Deep Habits: Macroeconomic and Asset Pricing Implications

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Abstract

Motivated by the idea of Ravn, Schmitt-Grohé and Uribe (2005), the deep habits models are developed in this paper. Under deep habits, households do not simply form habits from their overall consumption levels, but rather feel the need to catch up with Joneses on a good-by-good basis. This assumption alters not only the demand side, but also the supply side of the economy, which allows the models have more abundant implications on both asset market and business cycle properties.

Our empirical studies on the stylized facts of asset prices, business cycle facts and in particular the countercyclical markup evidence serve as the bases of our analysis. To explore the asset pricing implications of the model, we derive the explicit solutions of important financial variables based on the log-linearization of the models. Thus, the determinative factors of the Sharpe ratio can be expressed by the deep parameters, and in turn the Sharpe ratio can be numerically analyzed. On the other hand, the dynamics of markup are discussed with three important effects. Allowing more realistic assumption, we add the deep habits into a Calvo-type sticky price model, where both the price stickiness and deep habits affect the markup behavior.

Being consistent with the literature, our discussions show that deep habits contribute to the countercyclical markup under both flexible and sticky price frameworks; adding habit formation and capital adjustment cost can help generate sizeable Sharpe ratio; the nonseparability between consumption and leisure can help explain the premium puzzles within certain scope. Moreover, when combining the deep habit with a preference nonseparable in consumption and leisure, the problems, such as negative labor response of most capital adjustment cost models, will disappear. In turn, it allows to explain the comovement in output, consumption, investment and labor.

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Men's natures are alike;
it is their habits that carry them far apart.
Confucius[1]

1 Introduction

There has been a growing recognition in the economics literature of the role of habit formation,[2] either because of its psychological intuition or its fitness to data. When habits are formed, households would like to 'catch up with Joneses' by force of these habits. Their overall happiness or satisfaction over consumption replies on not only the current level, but also the comparison with certain benchmark level. Studies based on this idea showed that habit formation can reconcile, e.g. the observed equity premium with theory. See Abel (1990), Constantinides (1990) and Campbell and Cochrance (1999). In addition, habit formation has also been used in real business cycle (RBC) models, such as Lettau and Uhlig (2000), who examined consumption volatility, Ljungqvist and Uhlig (2000), who studied the fiscal policy, Fuhrer (2000), who focus on the monetary policy, etc.

While most literature payed attention to the habits which were formed at the level of consumption aggregate, Ravn, Schmitt-Grohé and Uribe (2005) raised the assumption that habits were also created at the level of individual consumption goods, such as clothes, cars, music, etc. Households who have much consumption on a particular good today are more likely to buy this kind of good in the future by force of the deep habits. They consider this assumption is more compelling because it not only holds the properties of standard habit formation but also influences the firm's pricing strategy with a countercyclical markup.

This paper is based on the assumption of deep habits. We focus on a technology-shock driven RBC model incorporated with the deep habits, which can be further extended with capital adjustment cost, preference nonseparable in consumption and leisure, and Calvo-type sticky prices according to most literature. Through qualitative and quantitative analysis, we examine both the macroeconomic and asset market implications. We find that being consistent with other literature, deep habits contribute most to the counter-

[1]Confucius, Analects, Wikiquote.org

[2]Habit formation in this paper refers to the external habit, or the catch-up-with-Joneses of Abel (1990).

cyclical markup under both flexible and sticky price frameworks; adding habit formation and capital adjustment cost can help generate sizeable Sharpe ratio; the nonseparability between consumption and leisure can help explain the premium puzzles within certain scope. Moreover, when combining the deep habit with a preference nonseparable in consumption and leisure, the problems, such as negative labor response of most capital adjustment cost models, will disappear. In turn, the comovement in output, consumption, investment and labor is generated and explained.

The methodology adopted in this paper to solve dynamic stochastic general equilibrium (DSGE) is following Campbell (1994) and Uhlig (1999). In particular, the Toolkit by Uhlig (1999) are very useful in both computation and analysis. With the necessary inputs, Toolkit solve the model automatically, and the results are just corresponding to the elasticities which we use in economic analysis. Moreover, in line with the Lagrangian method to solve DSGE, the Matlab codes to calculate the input of Toolkit are made available.

The remainder of the paper is organized in 7 sections. Section 2 takes a review on the related literature. Section 3 provides the facts as the foundation of the model analysis. In particular, we empirically study the relation between markup and business cycles. In section 4 we present the benchmark deep habits model and compare its equilibrium with standard habit formation models. Section 5 implements the computation and calibration. The results are analyzed in details. We consider three variations of the model in section 6, namely, adding the capital adjustment, considering the preference nonseparable in consumption and leisure, and relaxing to a sticky price framework. The economic intuitions are further discussed in section 7. Finally, we summarize the paper in section 8.

2 Literature

Among asset pricing literature, a lot of attentions have been paid to the intertemporal general equilibrium models. In these models, the prices and yields of the assets are linked to agent's choice on consumption and saving, through the general equilibriums. By analyzing the equilibriums, we can have a clear view on the economic intuitions of the variables which affect the predicted asset prices. Among these variables and parameters, indeed agent's preferences, in particular, the risk aversion and the intertemporal substitutions play an important role.

Traditionally, the preferences are assumed to be time separable, where only instantaneous consumptions are considered. However, most studies have shown that the asset pricing models with such preferences are unable to match the asset prices facts, such as the risk free rate, equity premium or Sharpe ratio. Two notable anomalies of the asset market are the equity premium puzzle and risk free rate puzzle. In Mehra and Prescott (1985), they found that the average annual excess return was 6.18 percentage over 1889-1978 of United States. However, the models with time separable utility predict too small premium, unless taking a very high but unrealistic risk aversion. They concluded this as the equity premium puzzle. Meanwhile, the predicted riskless interest rate are much higher, unless a weak risk aversion is considered. Weil (1989) pointed this as the risk free rate puzzle.

Because of the inability of the time separable preferences to match asset market facts, an enormous effort has been invested into models with non-separable preferences over time, such as habit formation models. Indeed, the habit formation can be traced back as Aristotelian idea[3] which describes essential feature of human behavior.[4] Households care about their overall satisfaction or happiness not only on their current consumptions, but also on some benchmark levels, i.e. the aggregate past consumption.[5] In other words, once habits are formed, agents tend to consume by force of habits. These habit-forming consumers are more risk averse and reluctant to change their consumptions from their habits. Accordingly the premium of these sudden change in consumption must be high.

The most successful application of habit formation is in asset pricing to reconcile some premium puzzles. Abel(1990) examined the equity premium by considering habit formation, either external or internal, in the form of the ratio of current consumption to the habit stock. Differently, Constantinides (1990) employed the additive habit formation, and showed that the premium puzzle could be resolved. Moreover, the habit formation in Campbell and Cochrance (1999) avoided many shortcoming of previous models, such as the negative consumption surplus, and high volatile interest rate. Their model matched the asset market data.

Since its success, habit formation has been more and more employed in RBC models. Usually, the RBC models consider the dynamic stochastic general

[3]Or even earlier, as the quotation from Confucius showed in the beginning of the paper.

[4]Messinis (1999) did a survey on the history of habit formation.

[5]Actually, as for external habits, the aggregate consumption serves as a benchmark level. As for internal habits, households' own past consumption is viewed as a benchmark.

equilibrium (DSGE), which allows to explain multivariate stochastic processes of aggregate time series, such as output, consumption, investment, etc. Actually, the RBC models can be turned into various asset pricing models. This linkage provide the possibility to analyze the macroeconomic and the asset market implications under the same framework.

Among the RBC models with habit formation, Jerman (1998) studied the one-sector model. They found the habit formation preference and capital adjustment cost helped on explaining equity premium. Boldrin, Christiano and Fisher (2001) studied the two-sector model with habit persistence preferences and limitations on inter-sectoral factor mobility. Both of these two paper applied the habit formation following Constantinides (1990). Lettau and Uhlig (2000) followed the habit formation in the form of Campbell and Cochrance (1999). Their studies showed that the consumption volatility puzzle took place of the asset pricing puzzle when habits are considered.

Along the history of habit formation, more recently Ravn, Schmitt-Grohé and Uribe (2005) developed the idea of the deep habits, where they believe "private agents do not simply form habits from their overall consumption levels, but rather from the consumption of individual goods". This assumption could have indistinguishable effects on the demand side of economy, and more important it changed the supply side in fundamental way. Their studies focused on the markup analysis, and found the deep rooted habits could generate the countercyclical markup, which was consistent with most empirical evidence such as Rotemberg and Woodford (1999) and Galí, Gertler and López-Salido (2002). Our models are based upon this deep habits assumption, and analyzed for its implications on both the asset market and macroeconomics.

As pointed by Ravn, Schmitt-Grohé and Uribe (2005), the brand-switching costs model by Klemperer (1995) and customer-marketing pricing model by Phelps and Winters (1970) also consider the firm's pricing strategy at the individual level. However, they consider the "discrete switches among supplier", while deep habits model consider "gradual substitution". Therefore, the deep habits are possible to incorporate both the brand-switching and customer-marketing.

Besides habit formation, other factors may also help explain the equity premium puzzles. Jerman (1998) and Francis and Ramey (2002) employed the capital adjustment cost, and showed large the equity premium as well as negative correlation between labor and the permanent component of productivity. The nonseparability between consumption and leisure choice, as stressed in Uhlig (2004), also helps explain the equity premium within certain scope. These factors are considered as variation to our models.

3 Stylized Facts

Before we proceed to the model description and further analysis, it is worthwhile to first have a look at some stylized facts and evidence. These facts can be regarded as the foundation of our analysis, and the models should try to match these facts. Some key business cycle facts are presented in subsection 3.1, and particularly, the markup evidence are analyzed in subsection 3.2. At last we show the facts on asset markets.

3.1 Key Business Cycle Facts

In macroeconomic analysis, the main object is to study the aggregate economic activities, which are usually measured by real Gross Domestic Product (GDP) and other key indicators, such as consumption expenditures, private investment, etc. If plotting these time series, we can see that not only do they grow in the long run, but also fluctuate over time. In order to separate the data into a long run growth trend and business cycle fluctuations, usually we need to filter the data. Among many filters, Hodrick- Prescott (HP) filter has enjoyed widespread popularity. With HP filter, we can specify the growth trend such that the deviations from that trend can be interpreted as business cycle fluctuations. Further details on HP filter can refer to Hodrick and Prescott (1997), and Ravn and Uhlig (1997).

Indicators	IDs
Real GDP	GDPC96
Consumption	PCENDC96+PCESVC96
Investment	FPIC1
Hours	AWHI
Labor Productivity	OPHNFB
Wage	COMPRNFB
Government Spending	GCEC1

Table 1: Data Description: Key Macroeconomic Indicators

To analyze the U.S. economy, we take the data from the Federal Reserve Bank of St. Louis Web Site. All the series are quarterly data, from 1964:Q1

	GDP	Cons	Inve	Hours	Lab Pro	Wage	Gov
GDP	1.5619						
Cons	0.8312	0.90749					
Inve	0.90459	0.78665	5.1172				
Hours	0.88939	0.71307	0.86718	1.8473			
Lab Pro	0.53103	0.50062	0.38744	0.12587	1.0318		
Wage	0.26497	0.37833	0.24036	0.098783	0.46713	0.93155	
Gov	0.13579	0.033562	-0.12407	0.050699	0.13471	0.11381	1.52

Table 2: Correlations and Standard Deviations (in percentage)

to 2004:Q4. More details, including the series names and IDs are listed in Table 1. Besides, the Matlab code concerning the HP-filter detrending and related analysis can be found in Appendix B.1.

After having detrended the data, we next analyze the business cycle component of the data and document the main stylized facts of business cycles, i.e. to study what are the main characteristics of business cycles. In general, we document the stylize facts from three aspects, namely, 1) volatility, 2) comovement and 3) timing.

Volatility is measured as the standard deviation of the time series. The diagonal elements of Table 2 shows the standard deviations. The essential features can be summarized as follows:

1. Real GDP has a volatility of about 2% around trend, or more precisely in our example, 1.6% when considering the HP-filtered series.
2. Consumption fluctuates less than output, say, 0.91% in our example.
3. Investment fluctuates much more than output.
4. The hours worked are about as volatile as output.
5. Labor productivity, the real wage and government expenditure are less volatile than output.

Comovements are measured as correlations, which is shown in Table 2 as the off-diagonal elements. Again, the main facts are summarized as follows:

1. Most macroeconomic series are procyclical (positive contemporaneous correlation with output)

2. Output, consumption, investment, and total hours worked are very highly correlated, say high correlation among these four indicators.

3. Wages and government expenditure have relative low correlations, which means un-correlated with output (acyclical)

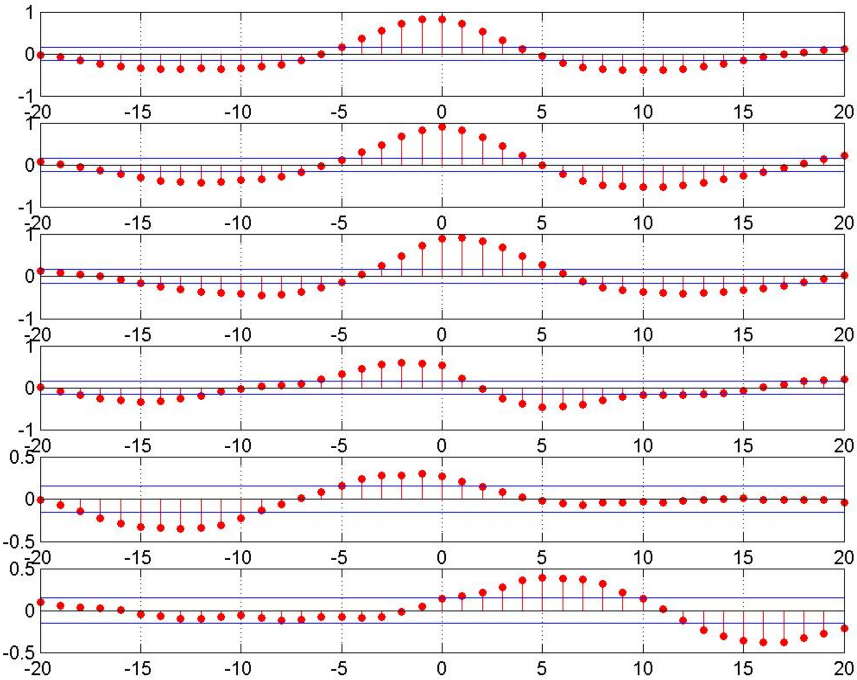


Figure 1: Cross correlation plots between output and 1)consumption, 2)investment, 3)hours, 4)labor productivity, 5)wage and 6)government expenditure, taking ± 20 lags

Timing refers to the cross correlations between the observed data and output. the be shown in the Figure 1 and Table 3, . It is clear that:

1. Consumption, investment and hours are coincident variables whose peaks (for procyclical variables) occur around the same time as the peak in GDP, i.e. the peak in the business cycle.

2. Labor productivity are leading variables whose peaks occur just before the peak in GDP, i.e., just before the peak in the business cycle.

Output	-3	-2	-1	0	1	2	3
Cons	0.5499	0.7188	0.8205	0.8312	0.7129	0.5383	0.3330
Inve	0.4767	0.6763	0.8326	0.9046	0.8219	0.6612	0.4566
Hour	0.2427	0.4809	0.7133	0.8894	0.9046	0.8188	0.6760
Lab Pro	0.5476	0.5879	0.5787	0.5310	0.2170	-0.0415	-0.2584
Wage	0.2744	0.2833	0.3033	0.2650	0.2063	0.1439	0.0811
Gov	-0.0811	-0.0208	0.0475	0.1358	0.1713	0.2160	0.2752

Table 3: Cross Correlations between Output and Other Variables

It is commonly accepted that any successful business cycle models must be consistent with the above stylized facts. In this paper, the models' results will be compared with these facts in our following analysis.

3.2 Markup and Business Cycle

Besides the above mentioned key indicators and facts of business cycle, there still exist others, among which, markup is the focus of this paper. In this section, we will try to provide some of the empirical issue about markup and business cycle, which would be regarded as the foundation of further discussion.

A large amount of literature has shown that markup of prices on marginal costs is countercyclical. For instance, the influential paper by Rotemberg and Woodford (1991,1999) argued the two ways of describing the feature of business cycles in symmetric (aggregate) cases. That is, the real marginal cost (MC/P) rises and equivalently, the markup of price over marginal cost (defined as P/MC) declines. An intuitive explanation can be such that since inputs are scarce, marginal cost should be an increasing function of output. A number of studies are also in favor of the countercyclical markup, such as the elasticity-of-demand models by Galí (1994), customer market models by Phelps and Winter (1970), and the implicit collusion model by Rotemberg and Woodford (1992). If the result holds, it would imply that "markup variations play a role in causing or at least amplifying cyclical fluctuations of economic activities". (Rotemberg and Woodford (1999))

However, as Rotemberg and Woodford (1999) pointed out the main challenge in constructing measures markup variation is to find suitable measures of marginal cost. "It is not easy to obtain measures of marginal cost of which one can be certain." A simple but most common measures of marginal cost in the literature consider labor income share. Figure 2 plots the labor share data as reported by Bureau of Labor Statistics (BLS), from 1964:Q1 to 2004:Q4.[6] Meanwhile, the NBER recessions are also plotted with the Matlab code in Appendix B.2. For each of these recessions, the first vertical line represents a business cycle peak while the second represents the trough.

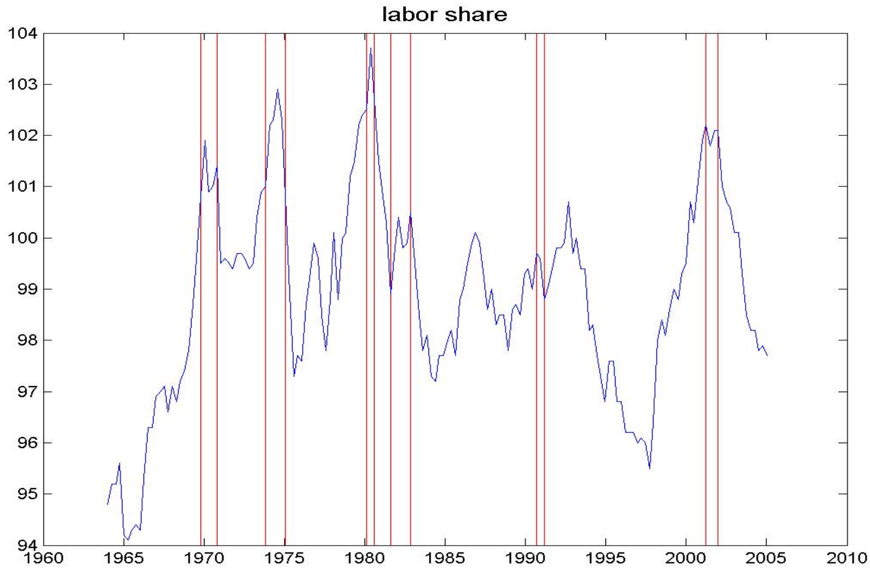


Figure 2: Labor Share Data Plot with NBER Recessions

For procyclical series, its peaks (troughs) ought to be aligned with the business cycle peaks (troughs). This means a procyclical labor share is indicated by a decline between peaks and troughs or alternatively, an increase between troughs and peaks. Unfortunately, in Figure 2, the procyclical labor share only shows for several periods, such as after 1975 and 1982 recessions, where labor share increases in the recoveries. Rotemberg and Woodford (1999) documented a number of reasons for the poor proxies for marginal cost, such as the existence of overhead labor, overtime premia, and adjustment costs for labor. If taking these consideration, they concluded that the labor marginal cost should be procyclical, and markup should be countercyclical.

Instead of further discussing the theoretical details of Rotemberg and Woodford (1999), we follow the empirical analysis by in Galí, Gertler and López-

[6]The data is obtained from BLS http://www.bls.gov/data; Find Productivity & Technology, Major Sector Productivity and Costs Index, and select Create Customized Tables (One Screen). Then select Nonfarm Business, Labor Share, and index, 1992=100.

Salido (2002)(GGL for short) where they show their inefficiency gap also mirrors countercyclical movements in markup. From a somewhat different perspective, they introduce the inefficiency gap which corresponds to the inverse of the markup of price over social marginal cost. The equations for the inefficient gap, wage markup and price markup are given in Appendix A.1 according to GGL. Since limitation, we proceed with different data, which are also described in Appendix A.1. At last the corresponding Matlab codes for the calculations and plotting are provided in Appendix B.3.

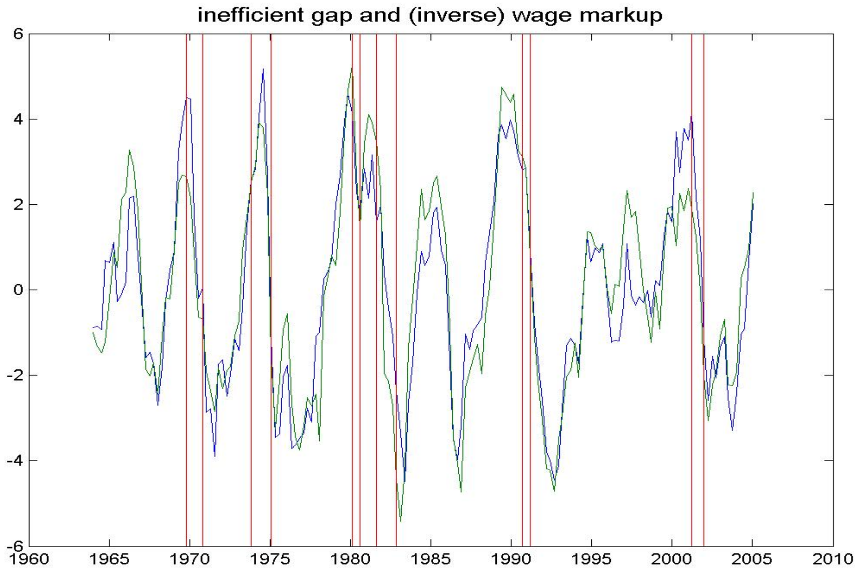


Figure 3: Inefficient Gap and (inverse) Wage Markup

Generally, the inefficient gap can be decomposed into wage and price markup components. In Figure 3, we plot the behavior of the inefficient gap against (inverse) wage markup,[7] which is consistent with the Figure 2 of GGL. From this plot, we see not only the comovement but also the procyclical behavior. To be specific, the procyclical behavior can be indicated by (approximate) coincidences of the peaks and troughs between the series and business cycles. In most cases, we see declines in contractions, and increases in recoveries. Combined with the comovement in inverse of wage markup and inefficient gap, we can conclude that the evidence shows a countercyclical wage markup,

[7]For facility and comparison reasons, we plot the inverse of wage markup, (i.e. minus the log wage markup).

	GDP Gap	Ineff. Gap	Wage Markup	Price Markup
GDP Gap	1.5619			
Ineff. Gap	0.47289	2.2971		
Wage Markup	-0.56377	-0.90948	2.3735	
Price Markup	0.25271	-0.13897	-0.28533	0.99645

Table 4: Inefficient Gap Method: Correlations and Standard Deviations

and this markup causes largely the inefficient gap.

To be precise, we provide the statistic which support the visual evidence of the plot. The diagonal of Table 4 are the standard deviations, and the off-diagonal are the correlations. Note we also add the HP-filter detrended real GDP as the usual indicator for output gap. Since we use the different dataset, the numbers in Table 4 are different from Table 1 of GGL. However, our conclusions from these statistics are consistent with theirs. The inefficient gap and wage markup are nearly volatile, and both fluctuate more than output gap. The positive correlation between output gap and inefficient gap confirms the procyclical behavior, while the negative correlation between output gap and wage markup confirms the countercyclical markup.[8] At last, the price markup is less volatile and less correlated with output gap.

GGL applied more alternatives for the robust analysis. However, the results are still consistent with our evidence and analysis here. To summarize, in this subsection, we review different empirical analysis on measuring the markup and its relation with business cycles. Our conclusion is that the markup of price over marginal cost is countercyclical. To build a good model, we should also take this fact into account.

3.3 Asset Markets

Finance theory holds that stocks are more "risky", where the risk is measured as standard deviation (or variance). Investors may require higher expected returns when investing in the volatile stock market than they do when investing in more stable assets, such as bounds or treasury bills. As a result, equity returns offer a risk premium relative to the returns available on bonds and treasury bills.

[8]Since we use different dataset, our statistics are not as high as GGL. In order to get a stronger countercyclical markup, it is better to consider more suitable data for the measurement.

Period	1871–	–2004	1871–	–1947	1848–	–2004
	Mean	std. Dev.	Mean	std. Dev.	Mean	std. Dev.
SP500	8.26	17.69	7.71	18.94	9.00	15.99
Risk-free Rate	1.03	0.07	1.04	0.08	1.02	0.03
Equity Premium	7.23	17.68	6.67	18.93	7.98	15.99
Sharpe Ratio	40.86		35.24		49.89	

Table 5: Asset Market Facts: Annual Data in percentage

Historical data provide a wealth of evidence documenting the fact that U.S. stock returns have been considerably higher than returns for riskless assets. For instance, Mehra (2003) reported a 6.9% equity premium at annual rate over past 110 years (average 7.9% stock return minus 1.0% riskless security). Similarly, Campbell (2004) document the annual excess return of 7.2% over 3-month treasury bills. With 15.6% of the volatility, the derived Sharpe ratio is 0.46 at an annual basis.

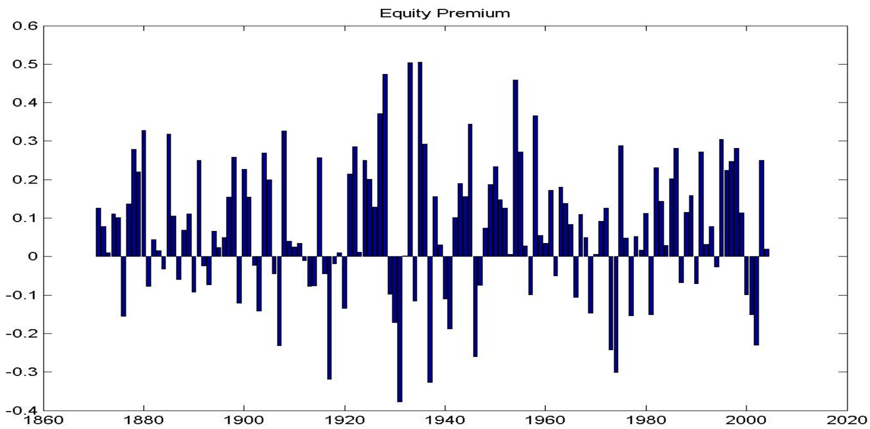


Figure 4: Equity Premium: 1871-2004, Annual Data

Table 5 summarize the facts of the asset market over different period, with the dataset from Shiller (2000).[9] Returns are calculated with real price and dividend of S&P500 index, and the risk-free rate is real one year interest rate. Thus the risk premium is the difference between equity return and riskless rate. The Sharpe ratio is measured as the ratio of mean excess return over standard deviation of equity return. The table shows that in early period, the

[9]Data can be found on Shiller's webpage: http://www.econ.yale.edu/ shiller/data

risk premium is relative low, in turn a relative low Sharpe ratio. Recent post-war data show a higher risk premium and higher Sharpe ratio. These results are consistent with other studies. Furthermore, the equity premium varies over time. It could be even negative in some periods as presented in Figure 4.

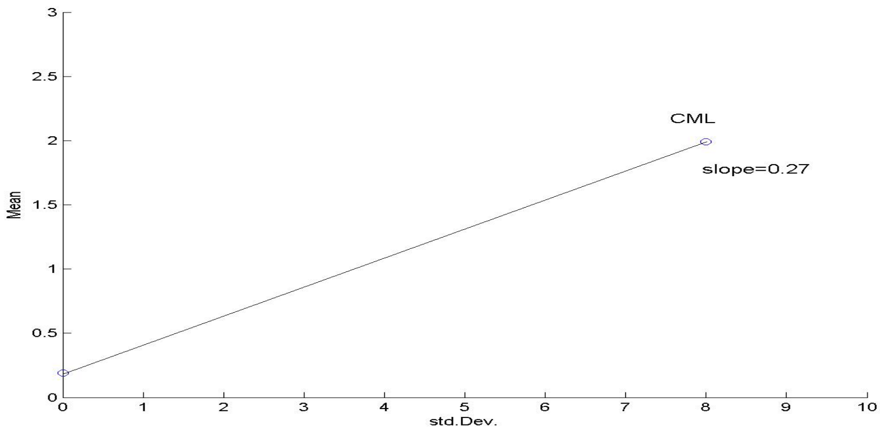


Figure 5: Mean-std.Dev. Frontier: from Lettau and Uhlig (2002)

Besides the first moment of the equity premium, Sharpe ratio (or price of risk), the ratio of the first and second moments, to certain degree, is more important. Lettau and Uhlig (2002) use quarterly data to plot the mean-standard deviation frontier as show in Figure 5. The capital market line (CML), starting from the T-bill point, goes through the S&P500 point. The slope of the CML is just the Sharpe ratio, 0.27. Since the calibrations of our model in this paper are corresponding to the quarterly post war U.S. data, the analysis on Sharpe ratio will take 0.27 as a fact.

4 The Model

The starting point is a standard real business cycle model under imperfect competition, incorporated with "deep habits". The model here is similar to the fully-fledged deep habits model of Ravn, Schmitt-Grohé and Uribe (2005), where they also consider 1)the preference shock, 2)the government sector, and 3)the fix cost of production. Our benchmark model is a technology-shock driven RBC model. The households and firms of the model are presented in subsection 3.1 and 3.2, respectively. Finally, the equilibrium is summarized and compared with standard superficial[10] habit formation models in subsection 3.3.

4.1 Household

Considering household $j \in [0,1]$, the preference is defined over (habit-adjusted) consumption x_t^j, and labor effort, h_t^j, as described by the concave utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(x_t^j, h_t^j) \tag{1}$$

where the variable x_t^j is a composite of habit-adjusted consumption of a continuum of differentiated goods indexed by $j \in [0,1)$. Specifically, the x_t^j can be written as:

$$x_t^j = \left[\int_0^1 (c_{it}^j - \theta s_{it-1})^{1-1/\eta} di\right]^{1/(1-1/\eta)} \tag{2}$$

where s_{it-1} represent the stock of external habit in consuming good i in period t. This stock of habit is assumed to depend on a weighted average of consumption in all past period. The s_{it} is defined and can be further written as law of motion:

$$\begin{aligned} s_{it} =& (1-\rho)\sum_{j=0}^{\infty} \rho^j c_{it-j} \\ =& (1-\rho)c_{it} + \rho s_{it-1} \end{aligned} \tag{3}$$

Here the parameter $\rho \in [0,1)$ measures the speed of adjustment of the external habit. When $\rho = 0$, the habit is measured as the past consumption. For any given level of consumption of the composite good, purchases of each variety i in period t must solve the dual problem of minimizing total expenditure, $\int_0^1 P_{it} c_{it} di$, subject to the aggregation constraint (2), where $P_{it} = \left[\int_0^1 P_{it}^{1-\eta} di\right]^{1/(1-\eta)}$ denotes the nominal price of a good of variety i at time t. The optimal level of c_{it}, i.e. the demand for variety i is then given by:

$$c_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} x_t + \theta s_{it-1} \tag{4}$$

[10]The terminology 'superficial' is used to distinguish with the deep habits, as indicated in Ravn, Schmitt-Grohé and Uribe (2005).

Households are assumed to have access to a complete set of nominal contingent claims. Their period-by-period budget constraint is given by:

$$x_t^j + i_t^j + \varpi_t + E_t r_{t,t+1} d_{t+1}^j = d_t^j + w_t h_t^j + u_t k_{t-1}^j + \Phi_t^j \tag{5}$$

where $r_{t,s}$ is a stochastic discount factor, defined so that $E_t r_{t,s} d_{t+1}$ is the nominal value in period t of a random nominal payment d_t in period s. Furthermore, ϖ_t is defined as $\varpi_t \equiv \theta \int_0^1 (P_{it}/P_t) s_{it-1} di$. The variable k_t denotes capital,[11] i_t denotes investment, w_t denotes wage, u_t denotes dividend, Φ_t denotes profits received from the ownership of firms.

The evolution of capital is given as:

$$k_t = (1+\delta)k_{t-1} + i_t \tag{6}$$

The investment good is assumed to be a composite good made with the aggregation function:

$$i_t^j = \left[\int_0^1 (i_{it}^j)^{1-1/\eta} di \right]^{1/(1-1/\eta)} \tag{7}$$

Again, for any given level of investment of the composite good, purchases of each variety i in period t must solve the dual problem of minimizing total investment expenditure, $\int_0^1 P_{it} i_{it}^j di$, subject to the above aggregation constraint (7). The optimal level of i_{it} is then given by:

$$i_{it}^j = \left(\frac{P_{it}}{P_t} \right)^{-\eta} i_t^j \tag{8}$$

On aggregate, let $p_{it} = \frac{P_{it}}{P_t}$, then the equation can be written as:

$$i_{it} = p_{it}^{-\eta} i_t \tag{9}$$

To complete the system, the households are also assumed to be subject to the non-Ponzi-game borrowing constraint. This condition stipulates that "in net present value terms, the agent should neither have capital left over at infinity or borrow anything at infinity" (Uhlig (1999)).

To summarize, the problem associated with households j is to choose the variable $x_t^j, h_t^j, i_t^j, d_{t+1}^j, k_t^j$ to maximize the utility function (1) subject to the budget constraint (5) capital evolution (6) and the non-Ponzi constraint,

[11] In this paper, we use the notations which are consistent with Toolkit by Uhlig (1999).

where given process $\varpi_t, w_t, u_t, r_{t,t+1}$ and Φ_t^j. To be specific, the correspondent first order conditions (FOCs) are as follows:

$$\lambda_t = U_x(x_t, h_t) \tag{10}$$
$$\lambda_t w_t = U_h(x_t, h_t) \tag{11}$$
$$\lambda_t = \beta E_t \lambda_{t+1}[1 - \delta + u_{t+1}] \tag{12}$$
$$\lambda_t = \beta R_t E_t \lambda_{t+1} \tag{13}$$

where we define $R_t = \frac{1}{E_t r_{t,t+1}}$, the gross one period risk free nominal interest rate. Note, the households' problem of deep habits model is actually the same as in standard superficial habit formation model. However, the equations concerning the habit stock will be further applied in firms' problem, which in essence differ this model from others.

4.2 Firm

We assume the representative firm produces output using a Cobb-Douglas production function with capital k_t and labor h_t as factor input. The production function is given by:

$$y_{it} = z_t F(k_{it-1}, h_{it}) \tag{14}$$

where the function F is assumed to be homogenous of degree one and concave. The variable y_{it} denotes the output of good i, and z_t denotes an aggregate technology, which is exogenous and stochastic:

$$\log z_t = \rho_z \log z_{t-1} + \epsilon_t \tag{15}$$

where ϵ_t is a white noise with standard deviation σ_ϵ. This exogenous process can be used to get closed-form solution for asset price as mentioned in Lettau (2003) and σ_ϵ should be rescaled to be consistent with data, as mentioned in Uhlig (2004). Later we will illustrate further in the asset pricing implication part.

Recall our analysis on households imply the aggregate demand for good i:,

$$c_{it} = (p_{it})^{-\eta} x_t + \theta^d s_{it-1} \tag{16}$$
$$i_{it} = (p_{it})^{-\eta} i_t \tag{17}$$

where $c_{it} \equiv \int_o^1 c_{it}^j dj$, $i_{it} \equiv \int_o^1 i_{it}^j dj$, $x_{it} \equiv \int_o^1 x_{it}^j dj$, and

$$s_{it} = (1 - \rho^d) c_{it} + \rho^d s_{it-1} \tag{18}$$

We rewrite the habit degree parameter as θ^d, ρ^d under the deep habit assumption. When $\theta^d = 0$, the model will be the same as the standard superficial habit formation models. Note that the firm actually faces the demand of good i in equation (16), which is composed by two parts, the price elastic (habit-adjusted consumption) aggregate demand and price inelastic habit stock. Thus the price elasticity of the demand of good i is a sum of weighted average of price elasticity η and price inelasticity 0. According to theory and practice, the more elastic the demand of good i, the more possible for firms to earn when cutting the price. This indeed implies a countercyclical markup. That is when aggregate demand increases, the price elasticity of good i increases as well, then the firms are more likely to cut the markup to have higher profit. This effect is called by Ravn, Schmitt-Grohé and Uribe (2005) as price-elasticity effect, which we will discuss in details later.

At last, in exchange economy, the firm must satisfy demand at the posted price. Formally,

$$z_t F(k_{it}, h_{it}) \geq c_{it} + i_{it} \tag{19}$$

Then, the objective of the firm is to choose contingent plans for $p_{it}, c_{it}, h_{it}, i_{it}$, and k_{it-1} so as to maximize the present discounted value of profits, given by:

$$E_0 \sum_{t=0}^{\infty} r_{0,t}[p_{it}(c_{it} + i_{it}) - w_t h_{it} - u_t k_{it-1}] \tag{20}$$

subject to production constraint by equation (19), the optimal consumption level by equation (16) and the evolution of habit stock by equation (18), given the processes $r_{0,t}, w_t, u_t, z_t, x_t$ and c_{it-1}. Thus, the firm i's optimization problem can written in Lagrangian as:

$$\begin{aligned} \mathcal{L}_f = E_0 \sum_{t=0}^{\infty} r_{0,t}\{ & p_{it}c_{it} + p_{it}^{1-\eta} i_t - w_t h_{it} - u_t k_{it-1} \\ & + mc_t[z_t F(k_{it}, h_{it}) - c_{it} - p_{it}^{-\eta} i_{it}] \\ & + \nu_t[p_{it}^{-\eta} x_t + \theta^d s_{it-1} - c_{it}] \\ & + \kappa_t[\rho^d s_{it-1} + (1-\rho^d) c_{it} - s_{it}]\} \end{aligned} \tag{21}$$

where mc_t, ν_t, κ_t are Lagrangian multipliers. To be specific, the mc_t is the marginal cost, ν_t is the shadow value of sales, κ_t is the shadow value of habit stock. If we take derivatives of the Lagrangian with respect to h_{it}, k_{it-1}, c_{it},

s_{it} and p_{it}, then we get the FOCs as follows:

$$w_t = mc_t z_t F_h(k_{it-1}, h_{it}) \tag{22}$$
$$u_t = mc_t z_t F_k(k_{it-1}, h_{it}) \tag{23}$$
$$0 = p_{it} - \nu_t - mc_t + \kappa_t(1-\rho^d) \tag{24}$$
$$0 = \theta^d E_t r_{t,t+1}\nu_{t+1} + \rho^d E_t r_{t,t+1}\kappa_{t+1} - \kappa_t \tag{25}$$
$$0 = c_{it} + (1-\eta)p_{it}^{-\eta} i_t + \eta p_{it}^{-\eta-1} mc_t i_t - \eta p_{it}^{-\eta-1}\nu_t x_t \tag{26}$$

Of particular interest, we rewrite the marginal cost and define the markup μ_t as the inverse of this marginal cost as follows:

$$\begin{aligned} mc_t = & \left(1 - \frac{1}{\eta}\frac{1}{(1-\theta s_{it-1}/y_{it})}\right) p_{it} \\ & + (1-\rho)\left(\frac{c_{it} - \theta^d s_{it-1}}{y_{it} - \theta^d s_{it-1}}\right)\left(E_t r_{t,t+1}(\theta^d \nu_{t+1} + \rho^d \kappa_{t+1})\right) \end{aligned} \tag{27}$$
$$\mu_t = 1/mc_t \tag{28}$$

The detailed derivations on the marginal cost and markup equations can be found in Appendix A.2. We will use the above equations to analyze their cyclical behavior in section 5.4.

4.3 Equilibrium

Now limit our attention to a symmetric equilibrium. we drop the superscript j of all variables, because all households are regarded as identical. we also drop the subscript i, due to symmetric assumption on goods (or firms which produce the goods). Moreover, in the equilibrium, all firms charge the same price, and the relative price of any variety of goods is unity, i.e. $p_{it} = 1$. To summarize, we collect all the equations which characterize the equilibrium as follows:

$$U_x(x_t, h_t) = \lambda_t \tag{29}$$
$$U_h(x_t, h_t) = \lambda_t w_t \tag{30}$$
$$\lambda_t = \beta E_t \lambda_{t+1}[1 - \delta + u_{t+1}] \tag{31}$$
$$\lambda_t = \beta R_t E_t \lambda_{t+1} \tag{32}$$
$$mc_t z_t F_h(k_{t-1}, h_t) = w_t \tag{33}$$
$$mc_t z_t F_k(k_{t-1}, h_t) = u_t \tag{34}$$

$$1 - \nu_t - mc_t + \kappa_t(1 - \rho^d) = 0 \tag{35}$$

$$\theta^d E_t r_{t,t+1} \nu_{t+1} + \rho^d E_t r_{t,t+1} \kappa_{t+1} - \kappa_t = 0 \tag{36}$$

$$c_t + (1 - \eta) i_t + \eta mc_t i_t - \eta \nu_t x_t = 0 \tag{37}$$

$$y_t = z_t F_h(k_{t-1}, h_t) \tag{38}$$

$$y_t = c_t + i_t \tag{39}$$

$$k_t = (1 + \delta) k_{t-1} + i_t \tag{40}$$

$$c_t = x_t + \theta s_{t-1} \tag{41}$$

$$s_t = (1 - \rho) c_t + \rho s_{t-1} \tag{42}$$

$$\mu_t = 1/mc_t \tag{43}$$

Totally, the stationary competitive equilibrium can be defined as a set of process of 15 variables, $\{c_t, h_t, x_t, s_t, y_t, i_t, k_t, R_t, \lambda_t, u_t, w_t, mc_t, \nu_t, \kappa_t, \mu_t\}$, satisfying the above 15 equations given the initial values of s_{-1}, k_{-1}, and exogenous stochastic process of z_t.[12]

It is of interest to compare the equilibrium conditions here with the one in standard (superficial) habit formation model. The standard superficial habit formation model shares the conditions (29)-(34), and (38)-(43) with the deep habits model. In particular, they have the same form of Euler equation which is in essence the basis of consumption-based CAPM. This implies the asset pricing implication of deep habits model can be generally interpreted as the standard habit formation model does. Moreover, the values of habits parameters θ, ρ should be similar and consistent with superficial habits models.

The equations (35)-(37) separate the deep habit model with others, because of the assumption of the existence of good-by-good habit. This assumption will generate the dynamic markup instead of the constant one in standard habit formation model. With the changes on pricing strategy, other variables will be also affected. Later, we will discuss in details.

5 Computation, Calibration and the Results

Having the system of equations, we now try to solve the model, i.e. find the analytical solution. Our solution technique follows Campbell (1994) and

[12]Of course some of these variables can be substituted out to facilitate the calculation by hand. However, our Matlab codes think the more the better to avoid some substitution mistakes.

Uhlig (1999), namely, first order approximation. The Toolkit by Uhlig is used to solve and calibrate the model, with the parameters in Ravn, Schmitt-Grohé and Uribe (2005). The Matlab codes to generate the Toolkit input are provided together with a brief description. Finally, the results are analyzed to shed light on the macroeconomic and asset pricing implications of the deep habits model.

5.1 Computation

Many solution algorithms are available for solving RBC models (Taylor and Uhlig (1990)). This paper follows Campbell (1994) and Uhlig (1999), which allows approximating all the relevant equations in log-linear form. Analytical solutions for the elasticities of the endogenous variables with respect to the state variables can be obtained. It would be of great use to analyze the approximate closed-form solutions for prices of a variety of financial assets so that find the implication of the relationship between asset prices and exogenous technology shocks (Lettau(2003)).

To implement the model, we use the Toolkit program by Uhlig. The necessary inputs of Toolkit are the steady states values and coefficient matrix of the log-linearized equations. The necessary equations to calculate the steady state are given in Appendix A.3, and it is quite clear that 15 equations are enough to solve 15 variables with parameters given.

Instead of log-linearizing by hand, we develop the programs to calculate the coefficient matrix by computer.[13] The Symbolic Math Toolbox is required, which allows the symbolic computation within Matlab. The following files are made available in Appendix B.4.

- deep_ss.m: To compute the steady state;
- deep_model.m: To build the model in Matlab;
- deep_deriv.m: To compute the analytical derivatives;
- deep_num.m: To compute the numerical derivatives;
- deep_run.m: To run the whole project.

The output gives the necessary Matrices AA,BB,CC etc. as the input of the Toolkit programs. If we pay attention to the deep_model.m file, you can

[13] The idea is stimulated from the Matlab codes of Schmitt-Grohé and Uribe (2004)

find out all the above mentioned equations for the equilibrium and (even) the Lagrangian associated with households and firms.[14] This method can largely reduce the chance of mistake and it is very convenient for modification or model extension. For example, we extend the model with capital adjustment cost by adding three more equations (constraints) and variables without changing others. This method is also approved by other examples. The corresponding codes can be provided on request.[15]

5.2 Calibration

To calibrate the model in Toolkit, we still need to specify the functional form and the parameter values. we calibrate the U.S. economy, and the time unit is meant to be quarterly. we assume the utility function separable in consumption and leisure as follows:

$$U(x_t, h_t) = \frac{x^{1-\sigma} - 1}{1-\sigma} + A\frac{(1-h)^{1-\gamma} - 1}{1-\gamma} \tag{44}$$

where σ, γ are the curvatures of the utility function with respect to x_t, h_t, satisfying $0 < \sigma \neq 1, 0 < \gamma \neq 1$. The parameter A ensure that at steady state the household devote 20% of their time to market activities (Prescott(1986)). We normalize the sum of labor and leisure to 1, i.e. $h + l = 1$. So actually, we refer the leisure l as $1 - h$. Later, we will consider the situation where consumption and leisure are non-separable.

The production function is Cobb-Douglas type:

$$z_t F(k_{t-1}, h_t) = z_t k_{t-1}^{\alpha} h_t^{1-\alpha} \tag{45}$$

where capital elasticity of output $\alpha = 25\%$.

With the quarterly data from 1967:Q1 to 2003:Q1, Ravn, Schmitt-Grohé and Uribe (2005) estimate the other parameters. we follow their estimation. The corresponding parameter values are listed in Table 6.

[14]Some equations in the code may have signs different from the ones in the paper. However, since Matlab calculates all the FOCs based on Lagrangian, the signs should be consistent as a whole.

[15]Actually, the steady state can also be calculated with Symbolic Math Toolbox. The function solve.m of Matlab can solve the system of equations of symbols. If so, a front end of Toolkit can be made.

Symbol	Value	Description
β	$1.04^{-\frac{1}{4}}$	Quarterly subjective discount rate
σ	2	Inverse of elasticity of intertemporal substitution(EIS)
γ	3.08	Preference parameter
α	0.25	Capital share
δ	0.025	Depreciation rate
η	5.3	Elasticity of substitution across varieties
h	0.2	Steady-state fraction of time devoted to work
θ, θ^d	0.86	Degree of (superficial and deep) habit formation
ρ, ρ^d	0.85	Persistence of habit stock
ρ_z	0.9	First-order serial correlation of technology
σ	0.712	Standard deviation of technology shock (in percent)

Table 6: Calibration Parameters

5.3 Aggregate Dynamics

After successfully implementing the model in Toolkit, we present the results here and try to interpret the underlying economy intuitions. In this subsection, we focus on the aggregate dynamic, namely, the quantitative response to shock, the standard deviation and the cross correlation of the HP-filtered series. These results can be compared with the stylized facts which we have discussed above.

Figure 6 shows the impulse-response of a number of endogenous variables to a one-percent increase in the exogenous productivity factor z_t. The technology improvement brings people more capital which can be used in production and investment. The investment and output increase as a result.[16] Consumption increases because the technology shock raises the marginal utility of consumption. However, the increase is relatively small, for the habit stock influence people's consumption choice. If consider the consumption and habit stock as a whole, we can find the habit-adjusted consumption increase as much as the consumption in the model without habits. The labor supply expends because the shock reduces the value of leisure in terms of consumption. Finally, the marginal cost rises, which implies a decrease in the markup. we will have a further discussion on these two variables.

Besides the graphical analysis, we also give the quantitative result. Table 7 shows the standard deviation and their correlation with GDP of several

[16]Investment rises by about 8 percent which is not yet shown in Figure 6.

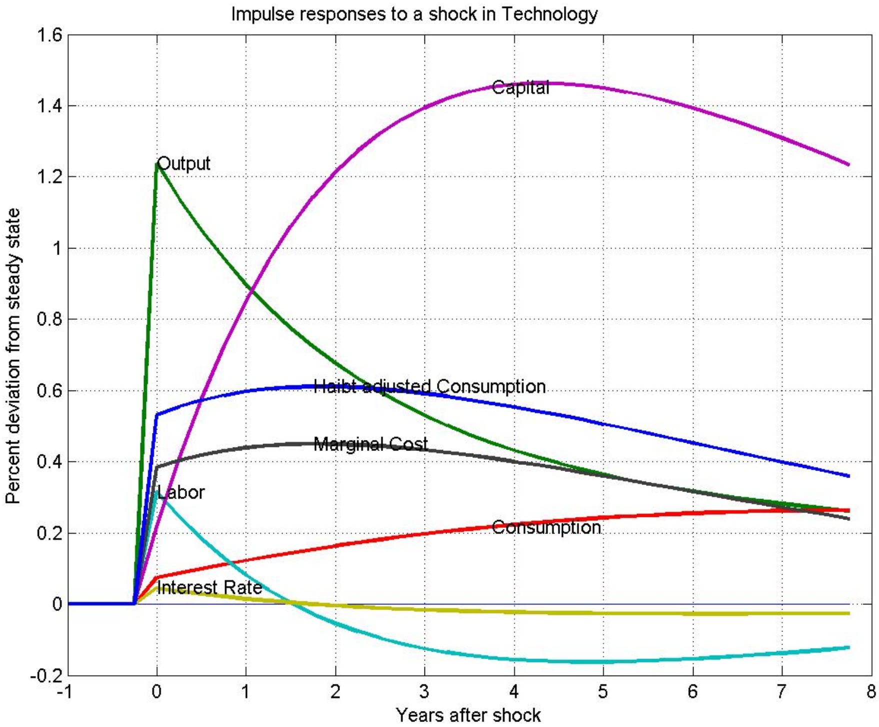


Figure 6: Deep Habits Model: Impulse-Response to Technology Shock

HP-filtered series. This table can be easily compared with the stylized facts we mentioned early. Through comparison, we can find that the model generate relative low volatility in output, consumption and labor. Especially, the standard deviation of consumption is around ten times lower than the facts. This is a common feature of all habit formation models, i.e. under-predicting the volatility in consumption. The standard deviation of investment is a bit higher. However, it is consistent with the fact that investment has largest volatility.

Most the correlations results are qualitatively and quantitatively consistent with the facts. The consumption, labor effort and investment are procyclical. The exception is wage. It seems that wage is highly correlated with output, which is inconsistent with the fact of relative small correlation (un-correlated with output). This procyclical effect is caused by considering the deep habit. Further discussions are presented in next subsection.

	std. Dev.	corr. w. Output
Output	1.2512	1.00
Consumption	0.0863	0.70
Investment	8.8921	1.00
Labor	0.3525	0.90
Wage	1.3662	0.97

Table 7: Deep Habits Model: Statistics

5.4 Dynamic Markup

The central feature of deep habits model is that markup behaves countercyclically as most empirical evidence show. In this subsection, we follow Ravn, Schmitt-Grohé and Uribe (2005), theoretically analyze the corresponding markup equation. Three effects are discussed which affect the dynamic of markup. Then the economic intuitions are discussed with the impulse-response plot.

Recall the equations (27) and (28) which represent the markup. It is convenient to write out the equilibrium markup:[17]

$$\begin{aligned}\mu_t =& [\left(1 - \frac{1}{\eta(1-\theta s_{t-1}/y_t)}\right) p_t \\ &+(1-\rho)\left(\frac{c_t - \theta^d s_{t-1}}{y_t - \theta^d s_{t-1}}\right)\left(E_t r_{t,t+1}(\theta^d \nu_{t+1} + \rho^d \kappa_{t+1})\right)]^{-1}\end{aligned} \tag{46}$$

Let's start from the case where $\theta^d = 0$ and $\rho^d = 0$, i.e. in absence of deep habit. Then, it becomes $\mu = \eta/(\eta-1)$. This means in no-habit or standard superficial habit formation model, the markup is just constant and equate to $\eta/(\eta-1)$.

However, the assumption of deep habit can bring time varying markup. Three factors affect the dynamic of markup, namely price-elasticity, intertemporal effect and demand composition.

Consider only the impact of first part of equation (46).[18] The coefficient represent the price elasticity, more precisely, as function of $(1-\theta s_{t-1}/y_t)$. If all other things constant, an increase in current aggregate demand y_t rises the

[17] The relative price in equilibrium should be $p_t = 1$.

[18] To get the markup which equates purely first part of equation (46), Ravn, Schmitt-Grohé and Uribe (2005) has tried the good-specific subsistence point model.

short-term price elasticity of demand which inducing a decline in equilibrium markup. When firms face an increase in the demand elasticity due to the increase in aggregate demand, they are more likely to decline the markup to get higher profit. This is called the price-elasticity effect of deep habits on markup.

Secondly, consider the only present value term, $E_t r_{t,t+1}(\theta^d \nu_{t+1} + \rho^d \kappa_{t+1})$. Any increase in the present value of future profit induced by current sale increase would leads a decline of markup. This is called the intertemporal effect of deep habits on markup. Furthermore, the markup is a decreasing function on these three variables, namely, $r_{t,t+1}, \nu_{t+1}$, and κ_{t+1}. If the discount factor $r_{t,t+1}$ is high, it means a low interest rate $R_t = \frac{1}{E_t r_{t,t+1}}$. Firms prefer to invest instead of putting the money in bank. Higher incentive of investment leads to a pricing strategy of lower markup. On the other hand, the large ν_{t+1}, κ_{t+1} mean high future profits. Firms increase the investment today to gain more market share in the future. This strategy can be achieved by charging lower markups today.

The last term $\left(\frac{c_t - \theta^d s_{t-1}}{y_t - \theta^d s_{t-1}}\right)$ is the demand composition. This composition does not have direct effect on markup. However, it can affect the markup by increasing the strength of intertemporal effect. If the consumption decomposition is low, then strength of intertemporal effect on markup is reduced. In our model, the $y_t = c_t + i_t$, where consumption is affected by habit while investment is not. In most cases, technology shock increase the output through capital accumulation channel, i.e. high i_t, and relative low c_t. Thus, in a technology-driven model, the intertemporal effect on markup is reduced much due to this demand composition.

Having finished the above theoretical analysis, we come to the impulse-response plot in Figure 7. For comparison, four variables of three models are plotted respectively. The four variables are consumption, wage, output and markup, while the three models are no-habit model, superficial habits model and deep habits model.[19]

The most obvious difference among the three models is the negative response of markup in deep habits model, while constant in the other two models. This

[19]Three models can be obtained by setting different values for habit parameters. Specifically, $\theta = \theta^d = 0, \rho = \rho^d = 0$ for no-habit model, $\theta = 0.86, \theta^d = 0, \rho = 0.85, \rho^d = 0$ for superficial habit model and $\theta = \theta^d = 0.86, \rho = \rho^d = 0.85$ for deep habit model, while all other parameters unchanged.

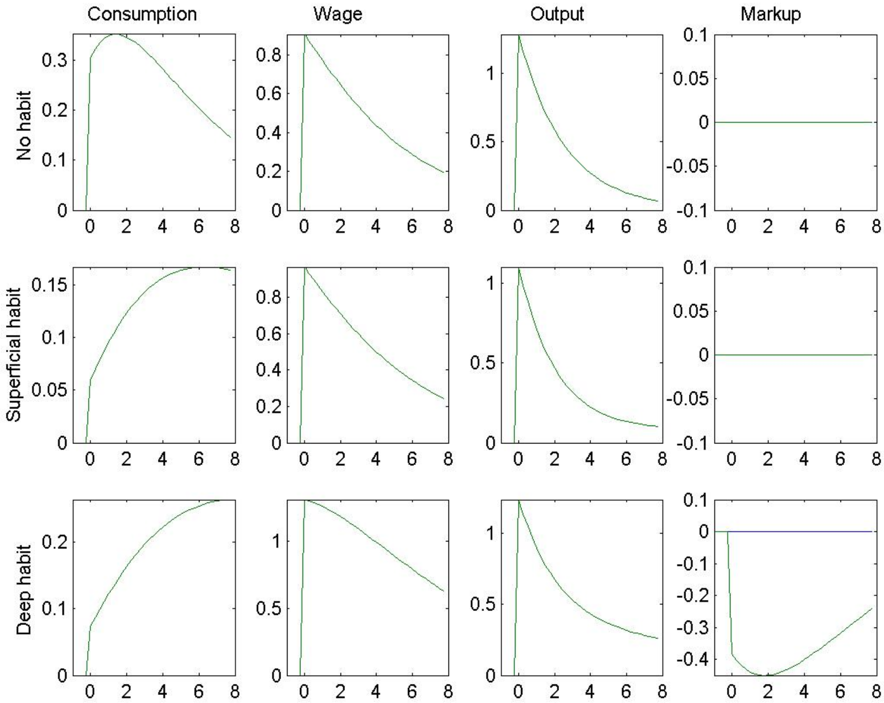


Figure 7: No Habits, Superficial Habits, and Deep Habits

difference is mainly due to the price elasticity effect under deep habits assumption. According to equation (46), as the aggregate demand y_t increases, the habitual effect $\theta^d s_{t-1}$ becomes less important. The price becomes more elastic, and in turn markup decreases at first. Later, the markup gradually increases, for agents form their habits, i.e. the habitual term $\theta^d s_{t-1}$ becomes more and more important.

In addition, the variable wage in deep habits model rises by 1.2% which is larger than the increases in the other two models, say, by around 0.9% percent. This extra increase is due to the reduction of the markup under deep habit assumption. Since firms cut the markups, the labor demand increases more than labor supply. As a result, the wage rises more under deep habits model than under either no-habit or superficial habits model.

As for the other two variables, the outputs increase approximately same amount among three models, while the consumptions rise much less under either superficial or habit model than no-habit model. These results are consistent with most studies on habits.

To summarize our analysis, the deep habits model can predict a countercyclical markup, a procyclical wage, while as other habit models, it under-predicts the consumption.

5.5 Asset Prices

With basically matching the macroeconomic facts, more and more economists emphasize the asset market implication of RBC model. In this subsection, following the analysis of Lettau (2003), Lettau and Uhlig (2002) and Uhlig (2004), we try to explore the asset pricing implication of deep habit models in details. The goal is to derive explicit solutions of important financial variables, such as risk premia and Sharpe ratio. The solutions can be written in terms of the deep parameters of the model, such as risk aversion and the elasticities of the endogenous variables.

The algorithm of Campbell (1994) and Uhlig (1999) tell that the endogenous variables (in logs) can be expressed as linear functions of the logs of the state variables. Here the state variables are capital and technology. The linear functions can be e.g. for the shadow value λ_t:

$$\hat{\lambda}_t = \eta_{\lambda k}\hat{k}_t + \eta_{\lambda z}\hat{z}_t \tag{47}$$

where we use the hat letters to denote the log-linear deviations, and the η_{xy} denotes the elasticity of variable x with respect to variable y. Similar forms of equations could be get for $\hat{c}_t, \hat{h}_t$, etc. As for the two state variables, next period capital can be written as in equation (48) and technology follows an AR(1) process in equation (49):

$$\hat{k}_{t+1} = \eta_{kk}\hat{k}_t + \eta_{kz}\hat{z}_t \tag{48}$$

$$\hat{z}_{t+1} = \rho_z\hat{z}_t + \epsilon_{t+1} \tag{49}$$

These elasticities are complicated functions of the deep parameters of the model, and their values influence the calculations of risk premium and Sharpe ratio.

Now, let's look at some basic asset pricing equations, where the above solution can be plug into to get the explicit solutions for some financial variables. The well-known asset pricing formula with gross return $R_{t,t+1}$ can be:

$$1 = E_t\left[\beta\frac{\Lambda_{t+1}}{\Lambda_t}R_{t,t+1}\right] \tag{50}$$

This equation is consistent with the Euler equation (12) in the model, where $R_{t,t+1} = 1 - \delta + u_{t+1}$. If considering the log-linear form of this equation, we can find the expression of risk premium:

$$\hat{r}^{rp}_{t,t+1} \equiv \hat{r}^{e}_{t,t+1} - \hat{r}^{f}_{t,t+1} = -\mathbf{cov}(\Delta\hat{\lambda}_{t+1}, \hat{r}_{t,t+1}) \tag{51}$$

where $\hat{r}^{e}_{t,t+1} = \mathbf{log} E_t R_{t,t+1}$ is the logarithm of the expected gross return.[20] $\hat{r}^{f}_{t,t+1}$ is the risk free rate. Thus, the excess return of an asset is the covariance of the asset return with the shadow value. More details on the derivation can be found in Appendix A.4.

According to the above analysis, the solutions of shadow value growth and asset returns are linear functions of stochastic shocks of technology. Plug the model solutions into the above equation, we get the expression of risk premium as the elasticities:

$$\hat{r}^{rp}_{t,t+1} = -\mathbf{cov}(\eta_{\lambda z}\epsilon_{t+1}, \eta_{rz}\epsilon_{t+1}) = -\eta_{\lambda z}\eta_{rz}\sigma^2_\epsilon \tag{52}$$

Sharpe ratio, measured as the ratio of risk premium over the standard deviation of the return, can also be explicitly expressed as follows:

$$SR_t = \left|\frac{\hat{r}^{rp}_{t,t+1}}{\sigma_{r,t}}\right| \leq SR^{max}_t = |\eta_{\lambda z}\sigma_\epsilon| \tag{53}$$

With the above two expressions, we can calculate out the risk premium and Sharpe ratio of the models. Unfortunately, the calculated values of most real business model are far away from the facts, i.e. 1.99% of equity premium and 0.27 of Sharpe ratio of quarterly data. For instance, Lettau (2003) reported a Sharpe ratio of 0.0263 and equity premium of 0.00068% with risk aversion of 10 and fix labor. In our this deep habit model, the reported Sharpe ratio is 0.013,[21] equity premium 0.00011%. Due to the above statements, we will focus on the determinants of Sharpe ratio instead of the numbers themselves.

Usually, given the standard deviation of exogenous shock, the only determinants of the Sharpe ratio is $\eta_{\lambda z}$, the elasticity of the shadow value with respect to the exogenous shock. To investigate the underlying intuition, we further decompose the shadow value following Uhlig (2004). The log-linearized equation of shadow value can be written as:

$$\hat{\lambda}_t = -\eta_{cc}\hat{c}_t + \eta_{cll}\hat{l}_t \tag{54}$$

[20] The notation $\hat{r}^{e}_{t,t+1} = logE_t R_{t,t+1} \neq E_t log R_{t,t+1}$ due to Jensen's inequality.

[21] As Lettau did, we calculate the $\sigma_\epsilon = (0.712^2) * (1 - \rho_z^2)/\alpha^2$, while the $\eta_{\lambda z} = 1.0622$ is the solution of our model.

where $\hat{l}_t$ denotes the log deviation of leisure. The risk aversion, η_{cc}, and cross derivative η_{cll} are defined as follows:

$$\eta_{cc} = -\frac{U_{cc}(\bar{c}, \bar{l})\bar{c}}{U_c(\bar{c}, \bar{l})} \tag{55}$$

$$\eta_{cll} = \frac{U_{cl}(\bar{c}, \bar{l})\bar{c}}{U_l(\bar{c}, \bar{l})} \tag{56}$$

Assuming asset returns, consumption and leisure are jointly log-normal distributed, then the growth of shadow value can be written as:

$$E_t[\Delta\hat{\lambda}_{t+1}] = -\eta_{cc}E_t[\Delta\hat{c}_{t+1}] + \eta_{cll}E_t[\Delta\hat{l}_{t+1}] \tag{57}$$

Further decomposing, the Sharpe ratio can be as follows:

$$\begin{aligned} SR_t^{max} &= \eta_{\lambda z}\sigma_\epsilon = (\eta_{cc}\sigma_{c,t} + |\eta_{cll}|\sigma_{l,t} \\ &= (\eta_{cc}\eta_{cz} + |\eta_{cll}|\eta_{lz})\sigma_\epsilon \end{aligned} \tag{58}$$

Up to now we decompose the Sharpe ratio and get the explicit solution of equation (58). Given the standard deviation of exogenous shock, the determinative factors are actually these ηs. In the following analysis, we will focus on this equation and these elasticities, which indeed determine the Sharpe ratio of our models.

	η_{cc}	$\times\eta_{cz}$	$+ \mid \eta_{cll} \mid$	$\times\eta_{lz}$	$= \mid \eta_{\lambda z} \mid$
no-habit	2	0.3047	0	-0.0957	0.6094
superficial habit	14.2857	0.0603	0	-0.0341	0.8608
deep habit	14.2857	0.0744	0	-0.0789	1.0622
	η_{xx}	$\times\eta_{xz}$	$+ \mid \eta_{xll} \mid$	$\times\eta_{lz}$	$= \mid \eta_{\lambda z} \mid$
superficial habit	2	0.4304	0	-0.0341	0.8608
deep habit	2	0.5311	0	-0.0789	1.0622

Table 8: Sharpe Ratio Determinants

In Table 8, we report the calculated values of the elasticities of our benchmark deep habits model. For comparison, we also give the values of the no-habit and superficial habit formation models. Since we take the additive utility function, it is separable in consumption and leisure, i.e. $\eta_{cll} = 0$. The cross derivative plays no role in affecting the Sharpe ratio. Later, we will discuss the effect of nonseparability between consumption and leisure, where $\eta_{cll} \neq 0$.

Generally, the models with (either superficial or deep) habit formation can have higher the Sharpe ratio than the models without habit, because they have much higher risk aversion. In no-habit model, the risk aversion is just the curvature of the utility function with respect to consumption, i.e. $\sigma = \eta_{cc} = 2$ in Table 8. However, in (superficial or deep) habit formation models, the risk aversions equate $\sigma/(1-\theta) = 14.29$, which is much higher. The habit-forming consumers are reluctant and dislike large and sudden changes in consumption. Therefore, the risk (or the premium) to hold risky assets which may force a sudden change in consumption will be large than the risk in time-separable utility model.

However, the relatively small η_{cz} in the habit models somewhat offset the high risk aversion effects on the $\eta_{\lambda z}$.[22] Consumption has fairly small elasticity and its volatility is much less than the data. This is the consumption volatility puzzle of most habit formation models. If considering the consumption and habit stock as a whole, i.e. $x_t = (c_t - \theta s_t)$, then this habit-adjusted consumption has the same curvature as the one in no-habit models, while still higher elasticity η_{xz} than η_{cz} in no-habit model, as shown by the last two rows in Table 8.

Finally, in deep habit model, the Sharpe ratio, precisely the $\eta_{\lambda z}$, is higher than the one in superficial habit model. This higher $\eta_{\lambda z}$ is due to relative high elasticities of consumption and labor (negative of leisure). Usually, under either superficial or deep habit formation, agents do not work that harder to build their capital, because they form the consumption habits and do not want to change a lot in future consumption. However, under deep habit, the pricing strategy of cutting markup generate a even higher labor demand, in turn, require even higher labor supply. This requirement drives lower leisure and higher consumption. Thus, we get a higher consumption elasticity of deep habits model in Table 8.

6 Variations

6.1 Capital Adjustment Cost

Many literature have pointed out capital adjustment cost can help explain the Sharpe ratio. It can "generate sizeable Sharpe ratio, in particular for high levels of relative risk aversion in consumption"(Uhlig(2004)). In literature, there are generally two kinds of capital adjustment costs. The common

[22] Usually, the higher the relative risk aversion, the lower the η_{cz}

version is that costs depend on the ratio of new investment to capital, e.g. Jermann (1998), Boldrin, Christiano and Fisher (2001) and Uhlig (2004). The other is less common however it allows the costs to depend on the ratio of current investment to previous investment, e.g. Christiano, Eichenbaum, and Evans (2003). In this section, we extend the above deep habit model with capital adjustment cost following the common version. Its implications on asset pricing are discussed.

The rigid investment is introduced by following capital accumulation equation:

$$k_t = (1-\delta)k_{t-1} + G\left(\frac{i_t}{k_{t-1}}\right)k_{t-1} \tag{59}$$

where the adjustment cost function $G(\cdot)$ is concave in investment which captures the difficulty of quickly changing capital stock. Specifically, the functional form can be:

$$G\left(\frac{i_t}{k_{t-1}}\right) = \frac{a_1}{1-1/\xi}\left(\frac{i_t}{k_{t-1}}\right)^{1-1/\xi} + a_2 \tag{60}$$

where ξ is the elasticity of investment with respect to Tobin's q. It is:

$$\xi = -\left(\left(\frac{i}{k}\right)\left(\frac{G''}{G'}\right)\right)^{-1} > 0 \tag{61}$$

The parameters a_1, a_2 are chosen so that $G(\delta) = \delta$ and $G'(\delta) = 1$. Specifically, the a_1 and a_2 are set as:

$$a_1 = (\delta)^{1/\xi}, \qquad a_2 = \frac{-\delta}{\xi - 1} \tag{62}$$

Actually, the only parameter we need to set is the elasticity ξ. No firm consensus about this value in literature. Jerman (1998) takes the value of 0.23, while King and Wolman (1996) use the value of 2.0. When $\xi = \infty$, it turns to the benchmark case, i.e. without capital adjustment cost, which we have discussed already. We are going to consider the three cases with three different parameter values. Finally, to incorporate the capital adjustment cost, we add equation (59) as a constraint to the Lagrangian of household with a Lagrangian multiplier τ. Actually, this τ is the shadow value of capital, or marginal efficiency of investment, which is discussed later on.

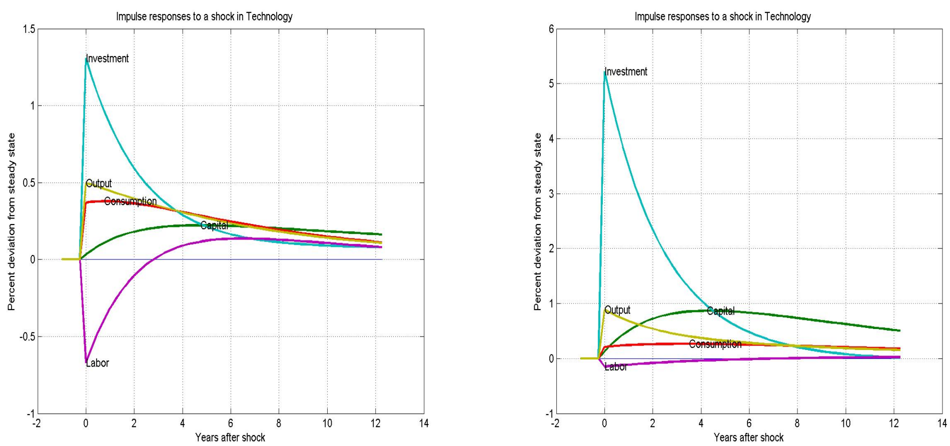
Table 9 lists the results of the elasticities. We can immediately find that the determinative factor of Sharpe ratio, $\eta_{\lambda z}$ jumps a lot (from 1.06 to 5.29).

	η_{cc}	$\times\eta_{cz}$	$+\lvert \eta_{cll} \rvert$	$\times\eta_{lz}$	$=\lvert \eta_{\lambda z} \rvert$
$\xi = 0.23$	14.2857	0.3705	0	0.1677	5.2924
$\xi = 2$	14.2857	0.2121	0	0.0374	3.0303
$\xi = \infty$	14.2857	0.0744	0	-0.0789	1.0622
	η_{xx}	$\times\eta_{xz}$	$+\lvert \eta_{xll} \rvert$	$\times\eta_{lz}$	$=\lvert \eta_{\lambda z} \rvert$
$\xi = 0.23$	2	2.6462	0	0.1677	5.2924
$\xi = 2$	2	1.5151	0	0.0374	3.0303
$\xi = \infty$	2	0.5311	0	-0.0789	1.0622

Table 9: Deep Habit Model with Capital Adjustment Cost

Moreover the lower the parameter ξ, the higher we get for $\eta_{\lambda z}$. Indeed, decreasing ξ implies more rigid investment and hence lower reaction of capital to a shock. Since less is invested, the consumption must react more to shocks. The η_{cz} increases with small ξ. Therefore, the higher Sharpe ratio will be generated by models when considering the rigid investment.

Although considering capital adjustment can increase $\eta_{\lambda z}$, however, the Sharpe ratio is still far from 0.27, the one in data. Lettau (2003) shows that given a moderate risk aversion, the Sharpe ratio is only 0.01 with extreme adjustment cost, $\xi = 0$. Unless the risk aversion is fairly high (e.g. 50 or above), the adjustment cost does play a role in generating sizeable Sharpe ratio. However, too large risk aversion is unrealistic.



(a) $\xi = 0.23$

(b) $\xi = 2$

Figure 8: Deep Habits Model with Capital Adjustment Costs

	$\xi=\infty$	$\xi=2$	$\xi=0.23$
Output	1.2512	0.7191	0.5276
Consumption	0.0863	0.1742	0.4123
Investment	8.8921	4.2628	1.3296
Labor	0.3525	0.1228	0.6946
Wage	1.3662	2.3744	4.8671
Markup	0.4289	1.5337	3.6793

Table 10: Deep Habit Model with Adjustment Cost:
Standard Deviation of HP-filtered Series(Simulation-based)

If we look at the plots in Figure 8 and standard deviations in Table 10, we can find some common features of the models with capital adjustment cost. The rigid investment makes consumption relative strong response, and higher volatile. These effect directly result in higher η_{cz}, in turn larger Sharpe ratio we mentioned above. However, much high adjustment cost can cause too little investment and output volatilities, which are far from the data. Perhaps, we need to scale up the technology shock to make the output fluctuations consistent with the data. (Uhlig (2004))

The labor response to technology negatively. This is contrary to the fact we have shown.[23] The higher the adjustment cost, the more the labor negatively reacts. This is counterfactual that in technology-shock driven models, the positive comovement between hours, investment, consumption, and output should exist. (Uhlig (2004)) Since the labor (or leisure) does not enter the Euler equation, (zero cross derivative), it has no direct effect on Sharpe ratio. Only indirect effect occurs by influencing the η_{cz} through the labor market. When considering the nonseparability between consumption and leisure, the case would be different. We discuss this later.

With capital adjustment cost, the wage and markup fluctuate much more than the ones without adjustment cost but under the same deep habit assumption. Remember, comparing to the no-habit or superficial habit model, the deep habit assumption can cause a dynamic markup and more volatile wage. Now considering the capital adjustment cost would cause less investment. Accordingly, firms change their pricing strategy even more. That is

[23] Actually, there has been a debate on the effect of technology shock to the hours worked, i.e. hours increase or fall in response to a positive innovation in productivity. For example, the studies by Galí (1999), Christiano et al (2003) and Uhlig (2003) are in favor of the decline in labor.

to cut more on markup, and in turn result a much higher wage.

To sum up, the capital adjustment cost can generate sizeable Sharpe ratio and consumption volatility. However, it also brings problems of too little investment and output volatilities. Meanwhile, the positive comovement in output, consumption, investment and labor can not be explained under the above discussed adjustment cost models. We would like to change the functional form of the utility, which may on the one hand improve the Sharpe ratio, on the other hand solve some of problems of capital adjustment cost.

6.2 Nonseparability between Consumption and Leisure

Thus far, we have assumed the additive utility function, which is separable in consumption and leisure, i.e. a zero cross derivative η_{cll}. With this zero cross derivative, we are limited to consider only consumption in affecting the Sharpe ratio, while labor (or leisure) plays no role. We believe, however, with appropriate value, the nonseparability between consumption and leisure has more abundant implications. In this subsection, we try to examine this effect in our models.

Consider our benchmark deep habits model. With all other things the same, we assume the utility function takes the form as follows:

$$U(x_t, h_t) = \frac{((x_t)^{\varsigma}(l)^{\gamma})^{1-\sigma} - 1}{1-\sigma} \tag{63}$$

where $\varsigma, \gamma, \sigma$ are parameters, x_t is still the habit-adjusted consumption, which equates $x_t = c_t - \theta s_{t-1}$. Leisure is defined as $l = 1 - h$. This multiplicative utility is nonseparable in consumption and leisure. The risk aversion and cross derivative are:

$$\eta_{cc} = (-\varsigma + \varsigma * \sigma + 1)/(1-\theta) \tag{64}$$

$$\eta_{cll} = \gamma(1-\sigma) \tag{65}$$

The cross derivative is no longer zero. It takes influence on the Sharpe ratio. We try our benchmark model with this nonseparable utility function, and give the results for $\eta_{\lambda z}$ in Table 11. We set $\varsigma = 1, \sigma = 2$ so that our results can be compared with the case of separable utility. In addition, we also set $\gamma = 18.64$ to ensure the steady-state fraction of time devoted to work, $\bar{h}$, equate 20%.

	η_{cc}	$\times\eta_{cz}$	$+\mid \eta_{cll} \mid$	$\times\eta_{lz}$	$=\mid \eta_{\lambda z} \mid$
Separable	14.2857	0.0744	0	-0.0789	1.0622
Nonseparable	14.2857	0.2080	18.6410	-0.1190	0.7533

Table 11: Benchmark Deep Habit Model with Different Utility Function

The results show that multiplicative utility actually decrease Sharpe ratio. This is mainly because the cross derivative η_{cll} amplifies the negative effect of η_{lz}, while in additive utility, the negative η_{lz} is multiplied by zero.

Consider again the equation for the Sharpe ratio determinant, $\eta_{\lambda z}$. In order to have a higher $\eta_{\lambda z}$, we can either raise η_{cz} or lower the absolute value of $|\eta_{lz}|$, given η_{cc} and η_{cll}. In first case, if η_{cz} is high enough, then $\eta_{\lambda z}$ is increased by the "positive effect" form consumption. In the second case, very small $|\eta_{lz}|$ can reduce the negative effect from leisure. (If η_{lz} becomes positive, the effect from leisure changes positive.) Therefore, the $\eta_{\lambda z}$ is affected by the two channels in the multiplicative utility function.

Adding capital adjustment cost can indeed affect both the two channels. In separable utility case, as we discussed and showed in Table 9, the adjustment cost increases η_{cz}, and makes η_{lz} positive. Thus, with the multiplicative utility function, we add the benchmark deep habits model with capital adjustment cost of the form in section 6.1. We choose a moderate adjustment cost $\xi = 2$, for we do not want too rigid investment which makes labor movement much counterfactual. Actually, if $\xi = 0.23$, the deep habit model will generate negative response of consumption and output to a technology shock. This implausible result is because the habit effect and too rigid investment make people too reluctant and conservative to new things. When technology improves, the deep rooted habitual behavior as well as the high adjustment cost lower the aggregate investment, consumption and output.

	η_{cc}	$\times\eta_{cz}$	$+\mid \eta_{cll} \mid$	$\times\eta_{lz}$	$=\mid \eta_{\lambda z} \mid$
$\xi = \infty$	14.2857	0.2080	18.6410	-0.1190	0.7533
$\xi = 2$	14.2857	0.6843	18.6410	-0.3031	4.1261

Table 12: Deep Habit Models with Nonseparable Utility between Consumption and Leisure with or without Adjustment Costs

Table 12 gives out the Sharpe ratio determinants of the benchmark deep habit model with moderate adjustment cost and nonseparable utility in consumption and leisure. (For short we call it DHNA model.) Unlike the model without adjustment cost, the much increased η_{cz} offset the relative small effect of negative η_{lz}. Therefore, $\eta_{\lambda z}$ increases as well. This confirms the conclusion that in principle, the nonseparability between consumption and leisure does help explain some of the asset pricing problems. Since $\eta_{\lambda z} = 4.1261$ is almost the largest number among all of our models,[24] our following analysis will focus on this DHNA model.

6.3 Comparison: Some Macroeconomic Implication

So far, we have focus on the asset market, esp. the Sharpe ratio. We have analyzed the capital adjustment cost and nonseparability between consumption and leisure, which help generate sizeable Sharpe ratio. Particularly in our DHNA model, a large $\eta_{\lambda z}$ is generated. In this section, we continue to analyze the macroeconomic implications of this DHNA model, where the deep habit effect is discussed. To emphasize the deep habit effect, we compare three similar models, namely, DHNA, SHNA and DHSA. The descriptions of the three models are as follows:

- DHNA: Deep Habit model with preference Nonseparable in consumption and leisure and Adjustment cost
- SHNA: Superficial Habit model with preference Nonseparable in consumption and leisure and Adjustment cost
- DHSA: Deep Habit model with preference Separable in consumption and leisure and Adjustment cost

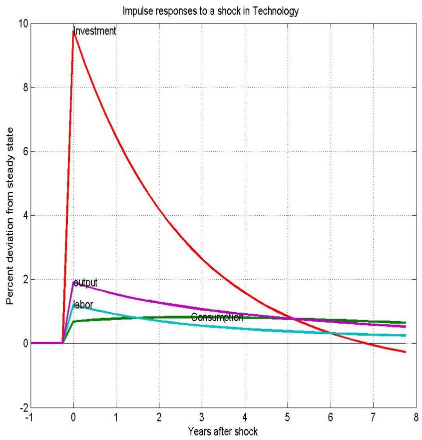
These three models have common features which ensure a fair comparison. They all incorporate the capital adjustment with $\xi = 2$, and they are all Sharpe ratio targeted. The $\eta_{\lambda z}$ are presented in Table 13, which are all relative higher than standard RBC models.

	η_{cc}	$\times\eta_{cz}$	$+ \mid \eta_{cll} \mid$	$\times\eta_{lz}$	$=\mid \eta_{\lambda z} \mid$
DHNA	14.2857	0.6843	18.641	-0.3031	4.1261
SHNA	14.2857	0.1559	20.3455	0.0569	3.385
DHSA	14.2857	0.2121	0	0.0374	3.0303

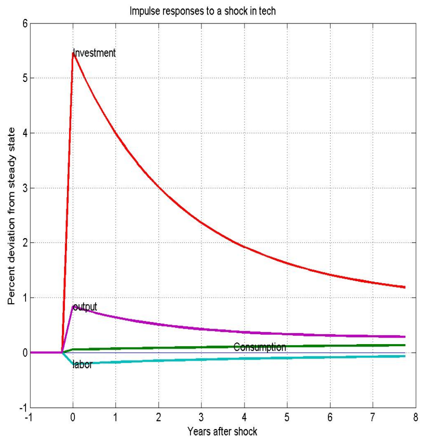
Table 13: Sharpe Ratio Determinants of Three Models

[24] Actually, $\eta_{\lambda z} = 5.2924$ in Table 9 is the largest in this paper. However, it considers too rigid investment with $\xi = 0.23$.

By comparing the first two models, we examine the macroeconomic implication of deep habits model. And through the comparison of the first and third models, we stress the importance of nonseparability between consumption and leisure when talking about the macroeconomic implications of deep habits model.



(a) DHNA



(b) SHNA

Figure 9: Comparison: Deep Habit cause Comovement

Figure 9 shows the comparison of the impulse-response plots to a technology shock of the first two models. The obvious difference is the labor's response. In SHNA model, labor negatively reacts to one percent increase in technology. This is the common feature of most adjustment cost models, like in panel (a) and (b) of Figure 8 in section 6.1. This is counterfactual to the facts we have shown. That is in technology-shock driven models, the positive comovement between hours, investment, consumption, and output should exist. However, in DHNA the four variables all positively react. This is mainly due to the deep habit assumption, which alters the pricing strategy of firms and further affect the labor market.

To have a clear view on the deep habit effect, we subplot 9 variables of the three models in Figure 10. From left to right, the 9 variables are output, capital, consumption, investment, wage, shadow value of wealth, labor, shadow value of capital, and markup. In SHNA model (dashed line of the subplot), the markup is constant, and adjustment cost causes the decrease of marginal

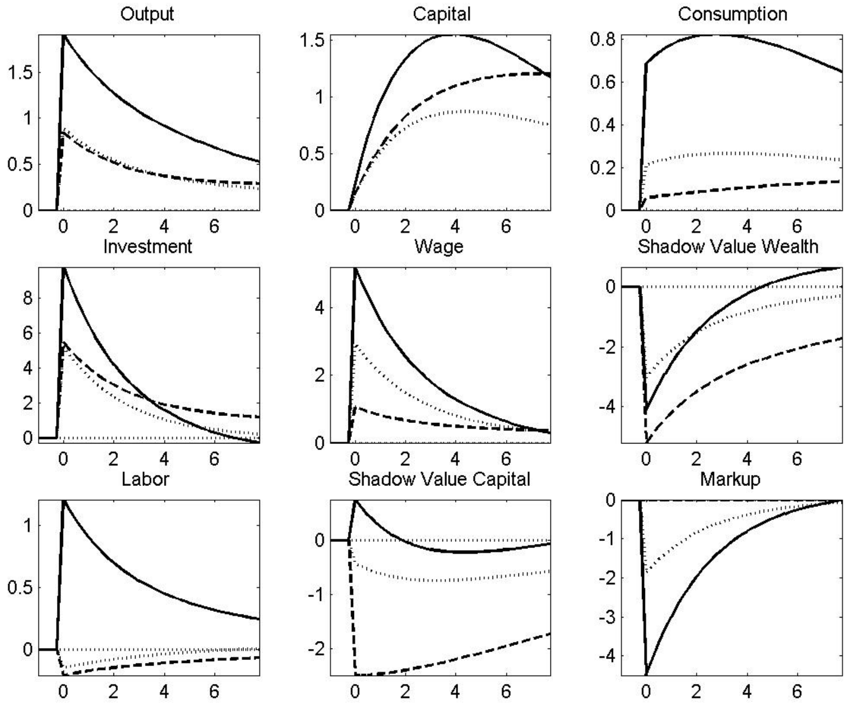


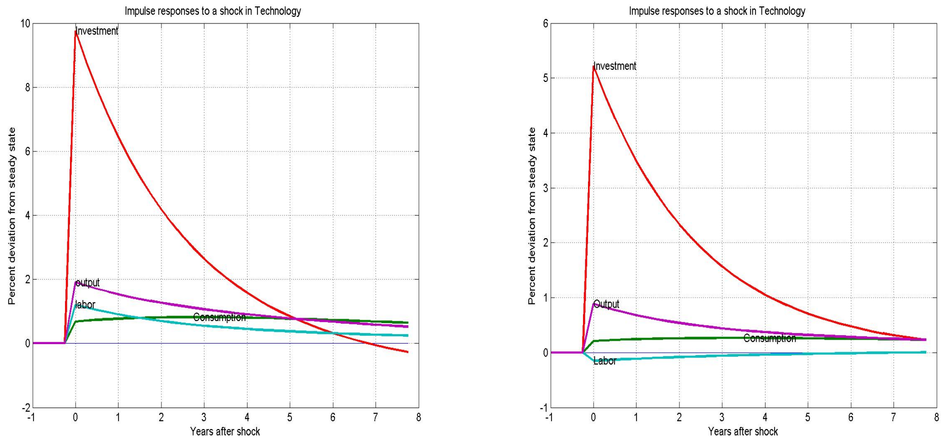
Figure 10: Comparison of three Models: Subplot
solid: DHNA dashed: SHNA dotted: DHSA

efficiency of investment,[25] i.e. the shadow value of capital in SHNA model decreases. With less investment, less labor is devoted. Thus, the labor in SHNA model falls. However, in DHNA model (solid line in subplot), technology improvement increase the demand of goods, and through the price-elasticity effect, firms cut the markup and change their pricing strategy. The decreased markup offsets the effect from adjustment cost. Hence the marginal efficiency of investment increases. This further affects the labor market and agent's choice on leisure. With a utility nonseparable in consumption and leisure, the labor increases as well.

One should notice that the deep habit model can generate positive labor reaction only when we consider the utility nonseparable in consumption and leisure, because this nonseparability have consequences for the endogenous

[25]The marginal efficiency of investment is defined as the Lagrangian multiplier before the constraint of equation (59). This multiplier is also the shadow value of capital.

choices in the macroeconomic models. The subplot of DHSA (dotted line) shows that although the markup decreases, the marginal efficiency of investment still goes down without the nonseparability effect. On impact, the labor negatively reacts and the comovement among the output, investment, consumption and labor disappears.



(a) DHNA (b) DHSA

Figure 11: Comparison: Importance of Nonseparability between Consumption and Leisure

To summarize, we examine the macroeconomic implications. By comparing three Sharpe ratio targeted models, we find that with utility nonseparable in consumption and leisure, the deep habit assumption can cause the positive comovement in output, investment, consumption and labor of technology-shock driven models. This finding is consistent with the facts in this paper.

6.4 Deep Habits and Sticky Prices

Up to now, we have focus my the discussion within the flexible price framework, i.e. in equilibrium, the relative price equates one. A more realistic assumption could be the sticky prices assumption. There has been an abundant of paper studying the macroeconomic implications with sticky prices. Since this assumption also influence the pricing strategy of firms, the marginal cost (related to markup) is hence no longer constant. The markups become time varying.

However, unlike the countercyclical markup in the deep habit model, most of these sticky price models predict a positive markup. For instance, see the study of Schmitt-Grohé and Uribe (2004). In this section, we first try to explain the intuitions for markup in a simple sticky price model. Then we incorporate the deep habit into a Calvo-type sticky price model to see the results of this combination.

The sticky prices model we present here is a similar to the model of Schmitt-Grohé and Uribe (2004). However, we make the model much simpler. We ignore the government sector and focus on zero long run inflation and cashless economy. Besides, we add the superficial habits and capital adjustment cost as mentioned before, for we want to compare with previous deep habit model. The descriptions of the model refer to Appendix A.5, and more details can be found in either Schmitt-Grohé and Uribe (2004) or the seminar paper Li and Zhang (2004).

If we have a look at the equilibrium equations, we can find most equations are the same as the equilibrium of our benchmark case, except for the firm part. We assume the Calvo-type sticky price, where $\vartheta \in [0, 1)$ of randomly picked firms is not allowed to change the nominal price of the good it produces. The remaining $(1 - \vartheta)$ firms choose prices optimally. Thus, the marginal cost is time varying instead of being a constant number of $(\eta - 1)/\eta$. The correspondent FOC with respect to chosen price $\tilde{P}_t$ is as follows:

$$E_t \sum_{s=t}^{\infty} r_{t,s} \vartheta^{s-t} (\frac{\tilde{P}_t}{P_s})^{-1-\eta} y_s \left[mc_s - \frac{\eta - 1}{\eta} \frac{\tilde{P}_t}{P_s} \right] = 0$$

If we further log-linearize this equation[26],[27] we would get the famous neo-Keynesian Phillips curve which involves inflation and marginal costs:

$$\hat{\pi}_t = \beta E_t[\hat{\pi}_{t+1}] + \chi \hat{mc}_t \tag{66}$$

where hat variables denote the log-deviations and χ is a positive constant number. Later we will use this equation in discussing the markup of a sticky price model.

[26]To log-linearize this equation, we still other equations, such as the price index equation in Appendix A.5.

[27]To solve the model, actually we did not log-linearize this equation. Instead, we follow the technique of Schmitt-Grohé and Uribe (2004). More details please refer to Appendix A.5.

How do the variables react to a technology shock in this sticky price model? Figure 12 shows the impulse response subplot. Again, as an improvement of technology, the investment and output both increase, the consumption increases as well. Due to the capital adjustment cost, labor negatively reacts. Among the three new variables, inflation and the nominal interest rate fall, while the markup of prices over marginal cost increases. We try to interpret the intuitions behind these three variables one by one.

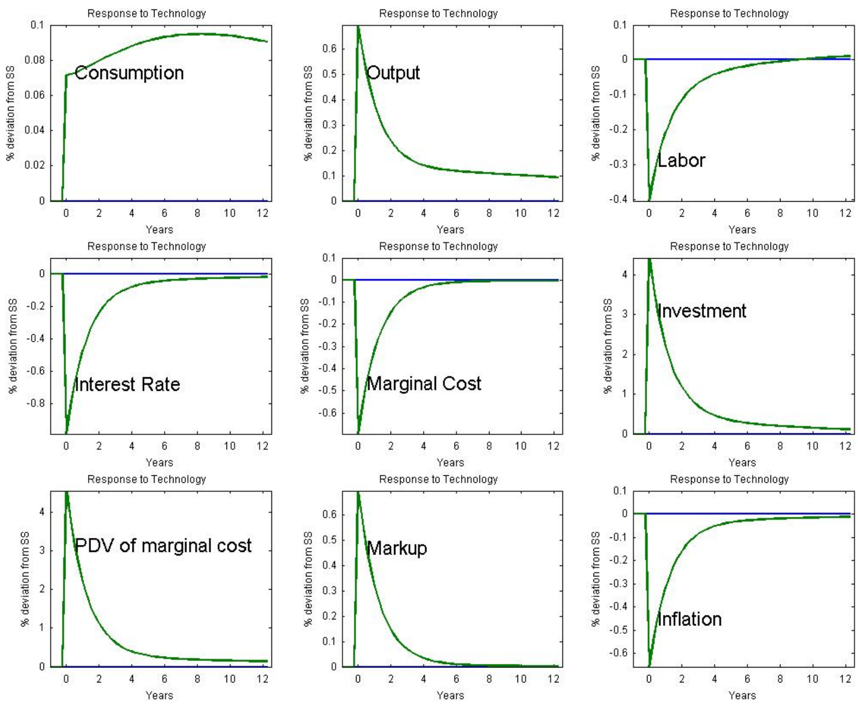


Figure 12: Sticky Price Model with Superficial Habits

A decrease in nominal interest rate is due to both the sticky price and rigid investment assumptions. Majority of firms[28] are unable to choose their price optimally in response to the shock. Meanwhile, the existence of capital adjustment cost makes firms less incentive to invest. Consequently, it gives less hours worked, and higher consumption. As the consumption demand increases, higher productions are needed to meet the demand. Therefore, the nominal interest rate declines to stimulate more production. Note since

[28]Here we set $\vartheta = 2/3$ according to most literature.

the prices are assumed to be sticky, actually a decline in nominal interest rate also leads to a decrease in real interest rate.

The reason why inflation falls in response to a positive technology shock, can be explained according to both the interest rule and Fisher relation. In our model, the monetary rule is assumed to be a simple Taylor form, i.e. $\mathbf{log}(R_t/R^*) = \alpha_\pi E_t \mathbf{log}(\pi_{t-1}/\pi^*)$. To set $\alpha_\pi = 1.5$ and log-linearize this equation, we get $\hat{R}_t = 1.5\hat{\pi}_t$. Together with the Fisher equation of $\hat{R}_t = \hat{\pi}_t + \hat{r}_t$, we get the inflation follows a first order process, i.e. $\hat{\pi}_t = 1.5\hat{\pi}_{t-1} + \hat{r}_t$. From our previous analysis, we know that the real interest rate $\hat{r}_t$ decrease below its steady state. Thus, in order to be non-explosive, the inflation must fall as well.

Having known the inflation falls in response to a positive productivity shock, we can also understand the behavior of marginal cost, in turn markup. Recall equation (66), the neo-Keynesian Phillips curve, which links the inflation and marginal cost. As long as the decline in current inflation is larger than the expected future inflation decline, marginal cost must fall. Accordingly, the markup increases in this simple Calvo-type sticky price model.

Now we are clear about the behavior and intuitions of the markup in the simple sticky price model. This result stands opposite to the dynamic markup in a flexible price model with deep habit, where the markup is countercyclical. Although it is hard to tell which one is much better on the whole, it is natural for people to think to incorporate the deep habit into the sticky price model. By doing this, we may probably on the one hand have a more realistic assumption of nominal rigidities, and on the other hand, improve the behavior of markup in sticky price model.

Ravn, Schmitt-Grohé and Uribe (2005) study the case of sticky prices and deep habits. They introduce sluggish price adjustment by assuming that firms face quadratic price adjustment cost following Rotemberg (1982). They derive the linearized Phillips which involves not only equation (66) but also a sum of future sales, discount rate and output, together multiplied by deep habit parameter θ^d. Thus, they conclude that "in the presence of deep habits, it is in principle possible that markups fall because output increases and the present value of future sales, may increase. So there is the possibility that adding deep habits to a sticky price model will result in the prediction that markups fall in response to a positive technology shock."

To further check their conclusion, we incorporate the deep habit into a Calvo-type sticky price model. Actually, this is to extend our previous sticky price model with deep habits. Capital adjustment cost and nonseparable utility in consumption and leisure are still considered. Other details on this model can be found in Appendix A.6. The calibration results are presented in Figure 13 and Table 14.

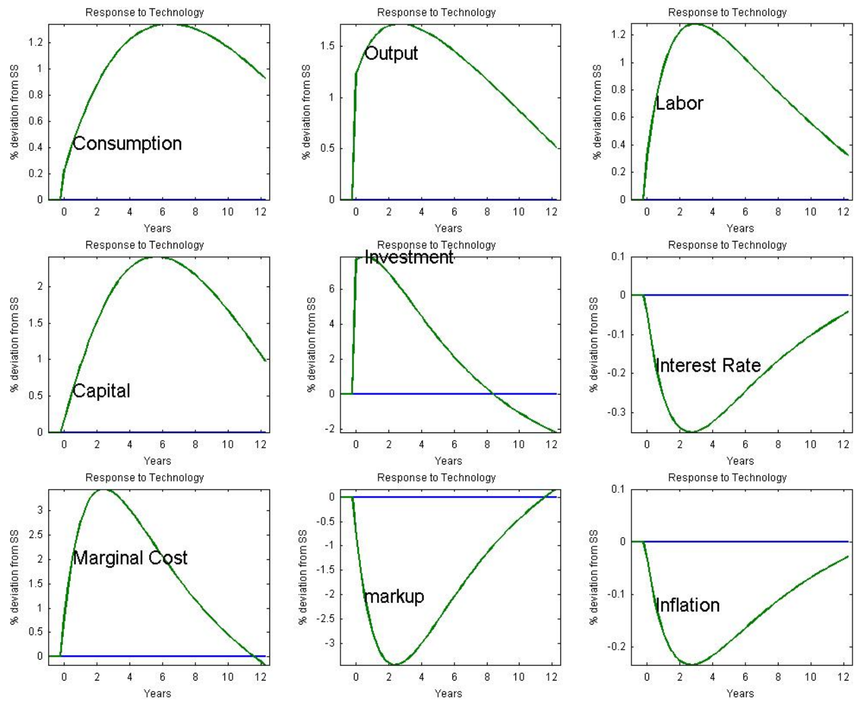


Figure 13: Sticky Price Model with Deep Habits

The impulse response plot can be a proof of the conclusion that incorporating deep habit into the sticky price model could improve the behavior of markup. Although the inflation still declines, the deep habits assumption allows firms to vary their pricing strategy. Therefore, we see an increase in marginal cost and a decreases in markup. This countercyclical markup is consistent with data. Moreover, we could also find the positive comovement in output, consumption, labor and investment to the technology shock, which is typical in the technology-driven RBC model.

Table 14 reports the standard deviation and correlation of the HP-filtered series based on simulation. It seems these numbers slightly improved com-

	std. Dev.	Corr. w. Output
Output	1.1293	1
Consumption	0.3108	0.77
Investment	6.9523	0.98
Labor	0.5721	0.79
Wage	2.2192	0.94
Interest Rate	0.1802	-0.83

Table 14: Statistics: Sticky Price Model with Deep Habits

pared with our benchmark flexible price model with deep habit. However, the volatilities of output and consumption are still not high enough to match the data. The wage is more volatile than data, and its correlation with output states procyclical rather than the acyclical property in data. Last, the determinative factor of Sharpe ratio is still too small to match the data.

To summarize the analysis in this subsection, we first illustrate the intuitions why sticky price model predicts a positive response of markup to the technology shock. A simple Calvo-type sticky price model with superficial habit is presented. Then we attempt to incorporate the deep habit into this Calvo-type sticky price model. The results show that this incorporation can improve the behavior of markup, i.e. a countercyclical markup is generated. Other macroeconomic facts, such as the positive comovement in output, labor, etc, are also matched. However, the model is still far from matching other facts, such as the Sharpe ratio and other asset market puzzles.

7 Discussion

Throughout our analysis, we have focused on the deep habits, which was first raised by Ravn, Schmitt-Grohé and Uribe (2005). This assumption, in our opinions, is more compelling, because it assume agents do not simply form habits from their overall consumption levels, but rather feel the need to catch up with the Joneses on a good-by-good basis. There are two important implications by this assumption, for it affects both the demand and supply sides of the economy. When agents form the habits from overall consumption levels, they would like to consume similar to their overall habits. The consumption habits change the Euler equation, in turn the asset pricing equations. Actually, this is what happens in most standard superficial habit formation models. On the other hand, agents form their habits good-by-good, which means if they have a higher consumption in particular good, they would like to consume this kind of good in the future by force of habit. When facing this type of habitual consumption demand, firms' optimization problem changes, where they have to take considerations on current consumption demand as well as the habit effect. Accordingly, their pricing strategies must become dynamic.

Having known the basic idea, we can see that the deep habits models not only incorporate the features of standard superficial habit model, but also brings more appropriate business cycle implications. Thus our discussion involves both the asset market and macroeconomics implications, based on our previous analysis and model results.

The models with time separable preferences have difficulties in reconcile the equity premium puzzle and risk free rate puzzle. These models predict either very high risk free rate or very low equity premium or Sharpe ratio are predicted. Their inability to match financial market characteristics as well as other business cycle properties leads to the use of time nonseparable preferences, i.e. habit formation models, where past consumption enters as a constraint.

In essence, adding this extra constraint on current consumption is to add an additional state variable, which helps explain the large premium. Agents form their habits and then consume according to their habits. They become more reluctant and dislike large and sudden changes in their consumption. Therefore, the premium of holding an equity which may force a sudden change in consumption would be large than the premium in time-separable utility model. With a higher equity premium, the Sharpe ratio of these models are higher as well.

This interpretation can be numerically seen from the Table 8 of section 5.5. With equation (58), we decompose the determinative factor $\eta_{\lambda z}$ of Sharpe ratio into the elasticities following Uhlig (2004). Without habit formation, the model shows a small η_{cc} and relative large η_{cz}. While in models with habit formation, the risk aversion η_{cc} is much larger, and the elasticity of consumption η_{cz} is pretty small. Agents with habit formation are too risk averse to change their consumption from their habit. The larger the risk aversion, the smaller the changes on consumption.

Generally, in the time-nonseparable preference, the risk aversion is calculated as $\frac{\gamma}{1-\theta}$, where $\theta < 1$ denotes the degree of habit formation. The relative risk aversion can be very large when taking very small number for $1 - \theta$. One of the examples can be the habit formation following Campbell and Cochrane (1999), where they actually taking risk aversion as $2.372/0.0498 = 47.635$. With this number, they find the models match the financial data pretty well.[29] For reasons, in this paper, we consider a moderate habit in form of Constantinides (1990), which generate the risk aversion $2/(1-0.86) = 14.2857$. With this number, we can have larger Sharpe ratio compared with time separable models. However, it is still far from matching the quarterly Sharpe ratio of 0.27 in data.

Habit formation may amplify investment and demand for capital goods. Moreover, Boldrin, Christiano and Fisher (2001) argue if there is perfectly elastic supply of capital there is no effect on the volatility of the return on equity. Many studies involve capital adjustment costs, which minimize the elasticity of the capital supply. Follow their studies, we adopt the typical form of capital adjustment costs which depend on the ratio of new investment to capital. The results show that rigid investment assumption can generate sizeable Sharpe ratio. In Table 9, the $\eta_{\lambda z}$ becomes 3-5 times larger from no adjustment cost to the capital adjustment cost case. Actually, this increase contribute most to the increase in η_{cz} given the same risk aversion. The costs in adjusting the capital indeed reduce the power from capital accumulation channel. It results in less response of investment, and higher response of consumption to the technology shock.

However, problems still exist within these capital adjustment cost models. For instance, the volatility of output predicted by the model is low, given the same standard deviation of technology in the model without adjustment cost.[30] More important, in the model with adjustment cost, the labor negatively reacts to a technology shock, which is contrary to the standard RBC model with an increase in labor. Although there has been a debate on the effect of technology shock on the working hour, the result still can not explain the comovement in output, consumption, investment and labor. This problem, however, is indeed solved when we have considered adjustment cost, deep habit, and the nonseparable utility in consumption and leisure together.

If we take a look once again at equation (58), we can find and guess that the cross-derivative η_{cll} could help in explaining the Sharpe ratio if we consider the utility nonseparable in consumption and leisure. At first, our benchmark

[29]Obviously, this large number of risk aversion must be accompanied by very small number for consumption. Hence, in their model, too low volatility in consumption.

[30]Maybe this is not actually a problem, if we scale up the technology shock to make output volatility consistent with data.

model with only the nonseparable utility in consumption and leisure refuses our tentative guess. Table 11 shows a decrease in $\eta_{\lambda z}$. However, if we further add the aforementioned adjustment cost, the $\eta_{\lambda z}$ actually rises as in Table 12. The reason is that the positive rise in η_{cz} mainly by force of the capital adjustment cost effect[31] offsets the negative power from considering leisure, η_{lz}. Thus the Sharpe ratio rises with a larger $\eta_{\lambda z}$.[32]

Some economists criticize this multiplicative specification. For instance, Lettau (2003) states that a positive (conditional) correlation of consumption and leisure lowers the conditional volatility of the marginal rate of substitution, which in turn lowers asset premia. However, in our model, the effects of both adjustment cost and nonseparability in consumption and leisure raise the η_{cz} much more so that the positive power from consumption, $\eta_{cc}\eta_{cz}$, overweighs much the negative power from leisure, $\eta_{cll}\eta_{lz}$. Thus, we conclude that given appropriate cross derivative, the nonseparability in consumption and leisure helps in explaining the Sharpe ratio. This conclusion especially holds where large elasticity of consumption, η_{cz} exists, like in our example with adjustment cost.

In addition, we find that there do exist a comovement in output, consumption, investment and labor in the above mentioned model of the combination of deep habit, adjustment cost and multiplicative specification. As panel (a) of Figure 9 shows the four variables including labor, all positively react to a positive productivity innovation. we analyze this comovement is mainly due to the deep habit assumption, which alters firm's pricing strategy, in turn the labor supply. However, we also emphasize the importance of nonseparability between consumption and leisure when generating this comovement. Therefore, adding the deep habit and multiplicative utility specification can solve the comovement problem which widely exists in capital adjustment cost models.

Actually, the central feature of deep habit model is the countercyclical dynamic markup, which is consistent with many empirical studies such as Rotemberg and Woodford (1999) and Galí, Gertler and López-Salido (2002). Unlike the constant markup in most flexible price model, the markup in the deep habit model is time varying and behaviors countercyclical in equilibrium. Similar to Ravn, Schmitt-Grohé and Uribe (2005), we analyze the

[31] Actually, the increase in η_{cz} is due to both the effects of adjustment cost and nonseparability in consumption and leisure, although the latter has a relative low influence.

[32] This number $\eta_{\lambda z} = 4.126$ is the largest among the comparative numbers, i.e. 0.7533 in Table 12 and 3.0303 in Table 9.

markup equation (46), and find the three factors which indeed affect the behavior of markup, namely, price elasticity effect, intertemporal effect and demand composition. By introducing the deep habit assumption, actually we introduce these three factors to drive the countercyclical markup in equilibrium.

Indeed, the price elasticity effect plays major role in affecting the countercyclical markup in our model. Under deep habits, firms must face certain good demand which is composed by both price elastic part of aggregate demand and price inelastic part of the habit stock. An increase of aggregate demand can increase the price elasticity of certain good. Therefore, with higher demand elasticity, firms are likely to cut their markup. A countercyclical markup is, therefore, generated on impact of this price-elasticity effect. The intertemporal effect means that when firms set their pricing strategy, they take into account future sales and profits via the formation of habit. If certain good is highly welcomed today, then it is more likely to be consumed in the future. To achieve high profit in the future, firms would rather to cut their markup today. However, this intertemporal effect is reduced by the demand composition in the technology driven model. In these models, investment is highly affected through the capital accumulation, while consumption is less affected due to the habit. Thus, the demand composition would be very low when technology shock happens. This low number in essence reduce the intertemporal effect on markup.

In sticky price models, the markup is also dynamic, however, positively react to the technology shock. we use the a simple Calvo-type sticky price model together with the new Phillips curve to illustrate this positive reaction. As long as the current inflation decline is larger than the expected future inflation decrease, the marginal costs must fall, which means, markups must rise. Then, the idea to incorporate the deep habit into sticky price model to improve the behavior of markup arises. We first attempt to add the deep habit into the Calvo-type sticky price model. The calibration results show indeed a countercyclical markup, which is consistent with the deduction of Ravn, Schmitt-Grohé and Uribe (2005).

Last but not least, we shall point out some of the limits and probable future directions of the analysis. First, our models consider solely the technology innovation, which is typically used in most RBC models. However, recently literature argues limited role for technology shocks as the source of aggregate fluctuation, such as the evidence in Galí and Rabanal (2004). Therefore, we'd better take other criterion on measuring the model's fitness to data or consider other sources of the fluctuation. In Ravn, Schmitt-Grohé and Uribe (2005), they analyze a full-fledged deep habit model with technology, preference and government purchase shocks. Secondly, throughout our analysis on asset pricing, we focus on, e.g. the maximum Sharpe ratio, where perfect correlations among variables are assumed. In data, as pointed by Uhlig (2004) the correlations are far from ± 1. When considering the actual correlations, the results may change. Finally, for convenience, we assume the external habit formation throughout the paper. The internal or other types of habit formation can be discussed in the future.

8 Summary and Conclusion

In this paper, we consider the deep habits by Ravn, Schmitt-Grohé and Uribe (2005). This assumption has more advantages than standard habit formation, for it assumes that household form their consumption habits not only at the aggregate level, but also at the individual level. This variety-by-variety formed habit affects both the demand and supply sides of the economy. Therefore, deep habits provide more asset pricing and macroeconomic implications.

Based on our empirical studies and stylized facts, we constructed our benchmark deep habit model, where both the Euler equation of households and pricing equation of firms were affected by habits consideration. We followed Campbell (1994) and Uhlig (1999) to solve the dynamic stochastic general equilibrium and implemented with Toolkit. Our Matlab codes, which could be regarded as a supplement to Toolkit were made available to calculate input matrix with Lagrangian method and symbolic computation. The calculated elasticities were applied to get the explicit solutions of the financial variables for asset prices analysis. We also analyzed the factors which affect the markup dynamics. As the extensions, we examined the effects of capital adjustment cost, nonseparability between consumption and leisure and the sticky prices assumption.

As in standard habit formation models, our analysis have shown that deep habits help explain the equity premium. The households, who consume according to their habits, are more risk avers and reluctant to the sudden change of their consumption. Hence, a high premium against the change is generated. Moreover, the deep rooted habits can further raise the Sharpe ratio through the labor market. Numerical comparison and analysis on the Sharpe ratio are presented.

In order to match or be close to the asset market facts, capital adjustment costs have been considered as well, for the rigid investments reduce the capital reaction, in turn increase the consumption reaction. Common problems of the capital adjustment models were summarized, including the low investment volatility, negative labor response, etc. These features are consistent with the findings of other literatures, e.g. Uhlig (2004).

A underlying avenue of nonseparability between consumption and leisure has been exploited. We found that it helped explain the equity premium within certain scope. For instance, when combined with the adjustment cost, it could generate higher Sharpe ratio. Moreover, when our deep habit model was incorporated with adjustment cost, and took the utility form nonseparable in consumption and leisure, the model could eliminate the negative response of labor and explain the comovement in output, investment, consumption and labor.

The countercyclical markups were also proved in our paper. Under deep habits, the price elasticity effect, intertemporal effect and demand composition all affected the dynamics of markup. Deep rooted habits force firms to take into account the formation of habit of each individual good demand when setting the pricing strategy. If relaxing the assumption from flexible price to sticky price, we still find that deep habits can improve the behavior of markup. These results are in line with Ravn, Schmitt-Grohé and Uribe (2005).

The limitation and potential future extension of our analysis are also pointed out. The correlation among the variables should be taken into account. Other types of shock can be added and considered as the source of the fluctuation. Beside the external habit in this paper, the internal deep habit formation can also be candidates of future studies.

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Appendix A: Technical Notes

A.1 Inefficient Gap Method according to GGL

In this subsection, we briefly describe the idea and calculation of inefficient gap of Galí, Gertler and López-Salido (2002)(GGL for short). The equation is given by:

$$gap_t = mrs_t - mpn_t = -(\mu_t^p + \mu_t^w) \tag{67}$$

where mrs_t and mpn_t denote, respectively, the (log) marginal product of labor and the (log) marginal rate of substitution between consumption and leisure. The inefficient gap is also the sum of minus aggregate price markup, μ_t^p, and the aggregate wage markup μ_t^w, which are defined as:

$$\begin{aligned}\mu_t^p &= p_t - (w_t - mpn_t)\\ \mu_t^w &= (w_t - p_t) - mrs_t\end{aligned}$$

where p_t is (log) price, and w_t (log) compensation. Assuming a technology with constant elasticity of output with respect to labor (say, α), then we get:

$$mpn_t = y_t - n_t$$

Moreover, assume the (log) marginal rate of substitution for a representative consumer can be written:

$$mrs_t = \sigma c_t + \varphi n_t$$

where y_t is (log) output, and c_t (log) consumption. Parameter σ refers the relative risk aversion and φ measures the curvature of the disutility of labor. In the baseline case, $\sigma = \varphi = 1$. Substitute out the mpn_t and mrs_t, we get:

$$\mu_t^p = (y_t - n_t) - (w_t - p_t) \tag{68}$$

$$\mu_t^w = (w_t - p_t) - (\sigma c_t + \varphi n_t) \tag{69}$$

Actually, equations (67)-(69) are exactly equations (6) (12) and (13) of GGL, except we ignore the $\bar{\xi}$ a low frequency preference shifter in last equation. Then GGL use the commercialized USECON data on the evidence analysis. Since limitation, however, we choose the St. Louis Fed collection data. The descriptions of the data and corresponding series IDs are:

- Y: Output, Nonfarm Business Sector (OUTNFB)
- N: Hours, Nonfarm Business Sector(HOANBS)

- W: Hourly Compensation, Nonfarm Business Sector(COMPNFB)
- C: Consumption Nondurables (PCND) + Service(PCESV)
- P: Consumer Price Index(CPIAUCSL)

With these data input, we first take log-deviations from their mean to get the small letter variables in above equations. Then, we use equation (68) to calculate the price markup, equation (69) for wage markup, and equation (67) for inefficient gap. At last, we detrend these three variables with HP-filter. Beside, we also detrend the (log) real GDP (GDPC96) to generate the output gap. These calculations are correspondent to the Matlab codes in Appendix B.3.

A.2 Derivation of Marginal Cost (Markup) Equation

Given equation (25), we derive the equation for κ_t expressed in the future terms (intertemporal effect):

$$\kappa_t = \theta^d E_t r_{t,t+1} \nu_{t+1} + \rho^d E_t r_{t,t+1} \kappa_{t+1} \tag{70}$$

Rearrange equation (26) to get ν_t. Note we substitute $c_{it} = p_{it}^{-\eta} x_t + \theta^d s_{it-1}$ for calculation convenience.

$$\nu_t = (p_{it}^{-\eta} x_t + \theta^d s_{it-1} + (1-\eta) p_{it}^{-\eta} i_t + \eta p_{it}^{-\eta-1} mc_t i_t)/\eta p_{it}^{-\eta-1} x_t \tag{71}$$

Substitute out the κ_t and ν_t of the equation (24) with the above two expressions. After rearranging, we get the following:

$$\begin{aligned}\left(1 + \frac{i_t}{x_t}\right) mc_t =& p_{it} - \left(\frac{1}{\eta} p_{it} + \frac{\theta}{\eta} \frac{s_{it-1}}{x_t} p_{it}^{\eta+1} + \frac{1-\eta}{\eta} \frac{i_t}{x_t} p_{it}\right) \\ &+ (1-\rho^d) E_t r_{t,t+1} (\theta^d \nu_{t+1} + \rho^d \kappa_{t+1})\end{aligned} \tag{72}$$

Then, we substitute $i_t = p_{it}^{\eta} i_{it}$ and $x_t = (c_{it} - \theta^d s_{it-1}) p_{it}^{-\eta}$. By doing so, we can get the y_{it} for the demand composition part, and price elasticity term. Finally, we drop the subscript i to get the marginal cost equation in equilibrium:

$$\begin{aligned} mc_t =& \left(1 - \frac{1}{\eta(1-\theta s_{t-1}/y_t)}\right) p_t \\ &+ (1-\rho) \left(\frac{c_t - \theta^d s_{t-1}}{y_t - \theta^d s_{t-1}}\right) \left(E_t r_{t,t+1} (\theta^d \nu_{t+1} + \rho^d \kappa_{t+1})\right)\end{aligned} \tag{73}$$

Defining the markup as the inverse of marginal cost, then it is the markup equation (46).

$$\begin{aligned} \mu_t =& [\left(1 - \frac{1}{\eta(1 - \theta s_{t-1}/y_t)}\right) p_t \\ &+ (1-\rho)\left(\frac{c_t - \theta^d s_{t-1}}{y_t - \theta^d s_{t-1}}\right)\left(E_t r_{t,t+1}(\theta^d \nu_{t+1} + \rho^d \kappa_{t+1})\right)]^{-1} \end{aligned} \tag{74}$$

A.3 Steady States of the Benchmark Deep Habit Model

we use the bar letters to denote the steady state values of the benchmark deep habit model in section 4, then the system of equations are given by:

$$\begin{aligned} \bar{\lambda} &= U_x(\bar{x}, \bar{h}) \\ \bar{\lambda}\bar{w} &= U_h(\bar{x}, \bar{h}) \\ 1 &= \beta[1 - \delta + \bar{u}] \\ 1 &= \beta\bar{R} \\ \bar{w} &= \overline{mc}\bar{z}F_h(\bar{k}, \bar{h}) \\ \bar{u} &= \overline{mc}\bar{z}F_k(\bar{k}, \bar{h}) \\ 0 &= 1 - \bar{\nu} - \overline{mc} + \bar{\kappa}(1 - \rho^d) \\ 0 &= \theta^d\bar{\nu} + (\bar{r} - \rho^d)\bar{\kappa} \\ 0 &= \bar{c} + (1-\eta)\bar{i} + \eta\overline{mc}\bar{i} - \eta\bar{\nu}\bar{x} \\ \bar{y} &= \bar{z}F_h(\bar{k}, \bar{h}) \\ \bar{y} &= \bar{c} + \bar{i} \\ \bar{i} &= \delta\bar{k} \\ \bar{x} &= (1-\theta)\bar{c} \\ \bar{s} &= \bar{c} \\ \bar{\mu} &= 1/\overline{mc} \end{aligned}$$

Given $\bar{z} = 1$ and other parameters in Table 6, we solve these 15 variables of the 15 equations. The calculations are correspondent to the Matlab file: deep_ss.m.

A.4 Derivation of Risk Premium and Sharpe Ratio

The typical asset pricing formula can be as follow:

$$\begin{aligned}1 =& E_t\left[\beta\frac{\Lambda_{t+1}}{\Lambda_t}R_{t,t+1}\right]\\ =& E_t\left[\beta\frac{\Lambda_{t+1}}{\Lambda_t}\right]E_t[R_{t,t+1}] + \mathbf{cov}\left[\beta\frac{\Lambda_{t+1}}{\Lambda_t}R_{t,t+1}\right]\\ =& \frac{1}{R^f_{t,t+1}}E_t[R_{t,t+1}] + \mathbf{cov}\left[\beta\frac{\Lambda_{t+1}}{\Lambda_t}R_{t,t+1}\right]\end{aligned}$$

where $R_{t,t+1}$ denotes the gross return, **cov** denote the covariance and the risk free rate is just $R^f_{t,t+1} = 1/E_t\left[\beta\frac{\Lambda_{t+1}}{\Lambda_t}\right]$. Now consider the log-linear form of that asset pricing equation, i.e. taking logs on both sides:

$$\begin{aligned}0 =& \mathbf{log}E_tR_{t,t+1} - \mathbf{log}R^f_{t,t+1} + \mathbf{cov}(\mathbf{log}\frac{\Lambda_{t+1}}{\Lambda_t}, \mathbf{log}R_{t,t+1})\\ =& \hat{r}^e_{t,t+1} - \hat{r}^f_{t,t+1} + \mathbf{cov}(\Delta\lambda_{t+1}, \hat{r}_{t,t+1})\end{aligned}$$

where $\hat{r}^e_{t,t+1}$ is the logarithm of expected gross return, $\hat{r}^f_{t,t+1}$ the logarithm of risk free rate, $\Delta\lambda_{t+1}$ the logarithm of stochastic discount factor, and $\hat{r}_{t,t+1}$ the logarithm of gross return. Now the risk premium is the minus covariance between (log) stochastic discount factor and (log) gross return.

The solutions for $\Delta\lambda_{t+1}$ and $\hat{r}_{t,t+1}$ of the model following Campbell (1994) and Uhlig (1999) can be as follows:

$$\begin{aligned}\hat{\lambda}_t =& \eta_{\lambda k}\hat{k}_t + \eta_{\lambda z}\hat{z}_t\\ \hat{\lambda}_{t+1} =& \eta_{\lambda k}\hat{k}_{t+1} + \eta_{\lambda z}\hat{z}_{t+1}\\ =& \eta_{\lambda k}(\eta_{kk}\hat{k}_t + \eta_{kz}\hat{z}_t) + \eta_{\lambda z}(\rho_z\hat{z}_t + \epsilon_{t+1})\\ \Delta\hat{\lambda}_{t+1} =& \eta_{\lambda k}(\eta_{kk} - 1)\hat{k}_t + (\eta_{\lambda k}\eta_{kz} + \eta_{\lambda z}(\rho^z - 1))\hat{z}_t + \eta_{\lambda z}\epsilon_{t+1}\\ \hat{r}_{t,t+1} =& \eta_{rk}\hat{k}_{t+1} + \eta_{rz}\hat{z}_{t+1}\\ =& \eta_{rk}\eta_{kk}\hat{k}_t + (\eta_{rk}\eta_{kz} + \eta_{rz}\rho^z)\hat{z}_t + \eta_{rz}\epsilon_{t+1}\end{aligned}$$

With these solutions, the risk premium and Sharpe ratio can be explicitly expressed by the elasticities and standard deviation of technology

$$\hat{r}^{rp}_{t,t+1} = -cov(\eta_{\lambda z}\epsilon_{t+1}, \eta_{rz}\epsilon_{t+1}) = -\eta_{\lambda z}\eta_{rz}\sigma^2_\epsilon$$

$$SR_t = \left|\frac{\hat{r}^{rp}_{t,t+1}}{\sigma_{r,t}}\right| \leq SR^{max}_t = \eta_{\lambda z}\sigma_\epsilon$$

Those two equations are the same as we used in section 5.5.

A.5 Sticky Price Model with Superficial Habit

Here we present a simple sticky price model with superficial habit and capital adjustment cost. Most of the equilibrium equations are quite similar with our benchmark model. we follow most of the notations of the benchmark model.

Households: the optimization in Lagrangian:

$$\begin{aligned}\mathcal{L}_h =& E_t \sum_{t=0}^{\infty} \beta^t [U_t(x_t, h_t) + \tau_t((1-\delta)k_{t-1} + G\left(\frac{i_t}{k_{t-1}}\right) k_{t-1} - k_t) \\ &+ \lambda_t(-x_t - i_t - \varpi_t - E_t r_{t,t+1} \frac{d_{t+1}}{P_t} + \frac{d_t}{P_t} + w_t h_t + u_t k_{t-1} + \Phi_t)]\end{aligned}$$

where λ_t, τ_t are Lagrangian multipliers, i.e. the shadow values of wealth and investment. Taking derivatives with respect to x_t, h_t, i_t, k_t, and d_{t+1} in this order, plus the external habit equations, we get:

$$\lambda_t = U_x(x_t, h_t) \tag{75}$$

$$w_t \lambda_t = U_h(x_t, h_t) \tag{76}$$

$$\lambda_t = \tau_t G(\cdot)' \tag{77}$$

$$\tau_t = \beta(\lambda_{t+1} u_{t+1} + \tau_{t+1}(1 - \delta - G(\cdot)' + G(\cdot)) \tag{78}$$

$$\lambda_t = \beta R_t E_t \left[\frac{\lambda_{t+1}}{\pi_{t+1}}\right] \tag{79}$$

$$x_t = c_t - \theta s_{t-1} \tag{80}$$

$$s_t = (1-\rho)c_t + \rho s_{t-1} \tag{81}$$

where $\pi_t = P_t/P_{t-1}$ denotes inflation, $R_t = \frac{1}{E_t r_{t,t+1}}$ denotes gross one period risk free nominal interest rate, and $G(\cdot)$ is the adjustment cost function in section 6.1.

Firms: as usual, we get the equations for wage and dividend:

$$u_t = mc_t z_t F_k(k_{t-1}, h_t) \tag{82}$$

$$w_t = mc_t z_t F_h(k_{t-1}, h_t) \tag{83}$$

Moreover, we assume the Calvo-type sticky prices, where $\vartheta \in [0, 1)$ of randomly picked firms is not allowed to change the nominal price of the good it produces. The remaining $(1 - \vartheta)$ firms choose prices optimally. Then, the optimization problem for the ϑ firms in Lagrangian is:

$$\begin{aligned}\mathcal{L}_f =& E_t \sum_{s=t}^{\infty} r_{t,s} P_s \vartheta^{s-t} [((\frac{\tilde{P}_t}{P_s})^{1-\eta} y_s - u_s k_{s-1} - w_s h_s) \\ &+ mc_s (z_s F(k_{s-1}, h_s) - (\frac{\tilde{P}_t}{P_s})^{-\eta} y_s)]\end{aligned}$$

where mc_t is the Lagrangian multiplier, i.e. the marginal cost. Take derivative with respect to $\tilde{P}_t$, then we get:

$$E_t \sum_{s=t}^{\infty} r_{t,s} \vartheta^{s-t} (\frac{\tilde{P}_t}{P_s})^{-1-\eta} y_s \left[mc_s - \frac{\eta - 1}{\eta} \frac{\tilde{P}_t}{P_s} \right] = 0$$

Instead of linearizing to get the notable neo-Keynesian Phillips curve which involves inflation and marginal costs (or the output gap), we follow the technique of Schmitt-Grohe and Uribe (2004) to add two more new variables, x_t^1, x_t^2 , and rewrite in a recursive fashion.

$$\begin{aligned}x_t^1 =& E_t \sum_{s=t}^{\infty} r_{t,s} \alpha^{s-t} (\frac{\tilde{P}_t}{P_s})^{-1-\eta} a_s mc_s \\ =& \tilde{p}_t^{-1-\eta} y_t mc_t + \alpha \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1}^{\eta} (\frac{\tilde{p}_t}{\tilde{p}_{t+1}})^{-1-\eta} x_{t+1}^1 \qquad (84) \\ x_t^2 =& E_t \sum_{s=t}^{\infty} r_{t,s} \alpha^{s-t} (\frac{\tilde{P}_t}{P_s})^{-1-\eta} a_s \frac{\tilde{P}_t}{P_s} \\ =& \tilde{p}_t^{-\eta} y_t + \alpha \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1}^{\eta-1} (\frac{\tilde{p}_t}{\tilde{p}_{t+1}})^{-\eta} x_{t+1}^2 \qquad (85) \\ x_t^2 =& \frac{\eta}{\eta - 1} x_t^1 \qquad (86)\end{aligned}$$

Actually, the x_t^1 is the present discounted value (PDV) of marginal cost, and x_t^2 is the PDV of relative price. Finally, to complete the equilibrium, we also need equations for price index, interest rate rule (simple Taylor rule), budget constraint, definition of output and exogenous process:

$$\begin{aligned}1 =& \alpha \pi_t^{-1+\eta} + (1 - \alpha) \tilde{p}_t^{1-\eta} \qquad (87) \\ \mathbf{log}(R_t / R^*) =& \alpha_\pi E_t \mathbf{log}(\pi_{t-1} / \pi^*) \qquad (88) \\ y_t =& c_t + i_t \qquad (89) \\ y_t =& z_t F(k_{t-1}, h_t) \qquad (90) \\ \mathbf{log} z_t =& \rho_z \mathbf{log} z_{t-1} + \epsilon_t \qquad (91)\end{aligned}$$

Now the system can be regarded as a process of 17 variables, c_t, h_t, s_t, x_t, λ_t, τ_t, w_t, u_t, mc_t, k_t, R_t, i_t, y_t, $\tilde{p}_t$, π_t, x_t^1, x_t^2, satisfying equation (75)–(91), given the initial values of s_{-1}, k_{-1} and exogenous stochastic process of z_t. The corresponding Matlab codes are also made available. Since limitation, we do not include these codes in Appendix of the paper. However, all the codes can be found in the CD-ROM.

A.6 Sticky Price Model with Deep Habit

When adding deep habit into the above sticky price model, only the optimization problem for the ϑ firms changes, for their pricing strategy affected by the deep habit. Then, its Lagrangian becomes:

$$
\begin{aligned}
\mathcal{L}_f =& E_t \sum_{s=t}^{\infty} r_{t,s} P_s \vartheta^{s-t} \{ [(\frac{\tilde{P}_t}{P_s}) c_s + (\frac{\tilde{P}_t}{P_s})^{1-\eta} i_s - u_s k_{s-1} - w_s h_s] \\
&+ mc_s [z_s F(k_{s-1}, h_s) - c_s + (\frac{\tilde{P}_t}{P_s})^{-\eta} i_s] \\
&+ \nu_s [(\frac{\tilde{P}_t}{P_s})^{-\eta} x_s + \theta^d s_{t-1} - c_s] + \kappa_s [\rho^d s_{t-1} + (1-\rho^d) c_s - s_s] \}
\end{aligned}
$$

Take derivative with respect to $\tilde{P}_t$. Then we use the same technique to rewrite the equation with four new variables:

$$x_t^1 = c_t + r_{t,t+1} \vartheta x_{t+1}^1 \tag{92}$$

$$x_t^2 = \tilde{p}_t^{-\eta} i_t + r_{t,t+1} \vartheta \pi_{t+1}^{\eta} \left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}} \right)^{-\eta} x_{t+1}^2 \tag{93}$$

$$x_t^3 = \tilde{p}_t^{-\eta-1} mc_t i_t + r_{t,t+1} \vartheta \pi_{t+1}^{\eta+1} \left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}} \right)^{-\eta-1} x_{t+1}^3 \tag{94}$$

$$x_t^4 = \tilde{p}_t^{-\eta-1} \nu_t x_t + r_{t,t+1} \vartheta \pi_{t+1}^{\eta+1} \left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}} \right)^{-\eta-1} x_{t+1}^4 \tag{95}$$

$$0 = x_t^1 + (1-\eta) x_t^2 + \eta x_t^3 - \eta x_t^4 \tag{96}$$

Keep other equations the same as in previous section, then we get the whole system of 19 variables, c_t, h_t, s_t, x_t, λ_t, τ_t, w_t, u_t, mc_t, k_t, R_t, i_t, y_t, $\tilde{p}_t$, π_t, x_t^1, x_t^2, x_t^3, x_t^4, satisfying equation (75)–(83),(87)–(91) and (92)–(96), given the initial values of s_{-1}, k_{-1} and exogenous stochastic process of z_t. For reasons, we do not include the Matlab codes in Appendix. Instead, the codes are provided in the CD-ROM.

Appendix B: Matlab Codes

B.1 HP Detrending and Related Analysis

```
% ***************************************************************************
%   This program compute the HP_filtered series for GDP, Consumption,
%   Investment, Hour, Labor Productivity, Wage and Government Spending
% ***************************************************************************

load macrodata.txt; %   data descriptions are corresponding to Table 1.
time=macrodata(:,1); data=macrodata(:,2:end); x=log(data);

%   The following HP_filter is taken from Toolkit.
HP_LAMBDA = 1600; LENGTH = max(size(x));
HP_mat = [1+HP_LAMBDA, -2*HP_LAMBDA, HP_LAMBDA, zeros(1,LENGTH-3);
-2*HP_LAMBDA,1+5*HP_LAMBDA,-4*HP_LAMBDA,HP_LAMBDA, ...
zeros(1,LENGTH-4); zeros(LENGTH-4,LENGTH);
zeros(1,LENGTH-4),HP_LAMBDA,-4*HP_LAMBDA,1+5*HP_LAMBDA,-2*HP_LAMBDA;
zeros(1,LENGTH-3), HP_LAMBDA, -2*HP_LAMBDA, 1+HP_LAMBDA ];
for iiiii=3:LENGTH-2;
HP_mat(iiiii,iiiii-2)=HP_LAMBDA;
HP_mat(iiiii,iiiii-1)=-4*HP_LAMBDA;
HP_mat(iiiii,iiiii)=1+6*HP_LAMBDA;
HP_mat(iiiii,iiiii+1)=-4*HP_LAMBDA;
HP_mat(iiiii,iiiii+2)=HP_LAMBDA;
end;
tr=HP_mat\x; hp=x-tr; hp=hp.*100;

%   Calculate the correlations and standard deviations in Table 2.
table=corrcoef(hp); n=1;
while (n<8)
table(n,n)=std(hp(:,n)); n=n+1;
end

%   Calculate the cross-correlations in Table 3.
table2=zeros(6,13); i=2;
while (i<8)
[xcf,lags,bounds]=crosscorr(hp(:,1),hp(:,i),6);
table2(i-1,:)=xcf'; i=i+1;
end

%   Plot the cross-correlations: Figure 1.
m=2;
while (m<8)
subplot(6,1,m-1); crosscorr(hp(:,1),hp(:,m),20)
xlabel(''); ylabel(''); title(''); m=m+1;
end
```

B.2 Labor Share Plot with NBER Recessions

```
% ***************************************************************************
%   This program plots the labor share data of BLS as in figure 2 in paper.
%
% ***************************************************************************
load laborshare.txt;
t=laborshare(:,1);
peaks=laborshare(:,2);
troughs=laborshare(:,3);
ser=laborshare(:,5);
startdate=1964.01;
titlestring='labor share';

%   The following is taken from Moeonch and Uhlig (2004)
enddate=startdate+fix(length(ser)/4)+mod(length(ser),4)/100;
unit=(enddate-startdate)/(length(ser)-1);
time = startdate:unit:enddate;
hndl = plot(time,ser);
title(titlestring, 'FontSize', 14);
hold on;
xx = get(gca,'xlim'); yy = get(gca,'ylim');
t = min(find(peaks));
while t < length(time),
year = floor(time(1)) + floor((t-1)/4);
if peaks(t) == 1,
x = [time(t) time(t)];
y = [yy(1) yy(2)];
year_str = num2str(year);
year_str = year_str(3:4);
month = mod(t,4); if month == 0, month = 4; end;
str = [year_str,'M',num2str(month)];
hndl = line(x,y,'Color',[1 0 0],'LineWidth',0.5);
hold on;
elseif troughs(t) ==1,
x = [time(t) time(t)];
y = [yy(1) yy(2)];
year_str = num2str(year);
year_str = year_str(3:4);
month = mod(t,4); if month == 0, month = 4; end;
str = [year_str,'M',num2str(month)];
hndl = line(x,y,'Color',[1 0 0],'LineWidth',0.5);
hold on;
end;
t = t + 1;
end;
```

B.3 Inefficient Gap and Markups according to GGL

```
% ************************************************************************
%   This program plot the inefficient gap and markup
%   calculate the correlations and standard deviations
%   corresponding to figure 3 and table 4.
% ************************************************************************

load ggl.txt t=ggl(:,1); peaks=ggl(:,2); troughs=ggl(:,3);
out=ggl(:,4); out=log(out)-log(mean(out)); %  nonfarm business
hour=ggl(:,5); hour=log(hour)-log(mean(hour)); %  nonfarm business
comp=ggl(:,6); comp=log(comp)-log(mean(comp)); %  nonfarm business
pric=ggl(:,7); pric=log(pric)-log(mean(pric)); %  nonfarm business
cons=ggl(:,8); cons=log(cons)-log(mean(cons)); %  nonfarm business
GDP_gap=log(ggl(:,10)); %   Output Gap by detrended real GDP

%   The following is the calculations of inefficient gap, wage markup,
%   and price markup according to GGL(2002)
mark_p=out-hour-comp+pric;
mark_w=comp-pric-cons-hour;
gap= -mark_p - mark_w;

%   I detrend the above calculated series with HP_filter
%   the function hp_filter can be found in Toolkit
[tr,mark_p]=hp_filter(mark_p); mark_p=mark_p*100; %   price markup
[tr,mark_w]=hp_filter(mark_w);mark_w=mark_w*100;%   wage markup
[tr,gap]=hp_filter(gap);gap=gap*100;%   inefficient gap
[tr,GDP_gap]=hp_filter(GDP_gap);GDP_gap=GDP_gap*100;%   Output gap

%   Calculate the numbers in Table 4
all=[GDP_gap,gap,mark_w,mark_p];
table4=corrcoef(all); n=1;
while (n<5)
table4(n,n)=std(all(:,n));
n=n+1;
end

%   The following plot the ineffecient gap and (inverse) wage markup
%   with the NBER recessions (by Moeonch and Uhlig (2004))
ser=[gap,-mark_w]; startdate=1964.01;
titlestring='inefficient gap and (inverse) wage markup';

%   The following is taken from Moeonch and Uhlig (2004)
enddate=startdate+fix(length(ser)/4)+mod(length(ser),4)/100;
unit=(enddate-startdate)/(length(ser)-1);
time = startdate:unit:enddate;
hndl = plot(time,ser);
title(titlestring, 'FontSize', 14);
hold on;
```

```
xx = get(gca,'xlim'); yy = get(gca,'ylim');
t = min(find(peaks));
while t < length(time),
year = floor(time(1)) + floor((t-1)/4);
if peaks(t) == 1,
x = [time(t) time(t)];
y = [yy(1) yy(2)];
year_str = num2str(year);
year_str = year_str(3:4);
month = mod(t,4); if month == 0, month = 4; end;
str = [year_str,'M',num2str(month)];
hndl = line(x,y,'Color',[1 0 0],'LineWidth',0.5);
hold on;
elseif troughs(t) ==1,
x = [time(t) time(t)];
y = [yy(1) yy(2)];
year_str = num2str(year);
year_str = year_str(3:4);
month = mod(t,4); if month == 0, month = 4; end;
str = [year_str,'M',num2str(month)];
hndl = line(x,y,'Color',[1 0 0],'LineWidth',0.5);
hold on;
end;
t = t + 1;
end;
```

B.4 The Benchmark Deep Habits Model

```
% *************************************************************************
% deep_run.m run the whole benchmark deep habits model
% This program calls three functions
% deep_ss.m deep_model.m deep_num.m
% *************************************************************************

clear; clc;
disp('Using Toolkit by Uhlig (1999) to calculate');
disp('The Benchmark Deep Habit Model of the Thesis: ');
disp('Deep Habits: Macroeconomic and Asset Pricing Implications');
disp('by Yu, LI, Humboldt Uni, 20.07.2005');
disp('Hit any key when ready...');
pause;

% Assign values to parameters and steady-state variables
[ THETA_D,RHOZ,A,GAMA,SIGMA,BETTA,DELTA,ETA,ALFA,THETA,RHO,
rb,r,rp,ub,u,up,kb,k,kp,mcb,mc,mcp,vb,v,vp,cb,c,cp,inveb,inve,invep,
xb,x,xp,kab,ka,kap,wb,w,wp,lab,la,lap,surb,sur,surp,hb,h,hp,zb,
z,zp,yb,y,yp,markb,mark,markp,xi,a1,a2,taub,tau,taup]=deep_ss;
```

```
% Compute analytical derivatives of equilibrium conditions
[AAA,BBB,CCC,DDD,FFF,GGG,HHH,JJJ,KKK,LLL,MMM,NNN] = deep_model;

% Compute numerical derivatives
deep_num

% Solve by Toolkit
sigmaz = 0.712;
Sigma = [ sigmaz^2];

VARNAMES = ['Interest Rate ',% 1
'Divident ',% 2
'Capital ',% 3
'Marginal Cost ',% 4
'Shadow Value v ',% 5
'Consumption ',% 6
'Investment ',% 7
'Haibt adjusted Consumption ',% 8
'Wage ',% 9
'Shadow Value lambda ',% 10
'Habit Stock ',% 11
'Labor ',% 12
'Shadow Value tau ',% 13
'Output ',% 14
'Markup ',% 15
'Shadow Value ka ',% 16
'Technology ',% 17
];

[l_equ,m_states] = size(AA);
[l_equ,n_endog ] = size(CC);
[l_equ,k_exog ] = size(DD);

PERIOD = 4; % number of periods per year,
GNP_INDEX = 14; % Index of output
IMP_SELECT = 1:(m_states+n_endog);% +k_exog);
HP_SELECT = 1:(m_states+n_endog+k_exog); % Selecting for HP
DO_SIMUL = 0; % Calculates Simulations
DO_MOMENTS = 0; % Calculates Moments
DO_IMPRESP = 1;
DO_STATE_RESP=0;
% IMP_SUBPLOT = 1;
DO_QZ = 1;
% HORIZON = 50;
DISPLAY_AT_THE_END = 0;

do_it;

% The following is to calculate the elasticities for sharpe ratio
```

```
e_cz = QQ(6); %   elas. of cons. w.r.t. tech
e_nz = QQ(12); %   elas. of labor w.r.t. tech
e_xz = QQ(8); %   elas. of habit adj. cons w.r.t. tech
e_laz = QQ(10); %   elas. of lambda w.r.t tech
%   If additive utility, then
e_cc = SIGMA/(1-THETA); %   RRA of consumption
e_cll = 0; %   cross derivative
%   If multiplicative utility, then
%  e_cc = (-CHI + CHI *SIGMA + 1)/(1-THETA);
%  e_cll = (1 - SIGMA)* GAMA;

%   Calculate eta_lambda z with three methods
%   they should get the same results
SR1 = e_cc*e_cz+abs(e_cll)*0.2/(1-0.2)*e_nz*(-1);
SR2 = e_laz;
SR3 = SIGMA*e_xz;
[SR1 SR2 SR3]

% ***************************************************************************
%   deep_ss.m
% ***************************************************************************

function[ THETA_D,RHOZ,A,GAMA,SIGMA,BETTA,DELTA,ETA,ALFA,THETA,RHO,
rb,r,rp,ub,u,up,kb,k,kp,mcb,mc,mcp,vb,v,vp,cb,c,cp,inveb,inve,invep,
xb,x,xp,kab,ka,kap,wb,w,wp,lab,la,lap,surb,sur,surp,hb,h,hp,zb,
z,zp,yb,y,yp,markb,mark,markp,xi,a1,a2,taub,tau,taup]=deep_ss;

%   This program computes the steady state values
%   The steady states equations are correspondent to Appendix A.3
%   By changing some of the parameters' values, we can generate the steady
%   state values of other models in the Variation part.
%   (c) Yu, LI, Humboldt Uni. 20.07.2005
GAMA = 3.08; %   curvature w.r.t. labor
SIGMA = 2; %   curvature w.r.t. habit-adj consumption
CHI = 1; %   para. when multiplicative utility
DELTA = 0.025; %   depreciation rate
ALFA = 0.25; %   capital share
ETA = 5.3; %   elas. cross varieties
RHO = 0.85; %   persistent habit stock
THETA = 0.86; %   habit degree
THETA_D = 0.86; %   deep habit degree
RHOZ = 0.9; %   tech persistence
z = 1; %   tech
h = 0.2; %   steady state labor
r = 1.04^(1/4); %   interest rate
BETTA = 1/r; %   quarterly discount factor
%   The following is para. associated with cap. adj. cost
xi = 9999999999; %   inf for no adj cost. alternatives: 0.23 or 2
a1 = DELTA^(1/xi);
```

```
a2 = -DELTA/(xi-1);

%   The following is to calculate the SS of each var.
u = r-1+DELTA; %   divident
%   I solve the equation for k with matlab, so it is a complicated result
k =exp((log(2*(ETA*u*r*THETA_D*DELTA-u*ETA*DELTA*THETA_D)/(-ETA*u*r-ETA*u*RHO
*THETA_D+ETA*u*r*THETA_D+ETA*ALFA*r*THETA_D*DELTA-ETA*DELTA*ALFA*THETA_D
+ETA*u*RHO+(ETA^2*u^2*r^2+ETA^2*u^2*RHO^2+4*u*ETA*DELTA*THETA_D^2*ALFA*RHO
-4*u*ETA*DELTA*THETA_D^2*ALFA+4*u*ETA*DELTA*THETA_D*ALFA*r-4*u*ETA*DELTA
*THETA_D*ALFA*RHO+2*ETA^2*u*r^2*ALFA*THETA_D*DELTA-2*ETA^2*u*r*DELTA*ALFA
*THETA_D-2*ETA^2*u^2*RHO*THETA_D^2*r+2*ETA^2*u*RHO*THETA_D^2*ALFA*r*DELTA-2*ETA^2
*u*RHO*THETA_D^2*DELTA*ALFA-2*ETA^2*u*r^2*THETA_D^2*ALFA*DELTA+2*ETA^2*u*r
*THETA_D^2*DELTA*ALFA+ETA^2*ALFA^2*r^2*THETA_D^2*DELTA^2-2*ETA^2*ALFA^2*r*
THETA_D^2*DELTA^2-2*ETA^2*ALFA*r*THETA_D*DELTA*u*RHO+2*ETA^2*DELTA*ALFA
*THETA_D*u*RHO-4*ETA*u*r*THETA_D^2*DELTA*ALFA*RHO+4*ETA*u*r*THETA_D^2
*DELTA*ALFA-4*ETA*u*r^2*THETA_D*DELTA*ALFA+4*ETA*u*r*THETA_D*DELTA
*ALFA*RHO+4*ETA^2*u^2*r*RHO*THETA_D-2*ETA^2*u^2*r^2
*THETA_D-2*ETA^2*u^2*r*RHO+ETA^2*u^2*RHO^2*THETA_D^2-2*ETA^2*u^2
*RHO^2*THETA_D+ETA^2*u^2*r^2*THETA_D^2+ETA^2*DELTA^2*ALFA^2
*THETA_D^2)^(1/2))/h)+log(h)*ALFA)/(-1+ALFA));
mc = u/ALFA * (1/h)^(1-ALFA) * k^(1-ALFA); %   marginal cost
v = (1-mc)/( (RHO-1)*THETA_D/((r-RHO)) + 1); %   shadow value
c = (1/h)^ALFA * h * k^ALFA - DELTA*k; %   consumption
inve = DELTA*k; %   investment
x = (1-THETA)*c; %   habit adj. consumption
ka = (1-mc-v)/(RHO-1); %   shadow value
w = mc*(1-ALFA)*(k/h)^ALFA; %   wage
sur = c; %   habit stock
mark = 1/mc; %   markup
y = c+inve; %   output
%   The following is additive utility case
la = x^(1-SIGMA)/x; %   lambda, shadow value
A = la*w/(1-h)^(-GAMA); %   para. of addtive
%   The following is multiplicative utility case
%  GAMA = w*CHI*(1-h)/x;
%  la = (x^CHI*(1-h)^GAMA)^(1-SIGMA)*CHI/x;
tau = la; %   shadow value of adj. cost

%   Taking logs
r=log(r); rb=r; rp=r;
u=log(u); ub=u; up=u;
k=log(k); kb=k; kp=k;
mc=log(mc); mcb=mc; mcp=mc;
v=log(v); vb=v; vp=v;
c=log(c); cb=c; cp=c;
inve=log(inve); inveb=inve; invep=inve;
x=log(x); xb=x; xp=x;
ka=log(ka); kab=ka; kap=ka;
w=log(w); wb=w; wp=w;
```

```
la=log(la); lab=la; lap=la;
sur=log(sur); surb=sur; surp=sur;
h=log(h); hb=h; hp=h;
z=log(z); zb=z; zp=z;
tau=log(tau); taub=tau; taup=tau;
mark=log(mark); markb=mark; markp=mark;
y=log(y); yb=y; yp=y;

% ***************************************************************************
%  deep_model.m
% ***************************************************************************

function [AAA,BBB,CCC,DDD,FFF,GGG,HHH,JJJ,KKK,LLL,MMM,NNN]=deep_model
%  This program generate the equations of our models

%  Define parameters
syms ALFA DELTA BETTA ETA SIGMA GAMA ALFA RHOZ THETA RHO A THETA_D
xi a1 a2

%  Define variables
%  Note: xb–x(t-1) x–x(t) xp–x(t+1)
syms cb c cp xb x xp hb h hp kb k kp rb r rp markb mark markp
syms db d dp wb w wp ub u up yb y yp zb z zp
syms inveb inve invep pb p pp surb sur surp wwb ww wwp
syms vb v vp kab ka kap mcb mc mcp taub tau taup lab la lap

%  Present the model
% ———————————————————————————————————————
%  Households
%
%  The following is for additive utility func. (t) and (t+1)
uf = ( (x)^(1-SIGMA)-1)/(1-SIGMA) + A*((1-h)^(1-GAMA)-1)/(1-GAMA);
ufp = ( (xp)^(1-SIGMA)-1)/(1-SIGMA) + A*((1-hp)^(1-GAMA)-1)/(1-GAMA);
%  The following is for multiplicative utility func. (t) and (t+1)
%  uf = (( (x)^chi*(1-h)^GAMA )^(1-SIGMA)-1)/(1-SIGMA);
%  ufp = (((xp)^chi*(1-hp)^GAMA )^(1-SIGMA)-1)/(1-SIGMA);
%  Households budget constraints (t) and (t+1)
con_la = -x - inve - ww - 1/r*d + ( db + w*h + u*kb);
con_lap = -xp - invep - wwp - 1/rp*dp + ( d + wp*hp + up*k);
%  Capital adj. cost func. (t) and (t+1)
G_adj = a1/(1-1/xi)*(inve/kb)^(1-1/xi) + a2;
G_adjp = a1/(1-1/xi)*(invep/k)^(1-1/xi) + a2;
%  Constraint for cap. adj. cost (t) and (t+1)
con_tau = (1-DELTA)*kb + G_adj * kb - k;
con_taup = (1-DELTA)*k + G_adjp * k - kp;
%  Lagrangian of the household
Lag = uf + la*con_la + (tau)*con_tau + BETTA*( ufp + lap*con_lap + (taup)*con_taup);
%  Taking derivative of Lag w.r.t. x, h, d, k, inve, tau
e_x = diff( Lag,'x');
```

```
e_h = diff( Lag,'h');
e_d = diff( Lag,'d');
e_k = diff( Lag,'k');
e_inve = diff( Lag,'inve');
e_tau = diff( Lag,'tau');



% ------------------------------------------------
% Firm
%
% Pro. Func. (t) and (t+1)
pf = z* kb^ALFA * h^(1-ALFA);
pfp = zp * k^ALFA * hp^(1-ALFA);
% temporal variable
a = c + p^(-ETA)*inve ;
ap = cp + pp^(-ETA)*invep ;
% Firms' profit
profit = p*a - w*h - u*kb;
profitp = pp*ap - wp*hp - up*k;
% pro. constraint
con_mc = pf - a;
con_mcp = pfp - ap;
% optimal consumptoion constraint
con_v = p^(-ETA)*x + THETA_D*surb -c;
con_vp = pp^(-ETA)*xp + THETA_D*sur -cp;
% habit stock constraint
con_ka = RHO*surb + (1-RHO)*c - sur;
con_kap = RHO*sur + (1-RHO)*cp - surp;
% Lagrangian of firms
Lag2 = profit + mc*con_mc + v*con_v + ka*con_ka +
1/r*( profitp + mcp*con_mcp + vp*con_vp + kap*con_kap);
% Taking derivatives of Lag w.r.t. c, sur, h, k, p
% In equilibrium of flexible case, relative price p=1
e_c = diff( Lag2,'c'); e_c = subs(e_c,[p,pp],[1,1]);
e_sur = diff( Lag2,'sur'); e_sur = subs(e_sur,[p,pp],[1,1]);
e_h_firm = diff( Lag2,'h'); e_h_firm = subs(e_h_firm,[p,pp],[1,1]);
e_k_firm = diff( Lag2,'kb'); e_k_firm = subs(e_k_firm,[p,pp],[1,1]);
e_p =diff( Lag2,'p'); e_p = subs(e_p,[p,pp],[1,1]);

% Other equations
e_buget = pf - (c + inve ); % budget constraint
e_y = -y + pf; % definiton of y
e_v = diff( Lag2,'v'); e_v = subs(e_v,[p,pp],[1,1]); % def. of habit adj. cons
e_ka = diff( Lag2,'ka'); e_ka = subs(e_ka,[p,pp],[1,1]); % habit stock equ.
e_mark = -mark + 1/mc; % def. of markup
e_z = -(zp) + RHOZ * (z); % tech

% Create function
```

```
%   ff–func. w.o. expectation; fe–func. w. expectation fs–func. exogeous
ff = [e_x,e_h,e_c,e_h_firm,e_k_firm,e_p,e_inve,e_buget,e_v,e_ka,e_tau,e_mark,e_y];
fe = [e_d,e_k,e_sur];
fs = [e_z];

%   Define the vector of variables
%   I put as many variables as possible into x
xxxb = [rb,ub,kb,mcb,vb,cb,inveb,xb,wb,lab,surb,hb,taub,yb,markb];
xxx = [r,u,k,mc,v,c,inve,x,w,la,sur,h,tau,y,mark];
xxxp = [rp,up,kp,mcp,vp,cp,invep,xp,wp,lap,surp,hp,taup,yp,markp];
yyy = [ka ];
yyyp = [kap ];
zzz = [z];
zzzp = [zp];



%  ————————————————————————————————————————
%  Make functions of the logarithm of the variables
ff = subs(ff, [xxxb,xxx,xxxp,yyy,yyyp,zzz,zzzp], exp([xxxb,xxx,xxxp,yyy,yyyp,zzz,zzzp]));
fe = subs(fe, [xxxb,xxx,xxxp,yyy,yyyp,zzz,zzzp], exp([xxxb,xxx,xxxp,yyy,yyyp,zzz,zzzp]));
fs = subs(fs, [xxxb,xxx,xxxp,yyy,yyyp,zzz,zzzp], exp([xxxb,xxx,xxxp,yyy,yyyp,zzz,zzzp]));

%  Compute analytical derivatives
[AAA,BBB,CCC,DDD,FFF,GGG,HHH,JJJ,KKK,LLL,MMM,NNN]=
deep_deriv(ff,fe,fs,xxxb,xxx,xxxp,yyy,yyyp,zzz,zzzp);

% **************************************************************************
%   deep_deriv.m
% **************************************************************************

function [AAA,BBB,CCC,DDD,FFF,GGG,HHH,JJJ,KKK,LLL,MMM,NNN]=
deep_deriv(ff,fe,fs,xxxb,xxx,xxxp,yyy,yyyp,zzz,zzzp);

%  Compute analytical derivatives of the first order approximation
AAA = jacobian(ff,xxx);
BBB = jacobian(ff,xxxb);
CCC = jacobian(ff,yyy);
DDD = jacobian(ff,zzz);

FFF = jacobian(fe,xxxp);
GGG = jacobian(fe,xxx);
HHH = jacobian(fe,xxxb);
JJJ = jacobian(fe,yyyp);
KKK = jacobian(fe,yyy);
LLL = jacobian(fe,zzzp);
MMM = jacobian(fe,zzz);

NNN = jacobian(fs,zzz);
```

```
% ***************************************************************************
%   deep_num.m
% ***************************************************************************

% Compute the numerical derivatives
AA = zeros(size(AAA));
BB = zeros(size(BBB));
CC = zeros(size(CCC));
DD = zeros(size(DDD));
FF = zeros(size(FFF));
GG = zeros(size(GGG));
HH = zeros(size(HHH));
JJ = zeros(size(JJJ));
KK = zeros(size(KKK));
LL = zeros(size(LLL));
MM = zeros(size(MMM));
NN = zeros(size(NNN));

AA(:) = eval(AAA(:));
BB(:) = eval(BBB(:));
CC(:) = eval(CCC(:));
DD(:) = eval(DDD(:));
FF(:) = eval(FFF(:));
GG(:) = eval(GGG(:));
HH(:) = eval(HHH(:));
JJ(:) = eval(JJJ(:));
KK(:) = eval(KKK(:));
LL(:) = eval(LLL(:));
MM(:) = eval(MMM(:));
NN(:) = eval(NNN(:));
```

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Declaration of Authorship

I hereby confirm that I have authored this master thesis independently and without use of others than the indicated resources. All passages, which are literally or in general matter taken out of publications or other resources are marked as such.

Yu, LI

Li yu

26th July 2005

Taiyuan, China