On Fiscal Policy Feedback Rules and Indeterminacy

Master Thesis

In pursuit of the degree

Master of Science

At the School of Business and Economics

Humboldt-University of Berlin

Submitted by

Jiao, Jie

(Registration No. 188765)

Supervisor: Prof. Harald Uhlig, Ph.D.

Berlin, 19 August 2005

Acknowledgement

I would express my gratitude to Prof. Harald Uhlig for his constructive suggestions, and Mr. Stefan Ried for the technical discussion.

And I am grateful to Zhisong Zhang and Lili Dai for their kind help.

Abstract

Slight departures from standard real business cycle models may lead the economy to display indeterminacy under perfect foresight. In a discrete-time version of the one-sector growth model, the economy may exhibit increasing returns-to-scale due to externality. In such an economic context, the paper involves the government with the following fiscal policy instruments, subsidies on capital income and labor income, and government debt, into consideration. The study shows that a constant labor income subsidy/tax rate and the debt level have no affect on the economy's stability property, because the labor subsidy rate and the debt adjustment parameter do not enter an inter-temporal tradeoff that affects the public's self-fulfilling rational expectations. By contrast, a constant capital subsidy rate may lead the economy to a saddle equilibrium, or endogenous fluctuations, or explosiveness. Joint influence of capital subsidy rate with the degree of increasing returns-to-scale, the inter-temporal elasticity of substitution in labor supply and the discount factor may lead the economy to various stability properties, saddle, sink or source. Thus the government's attempt of using a constant capital tax rate to avoid sunspot fluctuations may fail due to the model's property of global indeterminacy.

Contents

1. Introduction........1

2. Literature........3

3. Model........5

3.1 Household........5
3.2 Firm........7
3.3 Fiscal Authority........9
3.4 Model Solution........10
3.5 Results........18

4. Indeterminacy Analysis........20

4.1 Local Steady State Property........20
4.2 General Reduced Form........21
4.3 Reduced Form of Log-linearization........23
4.4 Parameters Relevant for Indeterminacy........25
4.5 Indeterminacy Analysis........27
4.6 Stabilization Methods........31

5. Policy Suggestions........33

5.1 Tax/Subsidy Rates........33
5.2 Government Debt........34
5.3 Welfare Implication........34

6. Conclusion and Discussion........36

7. Appendix........37

7.1 Benchmark Policy........37
7.2 Estimate λ........38
7.3 Conditions for indeterminacy........39

MATLAB Code........41

File 1: thesis.m........41
File 2: sk_beta.m........45
File 3: Estimate λ........46

References:........48

Figure:........50

1. Introduction

Studies have shown that economic models with externalities, imperfect competition, or incomplete markets are less likely subject to indeterminacy, sunspots or endogenous fluctuations. Therefore it has been interests for the government to intervene to address the source of market failure. Fiscal policy instruments, taxes or subsidies, have been studied by researchers to investigate how they may influence the economic stability. This paper will also work on the choice of government fiscal policy designation, and see how the government's attempt for stabilization may lead to a much richer set of endogenous dynamics.

The model of this paper is based on a discrete-time version of the one-sector growth model that was developed by Benhabib and Farmer (1994). They presented two situations for external effects in the production process that are not mediated by markets, externalities and monopolistic competition. Farmer and Guo (1994) show that this model compares favorably to a standard real business cycle model when applied to replicate some cyclical features of the postwar US economic data. This paper will focus on the case of externalities.

The idea of this paper mainly goes along with that of Guo and Lansing (2001), which incorporates fiscal policy choices to Benhabib-Farmer model of externality. In addition to their fiscal instruments, subsidies on capital income and labor income, this paper takes government debt into consideration. Furthermore, depreciation on capital is considered when the government subsidies or taxes on last period's capital income.

A productivity externality may lead to a situation where the social technology is linear in capital, and consequently create the wedge between the social and private marginal products of capital and labor. The analysis of the paper thus starts with a benchmark policy[1] that aims to equal the social and private returns in capital and labor. This attempt results in the fact that the subsidies on capital and labor income are constant

[1] This benchmark policy is introduced by Guo and Lansing (2001).

and equal the parameter of increasing returns-to-scale.

With the extension to Guo and Lansing's tasks, there are similarities as well as dissimilarities to their findings.

Firstly, the parameter that adjusts public debt does not have effect on the qualitative nature of the model's dynamics. This is because the debt adjustment parameter does not enter an inter-temporal tradeoff. In my opinion this is due to the fact that the debt is introduced in a balanced budget. Since the agent's expectations of future returns must become self-fulfilling, this inter-temporal tradeoff is crucial for generating multiple equilibria.

Secondly, the subsidy rate on capital income still plays an important role, for it affects the tradeoff between consumption and leisure at different dates. It shows that in the calibrated economy, the government should levy a sufficient high tax rate on capital income (higher than 8.69%) to achieve saddle path stability. In addition, global indeterminacy analysis tells that indeterminacy won't appear until the degree of increasing returns-to-scale is greater than 0.5937. These numbers are actually the same with the outcome of Guo and Lansing (2001).

Furthermore, I also investigate the joint influence of the inter-temporal elasticity of substitution in labor supply, the discount factor with capital subsidy rate on the model's global stability properties.

The remainder of the paper is organized as follows. Section 2 briefly introduces the existing literature done by researchers. Section 3 introduces and solves the model. Section 4 investigates the model's dynamics with constant subsidy rates. Section 5 presents policy suggestions. Section 6 concludes.

2. Literature

Earlier tasks on business cycle fluctuations due to self-fulfilling expectations haven been done by Azariadis (1981), Cass and Shell (1983) and ect. Since Woodford (1988) first take seriously the quantitative predictions of a business cycle model with self-fulfilling rational expectations, interest has been growing in such models with self-fulfilling shifts in private sector expectations.

In those models, a new source of impulse, the sunspot variable, generates the disturbance to expectations and thus leads to the multiplicity of equilibria. It offers new mechanism for propagating the effects of existing shocks, such as shocks to technology, and it "in effect acts as a coordinating device that generates changes in expectations that are self-fulfilling and fully consistent with rational expectations".[2] The possibility of multiple equilibira in real business cycle model due to increasing returns, externality and tax distortions has been studied by Benhabib and Farmer (1994), Farmer and Guo (1994), Schmitt and Uribe (1997) and Guo and Lansing (2001).

Studies on the new models give different policy implications. The mainstream equilibrium models, which take shifts to preferences and technology as the basic impulses to the business cycle, show that economic fluctuations represent the economy's efficient responses to shocks and cast doubt on the desirability of macroeconomic stabilization policy. In contrast, the new models suggest that institutional arrangements and policy rules designed to reduce fluctuations in output may be desirable.[3]

The studies by earlier researchers have shown that for appropriate parameter regions these models may have an indeterminate steady state near which there exist multiple equilibira. Initial versions by Benhabib and Farmer (1994) and Farmer and Guo (1994) investigate the requirement for the degree of increasing returns to scale to

---

[2] Evans G and McGough B *Indeterminacy and the stability puzzle in non-convex economies* 2004
[3] Christiano L and Harrison Sharon *Chaos, sunspots and automatic stabilizers* 1996

display sunspot equilibira. Chiristiano and Harrison (1996) show the relationship between the income tax regime, efficiency and determinacy. They find that when the tax rate is fixed at a constant, there are more than one equilibrium and the unique efficient one is determinate. Guo and Lansing (2001) also work on the income subsidy rates. They incorporate the fiscal policy into Benhabib-Farmer-Guo models and find the quantitative interaction between the degree of increasing returns to scale and subsidy rate on capital income to show the dynamics' determinant and indeterminant properties.

As a variation to Guo and Lansing's work, I extend the budget constraint based on their model. In addition to the subsidy rates, government debt held by the public is taken into the budget constraint. Furthermore, depreciation on capital is taken into consideration when subsidizing on it. Under such regime I will try to explore the model's local and global dynamic behavior.

3. Model

The model economy consists of three agents: the firm, the household and the government. The model is based on Benhabib and Farmer (1994) of increasing returns. Among the two variations of the model that exhibits increasing returns, the one with a productive externality and the other with monopolistic competition in the production of intermediate goods, this paper chooses the former just for simplicity.

3.1 Household

It is assumed that the economy is populated with a continuum of identical households who are consumers and producers as well. In a version of discrete time, each of them is endowed with one unite of time, and he maximizes a discounted stream of utilities over his infinite lifetime. The inter-temporal optimization problem faced by a representative consumer is given by:

$$\max \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \frac{A h_t^{1+\gamma}}{1+\gamma} \right) \tag{3.1}$$

where C is consumption, h is labor supply. As to the parameters, β, the discount factor, lies in the interval (0, 1), $\gamma \geq 0$ denotes the inverse of the inter-temporal elasticity of substitution in labor supply, $A > 0$ implies fraction of time spent on working. Consumption is taken in a logarithmic function since it is the only formulation of preferences which is consistent with stationary labor supply in a growing economy if a Cobb-Douglas production function is applied.

The household is subject to the following budget constraint:

$$T_t + q_t b_t + C_t + i_t = (1+s_{ht}) w_t h_t + (1+s_{kt})(r_t - \delta) k_{t-1} + b_{t-1} \tag{3.2}$$

The right hand side of equation (3.2) represents the income of the household at time t, where he derives income by providing labor h_t and physical capital k_t and obtaining wage w_t and capital rental rate r_t. In addition, he obtains at time period t the payback of government bond for time t-1. The left hand side of (3.2) denotes his spending, which includes consumption C_t, investment i_t, lump sum tax T_t and purchasing bond b_t. Fiscal authority employs these policy instruments: subsidy on labor income s_k, subsidy on capital s_h, and issuing government bond b. Variable q denotes the discount rate on the bond's face value when the consumer purchases the bond, and parameter $\delta \in [0, 1)$ is the depreciation rate. The case of $\delta = 1$ is excluded because it is not subject to indeterminacy in a regime of constant subsidy rates.[4] Furthermore the logarithmic utility and in section 3.2 the Cobb-Douglas production technology are applied in this model, with an assumption of 100 percent depreciation, the households can decide his consumption and investment based on the current state of the economy, since the total depreciation leads to offsetting income and substitution effects.

Investment adds to the stock of capital in the following law of motion:

$$k_t = (1-\delta)k_{t-1} + i_{t-1} \tag{3.3}$$

with k_0 given, and depreciation rate δ is taken as same as the above.

The optimality conditions for the household with respect to consumption, labor, capital and bond can be derived by formulating the Lagrangian and taking partial derivatives:

$$L = E_t \sum_{t=0}^{\infty} \beta^t (\log C_t - \frac{Ah_t^{1+\gamma}}{1+\gamma} - \lambda_t (T_t + q_t b_t + C_t + i_t - (1+s_{ht})w_t h_t - (1+s_{kt})(r_t - \delta)k_{t-1} - b_{t-1})) \tag{3.4}$$

The FONCs with respect to C_t, k_t, h_t, b_t are as follows:

4 Guo J.T and Lansing K.J *Fiscal Policy, Increasing Returns and Endogenous Fluctuations* 2001

$$\frac{1}{C_t} - \lambda_t = 0 \tag{3.5}$$

$$\beta^t(-\lambda_t) + \beta^{t+1}E_t\left[\lambda_{t+1}\left((1+s_{kt+1})(r_{t+1}-\delta) + (1-\delta)\right)\right] = 0 \tag{3.6}$$

$$-Ah_t^\gamma + \lambda_t(1+s_{ht})w_t = 0 \tag{3.7}$$

$$\beta^t(-\lambda_t)q_t + \beta^{t+1}E_t\left[\lambda_{t+1}\right] = 0 \tag{3.8}$$

Use equation (3.5) to substitute λ_t and λ_{t+1}, define R_t as the return on capital, the above equations can be converted to:

$$\beta\ E_t\left[\frac{C_t}{C_{t+1}}R_{t+1}\right] = 1 \tag{3.9}$$

$$\text{where}\quad R_t = (1+s_{kt})(r_t-\delta) + 1 - \delta \tag{3.10}$$

$$-AC_t h_t^\gamma + (1+s_{ht})w_t = 0 \tag{3.11}$$

$$-q_t + \beta\ E_t\left[\frac{C_t}{C_{t+1}}\right] = 0 \tag{3.12}$$

Equation (3.9) is the consumption Euler equation that describes the inter-temporal equilibrium on the goods market. Equation (3.11) equates the household's marginal rate of substitution between consumption and leisure to the after-subsidy real wage. The last equation can describe how the government bond's discount rate enter the inter-temporal equilibrium of consumption.

3.2 Firm

The economy is assumed to consist of identical competitive firms, and the number of the firms is normalized to one. The firm hires labor and invests capital in its production; it wants to maximize its profit:

$$\max_{h_t,k_t}\left(y_t - (r_t-\delta)k_{t-1} - w_t h_t\right) \tag{3.13}$$

where y_t is the output.

The Cobb-Douglas aggregate production function is given by:

$$Y = K^a H^b \quad a,\ b > 0 \tag{3.14}$$

Here K is the aggregate stock of capital, and H is aggregate labor input. To study the dynamics in the context of increasing returns to scale, it is assumed that a+b > 1, which implies that the technology exhibits increasing returns to scale.

Benhabib and Farmer (1994) have stated two theories of income distribution that reconcile increasing returns at the aggregate level with competitive behavior by individuals and firms, externality and monopolistic competition, which are consistent with the same aggregate dynamics. This paper, along with idea from Guo and Lansing (2001), focuses on the simpler and more familiar one, the case of externality, assuming that there are important external effects in the production technology that are not mediated by markets.

Thus the firm adopts the social technology redefined as:

$$y_t = z_t k_t^{\theta} h_t^{1-\theta} \tag{3.15}$$

where z_t is the state of technology which the firm takes as given. In a standard real business cycle model, technology z_t is governed by an exogenous stochastic process. Yet here the state of technology is given by:

$$z_t = \left(\widetilde{K}_t^{\theta} \widetilde{H}_t^{1-\theta}\right)^{\eta} \tag{3.16}$$

Here $\widetilde{K}_t$ and $\widetilde{H}_t$ represent the economy-wide average levels of capital and labor. From the representative firm's point of view, they are exogenous terms, and they represent external effects that are not traded in markets. In a symmetric equilibrium, same actions by all the firms lead to $\widetilde{K}_t = k_t$ and $\widetilde{H}_t = h_t$. For simplicity, defining $\alpha_1 = \theta(1+\eta)$ and $\alpha_2 = (1-\theta)(1+\eta)$, so that the production function is reduced to:

$$y_t = k_t^{\alpha_1} h_t^{\alpha_2} \tag{3.17}$$

Using the assumption for equation (3.14) that a+b > 1, which is $\alpha_1 + \alpha_2 > 1$ for equation (3.17), social technology exhibits increasing returns to scale when η>0.

Under the assumption that factor markets are perfectly competitive, solving the firm's maximization problem (3.13) subject to (3.17) implies:

$$\max_{h_t, k_t} profit = \left(k^{\alpha_1}{}_{t-1} h_t^{\alpha_2} - (r_t - \delta)k_{t-1} - w_t h_t\right) \tag{3.18}$$

Taking partial derivatives with respect to k_{t-1} and h_t gives:

$$r_t - \delta = \theta \frac{y_t}{k_{t-1}} \tag{3.19}$$

$$w_t = (1-\theta)\frac{y_t}{h_t} \tag{3.20}$$

3.3 Fiscal Authority

I construct the inter-temporal budget constraint for policy authority assuming that the government balances its budget every period with new debt, taxes and subsidies. This assumption means that the surplus or deficit from issuing government bonds and levying lump-sum tax/subsidy will be given to the household via subsidies on labor income and capital investment:

$$T_t + q_t b_t = s_{kt}(r_t - \delta)k_{t-1} + s_{ht} w_t h_t + b_{t-1} \tag{3.21}$$

The left hand side of the equation denotes the receipts of the government from lump-sum tax and issuing bonds. While the right hand side is the spending constructing of subsidies and payback on last time period's bond.

As to the bond, it is assumed that:

$$b_t = \lambda b_{t-1} \tag{3.22}$$

This indicates that the fiscal authority aims to decrease the government debt. Parameter λ lies in the interval (0, 1), which implies gradual reduction of debt. Although in real world λ most probably may not lie in this region, this assumption at least reveals the authority's policy goal, since no government wants its debt to grow without bounds.

One more point should be explained that lump-sum tax T_t and subsidies on capital and labor income can be interpreted as lump-sum subsidy and tax when they are negative. In this situation equation (3.21) still holds and is reasonable. This case is also applied to the household's budget constraint (3.2).

3.4 Model Solution

3.4.1 Characterizing Equations

To proceed on solving the model's dynamics, it is convenient to bring all the first-order necessary conditions, which have been derived in the previous sections, and the other characterizing equations together here, so that the following work will be easier.

$$\frac{1}{C_t} - \lambda_t = 0$$

$$\beta^t(-\lambda_t) + \beta^{t+1}E_t\left[\lambda_{t+1}\left((1+s_{kt+1})(r_{t+1}-\delta)+(1-\delta)\right)\right] = 0$$

$$-Ah_t^\gamma + \lambda_t\left(1+s_{ht}\right)w_t = 0$$

$$\beta^t(-\lambda_t)q_t + \beta^{t+1}E_t\left[\lambda_{t+1}\right] = 0$$

$$k_t = (1-\delta)k_{t-1} + i_t$$

$$C_t + i_t = y_t$$

$$y_t = k_{t-1}^{\alpha_1}h_t^{\alpha_2}$$

$$r_t - \delta = \theta \frac{y_t}{k_{t-1}}$$

$$w_t = (1-\theta)\frac{y_t}{h_t}$$

$$T_t + q_t b_t = s_{kt}(r_t - \delta)k_{t-1} + s_{ht} w_t h_t + b_{t-1}$$

$$b_t = \lambda b_{t-1}$$

3.4.2 Log-linearization

Up to now the optimality conditions for the household and the firm have been derived. Together with the policy rule, these necessary equations characterize the equilibrium. In order to analyze such a nonlinear dynamic discrete time stochastic model, it is standard to log-linearize these equations.

The main principle of log-linearization is to use a Taylor approximation around the steady state to replace all equations by approximations. Then, the variables are converted to be the percentage deviations from their steady state, and the functions become linear.

Uhlig (1999) has summarized the steps in a simple way. The essence of the method is as follows. Let X_t be the vector of variables, $\overline{X}$ be their steady state, and

$$x_t = \log X_t - \log \overline{X} \tag{3.23}$$

be the vector of log-deviations. The vector $100{*}x_t$ indicates the percentage that the variables differ from their steady state levels in time period t.[5] After Taylor expansion around its steady state, the variable is converted to:

$$X_t = \overline{X}e^{x_t} = \overline{X}(1+\hat{X}_t) \tag{3.24}$$

To prepare for the log-linearization, steady states of the variables are needed, which I

[5] This part is taken from Uhlig (1999)

put in calibration sector 3.4.3.1.

After log-linearization, the characteristic equations for the household are given by:

$$0 = -\hat{c}_t - \gamma\hat{h}_t + \hat{w}_t + \frac{\overline{s}_h}{1+\overline{s}_h}\overline{w}\hat{s}_{ht} \tag{3.25}$$

$$0 = E_t\left[\hat{c}_t - \hat{c}_{t+1} + \hat{R}_{t+1}\right] \tag{3.26}$$

$$0 = -\overline{R}\hat{R}_t + (1+\overline{s}_k)\overline{r}\hat{r}_t + (\overline{r} - \delta)\overline{s}_k\hat{s}_k \tag{3.27}$$

$$0 = E_t\left[\hat{c}_t - \hat{c}_{t+1} + \hat{q}_t\right] \tag{3.28}$$

Since the budget constraint for the representative household includes all the terms of the government budget constraint, for simplicity I subtract equation (3.21) from equation (3.2) to get an easier version for log-linearization. Thus, the budget constraint for household is log-linearized as:

$$0 = -\overline{c}\hat{c}_t - \overline{i}\hat{i}_t + \overline{y}\hat{y}_t \tag{3.29}$$

As to the law of motion for the investment adding to the stock of investment, it can be replaced by:

$$0 = \overline{i}\hat{i}_t + (1-\delta)\overline{k}\hat{k}_{t-1} - \overline{k}\hat{k}_t \tag{3.30}$$

The optimality functions for the firm are log-linearized as:

$$0 = -\theta\frac{\overline{y}}{\overline{k}}(\hat{y}_t - \hat{k}_{t-1}) + \overline{r}\hat{r}_t \tag{3.31}$$

$$0 = -\hat{y}_t + \hat{w}_t + \hat{h}_t \tag{3.32}$$

$$0 = -\hat{y}_t + \alpha_1\hat{k}_{t-1} + \alpha_2\hat{h}_t \tag{3.33}$$

Instead of the normal definition of the log-deviation for government debt $\hat{b}_t$, here I set $\hat{b}_t = \frac{b_t - \bar{b}}{\bar{y}}$, since its steady state $\bar{b}$ might be set to zero in later calculation. And the log-linearization for the government budget constrain is given by:

$$0 = -\bar{T}\hat{T}_t - \bar{y}\bar{q}\hat{b}_t - \bar{q}\bar{b}\hat{q}_t + \bar{s}_k \bar{r}k\hat{r}_t + \bar{s}_k \bar{k}(\bar{r} - \delta)\hat{k}_{t-1} + \bar{s}_h \bar{w}\bar{h}(\hat{w}_t + \hat{h}_t)$$
$$+ \bar{k}(\bar{r} - \delta)\hat{s}_{kt} + \bar{w}\bar{h}\hat{s}_{ht} + \bar{y}\hat{b}_{t-1} \qquad (3.34)$$

3.4.3 Calibration

To continue with the quantitative investigation of the model's local and global dynamics, suitable values for the parameters and steady states of the variables are needed.

3.4.3.1 Steady States for Variables

To prepare for the log-linearization, the steady states of the variables are necessary. Drop out the time subscripts of characterizing equations in sector 3.4.1, and denote the variables' steady state with a bar. After some algebraic manipulation, they are given by:

$$\bar{R} = \frac{1}{\beta}$$
$$\bar{q} = \beta$$
$$\bar{r} = \frac{\bar{R} - 1 + \delta}{1 + \bar{s}_k} + \delta$$

$$\bar{k} = \left(\frac{\frac{\bar{r} - \delta}{\theta} - \delta}{M} \right)^{\frac{\alpha_2}{(1+\gamma)(\alpha_1 - 1)}}$$

where define $M = \frac{1}{A}(1 + \bar{s}_h)(1 - \theta)(1 - \delta)^{\frac{(1+\gamma)\alpha_1}{\alpha_2}} (\frac{\bar{r} - \delta}{\theta})^{\frac{\alpha_2 - 1 - \gamma}{\alpha_2}}$

With calculated $\bar{k}$, the steady states for all the other variables are given by:

$$\bar{c} = M\bar{k}^{1+(1+\gamma)\frac{\alpha_1-1}{\alpha_2}}$$

$$\bar{i} = \delta\bar{k}$$

$$\bar{y} = \bar{k}\frac{\bar{r}-\delta}{\theta}$$

$$\bar{w} = (1-\theta)(1-\delta)^{\frac{\alpha_1}{\alpha_2}}(\frac{\bar{r}-\delta}{\theta})^{\frac{\alpha_2-1}{\alpha_2}}\bar{k}^{\frac{\alpha_1+\alpha_2-1}{\alpha_2}}$$

$$\bar{h} = (\frac{\bar{r}-\delta}{\theta})^{\frac{1}{\alpha_2}}(1-\delta)^{-\frac{\alpha_1}{\alpha_2}}\bar{k}^{\frac{1-\alpha_1}{\alpha_2}}$$

Up to now, with given parameter values, the steady states for the variables can be derived, except three special ones, subsidy rates on capital income and labor income and government debt.

3.4.3.2 Steady States for Subsidy Rates and Government Debt

The idea of this section is learnt from Guo and Lansing (2001). With this benchmark policy, the steady state for subsidies on capital and labor income can be decided.

The policy is to equal the social marginal products of capital and labor to those of the private sector. To fulfill this purpose, the subsidy rates on capital and labor income should both equal to the increasing-return-to-scale parameter η along all the time. This indicates the steady state values for these two subsidy rates are exactly the calibration for parameter η. The proof is left in Appendix 7.1. The steady state values of two subsidy rates are chosen as η for the baseline value. Their variations will be applied in section 4.1 to investigate the model's stability property.

If according to equation (3.22), the steady state for government debt will be zero. It is understandable under the assumption that the government aims to reduce its debt gradually; nevertheless this value is not realistic in real business world. I relate the steady state of debt to that of the output. Based on the US economic data from the year 1970 to 2005, I get a series of debt-to-GDP ratios, and in turn a mean value of these ratios, which equals 0.34. I now define the steady state of debt as 0.34 times of the steady state of the output, i.e. $\bar{b} = 0.34\bar{y}$.

3.4.3.3 Parameter Values

The values for parameters are chosen based on US economy data. Most are borrowed from other researcher's empirical studies, except the last one, gradual reduction on debt parameter, which is estimated by myself. Table 1 lists all the parameter values that are needed.

Parameter	Value
θ	0.3
β	0.962
A	2.876
γ	0
δ	0.067
η	2/3
λ	0.99

Table 1: Baseline Parameter Values

Parameter θ is set to be 0.3 according to Poterba (1997), which indicates capital share in US national income. Discount rate β is taken as 0.962, which implies in Poterba (1997, Table 1) an after-tax interest rate of 4 percent.[6] Parameter A is 2.876, this implies fraction of time spent on working is 0.3.[7] The inverse of inter-temporal elasticity of substitution in labor γ, equaling 0, means indivisible labor. Depreciation rate δ is estimated to be 0.067 by Benhabib and Lansing (2001) based on US data from 1954 to 1992.

The degree of returns-to-scale in the model is $1+\eta$. According to Basu and Fernald

---

[6] Guo J.T and Lansing K.J *Fiscal Policy, Increasing Returns and Endogenous Fluctuations* 2001

[7] Juster and Stafford *The Allocation of Time: Empirical Findings, Behavioral Models and Problems of Measurement* 1991

(1997), different types of data, the level of aggregation and the estimation method all have impacts on returns-to-scale estimates. They find that the uncorrected aggregate estimate of 1+η, which is larger, may be more appropriate for calibrating models like here. This is a support for the chosen parameter value, since the one-sector growth models require strong increasing returns for indeterminacy. In section 4.5.1, we will see how the variations of η, which equal the subsidy rates, affect the model's stability property.

In the model, external effects in the production process are not mediated by markets, so that the social technology differs from the technology faced by the representative agent. If the spillover effects of knowledge acquisition are great enough, the social technology is then linear in capital. Although each individual faces diminishing returns to the acquisition of knowledge, the society as a whole may grow without bound. Caballero and Lyons (1989) find evidence of important external effects in panel data. Their findings confirm that α_2 in the technology is about 1, and this is consistent with our chosen value of η and θ, where $\alpha_2 = (1-\theta)(1+\eta)$ is 1.16.

For the last one, I define λ as the parameter indicating the gradual reduction of government debt. However generally in the real world, it is not the case, so it is hard to estimate with the data on debt. Instead I choose to estimate the coefficient for the ratio of debt to output, i.e. to estimate λ. in the equation $\frac{b_t}{y_t} = \lambda \frac{b_{t-1}}{y_{t-1}}$. The data chosen is US economy data from the year 1970 to 2004. See the Appendix 7.2 for detailed estimation.

3.4.4 Solution

To solve the dynamic system, the recursive law of motion of the model can be numerically solved with the method of undetermined coefficients according to Uhlig's toolkit (1999). The idea goes as follows. Express all the variables in vectors separately as state endogenous, other endogenous (jump variables) and exogenous variables, and write the vectors in linear functions. In order to apply to the

MATLAB toolkit, it is standard to write the vectors in the following expression:

$$0 = Ax_t + Bx_{t-1} + Cy_t + Dz_t \tag{3.35}$$

$$0 = E_t\left[\ Fx_{t+1} + Gx_t + Hx_{t-1} + Jy_{t+1} + Ky_t + Lz_{t+1} + Mz_t\ \right] \tag{3.36}$$

$$z_{t+1} = Nz_t + \varepsilon_{t+1}; \quad E_t\left[\ \varepsilon_{t+1}\ \right] = 0 \tag{3.37}$$

where x_t is the vector of endogenous state variables, y_t, the vector of other endogenous variables, and z_t, the vector of the exogenous variables. It is especially assumed that the row dimension of coefficient matrix C is no smaller than its column dimension.

As to the model in this paper, x_t is defined as $\left[\ k_t, C_t, R_t, r_t, q_t\ \right]'$, and $y_t = \left[b_t, y_t, w_t, h_t, T_t, i_t,\right]'$. I choose subsidy rate on labor income s_{ht} and on capital income s_{kt} as exogenous variables.

What is to be found is the coefficient matrices P, Q, R, S for the recursive equilibrium law of motion:

$$x_t = Px_{t-1} + Qz_t \tag{3.38}$$

$$y_t = Rx_{t-1} + Sz_t \tag{3.39}$$

$$z_{t+1} = Nz_t + \varepsilon_{t+1} \tag{3.40}$$

With the calibrations of the steady states and parameters, the model could be solved. The equilibrium described by these rules can achieve the saddle point only when there is exactly one eigenvalue of matrix P smaller than unity in its absolute value. If more than one eigenvalue of P are smaller than unity in absolute value, there will be sunspot equilibrium. The system exhibits explosiveness when no eigenvalue is smaller than one in absolute value. Details will be explained in section 4.

3.5 Results

This section will look into the impulse responses of some important variables to the exogenous shocks, shocks on capital and labor income subsidy.

Generally, from Figure 1 to Figure 2, it can be shown that the system is unstable. The roots of the system are $0.9404 \pm 0.3939i$ with the absolute value of 1.0196, which is greater than one. This indicates that the steady state is a source, which is not welcomed in an economic sense.

To be more detailed, some specific variables will be analyzed separately.

First let's look at the consumption, see figure 1 (upper). Its impulse responses to the two shocks are similar. As to the capital subsidy shock, capital and consumption first undergo a very slight decrease at the time when the shock takes place. This may be because the shock on capital income subsidy encourages capital investment and thus extracts people's consumption. Then the consumption goes up afterwards. They reach the peak at the end of the 2nd year after the shock, and then soon decrease until below the steady state. Further on they keep on oscillating with increasing magnitude. As to the labor income subsidy shock, the consumption directly jumps up and reaches its peak about the same time with its impulse to the capital subsidy shock. But the magnitude is greater than that to the capital subsidy shock. This is understandable since generally the public has a higher proportion for his labor income than capital income.

The capital stock has the similar impulse response with consumption to the capital income and labor income subsidy shocks. See figure 1 (middle).

The interesting thing is with the government debt, see figure 1 (down). It does not go explosive, however by contrast to other variables, it converges. The debt reacts immediately to the shocks, and reaches its peak at the same time as the shocks take place. The difference is that it decreases with the shock of capital subsidy and increases with the shock of labor income subsidy. The capital income subsidy encourages the public to make money through investment, thus shifts the public's desire of purchasing government bonds. As for the labor income subsidy, when the government impose higher subsidy rate, the public has more money at hand, which could be spent on purchasing government bonds. One thing should be noticed that the magnitudes of both impulse responses are very small. After its maximum response, both reactions go back to the steady state gradually and slowly. They do not converge even until the 8th year after the shock. The reason that the impulse response of debt does not explode is due to the government's control. In the designation for the fiscal authority, it is assumed that the government debt is gradually reduced with the control parameter $\lambda \in (0,1)$.

Figure 2 presents the impulse responses of interest rate, investment and lump sum tax to the two exogenous shocks. Generally they have similar responses with that of the consumption. The common feature they share is that they are all explosive. And their cycles are all around 4 years. Comparatively the response of interest rate is much smoother, while investment fluctuates the most.

Figure 3 gives the impulse response of the variables to one percent deviation of the discount rate on the government bond. It can be shown that the discount rate on the face value of the bond has considerable impact. This is due to the reason that the discount rate influences the public's expectation in an inter-temporal tradeoff (see equation (3.28)).

4. Indeterminacy Analysis

Guo and Lansing (2001) have investigated the local determinacy and global indeterminacy properties in their model. It shows that subsidy rate on capital and some other parameters such as degree of returns-to-scale can influence the stability property of the model's dynamics. Railavo (2004) and Evans and McGough (2004) have also studied the stability property with changing parameters, they all find that specific parameter variations can lead to the model's different stability behavior. Therefore it is also my interest to look into how the changes of some economic-meaningful parameters will lead the model's dynamics to saddle, sink or source.

4.1 Local Steady State Property

It is my interest to find proper regimes for parameter λ that can lead to economic stability, however, to my disappointment, it does not influence the stability of the model's dynamics. See figure 4, the local steady state is always a source when varying λ within (0, 1). I will explain the reasons in section 4.4.

Thins are the same with labor income subsidy rate s_h. See figure 5.

Still, like the case in Guo and Lansing (2001), the subsidy on capital, s_k, is important. Figure 6 presents the comparison of the local stability properties. And the following table summarizes the changing stability property with different s_k when holding other parameters constant.

subsidy rate on capital	**eigenvalues**	**steady state**
$s_k < -0.0869$	$\lvert\mu_1\rvert < 1,\ \mu_2 < -1$	saddle
$s_k = -0.0869$	$\lvert\mu_1\rvert < 1,\ \mu_2 = -1$	saddle changes to sink
$-0.0869 < s_k < 0.6380$	$\lvert\mu_1\rvert < 1, \lvert\mu_2\rvert < 1$	sink
$s_k = 0.6380$	$\lvert\mu_1\rvert = \lvert\mu_2\rvert = 1$	sink changes to source
$s_k > 0.6380$	$\lvert\mu_1\rvert = \lvert\mu_2\rvert > 1$	source

Table 2: Stability Property Near Steady State

It is found that there are two critical points where the model changes from saddle to sink (s_k = -0.0869) with the chosen root of –1, and from sink further to source ($s_k = 0.638$) with both roots that have the absolute value of 1. When, indeterminacy (a sink), both eigenvalues lie inside the unit circle, i.e. -0.0869 < sk < 0.6380 in the paper's calibration, the economy is subject to multiple equilibria. Analyzing upon the numbers, the government should be careful with its tax rate on capital income. The government should levy a sufficiently high tax rate (higher than 8.69%) to ensure a saddle path to the unique equilibrium. Tax rate lower than this bound will lead the economy to multiple equilibrium. Alternatively, low subsidy rate on capital income will also result in indeterminacy. Very high subsidy rate (higher than 63.8%) may cause the economy to explosiveness, but this bound may be too high to be realistic in the real world. This result is actually the same with that of Guo and Lansing (2001). It seems that taking government bond into consideration in such a way does not affect the economy's stability property and the authority's policy rules. While the analysis here is just the case of local stability property, "such a policy may open the door to other forms of endogenous fluctuations – those arising from global indeterminacy".

4.2 General Reduced Form

To learn the stability property of the dynamics, it is the key to know whether the eigenvalue of the characterizing coefficient matrix is greater than one, and how many of the eigenvalues are greater than one. First I would like to present a general reduced form, which is the baseline for the later on algebraic manipulation.

The general reduced form goes as:

$$aE_t[c_{t+1}] + bE_t[k_{t+1}] = e_1 c_t + f_1 k_t + g_1 v_t \tag{4.1}$$

$$k_t = e_2 c_{t-1} + f_2 k_{t-1} + g_2 v_{t-1} \tag{4.2}$$

$$v_t = \rho v_{t-1} + u_t \tag{4.3}$$

where all the existing variables have the same meaning with the ones in the model of this paper, and v_t is a productivity shock. $E_t[c_{t+1}]$ and $E_t[k_{t+1}]$ are the forecasts by the private, where the expectations are formed in period t. Under the standard rational expectations assumption they are conditional on information dated t or earlier, and we have $E_t[k_{t+1}] = k_{t+1}$. So the equations can be rewritten in matrix expression:

$$\begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix} \begin{bmatrix} E_t[k_{t+1}] \\ E_t[c_{t+1}] \end{bmatrix} = \begin{bmatrix} e_1 & f_1 \\ e_2 & f_2 \end{bmatrix} \begin{bmatrix} k_t \\ c_t \end{bmatrix} + \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} v_t \tag{4.4}$$

or in abbreviation:

$$FE_t x_{t+1} = Hx_t + Gv_t \tag{4.5}$$

Here k_0 is given by the initial conditions of the economy, so variable k is predetermined. While variable c is different, c_0 is free to be determined by the choices of the household in the economy. If all the starting points (k_0, c_0) in the neighborhood of the steady state $(\bar{k}, \bar{c})$ with the paths that satisfies the equations (4.4) converge back to the steady state, the steady state $(\bar{k}, \bar{c})$ is completely stable. In this sense, there will be more than one equilibrium trajectory (k_t, c_t) with c_0 at any choice of the household near the steady state. This completely stable steady states, giving rise to a continuum of equilibria, is termed as indeterminate, and is called by Benhabib and Farmer (1994) that the stable manifold has dimension two.

While in the case of one-dimensional stable manifold in (k, c), the only one path that just starts on this manifold converge to the steady state, and all the other paths starting in the neighborhood will diverge. Thus there is a unique local equilibrium in the neighborhood of steady state. In this instance, for every k_0 in the neighborhood of $\bar{k}$ there will be a unique c_0 in the neighborhood of $\bar{c}$ that leads the path converging to $(\bar{k}, \bar{c})$. This c_0 is the one that places the economy on the stable branch of the saddle point $(\bar{k}, \bar{c})$ [8], the one that terms this stable steady state as determinant.

Blanchard and Kahn (1980) introduced the method to analyze the model's determinacy. They require that, for the model in state-space form, the number of roots inside the unite circle should equal the number of non-predetermined variables for a unique solution under rational expectations, which indicates determinacy.

To be more applicable in this general reduced form, let $J = F^{-1}H$. If one eigenvalue of matrix J is greater than one and the other is inside the unite circle, then there is a unique stationary solution, and the model achieves a saddle stability which is determinate. If both of the eigenvalues are inside the unite circle, there will be stationary solutions for any starting point which lead to an infinity of equilibira, the so-called indeterminate, and the economy may be subject to stochastic sunspot fluctuations. If both of the eigenvalues are greater than one, all solutions are explosive and the model is a source.

The necessary and sufficient conditions for indeterminacy are given as follows, and the proof is presented in Apendix 7.3.

$$\left|\det J\right| < 1 \tag{4.6}$$

$$\left|traceJ\right| < 1 + \det J \tag{4.7}$$

I will use these conditions to capture the parameters' influence on the model's dynamic property.

4.3 Reduced Form of Log-linearization

According to the first-order necessary conditions and the log-linearized equations obtained in part 3.4.1 and 3.4.2 respectively, it is possible to reduce the 10 equations into two equations in variable capital k and consumption c. Since it is intended to investigate the model's perfect-foresight dynamics under a regime of constant subsidy

8 Benhabib and Farmer (1994)

rates, i.e. $s_{kt} = s_k$ and $s_{ht} = s_h$ for all t, the equilibrium conditions change into the following forms:

$$Ac_t h_t^{\gamma} = (1 + s_h) w_t \tag{4.8}$$

$$1 = \beta E_t \left[\frac{C_t}{C_{t+1}} R_{t+1} \right] \tag{4.9}$$

$$R_t = (1 + s_k)(r_t - \delta) + 1 - \delta \tag{4.10}$$

$$C_t + i_t = y_t \tag{4.11}$$

$$k_t = i_t + (1 - \delta) k_{t-1} \tag{4.12}$$

$$y_t = k_t^{\alpha_1} h_t^{\alpha_2} \tag{4.13}$$

$$w_t = (1 - \theta) \frac{y_t}{h_t} \tag{4.14}$$

$$r_t - \delta = \theta \frac{y_t}{k_{t-1}} \tag{4.15}$$

Note that the little difference of the above characterizing equations from those in section 3.4.1 is that I drop the time index of subsidy rate on capital and labor income, which means they are constant. And their corresponding approximate log-linearization around the steady state equations goes as follows:

$$0 = \hat{c}_t + \gamma \hat{h}_t - \hat{w}_t \tag{4.16}$$

$$0 = E_t \left[\hat{c}_t - \hat{c}_{t+1} + \hat{R}_{t+1} \right] \tag{4.17}$$

$$0 = \hat{R}_t - (1 + s_k) \frac{\bar{r}}{\bar{R}} \hat{r}_t \tag{4.18}$$

$$0 = \frac{\bar{c}}{\bar{k}} \hat{c}_t + \hat{k}_{t+1} - \frac{\bar{y}}{\bar{k}} y_t - (1 - \delta) \hat{k}_t \tag{4.19}$$

$$0 = \theta \frac{\bar{y}}{\bar{k}} (\hat{y}_t - \hat{k}_t) - \bar{r}\hat{r}_t \tag{4.20}$$

$$0 = \hat{y}_t - \hat{w}_t - \hat{h}_t \tag{4.21}$$

$$0 = \hat{y}_t - \alpha_1 \hat{k}_t - \alpha_2 \hat{h}_t \tag{4.22}$$

With the steady state values in section 3.4.3.1, the above 7 log-linearization equations can be reduced further to two equations as the general form in section 4.1 with only two variables C_t and k_t. They can be expressed in matrixes as follows:

$$\begin{bmatrix} E_t \hat{k}_{t+1} \\ E_t \hat{c}_{t+1} \end{bmatrix} = \underbrace{\begin{bmatrix} \lambda_1 & \lambda_2 \\ \dfrac{\lambda_1 \lambda_3}{\lambda_4} & \dfrac{1 + \lambda_2 \lambda_3}{\lambda_4} \end{bmatrix}}_{J} \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix} \tag{4.23}$$

where the parameters are defined as:

$$\lambda_1 = 1 - \delta + \frac{(\frac{1}{\beta} - 1 + \delta)(1 + \gamma)\alpha_1}{(1 + s_k)\theta(\gamma + 1 - \alpha_2)} \tag{4.24}$$

$$\lambda_2 = \delta - \frac{(\frac{1}{\beta} - 1 + \delta)(1 + \gamma)}{(1 + s_k)\theta(\gamma + 1 - \alpha_2)} \tag{4.25}$$

$$\lambda_3 = \frac{(1 - \beta + \delta\beta)\left[(1 + \gamma)(\alpha_1 - 1) + \alpha_2\right]}{(\gamma + 1 - \alpha_2)} \tag{4.26}$$

$$\lambda_4 = 1 + \frac{(1 - \beta + \delta\beta)\alpha_2}{(\gamma + 1 - \alpha_2)} \tag{4.27}$$

According to the theories introduced in section 4.2, the stability property of the model's dynamics is dependant on the determinant and trace of the Jacobian matrix *J*.

4.4 Parameters Relevant for Indeterminacy

Looking at the expression for λ_1 to λ_4, unfortunately, parameter λ, indicating government's control on issuing public debt, does not appear in the Jacobian matrix, that is why it does not affect the model's determinacy as demonstrated in section 4.1. This is disappointing to some extent, since it is one of the goals of the paper to investigate what kind of debt level will lead to an indeterminant solution. The reason is that the debt adjustment parameter does not enter an inter-temporal tradeoff that is related with household's expectations. Since the agent's expectations of future returns must become self-fulfilling, this inter-temporal tradeoff is crucial for generating multiple equilibria.

In my opinion this is due to the fact that the debt is introduced in a balanced budget. The surplus or deficit from issuing government bonds and levying lump-sum tax/subsidy will be given at the current period to the household via subsidies on labor income and capital. The inflow from debt at the current period and the outlay of payback for last period's bond are balanced at present, so the debt does not influence the agent's expectation for next period. It is the agent's self-fulfilling rational expectations that are key important for the multiple equilibria. Railavo (2004) has investigated the indeterminacy property under the assumption of government deficit. He shows that in accompany with the monetary policy the deficit rule parameter and the Taylor rule parameter jointly lead to an interval for indeterminacy.

It is the same case with labor income subsidy rate.

Now the only fiscal policy instrument that shows up in this *J* matrix is the subsidy rate on capital income, s_k. Looking at equation (4.17) and (4.18), together with the expression of relevant steady state values, it is clear that only s_k enter the inter-temporal mechanism. Although now this Jacobian matrix is similar to that of Guo and Lansing (2001), I will still go on solving it, for here I consider the depreciation of last period's capital when levying subsidy or tax on it, i.e. $s_k(r_t - \delta)k_{t-1}$, while in their model, the government subsidizes directly on this period's capital income, $s_k r_t k_t$. This may result in different regimes of s_k for the model's determinacy behavior.

4.5 Indeterminacy Analysis

Section 4.1 takes a look at the local steady state property with given parameters; this section will try to investigate the model's global behavior. With the result of sector 4.3, the determinant and trace of *J* matrix in this model are:

$$\det J = \frac{\lambda_1}{\lambda_4} \qquad\qquad trJ = \lambda_1 + \frac{1+\lambda_2\lambda_3}{\lambda_4}$$

According to the theory introduced in previous sector, the sufficient and necessary conditions for indeterminacy in this model imply:

$$\left|\frac{\lambda_1}{\lambda_4}\right| < 1 \qquad\qquad \left|\lambda_1 + \frac{1+\lambda_2\lambda_3}{\lambda_4}\right| < 1 + \frac{\lambda_1}{\lambda_4}$$

During the following work, I will treat each of the above inequations as two:

$$\frac{\lambda_1}{\lambda_4} < 1 \qquad (4.28.1) \qquad\qquad \frac{\lambda_1}{\lambda_4} > -1 \qquad (4.28.2)$$

$$\lambda_1 + \frac{1+\lambda_2\lambda_3}{\lambda_4} < 1 + \frac{\lambda_1}{\lambda_4} \qquad (4.29.2) \qquad\qquad \lambda_1 + \frac{1+\lambda_2\lambda_3}{\lambda_4} > -1 - \frac{\lambda_1}{\lambda_4} \qquad (4.29.2)$$

Since it is the task to find out what region for capital subsidy rate will lead to multiple equilibira and what will ensure a saddle path, now the paper expresses s_k in other parameters according to the above inequations.

The critical points for inequations (4.28.1) to (4.29.2) are given by:

$$s_{k1} = \frac{(\frac{1}{\beta} - 1 + \delta)(1+\gamma)(1+\eta)}{(1+\gamma)\delta + \alpha_2(1-\delta)(1-\beta)} - 1 \qquad (4.30)$$

$$s_{k2} = -\frac{(\frac{1}{\beta} - 1 + \delta)(1+\gamma)(1+\eta)}{(1+\gamma-\alpha_2)(2-\delta) + \beta\alpha_2(\frac{1}{\beta} - 1 + \delta)} - 1 \qquad (4.31)$$

$$s_{k3} = -\frac{(\frac{1}{\beta}-1+\delta)^2(1+\gamma)\beta\left[\left(1+\eta-\frac{1}{\theta}\right)\alpha_2 - \frac{1}{\theta}(1+\gamma)(\alpha_1-1)\right]}{\delta(1+\gamma)(2-\delta)\beta(\frac{1}{\beta}-1+\delta)(1+\gamma-\alpha_2)} - 1 \tag{4.32}$$

$$s_{k4} = \frac{(\frac{1}{\beta}-1+\delta)(1+\gamma)\left[2\alpha_1 - \beta(\frac{1}{\beta}-1+\delta)(\alpha_1-1)\right]}{\beta(\frac{1}{\beta}-1+\delta)(\delta(1+\gamma)(1-\alpha_1)-2\alpha_2)-2(2-\delta)(1+\gamma-\alpha_2)} - 1 \tag{4.33}$$

Now it is very clear that the capital subsidy rate is expressed by other parameters. When varying some parameters, it will lead to different regions for s_k that satisfies the above necessary and sufficient conditions. Just one thing should be born in mind. The situation when the absolute value of s_k is greater than one should also be excluded, since the subsidy, or tax when negative, lies always in the interval [-1, 1].

In the following, I will try to find out the how the following parameters, increasing returns-to-scale η, inverse of inter-temporal elasticity of substitution in labor supply γ, and discount rate β, require for the indeterminacy to appear.

4.5.1 Stability Property With η and s_k

Figure 7 presents the region where indeterminacy appears with joint effort of η and s_k, where the blue line represents det(J) = 1, the green one for det(J) = -1, and the red one for tr(J) + det(J) = -1. The condition for tr(J) - det(J) = 1 is excluded from the figure, since this condition gives the values of s_k that are all outside the interval [-1, 1].

Synthesizing all the conditions, it shows that it is necessary to keep η greater than 0.5937 for the steady state to become a sink (indeterminacy), which is the same as in Guo and Lansing (2001). With η smaller than 0.5937, no indeterminacy will appear however varies s_k. The increase in the subsidy rate on capital income may turn the steady state into a source, while decrease could transform it into a saddle. In the paper's calibration, η is chosen to carry the value of 2/3 that is larger than 0.5937, hence the saddle-sink-source changes of the stability properties appear near the model's steady state. This also indicates that if the government wants to close the wedge between the social and private marginal products of capital by setting s_k at or near η = 2/3, it may destabilize the economy and introduce a set of endogenous dynamics.

4.5.2 Stability Property With γ and s_k

In the above calibration of the model, the value for the parameter γ (inverse of the inter-temporal elasticity of substitution in labor supply) is set to be zero, which indicates that the elasticity of the substitution in labor supply is infinite and that labor is indivisible. The result from the calibration shows that the two eigenvalues of the Jacobian matrix are both complex conjugates and the model is unstable near the steady state. Labor increases its divisibility with the increase of γ, so that the agents may have labor input to work and leisure at the same time. The household's utility increases if the value of γ increases, which indicates that the inter-temporal elasticity of substitution in labor supply decreases.

γ also appears in the Jacobian matrix (see equation (4.24) to (4.27)), so it might influence the model's dynamics as well. Figure 8 presents the stability property near the steady state with combination of different γ and s_k. The condition of $tr(J) - \det(J) = 1$ is excluded from the graph still due to the reason that it results in s_k all outside $[-1,1]$. It can be shown that it is necessary for γ to be lower than 0.04518 to make it possible for a sink. Given γ < 0.04518, increase in the capital subsidy rate would eventually transform the steady state into a source (explosive), while decrease in the capital subsidy rate would eventually transform the steady state into a saddle (determinate). At the point of γ equaling 0.04518, the capital subsidy rate is 0.6655 for both conditions of det(J) = 1 and det(J) + tr(J) = -1, which is a little bit smaller than the benchmark subsidy rate on capital income that is 2/3. This indicates that the benchmark subsidy rate on capital income eventually lead to the unstability of the model (source).

With the values of the other parameters and the steady state of capital and labor subsidy rates the same as in the benchmark calibration, the changes of γ would cause the changes to the stability properties of the steady state of the model. The following table 3 summarizes these changes.

From the table, one can see that when γ decreases passed γ_1, one of the eigenvalues of the Jacobian matrix crosses outside the unit circle and the other one remains outside the unit circle. So the stability property changes from a saddle equilibrium to a source. This can also be verified from Figure 8, that if holding s_k = 2/3, γ should equal γ_1 if satisfying critical point det(J) + tr(J) = -1.

Gama γ	**eigenvalues**	**steady state**
$0 < \gamma < \gamma_0$	Complex $\|\mu_1\| = \|\mu_2\| > 1$	source
$\gamma = \gamma_0$	Real $\mu_1 = \mu_2 = 1.0196$	source
$\gamma_0 < \gamma < \gamma_1$	Real $\mu_1 < -1$, $\mu_2 < -1$	source
$\gamma = \gamma_1$	Real $\mu_1 = -1$, $\mu_2 < -1$	source changes to saddle
$\gamma > \gamma_1$	Real $0 < \mu_1 < 1$, $\mu_2 > 1$	saddle

$\gamma_0 = 0.04522030677$, $\gamma_1 = 0.045220487$

Table 3: Stability properties near the steady state

4.5.3 Stability Property With β and s_k

Extensive study has been done on the relationship between the magnitude of the discount factor and the complicated behavior generated by the corresponding policy rules.[9] The studies by those authors have suggested that substantial discouting might be necessary to obtain complicated optimal behavior. Thus this subsection tries to find the bound of discount factor for indeterminacy to appear with the calibration of the model in this paper.

Figure 9 has presented the saddle-sink-source relations with joint influence of discount factor β and capital subsidy rate s_k. It is clear from the graph that the discount factor should be greater than 0.9225 so that indeterminacy may come forth. When $\beta > 0.9225$, an increase in the subsidy rate will lead to an explosive source,

[9] See Mitra T (1998) and Sorger (1994)

and a decrease in the subsidy rate, or even an increase in the capital tax rate will eventually result in a saddle equilibrium, while a moderate subsidy rate or tax rate region may introduce the multiple equilibria termed as sink.

4.6 Stabilization Methods

Multiplicity of equilibrium weakens our ability to predict. In order to make useful predictions, it is necessary to pin down to a particular economically meaningful equilibrium, or at least narrow set of equilibria. Thus economists have investigated the methods to determine a local unique equilibrium. Here I will summarize some of these works.

Some authors try to put in additional structure, e.g. the agent is assumed to select a specific expectational path out of a set of possible equilibria. As an example, McCallum (1983) imposes sufficient restrictions on the agents' expectational rule to ensure an unique perfect foresight equilibrium.

Georges (1995) uses quadratic adjustment costs applied to jump variables to select a locally unique equilibrium. The idea is to model structurally the additional restriction that some set of variables do not jump as a consequence of adjustment costs. In addition to the existed model, the newly constructed economy includes plausible small convex adjustment costs. If the new model has a unique equilibrium with sufficiently small adjustment costs, the arbitrary solution of the newly constructed model is thus chosen as the economically relevant perfect foresight equilibrium.

Other ways are explored to introduce additional institutional structure in order to close the model. Guo and Lansing (2001) have introduced a general method design a state-contingent capital subsidy/tax policy that selects a unique local equilibrium by ensuring saddle-point stability of the steady state. Since this is directly applied to the model in question, I will elaborate more on this method.

As for the indeterminacy properties in the previous sections, it is analyzed in the context of constant subsidy rate. But the analysis here is based on state-contingent rate, so s_k is replaced in the Euler Equation (4.9) with s_{kt}. Furthermore, the log-linearization will be taken around $(1+\bar{s}_k)$ instead of $\bar{s}_k$, for $\bar{s}_k$ might be negative when it represents tax rate. Set $(1+s_{kt}) = s_t^k$. Assuming that s_t^k is exogenous, and the reduced log-linearization set is now given by:

$$\begin{bmatrix} E_t\hat{k}_{t+1} \\ E_t\hat{c}_{t+1} \end{bmatrix} = \underbrace{\begin{bmatrix} \lambda_1 & \lambda_2 \\ \frac{\lambda_1\lambda_3}{\lambda_4} & \frac{1+\lambda_2\lambda_3}{\lambda_4} \end{bmatrix}}_{J} \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\lambda_5}{\lambda_4} \end{bmatrix} \hat{s}_{t+1}^k \tag{4.34}$$

where $\lambda_5 = \beta\left(\frac{1}{\beta} - 1 + \delta\right)$, and all the other parameters are same in equation (4.22).

The local control policy introduced by Guo and Lansing is aimed to find parameters that master the response of capital subsidy rate to the lagged variables capital and consumption. So let d_1 and d_2 be the control parameters, the control policy can be expressed as:

$$\hat{s}_{t+1}^k = d_1\hat{k}_t + d_2\hat{c}_t \tag{4.35}$$

Substituting (4.35) into (4.34), the standard reduced form of the log-linearization is:

$$\begin{bmatrix} E_t\hat{k}_{t+1} \\ E_t\hat{c}_{t+1} \end{bmatrix} = \underbrace{\begin{bmatrix} \lambda_1 & \lambda_2 \\ \frac{\lambda_1\lambda_3 + d_1\lambda_5}{\lambda_4} & \frac{1+\lambda_2\lambda_3 + d_{2\lambda_5}}{\lambda_4} \end{bmatrix}}_{JJ} \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix} \tag{4.36}$$

where $\det JJ = \frac{\lambda_1 + d_2\lambda_1\lambda_5 - d_1\lambda_2\lambda_5}{\lambda_4}$ and $trJJ = \lambda_1 + \frac{1+\lambda_2\lambda_3 + d_2\lambda_5}{\lambda_4}$.

Now it is possible to choose proper d_1 and d_2 according to *det(JJ)* and *tr(JJ)* so that the dynamic system ensures a saddle point stability. According to different criteria, an optimal combination of (d_1, d_2) can be chosen from the many candidate sets.

5. Policy Suggestions

Farmer (1999) has pointed out that, in general, a sequence of policy instruments are associated with a particular equilibrium. The government knows this sequence, controls with pinpoint accuracy, so that implement the equilibrium by applying specific rule. The government can't affect the set of equilibrium allocations but still retains considerable influence on the actual choice of equilibrium within the set. It guides the economy to any equilibrium allocation simply by reacting to events in a particular way. Up to now, the analysis based on the model may give some suggestions to the government on policies relating with economic equilibrium.

5.1 Tax/Subsidy Rates

With the calibration of the paper, it is shown that the economy may undergo different situations if the government varies its subsidy rate on capital income.

The choice of the tax rate or subsidy rate on labor income will not affect the economy's stability property.

The government should levy a sufficiently high tax rate (higher than 8.69%) on the capital income to ensure a saddle path to the unique equilibrium. Multiple equilibrium may emerge if the capital tax rate is set lower than this bound. In other ways, low or moderate subsidy rates on capital income will also lead the economy to indeterminacy. Explosiveness will not show up until the government raises its capital subsidy rate up to 63.8%. Frankly speaking nevertheless, it is almost impossible and meaningless to afford such a high subsidy. So the calibrated economy will not worry the problem of instability as long as the government takes a tax policy or moderate subsidy policies on capital income.

However, if the government wants to avoid sunspot fluctuations near the steady state simply by imposing high capital tax rate, global indeterminacy will lead the economy to other forms of endogenous fluctuations. The degree of increasing returns-to-scale, the inter-temporal elasticity of substitution in labor supply, and the discount factor may all subject the economy to multiple equilibria if they reach specific boundaries. So the government should be very careful with the choice of capital tax rate depending on the economy's intrinsic characteristics.

5.2 Government Debt

In the paper, the designation of the government debt has no influence on the economy's stability property.

The government aims to gradually reduce its public debt, and introduce the debt into the budget constraint in a balanced way. The surplus or deficit from issuing government bonds and levying lump-sum tax/subsidy will be given at the current period to the household via subsidies on labor income and capital. The balanced budget excludes the debt's influence on the economy's stability.

Some authors have done research on the way the debt affects the public's rational expectation and in turn lead to indeterminacy or crisis in the economy. Railavo (2004) investigated the stability property under the government's deficit rule. His work shows that when the deficit rule parameter is greater than zero, the economy may undergo indeterminacy depending on the Taylor rule parameter. Cole and Kehoe (1998) studies the values of government debt and the debt's maturity structure under which financial crisis brought on by a loss of confidence in the government can arise within a dynamic stochastic general equilibrium model. He shows that when the economy is inside the crisis zone, the government is motivated to gradually reduce its debt and exit the crisis zone because this leads to an economic boom and a reduction in the interest payment on the public's debt.

So the government won't worry about the debt's influence on the economy's stability property if it has a balanced budget. But unbalanced budget may cause multiple equilibria or crisis.

5.3 Welfare Implication

In the model, the government can make decisions on putting the economy in a fluctuating situation or stabilization. Welfare is usually a criterion for the government to choose among policy instruments. However, Guo and Lansing (2001) point out that the economies can't be ranked from the welfare standard, because in the model a fluctuating economy and its stabilized counterpart will both be Pareto-inferior due to the presence of the productive externality.

In the model, the fiscal policy controls the nature of the economy's stability property, and the sunspot shock may appear in the time of indeterminacy. Thus Guo and Lansing (2001) also mentioned that Monte Carlo simulations can't settle the welfare problem since the results will depend on the assumed fiscal policy and the assumed variance of a sunspot shock.

Normally the first-best allocations provide an important benchmark to judge the desirability of stabilization policy. But there is no enough information on the first-best allocation for the model, so the welfare problems are further complicated.

Conclusively in the case of endogenous fluctuations, there is no definitive answer to the question whether the government should stabilize the economy with specific fiscal policies.

6. Conclusion and Discussion

This section will summarize the findings above, and in addition, give an outlook for further research.

The paper studies a discrete-time version of the one-sector growth model with externality, where the government has the following fiscal policy instruments, subsidies on capital income and labor income, and government debt. The study shows that a constant labor income subsidy/tax rate and the debt level have no affect on the economy's stability property, because the debt adjustment parameter and the labor subsidy rate do not enter an inter-temporal tradeoff that affects the public's self-fulfilling rational expectations. The only fiscal instrument that affects the economy's stability property is the subsidy/tax rate on capital income. Specific constant capital subsidy rate may lead the economy to saddle, sink or source. Nevertheless, the government's attempt of using a constant capital tax rate to avoid sunspot fluctuations may fail due to the model's property of global indeterminacy. The degree of increasing returns-to-scale, the inter-temporal elasticity of substitution in labor supply and the discount factor may all result in global indeterminacy for constant capital subsidy rate.

In a Ricardian fiscal policy regime, where the government's inter-temporal budget constraint is always in balance, the debt level does not sway the public's expectational mechanism, so the debt level of the model in question does not affect the economy's stability property. While in a non-Ricardian regime, where the government's inter-temporal budget constraint may not be satisfied for arbitrary price levels, the joint effort of fiscal and monetary policy may influence the economy from a stability point of view.

7. Appendix

7.1 Benchmark Policy

Proposition: The social returns on capital and labor income equal to those in the private sector when

$$s_{ht} = s_{ht} = \eta \qquad \text{for all t}$$

Proof: From equation (3.17), the social marginal product of capital is given by:

$$\frac{\partial y_t}{\partial k_t} = \alpha_1 \frac{y_t}{k_t} \tag{7.1}$$

The social marginal product of labor income is given by:

$$\frac{\partial y_t}{\partial h_t} = \alpha_2 \frac{y_t}{h_t} \tag{7.2}$$

The after-subsidy private returns on capital and labor income are:

$$(1 + s_{kt})r_t \tag{7.3}$$

$$(1 + s_{ht})w_t \tag{7.4}$$

Substitute r_t and w_t with equation (3.19) and (3.20), and equal (7.3) and (7.4) with (7.1) and (7.2):

$$(1 + s_{kt})\frac{\theta y_t}{k_t} = \alpha_1 \frac{y_t}{k_t} = \theta(1+\eta)\frac{y_t}{k_t} \tag{7.5}$$

$$(1 + s_{ht})\frac{(1-\theta) y_t}{h_t} = \alpha_2 \frac{y_t}{h_t} = (1-\theta)(1+\eta)\frac{y_t}{h_t} \tag{7.6}$$

These two equations result in $s_{ht} = s_{ht} = \eta$.

7.2 Estimate λ

Parameter λ indicates the gradual reduction of government debt.

My first task is to find proper debt. Some of the government debt is held by the trust fund, including the ones for social security, unemployment insurance and employee pension. So the broadest measure of debt, the gross government debt, may not be the most important one, since not all of them can represent the past borrowing in the credit market.

What debt measure then is relevant in an economic sense is the debt held by the public. This is the part of the debt that has been sold on the credit market, and, that thus influences the interest rates and private investment decisions. In this sense I choose the debt held by the public between the year 1970 to 2004, and the data is taken quarterly.

As mentioned in section 3.4.3.3, I will choose the ratio of debt-to-GDP to estimate λ. Just imagine that an individual with a large income can afford higher burden. Comparably the amount of the debt may not be a good measure of the burden it places on the economy, but rather the debt relative to the overall economy may have more importance. The most common way is to express it as the percentage of the gross domestic product (GDP). So I also choose the data of US GDP from 1970's first quarter to 2004's last quarter.

The debt-to-GDP ratio is critical to financial stability. In the long run perpetual increase of debt-to-GDP ratio is an inherently unstable situation. Thus from the economical stability point of view, the government expects a λ in the interval [0, 1].

To estimate λ in $\frac{b_t}{y_t} = \lambda \frac{b_{t-1}}{y_{t-1}}$,

i.e.
$$\min_{\lambda} \sum \left(\frac{b_t}{y_t} - \lambda \frac{b_{t-1}}{y_{t-1}} \right)^2 \tag{7.7}$$

Taking derivative with respect to λ:

$$2\sum\left(\frac{b_t}{y_t}-\lambda\frac{b_{t-1}}{y_{t-1}}\right)\left(-\frac{b_{t-1}}{y_{t-1}}\right)=0 \tag{7.8}$$

So it gives:

$$\lambda=\frac{\sum_t \frac{b_t}{y_t}}{\sum_t \frac{b_{t-1}}{y_{t-1}}} \quad t = 2, 3, \ldots, 139 \tag{7.9}$$

Unluckily the data gives an estimate of 1.0082, which is slightly greater than 1. I then try an alternative way.

The data chosen gives an estimate that satisfies the tentative above. Running the MATLAB code, I get the plot of the 139 debt-to-GDP ratios, and it gives an estimate of λ of 0.99 which lies in the interval [0, 1]. There is an intercept of 0.25 in the estimation, which does not appear in the model's equation $\frac{b_t}{y_t}=\lambda\frac{b_{t-1}}{y_{t-1}}$. It can be barely omitted compared with the debt-to-GDP ratio since the ratio ranges from 5 to 35. See Figure 10.

The data is taken from FRED® (Federal Reserve Economic Data).

Data source: GDP http://research.stlouisfed.org/fred2/series/GDP/106
Debt http://research.stlouisfed.org/fred2/series/FYGFDPUN/

7.3 Conditions for indeterminacy

Proposition: The necessary and sufficient conditions for indeterminacy in a log-linearized dynamic system is given by:

$$|\det J|<1$$

$$|traceJ|<1+\det J$$

where J is the coefficient Jacobian matrix.

Proof: Let μ_1 and μ_2 be the eigenvalues of matrix J. The case of indeterminacy indicates that $|\mu_1| < 1$ and $|\mu_2| < 1$, thus we have $|\mu_1\mu_2| < 1$. Since the determination of a matrix is the product of the eigenvalues, i.e. $\det(J) = \mu_1\mu_2$, so the above analysis results in $|\det J| < 1$.

When $0 < \mu_1 < 1$ and $0 < \mu_2 < 1$, we have:

$$(1 - \mu_1)(1 - \mu_2) > 0 \tag{7.10}$$

With simple manipulation, equation (7.10) is equivalent to $\mu_1 + \mu_2 < 1 + \mu_1\mu_2$

Since the trace of a matrix is given by the sum of the eigenvalues, the above equation is the same as: $tr(J) < 1 + \det J$.

When $-1 < \mu_1 < 0$ and $-1 < \mu_2 < 0$, we have:

$$(1 + \mu_1)(1 + \mu_2) > 0 \tag{7.11}$$

Equation (7.11) result in $\mu_1 + \mu_2 > -1 - \mu_1\mu_2$, thus $tr(J) > -1 - \det J$.

For the case of $0 < \mu_1 < 1$ and $-1 < \mu_2 < 0$, either equation (7.10) or (7.11) still hold. So we have $|traceJ| < 1 + \det J$.

MATLAB Code

File 1: thesis.m

```
disp('On Fiscal Policy Feedback Rules and Indeterminacy');
disp('hit any key when ready');
pause;

% Setting parameters:
theta = .3;
beta = .962;
A = 2.876;
gama = 0;
delta = .067;
eta = 2/3;
sk_bar = eta;
sh_bar = eta;
lamda = .99;
sigma_sk = .712;
sigma_sh = .712;

% Calculating the steady state:
alpha1 = theta*(1+eta);
alpha2 = (1-theta)*(1+eta);
R_bar = 1/beta;
q_bar = beta;
r_bar = (R_bar-1+delta)/(sk_bar+1)+delta;
M =
((1+sh_bar)*(1-theta)*(1-delta)^(alpha1*(1+gama)/alpha2)*((r_bar-delta)/theta)^(1-(1+gama)/alp
ha2))/A;
k_bar = (((r_bar-delta)/theta-delta)/M)^(alpha2/((1+gama)*(alpha1-1)));
y_bar = (r_bar-delta)*k_bar/theta;
b_bar = 0.34*y_bar;
h_bar = ((r_bar-delta)*(1-delta)^(-alpha1)/theta)^(1/alpha2)*k_bar^((1-alpha1)/alpha2);
w_bar = (1-theta)*y_bar/h_bar;
c_bar = y_bar-delta*k_bar;
T_bar = sh_bar*w_bar*h_bar+sk_bar*(r_bar-delta)*k_bar+b_bar-q_bar*b_bar;
i_bar = delta*k_bar;
a = eta/(eta*theta+theta-1);
P = r_bar*k_bar-delta*k_bar+sk_bar*k_bar*(r_bar-delta)+(1-delta)*k_bar;

% Declaring the matrices.
VARNAMES = ['capital '
```

```
                'consumption '
                'debt '
                'return '
                'rent rate '
                'discount rate'
                'output '
                'wage '
                'labour '
                'lump sum tax '
                'investment '
                'sub on cap '
                'sub on labor '];

% Translating into coefficient matrices.
% for k(t) c(t)
AA = [ 0, -1
        0, 0
       0, -c_bar
       0, 0
       0, 0
       0, 0
       0, 0
       0, 0
       -k_bar, 0];

% for k(t-1) c(t-1):
BB = [ 0, 0
       0, 0
       0, 0
       -theta*y_bar/k_bar, 0
       0, 0
       alpha1, 0
       sk_bar*(r_bar-delta)*k_bar, 0
       0, 0
       k_bar*(1-delta), 0];

% for b(t) R(t) r(t) q(t) y(t) w(t) h(t) T(t) i(t)
CC = [ 0, 0, 0, 0, 0, 1, -g    ama, 0, 0
       0, -R_bar, r_bar*(1+sk_bar), 0, 0, 0,0, 0, 0
       0, 0, 0, 0, y_bar, 0,0, 0, -i_bar
       0, 0, -r_bar, 0, theta*y_bar/k_bar, 0,0, 0, 0
       0, 0, 0, 0, -1, 1,1, 0, 0
       0, 0, 0, 0, -1, 0,alpha2, 0, 0
       -q_bar*y_bar, 0, sk_bar*r_bar*k_bar, -q_bar*b_bar, 0, sh_bar*w_bar*h_bar, sh
```

```
    _bar*w_bar*h_bar, -T_bar, 0
    -1, 0, 0, 0, 0, 0,0, 0, 0
    0, 0, 0, 0, 0, 0,0, 0, i_bar];

% for sk(t) sh(t)
DD = [ 0, sh_bar/(1+sh_bar)
    sk_bar*(r_bar-delta), 0
    sk_bar*(r_bar-delta)*k_bar, sh_bar*w_bar*h_bar
    0, 0
    0, 0
    0, 0
    sk_bar*(r_bar-delta)*k_bar, sh_bar*w_bar*h_bar
    0, 0
    0, 0];

% For k(t+1) c(t+1)
FF = [0, -1
    0, -1];

% For k(t) c(t)
GG = [0, 1
    0, 1];

% For k(t-1) c(t-1)
HH = [ 0, 0
    0, 0];

% for b(t+1) R(t+1) r(t+1) q(t+1) y(t+1) w(t+1) h(t+1) T(t+1) i(t+1)
JJ = [ 0, 1, 0, 0, 0, 0, 0, 0, 0
    0, 0, 0, 0, 0, 0, 0, 0, 0 ];

% for b(t) R(t) r(t) q(t) y(t) w(t) h(t) T(t) i(t)
KK = [ 0, 0, 0, 0, 0, 0, 0, 0, 0
  0, 0, 0, -1, 0, 0, 0, 0, 0 ];

% for sk(t+1) sh(t+1)
LL = [0, 0
  0, 0];

% for sk(t) sh(t)
MM = [0, 0
    0, 0];

% AUTOREGRESSIVE MATRIX
```

```
NN =[ .9, 0
      0, 0.9];

Sigma = [sigma_sk^2, 0
         0, sigma_sh^2];

% Setting the options:
[l_equ,n_endog ] = size(CC);

% changing the above matrices due to the need to solve the model
% transform other endogenous variable to the endogenous state variable. (new declaration of the matrices)

AAnew = [AA,CC(:,1:4)];
BBnew = [BB,0*CC(:,1:4)];
CCnew = CC(:,5:n_endog);
FFnew = [FF,JJ(:,1:4)];
GGnew = [GG,KK(:,1:4)];
HHnew = [HH,0*JJ(:,1:4)];
JJnew = JJ(:,5:n_endog);
KKnew = KK(:,5:n_endog);
AA = AAnew;
BB = BBnew;
CC = CCnew;
FF = FFnew;
GG = GGnew;
HH = HHnew;
JJ = JJnew;
KK = KKnew;

[l_equ,m_states] = size(AA);
[l_equ,n_endog ] = size(CC);
[l_equ,k_exog ] = size(DD);

PERIOD = 4; % number of periods per year, i.e. 12 for monthly, 4 for quarterly
GNP_INDEX = 7; % Index of output among the variables selected for HP filter
IMP_SELECT = [1:11];% a vector containing the indices of the variables to be plotted
DO_SIMUL = 1; % Calculates simulations
SIM_LENGTH = 150;
DO_MOMENTS = 1; % Calculates moments based on frequency-domain methods
HP_SELECT = 1:(m_states+n_endog+k_exog); % Selecting the variables for the HP Filter calcs.
% Starting the calculations:
do_it;
% end of the file
```

File 2: sk_beta.m

% this is the code to investigate the global indeterminacy with β and s_k, codes are similar for η and γ with s_k

```
disp('On Fiscal Policy Feedback Rules and Indeterminacy');
disp('calculating the stability properties near the steady state');

% Setting parameters:
eta = 2/3;
theta = 0.3;
delta = .067;
gama = 0;

% declaring four matrices for processing
det1 = zeros(101,2);
det2 = zeros(101,2);
trdet1 = zeros(101,2);
trdet2 = zeros(101,2);
m = 1;
n = 1;
x = 1;
y = 1;

% Following code are based on the expression obtained for the necessary and
% sufficient conditions for indeterminacy
for beta = 0.9:0.001:1
ro = 1/beta-1;
alpha1 = theta*(1 + eta);
alpha2 = (1 - theta)*(1 + eta);
skh = (1/beta-1+delta)*(1 + gama)*(1 + eta)/((1 + gama)*delta + alpha2*(1 - delta)*(1 - beta)) - 1;
skh2 = -(ro+delta)*(1+gama)*(1+eta)/((1+gama-alpha2)*(2-delta)+beta*alpha2*(ro+delta))-1;
skf1 = (1/beta-1 + delta)*(1 + gama)*(2*alpha1 + beta*(1/beta - 1 + delta)*(1 - alpha1))/theta;
skf2 = (4-2*delta)*(alpha2-1-gama) + beta*(1/beta-1+delta)*(delta*(1 + gama)*(1-alpha1) -
2*alpha2);
skf = skf1/skf2 - 1;
skf3 =
beta*(1+gama)*(ro+delta)^2*((1+eta)*alpha2-((1+gama)*(alpha1-1)+alpha2)/theta+alpha2);
skf4 = beta*(ro+delta)*(1+gama-alpha2)*delta*(1+gama)*(alpha1-1);
skf22 = skf3/skf4-1;

% only save the capital income subsidy rate with the abosulte values
% smaller than one
```

```
if (abs(skh)<1)
det1(m,2) = beta;
det1(m,1) = skh;
m = m + 1;
end
if (abs(skh2)<1)
det2(x,2) = beta;
det2(x,1) = skh2;
x = x + 1;
end
if (abs(skf)<1)
trdet1(n,2) = beta;
trdet1(n,1) = skf;
n = n + 1;
end
if (abs(skf22)<1)
trdet2(y,2) = beta;
trdet2(y,1) = skf22;
y = y + 1;
end
end

det1new = det1(1:m-1,:);
det2new = det2(1:x-1,:);
trdet1new = trdet1(1:n-1,:);
trdet2new = trdet2(1:y-1,:);

% plot the graph showing the stability properties of the model near the steady state
plot(det1new(:,1),det1new(:,2),det2new(:,1),det2new(:,2),trdet1new(:,1),trdet1new(:,2),trdet2new(
:,1),trdet2new(:,2));
xlabel('Subsidy Rate on Capital Income Sk')
ylabel('Discount Rate beta')
title('Stability Properties Near Steady State of the model')

% end of the file
```

File 3: Estimate λ

```
% read data
A = xlsread('D:\Thesis\data\d-GDP ratio.xls');

% calculate debt-to-GDP ratio
```

```
GDP = A(:,2);
debt = A(:,3);
ratio = debt./GDP;
M = mean(ratio)    % mean of the ratio
t = (1:1:139);
x = ratio(t);
y = ratio(t+1);

% regression of ratio(t) = lamda*ratio(t-1)
plot(x,y,'*')
xlabel('ratio(t)')
ylabel('ratio(t+1)')
title('Estimate parameter of lamda')

% alternative way of using minimum squared error
%lamda = sum(y)/sum(x)
%plot(ratio,'*')
```

References:

Azariadis C. (1981). Self-fulfilling Prophecies, Journal of Economic Theory, 25, 380-369.

Azariadis C. (1993). Intertemporal Macroeconomics. Cambridge, MA: Blackwell Publishers.

Basu S. And J.G. Fernald. (1997). “Returns to Scale in US Production: Estimates and Implications”, Journal of Political Economy, 105, 249-283.

Benhabib, J. and R.E.A. Farmer (1994). “Indeterminacy and Increasing Returns”. Journal of Economic Theory, 63, 19-41.

Benhabib, J. and R.E.A. Farmer (1996). “Indeterminacy ans Sector-Specific Externalities”, Journal of Monetary Economics, 37, 421-443.

Blanchard O. And C. Kahn (1980). “The Solution of Linear Difference Models under Rational Expectations”, Econometrica, 48, 1305-1311.

Caballero R.J. and Lyons R.K. (1989). “The Role of External Economies in US Manufacturing”, mimeo, Columbia University, New York.

Cass D. And K. Shell (1983). Do Sunspots Matter? Journal of Political Economy, 91, 193-227.

Chiappori P.A., P.Y. Geoffard and R. Guesnerie (1992). “Sunspot Fluctuations Around a Steady State: The Case of Multidimensional, One-Step Forward Looking Economic Models”, Econometrica, Vol. 60, No. 5, 1097-1126.

Chrisitiano L.J. and S.G. Harrison (1996). Chaos, Sunspots, and Automatic Stabilizers. NBER Working Paper 5703

Cole H.L. and T.J. Kehoe (1998). “Self-Fulfilling Debt Crises” Review of Economic Studies, Blackwell Publishing, vol. 67(1), pages 91-116.

Davila J. (1997). “Sunspot Fluctuations in Dynamics with Predetermined Variables”. Economic Theory, 10, 483-495

Evans G.W. and B. McGough (2004). “Indeterminacy and the Stability Puzzle in Non-Convex Economies”. http://econpapers.repec.org/paper/oreuoecwp/2002-14.htm

Farmer R.E.A. Macroeconomics of Self-fulfilling Prophecies 2nd.edition, Cambridge, MIT Press, 1999

Farmer R.E.A and Guo J.T (1994). "Real Business Cycles and the Animal Spirits Hypothesis", Journal of Economic Theory, 63, 42-73

Georges C. (1995). "Adjustment Costs and Indeterminacy in Perfect Foresight Models", Journal of Economic Dynamics and Control, 19, 39-50.

Guo J.T. and K.J. Lansing (2001). "Fiscal Policy, Increasing Returns and Endogenous Fluctuations", Macroeconomic Dynamics, vol. 6(5), pages 633-64.

Juster T.F. and F.P. Stafford (1991). "The Allocation of Time: Empirical Findings, Behavioral Models and Problems of Measurement". Journal of Economic Literature, 29, 471-522

McCallum B.T. (1983). "On Non-unique in Rational Expectations Models: An Attempt at Perspective", Journal of Monetary Economics, 11, 139-168.

Mitra T. (1998). "On the Relationship Between Discounting and Complicated Behavior in Dynamic Optimization Models", Journal of Economic Behavior and Organization, 33, 421-434.

Poterba J.M. (1997). "The Rate of Return to Corporate Capital and Factor Shares: New Estimates Using Revised National Income Accounts and Capital Stock Data". NBER Working Paper 6263

Railavo J (2004). "Stability Consequence of Fiscal Policy Rules". http://www.bof.fi/eng/6_julkaisut/6.1_SPn_julkaisut/6.1.5_Keskustelualoitteita/0401.pdf

Schmitt-Grohe S. and M. Uribe (1997). Balanced-Budget Rules, Distortionary Taxes, and Aggregate Instability. Journal of Political Economy, 105, 976-1000.

Sorger G. (1994). "On the Structure of Ramsey Equilibrium: Cycles, Indeterminacy, and Sunspots", Economic Theory, Springer, vol. 4(5), pages 745-64.

Uhlig H. (1999). "A Toolkit for Analysing Nonlinear Dynamic Stochastic Models Easily". in Ramon Marimon and Andrew Scott, eds, Computational Methods for the Study of Dynamic Economies, Oxford University Press, Oxford (1999), 30-61.

Woodford M. (1988). Expectations, Finance, and Aggregate Instability. In M. Kohn and S.C. Tsiang, editors, Finance Constraints, Expectations, and Macroeconomics.

Woodford M. (1996). "Control for the Public Debt: A Requirement for Price Stability". NBER Working Paper 5684

Figure:

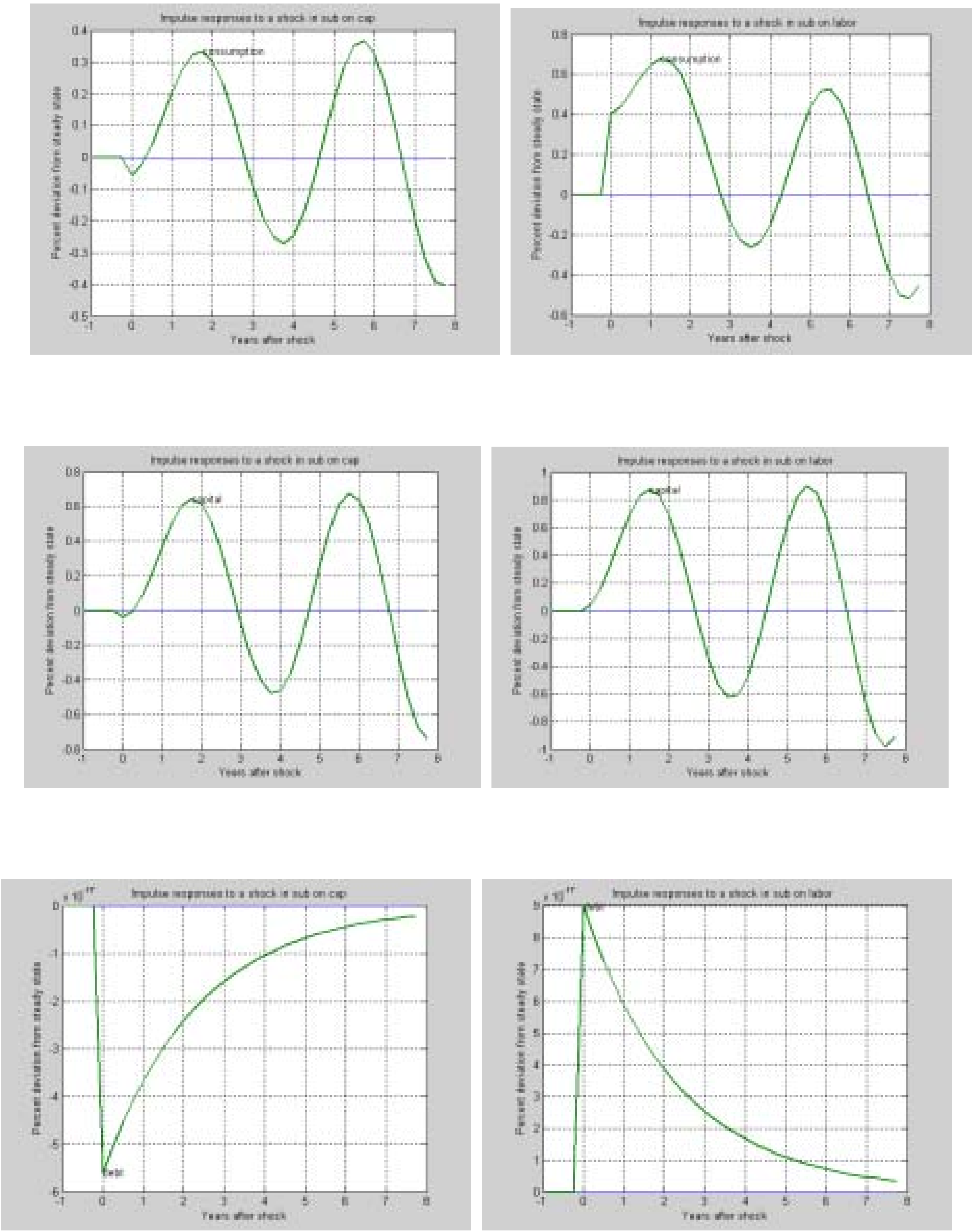


Figure 1: Impulse response of consumption (upper), capital (middle) and debt (down) to exogenous shocks (shocks on subsidy of capital income and labor income)

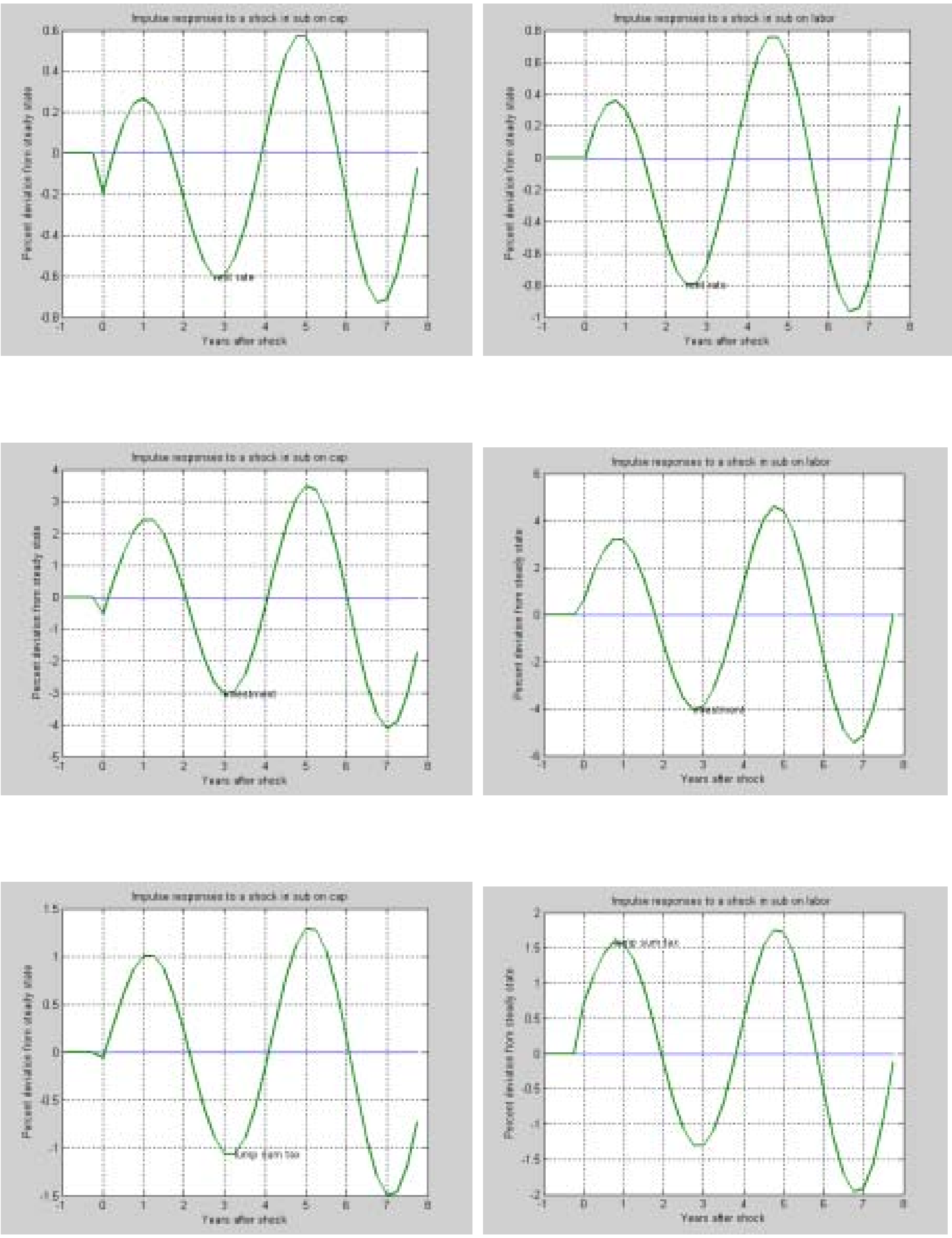


Figure 2: Impulse response of interest rate (upper), investment (middle) and lump sum tax (down) to exogenous shocks

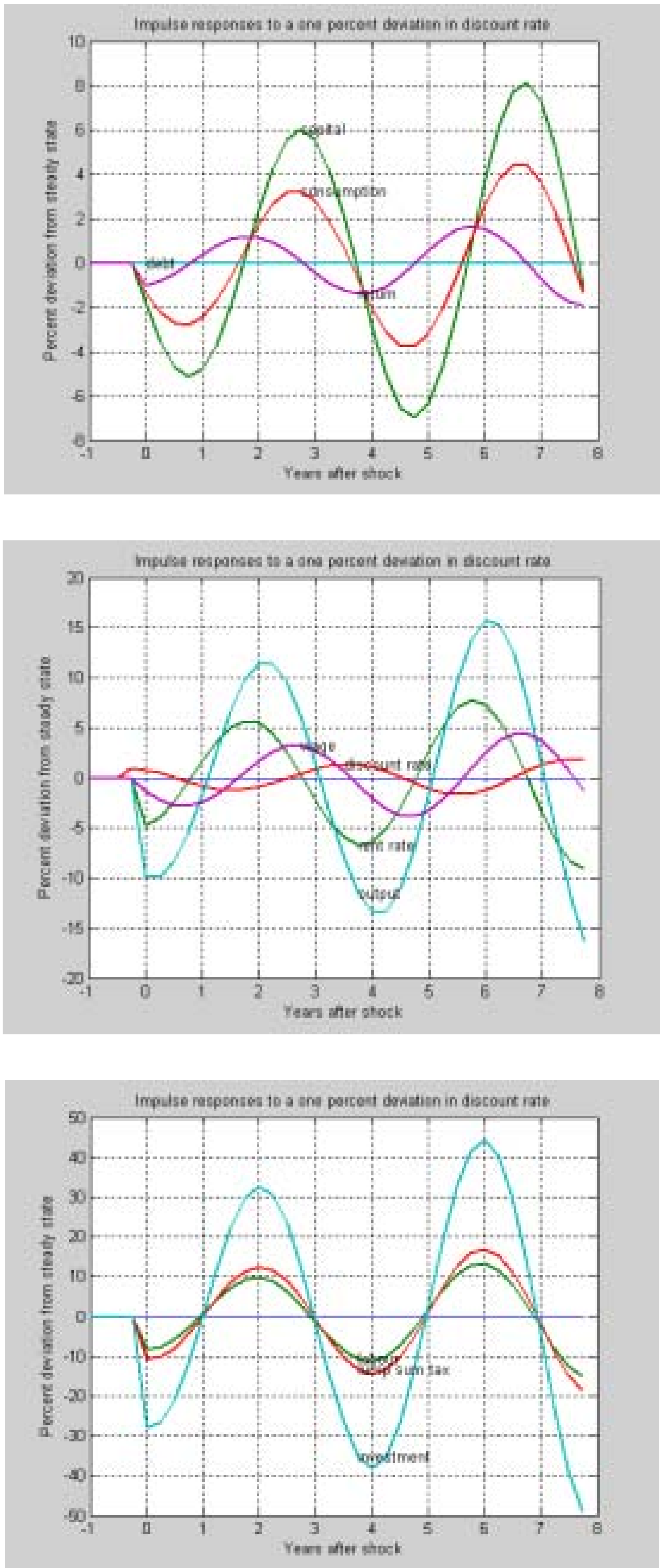
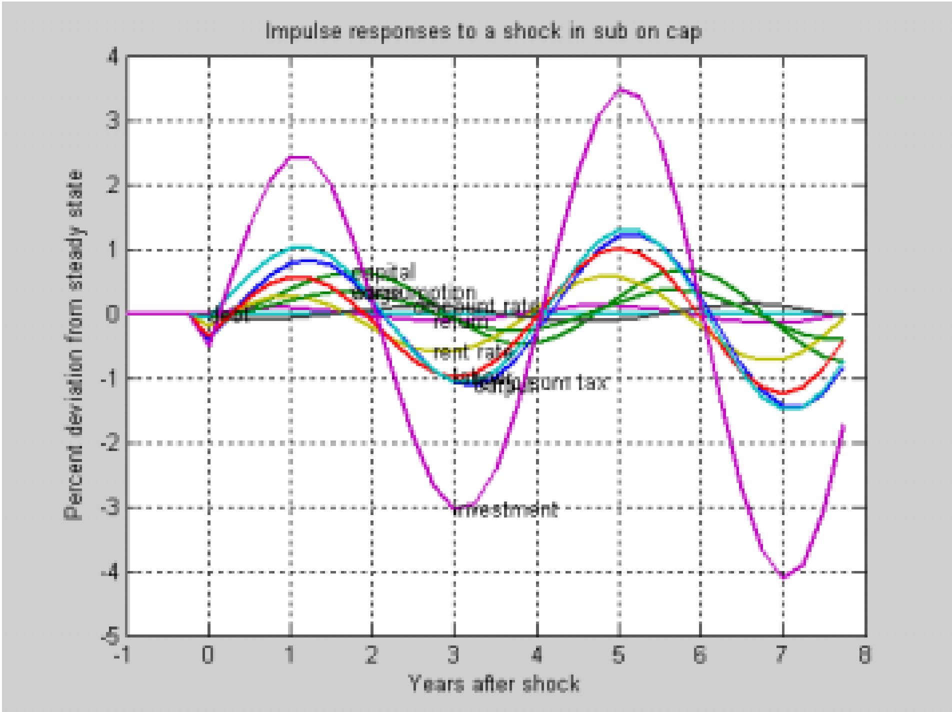
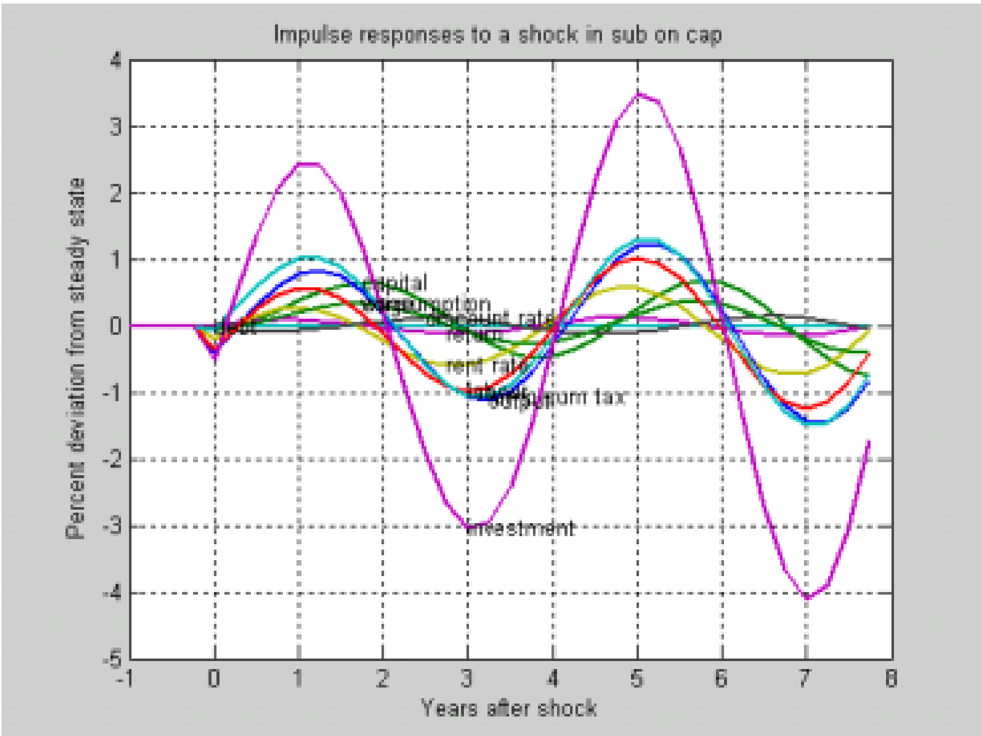


Figure 3: Impulse response to one percent deviation in discount rate

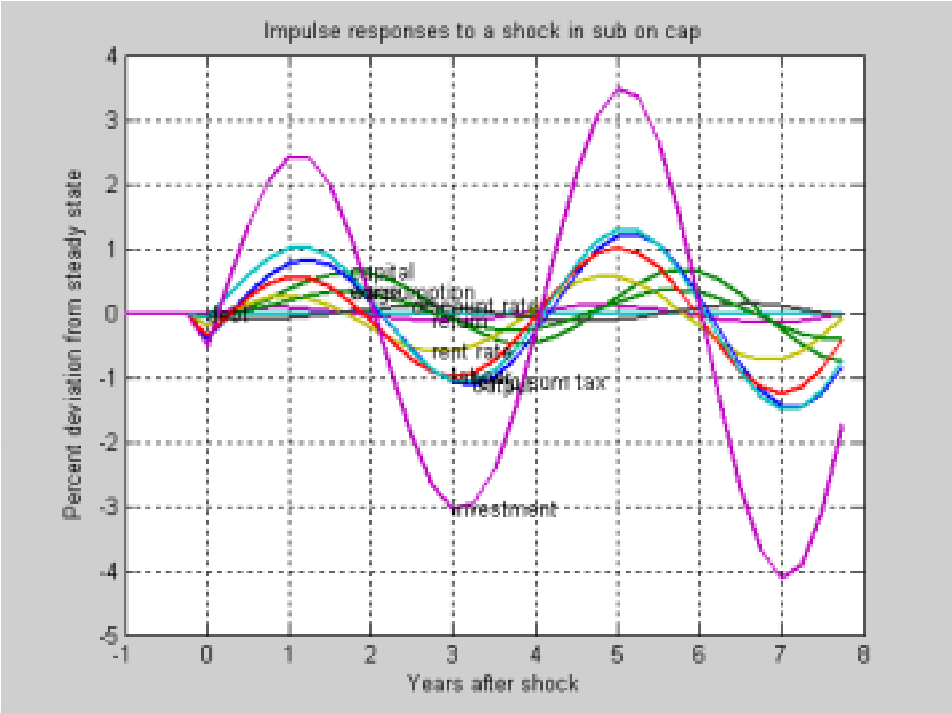


λ=0.99 (upper)

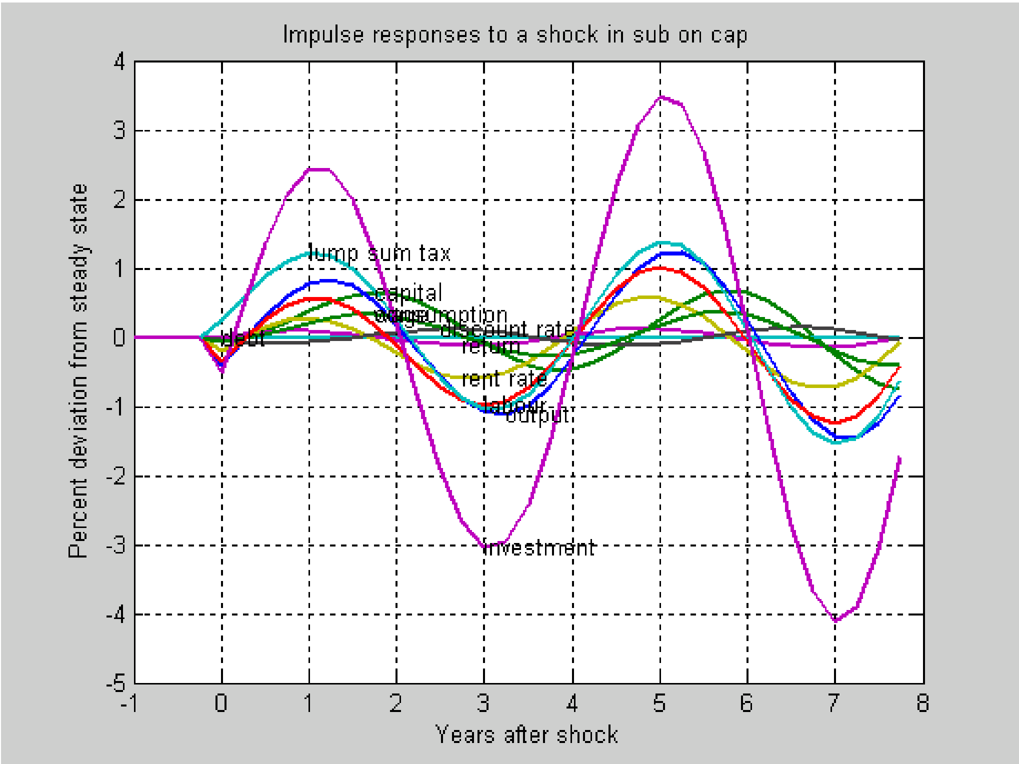


λ=0.1(down)

Figure 4: Stability with varying λ

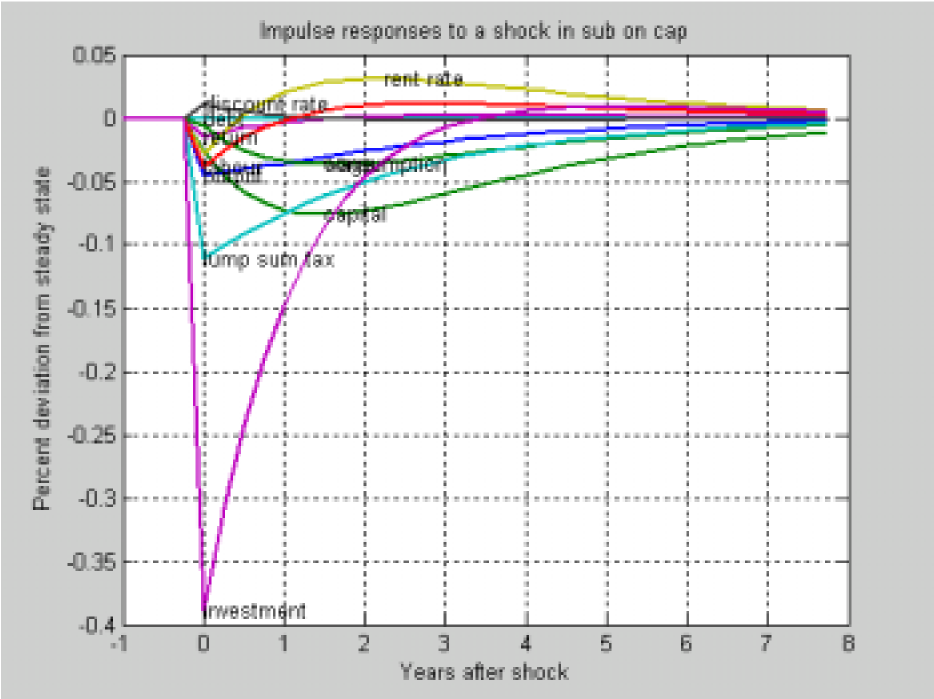


$s_h = 2/3$



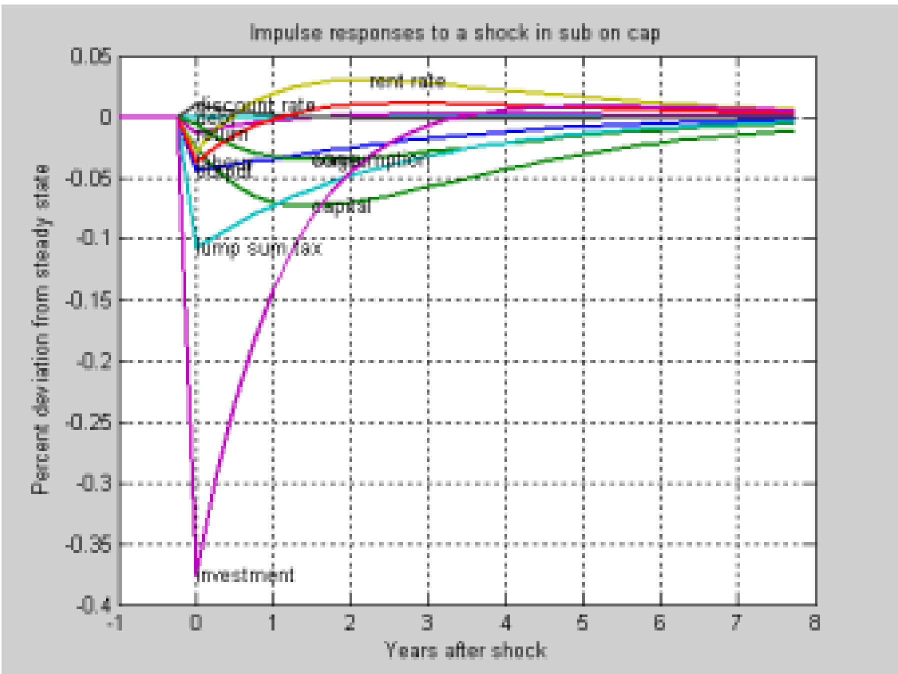
$s_h = 0.2$

Figure 5: stability with varying $\bar{s}_h$



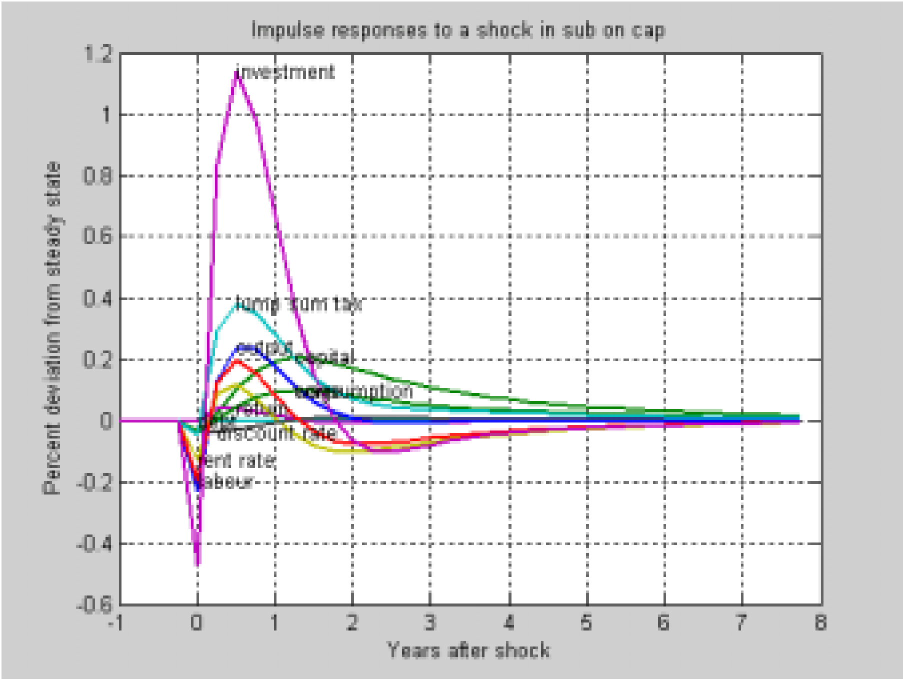
$s_k = -0.09$, saddle,

Two roots are 0.8258 and 1.0162



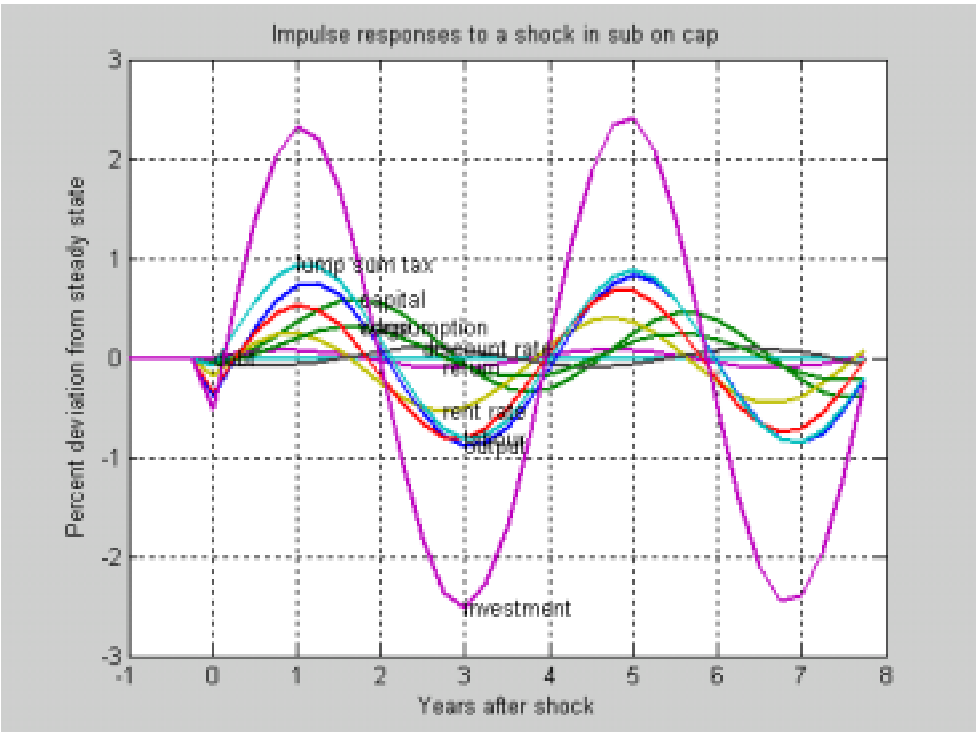
$s_k = -0.0869$, saddle changes to sink

Two roots are 0.8251 and 1



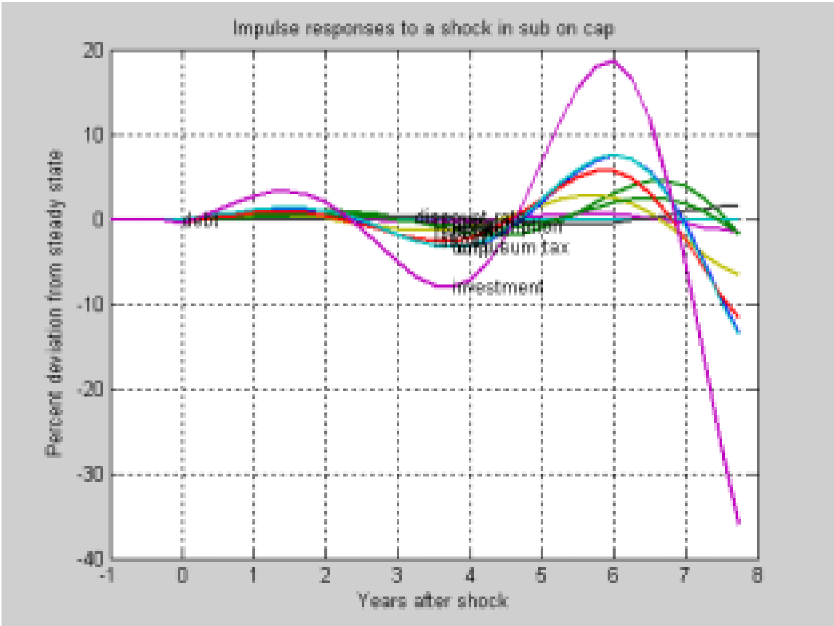
$s_k = 0.3$, sink

Two roots are 0.5891 + 0.2348i and 0.5891 - 0.2348i, with absolute value of 0.6342



$s_k = 0.638$, sink changes to source

Two roots are 0.9186 + 0.3951*i* and 0.9186 - 0.3951*i* with absolute value of 1



$s_k = 0.8$, source

Two roots are $1.0327 + 0.3748i$ and $1.0327 - 0.3748i$ with absolute value of 1.0986

Figure 6: Comparison of local stability properties

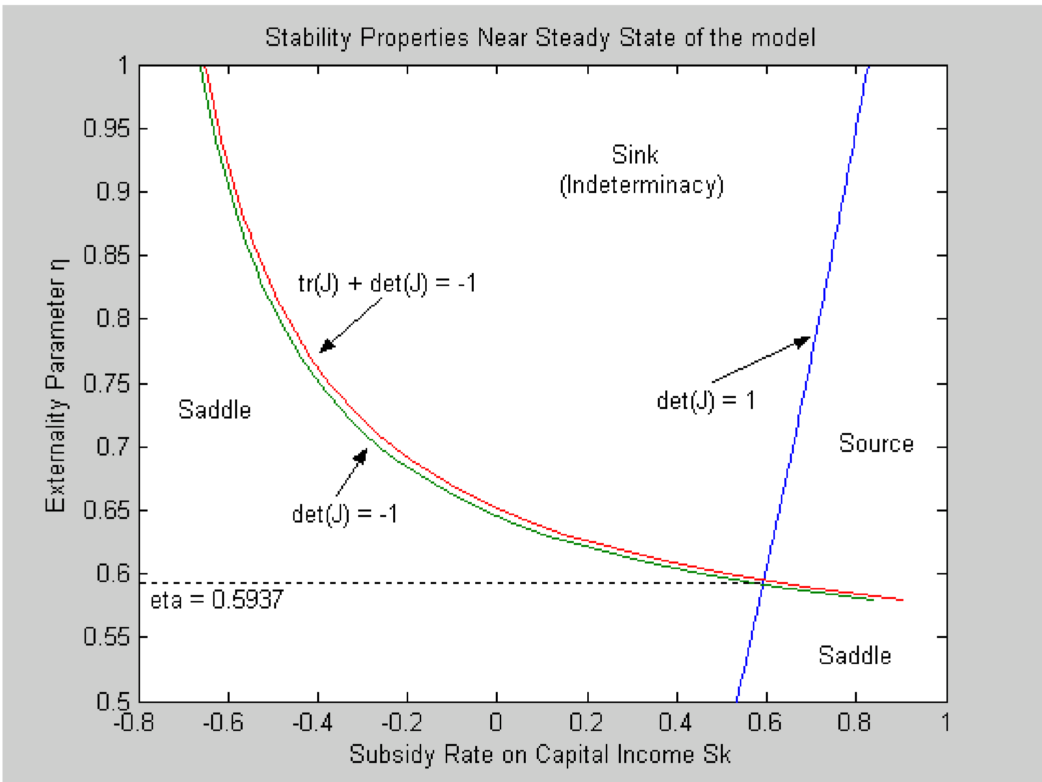


Figure 7: Stability property near steady state with changing η and s_k

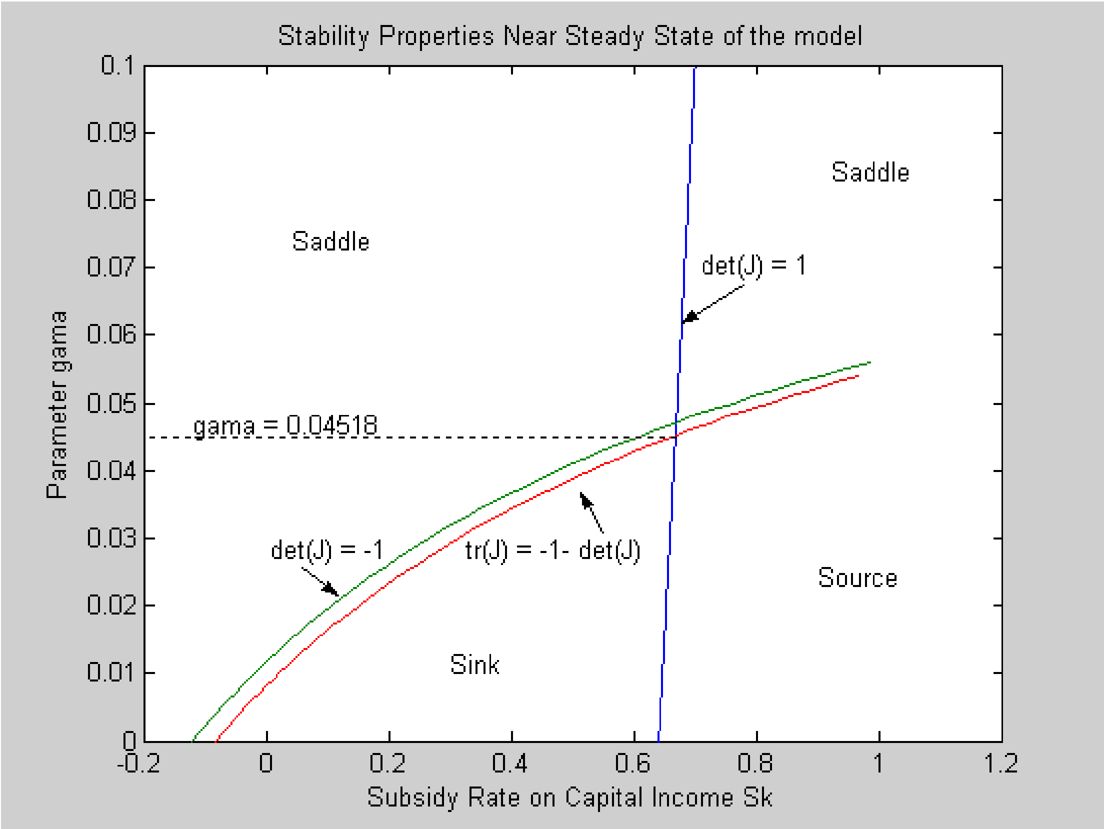


Figure 8: Stability property near steady state with γ and s_k

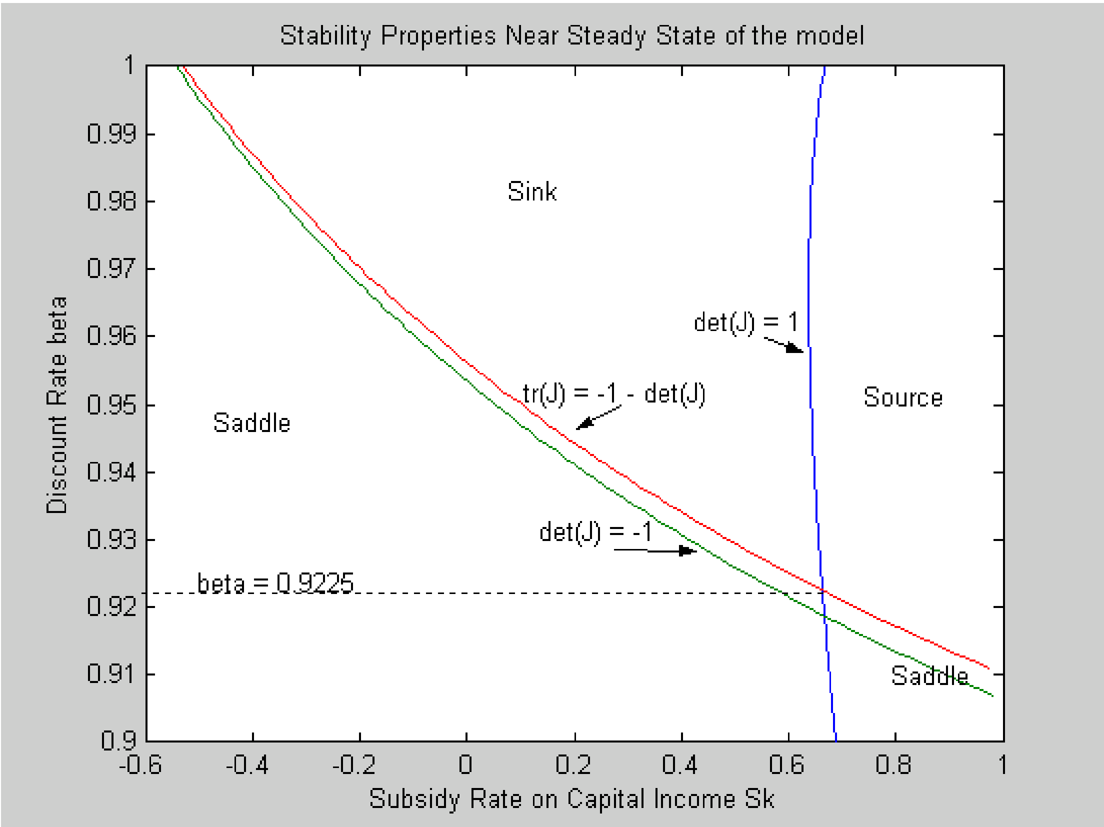


Figure 9: Stability property near steady state with β and s_k

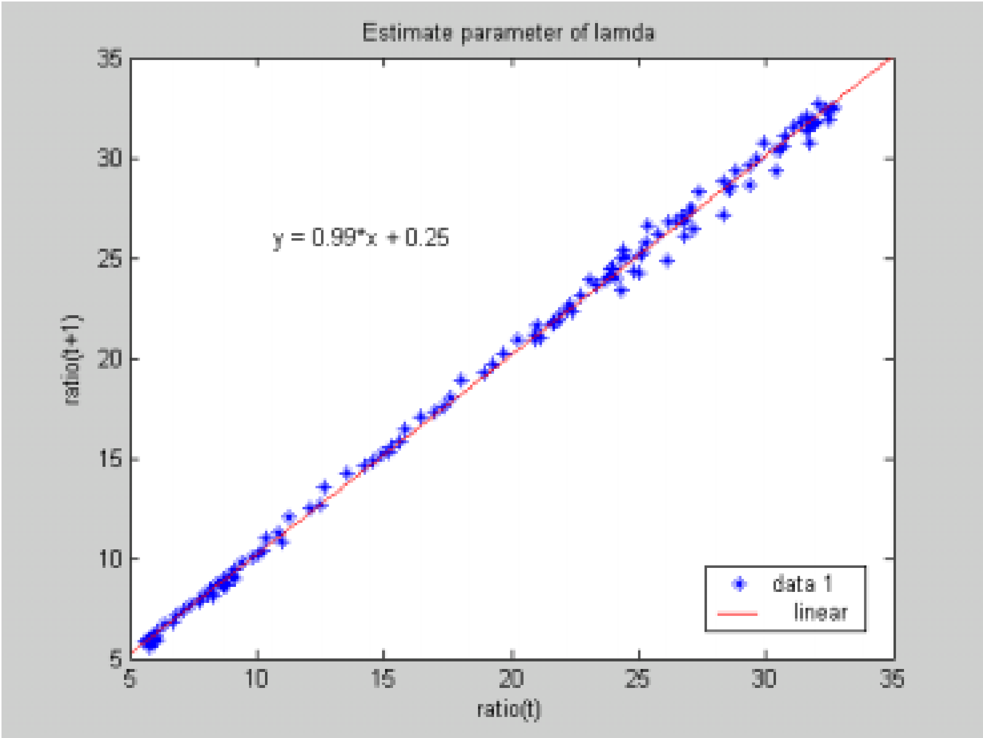


Figure 10: Linear regression on debt-to-GDP ratio – estimating parameter λ

Declaration

I hereby declare that I prepared this thesis independently. The only sources and materials used are those indicated.

Berlin, 19 August 2005

Jiao Jie