the fluctuation effects due to the (S,s) behavior via wage and interest rate.

I then implement this mechanism in a dynamic general equilibrium model to explore whether there is an investment driven business cycle and conclude that the investment has significant aggregate implication on the business cycle, which does not coincide with the result in section 6.4. Because their rigorous assumption of the market-clearing adjustment behavior.

6.3 Calibration-Parameters Choice

I calibrate the model at annual frequency. The value of the parameters used in the model are summarized in Table 3.

The adjustment cost ξ has a uniformly distributed cost, is independent draw from 0 to B, where the upper bound B 0.002 is chosen to match the observations by Domes and Dunne (1998).

1) Plants raising their real capital stocks by more than 30 percent (lumpy investors) comprise 25 percent of aggregate investment in the average year.

2) These investors constitute 8 percent of plants.

The capital share of output is 0.325 and the labor share of output is 0.58, as consistent with direct U.S estimates in King, Plosser and Rebelo (1988).

The rate of depreciation matches a long-run investment-to-capital ratio of 0.076 in Coo-

[2]due to the lack of enough time lags of the Euler equation the results are not 100 percent reliable

		Autocorrelation Table J=5 (HP-filtered series) corr(v(t+j),GNP(t))						
	$\sigma\%$	t-3	t-2	t-1	t	t+1	t+2	t+3
popdensity0	0.0023	-0.15	-0.24	-0.21	0.12	0.98	0.00	-0.40
popdensity1	0.0027	0.07	0.08	0.01	-0.19	-0.62	0.71	0.14
popdensity2	0.0033	0.12	0.20	0.19	-0.05	-0.70	-0.45	0.69
popdensity3	0.0022	-0.02	0.01	0.08	0.18	0.30	-0.64	-0.47
popdensity4	0.0055	-0.15	-0.25	-0.24	0.06	0.90	0.30	-0.53
capital0	0	0.14	0.24	0.22	-0.09	-0.97	0.08	0.37
capital1	0	-0.27	-0.27	0.05	0.97	0.26	-0.37	-0.32
capital2	0	0.06	0.08	0.04	-0.14	0.26	-0.78	0.27
capital3	0	0.23	0.23	-0.04	-0.80	-0.48	0.29	0.68
capital4	0	0.12	0.07	-0.14	-0.64	0.19	0.71	-0.28
capital5	0	0.17	0.18	0.01	-0.53	-0.47	0.70	-0.04
value0	0.0075	-0.27	-0.26	0.07	1.00	0.07	-0.24	-0.26
output0	0.0082	-0.27	-0.25	0.07	1.00	0.07	-0.25	-0.27
output1	0.0082	-0.27	-0.25	0.07	1.00	0.07	-0.25	-0.27
output2	0.0082	-0.27	-0.25	0.07	1.00	0.07	-0.25	-0.27
output3	0.0082	-0.27	-0.25	0.07	1.00	0.07	-0.25	-0.27
output4	0.0082	-0.27	-0.25	0.07	1.00	0.07	-0.25	-0.27
output5	0.0082	-0.27	-0.25	0.07	1.00	0.07	-0.25	-0.27
labor0	0.0006	0.25	0.21	-0.11	-0.99	-0.07	0.39	0.28
labor1	0.0006	0.25	0.21	-0.11	-0.99	-0.07	0.39	0.28
labor2	0.0006	0.25	0.21	-0.11	-0.99	-0.07	0.39	0.28
labor3	0.0006	0.25	0.21	-0.11	-0.99	-0.07	0.39	0.28
labor4	0.0006	0.25	0.21	-0.11	-0.99	-0.07	0.39	0.28
labor5	0.0006	0.25	0.21	-0.11	-0.99	-0.07	0.39	0.28
wage	0.0088	-0.27	-0.25	0.08	1.00	0.07	-0.26	-0.27
adjratio0	0.0067	0.28	0.28	-0.04	-1.00	-0.11	0.20	0.22
adjratio1	0.0070	0.25	0.23	-0.10	-0.99	-0.08	0.38	0.23
adjratio2	0.0035	-0.29	-0.31	-0.00	0.98	0.10	-0.09	-0.16
adjratio3	0.0367	-0.26	-0.26	0.05	0.94	0.27	-0.46	-0.21
adjratio4	0.0226	-0.12	-0.12	0.02	0.45	-0.30	0.57	-0.36
investment0	0.0077	-0.27	-0.26	0.07	1.00	0.07	-0.24	-0.26
investment1	0.0078	-0.27	-0.26	0.07	1.00	0.07	-0.25	-0.27
investment2	0.0079	-0.27	-0.26	0.07	1.00	0.07	-0.25	-0.27
investment3	0.0079	-0.27	-0.26	0.07	1.00	0.07	-0.25	-0.27
investment4	0.0080	-0.27	-0.26	0.07	1.00	0.07	-0.25	-0.27
investment5	0.0080	-0.27	-0.26	0.07	1.00	0.07	-0.25	-0.27
consumption	0.0088	-0.27	-0.25	0.08	1.00	0.07	-0.26	-0.27
employment	0.0008	0.30	0.38	0.15	-0.73	-0.70	-0.06	0.41
popdensity5	0.0042	0.15	0.24	0.23	-0.07	-0.92	-0.17	0.53
value1	0.0075	-0.27	-0.26	0.07	1.00	0.07	-0.24	-0.26
value2	0.0075	-0.27	-0.26	0.07	1.00	0.07	-0.24	-0.27
value3	0.0076	-0.27	-0.26	0.07	1.00	0.07	-0.24	-0.27
value4	0.0077	-0.27	-0.26	0.07	1.00	0.07	-0.25	-0.26
value5	0.0077	-0.27	-0.26	0.07	1.00	0.07	-0.24	-0.27
technology	0.0086	-0.27	-0.25	0.07	1.00	0.07	-0.26	-0.27

Table 2: Autocorrelation table J=5 (HP-filtered series),corr(v(t+j),GNP(t))

B	γ	ν	δ	ζ	β	ρ	σ_ϵ	$\bar{\alpha}_0$	$\bar{w}$	$\bar{Y}_0$	$\bar{Y}_1$
.002	.325	.580	.060	3.6142	.939	.9225	.0134	.95	5	2.6	2.9

Table 3: Parameters choice J=1

$\bar{\alpha}_0$	$\bar{\alpha}_1$	$\bar{\alpha}_2$	$\bar{\alpha}_3$	$\bar{\alpha}_4$	$\bar{\alpha}_5$	$\bar{Y}_0$	$\bar{Y}_1$	$\bar{Y}_2$	$\bar{Y}_3$	$\bar{Y}_4$	$\bar{Y}_5$
.059	.197	.337	.576	.782	1	3	3.5	4	4.5	5	5.5

Table 4: Parameters choice J=5

ley and prescott (1995).

The ζ for the preference for leisure means that 20 percent of available time is spent in market work King, Plosser and Rebelo (1988).

The discount factor β implies an average annual interest rate of 6.5 percent, which complies with long-run per-capita output growth of 1.6 percent per year King and Rebelo (1999).

The σ_ϵ and ρ of exogenous stochastic process for productivity is estimated from Solow residuals measured using Stock and Watson (1999) data on U.S. output, capital and total employment hours in 1953-1997.

6.4 The Empirical Study

In the empirical part of this paper the test of the correlation of investment and employment with the business cycle will be executed, therefore the conclusion is drawn that the investment has no significant aggregate implication.

Thomas [2002] adapts the neoclassical business cycle model allowing for the lumpy capital adjustments with individual plants. The different investment time changes can produce significantly large disturbances to the distribution of plants as to alter the path of aggregate investment demand relative to a business cycle model lacking establishment-level capital heterogeneity. The importance of market-clearing adjustment behavior may lead to the incorrect conclusion that the lumpy investment plays a substantial role in aggregate dynamics. But the inclusion of lumpy plant-level investment does not significantly alter the equilibrium predictions of the traditional neoclassical equilibrium business cycle model.

I then implement this mechanism in a dynamic general equilibrium model to explore whether there is an investment driven business cycle.

Title:	Real Private Fixed Investment, 1 Decimal
Series ID:	FPIC1
Source:	U.S. Department of Commerce: Bureau of Economic Analysis
Seasonal Adjustment:	Seasonally Adjusted Annual Rate
Frequency:	Quarterly
Units:	Billions of Chained 2000 Dollars
Date Range:	1947-01-01 to 2004-04-01

Table 5: Real private fixed investment

Title:	Real Gross National Product
Series ID:	GNPC96
Source:	U.S. Department of Commerce: Bureau of Economic Analysis
Seasonal Adjustment:	Seasonally Adjusted Annual Rate
Frequency:	Quarterly
Units:	Billions of Chained 2000 Dollars
Date Range:	1947-01-01 to 2004-04-01

Table 6: Real GNP

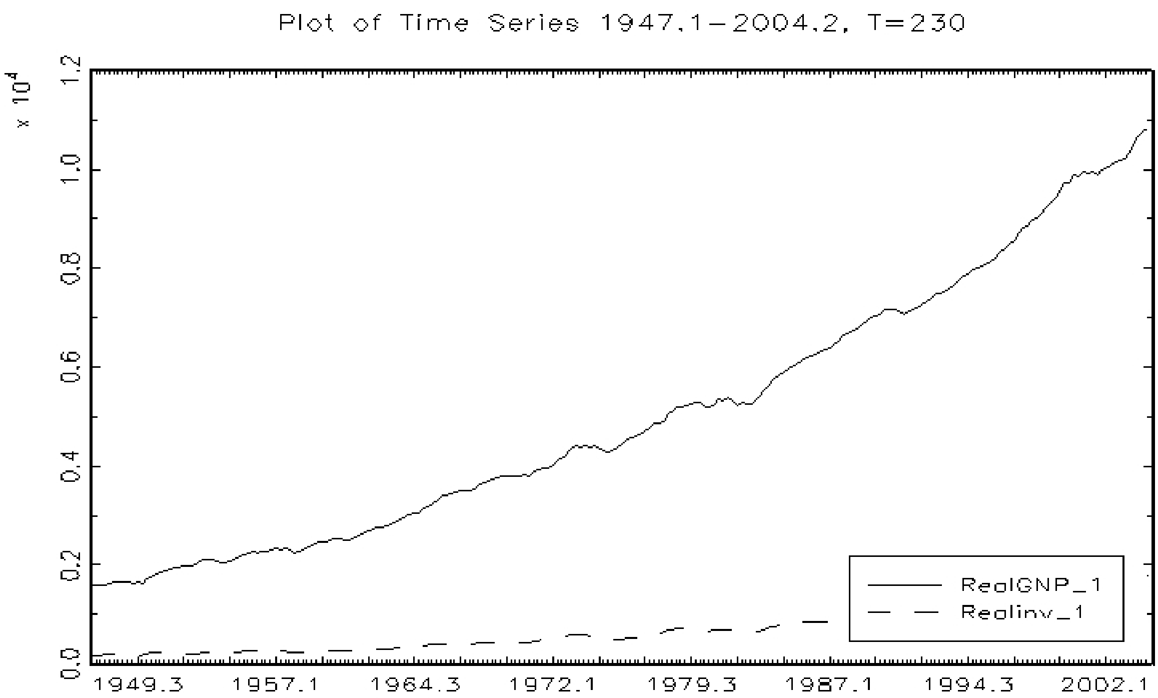

Figure 10: Relationship between real GNP and private fixed investment

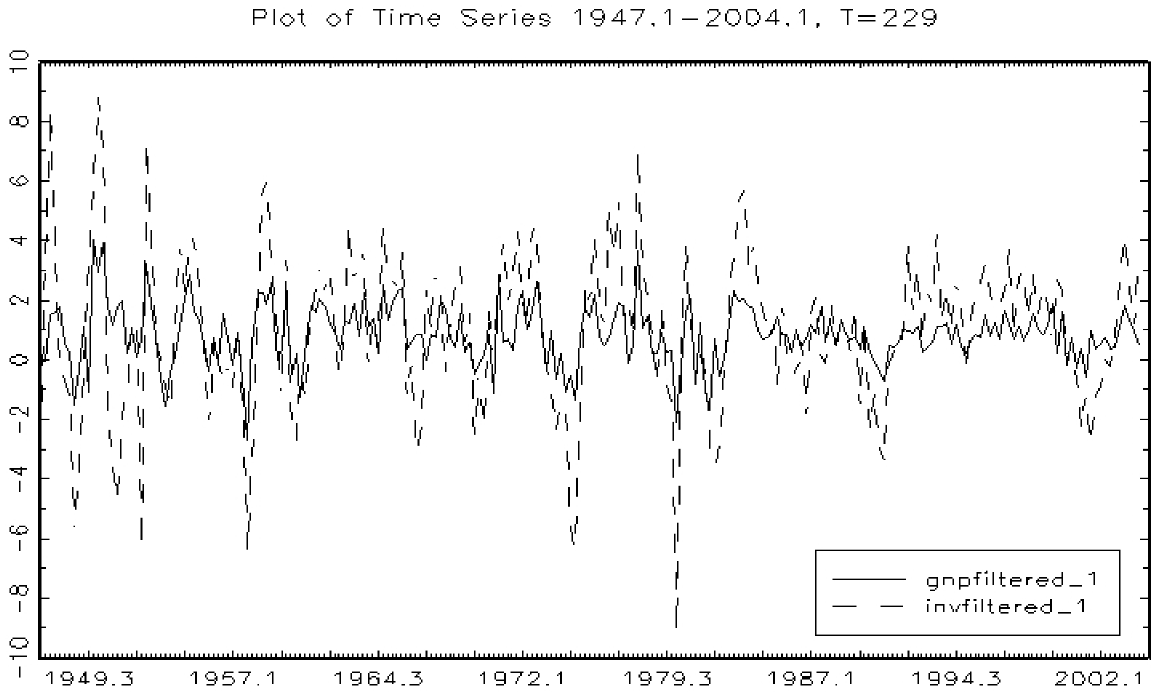

Figure 11: Relationship between real GNP and private fixed investment filtered

6.4.1 Mathematical Methods Applied-Linear Regression

In order to estimate the linear regression model as g(x,β)=β_1+β_2X....β_kLnX, where the β_1.....β_k are undetermined parameters I want to get.

Applying the Ordinary Least Squares (OLS) estimation of β:

$$S(\beta) \ = \ \sum_{t=1}^{T} (y_t - \beta_1 - \beta_2 x_t)^2$$

The minimum solution can be fulfilled by solving the First Order Necessary Condition of S(β) w.r.t β_1 ... β_k.

In order to establish the distribution rule behind the relationship of two random variables of the United State historical data: dependent variable (real gross national product) and explanatory variable (real private fixed investment), the OLS method is applied to estimate the coefficients β of the following model during the time period from 1947 quarter 1 to 2004 quarter 2.

$$GNP_i \ = \ \beta_0 + \beta_1 \times FPI_i$$

Coefficient and their significance test: Hypothesis test, two-side t-test.

The pair of hypotheses are:

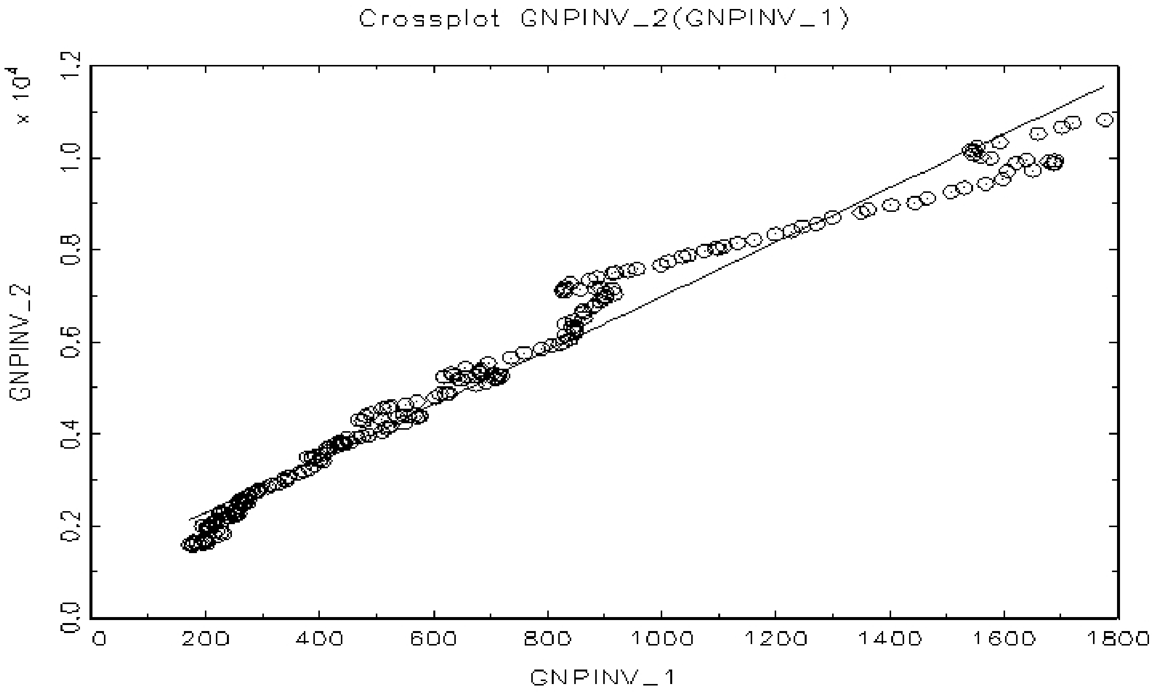

Figure 12: Relationship between real GNP and private fixed investment

$H_0 : \beta_1 = 0$ vs. $H_1 : \beta_1 \neq 0$

H_0 means there is no significant relationship between real gross national product and real private fixed investment.

Comparing the result $\frac{\widehat{\beta_1} - \beta_1}{Standard Deviation of X}$ applied to t-distribution with freedom degree (Number of observation - 1) with the critical value, the H_0 hypothesis can be accepted or rejected.

In the US case, the regression model is as following:

$$GNP = 1100,70 + 5,88 \times FPI$$

t-values = 19.9953, 85.2100
sigma = 458.0704
R-squared = 0.9696

This means that 1 percent change of real private fixed investment will lead to 5,88 percent change in GNP level. The critical value for the t-test $\alpha = 0.05$ freedom degree 229 is 1.645. The t-test result is 85.21, which is significantly larger than 1.645. Therefore I can reject the H_0 and conclude that there is a significant relationship between real private fixed investment level and real GNP.

ADF Test for series: $RealGNP_1$ sample range:[1947 Q4, 2004 Q2], T = 227 lagged differences: 2		
1%	5%	10%
-3.43	-2.86	-2.57

Table 7: Critical value for ADF test

The visual presentation of this regression line and the observation points refer to Figure 12, where the first variable is real GNP and the second is real private fixed investment. Figure shows us the strong correlated relationship between the long run dynamics of investment and business cycle in the model discussed in this paper.

6.4.2 Mathematical Methods Applied-Time Series Analysis and VAR Analysis

The visual presentations of these time series refer to figure 10 and the first difference as in figure 11 shows that they are stationary.

Augmented Dickey-Fuller (ADF) Test:

ADF tests are based on models of the form:

$$\Delta y_t = \rho y_{t-1} + \sum_{j=1}^{p-1} \alpha_j^* \Delta y_{t-j} + \epsilon_t \tag{70}$$

The pair of hypotheses are:

$H_0 : \rho = 0 \leftrightarrow \alpha = 1$ vs. $H_1 : \rho < 0$

The one-side t-test is based on the t-statistic of the coefficient ρ from an OLS estimation of equation 70.

H_0 is rejected if the t-statistic is smaller than the relevant critical value. If $\rho = 0$ (that is, under H_0) the series has a unit root and is non-stationary, whereas it is regarded as stationary if the null hypothesis is rejected.

value of test statistic: 3.20
Judgment from the above table 7 shows that t-test result 3.20 is significantly larger than the critical value at 1% level -3.43. I can not reject H_0 and conclude that there is a unit root and is not stationary.

In order to get the stable time series I make the first difference transformation $RealGNP_{d1}$ and get the following results:

value of test statistic: -6.1321

ADF Test for series:$RealFPI_1$ sample range:[1947 Q4, 2004 Q2], T = 227 lagged differences: 2		
1%	5%	10%
-3.43	-2.86	-2.57

Table 8: Asymptotic critical value for ADF test

Cointegration Rank	Test Statistics	5% Critical Value	1% Critical Value
lag=2			
0	104.25	20.16	24.69
1	3.79	9.14	12.53

Table 9: Johansen trace tests

Judgment from the above table 7 shows that t-test result -6.1321 is significantly smaller than the critical value at 1% level -3.43. I can reject H_0 and conclude that there is no unit root and is stationary. value of test statistic: 1.61
Judgment from the above table 8 shows that t-test result 1.6096 is significantly larger than the critical value at 1% level -3.43. I can not reject H_0 and conclude that there is a unit root and is not stationary.

After the first difference transformation the stable time series $RealFPI_{d1}$ and the following results are gotten:

value of test statistic: -4.7878
Judgment from the above table 8 shows that t-test result -4.7878 is significantly smaller than the critical value at 1% level -3.43. I can reject H_0 and conclude that there is no unit root and is stationary.

The above unit root tests show that, there are no unit roots in both first difference series $RealFPI_{d1}$ and $RealGNP_{d1}$. Therefore they are stationary and can be used in the causality test.

Johansen Trace Tests

Because Schwartz Criterion (SC) reports an optimal lag of 2, I report the test result as following table 9:

Lag 2 shows that cointegration rank of 1 is accepted at 5% critical value. Undoubtedly, this coincides with the cointegration relation and common trend shown on the time series plot - figure 10.

Causality Analysis:

Two vectors of endogenous variables y_{1t} and y_{2t}, are with dimensions K_1 andK_2 respectively, therefore $K = K_1 + K_2$. The vector y_{1t} is said to be Granger-causal for y_{2t} if it contains useful information for predicting the latter set of variables. For testing this property, a model of the form is considered:

$$\begin{bmatrix} y_{1_t} \\ y_{2_t} \end{bmatrix} = \sum_{i=1}^{p} \begin{bmatrix} \theta_{11,i} & \theta_{12,i} \\ \theta_{21,i} & \theta_{22,i} \end{bmatrix} \begin{bmatrix} y_{1_{t-i}} \\ y_{2_{t-i}} \end{bmatrix} + \begin{bmatrix} CD_t \end{bmatrix} + \begin{bmatrix} \epsilon_{1_t} \\ \epsilon_{2_t} \end{bmatrix} \quad (71)$$

In this model setup, y_{1t} is not Granger-causal for y_{2t} if and only if

$\theta_{21,i} = 0, i = 1, 2, \ldots p.$

Similarly, y_{2t} is not Granger-causal for y_{1t} if and only if

$\theta_{12,i} = 0, i = 1, 2, \ldots p.$

Therefore this null hypothesis is tested against the alternative that at least one of the $\theta_{21,i}$ is nonzero. A Wald test statistic, divided by the number of restrictions pK_1K_2 is used in conjunction with an $F(pK_1K_2, KT - n^*)$ distribution for testing the restrictions. Here KT is the total number of observations used for estimation and n^* is the total number of parameters in the system including the parameters of the deterministic term. F-version of this test is used because often leads to a better approximation of the desired size of the test. Of course, the role of y_{1t} and y_{2t} can be reversed to test Granger-causality from y_{2t} to y_{1t}.

The VAR(1) model and t-test results can be written as follows:

$$\begin{bmatrix} RealGNP_t \\ RealFPI_t \end{bmatrix} = \begin{bmatrix} 0.028 & 1.404 \\ 0.074 & 0.454 \end{bmatrix} \begin{bmatrix} RealGNP_{t-1} \\ RealFPI_{t-1} \end{bmatrix} + \begin{bmatrix} CD_t \end{bmatrix} + \begin{bmatrix} \epsilon_{1_t} \\ \epsilon_{2_t} \end{bmatrix} \quad (72)$$

T-test:

$$\begin{bmatrix} RealGNP_t \\ RealFPI_t \end{bmatrix} = \begin{bmatrix} 0.351 & 6.385 \\ 2.734 & 6.020 \end{bmatrix} \begin{bmatrix} RealGNP_{t-1} \\ RealFPI_{t-1} \end{bmatrix} + \begin{bmatrix} CD_t \end{bmatrix} + \begin{bmatrix} \epsilon_{1_t} \\ \epsilon_{2_t} \end{bmatrix} \quad (73)$$

The $\theta_{12,1}$ is larger than the t-test critical value of freedom degree 229 at $\alpha = 0.05$ 1.645, which means H_0 can be rejected and therefore conclude that there is significant influence real fixed private investment level on GNP.

The $\theta_{21,1}$ is smaller than the t-test critical value of freedom degree 229 at $\alpha = 0.05$ 1.645, which means H_0 can be rejected and therefore conclude that there is significant influence GNP on real fixed private investment level.

TEST FOR GRANGER-CAUSALITY:

Case 1:

H0: $RealFPI_{d1}$ do not Granger-cause $RealGNP_{d1}$

Test statistic l = 40.7698

pval-F(l; 1, 452) = 0.0000

Case 2:

H0: $RealGNP_{d1}$ do not Granger-cause $RealFPI_{d1}$

Test statistic l = 7.4724

pval-F(l; 1, 452) = 0.0065

All of the p-value are smaller than 0.05. Therefore using 5% as the significant level, all of the non-causality null hypotheses can be rejected. In other word, on the basis of these tests causal relation between the variables can be diagnosed with any certainty. However, stronger evidence of a Granger-causal relation from real GNP to real investment, because the p-value of the related test is larger than case one.

If there is a cointegration relation between two variables there must also be Granger-Causality in at least one direction. Similar to the very clear cointegration result, the causality tests also suggest a strong relation.

The cointegration analysis and a Granger-Causality analysis look at the data from different angles. In such a situation the view from one direction gives a similar picture as from another corner. The result of test shows that there is no conflict between the results from the cointegration analysis and the causality test.

Impulse response:

Impulse response analysis can be used to analyze the dynamic interactions between the endogenous variables of a VAR(p) process. In this analysis the exogenous and deterministic variables are treated as fixed and may therefore be dropped from the system. Saying in another way, the part of the conditional mean of the endogenous variables attributable to these variables is eliminated. The adjusted endogenous variables are now denoted by y_t. If the process y_t is stationary, it has a Wold moving average (MA) representation:

$$y_t = \sum_{i=0}^{\infty} \rho_i u_{t-i} \tag{74}$$

where $\rho_0 = I_K$ and the ρ_s can be computed recursively as:

$$\rho_s = \sum_{j=1}^{s} \rho_{s-i} A_i, s = 1, 2, ...\infty \tag{75}$$

with $\rho_0 = I_K$and $A_j = 0$ for $j > p$. The coefficients of this representation may be interpreted as reflecting the responses to impulses hitting the system. The (i,j)th elements of the matrices ρ_s, regarded as a function of s, trace out the expected response of $y_{i,t+s}$ to a unit change in y_{jt} holding constant all past values of y_t. The elements

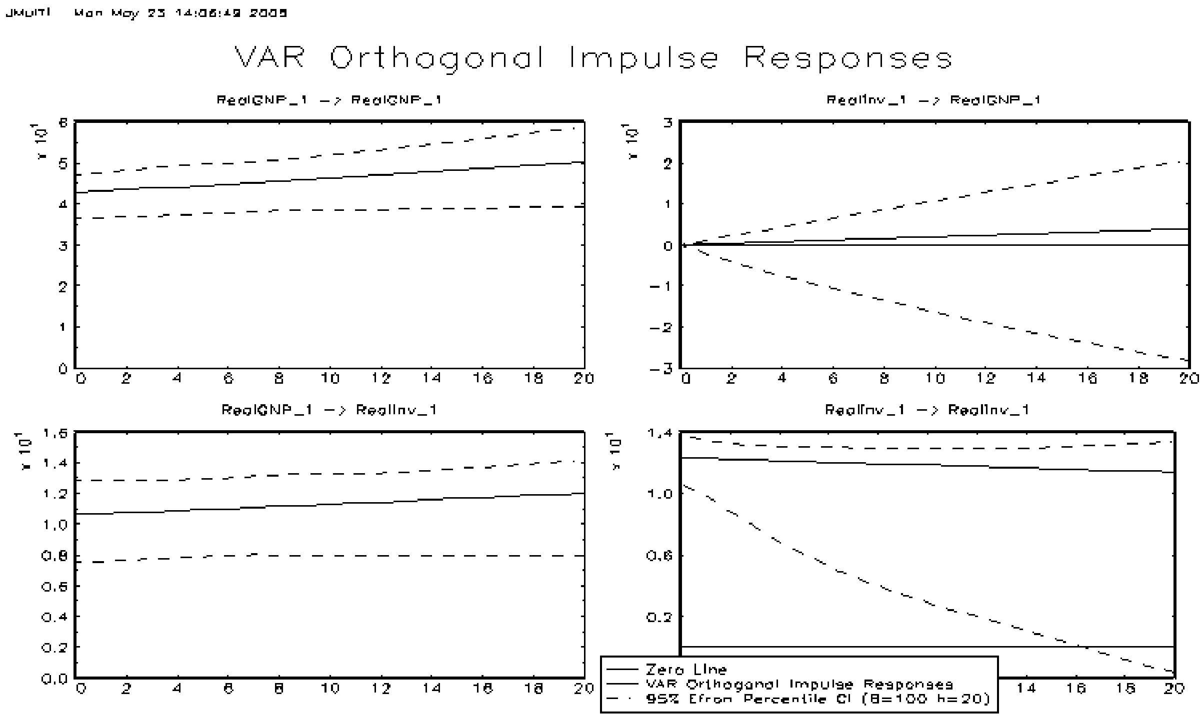

Figure 13: Impulse response between real GNP and real private fixed investment original

of ρ_s represent the impulse responses of the components of y_t with respect to the u_t innovations.

Given that the error shocks are instantaneously uncorrelated (orthogonal), the corresponding impulse responses are often referred as orthogonalized impulse responses.

From the four graphs of the impulse response analysis of the original time series (figure 6.4.2) and the those of first difference time series (figure 6.4.2), though origianl time series show no significant influence real private fixed investment on real gross national product and there is a significant influence real gross national product on real private fixed investment, first difference time series indicate the significant influence both real GNP on private fixed investment and vice versa significant case. I can conclude that there are significant influences both real GNP on private fixed investment and real private fixed investment on real GNP.

From the results of all methods mentioned above, I get the conclusions that, there are significant relationship between lumpy investment with business cycle and vice versa. Therefore individual fixed investment activities play significant role in aggregate dynamics.

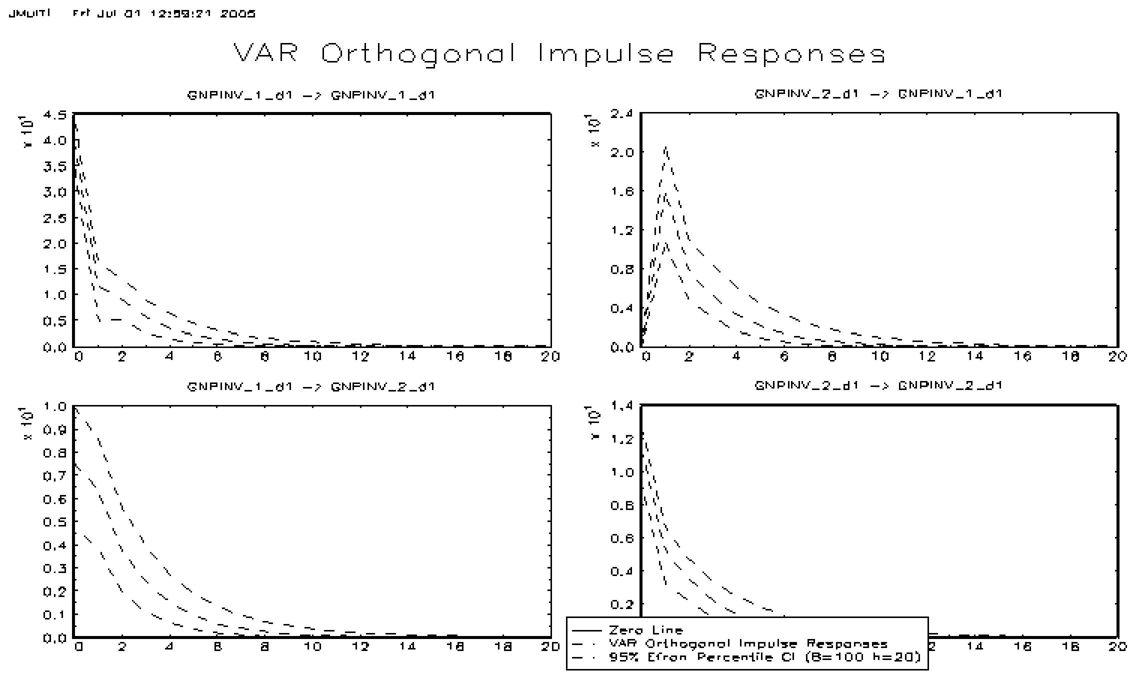

Figure 14: Impulse response between real GNP and real private fixed investment first difference

7 Variations

7.1 The Growth Accounting

X_t is the trend component and envolves with growth rate $\Theta_A = 1 + \gamma = g_A$. The technological prontier advances each period at rate $\gamma > 0$.

$$A_t = (1+\gamma)A_{t-1} \tag{76}$$

The steady state capital-output ratio (i.e. the measurement of capital intensity) is determined by:

$$\frac{k}{y} = \frac{s_k}{g_n + g_A + \delta} \tag{77}$$

The $\frac{k}{y}$ is the capital-output ratio, which is same over all vintage plants in the model discussed in this paper. It is a useful measure tool to study the equilibrium of long-run growth, in which every factor is growing together, at the same proportional rate is named as one of steady-state balanced growth. If everything is growing together, the relationships between key quantities in the economy are stable in an equilibrium . g_n is the labor force growth rate. g_A is the efficiency of labor growth rate and productivity growth rate. Technological progress increases AN, which we can treat as the amount of effective labor, or labor in "efficiency units" in the economy. Because output, capital, and effective labor all grow at the same rate, $(g_A + g_n)$, the steady state of the economy is also called a state of balanced growth.

As a simplifying assumption, the economy can keep its savings-investment rate s at any level it wishes by setting proper fiscal and monetary policies. By maximizing Steady-State Consumption per worker and setting the rate of change of consumption per worker equal to zero on the long-run growth path, the economy can achieve its "golden rule" savings rate s = γ , the power of capital in production function. The marginal product of capital $\gamma \frac{y_j}{k_j}$ on the steady-state growth path equals to $g_n + g_A + \delta$ and is constant over vintages, which means when steps to increase the savings rate to boost the capital stock by one unit, production increases amounts to $g_n + g_A + \delta$ units.

The steady-state equilibrium takes the following form: workers accumulate human capital at a constant rate. This determines the growth rate of the economy. The distribution of both machine quality and worker skills is invariant over time. The new vintages also take place at a constant rate. When a new vintage arrives, the most skilled worker abandons his machine and switches to the best one. Thereafter the second best worker gets the machine just abandoned by the most skilled one, and so on. This process continues until the lowest skilled worker scraps his machine.

The inequal income per head of two economic entities stems from three sources:

Endowments: One economy grows up from different soil: different physical and human capital more natural resources. This determines the comparative advantage of the economy.

Luck: A summary of exogenous shocks such as wars and famines or sectoral shocks influencing the economic policy.

Compensating differentials: The citizens' sacrifice contribution to a high per capita income of a developed economy is a reward for it.

If the inequlity comes from the first class, each economy's development path is determined by its initial comparative advantage.

The inequlity of the economy I am interested in is the long run one regardless of where it stems from. In the vintage capital model, the economy can not replace all its old capital at each date, instead they renew their capital step by step, and different technologies exist at the same time.

How does the labor force growth influence the balanced growth rate? The faster the growth rate of the labor force, the lower the economy's stead-state capital-output ratio will be. Because each new worker who joins the labor force must be equipped with enough capital to be productive and to on average match the productivity of his or her peers. The higher the growth rate of the labor force, the larger the share of current investment that must go to equip new members of the labor force with the capital they need to be productive. Thus the lower will be the amount of investment that can be devoted to building up the average ratio of capital to output.

A sudden and permanent increase in the rate of growth of the labor force will lower the level of output per worker, which is same over all vintage plants, on the steady-state growth path. How large will the long-run change in the level of output be, relative to what would have happened had population growth not increased?

The growth rate of Solow residual(also known as total factor productivity, TFP) of the one-sector neoclassical growth model is determined by $g_A = g_y - \gamma g_k - \nu g_n$, and in balanced growth path $g_y = g_k$. The new assumption of constant return to scale ensures a constant growth rate along the balanced-growth path.

This aggregate TFP measures neither provides the information on the specific sources or nature of the technical change nor distinguish between the different ways in which technology grows.

7.2 The Sector-Specific Productivity Growth and Factor-Specific Productivity Growth

Past 30 years technological change experiences of developed countries have witnessed the fact that the technological change originates from particular sectors of the economy and has favoured particular inputs of production. Specificly, there is an accelerated decline of price of equipment capital relative to the price of consumption goods and a substantial increase in the wage of highly educated labors with respect to less educated labors. Therefore the following specification are made by Greenwood Hercowitz and Krusell[1997]:

First, technology growth may differ across final output sectors.

Second, technology may affect differently on the productivity of different input factors.

The technical shock affects the production structure in an asymmetric way and induces two different growth features:

First, sector-specific productivity (SSP) growth. It increases the productivity of the sector that produces with new capital equipment, therefore its production becomes more beneficial.

Second, factor-specific productivity (FSP) growth. It favors skilled and educated labor disproportionally.

Based on the Solow growth accounting methodology in the one-sector neoclassical growth model, I specify a sector-specific productivity (SSP) accounting and its application to the capital-embodied technical change.

Following Greenwood Hercowitz and Krusell[1997] a two-sector model is introduced for extension. The economy is composed of two sectors, one is the consumption producer and another is new capital producer. They have the different production from this paper with constant return Gobb-Douglas technology. The plants in two sectors with the production function as the following:

$$\begin{aligned} Y_c &= A_c F(k_c, n_c) && (78) \\ Y_x &= A_x F(k_x, n_x) && (79) \end{aligned}$$

The economy total factor inputs can be expressed by sum across sectors:

$$\begin{aligned} k &= k_c + k_x && (80) \\ n &= n_c + n_x && (81) \end{aligned}$$

A_c is defined as SSP for consumption sector, or in other way, neutral, disembodied, or aggregate technical shock. A_x is defined as SSP for capital sector, or investment-specific technical shock. Since the identical factor substitution properties in the two sectors, its relation with the one sector economy with the exogenous changes in the relative price of investment goods can be derived in the following way:

$$y = c + i = c + \frac{xA_c}{A_x} = A_c[\frac{x}{A_x} + F(k_c, n_c)] = A_c[F(k_x, n_x) + F(k_c, n_c)] \quad (82)$$

The $q = \frac{A_x}{A_c}$ is defined as the productivity level (quality) embodied in new vintages of capital, which also can be used to connect the two-sector economy with one sector and accounting for quality improvements in new products:

$$y = c + \frac{x}{q} = A_c F(k, n) \quad (83)$$

Equation 83 gives us the definition of aggregate output, where A_c is defined as SSP for consumption sector,or in other way, neutral, disembodied, or aggregate technical shock.

Using the q defined previously, the relative productivity of the investment goods sector is also called capital-embodied (or investment-specific) technical change. For example, investment-specific technological changes include the introduction of more efficient software, more powerful computers or more convenient means of telecommunication and transportation. On the contrary, the improvements in accounting techniques or in the organization of production, marketing and management control can be taken as examples of neutral technological progress. The shock bringing about investment-specific technological advances is defined as q shock, whereas the shock bringing about the neutral technological progress is the z shock. The higher q implies a fall in the cost of producing a new unit of capital in terms of consumption.

In this way, equation 3 can be rewritten as the law of motion for capital in efficiency units:

$$k_{0,t+1} = (1 - \delta)k_{jt} + i_{jt}q \quad (84)$$

The average unit of productive capital in the economy at time t embodies a technology with productivity Q_t, defined as:

$$Q_t = \sum_{j=1}^{\infty}(1-\delta)^j q_{t-j}\frac{i_{t-j}}{k_t} \quad (85)$$

i and k denote investments and the capital stock in units of consumption. Q_t is the ratio between capital stock measured in efficiency units (adjusted for quality) and the capital stock not adjusted for quality. The technology gap is defined as: $\frac{q_t - Q_t}{Q_t}$, whose size can be thought of an indicator for shifts in the relative demand of skilled workers.

Following Michelacci and Lopez-Salido [2005], I introduce the parameters a_z and a_q, quantifying over the unit interval the extent to which firms can upgrade their neutral and investment-specific technology without replacing part of the current workforce. Jobs destruction occurs when their technology or capital stock become too obsolete relative to the current leading technology or the quality of new capital. The capital stock of the job is recovered and the worker may be employed in another job after the jobs are destroyed.

When the final output is produced in jobs which consist of firm-worker pairs and a worker can be employed in at most one job, the newly created jobs are always embodied with a leading technology z_t of that time. Old jobs may be incapable of upgrading their previously installed technologies. In another expression, old jobs can only adopt the current leading technology with probability $a_z \in [0,1]$, but with probability 1 - a_z adopt the current period job's neutral technology, z_{it}. The job technological gap is defined as the difference between the leading technology z_t and the job's neutral technology z_{it} , $\tau_{it} = z_t - z_{it}$.

The sector producing capital is perfectly competitive and it can produce one unit of quality adjusted capital at marginal cost e^{-qt} , which is also the price of a new capital unit at time t. An old job in operation can adapt its capital stock to reap the benefits of the most recent advance in capital quality only with probability $a_q \in [0,1]$. Otherwise, the job makes use of the capital stock inherited from the previous period with probability $1 - a_q$.

If a_z and a_q are both equal to zero, the model is a standard vintage model where technological progress is entirely embodied into new jobs. While if a_z and a_q are both equal to one, the model corresponds to a standard real-business-cycle model, where technological progress is new-jobs disembodied.

The usual approach to study the SSP is to define the aggregate output growth as a revenue-weighted sum of sectoral output growth rates, i.e. a Divisia index (see Jorgenson [2001] for more). Expressed in the Divisia-aggregator approach, the aggregate TFP growth can be written as the revenue-weighted sum of sectoral TFP growth.

7.3 Frictional Labor Markets

The model presented so far features an aggregate production technology, whose production structure is centralized, and competitive labor markets. When study of a slightly different economy is necessary, where a frictional model of the labor market requires departing from both attributes and moving towards a decentralized production structure and a labor market with imperfect coordination between workers and firms in the matching process, a function modelling the matching process can solve this problem. This model gives rise to frictional equilibrium unemployment and "frictional equilibrium inequality". In frictional inequality the wage dispersion is purely an artifact of frictions and it would disappear when without frictions. A useful way to think about this phenomenon is to introduce the concept of "return to labor market luck".

Pissarides [2000] uses the random matching model of the labor market. The existent frictions create a bilateral monopoly as a result of a meeting between a vacant firm and a worker. Wages are determined by bargaining over total output, hence more productive firms will pay more, which create wage dispersion among ex-ante equal workers.

The matching function is defined by the combination of two arguments, one is the number of workers looking for jobs, i.e. the masses of unemployment workers u, and the number of firms looking for workers, i.e. vacancies v. A worker may be either employed or unemployed but only unemployed workers search for jobs. As an assumption, the on-the-job search does not influence the equilibrium rate of unemployment. Vacant jobs and unemployed workers match to each other according to the prevailing matching technology. The unemployment in the steady state is caused by the unmatched job-worker pairs and new break up of existing jobs during the matching process.

n is the workers in the labor force. u denotes the unemployment rate, which is a fraction of unmatched workers and v is the number of vacant jobs as a fraction of the labor force, i.e. the vacancy rate. Only the un unemployed workers and vn job vacancies engage in matching. $\frac{v}{u} = \theta$ ratio is defined as the separate variable measuring the tightness of the labor market which is out of firm's control. The rate of the vacant jobs becoming filled is $q(\theta) = m(1, \frac{1}{\theta})$. The supply of jobs (adjusting employment) of the firm is with linear costs of adjustment that depend on the tightness of the market.

The firm's labor force changes according to its vacancy rate:

$$\dot{N} = q(\theta)V - sN \tag{86}$$

sN is the rate at which the firm loses its workers. $q(\theta)$ is the rate of return of each vacancy to a worker. V is the number of the firm's vacancies and is within each firm's choice. $\dot{N} = g_n N$.

In the steady state without growth, i.e. $\dot{N} = 0$, the equation 86 changes to:

$$\frac{V}{N} = \frac{s}{q(\theta)} \tag{87}$$

With $V = \theta un$ and $N = (1-u)n$ the equation 86 converts to:

$$\frac{\theta u}{1-u} = \frac{s}{q(\theta)} \tag{88}$$

If the labor force grows at rate g_n, in the steady state total employment and unemployment also grow at g_n. The flow to unemployment consists of workers who have lost their jobs, $s(1-u)n$ and of new entrants, $g_n n$. The flow out of unemployment consists of workers who find jobs, $q(\theta)\theta un$, where $q(\theta)\theta = p(\theta)$ is the probability of an unemployed worker finding a job and increasing in θ. In the steady state inflows exceeds outflows by $g_n un$. In equilibrium:

$$s(1-u)n + g_n n - q(\theta)\theta un = g_n un \tag{89}$$

$$u = \frac{s + g_n}{s + g_n + q(\theta)\theta} \tag{90}$$

Growth in the labor force raises both unemployment and vacancies, with labor-augmenting technical progress and labor-force growth, the capital stock, real wages, and the capital to labor ratio grow in the steady state, but the rates of unemployment and job vacancies are constant. Faster labor force growth implies higher rates of unemployment and job vacancies but faster rate of technical progress brings about a lower rate of unemployment and a higher rate of vacancies.

The increase in monetary growth raises inflation in the steady state, given real interest rate, which raises the nominal interest rate. The raise in savings increases the capital to labor ratio and reduces the real rate of interest. The fall in the real rate of interest raises labor market tightness, θ, which will reduce the unemployment in equilibrium. The rise in the rate of monetary growth has the effects: capital rises, unemployment falls, real wage rise, and so does the number of the job vacancies.

The homogeneous long-run equilibrium model that is consistent with the existence of a constant unemployment rate when there is real in a standard neoclassical model of economic growth, where on a balanced-growth path the rate of unemployment is constant. The constant rate of unemployment is in equilibrium a function of all the real and nominal rates of growth.

I am concerned with how does technological change affect unemployment in this model and wage inequality.

There are two distinct approaches characterizing how equilibrium unemployment reacts qualitatively to variations of the rate of technological change within a matching model with vintage capital.

Aghion and Howitt [1994] and Mortensen and Pissarides [1998] pioneered the research on the relation between embodied productivity growth and unemployment in a frictional labor market. In their models new capital adjustment is frictionless, thus vacancies all consist of the newest capital.

Aghion and Howitt [1994] discussed the new and more productive equipment entry in the economy exclusively through the creation of new matches. The entry has a Schumpeterian "creative-destruction" effect caused by not upgradable existing matches. New capital competes with old capital by making it more obsolete and tends to destroy existing matches, because workers are better off by being separated from their old matches to search for the new firms endowed with the most productive technology. Due to a higher job-separation rate, unemployment tends to go up as growth accelerates.

Mortensen and Pissarides [1998] proposes an alternative view that the new technologies enter into existing firms through a costly "upgrading" process of old capital without inducing the destruction of the match. Because upgrading of the existing machine is costly, while destroying the job and opening a vacancy with the new capital requires only the search costs, it remains true for the frictionless case. In the extremely costless upgrading case, though the new technology is carried in equipment, it is the model of disembodied technological change. Faster growth does not influence on the job-separation rate but job creation affects. When the upgrading cost is in small value, unemployment falls even in faster growth. This is attributed to the capitalization effect, where investors are encouraged to create more vacancies, knowing that they can incorporate and benefit from future technological advances at low cost.

In the search model, high wage dispersion makes workers very demanding and increases unemployment spans. Therefore in equilibrium high wage dispersion could only coexist with long unemployment durations. I analyse the random matching model to find how technological progress affects on frictional inequality.

Mortensen and Pissarides [1998] looks at equilibrium outcomes for job creation and job destruction when new technology is embodied in new machines. Their focus of analysis is the resulting relationship between unemployment and the rate of growth in productivity led by technical progress. This study satisfies the further requirement of study the relationship of unemployment rate and economic growth rate, which is lack of evidence in previous empirical study as stated in section 7.4.3. When new technologies arrive the old jobs are destroyed, but the old jobs are replaced by new more productive employment opportunities subsequently and new technology stimulates job creation at a given wage. The fact that new jobs are created when old ones are destroyed does not necessarily imply that the economy will settle at a higher or even the same level of employment. They resolve the empirical conflict by a more general equilibrium model in which the number of jobs is determined by the interaction of job-creation and job-destruction decisions.

The new jobs embody the most advanced technology. The productivity of a new job grows at a constant rate, the rate of technical progress, γ, which is also the job creation rate. Once a job is created, the firm has three choices at his disposal:

It continues to produce with the technology embodied in the job as it was created; or

Job updating: the firm pays a fixed renovation cost to update its previous technology and continues producing without changing its workers; or

Job destruction: the firm shuts the business down and exits production. It either leaves the market or reenters with a new job vacancy and its worker becomes unemployed.

Jobs are destroyed and replaced by new ones, when technological progress brings about structural changes that make it unprofitable for existing job matches to continue operating.

The decline and growth of the sectors are induced by technical change. The jobs in the declining sectors shut down and new unrelated jobs in the expanding sectors take their place. Workers can search for new jobs in the expanding sectors after they lose them in the declining sectors. In the similar way, migration takes place after the employers losing their place from the declining sectors to the expanding ones. Under the assumption of perfectly mobile factor, there is also the possibility that the Schumpeterian notion of "creative destruction" occurs, where the destruction of outdated job matches and the creation of new ones is cheaper than the migration of the job and worker pair to an expanding sector. Under the employers never updating assumption, when the technological progress takes place at a higher rate, the useful life of a job becomes shorter and the faster growth leads to higher unemployment. Because an increase in growth may reduce the duration of a job match, which raises the equilibrium level of unemployment directly by raising the job-separation rate and indirectly by reducing the job vacancies' creation and reducing the job-finding rate.

Another effect of growth on unemployment is capitalization effect, which means an increase in growth raises the return rate from creating a promising firm and increases the capitalized value of those returns. Therefore more firms are encouraged to enter. Under the employers continually updating assumption, job creation is positively affected by the rate of technological progress.

In Mortensen and Pissarides [1998]'s general model, the direction of the effect of productivity growth on unemployment depends only on the size of the cost of updating. There is a critical renovation cost. When the actual cost of updating a job's technology is below the critical value, faster growth decreases unemployment and unemployment increases in the opposite case. When renovation costs are sufficiently high, the effect of growth on job creation is negative.

Their model implies of match-specific heterogeneity and factor mobility for the relation between unemployment and productivity growth. They show that when there is an idiosyncratic component to the output of a job-worker pair, employers with high idiosyncratic output (signifying a good employer-employee match) which is a valuable resource the firm preserves updating its technology from time to time. Those with low idiosyncratic output eventually destroy their jobs. The frequency of renovation increases with the idiosyncratic component of match product. They study an economy with two sectors at each extreme of the renovation cost range to specify the implications of factor mobility. As a result, jobs are destroyed gradually in one sector due to technology obsolescence, while firms in other sector continuously update technology. When mobility of both labor and capital is costless between the sectors, higher equal productivity growth in both sectors leads to a shift of resources from the high renovation costs sector to the low costs one. Because of the changing composition of employment, it is possible for the economy's overall destruction rate to go down at higher productivity growth. Job-creation rates can also be higher overall, despite in the creative destruction sector the creation rates is lower.

On the contrary to Aghion and Howitt [1994] and Mortensen and Pissarides [1998] the key new feature of Hornstein, Krusell and Violante [2002] implies the existence of vacancy heterogeneity, i.e., vacancies differ with respect to the quality of the equipment on the job.

This vacancy heterogeneity is important for two reasons.

First, the vintage structure is a purely frictional phenomenon in the model: When the capital matches with a worker, it ages until results a break-up from the capital becoming too obsolete relative to the worker's outside option. When the matching process becomes more and more instantaneous and the friction becomes weaker, the separation occurs earlier until at the end of frictionless matching all capital is new and vintage effects disappear. Although their analysis has several common features as Aghion and Howitt [1994] and Mortensen and Pissarides [1998] studies, they model capital adjustment procedure differently. They model the capital adjustment is costly and once capital has been purchased, it is natural to use it until their can alternatively work with newer capital elsewhere more efficiently or due to its obsolescence. Therefore the capital unit has a natural life-cycle. Labor market frictions make the life-cycle of capital longer because it is costly matching for a worker to find new capital to work with. There may be an unproductive period of unemployment for a worker to endure. In contrast to the frictionless model, capital is used for a strictly positive time period before being scrapped. The model is the extension of the standard competitive vintage capital as Solow [1960] growth model to an economy with labor market frictions. They study how technology change affects equilibrium unemployment by extending the Solow [1960] vintage capital model to an economic environment with frictions in the labor market, since a non-degenerate vintage distribution survives even when matching frictions vanish in the limit, as long as investment in a new machine is costly. It shows that a growth of capital-embodied technological change rate together with different labor market institutions can explain over half of the different rise in the unemployment rate between the United States and European. In the model, shocks and policies interact through a reduction of firms' labor demand, which is much sharper when institutions are rigid as in Europe. The firm's outside option exists, which reduces the match surplus proportionally to the firm's meeting rate. Therefore the embodied productivity growth rate changes have an impact on the equilibrium meeting rates, which affect the surplus through this new option. Moreover the technical progress changes rate affect the equilibrium age distribution of vacancies through the worker's outside searching option.

Heterogeneity vacancy makes it possible to analyse the chief features of the equilibrium. The equilibrium represents the economy with job creation curve and job destruction curves in the two-dimensional space defined by the age of capital at destruction and the labor market tightness. The clear shifts of the two curves following a permanent rise in the rate of embodied productivity allows qualitative description of the response of unemployment, inequality, and income shares. An economy with unemployment benefits is more likely to respond to such a faster productivity growth rate with a rise in unemployment duration. A laissez-faire type economy is more likely to respond to faster productivity growth rate through the life-length reduction of capital and more job separations.

Michelacci and Lopez-Salido [2005]'s model is just a version of the Solow [1960] growth model where the labor market is subject to search frictions and technology adoption is sluggish, so that the existing productive units may fail to adopt the most advance technology. The neutral technology change to the matching process leads to an increase in job destruction, job reallocation and unemployment, accompanied with output, consumption and investment gradually increase.

Their analysis supports the view that neutral technological progress prompts waves of Schumpeterian creative destruction, where technologically obsolete productive units are gotten ride of from the productive system. After an explicit capital vintage structure is introduced into the model, a substantial proportion of old jobs upgrade their capital equipment and reap the benefits in the quality of new equipment of the most advanced technology progress. As the case in the standard neoclassical growth model, investment-specific technology shocks lead to an economic activity expansion. The past twenty years have been marked by very rapid capital-embodied technological change, and the influence on workers can be expressed in their acquirement of most advanced skills induced by investment-specific technological change, such as operation of a more powerful computer or more efficient means of telecommunication and robotization of assembly lines. Instead renewing their employees in the technology renewed job, firms can upgrade their capital equipment step by step.

Conversely, the neutral technological changes only influences some specific workers, who can get accustomed to the new routines and discipline associated with changes in accounting techniques or in the organization of production, marketing and management control. Therefore the neutral technological changes at least partly require replacing the firm's current labor force with more suitable employees.

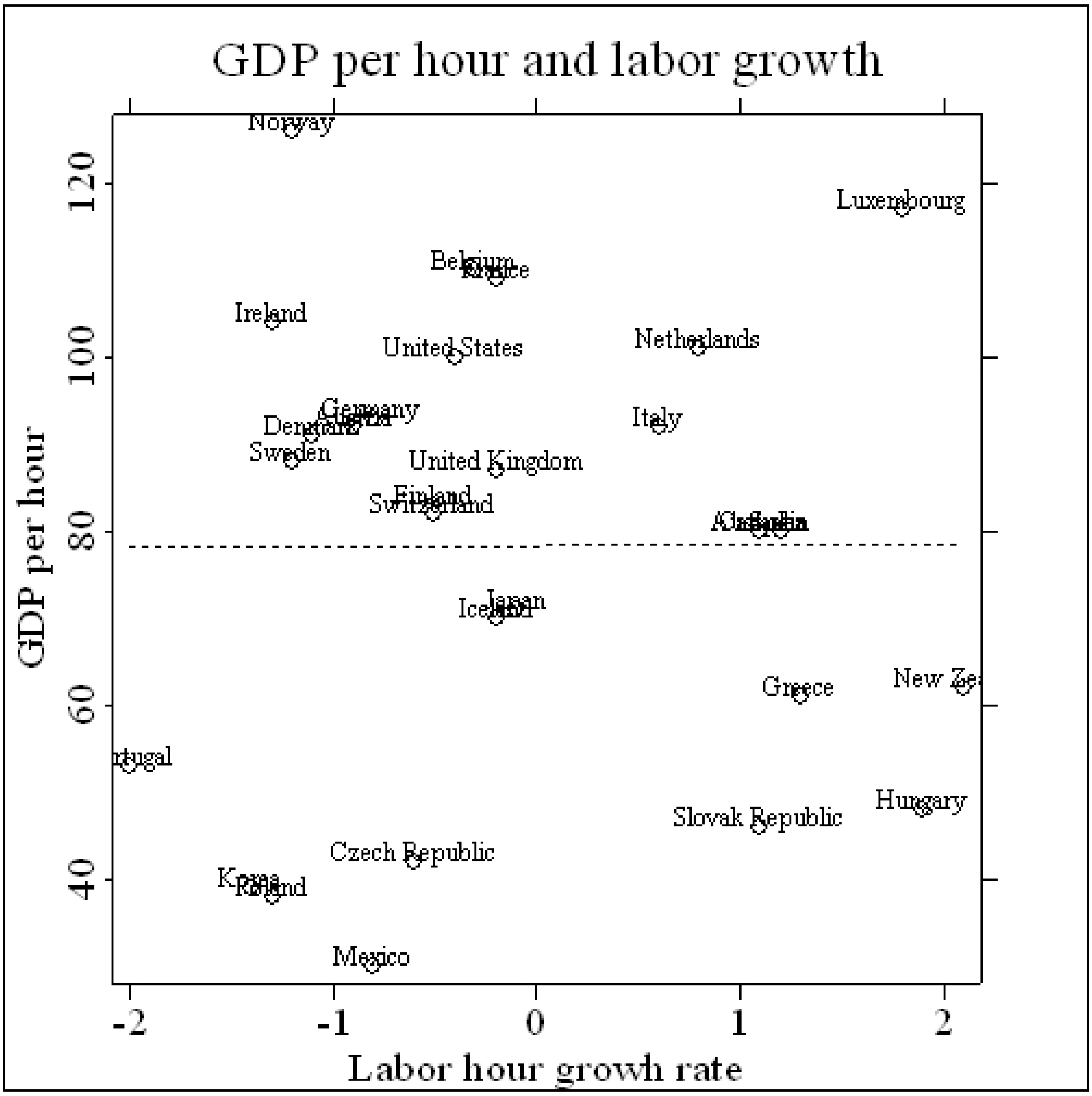

Figure 15: GDP per hour and labor hour growth rate in 2003

7.4 The OECD Empirical Evidence

7.4.1 The OECD Population Growth

The GDP per capita OECD estimated data dated February 2005 and Growth of Total Hours Worked data from OECD Productivity Database dated February 14, 2005 are used to get the figure 15. It helps us measuring whether this principle works in the real world. Does a high growth rate of total hours worked play a role in making countries relatively poor not just in economists' model but in reality? It turns out that it is important sometimes. As GDP per hour worked levels and total labor hours growth figure shows, of the twenty-nine OECD countries (exclude Turkey) in the world, six countries are with GDP per hour worked levels larger than that of the US level, Belgium, France, Ireland, Luxembourg, Netherlands and Norway, they are all with negative labor hour growth rate except Luxembourg. Three countries Hungary, New Zealand and Slovak Republic have half of the GDP per hour worked level as the US level, and they are all with very highly positive total labor hours growth rate. When considering the overall relationship between hours worked per week per worker and output per person, data based on the OECD finds no real relationship: some countries where workers work fewer hours have higher labor productivity than those who work more hours, and some have lower. Australian workers, for example, work more hours than Canadian workers and also have higher productivity; but work more hours than Italian workers and have lower productivity. Anyway, the additional investment requirements imposed by rapid labor force growth are a powerful reducer of capital intensity and a powerful obstacle to rapid economic growth.

The economic idea was introduced into economics late by Thomas R. Malthus, who was to become the first academic professor of economics at the East India Company's Haileybury College, that is one of the oldest ideas in economics that increase in technology inevitably run into natural resource scarcity, and so lead to increases in the numbers of people but not in their standard of living of productivity.

In Malthus's world inventions and higher living standards led to increases in the rate of population growth. The faster rates of population growth incresed natural resource scarcity and reduced productivity until once again people were so poor and malnourished that population growth was roughly zero.

The OECD countries convergence, the benefit comes from the population growth rates fall. Their healthy investment rates contribute to their becoming richer. All these factors boosted their steady-state capital output ratios. The main sources of variation in output per worker comes from the divergence of their respective steady state capital output ratio. It also led by their openness to new technologies enhancing the efficiency of labor and the labors' education level. The education level ensures the labor to invent new and adopt foreign born knowledge.

The solow model prediction works in the same way with nations that whether economies

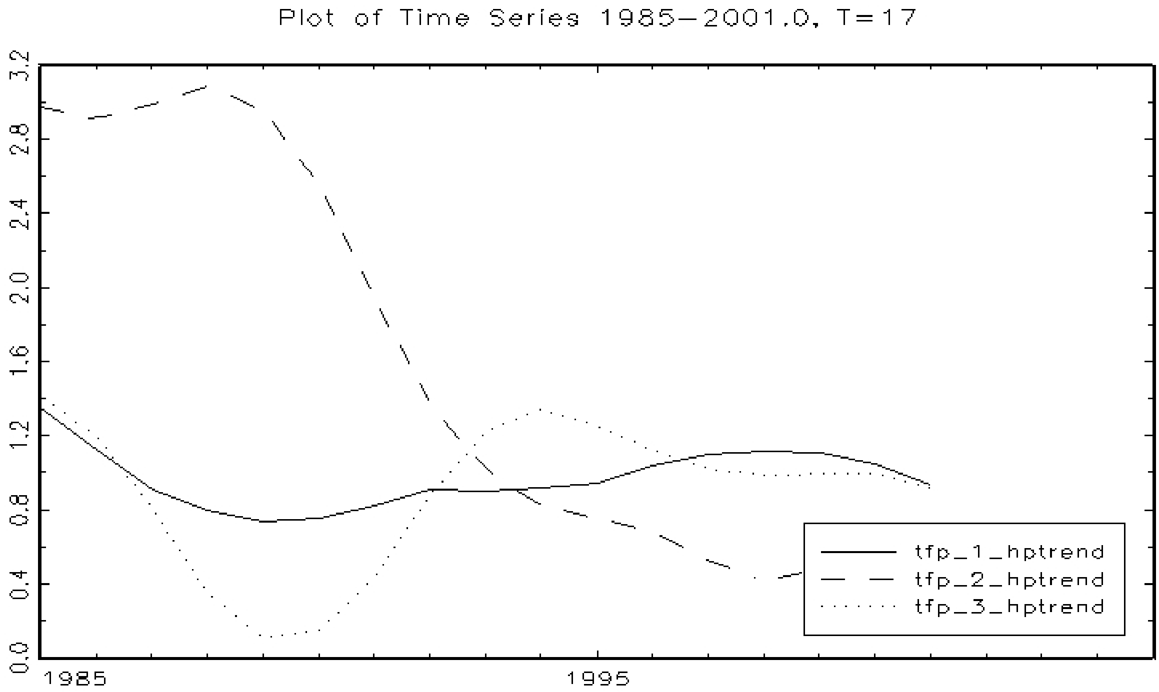

Figure 16: TFP growth of three main OECD countries

converge depends on what lead their difference in the first place. On one hand if two economies with the same steady states induced by historical coincidence start off with different capital stocks, then we could expect them to converge. The economy with the smaller capital stock will grow more quickly. On the other hand if two economies have different steady states due to their different rates of savings, then instead of their convergence we could expect their approach to their own steady state. In case of economies with similar cultures and policies, such as OECD countries, studies find that they converge to one another at a rate of about 2 percent per year. The gap between rich and poor economies closes by about 2 percent each year. Conditional convergence works in OECD countries that they appear to be converging to their own steady states, which in turn are determined by saving population growth and education.

7.4.2 The OECD Total Factor Productivity

Based on analysis of evidence in mid-1970s, Hornstein, Krusell and Violante [2004], there is a phenomenon known as a productivity slowdown. Use the TFP growth data from OECD Productivity database, 17 December 2004, to plot the trend of these productivity features. As shown on the figure 16, the first solid line is the TFP growth rate (HP-filtered data) for the United States from 1985-2001, and the second dash line is the time series for Japan in the same time period and the third one is that of United Kingdom.

The slowdown is most significant in the Japanese data and less straight in the United States and United Kingdom. Therefore the slowdown is demonstrated to be a reality. But why?

It has evidence for the acceleration of investment SSP and a slowdown of consumption SSP. New investment can not attain their full potential when they are introduced, but rather their productivity may stay temporarily below the productivity of older capital that was introduced some time ago with outdated technology. This feature is attributed to learning effects. This process is called learning-by-doing(LBD).

When the technical change is drastic, these learning effects can be very important. Especially, the advent of microelectronics led to a technical revolution and a radical shift in the technological paradigm, in the usual term as a new "General Purpose Technology"(GPT) or Major Technological Change. It is named to describe the major innovations with the potential for pervasive application in a wide range of sector in the economy.

Similar to other past GPTs, the information technology(IT) has affected productivity in a general way, and it should be responsible for this slowdown. When LBD is important in improving the efficiency of the production technology by abandoning the outdated but extensively used technology to include a new method of production, it will lead to a slowdown in labor productivity.

When large organizational capital investments are made the outcome with miss measurement is serious, which will underestimate the TFP growth. When the organizational capital stock has been built, the input of miss measurement dominates, which will overestimate the TFP growth. If the IT adoption phase coincides with associated investments in organizational capital, these investments can not be revealed in the official statistics, which will lead to a miss measurement in statistical artifact. The significant slowdown can be led by this statistical artifact.

This decline of productivity growth rate can reduce employment in the matching framework through the standard "capitalization effect". Suppose when a plant makes the decision of creating a job, the plant will compare the set-up cost with the discounted present value of profits. In a growing economy, where technical change is disembodied and benefits all plants equally, a productivity slowdown increases the "effective rate" at which profits are discounted therefore the discounted profits will decrease. This discourages the creation of new jobs.

7.4.3 TFP Growth Rate and Unemployment Ratio

The current contradictory literature, where faster growth reduces unemployment in Pissarides [1990] Chp. 2, but increases unemployment in Aghion and Howitt [1994], induce the following test of the relationship of the unemployment rate and TFP growth rate of the United States annual data from 1985 to 2003 from Source: U.S. Department of Labor: Bureau of Labor Statistics with the data name: UNRATE and OECD Productivity

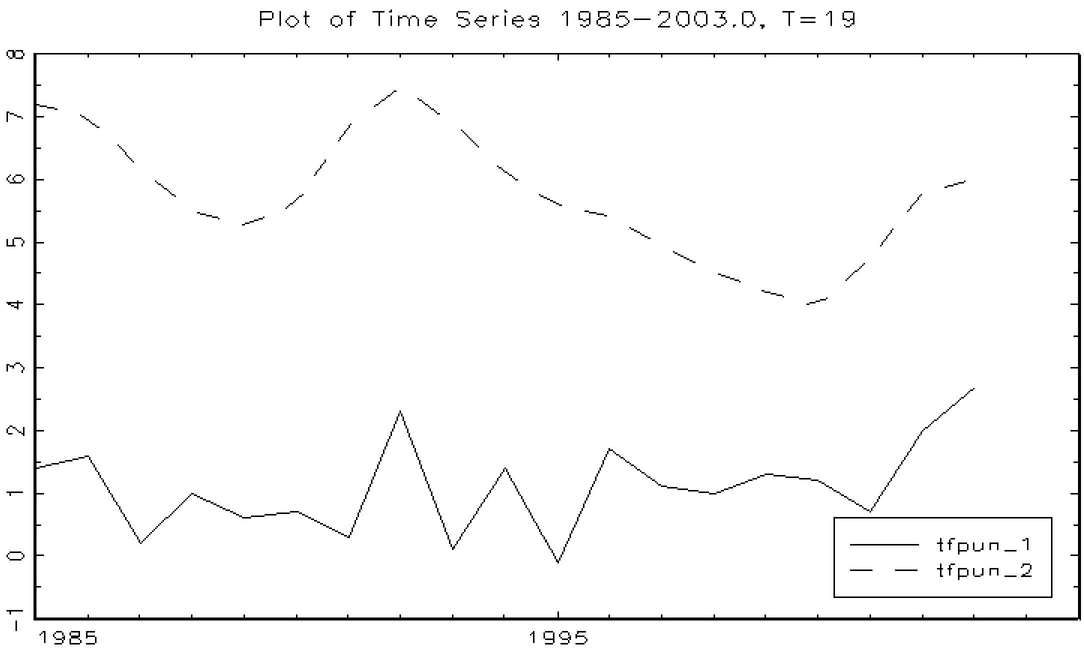

Figure 17: Plots of TFP growth rate and unemployment ratio

database, 17 December 2004 respectively. The two series are ploted in figure 17 and the regression relationship is shown in the figure 18. There is no significant relationship between TFP growth rate and unemployment ratio.

In order to establish the distribution rule behind the relationship of two random variables of the United State historical data: dependent variable (Unemployment ratio) and explanatory variable (TFP growth rate), the OLS method is applied to estimate the coefficients β of the following model during the time period from 1985 to 2003.

$$Unemployment_i \;=\; \beta_0 + \beta_1 \times TFPgrowth_i$$

Coefficient and their significance test: Hypothesis test, two-side t-test.

The pair of hypotheses are:

$H_0 : \beta_1 = 0$ vs. $H_1 : \beta_1 \neq 0$

H_0 means there is no significant relationship between unemployment ratio and TFP growth rate.

Comparing the result $\frac{\widehat{\beta_1} - \beta_1}{StandardDeviationofX}$ applied to t-distribution with freedom degree (Number of observation - 1) with the critical value, the H_0 hypothesis can be accepted or rejected.

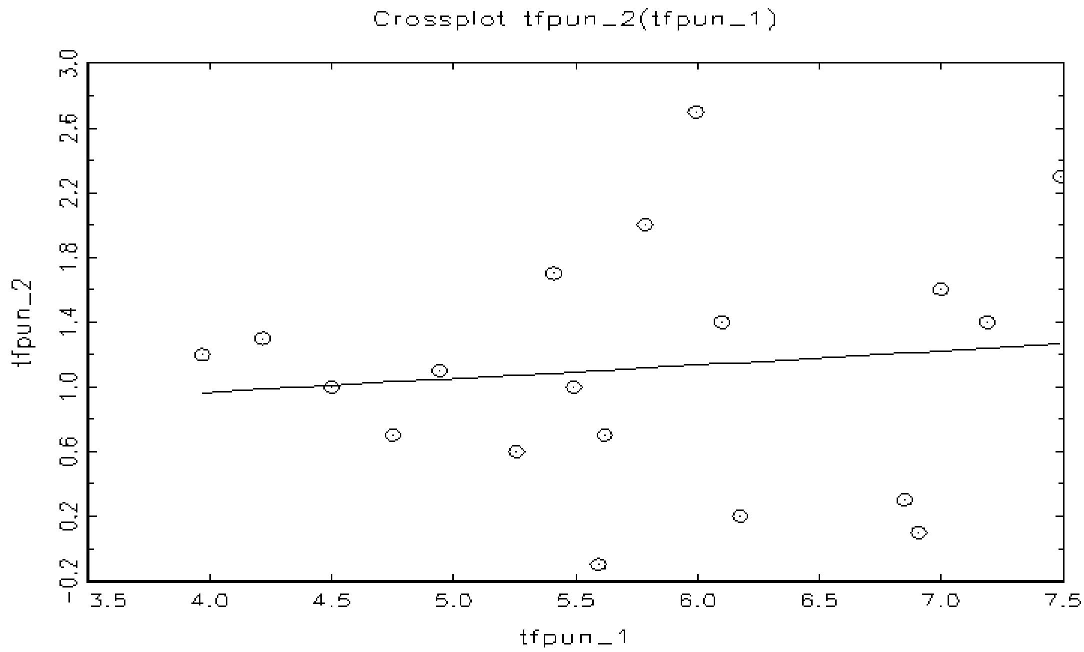

Figure 18: Regression of TFP growth rate and unemployment ratio

In the US case, the regression model is as following:

$$Unemployment \;=\; 0.6271 + 0.085 \times TFPgrowth$$

t-values = 0.6079, 0.4808
sigma = 0.7666
R-squared = 0.0134

This means that 1 percent increase of TFP growth rate will lead to 0.085 percent increase in unemployment ratio. The critical value for the t-test $\alpha = 0.05$ freedom degree 18 is 1.734. The t-test result is 0.4808, which is significantly smaller than 1.734. Therefore I can accept the H_0 that there is not a significant relationship between unemployment ratio and TFP growth rate.

Further VAR and causality tests can not be excuted due to the non-stability of the original two series and especially the first difference series of the unemployment series are not stable too in the unit roots tests.

The lack of significant relationship between TFP growth rate and unemployment ratio is due to the data defect of the multifactor productivity (MFP). This conclusion coincides with previous empirical study result of Layard, Nickell and Jackman [1991] of unemployment rates among OECD countries, which did not consider the rate of economic growth as a possible explanatory variable. And so does the seminal theoretical work of Phelps [1968] result of independence of natural rate of unemployment and the rate of productivity growth. This result necessitates the further study of the relationship of these two variables from other approaches.

Multifactor productivity measurement helps us to resolve the direct growth contributions from labour, capital, intermediate inputs and technology. It is an important tool for reviewing past growth patterns and for assessing the potential for future economic growth.

However, there are some defects of this measurement. One has to be aware that not all technical change can be expressed in the way of MFP growth. Because there is an important distinction between embodied and disembodied technological change. The former represents advances in the design and quality of new vintages of capital and intermediate inputs: machinery and equipment embody the fruits of research performed by the capital goods-producing industry, and other sectors obtain access to the outcome of this research through the purchase of new capital equipment or intermediate goods, whose effects are attributed to the respective factor if the factor is remunerated accordingly. Disembodied technical change comes without cost but relates to the advances in science. Usually it comes in the form of general knowledge, blueprints, network effects or spillovers from other factors of production including better management and organisational change.

Their distinction can be identified through market mechanism: the diffusion of embodied technical change is dependent on market transactions: investment in the improved capital or intermediate good will be undertaken until its marginal contribution to revenue generation just equals its user cost, itself dependent on the market price of the capital good. The diffusion of disembodied technical change is not necessarily associated with market transactions: information may circulate freely and its use by one person does not normally restrict its use by another one.

In static models of production capital is an exogenous input. In a dynamic context, this is not the case and feedback effects exist between productivity change and capital: suppose that technical change allows more output to be produced per person. The static MFP residual measures just this effect of technical change. Additional output per person may lead to additional savings and investment, and to a rise in the capital-labour ratio. Then, a traditional growth accounting measure would identify this induced effect as a growth contribution of capital, although it can be traced back to an initial shift in technology. The MFP residual correctly measures the shift in production possibilities but does not capture the induced effects of technology on growth.

Furthermore, in empirical studies, measured MFP growth is not necessarily caused by technological change: other non-technology factors, such as adjustment costs, scale and cyclical effects, pure changes in efficiency and measurement errors will also be picked up by the residual. Therefore MFP measures tend to understate the eventual importance of productivity change in stimulating the growth of output.

8 Discussion

8.1 Technology Change and its Influence on Labor

The study of sector-specific productivity growth and factor-specific productivity growth effects on frictional labor markets necessitates the extension of vintage capital model in this paper to include more variations.

This paper coincides the opinion with Hornstein, Krusell and Violante [2002] that in their case of vintage capital growth models without frictions in the labor markets, and the view for costly capital investment activity. This alternative approach implies that vacant production units with installed capital can have positive value in equilibrium since the cost of the machine is sunk, and that vacant machines of different vintages coexist in the labor market, even with free entry.

Though this paper shares their common characters of frictionless labor markets, it takes more advantage of a frictional capital adjustment framework therefore is significantly different. Therefore it could meet some limit, when study of capital-embodied technological change influence on the frictional labor market is necessary. The result from the vintage structure does well in explaining the capital flow among coexisting sectors.

This paper explains differently the neutral technological progress prompts waves of Schumpeterian creative destruction by pruning technologically obsolete productive units from the productive system, due to effects of the state dependent adjustment ratio. But it supports the view of Michelacci and Lopez-Salido [2005] that after an explicit capital vintage structure is introduced into the model, a substantial proportion of old jobs upgrade their capital equipment and reap the benefits in the quality of new equipment of the most advanced technology progress.

OECD evidence for growth source indicate that when $\frac{k}{y}$, the capital-output ratio, is used as a measure tool to study the equlibrium of long-run growth, in which every factor is growing together, at the same proportional rate is named as one of steady-state balanced growth. When this ratio is equal to $g_n + g_A + \delta$, the economy reaches its golden rule. The vintage capital renew each period at ratio g_A, the rate of technology progress and the job creation. This technology progress change influence the labor market differently depending on assumptions of the market structure.

8.2 Policy Implication to Savings, Investment and Education

In the long run, if an economy begins at a steady state with a higher capital-output ratio than the golden rule steady state, then consumption per worker can be increased by reducing the savings rate. Nevertheless, in the short run by decreasing on savings and increasing the funds available for consumption, consumption per worker can be increased.

If the economy begins at a steady state with a lower capital-output ratio than in the golden rule, then the policy maker must take steps to raise the savings rate in order to reach the golden rule steady state. In the long run, this increase in the savings rate will increase the steady-state level of consumption per worker. Moreover, the increase in the savings rate reduces the consumption in the short run. When the economy begins above the golden rule, reaching the golden rule always produces higher consumption. But when the economy begins below the golden rule, reaching the golden rule requires reducing the level of consumption now and in the near future increases consumption.

A policy maker has to consider whether the long run increase consumption outweighs the short run reduce in consumption, when he tries to consider whether to try to move the economy toward the golden rule steady state. Only when this tradeoff between the near future and the distant future is worthwhile, can he make the decision.

The determinants of the steady-state capital-output ratio can account for up to half of source of the divergence in national economies' levels of productivity per worker in the world today.

Long-run growth is the most important aspect of how the economy performs. Living standards and economic productivity levels in the United States today are about four times what they are today in Mexico-and about five times what they were at the end of the nineteenth century due to their rapid, sustained long-run economic growth. Good and bad policies can accelerate or slowdown this growth. Immediately before World War II the East Asian regions had output per worker levels less than one-tenth of the United States. Today Singapore's GDP per capita is 90%; Hong Kong's is 70%; Taiwan's is 50%, and South Korea's is 45% of the U.S. level. Almost all of this difference is due to differences in growth policies working through two channels.

The first is the impact of policies on the economy's technology that multiplies the efficiency of labor.

The second is their impact on the economy's capital intensity: the stock of machines, equipment, and buildings.

Its aim is to build up the growth model that economists use to analyze how much growth is generated by the advance of technology and how much by investment to boost capital intensity on the other.

The reason that Americans today are more productive than their predecessors of a century ago is better technology. Better technology leads to a higher efficiency of labor– the skills and education of the labor force, the ability of the labor force to handle modern machine technologies, and the efficiency with which the economy's businesses and markets function.

Capital intensity plays a large part as the second factor. The more capital the average worker has at his or her disposal to amplify productivity, the more prosperous the economy will be. There are two principal determinants of capital intensity. The first is the investment effort made by the economy: the share of total production (real GDP) is saved and invested to boost the capital stock. The second is the economy's investment requirements: how much new investment is needed to equip new workers with the standard capital level, in order to keep up with new technology, and to replace worn machines and buildings. The ratio between the investment effort and the investment requirements of the economy determines the economy's capital intensity.

The world's most industrialized and prosperous economies are the most industrialized and prosperous because they have attained very high levels of manufacturing productivity: their productivity advantage in unskilled service industries is much lower than in capital and technology-intensive manufactured goods. A poor country will have a high relative price of the capital equipment it needs to acquire in order to turn its savings into its additional productive capital stock. The higher relative price of machinery in developing countries means that poor countries get less investment, therefore a smaller share of total investment in real GDP from any given effort at saving some fixed share of their incomes. The successful East Asian economies and OECD economies have a number of similarities in economic policy and structure. Resource allocation decisions are left to the market. Governments regard the encouragement of entrepreneurship and enterprise as a major goal. High savings and investment rates are encouraged by a number of different government policies. On the contrary to OECD economies, governments in East Asia have been more aggressive in pursuing industrial policy to encourage industry investment.

Therefore, a government should adopt policies that boost national savings and improve the ability to translate saving into productive investment and accelerate the demographic transition.

Besides the principal cause of the extraordinary divergence in output per worker between countries today - differences in their respective steady-state capital-output ratios, the other two secondary causes leading to divergence should not be ignored: first, openness to creating and adapting the technologies that enhance the efficiency of labor as measured by levels of development two generations ago, and, second, the level of education today.

The endogenous growth theorists, led by stanford's Paul Romer, argue that it is a mistake to separate the determinants of the efficiency of labor from investment- that investments both raise the capital-worker ratio and increase the efficiency of labor since workers learn about the new technology installed with the purchase of new modern capital goods. Under this theory, government policies to boost national savings and investment rates are weakened.

As the vintage characters of the labor force in the model, men are distinguished by age and training. The recently trained labor with current vintage can product more efficiently than those of earlier vintages. In order to enhance the efficiency of labor, more investment should be injured into education, especially to the education of women. Though investment in education do not increase the saving rate, the education of women pays two fold benefits: The investment in female labor not only makes them more productive, but the educated women are more likely to have well educated children, which will in turn increase the career chance of their next cohort and get a lower born rate. Theoretically this lower population growth rate will bring about higher per capita income. Therefore education can be taken as another way of saving.

Productivity two generations ago is a good indicator the level of technological knowledge that had been acquired as of half a century ago. The level of education today captures the country's ability to invent and acquire further technological expertise today. Without education, inventing new and adopting foreign-born technological knowledge is simply not possible.

To some extent that education is an important kind of investment. A good education is much harder to provide in a poorer country. Because even the primary education requires at least a teacher, some books and a classroom, which are relatively cheap and easy for a rich country to provide, but expensive for a poor country.

The ability of providing proper training to their workers determines employers' operation quality. This non-economic benefit can be translated into monetary values. Human capital investment in the economy is associated with significant labour-market gains for the individuals.

Incentives for employers to invest in training can be spurred in several ways:

The principal incentive for firms to spend on training is whether such activity may increase profits. If training results in significant productivity gains and furthermore these productivity gains are not fully acquired by the trained workers in pursue of higher wages, this form of training will be rewarded with higher profits. The increased profits of the firms can be ensured by choosing proper nature of the training. On the one hand, training in firm-specific skills is unlikely to result in higher wages as the acquired skills are not readily exportable to other firms. On the other hand, training in general skills takes the risk of the productivity gains being appropriated by the trained workers as their value to other employers has risen and the resultant threat of hunting may force the employer' training sponsor to increase wages. Even if on the usual occasion training involves the acquisition of general and firm-specific skills at the same time, there are various mechanisms that can reduce the risk of hunting and introduce an element of cost-sharing between firm and worker. Empirical analysis determines the relationship between employer-sponsored training and profits.

The evidence available suggests that training tends to increase productivity, wages and profits. For example, a recent study based on UK data suggests that a 5 percentage point increase in training incidence could lead to an increase in the level of labour productivity by 4 per cent (see Dearden et al., [2000]). An OECD [1999] econometric study that controls for a wide range of individuals' characteristics has identified the important influence of training on wage determination in many countries, confirming results obtained in national studies. The few studies that look at the impact of training on productivity and wages jointly suggest that training has strong positive effects on profits. To the extent that these studies are representative, their findings suggest that employer-sponsored training is profitable and that employers have an incentive to offer training to their employees.

In order to overcome lack of supply of enterprise training, many governments have been prompted to intervene in the training market. Such requirements include interventions that employers spend a certain proportion of their wage bill on training and giving employees the right to training:

First, government provides the mandatory requirement of training spending. For example, in France, companies with ten or more employees have to either spend no less than 1.5 percent of their total wage payment on training, or pay a corresponding levy. Most of the programmes focus on already well-educated workers in the large or middle-scaled firms. Therefore workers in large enterprise have a higher access rate to training than workers in small and medium-sized enterprise. In the 1990s Korea and Australia both had similar training levies in operation, but they have now been abolished. A survey of the levy in Australia suggested that it had increased spending on training. On the contrary, in Korea the levy had not been effective in stimulating training expenditure in small and medium-sized enterprises which preferred to pay the levy rather than spend on training. The Australian levy left the distribution of training across different categories of workers relatively unchanged, when most of the training went to higher educated and more skilled workers, as it does in the absence of a levy in France.

Secondly, government grants employees rights to obtain the training from employer. For example, France, Belgium and Denmark grant workers a right to get on job training leave under certain conditions. This option puts the responsibility on the individual to choose whether to get training or not and the type, rather than the firm. So it is an active process and is more likely to a better training result than the passive one. In France, beneficiaries of the programme must have a permanent work contract, therefore temporary workers are been excluded. Similarly in Belgium the scheme is restricted to full-time workers.

9 Conclusion Remarks

This paper adapts the neoclassical business cycle model to allow for lumpy capital adjustments within each plant, and the vintage model occurs thereafter. In the vintage capital model, the economy can not replace all its old capital at each date, instead they renew their capital step by step, and different technologies exist at the same time. If technology resides in machines and if a firm or worker must use just one technology at a time, a variety of machines will be in use, and workers' productivities will differ. The vintage capital leads the inequality and it can be used to explain today huge inequality in per capita outputs among countries. Income disparity is not a consequence of different initial conditions, but the result of different investment choices made by each economy.

Being the pioneer of its kind, this paper has studied a vintage model with endogenous adjustment ratio and analyzed how the aggregate technical shocks affect aggregate employment by changing the fraction of plants that choose to adjust. Using this model and the solution approach specified in Uhlig [1999] I have concluded that the technology shock leads the labor productivity increase and total labor force number decrease correspondingly. The numerical analysis of the vintage model reveals the fact that the vintage capital does well in explaining the economic inequality. The neutral technological shocks increase job reallocation and reduce aggregate employment. The increasing adjustment hazard influences the job reallocation by affecting the marginal product of capital for all vintages, therefore the capital flows from the updated sector 0 to old sector 1. Then labors in the old sector with old technology are endowed with more capital, which boosts their relative wages, the labor forces in old sector will increase. Aided with the adjustment ratio, the previous vintage model gets a new explanation.

On the contrary to the real business cycle model, if the plants can not upgrade their neutral and investment-specific technology without replacing part of current workforce, the model is a standard vintage model, where technological progress is entirely embodied into new jobs. Due to the limitation of frictionless assumption of the neoclassical business cycle model, papers follow Pissarides [2000] random matching model to imitate the bargain between a vacant firm and worker. Though the total result of how the economy reacts to the shocks depends on difference assumptions, this paper takes more advantage and is significantly different from them by introducing a frictional capital adjustment framework.

Business cycle simulations and empirical evidences show that the up-to-date variables are most reliable to report the business cycle and individual fixed investment play a strong role in aggregate dynamics. Simulations show that the right magnitude of fluctuations is obtained when the vintage capital with up-to-date technology, while the correlation structure of the production and demand components match the empirical business cycle patterns.

Though there are lack of empirical evidences supporting the negative correlation between labor hour growth rate and per capita GNP, the OECD convergence benefit from the population growth rate fall and healthy investment rates. OECD TFP slowdown explicitly exist in Japan, and the United State, which is an indictor of the relatively decline of the respective economy. The definite explanation of the lack of evidence between TFP growth and unemployment rate depends on various assumptions.

Whether the long run economy growth benefits from the savings policy depends on its starting point of capital-output ratio compared with the golden rule steady state. When the policy maker wants to move toward the golden rule steady state, he must consider the consumption benefit to worker trade off between long run and short run.

The growth policies affect differently on economies through technology and capital intensity two channels. The policy maker should impose strength on boosting national savings as well as translation from savings to productive investment, furthermore enhance the labor efficiency by open to creating and adapting the technologies and increasing the level of education.

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10 Appendix MATLAB® *Codes*

J=1 Case:

```
% VERSION 2.0, MARCH 1997, COPYRIGHT H. UHLIG.
% EXAMPL1.M calculates through Hansens benchmark real business
% cycle model in H. Uhlig, "A toolkit for solving nonlinear dynamic stochastic models easily".
% First, parameters are set and the steady state is calculated. Next, the matrices are
% declared. In the last line, the model is solved and analyzed by calling DO_IT.M

% Copyright: H. Uhlig. Feel free to copy, modify and use at your own risk.
% However, you are not allowed to sell this software or otherwise impinge
% on its free distribution.
% Editted by Lijia Mo

disp('Thesis: An analysis of the dynamics of vintage capital');
disp(' see Thomas, K. Julia, "Is lumpy investment relevant for the business cycle?");
disp(' Journal of Political Economy, 3 (2002), 508-534.');

disp('Hit any key when ready...');
pause;

% Setting parameters:
  B          =  .002;     %upper bound of the adjustment cost
  v          =  .58;      % exponential of labor in production function
  gamma      =  .42;      % exponential of capital in production function
  Z_bar      =  1;        % Normalization
  delta      =  .06;      % Depreciation rate for capital
  betta      =  .939;     % Discount factor beta
  zeta       =  3.6142;   % efficient in utility function of leisure
  psi        =  .9225;    % autocorrelation of technology shock
  sigma_eps  =  .0134;    % Standard deviation of technology shock. Units: Percent.
  afa0_bar   =  0.95;
  w_bar      =  5;

% Calculate steady state values:
  C_bar        =  w_bar/zeta; ^meansexponenctialpowerforthefollowing
  R0_barupper  =  (1-delta)^2*(1-afa0_bar)*betta*afa0_bar+(1/betta)*(1-delta)^((1-gamma-v)/(1-v))..
                  ..-(1-delta)^((1-gamma-v)/(1-v))*betta*(1-afa0_bar)*(1-delta)^2;
  R0_barlower  =  betta*(1-delta)*(1-afa0_bar)+(1-delta)^((1-gamma-v)/(1-v));
  R0_bar       =  R0_barupper/R0_barlower;
  YN0_bar      =  w_bar/v;
  YN1_bar      =  w_bar/v;
  YK0_bar      =  (R0_bar-(1-delta) * afa0_bar)/gamma;
  R1_bar       =  (1/betta - R0_bar)/ (betta * (1-delta)* (1 - afa0_bar));
  YK1_bar      =  (R1_bar-(1-delta))/gamma;
  K0_bar       =  -2*C_bar/(3*delta-1-delta^2-YK0_bar-(1-delta)*YK1_bar);
  Y0_bar       =  2.6;
  i0_bar       =  Y0_bar-C_bar;
  K1_bar       =  (1-delta)*K0_bar;
  Y1_bar       =  2.9;
  i1_bar       =  Y1_bar-C_bar;
  N0_bar       =  Y0_bar/YN0_bar;
  N1_bar       =  Y1_bar/YN1_bar;
  theta0_bar   =  1/(2-afa0_bar);
  theta1_bar   =  1-theta0_bar;
  N_bar        =  theta0_bar*N0_bar + theta1_bar* N1_bar + (B/2)*theta0_bar*afa0_bar^2 + (B/2)*theta1_bar;
  P1_bar       =  (2*w_bar*B*afa0_bar-w_bar*B*betta*afa0_bar^2+2*i0_bar-2*betta*Y0_bar+..
                  2*betta*w_bar*N0_bar)/(2*betta-2);
  P0_bar       =  (2*betta*(Y0_bar-w_bar*N0_bar-afa0_bar*i0_bar)+2*betta*(1-afa0_bar)*P1_bar-..
                  ..w_bar*B*betta*afa0_bar^2)/(2-2*afa0_bar*betta);
  g0_bar       =  Y0_bar-w_bar*N0_bar-afa0_bar*i0_bar+afa0_bar*P0_bar+(1-afa0_bar)*P1_bar-1/2*w_bar*B*afa0_bar^2;
  a            =  gamma*betta^2*(1-delta)*(1-afa0_bar)*YK1_bar;
  b            =  -betta^2*R1_bar*(1-delta)*(1-afa0_bar);
  d            =  -w_bar*N0_bar/g0_bar;
  e            =  -(w_bar*N0_bar+B*w_bar*afa0_bar^2)/g0_bar;
  f            =  -(i0_bar+P1_bar-P0_bar+B*afa0_bar*w_bar)/g0_bar;
```

```
% Declaring the matrices.
 VARNAMES = [   'popdensity_0    ',
                'dummy_1         ',
                'capital_0       ',
                'capital_1       ',
                'dummy_0         ',
                'value_0         ',
                'output_0        ',
                'output_1        ',
                'labor_0         ',
                'labor_1         ',
                'wage            ',
                'adjratio        ',
                'investment_0    ',
                'investment_1    ',
                'consumption     ',
                'employment      ',
                'popdensity_1    ',
                'value_1         ',
                'technology      '];

% Translating into coefficient matrices.
% The equations are, conveniently ordered: see section 5.3
% 1) 0 = - y0(t)+ z(t)+ gamma * K0(t)+ v * n0(t)
% 2) 0 = - y1(t)+ z(t)+ gamma * K1(t)+ v * n1(t)
% 3) 0 = 1/(1-v)* z(t) + gamma/(1-v)* K0(t)- n0(t) - 1/(1-v)* w(t)
% 4) 0 = 1/(1-v)* z(t) + gamma/(1-v)* K1(t)- n1(t) - 1/(1-v)* w(t)
% 5) 0 = (1-delta) * K0_bar * K0(t) + i0_bar * i0(t)+(1-delta) * K1_bar * K1(t)+ i1_bar * i1(t)- K0_bar * K0(t+1)
% 6) 0 = (1-delta) * K0_bar * K0(t) - K1_bar * K1(t+1)
% 7) 0 = w(t) - c(t)
% 8) 0 = theta0_bar * afa0(t) + afa0_bar * theta0(t) + theta1(t) - theta0(t+1)
% 9) 0 = theta0(t) + theta1(t)
%10) 0 = -N_bar * N(t) + (n0_bar+(B*afa0_bar^2)/2) * theta0(t) + (B/2 + n1_bar) * theta1(t) + n0_bar * theta0_bar * n0(t) + n1_bar * theta1_bar * n1(t) + B * afa0_bar * theta0_bar * afa0(t)
%11)0 = Y0_bar * theta0_bar * Y0_bar(t) + Y1_bar * theta1_bar * Y1_bar(t) - c_bar * c(t) + (Y0_bar - i0_bar * afa0_bar) * theta0(t) + (Y1_bar - i1_bar) * theta1(t) - i0_bar * theta0_bar * afa0(t) - i0_bar * afa0_bar * theta0_bar * i0(t) - i1_bar * theta1_bar * i1(t)
%12)0 = Y0_bar * Y0(t) - C_bar * C(t) - i0_bar * i0(t)
%13)0 = Y1_bar * Y1(t) - C_bar * C(t) - i1_bar * i1(t)
%14)0 = -B * w_bar * afa0_bar * w(t) - B * w_bar * afa0(t) - i0_bar * i0(t) - P1_bar * P1(t) + P0_bar * P0(t)
%15)0 = D1(t - 1) - C(t - 1)
%16)0 = Theta0(t) - D2(t - 1)
%17)0 = E_t[c(t) - betta * R0_bar * c(t + 1) - betta^2 * R1_bar * (1 - delta) * (1 - afa0_bar) * c(t + 2) + betta * (1 - delta) * afa0(t + 1) + gamma * betta * Y0_bar/K0_bar * (y0(t + 1) - k0(t + 1)) + gamma * betta^2 * (1 - delta) * (1 - afa0_bar) * Y1_bar/K1_bar * (y1(t + 2) - k1(t + 2))]
%18)0 = E_t[c(t) - c(t + 1) - P0(t) + P0_bar * afa0_bar/g0_bar * P0(t + 1) + P1_bar * (1 - afa0_bar)/g0_bar * P1(t + 1) - i0_bar * afa0_bar/g0_bar * i0(t + 1) + Y0_bar/g0_bar * Y0(t + 1) - (w_bar * N0_bar + B * w_bar * afa0_bar)/g0_bar * w(t + 1) - w_bar * N0_bar/g0_bar * N0(t + 1) - (i0_bar + P1_bar - P0_bar + B * afa0_bar * w_bar)/g0_bar * afa0(t + 1)]
%19)z(t + 1) = psiz(t) + epsilon(t + 1)
%CHECK : 19equations, 19variables.
%Endogenousstatevariables"x(t)" : theta(j - 1t), k(jt), D(t), P(j - 1t)
%Endogenousothervariables"y(t)" : c(t), y(jt), n(jt), w(t), i(jt), afa(jt), N(t), theta(jt), P(jt)
%Exogenousstatevariables"z(t)" : z(t).
%Switchtothatnotation.Findmatricesforformat
%0 = AAx(t) + BBx(t - 1) + CCy(t) + DDz(t)
%0 = E_t[FFx(t + 1) + GGx(t) + HHx(t - 1) + JJy(t + 1) + KKy(t) + LLz(t + 1) + MMz(t)]
%z(t + 1) = NNz(t) + epsilon(t + 1)withE_t[epsilon(t + 1)] = 0,


% for k(t):
```

```
AA = [  0,                        0,   gamma,          0,                   0,   0
        0,                        0,   0,              gamma,               0,   0
        0,                        0,   gamma/(1-v),    0,                   0,   0
        0,                        0,   0,              gamma/(1-v),         0,   0
        0,                        0,   -K0_bar,        (1-delta)*K1_bar,    0,   0
        0,                        0,   0,              -K1_bar,             0,   0
        0,                        0,   0,              0,                   0,   0
        afa0_bar,                 -1,  0,              0,                   0,   0
        1,                        0,   0,              0,                   0,   0
        N0_bar+(B*afa0_bar^2)/2,  0,   0,              0,                   0,   0
        Y0_bar-i0_bar*afa0_bar,   0,   0,              0,                   0,   0
        0,                        0,   0,              0,                   0,   0
        0,                        0,   0,              0,                   0,   0
        0,                        0,   0,              0,                   0,   P0
        0,                        0,   0,              0,                   1,   0
        1,                        0,   0,              0,                   0,   0 ];
```

```
% for k(t-1):
```

```
BB = [  0,  0,   0,                 0,                 0,  0
        0,  0,   0,                 0,                 0,  0
        0,  0,   0,                 0,                 0,  0
        0,  0,   0,                 0,                 0,  0
        0,  0,   (1-delta)*K0_bar,  (1-delta)*K1_bar,  0,  0
        0,  0,   (1-delta)*K0_bar,  0,                 0,  0
        0,  0,   0,                 0,                 0,  0
        0,  0,   0,                 0,                 0,  0
        0,  0,   0,                 0,                 0,  0
        0,  0,   0,                 0,                 0,  0
        0,  0,   0,                 0,                 0,  0
        0,  0,   0,                 0,                 0,  0
        0,  0,   0,                 0,                 0,  0
        0,  0,   0,                 0,                 0,  0
        0,  0,   0,                 0,                 0,  0
        0,  -1,  0,                 0,                 0,  0 ];
```

```
% for y(t):
```

```
CC1 = [ -1,                   0,                   v,                   0,                   0
        0,                    -1,                  0,                   v,                   0
        0,                    0,                   -1,                  0,                   -1/(1-v)
        0,                    0,                   0,                   -1,                  -1/(1-v)
        0,                    0,                   0,                   0,                   0
        0,                    0,                   0,                   0,                   0
        0,                    0,                   0,                   0,                   1
        0,                    0,                   0,                   0,                   0
        0,                    0,                   0,                   0,                   0
        0,                    0,                   N0_bar*theta0_bar,   N1_bar*theta1_bar,   0
        Y0_bar*theta0_bar,    Y1_bar*theta1_bar,   0,                   0,                   0
        Y0_bar,               0,                   0,                   0,                   0
        0,                    Y1_bar,              0,                   0,                   0
        0,                    0,                   0,                   0,                   -B*w_bar*afa0_bar
        0,                    0,                   0,                   0,                   0
        0,                    0,                   0,                   0,                   0];
```

```
h=-P1_bar;
```

```
CC2 =
```

```
[0,                    0,                          0,                  0,      0,      0,              0
0,                     0,                          0,                  0,      0 ,     0,              0
0,                     0,                          0,                  0,      0 ,     0,              0
0,                     0,                          0,                  0,      0,      0,              0
0,                     i0_bar,                     i1_bar,             0,      0 ,     0,              0
0,                     0,                          0,                  0,      0,      0,              0
0,                     0,                          0,                  -1,     0 ,     0,              0
theta0_bar,            0,                          0,                  0,      0 ,     1,              0
0,                     0,                          0,                  0,      0 ,     1,              0
B*afa0_bar*theta0_bar, 0,                          0,                  0,      -N_bar, B/2+N1_bar,     0
-i0_bar*theta0_bar,    -i0_bar*afa0_bar*theta0_bar, -i1_bar*theta1_bar, -C_bar, 0 ,     Y1_bar-i1_bar,  0
0,                     -i0_bar,                    0,                  -C_bar, 0 ,     0,              0
0,                     0,                          -i1_bar,            -C_bar, 0 ,     0,              0
-B*w_bar,              -i0_bar,                    0,                  0,      0 ,     0,              h
0,                     0,                          0,                  1,      0 ,     0,              0
0,                     0,                          0,                  0,      0,      0,              0 ];
```

```
CC = [CC1, CC2];

% for z(t)

DD = [ 1, 1, 1/(1-v), 1/(1-v), 0, 0, 0, 0, 0, 0, 0, 0,0, 0, 0]^T;


% Kt+1

 FF = [   0,   0,   0,   -gamma*betta^2*(1-delta)*(1-afa0_bar)*YK1_bar,   0,   0
          0,   0,   0,   0,                                               0,   P0_bar*afa0_bar/g0_bar ];

% Kt

 GG = [   0,   0,   -gamma*betta*YK0_bar,   0, 0,   0
          0,   0,   0,                      0,      0,   -1 ];

% Kt-1

 HH = [   0,   0,   0,   0,   1,   0
          0,   0,   0,   0,   0,   0 ];

% y t+1

 JJ = [   0,              a,   0,   0,   0,   0,   0,                           0,   b,    0,   0,   0
          Y0_bar/g0_bar,  0,   d,   0,   e,   f,   -i0_bar*afa0_bar/g0_bar,     0,   -1,   0,   0,   P1_bar*(1-afa0_bar)/g0_bar];

% y t

 KK = [   gamma*betta*YK0_bar,   0,   0,   0,   0,   betta*(1-delta),   0,   0,   -betta*R0_bar,   0,   0,   0
          0,                     0,   0,   0,   0,   0,                 0,   0,   1,               0,   0,   0 ];

 LL = [   0
          0];

 MM = [   0
          0];

NN = [psi];

Sigma = [ sigma_eps^2];

% Setting the options:

 [l_equ,m_states]   =   size(AA);
 [l_equ,n_endog]    =   size(CC);
 [l_equ,k_exog]     =   size(DD);
```

```
PERIOD       = 1; % number of periods per year, i.e. 12 for monthly, 4 for quarterly
GNP_INDEX    = 7; % Index of output among the variables selected for HP filter
IMP_SELECT   = [3:4,13:14,9:11,7:8,16];% a vector containing the indices of the variables to be plotted
DO_PLOTS     = 1;
DO_QZ        = 1;
DO_SIMUL     = 1; % Calculates simulations
SIM_LENGTH   = 150;
DO_MOMENTS   = 1; % Calculates moments based on frequency-domain methods
HP_SELECT    = 1:(m_states+n_endog+k_exog); % Selecting the variables for the HP Filter calcs.

% Starting the calculations:
do_it;
```

J=5 case: Please contact author molijia@hotmail.com

Declaration of Authorship

I hereby confirm that I have authored this master thesis independently and without use of others than the indicated resources. All passages, which are literally or in general matter taken out of publications or other resources are marked as such.

Lijia Mo

Berlin, 6th July 2005