The Inflow of Eastern European Workers to the German Labor Market: Consequences and Policy Issues

Master Thesis

written by

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Abstract

In this paper the framework for analyzing the impact of the immigration on the labor market (e.g. unemployment, wages) and main economic variables as total output, output per capita etc. is developed. I will apply this model to German economy and draw the outcomes and conclusions. In order to check whether model works properly, I will calibrate it also for the UK economy and compare the results with known data.

The model predicts that the prospective immigration to Germany due to the enlargement of the European Union will reduce slightly the welfare of high- and low-skilled labor in the short-run, while in the long-run high-skilled agents will be a little bit better off and low-skilled households will be slightly worse off comparing to the situation before the immigration. The model presupposes the negligible increase of unemployment rate of both types of agents and wages of high-skilled labor, but wages of unskilled labor is forecasted tenuously to fall. Output per capita declines due to the unskilled workers dominant immigration. Total output in the short-run decreases, but in the long-run exceeds its initial steady state value, since the total employment increases.

Key words: immigration, skill differences, CES production function, divisible labor.

Contents

1. Introduction **8**

2. Literature **10**

3. Facts and Theoretical Considerations **14**

4. The Model **19**

4.1. Households . . . 20

4.1.1. High-Skilled Labor . . . 21

4.1.2. Low-Skilled Labor . . . 23

4.2. Firms . . . 24

4.3. Government . . . 25

4.4. Dynamic of Employment . . . 26

5. Model Analysis **27**

5.1. First Order Necessary Conditions . . . 27

5.2. Equations, characterizing Equilibrium . . . 29

5.3. Steady State Values . . . 31

5.4. Log-linearized Equations . . . 33

5.5. Calibration of the Model . . . 35

6. Model Results and Answer **37**

6.1. Short-run effects and adjustments . . . 39

6.2. Permanent and long-term effects . . . 45

7. Variations **49**

8. Discussion **58**

9. Summary and Concluding Remarks **60**

References **62**

A. Utility Function **65**

B. The Case: $\rho = 1$ **66**

C. Other Scenarios **67**

D. Calibration for the UK Economy **69**

E. Matlab Codes **71**

E.1. Benchmark and 1st scenario 71

E.2. Case when $\rho = 1$ 81

E.3. 2nd and 3rd scenarios in Variations of the Model 91

E.4. UK economy 101

List of Tables

1. The expected number of immigrants to the UK before the May 1st, 2004. 15
2. The nationalities of the applicants of Worker Registration Scheme. *Source: Home Office (UK) and UK Immigration News (2005).* 15
3. The expected **total** number of immigrants to Germany. *Source: Leibfritz, O'Brien and Dumont (2003).* 16
4. The correlation table between GDP, immigration and labor market. Annual data of 1987/1992 - 2001/04. *Source of data: OECD.* Author's calculations. 17
5. The cross correlation table with (1) and other variables ($corr((1)_{t+j}, y_t)$). Annual data of 1987/1992 - 2001/04. *Source of data: OECD.* Author's calculations. 18
6. The cross correlation table with (2) and other variables ($corr((2)_{t+j}, y_t)$). Annual data of 1987/1992 - 2001/04. *Source of data: OECD.* Author's calculations. 18
7. The cross correlation table with (3) and other variables ($corr((3)_{t+j}, y_t)$). Annual data of 1987/1992 - 2001/04. *Source of data: OECD.* Author's calculations. 18
8. The calibration of the model for German economy: values of parameters. 35
9. The calibration of the model for German economy: the steady state values of the variables (which are known). 36
10. The selected steady state values. 37
11. The values of the impulse responses for the GDP of the UK (in per cent). 37
12. The short-run effects on output, labor income tax and unemployment (the changes are in per cent from steady state). 39
13. The short-run effects on the capital market (the changes are in per cent from steady state). 40
14. The short-run effects on number of workers and time worked (the changes are in per cent from steady state). 41
15. The short-run effects on consumption and wages (changes are in per cent from steady state). 42
16. The permanent effects of immigration to the labor market (the changes are in per cent from steady state). 45

17. The long-run effects of the immigration on capital market (the changes are in per cent from steady state). 45
18. The long-run effects on output, labor income tax and unemployment (the changes are in per cent from steady state). 46
19. The long-run effects on number of workers and time worked (the changes are in per cent from steady state). 47
20. The long-run effects on the consumption and wages (the changes are in per cent from steady state). 47
21. Cross correlation Table (HP-filtered series, moments based), corr(v(t+j),GDP per capita(t)). Last row shows j. 48
22. The impact of parameter ρ to the main variables (in per cent). 49
23. The impact of parameter η on the main variables (in per cent). 51
24. The impact of the stochastic process on the main variables in the short-run (in per cent). 54
25. The impact of the stochastic process on the main variables in the long-run (in per cent). 54
26. The impact of parameter r on total output and labor income tax rate at time t=0 (in per cent). 57
27. The impact of parameter r on total output and labor income tax rate at time t=10 (in per cent). 57
28. The impact (in per cent) of parameter r on consumption, wages and working hours in t=0 in the benchmark case. 58
29. Other values of the parameter ρ. The value of 0.91 is taken by the author not from Kemnitz (2003). It corresponds to 1.1 elasticity of substitution between low- and high-skilled labor. 67
30. Other values of the parameter η. The value of 0 is taken by the author. Other values (0.08 and 0.12) are from Pohl (2005). 67
31. The calibration of the model for the UK economy: values of parameters. 69
32. The calibration of the model for UK economy: the steady state values of the variables (which are known). 69
33. The selected steady state values. 70

List of Figures

1. The impulse responses to a shock in immigrants for the UK. 38
2. The impulse responses of output, labor income tax and unemployment due to shock in labor supply. 39
3. The impulse responses of capital and its related variables. 40
4. The impulse responses of numbers of hours worked (per worker and total). 41
5. The impulse responses of wages and consumption of high- and low-skilled labor. 43
6. The impulse responses of total labor in case of the first scenario. 53
7. The impulse responses of total labor in case of the second scenario. . . . 53
8. The impulse responses of total labor in case of the third scenario. 53
9. The impulse responses for the high-skilled agents in case of 2nd (left) and 3rd (right) scenarios. 56
10. The impulse responses for the low-skilled agents in case of 2nd (left) and 3rd (right) scenarios. 56
11. Other impulse responses in case of 2nd (left) and 3rd (right) scenarios. . 56

1. Introduction

"*Fears have often been voiced that immigration would cause a rise in unemployment among the native population*" (Leibfritz, O'Brien, Dumont, 2003).

The immigration was always a "hot" topic among the politicians and the economists. After the enlargement of the European Union (EU) on the 1^{st} of May in 2004 this issue became more actual. It was expected that millions of workers from Eastern and Central Europe would surge the old members of the EU. These countries were worrying about their labor market and the balance of the state budget, so the transition period of up to 7 years (according to $2+3+2$ formula) was set for the liberalization their labor market. Some countries, Denmark, Sweden, Ireland, the Netherlands and the United Kingdom, didn't set any restriction for the workers from the new states of the EU. The exiting qualms about the immigrants originated since the effects of the migration within the EU on economy, labor market of the host country were almost unidentified (Bowen, Wu, 2004).

In this paper I will create the framework for analyzing the impact of the immigration on labor market and main economic variables as GDP, GDP per capita etc., also I will apply this model to German economy and draw the outcomes and conclusions. In order to check whether model works properly, I will calibrate it also for UK economy and compare the results with real (known and available) data.

In order to reach this aim, I will set up the appropriate model, then following the procedure documented in Uhlig (1999) which is apposite to be used in solving dynamic stochastic general equilibrium models in discrete time framework, I will obtain the impulse responses of various economic indicators in per capita and aggregate terms to the shock of labor supply due to the immigration[1]. The latter procedure consists of several steps and leads to the calculation of impulse responses, simulations and second moments. First of all, one gathers the equations that define the equilibrium of the model including constraints, identities, first order conditions, exogenous processes and other necessary equations. Second, I will solve for the steady state, i.e. I will provide the formulas for all variables. In the third step I will obtain the log-linearized equations and finally using

[1] I won't analyze the impact of 1 % deviations of the state variables because it is beyond the scope of my topic (actually labor in the model is one of the state variable which is affected by the shock, so only the deviations of that variable will be considered in manner of *impulse responses to a shock in immigrants*).

the method of undetermined coefficients and *the Toolkit*[2] I will get impulse responses and second moments.

I got that the developed model pretty well replicates total GDP growth for the economy of the United Kingdom, however due to the scarcity of the newest trends of the unemployment and consumption among the different skill groups in the UK, I could not ascertain whether the results I achieved are also adequate to the impact of the immigration from the new European Union countries on these economic variables. The model predicts that the plausible immigration to Germany will reduce slightly the welfare[3] of high- and low-skilled agents in the short-run, while in the long-run high-skilled agents will be a little bit better off and low-skilled labor will be slightly worse off comparing to the situation before the immigration. The model presupposes the negligible increase of the unemployment rate of both types of agents and wages of the high-skilled labor, but wages of the unskilled labor are forecasted tenuously to fall.

The results I derived are corresponding to ones which might be found in the existing literature regarding the impact of the immigration to the host country (see Leibfritz, O'Brien, Dumont, 2003). The contribution of the thesis to this research area is twofold. First, the model of this paper can be applied to the economies of other countries in order to extract the impact of the immigration[4] (or other sudden increase of the population, or even of the emigration). Second, using the Matlab code and *the Toolkit* it is easy to get the desirable results as the impulse responses, simulations or second moments.

The paper is organized as follow. Section 2 lists, discusses and relates to the relevant existing literature. In Section 3 I will provide the key facts and essential theoretical aspects which my model tries to capture. Section 4 describes the model. There I will introduce, precisely define, motivate and explain my model. In Section 5 I will analyze the model. In Section 6, I present the results of the model and the intuition for the achieved insights. Section 7 tests the different variation and the assumption of the model. In Section 8 I will summarize my answer, my arguments and the intuition behind it, based on the analysis provided. Section 9 concludes.

[2] More about *Toolkit* see Uhlig (1999).

[3] The welfare is measured by the weighted sum of the consumption and leisure.

[4] One needs to change the calibration and to select the desirable stochastic process.

2. Literature

In this section I will review the literature about the effects of the immigration and will introduce to the models and the main ideas of some authors. Because there are only several papers which analyzed the influence of the immigration to the labor market of Germany in the existing literature, hence I also will analyze the papers which models are related to the topic.

Probably the first migration which was analyzed was 1630-1640 in New England. According to Kochin (1996) the sudden end of immigration caused the Fall of the Cow, i.e. the first depression in the history of America. The author maintains that the waves of migrants cause the demand of the investment goods, such as the houses and cattle. If the immigration suddenly stops, the demand of the investment goods dramatically decreases. Consequently the economy becomes depressed. During the Fall of the Cow the price of the cow fell down more than four times [5]. In addition Kochin (1996) provides other examples like Mandatory Palestine in 1920s, New Zealand in 1870s, Canada in 1940s and concludes that the immigration does not increase the unemployment but causes the boom of the economy. If so, the old EU member should not be afraid about the workers from the new EU members. Probably, it is not going to work for the EU, and the concerns of German, Belgium etc. governments are real, because the new workers (or immigrants) won't boom the economy, but they will compete with the native labor force in the labor market: so the unemployment rate will increase and wages will tend to decrease.

Leibfritz, O'Brien, Dumont (2003) suggest that the migration changes both the size of the labor force and its skill composition, hence there are likely to be changes in both average wages and in the wage structure, with possible consequences for inequality. For instance, if immigration is predominantly low skilled compared with the existing population, wages of unskilled workers can be expected to fall, but there may be a resulting increased demand for skilled labor; their wages would tend to rise. These predictions correspond to the results I achieved. Leibfritz, O'Brien, Dumont (2003) provide a broad overview of the main studies on immigration, unemployment and wages in the different countries. Most of authors found that immigration slightly decreased wages and increased unemployment. On the other hand Leibfritz, O'Brien, Dumont (2003) argue that according to the results of detailed empirical studies "it is impossible

[5] Actually it was the reason why that depression was called the Fall of the Cowl.

to establish a systematic relationship between immigration and unemployment".

The model which incorporates the immigration and overlapping generation (OLG) model was created by Storesletten (2000). The paper investigates whether the immigration policy of the USA might be used alone to keep the government budget balanced keeping the tax rates and government expenditure constant. The author assumes that the government can select which immigrants to accept according to their age, education, family status. Storesletten (2000) found that the government should change its immigration rules and to accept annually 0.62 % of total population (or about 1.6 million) new immigrants which are high-skilled and 40-44 years old. The author assumes that the natives and immigrants are not identical, their labor productivity and fertility are different. In my model I assumed that the immigrants are identical to natives because according to the statistical data in whole European Union the fertility rate is almost the same (the differences are small comparing to the difference between the fertility rates in the USA and Mexico). I also assumed that the productivity of the immigrants and natives with the same education is equal, because the productivity depends only on the education, but not on the nationality. The model used by Storesletten (2000) is not suitable in my case because the EU members cannot select which immigrants to accept according to their age, education or work experience, the flow of the immigrant would not be so high (during one year the immigration to Great Britain, which opened its labor market to the workers from the new members of the EU, was only 0.4 % of total labor force, and the immigrants were almost low-skilled). Because of these reasons the model of this paper should not be the OLG model.

Merz (1997) developed a microeconomic structure decentralizing a generalized social planner's version of Mortesen and Pissarides (MP) model with heterogeneous job-matches and persistent unemployment. The model is specific because it incorporates the unemployment rate and the hours worked by the household representative. The analyzed model is not totally suitable for my problem because Merz (1997) assumed that the total labor force was constant over the time and the labor was not divided into the high-skilled and low-skilled. From Merz (1995, 1997) I "took" the dynamic of employment.

Manacorda and Petrongolo (1999) provide the analysis of the unemployment and wage differentials by skill of some country of OECD. The authors accent the high unemployment rate in Germany especially among the unskilled people, the high ratio of skilled people and the whole population and the constant ratio of the wages of skilled and unskilled workers (≈ 1.4)[6]. In the model two-input Cobb-Douglas production function is used. The inputs are low-skilled labor and high-skilled labor. The model does not fit for my thesis since I use three-input production function: the high-skilled labor, the low-skilled labor and the capital. The paper concludes that the skill mismatch is able to explain the current structural disturbances in the labor market.

Kemnitz (2003) developed the paper which explores the effects of high and low skilled immigration to a host country (Germany) with unionized low skilled labor and an unemployment insurance scheme. In the model two-input (high- and low-skilled labor) CES production function is used. The author emphasizes the importance of the elasticity of substitution between high- and low-skilled labor, while I found that the impact of this parameter is not so crucial. Kemnitz (2003) argued that according to the empirical studies the elasticity of substitution between high- and low-skilled labor exceeds unity[7], and then inputs are close substitutes, hence low skilled immigration leads to a more than proportional increase in low skilled employment, while high skilled immigration reduces low skilled employment. However, when inputs are close complements, high skilled immigration promotes low skilled employment and low skilled immigration increases the low skilled unemployment rate. To obtain the results Kemnitz (2003) used the static model (the method was *the total differential*), while I found that the impact of the immigration is long-lasting and it might be different in the short- and long-run. So the dynamic programming provides better and more trustful results.

The model which explains the rising total unemployment rate, the unemployment among low-skilled labor and wage rigidities in Europe was generated by Moreno-Galbis and Sneessens (2004). The main idea of the paper is that the high-skilled labor in the labor market also compete with low-skilled labor for low-skilled jobs. This notion doesn't work in Germany because of the high reservation wage. According to Siebert (2004) the reservation wage in Germany is 120 % of the previous salary among the unemployed and it does not decline when the duration of being unemployed increases. So the high-skilled unemployed household would not accept low-skilled or low-paid job.

[6]The authors define skilled and unskilled labor quite differently than I did, so in my model this ratio is also different.

[7]But Kemnitz (2003) did not provide any value of the elasticity of substitution between high- and low-skilled labor. Bassanini, Rasmussen and Scarpetta (1999) and Iregui (1999) used 0.9 as the elasticity of substitution between high- and low-skilled labor (I will use this value as the benchmark in my paper) for calibrating their model for German, British, Swedish and the USA economies. In the model's variations I will consider about the cases when the elasticity of substitution between high- and low-skilled labor is equal to 1 (Wapler, 2001, Moreno-Galbis and Sneessens, 2004) and bigger than 1 (e.g. 1.1) as Kemnitz (2003) suggests.

Moreno-Galbis and Sneessens (2004) use three different kind of shocks (the increase of the ratio between high-skilled labor and total labor, the aggregate technological shock, the embodied technological shock) in order to test the properties of their model. The authors conclude that their model is able to reproduce the situation in Belgium over the last 30 years.

Brücker and Kohlhaas (2004) analyzed the impact of migration on the wages and employment. The analysis is undertaken with a dynamic multi-regional CGE model of Germany and the rest of EU15. The LEAN model was employed[8]. The authors conclude that a higher share of low-skilled workers could cause higher unemployment and besides reduce the average productivity and GDP as it corresponds to my findings.

Canova and Ravn (2000) replicated the macroeconomic effects of German unification. They assumed that the unification for Germany implied a sudden increase of 26 % in low-skilled labor. The authors did not analyze the dynamic of the unemployment but focused on the consumption and the welfare analysis. The results they got (also as the results of Kemnitz (2003)) strongly depend on the assumption of the elasticity of substitution between high- and low-skilled labor. Since the latter model is quite well congenial for the German labor market, thus its part is engaged in my thesis.

The studies of the impact of the immigration on the labor market in the USA suggests that increases in the new immigrant share of workers within high and medium skill jobs actually have slightly positive wage effects, suggesting there may be complementarities between native workers and newly arrived immigrants in the top skill categories. The increases in the new immigrant share of unskilled workers have slightly negative wage effects (Orrenius, Zavodny, 2003), meaning that the native workers and newcomers can be substituted in the labor market. Since it is expected that mainly unskilled workers will arrive to the old members of the EU, I considered only about the second case and obtained adequate results.

To conclude, the main effect of immigration is distributional. According to Arackal (2000) the skilled immigrants now make up only 7 per cent of the total immigrant flow. So when unskilled labor increases, the wage offered to them declines. The welfare of low-skilled agents decreases. But since labor costs account for 70 per cent of production costs on average, firms profit from the flow of low skilled immigrants. The aggregate effect on national income as well as GDP depends on which of the above results dominate.

[8] Its structure was not provided in the paper.

3. Facts and Theoretical Considerations

Primarily let's answer to the question why the immigration exists, i.e. why do the host countries accept the immigrants and why do people migrate? Coleman and Rowthorn (2004) answer to the first part of this question and provide several reasons. First, the most general pro-immigration case derives from the old mercantilist assumption that larger populations are better than smaller ones and that population growth is therefore welcome. Thus the immigration is the external growth of the population. Second, the immigration can solve the problems of PAYG pension system in the declining or aging populations. Third, the large-scale immigration especially from the poor countries with low wages and expectations concerning conditions of work will be needed to fill "dirty" jobs that are difficult to mechanize and that the domestic labor force will not undertake. The answer to second part of that question is very simple, according to Leibfritz, O'Brien, Dumont (2003), the potential migrants attempt to increase (or maximize) their utility over their remaining lifetime.

Differently from Canova and Ravn (2000) in the model I won't assume that the skill composition of labor after the immigration dominated by the unskilled workers converges to the initial steady state. Following the recent literature (Kemnitz, 2003, Brücker and Kohlhaas, 2004) I will analyze the impact of the proposed immigration presuming that the composition of skills is exogenously given and constant (it might be changed only by the immigration if the skill composition of newcomers differs from one of the existing population).

For many decades the main problem of German economy was high unemployment rate which even did not decrease during economic booms. Very big part of the whole unemployment is the structural unemployment (Steiner, 2004). More than 36.5 % of all jobless are the long-term unemployed (in April 2005)[9]. The reason for the big structural unemployment are the high wage levels and especially too little wage differentiation between skill groups, high reservation wage, strict regulation of the labor market and that the big part of the total jobless consists of the low-skilled labor (Steiner, 2004). This implies persistent unemployment thus employing the dynamic of employment from Merz (1997) is the sound argument for replicating German economy in the model. The labor market conditions signify that the immigration might have negative impact to the labor market especially among unskilled people.

[9] Author's calculation according to the data of Federal Statistical Office Germany.

Between 1970 and 2000, employment in the US rose by 30 percent, from 78 million to 136 million, compared to 3 % in Germany, from 27 million to 36 million (adjusted for unification). The standard explanation for slow employment growth in Germany is an over-regulated labor market (Oezcan, 2004), thus in the further analysis I will assume that the labor force in Germany can grow only due to the immigration.

The research shows that "there is no strong evidence of wage discrimination against immigrants, but there may be discriminatory forces in the employment process" (Zimmermann, 2001), thus it is consistent to assume that in the model's economy the immigrants get the same wage if employed.

The impact of the immigrants to Germany's economy should be similar to the experience of UK. The fact that it is difficult to predict the amount of the immigrants shows the experience of the United Kingdom. Before the May 1^{st}, 2004 there were several research made to forecast the **annual** scale of immigration from the new EU countries (see Table 1).

Source	Number of immigrants
Management Issues (2004b)	54,000
UK government (*Source: Management Issues, 2004*)	5,000 - 13,000
Migration Watch UK (*Source: CNN, 2004*)	40,000

Table 1: The expected number of immigrants to the UK before the May 1st, 2004.

But actually after the enlargement of the European Union more than 175 thousand people (mostly from Poland, Lithuania and Slovakia, see Table 2 below) came to work to UK (so-called Accession Eight (A8) migrants) during first 11 months. That number was more than 10 bigger than British government anticipated.

Polish	Lithuanian	Slovakian	Czech	Latvian	Hungarian	Estonian	Others
56 %	15 %	11 %	7 %	7 %	3 %	2 %	< 1 %

Table 2: The nationalities of the applicants of Worker Registration Scheme. *Source: Home Office (UK) and UK Immigration News (2005).*

Over half of those immigrants worked in administration, business and management, or hospitality. The number of A8 migrants applying for tax-funded income-related benefits, child benefit, tax credits and housing support remained "very low", the Home Office said (UK Immigration News, 2005).

"The expansion of the EU's borders led to a surge in both legal and illegal immigration. According to shadow home secretary David Davis, not all workers registered and coupled with official statistics showing that between May and August, 2004 542,000 visited the UK, an increase of 222,000 from last year" (Management Issues, 2004), and it is only during 3 months. Thus during 12 months the number might be fourfold bigger.

According to Tony McNulty, Secretary of State for Home Affairs (according to other sources, Immigration Minister), the immigrants created the value added for 500 million pounds (with reference to the third quarterly Accession Monitoring Report), it is just 0.043 % of GDP of 2004. More than 82 % of all new workers are 18-34 years old. At least 96% were working full-time and up to a third may have already been in the UK before expansion. Approximately 1200 immigrants applied for the the unemployment relief or pension but only 24 applications were were approved for further consideration, so it means that the concerns that the immigrants would come in order to get the benefits were wrong, i.e. they came to work. According to Mr. T. McNulty, the ratio of the new workers and total labor force is just 0.4 %, so their impact to the placement, unemployment and wages is not significant.

We can also conclude that almost all the immigrants were occupied by the low-skilled jobs, because they, being 0.4 % of total labor force, created just 0.043 % of GDP.

How many migrants are expected to come to Germany? According to some researches the number of the immigrants should vary 4-10 % (see Table 3) of an existing German labor force during 15 years (Leibfritz, O'Brien, Dumont, 2003).

Source	Number of immigrants
IFO	5.7-7.3 mln.
DIW	2.9 - 3.7 mln.

Table 3: The expected **total** number of immigrants to Germany. *Source: Leibfritz, O'Brien and Dumont (2003).*

According to IFO over a fifteen-year period after free accession, between 5.7-7.3 million immigrants from the new members of the EU will come to Germany. It corresponds to a 8.25-10.32 % of the total employed persons in 2004. DIW announced other predictions which are twice smaller than ones of IFO (Leibfritz, O'Brien, Dumont, 2003). Despite the UK is more superior among the immigrants than Germany[10], new immigrants are only 0.4 % of the total labor force in the UK, thus I value these predictions (see Table 3) very critically. As a benchmark I will assume that the number of immigrants which

[10]In the UK the unemployment rate is lower and the GDP growth is bigger than in Germany, also it is easier to find a job there.

will come to Germany will be only 0.4 % of German labor force in 2004.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
(1)	1								
(2)	**0.79**	1							
(3)	**0.73**	0.36	1						
(4)	**0.79**	0.45	0.36	1					
(5)	**0.79**	0.48	0.45	**0.97**	1				
(6)	-0.60	-0.47	-0.27	-0.52	-0.31	1			
(7)	**0.68**	0.51	0.33	0.34	0.51	**-0.70**	1		
(8)	0.14	0.22	-0.04	-0.17	-0.01	**-0.69**	**0.77**	1	
(9)	**-0.76**	-0.57	-0.42	-0.43	-0.59	0.70	**-0.96**	-0.59	1

Where (as well as in Table 5, Table 6 and Table 7):
(1) - the annual change (in per cent) of the inflows of foreign population into Germany;
(2) - the annual change (in per cent) of the inflows of foreign workers into Germany;
(3) - the annual change (in per cent) of the net migration per 1 000 population in Germany;
(4) - the annual change (in per cent) of the GDP (expenditure approach) in constant prices;
(5) - the annual change (in per cent) of the GDP per capita (in the national currency);
(6) - the annual change (in per cent) of the total population in Germany;
(7) - the annual change (in per cent) of the total employment in Germany ;
(8) - the annual change (in per cent) of the economically active population in Germany;
(9) - the annual change (in per cent) of the unemployment rate in Germany.

Table 4: The correlation table between GDP, immigration and labor market. Annual data of 1987/1992 - 2001/04. *Source of data: OECD.* Author's calculations.

In the correlation table (see Table 4) we can see that the inflows of the immigrants are positively correlated with the GDP (in both aggregate and per capita terms), the total employment in Germany and negatively with the unemployment rate. The highly correlated/uncorrelated values are highlighted (in **bold** style). It might imply two things. First, that these numbers are just coincidences, because there is a clear tendency of the rise immigration throughout the Western world (Cohen-Goldner, Paserman, 2004), also in the previous decades we observed the increasing GDP and unemployment not only in Germany, but in most countries of the EU. It suggests that the immigrants may prefer to come to that country which economy is growing. If so, then they increase the number of total workers and unemployment rate. Or second, that the dynamic of the total employment in Germany could depend on the immigration, while the immigrants having smaller bargaining power agree to work for a smaller salary, thus it is the reason why they are employed. The unemployment rate increases since maybe not all immigrants could find a job or some of the native workers were "crowded out" by the newcomers.

(1)	t-5	t-4	t-3	t-2	t-1	t	t+1	t+2	t+3	t+4	t+5
(1)	**-0.65**	**0.72**	**0.63**	0.04	0.23	1	0.23	0.04	**0.63**	**0.72**	**-0.65**
(4)	-0.45	-0.23	-0.25	-0.43	-0.16	**0.79**	0.07	**-0.65**	-0.41	-0.24	-0.35
(5)	-0.46	-0.22	-0.20	-0.39	-0.12	**0.79**	0.29	-0.53	-0.29	-0.10	-0.29
(7)	-0.14	-0.44	-0.12	-0.11	0.27	**0.68**	**0.82**	0.38	0.29	**0.89**	**0.61**
(9)	0.21	0.29	0.00	0.26	-0.05	**-0.76**	**-0.74**	-0.14	-0.29	**-0.89**	-0.35

Table 5: The cross correlation table with (1) and other variables ($corr((1)_{t+j}, y_t)$). Annual data of 1987/1992 - 2001/04. *Source of data: OECD.* Author's calculations.

(2)	t-5	t-4	t-3	t-2	t-1	t	t+1	t+2	t+3	t+4	t+5
(2)	0.28	**0.79**	-0.18	0.17	0.00	1	0.00	0.17	-0.18	**0.79**	0.28
(4)	-0.52	0.22	0.07	-0.30	-0.27	0.45	0.18	**-0.71**	-0.49	-0.32	0.13
(5)	-0.55	0.20	0.14	-0.25	-0.21	0.48	0.38	**-0.60**	-0.41	-0.26	0.17
(7)	-0.11	-0.07	0.27	-0.07	0.12	0.51	**0.72**	0.43	-0.12	0.40	**0.60**
(9)	0.25	-0.06	-0.41	0.21	0.13	-0.57	**-0.71**	-0.32	0.17	-0.25	**-0.62**

Table 6: The cross correlation table with (2) and other variables ($corr((2)_{t+j}, y_t)$). Annual data of 1987/1992 - 2001/04. *Source of data: OECD.* Author's calculations.

(3)	t-5	t-4	t-3	t-2	t-1	t	t+1	t+2	t+3	t+4	t+5
(3)	-0.16	0.43	-0.13	0.00	-0.20	1	-0.20	0.00	-0.13	0.43	-0.16
(4)	0.19	-0.37	-0.48	-0.08	-0.14	0.36	-0.02	-0.33	-0.21	0.00	-0.20
(5)	0.19	-0.33	-0.46	-0.05	-0.12	0.45	0.11	-0.29	-0.17	0.09	-0.13
(7)	0.14	-0.47	-0.32	-0.05	0.39	0.33	0.50	-0.02	0.00	0.33	0.43
(9)	-0.11	0.39	0.28	0.06	-0.26	-0.42	-0.47	0.18	0.05	-0.53	-0.34

Table 7: The cross correlation table with (3) and other variables ($corr((3)_{t+j}, y_t)$). Annual data of 1987/1992 - 2001/04. *Source of data: OECD.* Author's calculations.

The increase of employment boosts the economy... Due to the scarcity of data we are not able to answer to this question. In the most plausible case the both discussed effects are involved. The further analysis will show that the immigration will increase the total employment (or the total number of workers) and unemployment rate, however suppress total output (during the first year).

In Tables 5, 6 and 7 (see the page 18) I provide the cross correlations between the immigrants and some economic variables. These tables don't imply any comprehensible relationship between the immigration and the proposed economic indicators: e.g. the values of correlations between the the inflows of foreign population into Germany (or (1)) and economic variables are almost always higher than ones between the inflows of foreign workers into Germany and economic variables, especially correlation with GDP or GDP per capita. It might be explained from another aspect, that the immigrants (the workers with whole families) tend to come to the host country which economy is booming, while it would be easier for them to find jobs, support the family etc.. Tables 5 and 6 imply that the immigration occurs 3-4 year later after the initial immigration, like first some members of family immigrate to Germany and 3-4 later their relatives come also. First two table in page 18 supports the proposition that the immigration increase the unemployment rate, but increase the total employment (in units) even one period later (what corresponds to the model of this paper). But Table 7 does not provide any evidences that support these statements. All three tables in page 18 doesn't support the statement that the immigration increases/decreases the total output, output per capita or the total employment in the next periods (correlations are negative, but they are smaller than even 0.50 in absolute value). On the other hand, we should remember that the small value of correlation means that there is no *linear* dependence between two variables, but it does not mean that these variables are independent.

4. The Model

The model of the paper is mainly based on the model of Canova and Ravn (2000) and Merz (1997). From the first paper I took the houeholds' utility function and the production function, from the second paper I took the equation of the dynamic of employment. The model was developed such that it would evidently disclose the impact of immigrants on the labor market (e.g. unemployment, wages, consumption etc.) and on whole economy (output, capital stock, taxes etc.).

4.1. Households

The stylized economy is populated by the large number of identical consumer-workers households, each of which will live and grow forever and each with identical preferences (utility function). There are in the economy two types of labor: high-skilled and low-skilled. High-skilled people work, consume and own all capital of the economy. So the low skilled households just work and consume [11].

The types were determined according to OECD specification and Bassanini, Rasmussen, Scarpetta (1999). The low-skilled individuals are those who attained below upper secondary, upper secondary and post secondary education level. The high-skilled agents have achieved the tertiary education level.

I assumed that the unemployment relief is paid only to German unemployed [12]. All newcomers are employed, otherwise they would come back to their origin country, because the German government will not pay any unemployment relief to them, so in their home country they could get a job or even unemployment benefits instead of being without any income in Germany. The unemployed native households receive the unemployment relief which is the product of the steady state salary after income tax and *the out-of-work benefit (replacement ratio)* r[13]. For the simplicity I assumed that the immigrants have the same preferences and the utility function as the native people. Both types of households have the same utility function with two inputs: the consumption and the leisure:

for high-skilled labor: $logc_t^h + A \ln l_t^h$,

for low-skilled labor: $logc_t^l + A \ln l_t^l$.

A is the preference parameter, and it is the positive number, c_t^h and c_t^l is the consumption of each high- and low-skilled agent correspondingly.

[11] The weekly net income of the low-skilled employee which is single, has no children and receives 7 euros per hour exceeds her/his claims on the welfare benefits by only about 60 euros assuming that there are no costs of working (Bonin, Kempe, Schneider, n. d.).

[12] If we look at Great Britain experience we will see that during one year (from May 1, 2004 till May 1, 2005 more than 175 thousand immigrants came from the eastern and central Europe to Great Britain, but only 24 of them got some social benefits. So almost all the immigrants came to work, but not to ask for benefits.

[13] I assumed that the unemployment relief depends on the steady state salary in the market, but in Germany it is related to the last salary the unemployed people received. This assumption won't change the results significantly due to the wage rigidities.

The total time endowment is normalized to one with agents being able to divide up their available time between hours spent working (h_t^h and h_t^l for high- and low-skilled agents correspondingly) and enjoying leisure (l_t^h and l_t^l).

Each household belongs to a very large extended family which contains a continuum of members. For simplicity I assumed that the family has either low- or high-skilled members. The members of the family are maximizing the utility of whole family, but not their own utility. Members in each family perfectly insure each other against variations in labor income due to employment or unemployment (Merz, 1995). So consumption of employed and unemployed members of the family is the same (for more details see Appendix A). In this case the utility function transforms into:

for high-skilled labor: $logc_t^h + n_t^h A \ln l_t^h$,

for low-skilled labor: $logc_t^l + n_t^l A \ln l_t^l$,

where n_t^h and n_t^l are the employment rates of the high- and low-skilled labor.

The total number of the households at time t is denoted as L_t. Assume that there is no endogenous growth of labor. The labor can only be increased due to the immigration. The dynamic of labor:

$$L_t = L_{t-1} + i_t, \tag{1}$$

$$\text{where } \; i_t = \epsilon_t, \tag{2}$$

when i_t is the number of immigrants at time t and $\epsilon_t \sim N(0, \sigma^2)$.

4.1.1. High-Skilled Labor

The number of high-skilled labor is denoted as L_t^h at time t. The dynamic of the number of high-skilled labor is:

$$L_t^h = L_{t-1}^h + \eta i_t. \tag{3}$$

η denotes the share of high-skilled immigrants among all immigrants. The ratio between high-skilled agents and whole population is:

$$q_t^h = \frac{L_t^h}{L_t}. \tag{4}$$

The number of employed high-skilled people is N_t^h, so there are $L_t^h - N_t^h$ unemployed high-skilled people in the model's economy at time t. The employment ratio among the high-skilled labor is:

$$n_t^h = \frac{N_t^h}{L_t^h}. \tag{5}$$

Thus the unemployment rate is:

$$u_t^h = 1 - n_t^h. \tag{6}$$

The budget constraint of all high-skilled labor is:

$$\underbrace{(1+\tau^c)C_t^h + X_t}_{expenditure} = \underbrace{(1-\tau^k)K_{t-1}d_t + (1-\tau_t^l)N_t^h w_t^h h_t^h + r(1-\bar{\tau}^l)(L_t^h - N_t^h)\bar{w}^h \bar{h}^h}_{income}.$$

C_t^h denotes total consumption of the high-skilled people. K_{t-1} and X_t are total capital stock at time t-1 and total investment at time t. h_t^h denotes the working time of high-skilled labor. τ^c, τ_t^l, τ^k are consumption tax (e.g. value added tax), labor income tax and capital tax on dividends. d_t is the dividend rate at time t, w_t^h is wage of high-skilled labor at time t. In order to express the budget constraint per each high-skilled household we need to set:

$$c_t^h = \frac{C_t^h}{L_t^h}, \quad x_t = \frac{X_t}{L_t}, \quad k_t = \frac{K_t}{L_t},$$

where c_t^h is consumption of each high-skilled household, x_t and k_t is investment and capital stock per capita (including all households). Then the budget constraint of each high-skilled household is:

$$(1+\tau^c)c_t^h + \frac{x_t}{q_t^h} = (1-\tau_t^l)h_t^h w_t^h n_t^h + r(1-\bar{\tau}^l)\bar{h}^h \bar{w}^h u_t^h + (1-\tau^k)\frac{k_{t-1}}{q_t^h}\frac{L_{t-1}}{L_t}d_t, \tag{7}$$

The capital accumulation equation is:

$$K_t = (1-\delta)K_{t-1} + X_t,$$

where δ is the annual depreciation rate. In per capita terms this equation is:

$$k_t = (1-\delta)k_{t-1}\frac{L_{t-1}}{L_t} + x_t. \tag{8}$$

High-skilled household representative has such a maximization problem:

$$\begin{aligned}
&\max_{\{c_t^h, h_t^h, k_t\}} E_0 \sum_{t=1}^{\infty} \beta_t (log c_t^h + n_t^h A \ln l_t^h) \\
\text{s.t. } l_t^h + h_t^h &= 1, \\
(1+\tau^c)c_t^h + \frac{x_t}{q_t^h} &= (1-\tau_t^l)h_t^h w_t^h n_t^h + r(1-\bar{\tau}^l)\bar{h}^h \bar{w}^h u_t^h + (1-\tau^k)\frac{k_{t-1}}{q_t^h}\frac{L_{t-1}}{L_t} d_t, \\
k_t &= (1-\delta)k_{t-1}\frac{L_{t-1}}{L_t} + x_t.
\end{aligned}$$

A is the preference parameter, and it is the positive number.

4.1.2. Low-Skilled Labor

The number of low-skilled labor is denoted as L_t^l at time t. The dynamic of the number of low-skilled labor is:

$$L_t^l = L_{t-1}^l + (1-\eta)i_t. \tag{9}$$

The ratio between low-skilled agents and whole population is:

$$q_t^l = \frac{L_t^l}{L_t}. \tag{10}$$

The number of employed low-skilled people is N_t^l, so there are $L_t^l - N_t^l$ unemployed low-skilled people in the model's economy at time t. The employment ratio among the low-skilled labor is:

$$n_t^l = \frac{N_t^l}{L_t^l}. \tag{11}$$

Thus the unemployment rate is:

$$u_t^l = 1 - n_t^l. \tag{12}$$

The budget constraint of all low-skilled labor is:

$$\underbrace{(1+\tau^c)C_t^l}_{expenditure} = \underbrace{(1-\tau_t^l)N_t^l w_t^l h_t^l + r(1-\bar{\tau}^l)(L_t^l - N_t^l)\bar{w}^l \bar{h}^l}_{income}.$$

C_t^l denotes total consumption of low-skilled people. w_t^l is wage of low-skilled labor at time t. h_t^l denotes the working time of low-skilled labor. In order to express the budget constraint per each low-skilled household we need to set:

$$c_t^l = \frac{C_t^l}{L_t^l},$$

where c_t^l is consumption of each low-skilled household. Then the budget constraint of each low-skilled household is:

$$(1+\tau^c)c_t^l = (1-\tau_t^l)h_t^l w_t^l n_t^l + r(1-\bar{\tau}^l)\bar{h}^l \bar{w}^l u_t^l. \tag{13}$$

The low-skilled household representative has such a maximization problem:

$$\max_{\{c_t^l, h_t^l\}} E_0 \sum_{t=1}^{\infty} \beta_t (\ln c_t^l + n_t^l A \ln l_t^l)$$

$$\begin{aligned} \text{s. t. } (1+\tau^c)c_t^l &= (1-\tau_t^l)h_t^l w_t^l n_t^l + r(1-\bar{\tau}^l)\bar{h}^l \bar{w}^l u_t^l, \\ l_t^l + h_t^l &= 1. \end{aligned}$$

4.2. Firms

In the model's economy there are infinitely many identical competitive firms renting factors of production (capital and labor) from the households. The aggregate (of all firms) production function is Cobb-Douglas production function with constant returns to scale to these factors. It is reasonable to use such a production function because it is the only production function where the factor income shares are independent of relative factor prices (Minneapolis Fed, n. d.)[14]. The factor labor is a constant elasticity of substitution (CES) aggregate of high-skilled hours (H_t^h) and low-skilled hours (H_t^l), the elasticity of substitution is $\frac{1}{\rho}$:

$$\begin{aligned} H_t^h &= N_t^h h_t^h, \quad (14) \\ H_t^l &= N_t^l h_t^l. \quad (15) \end{aligned}$$

The high-skilled labor is more productive than low-skilled labor, the ratio of their

[14] Empirical evidences show that the real wages in the USA has increased 10-20 times, while the rental price of capital and the factor income shares remain almost unchanged (Minneapolis Fed, n. d.).

productivities is denoted by ϕ when $\phi > 1$. The aggregate production function is:

$$Y_t = \left[\phi(H_t^h)^{1-\rho} + (H_t^l)^{1-\rho}\right]^{\frac{\alpha}{1-\rho}} K_{t-1}^{1-\alpha}. \tag{16}$$

The production function in per capita terms:

$$y_t = \left[\phi(n_t^h q_t^h h_t^h)^{1-\rho} + (n_t^l q_t^l h_t^l)^{1-\rho}\right]^{\frac{\alpha}{1-\rho}} \left(k_{t-1}\frac{L_{t-1}}{L_t}\right)^{1-\alpha}. \tag{17}$$

The maximization problem of the firms:

$$\max_{\{H_t^h, H_t^l, K_{t-1}\}} \underbrace{\left[\phi(H_t^h)^{1-\rho} + (H_t^l)^{1-\rho}\right]^{\frac{\alpha}{1-\rho}} K_{t-1}^{1-\alpha}}_{=Y_t} - w_t^h H_t^h - w_t^l H_t^l - d_t K_{t-1}.$$

4.3. Government

The government in the model's economy collects three kinds of taxes: consumption tax τ^c (e.g. value added tax), labor income tax τ_t^l and tax on dividends τ^k. The expenditure of the government is divided into two part: the governing expenses G_t (e.g. wages for the officials etc.) and the subsidies for jobless (unemployment reliefs). In my thesis I assume that the government expenditure per capita g_t are constant (e.g. bigger population needs more police, officials etc., so there are more expenses). The government changes only the personal income tax (other taxes are constant) in order to keep the budget deficit equal to zero. This case might be interesting due to the increasing pressure to the government to decrease the income tax rate.

How the government of Federal Republic of Germany can control the immigration process from Eastern European? Actually, it cannot do anything except to postpone this process for some years. The government cannot select which type of people will immigrate (high- or low-skilled). The government cannot force the immigrants from new European Union countries to pay higher taxes, because according to the EU laws the citizens of the other EU member states have the same rights as the native. The government cannot select the number of the immigrants, their skills, age, type of immigration (permanent or temporal immigration). Because of these reasons the government cannot sustain the fiscal policy through immigration. It might be done using the experience of Canada, Australia, the USA ("green cards") etc..

The government budget constraint:

$$\underbrace{\tau^c(C_t^h + C_t^l) + \tau^k K_{t-1} d_t + \tau_t^l(H_t^h w_t^h n_t^h + H_t^l w_t^l n_t^l)}_{income} = \underbrace{G_t + r[(L_t^h - N_t^h)\bar{w}^h \bar{h}^h + (L_t^l - N_t^l)\bar{w}^l \bar{h}^l](1 - \bar{\tau}^l)}_{expenditure}.$$

In per capita terms:

$$\tau^c(c_t^h q_t^h + c_t^l q_t^l) + \tau^k d_t k_{t-1} \frac{L_{t-1}}{L_t} + \tau_t^l(w_t^h h_t^h n_t^h q_t^h + w_t^l h_t^l n_t^l q_t^l) = g_t + r[\bar{w}^h \bar{h}^h u_t^h q_t^h + \bar{w}^l \bar{h}^l u_t^l q_t^l](1 - \bar{\tau}^l). \tag{18}$$

4.4. Dynamic of Employment

The equations of the dynamic of employment each period are mainly based on the model of Merz (1997). The number of the current workers depends on the job destruction rate δ_N which is the same for both types of employees and on the number of the new job-matches:

$$N_t^h = (1 - \delta_N) N_{t-1}^h + M_{t-1}^h, \tag{19}$$
$$N_t^l = (1 - \delta_N) N_{t-1}^l + M_{t-1}^l, \tag{20}$$

where M_t^h and M_t^l represents the number of new job-matches for high- and low-skilled labor that are formed at time period t-1. As in Merz (1997) the new job-matches at time t-1 follow Cobb-Douglas function that uses two inputs: vacancies posted (V_{t-1}^h and V_{t-1}^h) and total number of unemployed persons ($L_{t-1}^h - N_{t-1}^h$ and $L_{t-1}^l - N_{t-1}^l$):

$$M_{t-1}^h = (V_{t-1}^h)^{1-\lambda}(L_{t-1}^h - N_{t-1}^h)^\lambda, \tag{21}$$
$$M_{t-1}^l = (V_{t-1}^l)^{1-\lambda}(L_{t-1}^l - N_{t-1}^l)^\lambda, \tag{22}$$

where $0 \leq \lambda \leq 1$. The parameter λ might be interpreted as the households' bargaining power (Merz, 1997). I assumed that the posted vacancies are constant over the time (firms issue constant number of vacancies each period):

$$V_{t-1}^h = \bar{V}^h, \tag{23}$$
$$V_{t-1}^l = \bar{V}^l. \tag{24}$$

Consequently total number of each kind of workers at time t is represented as the process:

$$N_t^h = (1-\delta_N)N_{t-1}^h + (\bar{V}^h)^{1-\lambda}(L_{t-1}^h - N_{t-1}^h)^{\lambda}, \quad (25)$$

$$N_t^l = (1-\delta_N)N_{t-1}^l + (\bar{V}^l)^{1-\lambda}(L_{t-1}^l - N_{t-1}^l)^{\lambda}. \quad (26)$$

5. Model Analysis

In this section, I will present the necessary steps to implement the benchmark model in *Toolkit*, i.e. finding first order conditions, collecting the equations that characterize the equilibrium, solving for the steady state, log-linearization of the equations and calibration.

5.1. First Order Necessary Conditions

High-skilled household

After combining the constraints and we can set up the *Lagrangian* function for high-skilled household representative:

$$\begin{aligned} L_{\{c_t^h, h_t^h, k_t\}} &= E_0\{\sum_{t=1}^{\infty}\beta^t[\ln c_t^h + n_t^h A\ln(1-h_t^h) - \lambda_t((1+\tau^c)c_t^h + \frac{k_t}{q_t^h} - (1-\delta)\frac{k_{t-1}}{q_t^h}\frac{L_{t-1}}{L_t} - \\ &- (1-\tau_t^l)h_t^h w_t^h n_t^h - r(1-\bar{\tau}^l)\bar{h}^h\bar{w}^h u_t^h - (1-\tau^k)d_t\frac{k_{t-1}}{q_t^h}\frac{L_{t-1}}{L_t})]\}. \end{aligned}$$

FONCs:

$$\frac{\partial L}{\partial c_t^h}: \Rightarrow \frac{1}{c_t^h} = \lambda_t(1+\tau^c), \quad (27)$$

$$\frac{\partial L}{\partial h_t^h}: \Rightarrow \frac{A}{1-h_t^h} = \lambda_t(1-\tau_t^l)w_t^h, \quad (28)$$

$$\frac{\partial L}{\partial k_t}: \Rightarrow \lambda_t = \beta E_t[\lambda_{t+1}R_{t+1}], \quad (29)$$

$$\text{where } R_t = \frac{L_{t-1}^h}{L_t^h}[1-\delta+(1-\tau^k)d_t], \quad (30)$$

$$\begin{aligned}\frac{\partial L}{\partial \lambda_t}: \quad \Rightarrow \quad & (1+\tau^c)c_t^h + \frac{k_t}{q_t^h} - (1-\delta)\frac{k_{t-1}}{q_t^h}\frac{L_{t-1}}{L_t} = \\ = \quad & (1-\tau_t^l)h_t^h w_t^h n_t^h + r(1-\bar{\tau}^l)\bar{h}^h \bar{w}^h u_t^h + (1-\tau^k)d_t \frac{k_{t-1}}{q_t^h}\frac{L_{t-1}}{L_t}.\end{aligned}$$

Combining first order conditions with respect to c_t^h and k_t we get *Euler equation*:

$$\frac{1}{c_t^h} = \beta E_t \left[\frac{R_{t+1}}{c_{t+1}^h}\right]. \tag{31}$$

Combining first order conditions with respect to c_t^h and h_t^h we get:

$$\frac{A}{1-h_t^h} = \frac{(1-\tau_t^l)w_t^h}{(1+\tau^c)c_t^h}. \tag{32}$$

Low-skilled household

The *Lagrangian* function for low-skilled household representative:

$$L_{\{c_t^l, h_t^l\}} = E_0 \left\{ \sum_{t=1}^{\infty} \beta^t \left[\ln c_t^l + n_t^l A \ln(1-h_t^l) - \lambda_t \left((1+\tau^c)c_t^l - (1-\tau_t^l)h_t^l w_t^l n_t^l - r(1-\bar{\tau}^l)\bar{h}^l \bar{w}^l u_t^l \right) \right] \right\}.$$

FONCs:

$$\begin{aligned}\frac{\partial L}{\partial c_t^l}: \ \Rightarrow \frac{1}{c_t^l} \quad &= \quad \lambda_t(1+\tau^c), & (33)\\ \frac{\partial L}{\partial h_t^l}: \ \Rightarrow \frac{A}{1-h_t^l} \quad &= \quad \lambda_t(1-\tau_t^l)w_t^l, & (34)\\ \frac{\partial L}{\partial \lambda_t}: \ \Rightarrow (1+\tau^c)c_t^l \quad &= \quad (1-\tau_t^l)h_t^l w_t^l n_t^l + r(1-\bar{\tau}^l)\bar{h}^l \bar{w}^l u_t^l. & (35)\end{aligned}$$

Combining two first equations and dividing by the third equation we get:

$$\frac{A}{1-h_t^l} = \frac{(1-\tau_t^l)w_t^l}{(1+\tau^c)c_t^l}. \tag{36}$$

Firms

The maximization problem of the firms:

$$\max_{\{H_t^h,H_t^l,K_{t-1}\}} \underbrace{\left[\phi(H_t^h)^{1-\rho}+(H_t^l)^{1-\rho}\right]^{\frac{\alpha}{1-\rho}} K_{t-1}^{1-\alpha}}_{=Y_t} - w_t^h H_t^h - w_t^l H_t^l - d_t K_{t-1}.$$

$$\frac{\partial}{\partial H_t^h} : \ \Rightarrow w_t^h = \frac{\alpha\phi Y_t}{\left[\phi(H_t^h)^{1-\rho}+(H_t^l)^{1-\rho}\right](H_t^h)^{\rho}}, \tag{37}$$

$$\frac{\partial}{\partial H_t^l} : \ \Rightarrow w_t^l = \frac{\alpha Y_t}{\left[\phi(H_t^h)^{1-\rho}+(H_t^l)^{1-\rho}\right](H_t^l)^{\rho}}, \tag{38}$$

$$\frac{\partial}{\partial K_{t-1}} : \ \Rightarrow d_t = (1-\alpha)\frac{Y_t}{K_{t-1}} \quad \Rightarrow \quad d_t = (1-\alpha)\frac{y_t}{k_{t-1}}\cdot\frac{L_t}{L_{t-1}}. \tag{39}$$

For the case when $\rho = 1$ (when we have division by 0) the first order conditions as well the corresponding steady state equations and log-linearized equations are in Appendix B.

5.2. Equations, characterizing Equilibrium

From the maximization problem of high-skilled agent:

$$\frac{A}{1-h_t^h} = \frac{(1-\tau_t^l)w_t^h}{(1+\tau^c)c_t^h},$$

$$\frac{1}{c_t^h} = \beta E_t\left[\frac{R_{t+1}}{c_{t+1}^h}\right],$$

$$R_t = \frac{L_{t-1}^h}{L_t^h}\left[1-\delta+(1-\tau^k)d_t\right],$$

$$(1+\tau^c)c_t^h + \frac{x_t}{q_t^h} = (1-\tau_t^l)h_t^h w_t^h n_t^h + r(1-\bar{\tau}^l)\bar{h}^h\bar{w}^h u_t^h + (1-\tau^k)\frac{k_{t-1}}{q_t^h}\frac{L_{t-1}}{L_t}d_t,$$

$$x_t = k_t - (1-\delta)k_{t-1}\frac{L_{t-1}}{L_t}.$$

From the maximization problem of low-skilled agent:

$$\frac{A}{1-h_t^l} = \frac{(1-\tau_t^l)w_t^l}{(1+\tau^c)c_t^l},$$

$$(1+\tau^c)c_t^l = (1-\tau_t^l)h_t^l w_t^l n_t^l + r(1-\bar{\tau}^l)\bar{h}^l\bar{w}^l u_t^l.$$

From the maximization problem of the firms:

$$y_t = \left[\phi(n_t^h q_t^h h_t^h)^{1-\rho} + (n_t^l q_t^l h_t^l)^{1-\rho}\right]^{\frac{\alpha}{1-\rho}} \left(k_{t-1}\frac{L_{t-1}}{L_t}\right)^{1-\alpha},$$

$$w_t^h = \frac{\alpha\phi Y_t}{\left[\phi(H_t^h)^{1-\rho} + (H_t^l)^{1-\rho}\right](H_t^h)^{\rho}},$$

$$w_t^l = \frac{\alpha Y_t}{\left[\phi(H_t^h)^{1-\rho} + (H_t^l)^{1-\rho}\right](H_t^l)^{\rho}},$$

$$d_t = (1-\alpha)\frac{y_t}{k_{t-1}} \cdot \frac{L_t}{L_{t-1}}.$$

Government budget constraint:

$$\tau^c(c_t^h q_t^h + c_t^l q_t^l) + \tau^k d_t k_{t-1}\frac{L_{t-1}}{L_t} + \tau_t^l(w_t^h h_t^h n_t^h q_t^h + w_t^l h_t^l n_t^l q_t^l) = g_t + r_t[\bar{w}^h\bar{h}^h u_t^h q_t^h + \bar{w}^l\bar{h}^l u_t^l q_t^l](1-\bar{\tau}^l).$$

Dynamic of employment:

$$N_t^h = (1-\delta_N)N_{t-1}^h + (\bar{V}^h)^{1-\lambda}(L_{t-1}^h - N_{t-1}^h)^{\lambda},$$

$$N_t^l = (1-\delta_N)N_{t-1}^l + (\bar{V}^l)^{1-\lambda}(L_{t-1}^l - N_{t-1}^l)^{\lambda}.$$

Definitions:

$$\begin{aligned} q_t^h &= \frac{L_t^h}{L_t}, \\ q_t^l &= \frac{L_t^l}{L_t}, \\ n_t^h &= \frac{N_t^h}{L_t^h}, \\ n_t^l &= \frac{N_t^l}{L_t^l}, \end{aligned}$$

$$
\begin{aligned}
u_t^h &= 1 - n_t^h, \\
u_t^l &= 1 - n_t^l, \\
H_t^h &= N_t^h h_t^h, \\
H_t^l &= N_t^l h_t^l, \\
L_t^h &= L_{t-1}^h + \eta i_t, \\
L_t^l &= L_{t-1}^l + (1-\eta) i_t, \\
L_t &= L_t^h + L_t^l, \\
K_t &= L_t k_t, \\
Y_t &= L_t y_t.
\end{aligned}
$$

Exogenous stochastic process:

$$i_t = \epsilon_t, \ \text{ where } \ \epsilon_t \sim N(0, \sigma^2) \ \ i.i.d..$$

5.3. Steady State Values

The values of these variables or parameters are given (known):

τ^k - the tax on the dividends;

τ^c - the consumption tax;

$\bar{\tau}^l$ - the personal income tax;

$\bar{u}^h$ - the unemployment rate among the high-skilled labor;

$\bar{u}^l$ - the unemployment rate among the low-skilled labor;

$\bar{q}^h$ - the ratio between the high-skilled labor and the total population;

$\bar{q}^l$ - the ratio between the low-skilled labor and the total population;

$\bar{L}$ - the total labor;

r - the out-of-work benefits (parameter of unemployment relief);

δ - the annual depreciation rate;

β - the subjective discount factor;

A - the preference parameter.

In the equations the entries with a *"bar"* are the steady state values of the corresponding variables. The steady state values are (in the order of being expressed):

$$\bar{R} = \frac{1}{\beta}; \tag{40}$$

$$\bar{d} = \frac{\bar{R} - 1 + \delta}{1 - \tau^k}; \tag{41}$$

$$\bar{h}^l = \frac{1}{1 + (\bar{n}^l + r\bar{u}^l)A}; \tag{42}$$

$$\bar{L}^h = \bar{q}^h \bar{L}; \tag{43}$$

$$\bar{L}^l = \bar{q}^l \bar{L}; \tag{44}$$

$$\bar{N}^h = \bar{n}^h \bar{L}^h; \tag{45}$$

$$\bar{N}^l = \bar{n}^l \bar{L}^l; \tag{46}$$

$$\bar{n}^h = 1 - \bar{u}^h; \tag{47}$$

$$\bar{n}_l = 1 - \bar{u}^l; \tag{48}$$

$$\bar{V}^h = \delta_N^{\frac{1}{1-\lambda}} \bar{N}^h \left(\frac{\bar{n}^h}{1 - \bar{n}^h} \right)^{\frac{\lambda}{1-\lambda}}; \tag{49}$$

$$\bar{V}^l = \delta_N^{\frac{1}{1-\lambda}} \bar{N}^l \left(\frac{\bar{n}^l}{1 - \bar{n}^l} \right)^{\frac{\lambda}{1-\lambda}}; \tag{50}$$

$$\bar{h}^h : \quad \frac{\alpha\phi \left[\frac{(1-\tau^l)(1-\bar{h}^h)}{A} - (1 - \tau^l)(\bar{n}^h + r\bar{u}^h)\bar{h}^h \right]}{\left[\phi(\bar{n}^h \bar{q}^h \bar{h}^h)^{1-\rho} + (\bar{n}^l \bar{q}^l \bar{h}^l)^{1-\rho} \right] (\bar{n}^h \bar{q}^h \bar{h}^h)^{\rho}} = \frac{1 - \alpha}{\bar{q}^h} \left(1 - \tau^k - \frac{\delta}{\bar{d}} \right); \tag{51}$$

$$\bar{H}^h = \bar{N}^h \bar{h}^h; \tag{52}$$

$$\bar{H}^l = \bar{N}^l \bar{h}^l; \tag{53}$$

$$\bar{y} = \left[\phi(\bar{n}^h \bar{q}^h \bar{h}^h)^{1-\rho} + (\bar{n}^l \bar{q}^l \bar{h}^l)^{1-\rho} \right]^{\frac{1}{1-\rho}} \left(\frac{1 - \alpha}{\bar{d}} \right)^{\frac{1-\alpha}{\alpha}}; \tag{54}$$

$$\bar{k} = (1 - \alpha)\frac{\bar{y}}{\bar{d}}; \tag{55}$$

$$\bar{K} = \bar{L}\bar{k}; \tag{56}$$

$$\bar{Y} = \bar{L}\bar{y}; \tag{57}$$

$$\bar{x} = \delta\bar{k}; \tag{58}$$

$$\bar{w}^h = \frac{\alpha\phi\bar{Y}}{\left[\phi(\bar{H}^h)^{1-\rho} + (\bar{H}^l)^{1-\rho}\right](\bar{H}^h)^\rho}; \tag{59}$$

$$\bar{w}^l = \frac{\alpha\bar{Y}}{\left[\phi(\bar{H}^h)^{1-\rho} + (\bar{H}^l)^{1-\rho}\right](\bar{H}^l)^\rho}; \tag{60}$$

$$\bar{c}^h = \frac{(1-\bar{\tau}^l)\bar{w}^h(1-\bar{h}^h)}{A(1+\tau^c)}; \tag{61}$$

$$\bar{c}^l = \frac{(1-\bar{\tau}^l)\bar{w}^l\bar{h}^l(\bar{n}^l + r\bar{u}^l)}{1+\tau^c}. \tag{62}$$

5.4. Log-linearized Equations

In the further equations the entries with a *"hat"* are the log-deviations of the corresponding variables from their steady state, which are interpreted as the approximate percentage deviations. The log-linearized equations are in the same order as in the list of equations which determine the equilibrium:

$$0 = \hat{c}_t^h - \hat{w}_t^h + \frac{\bar{h}^h}{1-\bar{h}^h}\hat{h}_t^h + \frac{\bar{\tau}^l}{1-\bar{\tau}^l}\hat{\tau}_t^l, \tag{63}$$

$$0 = E_t\left[\hat{c}_t^h + \hat{R}_{t+1} - \hat{c}_{t+1}^h\right], \tag{64}$$

$$0 = -\hat{R}_t + \hat{L}_{t-1}^h - \hat{L}_t^h + \frac{(1-\tau^k)\bar{d}}{\bar{R}}\hat{d}_t, \tag{65}$$

$$\begin{aligned} 0 = & - (1+\tau^c)\bar{c}^h\hat{c}_t^h - \frac{\bar{x}}{\bar{q}^h}\hat{x}_t + \left[\frac{\bar{x}}{\bar{q}^h} - (1-\tau^k)\frac{\overline{dk}}{\bar{q}^h}\right]\hat{q}_t^h + (1-\bar{\tau}^l)\bar{h}^h\bar{w}^h\bar{n}^h\hat{h}^h + \\ & + (1-\bar{\tau}^l)\bar{h}^h\bar{w}^h\bar{n}^h\hat{w}^h + (1-\bar{\tau}^l)\bar{h}^h\bar{w}^h\bar{n}^h\hat{n}^h - \bar{\tau}^l\bar{h}^h\bar{w}^h\bar{n}^h\hat{\tau}_t^l + \\ & + r(1-\bar{\tau}^l)\bar{h}^h\bar{w}^h\bar{u}^h\hat{u}^h + (1-\tau^k)\frac{\overline{dk}}{\bar{q}^h}\hat{d}_t + (1-\tau^k)\frac{\overline{dk}}{\bar{q}^h}\hat{k}_{t-1} + \\ & + (1-\tau^k)\frac{\overline{dk}}{\bar{q}^h}\hat{L}_{t-1} - (1-\tau^k)\frac{\overline{dk}}{\bar{q}^h}\hat{L}_t, \end{aligned} \tag{66}$$

$$0 = -\bar{x}\hat{x}_t + \bar{k}\hat{k}_t - (1-\delta)\bar{k}\hat{k}_{t-1} - (1-\delta)\bar{k}\hat{L}_{t-1} + (1-\delta)\bar{k}\hat{L}_t, \tag{67}$$

$$0 = \hat{c}_t^l - \hat{w}_t^l + \frac{\bar{h}^l}{1-\bar{h}^l}\hat{h}_t^l + \frac{\bar{\tau}^l}{1-\bar{\tau}^l}\hat{\tau}_t^l, \tag{68}$$

$$0 = -\frac{(1+\tau^c)\bar{c}^l}{(1-\bar{\tau}^l)\bar{h}^l\bar{w}^l\bar{n}^l}\hat{c}_t^l + \hat{h}_t^l + \hat{w}_t^l + \hat{n}_t^l - \frac{\bar{\tau}^l}{1-\bar{\tau}^l}\hat{\tau}_t^l + \frac{\bar{u}^l r}{\bar{n}^l}\hat{u}_t^l, \tag{69}$$

$$
\begin{aligned}
0 = \; & - \; \hat{y}_t + \frac{\phi\alpha(\bar{n}^h\bar{q}^h\bar{h}^h)^{1-\rho}}{\bar{a}}\hat{n}_t^h + \frac{\phi\alpha(\bar{n}^h\bar{q}^h\bar{h}^h)^{1-\rho}}{\bar{a}}\hat{q}_t^h + \frac{\phi\alpha(\bar{n}^h\bar{q}^h\bar{h}^h)^{1-\rho}}{\bar{a}}\hat{h}_t^h + \frac{\alpha(\bar{n}^l\bar{q}^l\bar{h}^l)^{1-\rho}}{\bar{a}}\hat{n}_t^l + \\
& + \; \frac{\alpha(\bar{n}^l\bar{q}^l\bar{h}^l)^{1-\rho}}{\bar{a}}\hat{q}_t^l + \frac{\alpha(\bar{n}^l\bar{q}^l\bar{h}^l)^{1-\rho}}{\bar{a}}\hat{h}_t^l + (1-\alpha)\hat{k}_{t-1} + (1-\alpha)\hat{L}_{t-1} - (1-\alpha)\hat{L}_t, \qquad (70)
\end{aligned}
$$

where:

$$
\bar{a} = \phi(\bar{n}^h\bar{q}^h\bar{h}^h)^{1-\rho} + (\bar{n}^l\bar{q}^l\bar{h}^l)^{1-\rho};
$$

$$
0 = -\left[(1-\rho)\phi(\bar{H}^h)^{1-\rho} + \frac{\rho\alpha\phi\bar{Y}}{\bar{w}^h(\bar{H}^h)^\rho}\right]\hat{H}_t^h - (1-\rho)(\bar{H}^l)^{1-\rho}\hat{H}_t^l + \frac{\alpha\phi\bar{Y}}{\bar{w}^h(\bar{H}^h)^\rho}\hat{Y}_t - \frac{\alpha\phi\bar{Y}}{\bar{w}^h(\bar{H}^h)^\rho}\hat{w}_t^h; \tag{71}
$$

$$
0 = -\left[(1-\rho)(\bar{H}^l)^{1-\rho} + \frac{\rho\alpha\bar{Y}}{\bar{w}^l(\bar{H}^l)^\rho}\right]\hat{H}_t^l - (1-\rho)\phi(\bar{H}^h)^{1-\rho}\hat{H}_t^h + \frac{\alpha\bar{Y}}{\bar{w}^l(\bar{H}^l)^\rho}\hat{Y}_t - \frac{\alpha\bar{Y}}{\bar{w}^l(\bar{H}^l)^\rho}\hat{w}_t^l; \tag{72}
$$

$$
0 = -\hat{d}_t + \hat{y}_t - \hat{k}_{t-1} + \hat{L}_t - \hat{L}_{t-1}; \tag{73}
$$

$$
\begin{aligned}
0 \; = \; & -\tau^c\bar{c}^h\bar{q}^h\hat{c}_t^h - \tau^c\bar{c}^l\bar{q}^l\hat{c}_t^l - \tau^k\overline{dk}\hat{d}_t - \tau^k\overline{dk}\hat{k}_{t-1} - \tau^k\overline{dk}\hat{L}_{t-1} + \tau^k\overline{dk}\hat{L}_t - \\
- \; & (\bar{\tau}^l\bar{w}^h\bar{h}^h\bar{n}^h\bar{q}^h + \bar{\tau}^l\bar{w}^l\bar{h}^l\bar{n}^l\bar{q}^l)\hat{\tau}_t^l - \bar{\tau}^l\bar{w}^h\bar{h}^h\bar{n}^h\bar{q}^h\hat{w}_t^h - \bar{\tau}^l\bar{w}^l\bar{h}^l\bar{n}^l\bar{q}^l\hat{w}_t^l - \\
- \; & \bar{\tau}^l\bar{w}^h\bar{h}^h\bar{n}^h\bar{q}^h\hat{h}_t^h - \bar{\tau}^l\bar{w}^l\bar{h}^l\bar{n}^l\bar{q}^l\hat{h}_t^l - \bar{\tau}^l\bar{w}^h\bar{h}^h\bar{n}^h\bar{q}^h\hat{n}_t^h - \bar{\tau}^l\bar{w}^l\bar{h}^l\bar{n}^l\bar{q}^l\hat{n}_t^l + \\
+ \; & r\bar{w}^h\bar{h}^h\bar{u}^h\bar{q}^h\hat{u}_t^h + r\bar{w}^l\bar{h}^l\bar{u}^l\bar{q}^l\hat{u}_t^l - \left[\tau^c\bar{c}^h\bar{q}^h + \bar{\tau}^l\bar{w}^h\bar{h}^h\bar{n}^h\bar{q}^h - r(1-\bar{\tau}^l)\bar{h}^h\bar{w}^h\bar{u}^h\bar{q}^h\right]\hat{q}_t^h - \\
- \; & \left[\tau^c\bar{c}^l\bar{q}^l + \bar{\tau}^l\bar{w}^l\bar{h}^l\bar{n}^l\bar{q}^l - r(1-\bar{\tau}^l)\bar{h}^l\bar{w}^l\bar{u}^l\bar{q}^l\right]\hat{q}_t^l; \qquad (74)
\end{aligned}
$$

$$
\begin{aligned}
0 \; &= \; -\hat{N}_t^h + \left[1 - \delta_N - \lambda\left(\frac{\bar{V}^h}{\bar{L}^h - \bar{N}^h}\right)^{1-\lambda}\right]\hat{N}_{t-1}^h + \frac{\lambda}{\bar{n}^h}\left(\frac{\bar{V}^h}{\bar{L}^h - \bar{N}^h}\right)^{1-\lambda}\hat{L}_{t-1}^h, \qquad (75) \\
0 \; &= \; -\hat{N}_t^l + \left[1 - \delta_N - \lambda\left(\frac{\bar{V}^l}{\bar{L}^l - \bar{N}^l}\right)^{1-\lambda}\right]\hat{N}_{t-1}^l + \frac{\lambda}{\bar{n}^l}\left(\frac{\bar{V}^l}{\bar{L}^l - \bar{N}^l}\right)^{1-\lambda}\hat{L}_{t-1}^l, \qquad (76)
\end{aligned}
$$

$$
\begin{aligned}
0 \; &= \; -\hat{q}_t^h + \hat{L}_t^h - \hat{L}_t; \qquad (77) \\
0 \; &= \; -\hat{q}_t^l + \hat{L}_t^l - \hat{L}_t; \qquad (78) \\
0 \; &= \; -\hat{n}_t^h + \hat{N}_t^h - \hat{L}_t^h; \qquad (79) \\
0 \; &= \; -\hat{n}_t^l + \hat{N}_t^l - \hat{L}_t^l; \qquad (80) \\
0 \; &= \; \bar{u}^h\hat{u}_t^h + \bar{n}^h\hat{n}_t^h; \qquad (81) \\
0 \; &= \; \bar{u}^l\hat{u}_t^l + \bar{n}^l\hat{n}_t^l; \qquad (82)
\end{aligned}
$$

$$
\begin{aligned}
0 &= -\hat{H}_t^h + \hat{N}_t^h + \hat{h}_t^h; & (83)\\
0 &= -\hat{H}_t^l + \hat{N}_t^l + \hat{h}_t^l; & (84)\\
0 &= -\hat{L}_t^h + \hat{L}_{t-1}^h + \frac{\eta}{\bar{q}^h}\hat{i}_t; & (85)\\
0 &= -\hat{L}_t^l + \hat{L}_{t-1}^l + \frac{1-\eta}{\bar{q}^l}\hat{i}_t; & (86)\\
0 &= -\hat{L}_t + \bar{q}^h\hat{L}_t^h + \bar{q}^l\hat{L}_t^l; & (87)\\
0 &= -\hat{K}_t + \hat{L}_t + \hat{k}_t; & (88)\\
0 &= -\hat{Y}_t + \hat{L}_t + \hat{y}_t. & (89)
\end{aligned}
$$

Exogenous stochastic process:

$$\hat{i}_t = \hat{\epsilon}_t, \tag{90}$$

where the exogenous variable i_t was log-linearized around L_t, so $\hat{i}_t$ expresses the percentage of immigrants comparing to the total labor in the steady state (not the number of the immigrants).

5.5. Calibration of the Model

The model was calibrated to match the most recent annual German data and to use standard parameter values whenever possible. The values of parameters for German economy are given in Table 8 and the steady state values of the variables (which are known) in Table 9.

α	β	δ	τ^c	τ^k	r	A	ρ	η	ϕ	λ	β_N
0.64	0.96	0.08	0.16	0.137	0.67	2.42	$\frac{10}{9}$	0.07	2	0.05	0.064

Table 8: The calibration of the model for German economy: values of parameters.

Labor share α is set to 0.64 (following Canova, Ravn (2000)). The subjective discount factor β is 0.96 in order the steady state value of interest rate would be 4 %. Depreciation rate δ was calibrated to 0.08, what is between 7.5 % used by Burda and Hunt (2001) and benchmark 10 % (Canova, Ravn, 2000, Moreno-Galbis and Sneessens, 2004 etc.), in order to be consistent to the recent German data (i.e. investment-output and capital-output ratios, see Table 10). Tax on consumption in the model is the value added tax (VAT), so it was calibrated to its current (in year 2005) value, i.e. 16 %. Tax on dividends is set to 0.137[15]. This implies the steady state value of capital-output ratio

[15]It was calculated according to the formula: tax on dividends = overall personal income tax + corporate

would be 2.55 as in the model of German economy of Canova and Ravn (2000) (see Table 10). The replacement ratio r (out of work benefits) is 0.67 because the unemployment benefits is 67 % of the previous net income after tax (for an unemployed with at least one child; 60 % for singles)[16]. The parameter A was chosen to be 2.42 in order the unskilled agents use 30 % of their non-sleeping time to work (see Table 10). The inverse of elasticity of substitution between low- and high-skilled labor is $\frac{10}{9}$ (it corresponds to 0.9 elasticity of substitution between low- and high-skilled labor) as a benchmark as in Bassanini, Rasmussen and Scarpetta (1999) and Iregui (1999)[17]. In my paper I assumed that the ratio between high-skilled immigrants and the total immigrant η is 7 %, as it consistent with empirical evidences[18]. Because of possible differences in the definitions of low- and high-skilled labor in variations of the model I will try other values of this parameter. The productivity difference between high-and low-skilled hours ϕ is set to 2 according Canova and Ravn (2000). The households' bargaining power λ is 0.5, and job destruction rate β_N is equal to 0.064 as in Merz (1999).

$\bar{L}$	$\bar{u}^h$	$\bar{u}^l$	$\bar{q}^h$	$\bar{q}^l$	$\bar{\tau}^l$
42806000	0.049	0.115	0.50	0.50	0.196

Table 9: The calibration of the model for German economy: the steady state values of the variables (which are known).

Total labor force $\bar{L}$ is calibrated to the number of economically active population in 2005 (according to Federal Statistical Office Germany). The unemployment rates among the different skills groups ($\bar{u}^h$ and $\bar{u}^l$) are determined with reference to Bassanini, Rasmussen and Scarpetta (1999). The shares of skilled and unskilled agents are set according to Bassanini, Rasmussen, Scarpetta (1999) and Canova, Ravn (2000). Labor income tax is set to 19.6 %, it is the average personal income tax without social security contributions in Germany in 2004 (according to OECD).

The calibration of the model implies that the investment-output ratio is 0.20 (see Table 10) what corresponds to the data of National Institute of Economic and Social Research

income tax rate − corporate income tax on distributed profits rate. *Source: OECD.*

[16] A person unemployed can receive this benefit, if he and his employer have paid contributions of at least twelve months in the two years prior to getting unemployment; this requirement has been tightened from a three year period which applied until 2003 (Siebert, 2004).

[17] The value of the elasticity of substitution between low- and high-skilled labor for German economy is quite different in many papers, in the variations of the model I also will use other values of this parameter.

[18] Source: Arackal (2000). Also Bonin (2005) states that the share of high-skilled foreign workers is smaller than 10 per cent.

(UK). The values of parameters imply that high-skilled hours is 0.244 or slightly less than Canova and Ravn (2000) obtained (i.e. 0.25, it corresponds to 36 working hours per week).

$\frac{\bar{x}}{\bar{y}}$	$\frac{\bar{k}}{\bar{y}}$	$\bar{h}^h$	$\bar{h}^l$	$\bar{R}$
0.20	2.55	0.244	0.30	1.04

Table 10: The selected steady state values.

The main scenario of the stochastic process or the inflow of the immigrants implies that the immigrants are coming only once, the amount of them is 0.4 per cent of total German labor force in the steady state. This number corresponds to the percentage of the new registered immigrants during 11 months after May 1, 2004 comparing to the total British labor force. This stochastic process will be implemented in *Toolkit* multiplying the DD matrix by 0.4[19]. The calibration of the stochastic process implies more than 170,000 immigrants, and it quite near to 200,000 used by Brücker, Kohlhaas (2004)[20].

Other scenarios are in Appendix C and they will be considered in the section of model's variations.

6. Model Results and Answer

The model described and developed above, and calibrated to the economy of the United Kingdom (see Appendix D) implies 0.021 % increase in GDP during the first year (see Figure 1 and see Table 11).

Years after shock	0	1	2	3	4
In %	0.021	0.079	0.112	0.135	0.154

Table 11: The values of the impulse responses for the GDP of the UK (in per cent).

The reasons why I did not get 0.043 % might be various. First, probably not all immigrants have applied for the Worker Registration Scheme. According to shadow home secretary David Davis, not all workers registered and coupled with official statistics showing that between May and August, 2004 542,000 visited the UK, an increase of

[19]More about *Toolkit* see Uhlig (1999).
[20]They assumed the annual immigration of 200,000 to Germany.

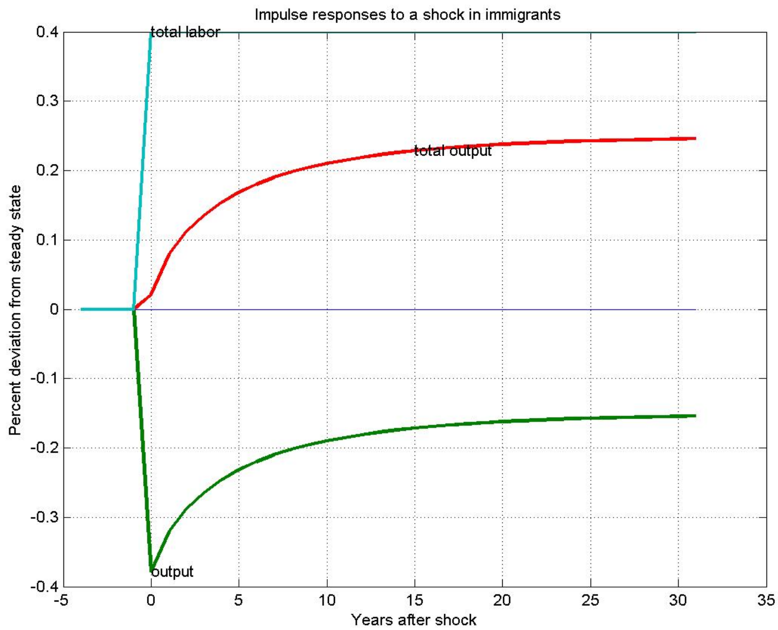

Figure 1: The impulse responses to a shock in immigrants for the UK.

222,000 from last year (Management Issues, 2004), and it is only during 3 months. Thus during 12 months the number might be fourfold bigger. Second, the model's calibration is not so precise since it doesn't match the data of economy in 2004 but in 2001-2003. Third, the Home Office (UK Immigration News, 2005) predicts that up to $\frac{1}{3}$ of all registered immigrants have been in the UK before the EU expansion. Thus the results of the model are reliable till we can trust the data regarding the number of *new* immigrants. Because of the scarcity of the newest data of the economy of the UK, I could not check whether the model matches the real unemployment dynamic among the different skills groups as well as the consumption, wages, working hours. From Figure 1 we can see that output per capita decreased by 0.38 % because most of immigrants were unskilled.

Further in this section I will enclose the short-term and long-term effects (impulse responses) of the inflow of the immigrants on German labor market and whole economy (considering the main scenario) and the intuition for the achieved insights. All relevant Matlab codes (including one for the UK economy) are in Appendix E.

6.1. Short-run effects and adjustments

The short-run effects on output, labor income tax and unemployment are provided in Table 12 (see Figure 2 for short- and long-run effects, HS denotes high-skilled, and LS means low-skilled).

Years after shock	0	1	2	3	4	5
y	-0.496	-0.422	-0.373	-0.337	-0.310	-0.288
Y	-0.096	-0.022	0.027	0.063	0.090	0.112
u^h	1.087	0.377	0.154	0.083	0.061	0.054
u^l	5.726	4.132	3.034	2.276	1.753	1.392
τ^l	2.037	1.607	1.342	1.162	1.033	0.936

Table 12: The short-run effects on output, labor income tax and unemployment (the changes are in per cent from steady state).

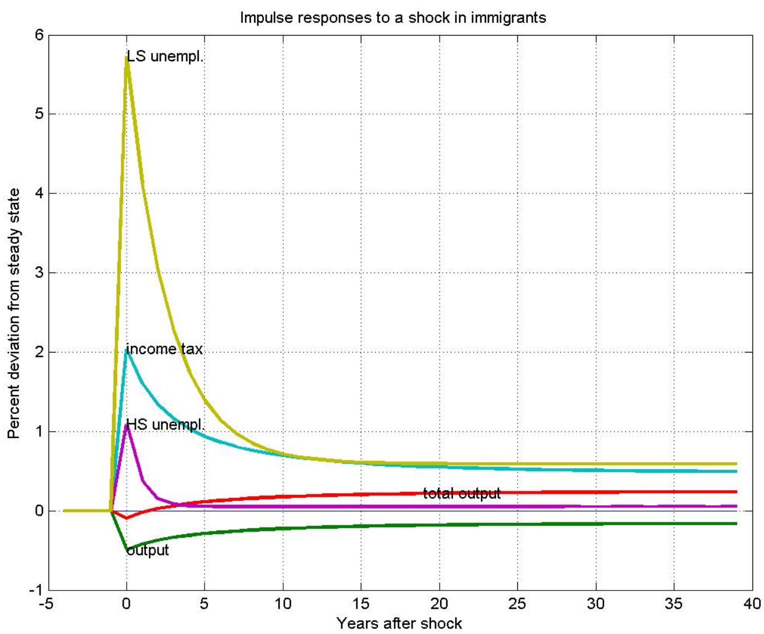

Figure 2: The impulse responses of output, labor income tax and unemployment due to shock in labor supply.

Output per capita (in Figures just *output*) after the shock of labor supply decreases nearly be half per cent, later it increases but does not reach its steady state value due to decreased average productivity of labor[21]. Total output (in Figures *total output*) initially

[21] 93 % of all the immigrants are unskilled.

after the shock decreases by 0.1 % but in the second year after the shock exceeds its steady state value and increases further. Unemployment rates and tax on labor income increase due to immigration during the first year, but later decrease. Unemployment rate of the skilled labor first increases by 1.087 %, so its value in that period is $0.049 \cdot 1.01087 \approx 0.0495$. Unemployment rate among the unskilled agents increases by 5.726 %, so its value in that period is $0.115 \cdot 1.05726 \approx 0.121$. So the increases in the high- and low-skilled labor unemployment rates are negligible.

Years after shock	0	1	2	3	4	5
k	-0.458	-0.482	-0.485	-0.477	-0.462	-0.443
K	-0.058	-0.082	-0.085	-0.077	-0.062	-0.043
R	-0.067	0.004	0.012	0.017	0.020	0.020
d	-0.096	0.036	0.109	0.148	0.167	0.174
x	-1.121	-0.756	-0.530	-0.383	-0.287	-0.224

Table 13: The short-run effects on the capital market (the changes are in per cent from steady state).

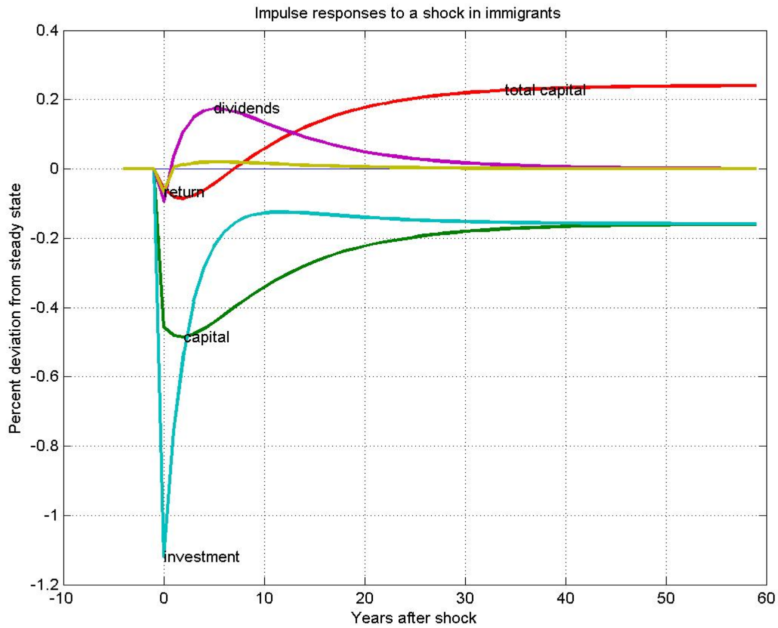

Figure 3: The impulse responses of capital and its related variables.

Due to the inflow of the Eastern European workers capital stock per capita decreases (see Table 13 and Figure 3). Total capital stock after the shock initially decreases but after 7-8 years it reaches its steady state value. Return and dividend first also decrease, but in the next period exceed their steady state values. Investment per capita immediately after the shock drops down by more than 1 %, later it increases but never reaches its steady state value because the ratio between high-skilled and total labor decreases (93 % of the immigrant are low-skilled).

The short-run impact on total number of high- and low-skilled workers, working time of each labor group and total hours worked by each group of workers is visible in Table 14.

Years after shock	0	1	2	3	4	5
h^h	-0.284	-0.179	-0.110	-0.065	-0.036	-0.017
h^l	0.123	0.077	0.045	0.023	0.008	-0.002
N^h	0	0.037	0.048	0.052	0.053	0.053
N^l	0	0.207	0.350	0.448	0.516	0.563
H^h	-0.283	-0.142	-0.062	-0.014	0.017	0.036
H^l	0.123	0.285	0.395	0.472	0.525	0.561

Table 14: The short-run effects on number of workers and time worked (the changes are in per cent from steady state).

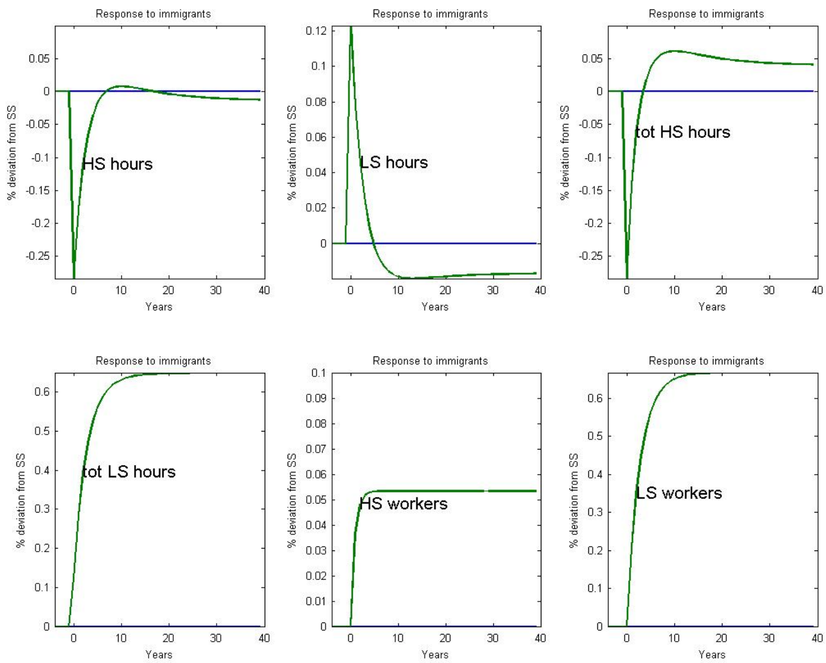

Figure 4: The impulse responses of numbers of hours worked (per worker and total).

The number of employed high- and low-skilled agents increase after the immigration (see Table 14 and Figure 4) and reaches new steady state only in the long-run. After the shock to labor supply the number of hours, each high-skilled agent works, decreases, but one period later starts increasing. Working hours of low-skilled agents due to the shock increase, but later start decreasing and 4-5 year later after the shock reach the steady state value. The total working hours by skilled labor because of immigration first decreases but 4 years later exceeds the initial steady state value. The total working hours of low-skilled agents in the short-run monotonous increases.

Consumptions of both groups of individuals after the inflow of the immigrants first decrease: by 0.2 % for high-skilled agents and by 0.8 % for low-skilled agents but from the next period start increasing (see Table 15). Wages of skilled workers initially increase by 0.2 %, but in the next period start decreasing. Wages of low-skilled agents monotonically decline, 5 years after the shock their value is more than half per cent lower than in steady state.

Years after shock	0	1	2	3	4	5
c^h	-0.202	-0.198	-0.185	-0.168	-0.148	-0.128
c^l	-0.798	-0.763	-0.748	-0.738	-0.727	-0.716
w^h	0.203	0.136	0.106	0.094	0.092	0.095
w^l	-0.249	-0.338	-0.402	-0.445	-0.472	-0.489

Table 15: The short-run effects on consumption and wages (changes are in per cent from steady state).

Now let's analyze everything *step-by-step*. Total output decreases during the time period when immigrants come because total hours worked by high-skilled agents decrease more than total hours worked by low skilled agents increase[22], and the change in total capital at that period has no impact to output since it will affect the production in the next period (see Figure 2). The decrease in output per capita is bigger than the fall of total capital due to decreased average productivity of labor (the ratio of high-skilled agents to the total population falls down, see Table 16). In the next period the growth of total output and output per capita is related to the increased number of total hours worked by high-skilled labor (output per capita and number of hours worked by each high-skilled agent are very strong positively correlated while the number of hours worked by each low-skilled agent are very strong negatively correlated (due to the substitution effect), see Table 21).

The dividends fall down in the time t=0 because total output decreases, but the

[22]High-skilled labor is ϕ times more productive than low-skilled agents.

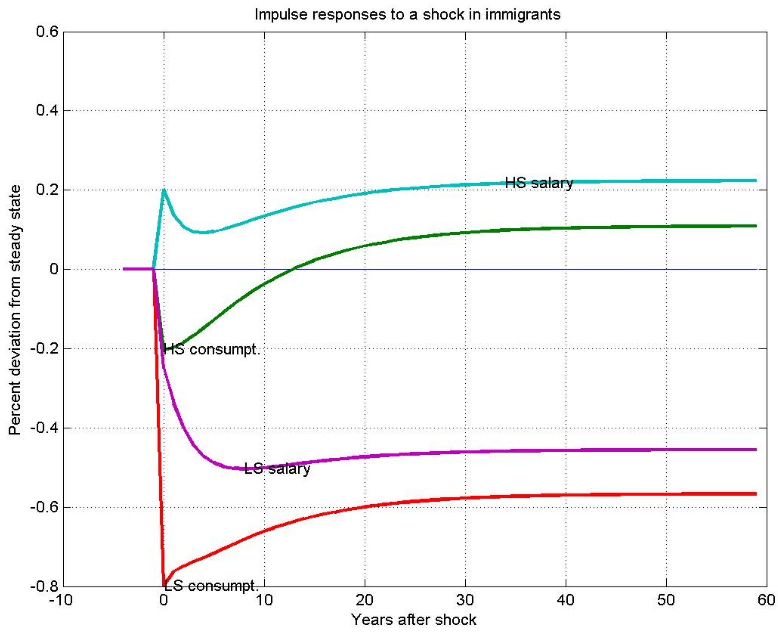

Figure 5: The impulse responses of wages and consumption of high- and low-skilled labor.

change in capital is 0 (capital used in production in that period is from the last period (so it is equal to steady state value)), thus dividends also decreases (see Figure 3). Consequently high-skilled agents have less incentives to invest in capital, so investment per capita decreases. Therefore total capital declines. The decreased amount of capital stock, of course, negatively affects total output in the next period. But 5 years dividends increases more than 17 % above their steady state value because starting from the next period after the immigration, the more workers are hired so the demand for capital increases. The raise of dividends initiates the growth of investment (more incentives for high-skilled labor to invest) what automatically caused the increase in total capital and capital per capita.

The positive shock to amount of labor pushes up the total number of employees, i.e. both numbers of high- and low-skilled workers increase (see Figure 4) starting from period t=1.

The immigrants in period t=0 just increase the unemployment rates [23], thus the government needs to pay more unemployment reliefs, which are bigger for high-skilled unemployed. Hence the government slightly increases labor income tax rate by approxi-

[23] According to dynamic of employment the increase in unemployed affects the number of workers in the next period

mately 2 % (from 0.196 to 0.196*1.02037=0.20)[24]. It causes the decrease of high-skilled agents *net* wages, so they start to work less and have more leisure time. Thereby total high-skilled hours in period t=0 falls down, thus their wages increase[25] (see Figures 5 and 4). Then the firms substitute high-skilled labor with low-skilled workers. So working hours of each high-skilled agent decreases, and working hours of each low-skilled agents increases. The latter work more in order to compensate the increased labor income tax, but then their salary decreases. Starting from the next period there are less unemployed in the economy, so the authority decreases the rate of labor income tax, consequently high-skilled agents demand smaller wages and the firms substitute them back with unskilled labor: each high-skilled worker starts working more and each low-skilled agent works less. The monotonous fall in wages of low-skilled labor is caused by the increase in total number of hours worked by unskilled agents.

One of the reasons why consumption of high-skilled declines less than one of low-skilled agents is that the unemployment rate of high-skilled labor increases less than unemployment rate of low-skilled labor (consumption doesn't depend on the employment status of agent, i.e. family members insure themselves (see Appendix A)). Low skilled agents consume less because their wages decrease (despite their working hours per week increase), and labor income tax, as well as unemployment rate among low-skilled agents, increases. Consumption of high-skilled agents also decrease, despite their salary increase. The reasons are decreased dividends and number of working hours, increased labor income tax and unemployment rate among the high-skilled agents. We observe the decreased consumption level later on in the short-run since skilled agents then investment more (see Table 13).

Therefore in the short-run the economic effect of immigration are likely to be felt in the labor market and on the public finance (as Leibfritz, O'Brien, Dumont (2003) suggests). Concluding we can state that in the short-run the welfare of the low-skilled labor declines due to the immigration because the consumption of these agents decreases and working time increases (so the leisure decreases). The impact of the immigration on the welfare of the high-skilled agents is ambiguous while their consumption declines but leisure increases.

[24] The government at time t=0 also faces the decreased collected taxes from capital (dividends decrease), consumption (consumption of both groups of agents fall down).

[25] The profit maximization problem of the firms implies that total hours worked and salary are negatively related (if one increases, another decreases).

6.2. Permanent and long-term effects

The permanent effects of the immigration primarily are ones to the labor market: total number of high-, low-skilled workers, total labor and the shares of the high- and low-skilled labor are altered (see Table 16).

L	L^h	L^l	q^h	q^l
0.4	0.056	0.744	-0.344	0.344

Table 16: The permanent effects of immigration to the labor market (the changes are in per cent from steady state).

Thus the immigration changes the structure of labor market in the host economy.

The time is needed to absorb the initial disturbance in the model. As in Canova and Ravn (2000) it takes approximately 40 years for most variables to converge to the initial or new steady state.

The long-run effects on capital and its related variables are shown in Table 17.

Years after shock	10	20	30	40	50
k	-0.342	-0.223	-0.181	-0.166	-0.161
K	0.058	0.177	0.219	0.234	0.239
R	0.016	0.006	0.002	0.001	0
d	0.134	0.050	0.017	0.006	0.002
x	-0.127	-0.140	-0.152	-0.157	-0.158

Table 17: The long-run effects of the immigration on capital market (the changes are in per cent from steady state).

We can state that in the long-run (40-50 year after the inflow of immigrants) return and dividend rate finally converge to the steady state. Capital per capita and investment per capita converge to the new steady state. The new values are smaller than before the immigration. Total capital stock exceeds its steady state value in 8 years after the shock to labor supply. This means that probably capital stock per each **high-skilled** agent[26] (only they own the capital) in the long-run converges to the previous value (that is why total capital stock in the long-run tends to grow) or even exceeds it.

The long-run effects on output, labor income tax and unemployment are provided in Table 18.

[26] Author: in the model k_t denotes capital per capita, i.e. the capital stock per each (low- and high-skilled) agent.

Years after shock	10	20	30	40	50
y	-0.226	-0.181	-0.166	-0.161	-0.160
Y	0.174	0.219	0.234	0.239	0.240
u^h	0.051	0.051	0.051	0.051	0.051
u^l	0.716	0.594	0.591	0.591	0.591
τ^l	0.693	0.550	0.508	0.494	0.489

Table 18: The long-run effects on output, labor income tax and unemployment (the changes are in per cent from steady state).

Output and output per capita converge to the new steady state in the long-run. Total output is 0.240 % higher in the long-run than in the initial steady state, since the number of workers increase. Output per capita is 0.160 % lower in the long-run than in the initial steady state (the reason, as I discussed before, is the changed composition of labor structure). Unemployment rates of high- and low-skilled labor converge to the new steady state. Because of the inflow of immigrants to the German labor market, unemployment rate among high-skilled agents increases just by 0.051 %, so if initially it was 0.049, thus after the shock it is 0.04902. We can conclude that the proposed immigration doesn't influence unemployment among the high-skilled labor in the long-run. For low-skilled labor unemployment rate before the shock was 0.115, due to the inflow of immigrants, the rate increases by 0.591 %, so the unemployment rate of low-skilled individuals after the shock is 0.1157. We can state that immigration slightly increases the unemployment among low-skilled labor in the long-run. The tax on labor income in the long-run increases by 0.489 %. So in the long-run $\tau^l = 0.196 * 1.00489 = 0.197$. The government needs slightly to increase the tax rate in the long-run because its income from the consumption and labor taxes might decrease (wages and consumption of high-skilled labor increase, so the government incomes too, but wages and consumption of low-skilled labor decrease, so the government incomes also declines) and the expenses of the government increases, it needs to pay more unemployment reliefs because there are more unemployed agents in the economy.

The long-run impact on total number of high- and low-skilled workers, working time of agents of each labor group and total hours worked by each group of workers are presented in Table 19.

The numbers of employed high- and low-skilled agents converge to the new steady state, i.e. in the long-run they are constant. The working hours of each high-skilled worker and low-skilled workers converge almost to the initial steady state 10 years after

Years after shock	10	20	30	40	50
h^h	0.008	-0.004	-0.010	-0.013	-0.013
h^l	-0.019	-0.019	-0.017	-0.017	-0.017
N^h	0.053	0.053	0.053	0.053	0.053
N^l	0.651	0.667	0.667	0.667	0.667
H^h	0.061	0.050	0.043	0.041	0.040
H^l	0.632	0.648	0.650	0.650	0.650

Table 19: The long-run effects on number of workers and time worked (the changes are in per cent from steady state).

the shock (the number are just slightly smaller, see Table 19). Because the quantities of both types of workers in the long-run are constant, the total hours worked by each type of workers follow the dynamics of the individual hours worked. Total hours worked by the high-skilled agents are almost equal to the steady state value, but total hours worked by unskilled labor a little bit increase and reach the new steady state which is 0.65 % higher than the initial situation due to the inflow of immigrants.

Consumption of high-skilled agents in the long-run is higher than its steady state value (see Table 20). Consumption of unskilled labor is lower than in the steady state. From Table 20 we can see that the immigration in the long-run affects wages of both groups of the labor differently: high-skilled wages increase by 0.2 %, wages of the low-skilled workers in the long-run are 0.46 % lower than in the steady state. Thus we can conclude that in the long-run the welfare of high-skilled agents increase (consumption increases and working time declines (so the leisure increases)), but the welfare of low-skilled individuals decreases since their consumption declines and leisure almost converges to the steady state.

Years after shock	10	20	30	40	50
c^h	-0.038	0.059	0.092	0.104	0.108
c^l	-0.662	-0.600	-0.578	-0.570	-0.567
w^h	0.134	0.192	0.213	0.220	0.223
w^l	-0.501	-0.473	-0.461	-0.457	-0.456

Table 20: The long-run effects on the consumption and wages (the changes are in per cent from steady state).

The correlation table (see Table 21) implies that there are no strong linear relationship between output and other variables (except the return and immigrants) when they are taken from the earlier or later periods than output per capita. Due to the inflow of immigrants capital and output (in per capita terms) fall down: immigrants and output are negatively correlated, output and capital are positively correlated, thus the immigrants and capital are also negatively correlated (see Table 21). From Table 21 we can see the influence of immigration on the public finance: the immigration affects output per capita (it decreases), the correlation between output per capital and labor income tax in the model's economy is -1 (very strong negative correlation). So if output per capita decreases (it is the consequence of immigration because unskilled immigrants dominate), labor income tax increases. Consequently in the analyzed model the inflow of foreign workers negatively affects the public finance, i.e. the authority in order to apply the undertakings (i.e. to pay the unemployment reliefs) has to increase the tax rate. The quantity of immigrants in period t=1 is positively correlated with output in t=0, since the immigration affects number of workers one period later.

capital	-0.01	-0.13	-0.27	-0.35	-0.06	**0.98**	0.21	-0.15	-0.23	-0.17	-0.08
output	-0.03	-0.14	-0.25	-0.27	0.04	**1**	0.04	-0.27	-0.25	-0.14	-0.03
investment	-0.04	-0.13	-0.22	-0.19	0.12	**0.99**	-0.09	-0.34	-0.26	-0.11	0.01
income tax	0.03	0.14	0.24	0.24	-0.07	**-1**	0.01	0.29	0.25	0.12	0.02
dividends	-0.08	-0.1	-0.06	0.07	0.35	**0.74**	-0.5	-0.53	-0.23	0.02	0.14
return	-0.05	-0.09	-0.1	-0.01	0.25	**0.77**	**-0.6**	-0.32	-0.08	0.05	0.09
HS consumpt.	-0.02	-0.14	-0.28	-0.32	-0.02	**0.99**	0.17	-0.19	-0.25	-0.17	-0.07
LS consumpt.	-0.02	-0.13	-0.27	-0.31	-0.02	**0.99**	0.11	-0.21	-0.23	-0.15	-0.05
HS salary	0.04	0.13	0.21	0.2	-0.11	**-0.98**	0.13	0.35	0.24	0.08	-0.02
LS salary	0.03	-0.11	-0.31	-0.46	-0.25	**0.85**	0.4	0.05	-0.14	-0.18	-0.14
HS unempl.	0.05	0.11	0.16	0.1	-0.19	**-0.9**	0.38	0.39	0.16	-0.01	-0.08
LS unempl.	0.04	0.14	0.23	0.21	-0.1	**-0.99**	0.05	0.33	0.27	0.12	0
HS hours	-0.05	-0.13	-0.21	-0.18	0.14	**0.98**	-0.11	-0.37	-0.27	-0.1	0.02
LS hours	0.05	0.14	0.21	0.17	-0.14	**-0.98**	0.1	0.37	0.28	0.11	-0.01
immigrants	0.05	0.14	0.11	0.04	-0.21	**-0.76**	**0.62**	0.25	0.03	-0.06	-0.07
	-5	-4	-3	-2	-1	0	1	2	3	4	5

Table 21: Cross correlation Table (HP-filtered series, moments based), corr(v(t+j),GDP per capita(t)). Last row shows j.

The contradiction with Table 4 where I presented the correlations of the empirical data is that in Table 21 we observe negative relationship between the immigrants and output per capita as in Table 4 it is positive. The reason might be different definitions of the term "per capita": in Table 4 it means per each agent of population, while in Table 21 (as well as in whole analysis) it means per each economically active household.

7. Variations

There might be several variations in the model's calibration of the German economy. They are clearly described in Appendix C. I will consider the outcomes at the time period 0 (as short-run results) and 10 (as long-run results)[27].

First I will analyze the impact of the inverse of elasticity of substitution between low- and high-skilled labor ρ remaining all other parameters unchanged. Let's consider about these variables: Y_t, y_t, x_t, K_t, k_t, τ_t^l, c_t^h, c_t^l, w_t^h, w_t^l, u_t^h, u_t^l, h_t^h and h_t^l.

Variable	Short-run			Long-run		
ρ	$\frac{10}{9}$	1	0.91	$\frac{10}{9}$	1	0.91
Y_t	-0.096	-0.095	-0.095	0.174	0.175	0.175
y_t	-0.496	-0.495	-0.495	-0.226	-0.225	-0.225
x_t	-1.121	-1.114	-1.108	-0.127	-0.126	-0.125
K_t	-0.058	-0.057	-0.057	0.058	0.060	0.061
k_t	-0.458	-0.457	-0.457	-0.342	-0.34	-0.339
τ_t^l	2.037	2.046	2.054	0.693	0.693	0.692
c_t^h	-0.202	-0.216	-0.228	-0.038	-0.053	-0.065
c_t^l	-0.798	-0.772	-0.751	-0.662	-0.622	-0.589
w_t^h	0.203	0.190	0.180	0.134	0.118	0.104
w_t^l	-0.249	-0.22	-0.196	-0.501	-0.460	-0.427
u_t^h	1.087	1.087	1.087	0.051	0.051	0.051
u_t^l	5.726	5.726	5.726	0.716	0.716	0.716
h_t^h	-0.284	-0.286	-0.287	0.008	0.004	0.000
h_t^l	0.123	0.125	0.126	-0.019	-0.016	-0.014

Table 22: The impact of parameter ρ to the main variables (in per cent).

Since the parameter ρ is related to labor it has almost no influence to total capital, capital per capita, investment, total output and output per capita[28] (there is a comparison of the impact of different values of the parameter ρ, including its value according to the main scenario, on other variables in Table 22). We can see that parameter ρ doesn't influence unemployment rates because they are unrelated[29]. We observe that when the

[27]The reader can argue that we need to consider as a long-run not 10 year, but, for instance, 20 years when the trend of the impulse responses is more stable. But on another hand, 20 years is very long time period: several business cycles and nearly one generation. Since the long-run effects were analyzed in the previous section, here I will restrict to the comparison between different cases.

[28]Because high-skilled labor is partially substituted by low-skilled labor, output has almost no influence (total and per capita in long- and short-run).

[29]Because the parameter ρ has no influence to numbers of total workers while they have their own dynamic process, thus I did not include total workers in Table 22.

parameter ρ is smaller (or the elasticity of substitution between low- and high-skilled labor is bigger), more high-skilled labor working hours are substituted with low-skilled labor working hours. Thus, the government collecting less labor income tax (the income of the high-skilled workers decreases due to the substitution effect) increases the labor income tax rate (the smaller ρ is, the higher labor income rate tax is). Moreover when ρ is smaller, high-skilled workers after the immigration work less hours per week (each of them), so their gross income from working activities decreases, and besides the government increases labor income tax rate, consequently net income from working activities even decreases much more. So does their consumption (the smaller ρ is, the smaller consumption of the high-skilled labor is). Now let's look at the low-skilled agents. Bigger value of the elasticity of substitution between low- and high-skilled labor causes higher increase of total working hours of the low-skilled labor. This stipulate the increase of working hours of each low-skilled worker. Thus higher value of the elasticity of substitution between low- and high-skilled labor (or the lower ρ) determines larger number of the working hours of each low-skilled agent. Equation (36) implies the trade-off between wage and working time of the low-skilled agent (if h_t^l increases, w_t^l declines). Hence smaller ρ causes higher wages for low-skilled agent (or smaller decrease in wages due to the immigration) as well as consumption (see Table 22). All these propositions hold in both short- and long-run.

Now consider the second variation of the model's calibration for the German economy, i.e. parameter η. In the analysis beside the benchmark (0.07) these values are considered: 0, 0.08 and 0.12 (see Appendix C for more information). Other parameters remain unchanged. In this case let's consider these variables: Y_t, y_t, x_t, K_t, k_t, τ_t^l, c_t^h, c_t^l, w_t^h, w_t^l, H_t^h, H_t^l, h_t^h and h_t^l.

The difference from the previous case is that here the change of the value of the parameter affects unemployment rates. When the immigrants are only unskilled (η=0), unemployment rate of high-skilled labor is not affected in the short- and long-run (see Table 23). If the share of high-skilled agents among all immigrants increases, unemployment rate of skilled labor raises in the short-run (except the time period t=0) and in the long-run . Unemployment rate of low-skilled agents follows the opposite dynamic, because if η increases, the share of low-skilled agents among all immigrants falls down. When the immigrants come at time period 0, they do not increase the number of workers (the impulse responses of the number of workers are 0 (see Table 14 and notice that in the short-run (t=0) the impulse responses of total working time of high- and

Variable	Short-run				Long-run			
η	0	0.07	0.08	0.12	0	0.07	0.08	0.12
Y_t	-0.102	-0.096	-0.095	-0.091	0.152	0.174	0.177	0.189
y_t	-0.502	-0.496	-0.495	-0.491	-0.248	-0.226	-0.223	-0.210
x_t	-1.141	-1.121	-1.118	-1.106	-0.151	-0.127	-0.123	-0.109
K_t	-0.059	-0.058	-0.057	-0.057	0.077	0.058	0.061	0.073
k_t	-0.459	-0.458	-0.457	-0.457	-0.361	-0.342	-0.339	-0.327
τ_t^l	1.984	2.037	2.044	2.075	0.748	0.693	0.685	0.654
c_t^h	-0.166	-0.202	-0.207	-0.228	-0.011	-0.038	-0.041	-0.056
c_t^l	-0.812	-0.798	-0.796	-0.789	-0.749	-0.662	-0.649	-0.600
w_t^h	0.219	0.203	0.201	0.191	0.172	0.134	0.128	0.106
w_t^l	-0.270	-0.249	-0.246	-0.234	-0.575	-0.501	-0.490	-0.447
u_t^h	0	1.087	1.242	1.863	0	0.051	0.058	0.087
u_t^l	6.157	5.726	5.664	5.418	0.770	0.716	0.708	0.677
H_t^h	-0.306	-0.284	-0.280	-0.268	0.005	0.061	0.069	0.101
H_t^l	0.135	0.123	0.121	0.114	0.678	0.632	0.626	0.600
h_t^h	-0.306	-0.284	-0.280	-0.268	0.005	0.008	0.008	0.010
h_t^l	0.135	0.123	0.121	0.114	-0.022	-0.019	-0.018	-0.016

Table 23: The impact of parameter η on the main variables (in per cent).

low-skilled agents is equal to the impulse responses of the working time of high- and low-skilled agents correspondingly, the reason is specific employment dynamic process which was described before)). Immigration just increases unemployment rate, hence the government needs to pay more unemployment reliefs, which are bigger to the high-skilled unemployed. It is the reason why in the short-run the labor income tax rate increases more when more high-skilled immigrants come (or when η is higher). The raised tax rate causes the decrease of *net* wages of high-skilled agents, so they tend to work less and have more leisure. Thus the firms substitute high-skilled labor with low-skilled workers. So working hours of each high-skilled agent decreases, when η increases, and for low-skilled agents on the contrary. The latter work more in order to compensate the increased labor income tax, but then their salary as well as consumption decreases. Larger increase in tax rate and unemployment rate (consumption doesn't depend on the employment status of the agent, i.e. the family members insure themselves (see Appendix A)) of course cause higher decrease in consumption of high-skilled agent (see Table 23).

When the parameter η increases, the working time of high-skilled agents decreases less comparing to the steady state in the short-run and increases more in the long-run. As the working hours of high-skilled labor is highly correlated (see Table 21)with output per capita (so with total output also), hence output follows the dynamic of individual working hours of the high-skilled agents, thus when η increases, output per capita and in aggregate terms increase.

As I discussed above, that the family members insure themselves, thus when the value of the parameter η is bigger, there are more unemployed high-skilled agents in the short-run, so in order to smooth the consumption over the time, high-skilled agents decrease investment (when η is higher, then investment declines more). The decreased investment certainly affects the capital stock, and finally the output (when η is higher, the decrease of the output is bigger). This holds only in the short-run. In the long-run high-skilled agents build more capital, so if η is higher, investment and capital stock are higher (more high-skilled agents can build more capital). Consequently, total output and output per capita are bigger, when η is higher.

Now consider the last variation of the model, i.e. the stochastic process. In the analysis beside the benchmark (the ratio between the new-comers and the total German labor force in the steady state is 0.4 per cent) other 3 scenarios are considered: sudden increase of the working force by 1 % (it is a standard case in *the Toolkit*), the second-order autoregressive process (with only permanent immigration; the immigration lasts for 10-12 years) and the second-order autoregressive process (with partially temporal immigration: the immigrants arrive when $t \in [0;4]$, some of them will depart when $t \in [5;9]$, since the disturbances in the later periods ($t > 9$) are relatively small, I assume that the net immigration is 0 and has no impact on the other variables) (see Appendix C for more information). The values of other parameters remain unchanged. In this case let's consider these variables: Y_t, y_t, x_t, K_t, k_t, τ_t^l, c_t^h, c_t^l, w_t^h, w_t^l, H_t^h, H_t^l, h_t^h and h_t^l. The corresponding dynamics of total labor, total high-skilled labor and total low-skilled labor are shown in Figures 6, 7 and 8 (in the page 53).

If we compare the 1st scenario with the benchmark, we will see that all values of the impulse responses are 2.5 times bigger in the 1st scenario. It is what we should expect, since the shock in the 1st scenario is 2.5 times bigger (1/0.4=2.5).

The second scenario of the stochastic process is quite interesting in that extent that in the time period t=0 the same amount of immigrants comes (0.4 % of existing labor force) as in the benchmark, so the unemployment rates in both cases in the short-run (t=0) increase by the same amount (see Table 24). But surprisingly the impulse responses of other variables are different. Why? Because in the case of the 2nd scenario after the immigrants start arriving to the host country in t=0 the agents know that at time period

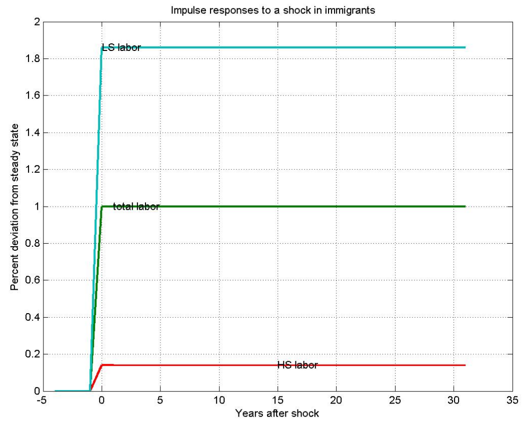

Figure 6: The impulse responses of total labor in case of the first scenario.

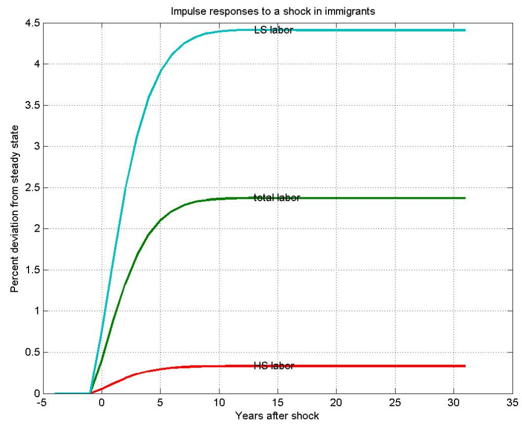

Figure 7: The impulse responses of total labor in case of the second scenario.

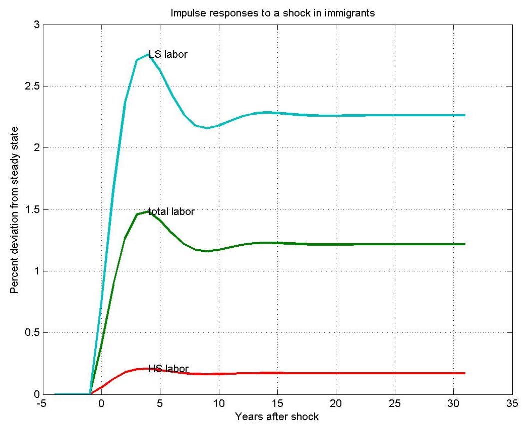

Figure 8: The impulse responses of total labor in case of the third scenario.

Variable	benchmark	$1st$ scen.	$2nd$ scen.	$3rd$ scen.
Y_t	-0.096	-0.239	0.103	0.019
y_t	-0.496	-1.239	-0.297	-0.381
x_t	-1.121	-2.802	0.427	-0.224
K_t	-0.058	-0.144	0.066	0.014
k_t	-0.458	-1.144	-0.334	-0.386
τ_t^l	2.037	5.092	1.91	1.963
c_t^h	-0.202	-0.505	-0.592	-0.428
c_t^l	-0.798	-1.996	-0.557	-0.659
w_t^h	0.203	0.507	-0.071	0.044
w_t^l	-0.249	-0.623	-0.032	-0.123
u_t^h	1.087	2.717	1.087	1.087
u_t^l	5.726	14.314	5.726	5.726
H_t^h	-0.284	-0.709	0.172	-0.019
H_t^l	0.123	0.308	0.138	0.132
h_t^h	-0.284	-0.709	0.172	-0.019
h_t^l	0.123	0.308	0.138	0.132

Table 24: The impact of the stochastic process on the main variables in the short-run (in per cent).

Variable	benchmark	$1st$ scen.	$2nd$ scen.	$3rd$ scen.
Y_t	0.174	0.435	0.897	0.546
y_t	-0.226	-0.565	-1.466	-0.626
x_t	-0.127	-0.317	-1.121	-0.214
K_t	0.058	0.146	0.225	0.190
k_t	-0.342	-0.854	-2.139	-0.982
τ_t^l	0.693	1.732	4.684	1.883
c_t^h	-0.038	-0.094	-0.351	-0.101
c_t^l	-0.662	-1.654	-3.980	-1.922
w_t^h	0.134	0.335	0.770	0.380
w_t^l	-0.501	-1.252	-2.860	-1.494
u_t^h	0.051	0.127	0.374	0.149
u_t^l	0.716	1.789	6.402	1.49
H_t^h	0.061	0.153	0.247	0.227
H_t^l	0.632	1.581	3.513	1.914
h_t^h	0.008	0.019	-0.064	0.071
h_t^l	-0.019	-0.047	-0.050	-0.072

Table 25: The impact of the stochastic process on the main variables in the long-run (in per cent).

t=1 more immigrants will come (the stochastic process is known to the agents), thus the existing labor behaves differently than in the benchmark case. Expecting the raise in taxes in the next period, high-skilled agents at time period t=0 work more hours in order to build more capital (get more dividends later) and work less in the next periods (see Figures 9 and 11). Since they work more, their salary decreases (according to the FOCs of the firms). Low-skilled agents also, knowing that labor income tax will increase in the next period more, work more than in the benchmark case in t=0. The government needs to increase the labor income tax rate but not so much than in the benchmark case, since it collect more labor income tax (the agents work more). While labor works more, total output increases (see Table 24).

Despite the inflow of the immigrants in $t = 0$ is the same in 2nd and 3rd scenarios, the results are different, since labor knows that some immigrants will depart later, so labor in 3rd scenario works less than in 2nd. Consequently output in the short-run in case of 2nd scenario is higher and taxes smaller etc.. In the long-run we observe that more immigrants cause[30] higher increase in high-skilled wages and bigger reduction in wages of the low-skilled workers. Since more immigrants cause bigger unemployment rate, thus in the second scenario we have the highest labor income tax. As I discussed before, the increase in tax rate retrenches the consumption of both types of labor. Hence in the case of the 2nd scenario consumption is the lowest in the long-run. In the long-term capital stock per capita, output per capita and investment per capita also decline more when the quantity of immigrants increases, besides capital and output in the aggregate values increase when the scale of immigration expands. In the long-run the welfare of agents as well as output per capita are likely to be the highest in the benchmark case, then in the 1st scenario, 2nd, and in the 3rd scenario the welfare of the agents and output per capita are the lowest (see Table 25). We cannot be so precise in the short-run, since the results are onerous to gauge (see Table 24), i.e. in some cases consumption is higher, in other cases leisure is higher. We can draw the conclusion that the immigration dominated by the unskilled workers is not beneficial for the population of the host country (i.e. Germany), even high-skilled agents can obtain consumption level, which exceeds the initial steady state value, more than 10 year after the start of immigration (see Table 20).

According to the outcomes of the shock in immigrants, we observe that the impulse responses of total output as well as of other variables extremely depend on the the

[30] The smallest number of immigrants is in the benchmark case, then in the 1st scenario, further in 3rd, and biggest number of the immigrants is in the case of the 2nd scenario.

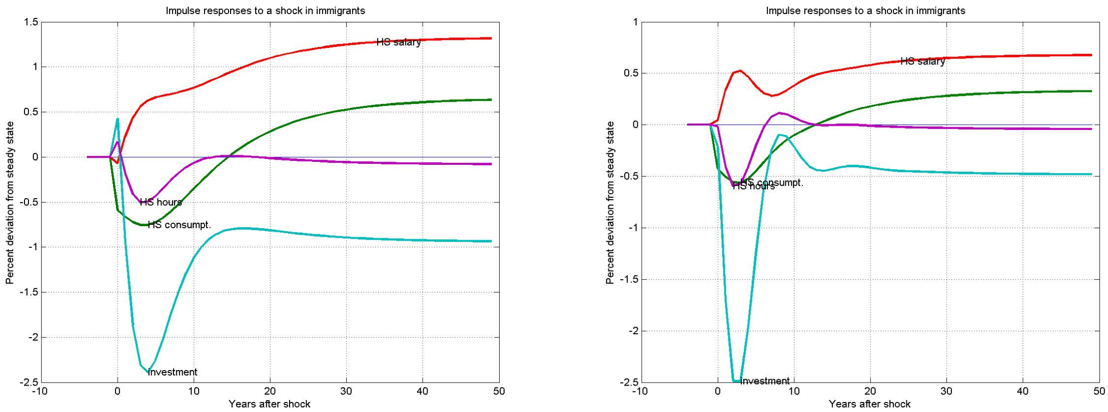

Figure 9: The impulse responses for the high-skilled agents in case of 2nd (left) and 3rd (right) scenarios.

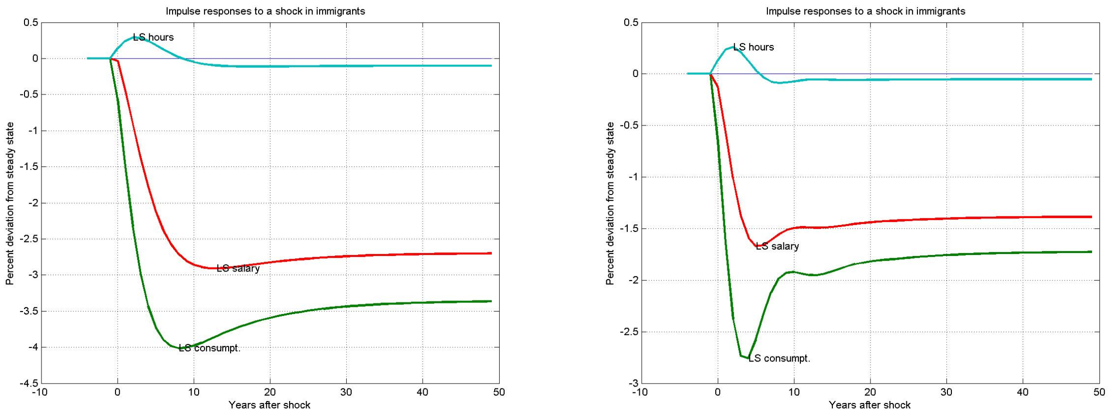

Figure 10: The impulse responses for the low-skilled agents in case of 2nd (left) and 3rd (right) scenarios.

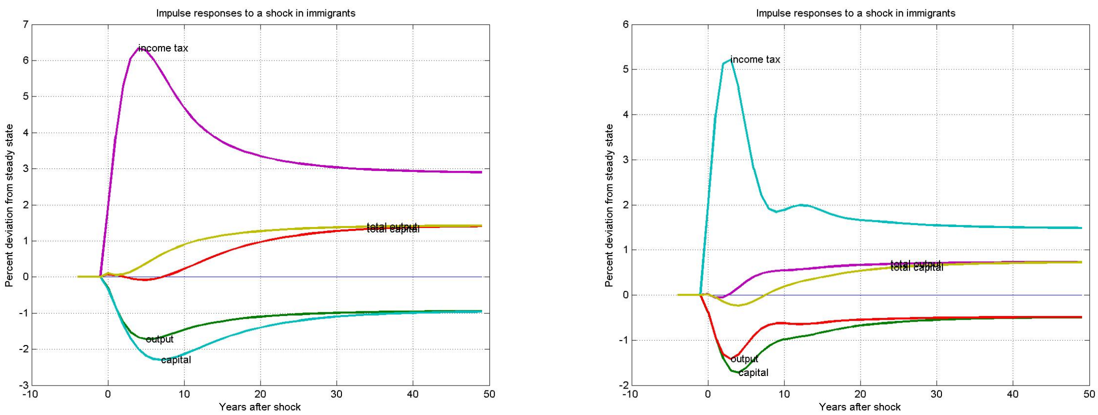

Figure 11: Other impulse responses in case of 2nd (left) and 3rd (right) scenarios.

expenses of the government. The government pays unemployment benefits which depend on the parameter r (*replacement ratio*). Let's check how sensitive are the outcomes in the short-run in the benchmark case and other three scenarios of the stochastic process.

r	benchmark		1*st* scen.		2*nd* scen.		3*rd* scen.	
	Y	τ^l	Y	τ^l	Y	τ^l	Y	τ^l
0.33	0.010	1.356	0.024	3.389	0.114	1.291	0.070	1.318
0.40	-0.012	1.495	-0.030	3.739	0.111	1.418	0.060	1.451
0.45	-0.027	1.596	-0.068	3.989	0.110	1.509	0.052	1.546
0.50	-0.043	1.696	-0.107	4.239	0.108	1.600	0.045	1.640
0.55	-0.058	1.796	-0.146	4.490	0.107	1.691	0.037	1.735
0.60	-0.074	1.896	-0.185	4.740	0.105	1.783	0.029	1.830
0.67	-0.096	2.036	-0.239	5.092	0.103	1.91	0.019	1.963

Table 26: The impact of parameter r on total output and labor income tax rate at time t=0 (in per cent).

r	benchmark		1*st* scen.		2*nd* scen.		3*rd* scen.	
	Y	τ^l	Y	τ^l	Y	τ^l	Y	τ^l
0.33	0.204	0.556	0.515	1.391	1.094	3.700	0.622	1.544
0.40	0.198	0.584	0.495	1.461	1.054	3.902	0.607	1.614
0.45	0.196	0.604	0.484	1.511	1.025	4.046	0.596	1.663
0.50	0.189	0.624	0.473	1.561	0.996	4.191	0.584	1.713
0.55	0.185	0.644	0.462	1.611	0.967	4.335	0.573	1.763
0.60	0.180	0.665	0.451	1.661	0.938	4.480	0.562	1.813
0.67	0.174	0.693	0.435	1.732	0.897	4.684	0.546	1.883

Table 27: The impact of parameter r on total output and labor income tax rate at time t=10 (in per cent).

In the short-run (t=0) and in the long-run (t=10) the dynamic of output and labor income tax rate in all four cases is the same (see Tables 26 and 27): when replacement ratio increases, total output declines and labor income tax increases. Thus we can state that, no matter what the stochastic process is, the total output is bigger and taxes are smaller, when r is smaller. Consequently the sensitivity analysis for one case will represent the nature of the dynamic of the economic indicators for all four cases. Let's assume benchmark case in a short-run. Output tends to increase when r declines, since then both types of agents work more (see Table 28) in order to compensate the decrease of their income from unemployment benefits of whole family, but then their wages decrease. When the government cuts unemployment reliefs, the average consumption of each member of the *unskilled* family slightly decreases, but the consumption of the skilled agents slightly increases. Table 28 shows that wages are higher when the *"out-of-work"* ratio is higher. It might be interpreted that bigger r increases the reservation wage.

Variable	$r = 0.67$	$r = 0.33$
c_t^h	-0.202	-0.126
c_t^l	-0.798	-0.818
w_t^h	0.203	0.161
w_t^l	-0.249	-0.344
h_t^h	-0.284	-0.134
h_t^l	0.123	0.320

Table 28: The impact (in per cent) of parameter r on consumption, wages and working hours in t=0 in the benchmark case.

8. Discussion

In this paper I developed a model which is quite suitable to analyze the economic effect of immigration on the labor market, public finance and the whole economy and explored the impact of the proposed immigration to the German labor market and economy. The model is able to reproduce the persistence which is characteristic for the German economy. Throughout the analysis in this paper we have seen that because of the proposed immigration in the long-run (i.e. after 10 year and more) the welfare of the high-skilled agents increase (the consumption increases and working time declines (so the leisure increases)), but the welfare of the low-skilled individuals decreases since their consumption declines and leisure almost converges to the steady state. The short-run effects of the immigration is not beneficial for the both types of agents, because their welfare declines. The negative effects of the immigration are bigger for the low-skilled agents, since the unskilled workers dominate among the immigrants. This result is consistent with the corresponding literature (Leibfritz, O'Brien, Dumont, 2003). The effects on the labor market are the increased unemployment rates of both labor types, decreased wages of the unskilled workers and bigger wages of the skilled workers. Nevertheless the changes of the economic variables are small due to the conditionally small amount of the newcomers comparing to the existing German labor force. The behavior of household representative is determined by the average of the *net* income falling to each member of the large extended family including wages, unemployment benefits and dividends in case of skilled labor. The variation of each source of income automatically changes the decision of agent how long to work or how much to invest since the individual wants to smooth consumption and leisure over the time.

Following the benchmark model, the immigration will decrease GDP by nearly 0.1 % and GDP per capita by 0.5 %. The generally received measurement of the welfare of the population is GDP per capita not total GDP. The analyzes showed that in all scenarios GDP per capita (output per capita, in Figures just *"output"*) due to the possible immigration declines, since the most of the immigrants are considered to be unskilled as the empirical evidences indicate (Arackal, 2000). Thus we can make a conclusion that the immigration won't increase the welfare of an existing population in Germany (but almost for sure the welfare of the immigrants will increase due to the existing wages difference in Germany and new members of the EU), but will decrease.

The model successfully combines the individual hours worked and unemployment while in many cases the authors confine themselves strictly to one of them (Canova and Ravn, 2000, Moreno-Galbis and Sneessens, 2004 etc.). The most of deviations of total output are determined by the changes in individual hours worked (especially by high-skilled agents because they are $\phi = 2$ times more productive than low-skilled ones), keeping in mind the fixed working week[31], this implication might be explained by the changes in the overtime or additional day-offs, since the impulse responses of individual working hours are smaller than 0.5 % in all cases. Despite the model predicts that the government will increase tax rate in case of the immigration, certainly it won't happen, just the positive impulse responses of labor income tax show the increasing tightness of fiscal budget and that due to the newcomers the prerequisites for decreasing labor income tax rate won't be created.

The theoretical importance of the analysis is related to the contradiction of Kemnitz (2003) proposition of magnitude of the elasticity of substitution between high and low skilled labor. I found that the impact of this parameter is not significant (see Table 22), when it varies inside the range consistent with empirical evidences.

The calibration of the model implies that the ratio between wages of high- and low-skilled agents is higher than 2 (as in Canova and Ravn (2000)), this might contradict to Manacorda and Petrongolo (1999), since they argue that it is 1.4, but they used different definitions of the high- and low-skilled labor, so both outcomes might be possible. According to the calibration of the model of Canovaa and Ravn (2000), the high-skilled

[31]It is 35 work hours per week in Germany.

agent sleeps less than 4 (≈3.43) hours per day[32], since I was following that model (in case of utility and production functions), the model of this paper implies similar results. Furthermore, there might be various interpretations regarding holidays, weekends etc..

The German authority in order to relief the impact of immigration could try different legal methods (e.g. tax deductions etc.) to attract more high-skilled immigrants or consider about decreasing *the out-of-work benefit*. The latter would reduce the possible tightness of the fiscal budget, diminish the reservation salary and finally would increase the incentives for the unemployed to find a job. Moreover the important issue is the number of the immigrants. I showed in this paper that the less immigrants come, the better is. Thus the authority of Germany could attempt to create some barriers which wouldn't violate the so called free labor movement (e.g. to introduce high labor income or other taxes for single, since most of the immigrants coming to the UK are single etc.).

The main disadvantage of the method used in the analysis is considered to be a linear approximation of the equations characterizing equilibrium. Nevertheless in our case this method is expedient, since the deviations from steady state are very small (usually less than 1 %), and employing this method we have an opportunity to explore the dynamics of variables.

9. Summary and Concluding Remarks

Being the first one of its kind, this paper analyzes the impact of the prospective inflow of Eastern European workers on the German labor market (e.g. unemployment, wages and labor composition) and main economic variables as GDP, GDP per capita etc.. The model was developed following Canova and Ravn (2000) and Merz (1995, 1997). I showed that in the long-term high-skilled labor will be better off, since the welfare of low-skilled labor will slightly decrease. The analysis showed that the proposed immigration tenuously would increase unemployment rates, total output, but due to the fall of the average productivity of labor, output per capita as measurement of the welfare would slightly decrease. The model suggests that the prospective immigration would increase the tightness of fiscal budget which might be mitigated through the decreasing the unemployment benefits. The deeper analysis showed that the elasticity of substitution between high- and low-skilled labor did not have significant influence. If

[32] 36/((24-3.43)*7)=0.25 (Author's calculation).

the share of high-skilled immigrants increases, the fall of output per capita is smaller, consequently the welfare of an existing German population would decrease less. I found that the impact of the immigration strongly depends on the nature of it. The amount of immigrants is important as well as the expected dynamic of the inflow. I do not claim that the acquired results could perfectly capture the dynamics of German economy in case of the proposed immigration, since the inflow of immigrants is difficult to predict (as in case of the UK) and it[33] is beyond the scope of this paper.

The results (impulse responses) were obtained employing the procedure described in Uhlig (1999). The latter method consists of several steps. First of all, I collected the equations that define the equilibrium of the model including constraints, identities, first order conditions, exogenous processes and other necessary equations. Second, I solved for the steady state, i.e. I provided the formulas for all variables. In the third step I obtained the log-linearized equations and finally using the method of undetermined coefficients and the *Toolkit* I got impulse responses and second moments (for Matlab code see Appendix E).

As the potential extensions of the developed model I see the improving the dynamic of employment (not to assume that vacancies posted are constant), the introducing growth rate of population (for different labor groups it might be also different if the empirical data supports that) and the constant growth of GDP (total output). Another logical expansion of the model is to let the government change other taxes not only labor income tax. Or let the government borrow money in the bond market. If so, the government could use the credit market opportunities in order to apply its undertakings.

In addition, the good candidate for an extension of the model could be the upgrading the current model to the overlapping generations (OLG) model in spite of the fact that the *"migration can mitigate, but not solve the demographic problem in the receiving countries"* (Brücker, Kohlhaas, 2004). As the realistic opportunity of the perfection of the model is the improving the stochastic process in a manner of the temporal immigration.

All above sketched possible extensions of the developed model in this paper could be considered as the prospective research proposals.

[33]The dynamic of immigration process.

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A. Utility Function

Let's analyze the general case which fits for both high- and low-skilled families. The members of the family are maximizing the utility of whole family:

$$\begin{aligned}
U_t &= n_t(logc_t^e + A\ln l_t) + u_t logc_t^u, \\
\text{s.t. } l_t &+ h_t = 1, \\
(1 &+ \tau^c)(n_t c_t^e + u_t c_t^u) + \frac{x_t}{q_t} = (1-\tau_t^l)h_t w_t n_t + r(1-\bar{\tau}^l)\bar{h}\bar{w}u_t + (1-\tau^k)\frac{k_{t-1}}{q_t}\frac{L_{t-1}}{L_t}d_t, \\
k_t &= (1-\delta)k_{t-1}\frac{L_{t-1}}{L_t} + x_t,
\end{aligned}$$

where c_t^e and c_t^u denote the consumption of employed and unemployed members of the family. The explanation of other variables is in the main part of the paper. *Lagrangian* function:

$$\begin{aligned}
L_{\{c_t^e, c_t^u, h_t, k_t\}} &= E_0\{\sum_{t=1}^{\infty}\beta^t[n_t(logc_t^e + A\ln(1-h_t)) + u_t logc_t^u - \lambda_t((1+\tau^c)(n_t c_t^e + u_t c_t^u) + \frac{k_t}{q_t^h} - \\
&- (1-\delta)\frac{k_{t-1}}{q_t^h}\frac{L_{t-1}}{L_t} - (1-\tau_t^l)h_t^h w_t^h n_t^h - r(1-\bar{\tau}^l)\bar{h}^h\bar{w}^h u_t^h - (1-\tau^k)d_t\frac{k_{t-1}}{q_t^h}\frac{L_{t-1}}{L_t})]\}.
\end{aligned}$$

FONCs:

$$\begin{aligned}
\frac{\partial L}{\partial c_t^e} : \ \Rightarrow \frac{1}{c_t^e} &= \lambda_t(1+\tau^c), \\
\frac{\partial L}{\partial c_t^u} : \ \Rightarrow \frac{1}{c_t^u} &= \lambda_t(1+\tau^c),
\end{aligned}$$

Thus $c_t^e = c_t^u = c_t$, then the utility function of each member of the family is:

$$logc_t + n_t A\ln l_t.$$

B. The Case: $\rho = 1$

In case $\rho = 1$, we have the division by 0. After the applying L'Hopital rule we obtain these equations:

$$\begin{aligned}
\text{instead of (16): } Y_t &= \left[\left(H_t^h\right)^\phi H_t^l\right]^{\frac{\alpha}{\phi+1}} K_{t-1}^{1-\alpha}, && (91)\\
\text{instead of (17): } y_t &= \left[\left(h_t^h n_t^h q_t^h\right)^\phi h_t^l n_t^l q_t^l\right]^{\frac{\alpha}{\phi+1}} \left(k_{t-1}\frac{L_{t-1}}{L_t}\right)^{1-\alpha}, && (92)\\
\text{instead of (38): } w_t^h &= \frac{\phi}{\phi+1}\cdot\frac{\alpha Y_t}{H_t^h}, && (93)\\
\text{instead of (39): } w_t^l &= \frac{1}{\phi+1}\cdot\frac{\alpha Y_t}{H_t^l}. && (94)
\end{aligned}$$

When $\rho = 1$, the production function, I use, is identical to one used in Moreno-Galbis and Sneessens (2004). Their production function (keeping my notation):

$$Y_t = zK_t^{1-\mu}\left[(H_t^h)^{\theta_t^h}(H_t^l)^{\theta_t^l}\right]^\mu, \quad \text{when} \quad \theta_t^h + \theta_t^l = 1.$$

In my case:

$$Y_t = K_{t-1}^{1-\alpha}\left[(H_t^h)^{\frac{\phi}{\phi+1}}(H_t^l)^{\frac{1}{\phi+1}}\right]^\alpha, \quad \text{and} \quad \frac{\phi}{\phi+1} + \frac{1}{\phi+1} = 1.$$

The steady state values of relevant equations:

$$\begin{aligned}
\text{instead of (54): } \bar{y} &= \left[\left(\bar{h}^h\bar{n}^h\bar{q}^h\right)^\phi \bar{h}^l\bar{n}^l\bar{q}^l\right]^{\frac{\alpha}{\phi+1}}\left(\frac{1-\alpha}{\bar{d}}\right)^{\frac{1-\alpha}{\alpha}}, && (95)\\
\text{instead of (59): } \bar{w}^h &= \frac{\phi}{\phi+1}\cdot\frac{\alpha\bar{Y}}{\bar{H}^h}, && (96)\\
\text{instead of (60): } \bar{w}^l &= \frac{1}{\phi+1}\cdot\frac{\alpha\bar{Y}}{\bar{H}^l}. && (97)
\end{aligned}$$

The log-linearized equations are:

$$\begin{aligned}
\text{instead of (70): } 0 &= -\hat{y}_t + \frac{\phi\alpha}{\phi+1}\hat{h}_t^h + \frac{\phi\alpha}{\phi+1}\hat{n}_t^h + \frac{\phi\alpha}{\phi+1}\hat{q}_t^h + \frac{\alpha}{\phi+1}\hat{h}_t^l + \frac{\alpha}{\phi+1}\hat{n}_t^l + \\
&+ (1-\alpha)\hat{k}_{t-1} + (1-\alpha)\hat{L}_{t-1} - (1-\alpha)\hat{L}_t, && (98)\\
\text{instead of (71): } 0 &= -\hat{w}_t^h + \hat{Y}_t - \hat{H}_t^h, && (99)\\
\text{instead of (72): } 0 &= -\hat{w}_t^l + \hat{Y}_t - \hat{H}_t^l. && (100)
\end{aligned}$$

C. Other Scenarios

Other variations of the model are based on the different calibration. Three things are diverse:

ρ - the inverse of elasticity of substitution between low- and high-skilled labor;

η - the share of high-skilled immigrants among all the immigrants;

i_t - the stochastic process.

As discussed in the sections of Literature and Calibration of the Model, I will consider about these values of the parameter ρ:

Source	Wapler (2001), Moreno-Galbis and Sneessens (2004)	Kemnitz (2003)
ρ	1	< 1 (e.g. 0.91)

Table 29: Other values of the parameter ρ. The value of 0.91 is taken by the author not from Kemnitz (2003). It corresponds to 1.1 elasticity of substitution between low- and high-skilled labor.

For the case $\rho = 1$ we need to replace some equations in order to avoid the division by 0 and get feasible results (see Appendix B).

For the share of high-skilled immigrants among all the immigrants (η) I considered these values in the variations of the model:

Source	-	Pohl (2005)	Pohl (2005)
ρ	0	0.08	0.12

Table 30: Other values of the parameter η. The value of 0 is taken by the author. Other values (0.08 and 0.12) are from Pohl (2005).

In the model's variations I will consider these different stochastic processes:

1. the immigrants are coming only once, the amount of them is 1 per cent of total German labor force in the steady state (it is a standard case in the *Toolkit*);

2. the second-order autoregressive process when in the first year the amount of immigrants is 0.4 per cent of total German labor force, in the next time period the number of immigrants reaches the maximum (or almost 0.5 % of total German

labor force in the steady state), and later converges to zero, whole immigration lasts 10-12 year until the process converges to 0; let's assume that the second-order autoregressive process is:

$$i_t = 1.221257 i_{t-1} - 0.39 i_{t-2} + d_t; \tag{101}$$

3. the second-order autoregressive process when in the first year the amount of immigrants is 0.4 per cent of total German labor force, in the next time period the number of immigrants reaches the maximum (or almost 0.5 % of total German labor force in the steady state), after 4 years the process becomes negative (it corresponds to temporal immigration: some immigrants come back home), the minimum is reached at time period 6 (≈-0.10) and from time period 9 the net immigrants is almost 0; the corresponding second-order autoregressive process is:

$$i_t = 1.221257 i_{t-1} - 0.55 i_{t-2} + d_t. \tag{102}$$

In order to implement the last two scenarios one needs to add the new variable into the variable list (more about *Toolkit* see Uhlig (1999)), change the matrices DD, LL and MM. i.e. to add the second column which consists only of zeros, also change matrix NN. For the second case NN should be[34]:

$$\begin{matrix} [1.221257, & -0.39 \\ 1, & 0]. \end{matrix}$$

For the third case NN should be:

$$\begin{matrix} [1.221257, & -0.55 \\ 1, & 0]. \end{matrix}$$

Also the correlation matrix (*Sigma*) of the shock must be changed to:

$$\begin{matrix} [1, & 0 \\ 0, & 0000001]. \end{matrix}$$

Actually the number in the second line on the right also should be 0, but then the *Toolkit* cannot do the simulations, so I took very small number, which is close to 0.

[34]This stochastic process is made referring with Weder (2001), because it nicely replicates the desirable process (first, the amount of immigrants increases, later it decreases).

D. Calibration for the UK Economy

For the economy of the United Kingdom the model was calibrated to match the most recent annual country data and to use standard parameter values whenever possible. The values of calibration are quite similar to those used to calibrate German economy. The values of parameters for the UK economy are given in Table 31 and steady state values of the variables (which are known) in Table 32.

α	β	δ	τ^c	τ^k	r	A	ρ	η	ϕ	λ	β_N
$\frac{2}{3}$	0.96	0.08	0.175	0.175	0.33	2.52	0.8	0.07	2	0.05	0.064

Table 31: The calibration of the model for the UK economy: values of parameters.

The labor share α is set equal to $\frac{2}{3}$ (as in Riley, Young (2003)). The subjective discount factor β is 0.96 in order the steady state value of interest rate would be 4 %. Depreciation rate δ was calibrated to 0.08 in order to be consistent to the UK data (i.e. investment-output and capital-output ratios, see Table 33). Tax on consumption in the model is the value added tax (VAT), so it was calibrated to its current (in year 2005) value, i.e. 17.5 %. Tax on dividends is set to 0.175[35]. The replacement ratio r (out of work benefits) is 0.33 (according to OECD). The parameter A is chosen to be 2.52 in order the unskilled agents use 30 % of their non-sleeping time to work (see Table 33). The inverse of elasticity of substitution between low- and high-skilled labor is 0.8 (it corresponds to 1.25 elasticity of substitution between low- and high-skilled labor) as in Abrego, Whalley (2000). In my paper I assumed that the ratio between high-skilled immigrants and the total immigrant η is equal to 7 %, as it consistent with empirical evidences (Arackal, 2000). The productivity difference between high-and low-skilled hours ϕ is set to 2 following Canova and Ravn (2000) and assuming that this calibration also fits to the UK economy. The households' bargaining power λ is 0.5, and job destruction rate β_N is set to 0.064 as in Merz (1999).

$\bar{L}$	$\bar{u}^h$	$\bar{u}^l$	$\bar{q}^h$	$\bar{q}^l$	$\bar{\tau}^l$
31936000	0.037	0.111	0.463	0.537	0.159

Table 32: The calibration of the model for UK economy: the steady state values of the variables (which are known).

The total labor force $\bar{L}$ is calibrated to the number of economically active population

[35] It was calculated according to the formula: tax on dividends = overall personal income tax + corporate income tax rate − corporate income tax on distributed profits rate. *Source: OECD.*

in 2005 (according to OECD). The unemployment rates among the different skills groups $\bar{u}^h$ and $\bar{u}^l$) as well as the shares of skilled and unskilled agents were determined according to Bassanini, Rasmussen and Scarpetta (1999). The labor income tax is set to 15.9 %, it is the average personal income tax without social security contributions in Great Britain in 2004 (according to OECD).

Calibration of the model implies that investment-output ratio is 0.18 and capital-output ratio is 2.26 (see Table 33) what nearly corresponds to the data of National Institute of Economic and Social Research (UK)[36]. The values of parameters imply that high-skilled hours is 0.243.

$\frac{\bar{x}}{\bar{y}}$	$\frac{\bar{k}}{\bar{y}}$	$\bar{h}^h$	$\bar{h}^l$	$\bar{R}$
0.18	2.26	0.243	0.30	1.04

Table 33: The selected steady state values.

The stochastic process or the inflow of the immigrants implies that the immigrants are coming only once, the amount of them is 0.4 per cent of total UK labor force in the steady state. This number corresponds to the percentage of the new registered immigrants during 11 months after May 1, 2004 comparing to the total British labor force. This stochastic process will be implemented in *Toolkit* multiplying the DD matrix by 0.4[37].

[36] According to National Institute of Economic and Social Research these values should be 0.17 and 2.30 correspondingly.

[37] More about *Toolkit* see Uhlig (1999).

E. Matlab Codes

E.1. Benchmark and 1st scenario

```
% Master Thesis "The Inflow of Eastern European Workers to German Labor
% Market: Consequences and Policy Issues"
% MATLAB code for German economy when rho<>1 (benchmark and 1st scenario)
% Author: Sigitas KARPAVICIUS
% Humboldt-Universität zu Berlin, SS2005
% Revised on 24th August, 2005

%disp('Hit any key when ready...');
%pause;


% Setting parameters: (constant values):

alpha      = 0.64;    % Labor share
beta       = 0.96;    % Discount factor
delta      = 0.08;    % Depreciation rate
tau_c      = 0.16;    % VAT
tau_k      = 0.137;   % Tax rate on dividends
r          = 0.67;    % Unemployment benefits
A          = 2.42;    % Preference parameter
rho        = 10/9;    % Inverse of elasticity of substitution between high- and low-skilled labor
eta        = 0.07;    % Share of high-skilled workers among the immigrants
phi        = 2;       % Productivity difference between high- and low-skilled workers
lambda     = 0.5;     % Bargaining power of unemployed
betan      = 0.064;   % Job destruction rate

% Values in the steady state:

%---------------------------Given---------------------------------------

L_bar      = 42806000;% Labor in mln. in 2003
uh_bar     = 0.049;   % Unemployment among high-skilled labor
ul_bar     = 0.115;   % Unemployment among low-skilled labor
qh_bar     = 0.500;   % Ratio of high-skilled labor and all the labor
ql_bar     = 0.500;   % Ratio of low-skilled labor and all the labor
tau_l_bar  = 0.196;   % Labor income tax rate

%------------------------Calculated--------------------------------------

% R_bar - Interest factor
R_bar = 1/beta;
% d_bar - Dividends rate
d_bar = (R_bar - 1 + delta)/(1-tau_k);
% nh_bar - Employment among high-skilled labor in 2002
nh_bar = 1- uh_bar;
% nl_bar - Employment among low-skilled labor in 2002
nl_bar = 1- ul_bar;
% Total number of high-skilled economically active agents
Lh_bar = L_bar * qh_bar;
% Total number of low-skilled economically active agents
Ll_bar = L_bar * ql_bar;
% Nh_bar - Total number of high-skilled workers
Nh_bar = nh_bar * Lh_bar;
% Nl_bar - Total number of low-skilled workers
Nl_bar = nl_bar * Ll_bar;
% hh_bar - Ratio of working time and total time of high-skilled labor
% alpha*phi/((phi*(nh_bar*qh_bar*hh_bar)^(1-rho)+(nl_bar*ql_bar*hl_bar)^(1-rho))*
%  *(nh_bar*qh_bar*hh_bar)^rho)*((1-tau_l_bar)*(1-hh_bar)/A-(1-tau_l_bar)*
```

```
%  *(nh_bar+r*uh_bar)*hh_bar)=(1-alpha)/qh_bar*(1-tau_k-delta/d_bar);
hh_bar = 0.244;
% hl_bar - Ratio of working time and total time of low-skilled labor
hl_bar = 1/(1+A*(nl_bar+r*ul_bar));
% Hh_bar - Total amount of time worked by high-skilled agents
Hh_bar = Nh_bar*hh_bar;
% Hl_bar - Total amount of time worked by low-skilled agents
Hl_bar = Nl_bar*hl_bar;
% y_bar - Output per capita (high- and low-skilled labor)
y_bar
=(phi*(nh_bar*qh_bar*hh_bar)^(1-rho)+(nl_bar*ql_bar*hl_bar)^(1-rho))^(1/(1-rho))*((1-alpha)/d_bar)^((1-alpha)/alpha);
% k_bar - Capital stock per capita (high- and low-skilled labor)
k_bar = (1-alpha)*y_bar/d_bar;
% K_bar - Total capital stock
K_bar = L_bar*k_bar;
% Y_bar - Total output
Y_bar = L_bar*y_bar;
% x_bar - Investment per capita (low- and high-skilled)
x_bar = delta*k_bar;
% wh_bar - wage of high-skilled worker per one unit of her/his working time
wh_bar = alpha*phi*Y_bar/((phi*Hh_bar^(1-rho)+Hl_bar^(1-rho))*Hh_bar^rho);
% wl_bar - wage of low-skilled worker per one unit of her/his working time
wl_bar = alpha*Y_bar/((phi*Hh_bar^(1-rho)+Hl_bar^(1-rho))*Hl_bar^rho);
% ch_bar - consumption of each high-skilled worker
ch_bar = wh_bar*(1-tau_l_bar)*(1-hh_bar)/(A*(1+tau_c));
% cl_bar - consumption of each low-skilled worker
cl_bar = (1-tau_l_bar)*wl_bar*hl_bar*(nl_bar+ul_bar*r)/(1+tau_c);
% Vh_bar - vacancies for high-skilled jobs posted
Vh_bar = betan^(1/(1-lambda))*Nh_bar*(nh_bar/(1-nh_bar))^(lambda/(1-lambda));
% Vl_bar - vacancies for low-skilled jobs posted
Vl_bar = betan^(1/(1-lambda))*Nl_bar*(nl_bar/(1-nl_bar))^(lambda/(1-lambda));


% Declaring the matrices.

VARNAMES = ['capital      ',    % 1
            'total labor  ',    % 2
            'HS labor     ',    % 3
            'LS labor     ',    % 4
            'HS workers   ',    % 5
            'LS workers   ',    % 6
            'output       ',    % 7
            'investment   ',    % 8
            'income tax   ',    % 9
            'dividends    ',    % 10
            'return       ',    % 11
            'HS consumpt. ',    % 12
            'LS consumpt. ',    % 13
            'HS salary    ',    % 14
            'LS salary    ',    % 15
            'HS ratio     ',    % 16
            'LS ratio     ',    % 17
            'HS employment',    % 18
            'LS employment',    % 19
            'HS unempl.   ',    % 20
            'LS unempl.   ',    % 21
            'total output ',    % 22
            'total capital',    % 23
            'HS hours     ',    % 24
            'LS hours     ',    % 25
            'tot HS hours ',    % 26
            'tot LS hours ',    % 27
            'immigrants   '];   % 28


% Translating into coefficient matrices.
```

```
% The loglinearized equations are, conveniently ordered:
% Equ. 1)  0 = tau_l_bar/(1-tau_l_bar)*tau_l(t) + cl(t) - wl(t) + hl_bar/(1-hl_bar)*hl(t)
% Equ. 2)  0 = - tau_l_bar/(1-tau_l_bar)*tau_l(t) -
%              - (1+tau_c)*cl_bar/((1-tau_l_bar)*hl_bar*wl_bar*nl_bar)*cl(t) +
%              + wl(t) + nl(t) + r*ul_bar/nl_bar*ul(t) + hl(t)
% Equ. 3)  0 = (1-tau_k)*d_bar/R_bar*d(t) - R(t) - Lh(t) + Lh(t-1)
% Equ. 4)  0 = -(1-delta)*k_bar*k(t-1) - (1-delta)*k_bar*L(t-1) - x_bar*x(t) +
%              +k_bar*k(t) + (1-delta)*k_bar*L(t)
% Equ. 5)  0 = (1-alpha)*k(t-1) + (1-alpha)*L(t-1) - (1-alpha)*L(t) - y(t) +
%            + phi*alpha*(nh_bar*qh_bar*hh_bar)^(1-rho)/a*qh(t) +
%            + alpha*(nl_bar*ql_bar*hl_bar)^(1-rho)/a*ql(t) +
%            + phi*alpha*(nh_bar*qh_bar*hh_bar)^(1-rho)/a*nh(t) + alpha*(nl_bar*ql_bar*hl_bar)^(1-rho)/a*nl(t)+
%            + phi*alpha*(nh_bar*qh_bar*hh_bar)^(1-rho)/a*hh(t) + alpha*(nl_bar*ql_bar*hl_bar)^(1-rho)/a*hl(t)
% Equ. 6)  0 =  - k(t-1) - L(t-1) + L(t) + y(t) - d(t)
% Equ. 7)  0 = - alpha*phi*Y_bar/(wh_bar*Hh_bar^rho)*wh(t) + alpha*phi*Y_bar/(wh_bar*Hh_bar^rho)*Y(t) -
%            - ((1-rho)*phi*Hh_bar^(1-rho)+rho*alpha*phi*Y_bar/(wh_bar*Hh_bar^rho))*Hh(t) -
%            - (1-rho)*Hl_bar^(1-rho)*Hl(t)
% Equ. 8)  0 = - alpha*Y_bar/(wl_bar*Hl_bar^rho)*wl(t) + alpha*Y_bar/(wl_bar*Hl_bar^rho)*Y(t) -
%            - (1-rho)*phi*Hh_bar^(1-rho)*Hh(t) - ((1-rho)*Hl_bar^(1-rho)+rho*alpha*Y_bar/(wl_bar*Hl_bar^rho))*Hl(t)
% Equ. 9)  0 = nh_bar*nh(t) + uh_bar*uh(t)
% Equ. 10) 0 = nl_bar*nl(t) + ul_bar*ul(t)
% Equ. 11) 0 = - nh(t) + Nh(t) - Lh(t)
% Equ. 12) 0 = - nl(t) + Nl(t) - Ll(t)
% Equ. 13) 0 = - qh(t) + Lh(t) - L(t)
% Equ. 14) 0 = - ql(t) + Ll(t) - L(t)
% Equ. 15) 0 = qh_bar*Lh(t) + ql_bar*Ll(t) - L(t)
% Equ. 16) 0 = Lh(t-1) - Lh(t) + eta/qh_bar*i(t)
% Equ. 17) 0 = Ll(t-1) - Ll(t) + (1-eta)/ql_bar*i(t)
% Equ. 18) 0 = - Nh(t) + lambda/nh_bar*(Vh_bar/(Lh_bar-Nh_bar))^(1-lambda)*Lh(t-1) +
%            + (1-betan-lambda*(Vh_bar/(Lh_bar-Nh_bar))^(1-lambda))*Nh(t-1)
% Equ. 19) 0 = - Nl(t) + lambda/nl_bar*(Vl_bar/(Ll_bar-Nl_bar))^(1-lambda)*Ll(t-1) +
%            + (1-betan-lambda*(Vl_bar/(Ll_bar-Nl_bar))^(1-lambda))*Nl(t-1)
% Equ. 20) 0 = (1-tau_k)*k_bar*d_bar/qh_bar*k(t-1) + (1-tau_k)*k_bar*d_bar/qh_bar*L(t-1) -
%            - (1-tau_k)*k_bar*d_bar/qh_bar*L(t) - x_bar/qh_bar*x(t) - tau_l_bar*wh_bar*hh_bar*nh_bar*tau_l(t) +
%            + (1-tau_k)*k_bar*d_bar/qh_bar*d(t) - (1+tau_c)*ch_bar*ch(t) + (1-tau_l_bar)*wh_bar*hh_bar*nh_bar*wh(t)+
%            + x_bar/qh_bar-(1-tau_k)*k_bar*d_bar/qh_bar*qh(t) + (1-tau_l_bar)*wh_bar*hh_bar*nh_bar*nh(t) +
%            + r*(1-tau_l_bar)*wh_bar*hh_bar*uh_bar*uh(t) + (1-tau_l_bar)*wh_bar*hh_bar*nh_bar*hh(t)
% Equ. 21) 0 = - tau_k*d_bar*k_bar*k(t-1) - tau_k*d_bar*k_bar*L(t-1) + tau_k*d_bar*k_bar*L(t) -
%            - (tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar+tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar)*tau_l(t) -
%            - tau_k*d_bar*k_bar*d(t) - tau_c*ch_bar*qh_bar*ch(t) - tau_c*cl_bar*ql_bar*cl(t) -
%            - tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar*wh(t) - tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar*wl(t) -
%            - tau_c*ch_bar*qh_bar-tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar+r*hh_bar*wh_bar*qh_bar*uh_bar*(1-tau_l_bar)*qh(t)-
%            - tau_c*cl_bar*ql_bar-tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar+r*hl_bar*wl_bar*ul_bar*ql_bar*(1-tau_l_bar)*ql(t)-
%            - tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar*nh(t) - tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar*nl(t) +
%            + r*uh_bar*qh_bar*hh_bar*wh_bar*uh(t) + r*hl_bar*wl_bar*ul_bar*ql_bar*ul(t) -
%            - tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar*hh(t) - tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar*hl(t)
% Equ. 22) 0 = y(t) + L(t) - Y(t)
% Equ. 23) 0 = k(t) + L(t) - K(t)
% Equ. 24) 0 = hh(t) + Nh(t) - Hh(t)
% Equ. 25) 0 = hl(t) + Nl(t) - Hl(t)
% Equ. 26) 0 = - tau_l_bar/(1-tau_l_bar)*tau_l(t) - ch(t) + wh(t) - hh_bar/(1-hh_bar)*hh(t)
% Equ. 27) 0 = E_t [ - ch(t+1) + R(t+1) +  ch(t) ]
% Equ. 28) z(t+1) = i(t+1)
%
% Additional variable introduced for simplicity:
a =
phi*(nh_bar*qh_bar*hh_bar)^(1-rho)+(nl_bar*ql_bar*hl_bar)^(1-rho);
%
% Endogenous state variables "x(t)": k(t), L(t), Lh(t), Ll(t), Nh(t), Nl(t).
% Endogenous other variables "y(t)": y(t), x(t), tau_l(t), d(t),  R(t),
% ch(t), cl(t), wh(t), wl(t), qh(t), ql(t), nh(t), nl(t), uh(t), ul(t),
% Y(t), K(t), hh(t), hl(t), Hh(t), Hl(t).
% Exogenous state variables  "z(t)": i(t).
% CHECK: 28 equations, 28 variables.
% Switch to that notation.  Find matrices for format:
% 0 = AA x(t) + BB x(t-1) + CC y(t) + DD z(t),
```

```
% 0 = E_t [ FF x(t+1) + GG x(t) + HH x(t-1) + JJ y(t+1) + KK y(t) + LL z(t+1) + MM z(t)],
% z(t+1) = NN z(t) + i(t+1) with E_t [ i(t+1) ] = 0.

% For k(t)          L(t)                    Lh(t)         Ll(t)            Nh(t)           Nl(t):
AA = [ 0,            0,                      0,            0,               0,              0       % Equ. 1)
       0,            0,                      0,            0,               0,              0       % Equ. 2)
       0,            0,                     -1,            0,               0,              0       % Equ. 3)
       k_bar,    (1-delta)*k_bar,            0,            0,               0,              0       % Equ. 4)
       0,          -1+alpha,                 0,            0,               0,              0       % Equ. 5)
       0,            1,                      0,            0,               0,              0       % Equ. 6)
       0,            0,                      0,            0,               0,              0       % Equ. 7)
       0,            0,                      0,            0,               0,              0       % Equ. 8)
       0,            0,                      0,            0,               0,              0       % Equ. 9)
       0,            0,                      0,            0,               0,              0       % Equ. 10)
       0,            0,                     -1,            0,               1,              0       % Equ. 11)
       0,            0,                      0,           -1,               0,              1       % Equ. 12)
       0,           -1,                      1,            0,               0,              0       % Equ. 13)
       0,           -1,                      0,            1,               0,              0       % Equ. 14)
       0,           -1,                    qh_bar,       ql_bar,            0,              0       % Equ. 15)
       0,            0,                     -1,            0,               0,              0       % Equ. 16)
       0,            0,                      0,           -1,               0,              0       % Equ. 17)
       0,            0,                      0,            0,              -1,              0       % Equ. 18)
       0,            0,                      0,            0,               0,             -1       % Equ. 19)
       0,    -(1-tau_k)*k_bar*d_bar/qh_bar,  0,            0,               0,              0       % Equ. 20)
       0,         tau_k*d_bar*k_bar,         0,            0,               0,              0       % Equ. 21)
       0,            1,                      0,            0,               0,              0       % Equ. 22)
       1,            1,                      0,            0,               0,              0       % Equ. 23)
       0,            0,                      0,            0,               1,              0       % Equ. 24)
       0,            0,                      0,            0,               0,              1       % Equ. 25)
       0,            0,                      0,            0,               0,              0    ]; % Equ. 26)


   % For k(t-1)                       L(t-1)                  Lh(t-1):
BB1 = [ 0,                              0,                      0                                            % Equ. 1)
        0,                              0,                      0                                            % Equ. 2)
        0,                              0,                      1                                            % Equ. 3)
 -(1-delta)*k_bar                -(1-delta)*k_bar,              0                                            % Equ. 4)
     1-alpha,                        1-alpha,                   0                                            % Equ. 5)
       -1,                             -1,                      0                                            % Equ. 6)
        0,                              0,                      0                                            % Equ. 7)
        0,                              0,                      0                                            % Equ. 8)
        0,                              0,                      0                                            % Equ. 9)
        0,                              0,                      0                                            % Equ. 10)
        0,                              0,                      0                                            % Equ. 11)
        0,                              0,                      0                                            % Equ. 12)
        0,                              0,                      0                                            % Equ. 13)
        0,                              0,                      0                                            % Equ. 14)
        0,                              0,                      0                                            % Equ. 15)
        0,                              0,                      1                                            % Equ. 16)
        0,                              0,                      0                                            % Equ. 17)
        0,                              0,              lambda/nh_bar*(Vh_bar/(Lh_bar-Nh_bar))^(1-lambda)   % Equ. 18)
        0,                              0,                      0                                            % Equ. 19)
 (1-tau_k)*k_bar*d_bar/qh_bar,  (1-tau_k)*k_bar*d_bar/qh_bar,   0                                            % Equ. 20)
 -tau_k*d_bar*k_bar,              -tau_k*d_bar*k_bar,           0                                            % Equ. 21)
        0,                              0,                      0                                            % Equ. 22)
        0,                              0,                      0                                            % Equ. 23)
        0,                              0,                      0                                            % Equ. 24)
        0,                              0,                      0                                            % Equ. 25)
        0,                              0,                      0                              ];           % Equ. 26)

   % For  Ll(t-1)                                               Nh(t-1)  :
BB2 = [ 0,                                                      0                      % Equ. 1)
        0,                                                      0                      % Equ. 2)
        0,                                                      0                      % Equ. 3)
        0,                                                      0                      % Equ. 4)
```

```
        0,                                                         0                           % Equ. 5)
        0,                                                         0                           % Equ. 6)
        0,                                                         0                           % Equ. 7)
        0,                                                         0                           % Equ. 8)
        0,                                                         0                           % Equ. 9)
        0,                                                         0                           % Equ. 10)
        0,                                                         0                           % Equ. 11)
        0,                                                         0                           % Equ. 12)
        0,                                                         0                           % Equ. 13)
        0,                                                         0                           % Equ. 14)
        0,                                                         0                           % Equ. 15)
        0,                                                         0                           % Equ. 16)
        1,                                                         0                           % Equ. 17)
        0,                                1-betan-lambda*(Vh_bar/(Lh_bar-Nh_bar))^(1-lambda) % Equ. 18)
 lambda/nl_bar*(Vl_bar/(Ll_bar-Nl_bar))^(1-lambda),                0                           % Equ. 19)
        0,                                                         0                           % Equ. 20)
        0,                                                         0                           % Equ. 21)
        0,                                                         0                           % Equ. 22)
        0,                                                         0                           % Equ. 23)
        0,                                                         0                           % Equ. 24)
        0,                                                         0                           % Equ. 25)
        0,                                                         0                     ];   % Equ. 26)


% For    Nl(t-1):
BB3 = [      0                                     % Equ. 1)
             0                                     % Equ. 2)
             0                                     % Equ. 3)
             0                                     % Equ. 4)
             0                                     % Equ. 5)
             0                                     % Equ. 6)
             0                                     % Equ. 7)
             0                                     % Equ. 8)
             0                                     % Equ. 9)
             0                                     % Equ. 10)
             0                                     % Equ. 11)
             0                                     % Equ. 12)
             0                                     % Equ. 13)
             0                                     % Equ. 14)
             0                                     % Equ. 15)
             0                                     % Equ. 16)
             0                                     % Equ. 17)
             0                                     % Equ. 18)
 1-betan-lambda*(Vl_bar/(Ll_bar-Nl_bar))^(1-lambda) % Equ. 19)
             0                                     % Equ. 20)
             0                                     % Equ. 21)
             0                                     % Equ. 22)
             0                                     % Equ. 23)
             0                                     % Equ. 24)
             0                                     % Equ. 25)
             0                               ];   % Equ. 26)

  BB = [BB1, BB2, BB3];


% For y(t): output   investment           income tax
CC1 = [       0,        0,          tau_l_bar/(1-tau_l_bar)                                                  % Equ. 1)
              0,        0,        -tau_l_bar/(1-tau_l_bar)                                                   % Equ. 2)
              0,        0,                   0                                                               % Equ. 3)
              0,    -x_bar,                  0                                                               % Equ. 4)
             -1,        0,                   0                                                               % Equ. 5)
              1,        0,                   0                                                               % Equ. 6)
              0,        0,                   0                                                               % Equ. 7)
              0,        0,                   0                                                               % Equ. 8)
              0,        0,                   0                                                               % Equ. 9)
              0,        0,                   0                                                               % Equ. 10)
```

```
             0,       0,              0                                                               % Equ. 11)
             0,       0,              0                                                               % Equ. 12)
             0,       0,              0                                                               % Equ. 13)
             0,       0,              0                                                               % Equ. 14)
             0,       0,              0                                                               % Equ. 15)
             0,       0,              0                                                               % Equ. 16)
             0,       0,              0                                                               % Equ. 17)
             0,       0,              0                                                               % Equ. 18)
             0,       0,              0                                                               % Equ. 19)
             0,   -x_bar/qh_bar,     -tau_l_bar*wh_bar*hh_bar*nh_bar                                  % Equ. 20)
             0,       0,       -(tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar+tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar) % Equ. 21)
             1,       0,              0                                                               % Equ. 22)
             0,       0,              0                                                               % Equ. 23)
             0,       0,              0                                                               % Equ. 24)
             0,       0,              0                                                               % Equ. 25)
             0,       0,             -tau_l_bar/(1-tau_l_bar)                          ];             % Equ. 26)

% For y(t):  dividends          return        HS consumption     LS consumption
CC2 = [         0,                 0,             0,                 1                               % Equ. 1)
                0,                 0,             0,    -(1+tau_c)*cl_bar/((1-tau_l_bar)*hl_bar*wl_bar*nl_bar) % Equ. 2)
       (1-tau_k)*d_bar/R_bar,     -1,             0,                 0                               % Equ. 3)
                0,                 0,             0,                 0                               % Equ. 4)
                0,                 0,             0,                 0                               % Equ. 5)
               -1,                 0,             0,                 0                               % Equ. 6)
                0,                 0,             0,                 0                               % Equ. 7)
                0,                 0,             0,                 0                               % Equ. 8)
                0,                 0,             0,                 0                               % Equ. 9)
                0,                 0,             0,                 0                               % Equ. 10)
                0,                 0,             0,                 0                               % Equ. 11)
                0,                 0,             0,                 0                               % Equ. 12)
                0,                 0,             0,                 0                               % Equ. 13)
                0,                 0,             0,                 0                               % Equ. 14)
                0,                 0,             0,                 0                               % Equ. 15)
                0,                 0,             0,                 0                               % Equ. 16)
                0,                 0,             0,                 0                               % Equ. 17)
                0,                 0,             0,                 0                               % Equ. 18)
                0,                 0,             0,                 0                               % Equ. 19)
(1-tau_k)*k_bar*d_bar/qh_bar,      0,   -(1+tau_c)*ch_bar,           0                               % Equ. 20)
       -tau_k*d_bar*k_bar,         0,   -tau_c*ch_bar*qh_bar,   -tau_c*cl_bar*ql_bar                 % Equ. 21)
                0,                 0,             0,                 0                               % Equ. 22)
                0,                 0,             0,                 0                               % Equ. 23)
                0,                 0,             0,                 0                               % Equ. 24)
                0,                 0,             0,                 0                               % Equ. 25)
                0,                 0,            -1,                 0                  ];           % Equ. 26)

% For y(t):  HS salary                                 LS salary
CC3 = [         0,                                       -1                           % Equ. 1)
                0,                                        1                           % Equ. 2)
                0,                                        0                           % Equ. 3)
                0,                                        0                           % Equ. 4)
                0,                                        0                           % Equ. 5)
                0,                                        0                           % Equ. 6)
-alpha*phi*Y_bar/(wh_bar*Hh_bar^rho),                     0                           % Equ. 7)
                0,                          -alpha*Y_bar/(wl_bar*Hl_bar^rho)          % Equ. 8)
                0,                                        0                           % Equ. 9)
                0,                                        0                           % Equ. 10)
                0,                                        0                           % Equ. 11)
                0,                                        0                           % Equ. 12)
                0,                                        0                           % Equ. 13)
                0,                                        0                           % Equ. 14)
                0,                                        0                           % Equ. 15)
                0,                                        0                           % Equ. 16)
                0,                                        0                           % Equ. 17)
                0,                                        0                           % Equ. 18)
```

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              0,                                    0                         % Equ. 19)
(1-tau_l_bar)*wh_bar*hh_bar*nh_bar,                 0                         % Equ. 20)
-tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar,      -tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar   % Equ. 21)
              0,                                    0                         % Equ. 22)
              0,                                    0                         % Equ. 23)
              0,                                    0                         % Equ. 24)
              0,                                    0                         % Equ. 25)
              1,                                    0                  ];     % Equ. 26)

% For y(t):                                   HS ratio
CC4 = [                                          0                                       % Equ. 1)
                                                 0                                       % Equ. 2)
                                                 0                                       % Equ. 3)
                                                 0                                       % Equ. 4)
                        phi*alpha*(nh_bar*qh_bar*hh_bar)^(1-rho)/a                       % Equ. 5)
                                                 0                                       % Equ. 6)
                                                 0                                       % Equ. 7)
                                                 0                                       % Equ. 8)
                                                 0                                       % Equ. 9)
                                                 0                                       % Equ. 10)
                                                 0                                       % Equ. 11)
                                                 0                                       % Equ. 12)
                                                -1                                       % Equ. 13)
                                                 0                                       % Equ. 14)
                                                 0                                       % Equ. 15)
                                                 0                                       % Equ. 16)
                                                 0                                       % Equ. 17)
                                                 0                                       % Equ. 18)
                                                 0                                       % Equ. 19)
                             x_bar/qh_bar-(1-tau_k)*k_bar*d_bar/qh_bar                   % Equ. 20)
 -tau_c*ch_bar*qh_bar-tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar+r*hh_bar*wh_bar*qh_bar*uh_bar*(1-tau_l_bar)   % Equ. 21)
                                                 0                                       % Equ. 22)
                                                 0                                       % Equ. 23)
                                                 0                                       % Equ. 24)
                                                 0                                       % Equ. 25)
                                                 0                               ];      % Equ. 26)

% For y(t):                                   LS ratio
CC5 = [                                          0                                       % Equ. 1)
                                                 0                                       % Equ. 2)
                                                 0                                       % Equ. 3)
                                                 0                                       % Equ. 4)
                           alpha*(nl_bar*ql_bar*hl_bar)^(1-rho)/a                        % Equ. 5)
                                                 0                                       % Equ. 6)
                                                 0                                       % Equ. 7)
                                                 0                                       % Equ. 8)
                                                 0                                       % Equ. 9)
                                                 0                                       % Equ. 10)
                                                 0                                       % Equ. 11)
                                                 0                                       % Equ. 12)
                                                 0                                       % Equ. 13)
                                                -1                                       % Equ. 14)
                                                 0                                       % Equ. 15)
                                                 0                                       % Equ. 16)
                                                 0                                       % Equ. 17)
                                                 0                                       % Equ. 18)
                                                 0                                       % Equ. 19)
                                                 0                                       % Equ. 20)
 -tau_c*cl_bar*ql_bar-tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar+r*hl_bar*wl_bar*ul_bar*ql_bar*(1-tau_l_bar)   % Equ. 21)
                                                 0                                       % Equ. 22)
                                                 0                                       % Equ. 23)
                                                 0                                       % Equ. 24)
                                                 0                                       % Equ. 25)
                                                 0                               ];      % Equ. 26)
```

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% For y(t):         HS employment                          LS employment
CC6 = [                   0,                                     0                    % Equ. 1)
                          0,                                     1                    % Equ. 2)
                          0,                                     0                    % Equ. 3)
                          0,                                     0                    % Equ. 4)
    phi*alpha*(nh_bar*qh_bar*hh_bar)^(1-rho)/a,     alpha*(nl_bar*ql_bar*hl_bar)^(1-rho)/a  % Equ. 5)
                          0,                                     0                    % Equ. 6)
                          0,                                     0                    % Equ. 7)
                          0,                                     0                    % Equ. 8)
                        nh_bar,                                  0                    % Equ. 9)
                          0,                                  nl_bar                  % Equ. 10)
                         -1,                                     0                    % Equ. 11)
                          0,                                    -1                    % Equ. 12)
                          0,                                     0                    % Equ. 13)
                          0,                                     0                    % Equ. 14)
                          0,                                     0                    % Equ. 15)
                          0,                                     0                    % Equ. 16)
                          0,                                     0                    % Equ. 17)
                          0,                                     0                    % Equ. 18)
                          0,                                     0                    % Equ. 19)
           (1-tau_l_bar)*wh_bar*hh_bar*nh_bar,                   0                    % Equ. 20)
          -tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar,     -tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar  % Equ. 21)
                          0,                                     0                    % Equ. 22)
                          0,                                     0                    % Equ. 23)
                          0,                                     0                    % Equ. 24)
                          0,                                     0                    % Equ. 25)
                          0,                                     0                 ];  % Equ. 26)

% For y(t):     HS unemployment               LS unemployment              total output       tot. capital
CC7 = [              0,                             0,                            0,                 0   % Equ. 1)
                     0,                      r*ul_bar/nl_bar,                     0,                 0   % Equ. 2)
                     0,                             0,                            0,                 0   % Equ. 3)
                     0,                             0,                            0,                 0   % Equ. 4)
                     0,                             0,                            0,                 0   % Equ. 5)
                     0,                             0,                            0,                 0   % Equ. 6)
                     0,                             0,       alpha*phi*Y_bar/(wh_bar*Hh_bar^rho),    0   % Equ. 7)
                     0,                             0,         alpha*Y_bar/(wl_bar*Hl_bar^rho),      0   % Equ. 8)
                  uh_bar,                           0,                            0,                 0   % Equ. 9)
                     0,                          ul_bar,                          0,                 0   % Equ. 10)
                     0,                             0,                            0,                 0   % Equ. 11)
                     0,                             0,                            0,                 0   % Equ. 12)
                     0,                             0,                            0,                 0   % Equ. 13)
                     0,                             0,                            0,                 0   % Equ. 14)
                     0,                             0,                            0,                 0   % Equ. 15)
                     0,                             0,                            0,                 0   % Equ. 16)
                     0,                             0,                            0,                 0   % Equ. 17)
                     0,                             0,                            0,                 0   % Equ. 18)
                     0,                             0,                            0,                 0   % Equ. 19)
 r*(1-tau_l_bar)*wh_bar*hh_bar*uh_bar,              0,                            0,                 0   % Equ. 20)
   r*uh_bar*qh_bar*hh_bar*wh_bar,      r*hl_bar*wl_bar*ul_bar*ql_bar,             0,                 0   % Equ. 21)
                     0,                             0,                           -1,                 0   % Equ. 22)
                     0,                             0,                            0,                -1   % Equ. 23)
                     0,                             0,                            0,                 0   % Equ. 24)
                     0,                             0,                            0,                 0   % Equ. 25)
                     0,                             0,                            0,                 0 ];% Equ. 26)

% For y(t):   HS hours                                       LS hours
CC8 = [         0,                                     hl_bar/(1-hl_bar)                % Equ. 1)
                0,                                             1                        % Equ. 2)
                0,                                             0                        % Equ. 3)
                0,                                             0                        % Equ. 4)
 phi*alpha*(nh_bar*qh_bar*hh_bar)^(1-rho)/a,    alpha*(nl_bar*ql_bar*hl_bar)^(1-rho)/a  % Equ. 5)
                0,                                             0                        % Equ. 6)
                0,                                             0                        % Equ. 7)
```

```
              0,                                          0                     % Equ. 8)
              0,                                          0                     % Equ. 9)
              0,                                          0                     % Equ. 10)
              0,                                          0                     % Equ. 11)
              0,                                          0                     % Equ. 12)
              0,                                          0                     % Equ. 13)
              0,                                          0                     % Equ. 14)
              0,                                          0                     % Equ. 15)
              0,                                          0                     % Equ. 16)
              0,                                          0                     % Equ. 17)
              0,                                          0                     % Equ. 18)
              0,                                          0                     % Equ. 19)
(1-tau_l_bar)*wh_bar*hh_bar*nh_bar,                       0                     % Equ. 20)
-tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar,    -tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar          % Equ. 21)
              0,                                          0                     % Equ. 22)
              0,                                          0                     % Equ. 23)
              1,                                          0                     % Equ. 24)
              0,                                          1                     % Equ. 25)
       -hh_bar/(1-hh_bar),                                0   ];                % Equ. 26)

% For y(t):   tot HS hours                                              tot LS hours:
CC9 = [           0,                                                           0           % Equ. 1)
                  0,                                                           0           % Equ. 2)
                  0,                                                           0           % Equ. 3)
                  0,                                                           0           % Equ. 4)
                  0,                                                           0           % Equ. 5)
                  0,                                                           0           % Equ. 6)
-((1-rho)*phi*Hh_bar^(1-rho)+rho*alpha*phi*Y_bar/(wh_bar*Hh_bar^rho)),   -(1-rho)*Hl_bar^(1-rho) % Equ. 7)
-(1-rho)*phi*Hh_bar^(1-rho),       -((1-rho)*Hl_bar^(1-rho)+rho*alpha*Y_bar/(wl_bar*Hl_bar^rho)) % Equ. 8)
                  0,                                                           0           % Equ. 9)
                  0,                                                           0           % Equ. 10)
                  0,                                                           0           % Equ. 11)
                  0,                                                           0           % Equ. 12)
                  0,                                                           0           % Equ. 13)
                  0,                                                           0           % Equ. 14)
                  0,                                                           0           % Equ. 15)
                  0,                                                           0           % Equ. 16)
                  0,                                                           0           % Equ. 17)
                  0,                                                           0           % Equ. 18)
                  0,                                                           0           % Equ. 19)
                  0,                                                           0           % Equ. 20)
                  0,                                                           0           % Equ. 21)
                  0                                                            0           % Equ. 22)
                  0,                                                           0           % Equ. 23)
                 -1,                                                           0           % Equ. 24)
                  0,                                                          -1           % Equ. 25)
                  0,                                                           0      ];   % Equ. 26)

CC = [CC1, CC2, CC3, CC4, CC5, CC6, CC7, CC8, CC9];

% For exog. var. in time period (t):
%      i(t)
DD = [ 0                    % Equ. 1)
       0                    % Equ. 2)
       0                    % Equ. 3)
       0                    % Equ. 4)
       0                    % Equ. 5)
       0                    % Equ. 6)
       0                    % Equ. 7)
       0                    % Equ. 8)
       0                    % Equ. 9)
       0                    % Equ. 10)
       0                    % Equ. 11)
       0                    % Equ. 12)
       0                    % Equ. 13)
       0                    % Equ. 14)
```

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       0                       % Equ. 15)
       eta/qh_bar              % Equ. 16)
       (1-eta)/ql_bar          % Equ. 17)
       0                       % Equ. 18)
       0                       % Equ. 19)
       0                       % Equ. 20)
       0                       % Equ. 21)
       0                       % Equ. 22)
       0                       % Equ. 23)
       0                       % Equ. 24)
       0                       % Equ. 25)
       0                   ];  % Equ. 26)

DD=0.4*DD;     % We need to multiple DD by 0.4 in order to get 0.4 % shock.
               % In case of 1st scenario (see Variations of the model) we
               % do not need this equation.

% EXPECTATIONAL EQUATIONS:

% For  k(t+1)    L(t+1)     Lh(t+1)   Ll(t+1)    Nh(t+1)   Nl(t+1):
FF = [    0,        0,         0,        0,         0,         0 ];

% For     k(t)    L(t)       Lh(t)      Ll(t)    Nh(t)       Nl(t):
GG = [    0,        0,         0,        0,         0,         0 ];

% For   k(t-1)    L(t-1)     Lh(t-1)  Ll(t-1)    Nh(t-1)  Nl(t-1):
HH = [    0,        0,         0,        0,         0,         0 ];

% For y(t+1): output  investm.   income tax  divid.  return  HS consumt.  LS consump.
JJ1 = [          0,       0,         0,        0,      1,       -1,          0 ];

        %  HS sal.   LS sal.  HS ratio  LS ratio  HS employment  LS employment  HS unemployment
JJ2 = [     0,        0,         0,        0,          0,            0,              0 ] ;

       % LS umemployment total output   tot. capital   HS hours  LS hours  tot HS hours  tot LS hours
JJ3 =  [       0,           0,             0,            0,       0,           0,            0    ];

JJ = [JJ1, JJ2, JJ3];

% For y(t)  : output  investm.   income tax  divid.  return  HS consumt.  LS consump.  HS sal.   LS sal.
KK1 = [          0,       0,         0,         0,      0,        1,           0,         0,       0 ];

% For y(t)  : HS ratio  LS ratio  HS employment  LS employment  HS unemployment  LS umemployment  total output
KK2 = [          0,        0,          0,             0,              0,                0,             0   ];

% For y(t)  :  tot. capital   HS hours  LS hours  tot HS hours  tot LS hours
KK3 =  [             0,           0,       0,         0,            0];

KK = [KK1, KK2, KK3];

% For z(t+1):
LL =      [ 0 ];

% For z(t):
MM =      [ 0 ];

% AUTOREGRESSIVE MATRIX FOR z(t):
NN =       [ 0 ];

Sigma =    [ 1 ];


%Setting the options:

[l_equ,m_states] = size(AA);
[l_equ,n_endog ] = size(CC);
```

```
[l_equ,k_exog ]  = size(DD);


PERIOD     = 1; % The fundamental period in the model is one year.
GNP_INDEX  = 2; % Index of output among the variables selected for HP filter
IMP_SELECT = [1,7:15,20,21,24,25,28]; % A vector containing the indices of the variables to be plotted
HP_SELECT  = [1,7:15,20,21,24,25,28]; % Selecting the variables for the HP Filter calcs.
DO_SIMUL   = 0; % Calculates Simulations (if =1)
DO_MOMENTS = 0; % Calculates Moments (if =1)
DO_STATE_RESP = 0; % Do not calculate impulse responses to deviations of state variables
HORIZON      = 51;
% Starting the calculations:

do_it;
```

E.2. Case when $\rho = 1$

```
% Master Thesis "The Inflow of Eastern European Workers to German Labor
% Market: Consequences and Policy Issues"
% MATLAB code for German economy when rho=1
% Author: Sigitas KARPAVICIUS
% Humboldt-Universität zu Berlin, SS2005
% Revised on 20th August, 2005

%disp('Hit any key when ready...');
%pause;


% Setting parameters: (constant values):

alpha       = 0.64;    % Labor share
beta        = 0.96;    % Discount factor
delta       = 0.08;    % Depreciation rate
tau_c       = 0.16;    % VAT
tau_k       = 0.137;   % Tax rate on dividends
r           = 0.67;    % Unemployment benefits
A           = 2.42;    % Preference parameter
eta         = 0.07;    % Share of high-skilled workers among the immigrants
phi         = 2;       % Productivity difference between high- and low-skilled workers
lambda      = 0.5;     % Bargaining power of unemployed
betan       = 0.064;   % Job destruction rate

% Values in the steady state:

%--------------------------Given--------------------------------------

L_bar       = 42806000;% Labor in mln. in 2003
uh_bar      = 0.049;   % Unemployment among high-skilled labor
ul_bar      = 0.115;   % Unemployment among low-skilled labor
qh_bar      = 0.500;   % Ratio of high-skilled labor and all the labor
ql_bar      = 0.500;   % Ratio of low-skilled labor and all the labor
tau_l_bar   = 0.196;   % Labor income tax rate

%------------------------Calculated-------------------------------------

% R_bar - Interest factor
R_bar = 1/beta;
% d_bar - Dividends rate
d_bar = (R_bar - 1 + delta)/(1-tau_k);
% nh_bar - Employment among high-skilled labor in 2002
```

```
nh_bar = 1- uh_bar;
% nl_bar - Employment among low-skilled labor in 2002
nl_bar = 1- ul_bar;
% Total number of high-skilled economically active agents
Lh_bar = L_bar * qh_bar;
% Total number of low-skilled economically active agents
Ll_bar = L_bar * ql_bar;
% Nh_bar - Total number of high-skilled workers
Nh_bar = nh_bar * Lh_bar;
% Nl_bar - Total number of low-skilled workers
Nl_bar = nl_bar * Ll_bar;
% hh_bar - Ratio of working time and total time of high-skilled labor
% alpha*phi/((phi*(nh_bar*qh_bar*hh_bar)^(1-rho)+(nl_bar*ql_bar*hl_bar)^(1-rho))*
%  *(nh_bar*qh_bar*hh_bar)^rho)*((1-tau_l_bar)*(1-hh_bar)/A-(1-tau_l_bar)*
%  *(nh_bar+r*uh_bar)*hh_bar)=(1-alpha)/qh_bar*(1-tau_k-delta/d_bar);
hh_bar = 0.244;
% hl_bar - Ratio of working time and total time of low-skilled labor
hl_bar = 1/(1+A*(nl_bar+r*ul_bar));
% Hh_bar - Total amount of time worked by high-skilled agents
Hh_bar = Nh_bar*hh_bar;
% Hl_bar - Total amount of time worked by low-skilled agents
Hl_bar = Nl_bar*hl_bar;
% y_bar - Output per capita (high- and low-skilled labor)
y_bar = ((nh_bar*qh_bar*hh_bar)^phi*(nl_bar*ql_bar*hl_bar))^(1/(phi+1))*((1-alpha)/d_bar)^((1-alpha)/alpha);
% k_bar - Capital stock per capita (high- and low-skilled labor)
k_bar = (1-alpha)*y_bar/d_bar;
% K_bar - Total capital stock
K_bar = L_bar*k_bar;
% Y_bar - Total output
Y_bar = L_bar*y_bar;
% x_bar - Investment per capita (low- and high-skilled)
x_bar = delta*k_bar;
% wh_bar - wage of high-skilled worker per one unit of her/his working time
wh_bar = phi/(phi+1)*alpha*Y_bar/Hh_bar;
% wl_bar - wage of low-skilled worker per one unit of her/his working time
wl_bar = 1/(phi+1)*alpha*Y_bar/Hl_bar;
% ch_bar - consumption of each high-skilled worker
ch_bar = wh_bar*(1-tau_l_bar)*(1-hh_bar)/(A*(1+tau_c));
% cl_bar - consumption of each low-skilled worker
cl_bar = (1-tau_l_bar)*wl_bar*hl_bar*(nl_bar+ul_bar*r)/(1+tau_c);
% Vh_bar - vacancies for high-skilled jobs posted
Vh_bar = betan^(1/(1-lambda))*Nh_bar*(nh_bar/(1-nh_bar))^(lambda/(1-lambda));
% Vl_bar - vacancies for low-skilled jobs posted
Vl_bar = betan^(1/(1-lambda))*Nl_bar*(nl_bar/(1-nl_bar))^(lambda/(1-lambda));


% Declaring the matrices.

VARNAMES = ['capital      ',    % 1
            'total labor  ',    % 2
            'HS labor     ',    % 3
            'LS labor     ',    % 4
            'HS workers   ',    % 5
            'LS workers   ',    % 6
            'output       ',    % 7
            'investment   ',    % 8
            'income tax   ',    % 9
            'dividends    ',    % 10
            'return       ',    % 11
            'HS consumpt. ',    % 12
            'LS consumpt. ',    % 13
            'HS salary    ',    % 14
            'LS salary    ',    % 15
            'HS ratio     ',    % 16
            'LS ratio     ',    % 17
            'HS employment',    % 18
```

```
            'LS employment',    % 19
            'HS unempl.   ',    % 20
            'LS unempl.   ',    % 21
            'total output ',    % 22
            'total capital',    % 23
            'HS hours     ',    % 24
            'LS hours     ',    % 25
            'tot HS hours ',    % 26
            'tot LS hours ',    % 27
            'immigrants   '];   % 28


% Translating into coefficient matrices.
% The loglinearized equations are, conveniently ordered:
% Equ. 1)  0 = tau_l_bar/(1-tau_l_bar)*tau_l(t) + cl(t) - wl(t) + hl_bar/(1-hl_bar)*hl(t)
% Equ. 2)  0 = - tau_l_bar/(1-tau_l_bar)*tau_l(t) -
%              - (1+tau_c)/((1-tau_l_bar)*hl_bar*wl_bar*nl_bar)*cl(t) +
%              + wl(t) + nl(t) + r*ul_bar/nl_bar*uh(t) + hl(t)
% Equ. 3)  0 = (1-tau_k)*d_bar/R_bar*d(t) - R(t) - Lh(t) + Lh(t-1)
% Equ. 4)  0 = -(1-delta)*k_bar*k(t-1) - (1-delta)*k_bar*L(t-1) - x_bar*x(t) +
%              +k_bar*k(t) + (1-delta)*k_bar*L(t)
% Equ. 5)  0 = (1-alpha)*k(t-1) + (1-alpha)*L(t-1) - (1-alpha)*L(t) - y(t) +
%            + phi*alpha/(phi+1)*qh(t) + alpha/(phi+1)*ql(t) + phi*alpha/(phi+1)*nh(t) +
%            + alpha/(phi+1)*nl(t) + phi*alpha/(phi+1)*hh(t) + alpha/(phi+1)*hl(t)
% Equ. 6)  0 =  - k(t-1) - L(t-1) + L(t) + y(t) - d(t)
% Equ. 7)  0 = - wh(t) + Y(t) - Hh(t)
% Equ. 8)  0 = - wl(t) + Y(t) - Hl(t)
% Equ. 9)  0 = nh_bar*nh(t) + uh_bar*uh(t)
% Equ. 10) 0 = nl_bar*nl(t) + ul_bar*ul(t)
% Equ. 11) 0 = - nh(t) + Nh(t) - Lh(t)
% Equ. 12) 0 = - nl(t) + Nl(t) - Ll(t)
% Equ. 13) 0 = - qh(t) + Lh(t) - L(t)
% Equ. 14) 0 = - ql(t) + Ll(t) - L(t)
% Equ. 15) 0 = qh_bar*Lh(t) + ql_bar*Ll(t) - L(t)
% Equ. 16) 0 = Lh(t-1) - Lh(t) + eta/qh_bar*i(t)
% Equ. 17) 0 = Ll(t-1) - Ll(t) + (1-eta)/ql_bar*i(t)
% Equ. 18) 0 = - Nh(t) + lambda/nh_bar*(Vh_bar/(Lh_bar-Nh_bar))^(1-lambda)*Lh(t-1) +
%            + (1-betan-lambda*(Vh_bar/(Lh_bar-Nh_bar))^(1-lambda))*Nh(t-1)
% Equ. 19) 0 = - Nl(t) + lambda/nl_bar*(Vl_bar/(Ll_bar-Nl_bar))^(1-lambda)*Ll(t-1) +
%            + (1-betan-lambda*(Vl_bar/(Ll_bar-Nl_bar))^(1-lambda))*Nl(t-1)
% Equ. 20) 0 = (1-tau_k)*k_bar*d_bar/qh_bar*k(t-1) + (1-tau_k)*k_bar*d_bar/qh_bar*L(t-1) -
%            - (1-tau_k)*k_bar*d_bar/qh_bar*L(t) - x_bar/qh_bar*x(t) - tau_l_bar*wh_bar*hh_bar*nh_bar*tau_l(t) +
%            + (1-tau_k)*k_bar*d_bar/qh_bar*d(t) - (1+tau_c)*ch_bar*ch(t) + (1-tau_l_bar)*wh_bar*hh_bar*nh_bar*wh(t)+
%            + x_bar/qh_bar-(1-tau_k)*k_bar*d_bar/qh_bar*qh(t) + (1-tau_l_bar)*wh_bar*hh_bar*nh_bar*nh(t) +
%            + r*(1-tau_l_bar)*wh_bar*hh_bar*uh_bar*uh(t) + (1-tau_l_bar)*wh_bar*hh_bar*nh_bar*hh(t)
% Equ. 21) 0 = - tau_k*d_bar*k_bar*k(t-1) - tau_k*d_bar*k_bar*L(t-1) + tau_k*d_bar*k_bar*L(t) -
%            - (tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar+tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar)*tau_l(t) -
%            - tau_k*d_bar*k_bar*d(t) - tau_c*ch_bar*qh_bar*ch(t) - tau_c*cl_bar*ql_bar*cl(t) -
%            - tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar*wh(t) - tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar*wl(t) -
%            - tau_c*ch_bar*qh_bar-tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar+r*hh_bar*wh_bar*qh_bar*uh_bar*(1-tau_l_bar)*qh(t)-
%            - tau_c*cl_bar*ql_bar-tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar+r*hl_bar*wl_bar*ul_bar*ql_bar*(1-tau_l_bar)*ql(t)-
%            - tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar*nh(t) - tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar*nl(t) +
%            + r*uh_bar*qh_bar*hh_bar*wh_bar*uh(t) + r*hl_bar*wl_bar*ul_bar*ql_bar*ul(t) -
%            - tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar*hh(t) - tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar*hl(t)
% Equ. 22) 0 = y(t) + L(t) - Y(t)
% Equ. 23) 0 = k(t) + L(t) - K(t)
% Equ. 24) 0 = hh(t) + Nh(t) - Hh(t)
% Equ. 25) 0 = hl(t) + Nl(t) - Hl(t)
% Equ. 26) 0 = - tau_l_bar/(1-tau_l_bar)*tau_l(t) - ch(t) + wh(t) - hh_bar/(1-hh_bar)*hh(t)
% Equ. 27) 0 = E_t [ - ch(t+1) + R(t+1) +  ch(t) ]
% Equ. 28) z(t+1) = i(t+1)
%
%
% Endogenous state variables "x(t)": k(t), L(t), Lh(t), Ll(t), Nh(t), Nl(t).
% Endogenous other variables "y(t)": y(t), x(t), tau_l(t), d(t),  R(t),
% ch(t), cl(t), wh(t), wl(t), qh(t), ql(t), nh(t), nl(t), uh(t), ul(t),
```

```
% Y(t), K(t), hh(t), hl(t), Hh(t), Hl(t).
% Exogenous state variables  "z(t)": i(t).
% CHECK: 28 equations, 28 variables.
% Switch to that notation.  Find matrices for format:
% 0 = AA x(t) + BB x(t-1) + CC y(t) + DD z(t),
% 0 = E_t [ FF x(t+1) + GG x(t) + HH x(t-1) + JJ y(t+1) + KK y(t) + LL z(t+1) + MM z(t)],
% z(t+1) = NN z(t) + i(t+1) with E_t [ i(t+1) ] = 0.

% For k(t)          L(t)                    Lh(t)         Ll(t)             Nh(t)           Nl(t):
AA = [ 0,            0,                      0,            0,                0,              0       % Equ. 1)
       0,            0,                      0,            0,                0,              0       % Equ. 2)
       0,            0,                     -1,            0,                0,              0       % Equ. 3)
       k_bar,    (1-delta)*k_bar,            0,            0,                0,              0       % Equ. 4)
       0,          -1+alpha,                 0,            0,                0,              0       % Equ. 5)
       0,            1,                      0,            0,                0,              0       % Equ. 6)
       0,            0,                      0,            0,                0,              0       % Equ. 7)
       0,            0,                      0,            0,                0,              0       % Equ. 8)
       0,            0,                      0,            0,                0,              0       % Equ. 9)
       0,            0,                      0,            0,                0,              0       % Equ. 10)
       0,            0,                     -1,            0,                1,              0       % Equ. 11)
       0,            0,                      0,           -1,                0,              1       % Equ. 12)
       0,           -1,                      1,            0,                0,              0       % Equ. 13)
       0,           -1,                      0,            1,                0,              0       % Equ. 14)
       0,           -1,                    qh_bar,       ql_bar,             0,              0       % Equ. 15)
       0,            0,                     -1,            0,                0,              0       % Equ. 16)
       0,            0,                      0,           -1,                0,              0       % Equ. 17)
       0,            0,                      0,            0,               -1,              0       % Equ. 18)
       0,            0,                      0,            0,                0,             -1       % Equ. 19)
       0,    -(1-tau_k)*k_bar*d_bar/qh_bar,  0,            0,                0,              0       % Equ. 20)
       0,         tau_k*d_bar*k_bar,         0,            0,                0,              0       % Equ. 21)
       0,            1,                      0,            0,                0,              0       % Equ. 22)
       1,            1,                      0,            0,                0,              0       % Equ. 23)
       0,            0,                      0,            0,                1,              0       % Equ. 24)
       0,            0,                      0,            0,                0,              1       % Equ. 25)
       0,            0,                      0,            0,                0,              0    ]; % Equ. 26)


   % For k(t-1)                          L(t-1)                Lh(t-1):
BB1 = [ 0,                                0,                    0                                                   % Equ. 1)
        0,                                0,                    0                                                   % Equ. 2)
        0,                                0,                    1                                                   % Equ. 3)
 -(1-delta)*k_bar                 -(1-delta)*k_bar,             0                                                   % Equ. 4)
     1-alpha,                        1-alpha,                   0                                                   % Equ. 5)
       -1,                              -1,                     0                                                   % Equ. 6)
        0,                                0,                    0                                                   % Equ. 7)
        0,                                0,                    0                                                   % Equ. 8)
        0,                                0,                    0                                                   % Equ. 9)
        0,                                0,                    0                                                   % Equ. 10)
        0,                                0,                    0                                                   % Equ. 11)
        0,                                0,                    0                                                   % Equ. 12)
        0,                                0,                    0                                                   % Equ. 13)
        0,                                0,                    0                                                   % Equ. 14)
        0,                                0,                    0                                                   % Equ. 15)
        0,                                0,                    1                                                   % Equ. 16)
        0,                                0,                    0                                                   % Equ. 17)
        0,                                0,            lambda/nh_bar*(Vh_bar/(Lh_bar-Nh_bar))^(1-lambda)         % Equ. 18)
        0,                                0,                    0                                                   % Equ. 19)
 (1-tau_k)*k_bar*d_bar/qh_bar,  (1-tau_k)*k_bar*d_bar/qh_bar,   0                                                   % Equ. 20)
 -tau_k*d_bar*k_bar,               -tau_k*d_bar*k_bar,          0                                                   % Equ. 21)
        0,                                0,                    0                                                   % Equ. 22)
        0,                                0,                    0                                                   % Equ. 23)
        0,                                0,                    0                                                   % Equ. 24)
        0,                                0,                    0                                                   % Equ. 25)
        0,                                0,                    0                                  ];               % Equ. 26)
```

```
   % For  Ll(t-1)                                                Nh(t-1)  :
BB2 = [ 0,                                                        0                         % Equ. 1)
        0,                                                        0                         % Equ. 2)
        0,                                                        0                         % Equ. 3)
        0,                                                        0                         % Equ. 4)
        0,                                                        0                         % Equ. 5)
        0,                                                        0                         % Equ. 6)
        0,                                                        0                         % Equ. 7)
        0,                                                        0                         % Equ. 8)
        0,                                                        0                         % Equ. 9)
        0,                                                        0                         % Equ. 10)
        0,                                                        0                         % Equ. 11)
        0,                                                        0                         % Equ. 12)
        0,                                                        0                         % Equ. 13)
        0,                                                        0                         % Equ. 14)
        0,                                                        0                         % Equ. 15)
        0,                                                        0                         % Equ. 16)
        1,                                                        0                         % Equ. 17)
        0,                                  1-betan-lambda*(Vh_bar/(Lh_bar-Nh_bar))^(1-lambda) % Equ. 18)
 lambda/nl_bar*(Vl_bar/(Ll_bar-Nl_bar))^(1-lambda),               0                         % Equ. 19)
        0,                                                        0                         % Equ. 20)
        0,                                                        0                         % Equ. 21)
        0,                                                        0                         % Equ. 22)
        0,                                                        0                         % Equ. 23)
        0,                                                        0                         % Equ. 24)
        0,                                                        0                         % Equ. 25)
        0,                                                        0                    ];   % Equ. 26)


% For    Nl(t-1):
BB3 = [      0                                  % Equ. 1)
             0                                  % Equ. 2)
             0                                  % Equ. 3)
             0                                  % Equ. 4)
             0                                  % Equ. 5)
             0                                  % Equ. 6)
             0                                  % Equ. 7)
             0                                  % Equ. 8)
             0                                  % Equ. 9)
             0                                  % Equ. 10)
             0                                  % Equ. 11)
             0                                  % Equ. 12)
             0                                  % Equ. 13)
             0                                  % Equ. 14)
             0                                  % Equ. 15)
             0                                  % Equ. 16)
             0                                  % Equ. 17)
             0                                  % Equ. 18)
 1-betan-lambda*(Vl_bar/(Ll_bar-Nl_bar))^(1-lambda) % Equ. 19)
             0                                  % Equ. 20)
             0                                  % Equ. 21)
             0                                  % Equ. 22)
             0                                  % Equ. 23)
             0                                  % Equ. 24)
             0                                  % Equ. 25)
             0                              ];  % Equ. 26)

  BB = [BB1, BB2, BB3];


% For y(t): output   investment            income tax
CC1 = [       0,        0,         tau_l_bar/(1-tau_l_bar)                                                   % Equ. 1)
              0,        0,        -tau_l_bar/(1-tau_l_bar)                                                   % Equ. 2)
              0,        0,                    0                                                              % Equ. 3)
              0,    -x_bar,                   0                                                              % Equ. 4)
             -1,        0,                    0                                                              % Equ. 5)
```

```
                1,       0,                     0                                                              % Equ. 6)
                0,       0,                     0                                                              % Equ. 7)
                0,       0,                     0                                                              % Equ. 8)
                0,       0,                     0                                                              % Equ. 9)
                0,       0,                     0                                                              % Equ. 10)
                0,       0,                     0                                                              % Equ. 11)
                0,       0,                     0                                                              % Equ. 12)
                0,       0,                     0                                                              % Equ. 13)
                0,       0,                     0                                                              % Equ. 14)
                0,       0,                     0                                                              % Equ. 15)
                0,       0,                     0                                                              % Equ. 16)
                0,       0,                     0                                                              % Equ. 17)
                0,       0,                     0                                                              % Equ. 18)
                0,       0,                     0                                                              % Equ. 19)
                0,   -x_bar/qh_bar,       -tau_l_bar*wh_bar*hh_bar*nh_bar                                      % Equ. 20)
                0,       0,       -(tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar+tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar) % Equ. 21)
                1,       0,                     0                                                              % Equ. 22)
                0,       0,                     0                                                              % Equ. 23)
                0,       0,                     0                                                              % Equ. 24)
                0,       0,                     0                                                              % Equ. 25)
                0,       0,             -tau_l_bar/(1-tau_l_bar)                                ];             % Equ. 26)

% For y(t):  dividends            return        HS consumption     LS consumption
CC2 = [         0,                  0,            0,                  1                                      % Equ. 1)
                0,                  0,             0,   -(1+tau_c)*cl_bar/((1-tau_l_bar)*hl_bar*wl_bar*nl_bar) % Equ. 2)
       (1-tau_k)*d_bar/R_bar,      -1,             0,                 0                                      % Equ. 3)
                0,                  0,            0,                  0                                      % Equ. 4)
                0,                  0,            0,                  0                                      % Equ. 5)
               -1,                  0,            0,                  0                                      % Equ. 6)
                0,                  0,            0,                  0                                      % Equ. 7)
                0,                  0,             0,                 0                                      % Equ. 8)
                0,                  0,            0,                  0                                      % Equ. 9)
                0,                  0,            0,                  0                                      % Equ. 10)
                0,                  0,            0,                  0                                      % Equ. 11)
                0,                  0,            0,                  0                                      % Equ. 12)
                0,                  0,            0,                  0                                      % Equ. 13)
                0,                  0,            0,                  0                                      % Equ. 14)
                0,                  0,            0,                  0                                      % Equ. 15)
                0,                  0,            0,                  0                                      % Equ. 16)
                0,                  0,            0,                  0                                      % Equ. 17)
                0,                  0,            0,                  0                                      % Equ. 18)
                0,                  0,            0,                  0                                      % Equ. 19)
(1-tau_k)*k_bar*d_bar/qh_bar,       0,  -(1+tau_c)*ch_bar,            0                                      % Equ. 20)
       -tau_k*d_bar*k_bar,          0,  -tau_c*ch_bar*qh_bar,   -tau_c*cl_bar*ql_bar                         % Equ. 21)
                0,                  0,            0,                  0                                      % Equ. 22)
                0,                  0,            0,                  0                                      % Equ. 23)
                0,                  0,            0,                  0                                      % Equ. 24)
                0,                  0,            0,                  0                                      % Equ. 25)
                0,                  0,           -1,                  0                    ];                % Equ. 26)

% For y(t):  HS salary                                   LS salary
CC3 = [         0,                                          -1                        % Equ. 1)
                0,                                           1                        % Equ. 2)
                0,                                           0                        % Equ. 3)
                0,                                           0                        % Equ. 4)
                0,                                           0                        % Equ. 5)
                0,                                           0                        % Equ. 6)
               -1,                                           0                        % Equ. 7)
                0,                                          -1                        % Equ. 8)
                0,                                           0                        % Equ. 9)
                0,                                           0                        % Equ. 10)
                0,                                           0                        % Equ. 11)
                0,                                           0                        % Equ. 12)
                0,                                           0                        % Equ. 13)
```

```
                0,                                      0                          % Equ. 14)
                0,                                      0                          % Equ. 15)
                0,                                      0                          % Equ. 16)
                0,                                      0                          % Equ. 17)
                0,                                      0                          % Equ. 18)
                0,                                      0                          % Equ. 19)
(1-tau_l_bar)*wh_bar*hh_bar*nh_bar,                     0                          % Equ. 20)
-tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar,      -tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar  % Equ. 21)
                0,                                      0                          % Equ. 22)
                0,                                      0                          % Equ. 23)
                0,                                      0                          % Equ. 24)
                0,                                      0                          % Equ. 25)
                1,                                      0                  ];      % Equ. 26)

% For y(t):                                   HS ratio
CC4 = [                                          0                                   % Equ. 1)
                                                 0                                   % Equ. 2)
                                                 0                                   % Equ. 3)
                                                 0                                   % Equ. 4)
                                         phi*alpha/(phi+1)                           % Equ. 5)
                                                 0                                   % Equ. 6)
                                                 0                                   % Equ. 7)
                                                 0                                   % Equ. 8)
                                                 0                                   % Equ. 9)
                                                 0                                   % Equ. 10)
                                                 0                                   % Equ. 11)
                                                 0                                   % Equ. 12)
                                                -1                                   % Equ. 13)
                                                 0                                   % Equ. 14)
                                                 0                                   % Equ. 15)
                                                 0                                   % Equ. 16)
                                                 0                                   % Equ. 17)
                                                 0                                   % Equ. 18)
                                                 0                                   % Equ. 19)
                             x_bar/qh_bar-(1-tau_k)*k_bar*d_bar/qh_bar               % Equ. 20)
  -tau_c*ch_bar*qh_bar-tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar+r*hh_bar*wh_bar*qh_bar*uh_bar*(1-tau_l_bar)    % Equ. 21)
                                                 0                                   % Equ. 22)
                                                 0                                   % Equ. 23)
                                                 0                                   % Equ. 24)
                                                 0                                   % Equ. 25)
                                                 0                            ];     % Equ. 26)

% For y(t):                                   LS ratio
CC5 = [                                          0                                   % Equ. 1)
                                                 0                                   % Equ. 2)
                                                 0                                   % Equ. 3)
                                                 0                                   % Equ. 4)
                                           alpha/(phi+1)                             % Equ. 5)
                                                 0                                   % Equ. 6)
                                                 0                                   % Equ. 7)
                                                 0                                   % Equ. 8)
                                                 0                                   % Equ. 9)
                                                 0                                   % Equ. 10)
                                                 0                                   % Equ. 11)
                                                 0                                   % Equ. 12)
                                                 0                                   % Equ. 13)
                                                -1                                   % Equ. 14)
                                                 0                                   % Equ. 15)
                                                 0                                   % Equ. 16)
                                                 0                                   % Equ. 17)
                                                 0                                   % Equ. 18)
                                                 0                                   % Equ. 19)
                                                 0                                   % Equ. 20)
  -tau_c*cl_bar*ql_bar-tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar+r*hl_bar*wl_bar*ul_bar*ql_bar*(1-tau_l_bar)    % Equ. 21)
```

```
                                     0                                          % Equ. 22)
                                     0                                          % Equ. 23)
                                     0                                          % Equ. 24)
                                     0                                          % Equ. 25)
                                     0                                      ];  % Equ. 26)

% For y(t):         HS employment                     LS employment
CC6 = [                   0,                                0                  % Equ. 1)
                          0,                                1                  % Equ. 2)
                          0,                                0                  % Equ. 3)
                          0,                                0                  % Equ. 4)
                 phi*alpha/(phi+1),                   alpha/(phi+1)            % Equ. 5)
                          0,                                0                  % Equ. 6)
                          0,                                0                  % Equ. 7)
                          0,                                0                  % Equ. 8)
                       nh_bar,                              0                  % Equ. 9)
                          0,                              nl_bar               % Equ. 10)
                         -1,                                0                  % Equ. 11)
                          0,                               -1                  % Equ. 12)
                          0,                                0                  % Equ. 13)
                          0,                                0                  % Equ. 14)
                          0,                                0                  % Equ. 15)
                          0,                                0                  % Equ. 16)
                          0,                                0                  % Equ. 17)
                          0,                                0                  % Equ. 18)
                          0,                                0                  % Equ. 19)
          (1-tau_l_bar)*wh_bar*hh_bar*nh_bar,               0                  % Equ. 20)
         -tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar,     -tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar  % Equ. 21)
                          0,                                0                  % Equ. 22)
                          0,                                0                  % Equ. 23)
                          0,                                0                  % Equ. 24)
                          0,                                0                  % Equ. 25)
                          0,                                0               ]; % Equ. 26)

% For y(t):     HS unemployment              LS unemployment            total output       tot. capital
CC7 = [               0,                           0,                        0,                 0   % Equ. 1)
                      0,                     r*ul_bar/nl_bar,                0,                 0   % Equ. 2)
                      0,                           0,                        0,                 0   % Equ. 3)
                      0,                           0,                        0,                 0   % Equ. 4)
                      0,                           0,                        0,                 0   % Equ. 5)
                      0,                           0,                        0,                 0   % Equ. 6)
                      0,                           0,                        1,                 0   % Equ. 7)
                      0,                           0,                        1,                 0   % Equ. 8)
                   uh_bar,                         0,                        0,                 0   % Equ. 9)
                      0,                        ul_bar,                      0,                 0   % Equ. 10)
                      0,                           0,                        0,                 0   % Equ. 11)
                      0,                           0,                        0,                 0   % Equ. 12)
                      0,                           0,                        0,                 0   % Equ. 13)
                      0,                           0,                        0,                 0   % Equ. 14)
                      0,                           0,                        0,                 0   % Equ. 15)
                      0,                           0,                        0,                 0   % Equ. 16)
                      0,                           0,                        0,                 0   % Equ. 17)
                      0,                           0,                        0,                 0   % Equ. 18)
                      0,                           0,                        0,                 0   % Equ. 19)
 r*(1-tau_l_bar)*wh_bar*hh_bar*uh_bar,             0,                        0,                 0   % Equ. 20)
   r*uh_bar*qh_bar*hh_bar*wh_bar,      r*hl_bar*wl_bar*ul_bar*ql_bar,        0,                 0   % Equ. 21)
                      0,                           0,                       -1,                 0   % Equ. 22)
                      0,                           0,                        0,                -1   % Equ. 23)
                      0,                           0,                        0,                 0   % Equ. 24)
                      0,                           0,                        0,                 0   % Equ. 25)
                      0,                           0,                        0,                 0 ];% Equ. 26)

% For y(t):   HS hours                                   LS hours
CC8 = [         0,                                   hl_bar/(1-hl_bar)               % Equ. 1)
                0,                                           1                       % Equ. 2)
```

```
               0,                                            0                       % Equ. 3)
               0,                                            0                       % Equ. 4)
        phi*alpha/(phi+1),                             alpha/(phi+1)                 % Equ. 5)
               0,                                            0                       % Equ. 6)
               0,                                            0                       % Equ. 7)
               0,                                            0                       % Equ. 8)
               0,                                            0                       % Equ. 9)
               0,                                            0                       % Equ. 10)
               0,                                            0                       % Equ. 11)
               0,                                            0                       % Equ. 12)
               0,                                            0                       % Equ. 13)
               0,                                            0                       % Equ. 14)
               0,                                            0                       % Equ. 15)
               0,                                            0                       % Equ. 16)
               0,                                            0                       % Equ. 17)
               0,                                            0                       % Equ. 18)
               0,                                            0                       % Equ. 19)
(1-tau_l_bar)*wh_bar*hh_bar*nh_bar,                          0                       % Equ. 20)
-tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar,     -tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar          % Equ. 21)
               0,                                            0                       % Equ. 22)
               0,                                            0                       % Equ. 23)
               1,                                            0                       % Equ. 24)
               0,                                            1                       % Equ. 25)
        -hh_bar/(1-hh_bar),                                  0   ];                  % Equ. 26)

% For y(t):   tot HS hours                                                tot LS hours:
CC9 = [           0,                                                          0          % Equ. 1)
                  0,                                                          0          % Equ. 2)
                  0,                                                          0          % Equ. 3)
                  0,                                                          0          % Equ. 4)
                  0,                                                          0          % Equ. 5)
                  0,                                                          0          % Equ. 6)
                 -1,                                                          0          % Equ. 7)
                  0,                                                         -1          % Equ. 8)
                  0,                                                          0          % Equ. 9)
                  0,                                                          0          % Equ. 10)
                  0,                                                          0          % Equ. 11)
                  0,                                                          0          % Equ. 12)
                  0,                                                          0          % Equ. 13)
                  0,                                                          0          % Equ. 14)
                  0,                                                          0          % Equ. 15)
                  0,                                                          0          % Equ. 16)
                  0,                                                          0          % Equ. 17)
                  0,                                                          0          % Equ. 18)
                  0,                                                          0          % Equ. 19)
                  0,                                                          0          % Equ. 20)
                  0,                                                          0          % Equ. 21)
                  0                                                           0          % Equ. 22)
                  0,                                                          0          % Equ. 23)
                 -1,                                                          0          % Equ. 24)
                  0,                                                         -1          % Equ. 25)
                  0,                                                          0      ];  % Equ. 26)

CC = [CC1, CC2, CC3, CC4, CC5, CC6, CC7, CC8, CC9];

% For exog. var. in time period (t):
%      i(t)
DD = [ 0                    % Equ. 1)
       0                    % Equ. 2)
       0                    % Equ. 3)
       0                    % Equ. 4)
       0                    % Equ. 5)
       0                    % Equ. 6)
       0                    % Equ. 7)
       0                    % Equ. 8)
       0                    % Equ. 9)
```

```
      0                        % Equ. 10)
      0                        % Equ. 11)
      0                        % Equ. 12)
      0                        % Equ. 13)
      0                        % Equ. 14)
      0                        % Equ. 15)
      eta/qh_bar               % Equ. 16)
      (1-eta)/ql_bar           % Equ. 17)
      0                        % Equ. 18)
      0                        % Equ. 19)
      0                        % Equ. 20)
      0                        % Equ. 21)
      0                        % Equ. 22)
      0                        % Equ. 23)
      0                        % Equ. 24)
      0                        % Equ. 25)
      0                    ];  % Equ. 26)

DD=0.4*DD; % We need to multiple DD by 0.4 in order to get 0.4 % shock

% EXPECTATIONAL EQUATIONS:

% For  k(t+1)    L(t+1)     Lh(t+1)   Ll(t+1)    Nh(t+1)   Nl(t+1):
FF = [    0,        0,         0,        0,        0,         0];

% For     k(t)    L(t)       Lh(t)      Ll(t)    Nh(t)       Nl(t):
GG = [    0,        0,         0,        0,        0,         0];

% For   k(t-1)    L(t-1)     Lh(t-1)  Ll(t-1)    Nh(t-1)  Nl(t-1):
HH = [    0,        0,         0,        0,        0,         0];

% For y(t+1): output  investm.   income tax  divid.  return  HS consumt.  LS consump.
JJ1 = [          0,       0,         0,         0,      1,       -1,        0   ];

        %  HS sal.   LS sal.  HS ratio  LS ratio  HS employment  LS employment  HS unemployment
JJ2 = [     0,        0,         0,        0,          0,           0,              0 ] ;

       % LS umemployment total output   tot. capital   HS hours  LS hours  tot HS hours  tot LS hours
JJ3 =  [       0,           0,             0,             0,       0,          0,            0     ];

JJ = [JJ1, JJ2, JJ3];

% For y(t)  : output  investm.   income tax  divid.  return  HS consumt.  LS consump.  HS sal.   LS sal.
KK1 = [          0,       0,         0,         0,      0,        1,          0,         0,       0 ];

% For y(t)  : HS ratio  LS ratio  HS employment  LS employment  HS unemployment  LS umemployment  total output
KK2 = [          0,        0,          0,             0,            0,                0,             0   ];

% For y(t)  :  tot. capital   HS hours  LS hours  tot HS hours  tot LS hours
KK3 =  [             0,            0,       0,         0,          0      ];

KK = [KK1, KK2, KK3];

% For z(t+1):
LL =       [ 0 ];

% For z(t):
MM =       [ 0 ];

% AUTOREGRESSIVE MATRIX FOR z(t):
NN =       [ 0 ];

Sigma =    [ 1 ];


%Setting the options:
```

```
[l_equ,m_states] = size(AA);
[l_equ,n_endog ] = size(CC);
[l_equ,k_exog ]  = size(DD);


PERIOD     = 1; % The fundamental period in the model is one year.
GNP_INDEX  = 1; % Index of output among the variables selected for HP filter
IMP_SELECT = [24,25]; % A vector containing the indices of the variables to be plotted
HP_SELECT  = [1,7,12,15]; % Selecting the variables for the HP Filter calcs.
DO_SIMUL   = 0; % Calculates Simulations (if =1)
DO_MOMENTS = 0; % Calculates Moments (if =1)
DO_STATE_RESP = 0; % Do not calculate impulse responses to deviations of state variables

% Starting the calculations:

do_it;
```

E.3. 2nd and 3rd scenarios in Variations of the Model

```
% Master Thesis "The Inflow of Eastern European Workers to German Labor
% Market: Consequences and Policy Issues"
% MATLAB code for German economy when rho<>1
% Author: Sigitas KARPAVICIUS
% Humboldt-Universität zu Berlin, SS2005
% Revised on 9th August, 2005
% Implements the second-order autoregression process (2nd and 3rd scenarios
% in Variations of the Model)

%disp('Hit any key when ready...');
%pause;


% Setting parameters: (constant values):

alpha      = 0.64;    % Labor share
beta       = 0.96;    % Discount factor
delta      = 0.08;    % Depreciation rate
tau_c      = 0.16;    % VAT
tau_k      = 0.137;   % Tax rate on dividends
r          = 0.33;    % Unemployment benefits
A          = 2.42;    % Preference parameter
rho        = 10/9;    % Inverse of elasticity of substitution between high- and low-skilled labor
eta        = 0.07;    % Share of high-skilled workers among the immigrants
phi        = 2;       % Productivity difference between high- and low-skilled workers
lambda     = 0.5;     % Bargaining power of unemployed
betan      = 0.064;   % Job destruction rate

% Values in the steady state:

%---------------------------Given--------------------------------------

L_bar      = 42806000;% Labor in mln. in 2003
uh_bar     = 0.049;   % Unemployment among high-skilled labor
ul_bar     = 0.115;   % Unemployment among low-skilled labor
qh_bar     = 0.500;   % Ratio of high-skilled labor and all the labor
ql_bar     = 0.500;   % Ratio of low-skilled labor and all the labor
tau_l_bar  = 0.196;   % Labor income tax rate

%------------------------Calculated--------------------------------------

% R_bar - Interest factor
```

```
R_bar = 1/beta;
% d_bar - Dividends rate
d_bar = (R_bar - 1 + delta)/(1-tau_k);
% nh_bar - Employment among high-skilled labor in 2002
nh_bar = 1- uh_bar;
% nl_bar - Employment among low-skilled labor in 2002
nl_bar = 1- ul_bar;
% Total number of high-skilled economically active agents
Lh_bar = L_bar * qh_bar;
% Total number of low-skilled economically active agents
Ll_bar = L_bar * ql_bar;
% Nh_bar - Total number of high-skilled workers
Nh_bar = nh_bar * Lh_bar;
% Nl_bar - Total number of low-skilled workers
Nl_bar = nl_bar * Ll_bar;
% hh_bar - Ratio of working time and total time of high-skilled labor
% alpha*phi/((phi*(nh_bar*qh_bar*hh_bar)^(1-rho)+(nl_bar*ql_bar*hl_bar)^(1-rho))*
%  *(nh_bar*qh_bar*hh_bar)^rho)*((1-tau_l_bar)*(1-hh_bar)/A-(1-tau_l_bar)*
%  *(nh_bar+r*uh_bar)*hh_bar)=(1-alpha)/qh_bar*(1-tau_k-delta/d_bar);
hh_bar = 0.244;
% hl_bar - Ratio of working time and total time of low-skilled labor
hl_bar = 1/(1+A*(nl_bar+r*ul_bar));
% Hh_bar - Total amount of time worked by high-skilled agents
Hh_bar = Nh_bar*hh_bar;
% Hl_bar - Total amount of time worked by low-skilled agents
Hl_bar = Nl_bar*hl_bar;
% y_bar - Output per capita (high- and low-skilled labor)
y_bar =
(phi*(nh_bar*qh_bar*hh_bar)^(1-rho)+(nl_bar*ql_bar*hl_bar)^(1-rho))^(1/(1-rho))*((1-alpha)/d_bar)^((1-alpha)/alpha);
% k_bar - Capital stock per capita (high- and low-skilled labor)
k_bar = (1-alpha)*y_bar/d_bar;
% K_bar - Total capital stock
K_bar = L_bar*k_bar;
% Y_bar - Total output
Y_bar = L_bar*y_bar;
% x_bar - Investment per capita (low- and high-skilled)
x_bar = delta*k_bar;
% wh_bar - wage of high-skilled worker per one unit of her/his working time
wh_bar = alpha*phi*Y_bar/((phi*Hh_bar^(1-rho)+Hl_bar^(1-rho))*Hh_bar^rho);
% wl_bar - wage of low-skilled worker per one unit of her/his working time
wl_bar = alpha*Y_bar/((phi*Hh_bar^(1-rho)+Hl_bar^(1-rho))*Hl_bar^rho);
% ch_bar - consumption of each high-skilled worker
ch_bar = wh_bar*(1-tau_l_bar)*(1-hh_bar)/(A*(1+tau_c));
% cl_bar - consumption of each low-skilled worker
cl_bar = (1-tau_l_bar)*wl_bar*hl_bar*(nl_bar+ul_bar*r)/(1+tau_c);
% Vh_bar - vacancies for high-skilled jobs posted
Vh_bar = betan^(1/(1-lambda))*Nh_bar*(nh_bar/(1-nh_bar))^(lambda/(1-lambda));
% Vl_bar - vacancies for low-skilled jobs posted
Vl_bar = betan^(1/(1-lambda))*Nl_bar*(nl_bar/(1-nl_bar))^(lambda/(1-lambda));


% Declaring the matrices.

VARNAMES = ['capital       ',    % 1
            'total labor   ',    % 2
            'HS labor      ',    % 3
            'LS labor      ',    % 4
            'HS workers    ',    % 5
            'LS workers    ',    % 6
            'output        ',    % 7
            'investment    ',    % 8
            'income tax    ',    % 9
            'dividends     ',    % 10
            'return        ',    % 11
            'HS consumpt.  ',    % 12
            'LS consumpt.  ',    % 13
```

```
            'HS salary    ',    % 14
            'LS salary    ',    % 15
            'HS ratio     ',    % 16
            'LS ratio     ',    % 17
            'HS employment',    % 18
            'LS employment',    % 19
            'HS unempl.   ',    % 20
            'LS unempl.   ',    % 21
            'total output ',    % 22
            'total capital',    % 23
            'HS hours     ',    % 24
            'LS hours     ',    % 25
            'tot HS hours ',    % 26
            'tot LS hours ',    % 27
            'immigrants   ',    % 28
            'immig. lagged'];   % 29


% Translating into coefficient matrices.
% The loglinearized equations are, conveniently ordered:
% Equ. 1)  0 = tau_l_bar/(1-tau_l_bar)*tau_l(t) + cl(t) - wl(t) + hl_bar/(1-hl_bar)*hl(t)
% Equ. 2)  0 = - tau_l_bar/(1-tau_l_bar)*tau_l(t) -
%              - (1+tau_c)/((1-tau_l_bar)*hl_bar*wl_bar*nl_bar)*cl(t) +
%              + wl(t) + nl(t) + r*ul_bar/nl_bar*uh(t) + hl(t)
% Equ. 3)  0 = (1-tau_k)*d_bar/R_bar*d(t) - R(t) - Lh(t) + Lh(t-1)
% Equ. 4)  0 = -(1-delta)*k_bar*k(t-1) - (1-delta)*k_bar*L(t-1) - x_bar*x(t) +
%              +k_bar*k(t) + (1-delta)*k_bar*L(t)
% Equ. 5)  0 = (1-alpha)*k(t-1) + (1-alpha)*L(t-1) - (1-alpha)*L(t) - y(t) +
%            + phi*alpha*(nh_bar*qh_bar*hh_bar)^(1-rho)/a*qh(t) +
%            + alpha*(nl_bar*ql_bar*hl_bar)^(1-rho)/a*ql(t) +
%            + phi*alpha*(nh_bar*qh_bar*hh_bar)^(1-rho)/a*nh(t) + alpha*(nl_bar*ql_bar*hl_bar)^(1-rho)/a*nl(t)+
%            + phi*alpha*(nh_bar*qh_bar*hh_bar)^(1-rho)/a*hh(t) + alpha*(nl_bar*ql_bar*hl_bar)^(1-rho)/a*hl(t)
% Equ. 6)  0 =  - k(t-1) - L(t-1) + L(t) + y(t) - d(t)
% Equ. 7)  0 = - alpha*phi*Y_bar/(wh_bar*Hh_bar^rho)*wh(t) + alpha*phi*Y_bar/(wh_bar*Hh_bar^rho)*Y(t) -
%            - ((1-rho)*phi*Hh_bar^(1-rho)+rho*alpha*phi*Y_bar/(wh_bar*Hh_bar^rho))*Hh(t) -
%            - (1-rho)*Hl_bar^(1-rho)*Hl(t)
% Equ. 8)  0 = - alpha*Y_bar/(wl_bar*Hl_bar^rho)*wl(t) + alpha*Y_bar/(wl_bar*Hl_bar^rho)*Y(t) -
%            - (1-rho)*phi*Hh_bar^(1-rho)*Hh(t) - ((1-rho)*Hl_bar^(1-rho)+rho*alpha*Y_bar/(wl_bar*Hl_bar^rho))*Hl(t)
% Equ. 9)  0 = nh_bar*nh(t) + uh_bar*uh(t)
% Equ. 10) 0 = nl_bar*nl(t) + ul_bar*ul(t)
% Equ. 11) 0 = - nh(t) + Nh(t) - Lh(t)
% Equ. 12) 0 = - nl(t) + Nl(t) - Ll(t)
% Equ. 13) 0 = - qh(t) + Lh(t) - L(t)
% Equ. 14) 0 = - ql(t) + Ll(t) - L(t)
% Equ. 15) 0 = qh_bar*Lh(t) + ql_bar*Ll(t) - L(t)
% Equ. 16) 0 = Lh(t-1) - Lh(t) + eta/qh_bar*i(t)
% Equ. 17) 0 = Ll(t-1) - Ll(t) + (1-eta)/ql_bar*i(t)
% Equ. 18) 0 = - Nh(t) + lambda/nh_bar*(Vh_bar/(Lh_bar-Nh_bar))^(1-lambda)*Lh(t-1) +
%            + (1-betan-lambda*(Vh_bar/(Lh_bar-Nh_bar))^(1-lambda))*Nh(t-1)
% Equ. 19) 0 = - Nl(t) + lambda/nl_bar*(Vl_bar/(Ll_bar-Nl_bar))^(1-lambda)*Ll(t-1) +
%            + (1-betan-lambda*(Vl_bar/(Ll_bar-Nl_bar))^(1-lambda))*Nl(t-1)
% Equ. 20) 0 = (1-tau_k)*k_bar*d_bar/qh_bar*k(t-1) + (1-tau_k)*k_bar*d_bar/qh_bar*L(t-1) -
%            - (1-tau_k)*k_bar*d_bar/qh_bar*L(t) - x_bar/qh_bar*x(t) - tau_l_bar*wh_bar*hh_bar*nh_bar*tau_l(t) +
%            + (1-tau_k)*k_bar*d_bar/qh_bar*d(t) - (1+tau_c)*ch_bar*ch(t) + (1-tau_l_bar)*wh_bar*hh_bar*nh_bar*wh(t)+
%            + x_bar/qh_bar-(1-tau_k)*k_bar*d_bar/qh_bar*qh(t) + (1-tau_l_bar)*wh_bar*hh_bar*nh_bar*nh(t) +
%            + r*(1-tau_l_bar)*wh_bar*hh_bar*uh_bar*uh(t) + (1-tau_l_bar)*wh_bar*hh_bar*nh_bar*hh(t)
% Equ. 21) 0 = - tau_k*d_bar*k_bar*k(t-1) - tau_k*d_bar*k_bar*L(t-1) + tau_k*d_bar*k_bar*L(t) -
%            - (tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar+tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar)*tau_l(t) -
%            - tau_k*d_bar*k_bar*d(t) - tau_c*ch_bar*qh_bar*ch(t) - tau_c*cl_bar*ql_bar*cl(t) -
%            - tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar*wh(t) - tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar*wl(t) -
%            - tau_c*ch_bar*qh_bar-tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar+r*hh_bar*wh_bar*qh_bar*uh_bar*(1-tau_l_bar)*qh(t)-
%            - tau_c*cl_bar*ql_bar-tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar+r*hl_bar*wl_bar*ul_bar*ql_bar*(1-tau_l_bar)*ql(t)-
%            - tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar*nh(t) - tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar*nl(t) +
%            + r*uh_bar*qh_bar*hh_bar*wh_bar*uh(t) + r*hl_bar*wl_bar*ul_bar*ql_bar*ul(t) -
%            - tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar*hh(t) - tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar*hl(t)
% Equ. 22) 0 = y(t) + L(t) - Y(t)
```

```
% Equ. 23) 0 = k(t) + L(t) - K(t)
% Equ. 24) 0 = hh(t) + Nh(t) - Hh(t)
% Equ. 25) 0 = hl(t) + Nl(t) - Hl(t)
% Equ. 26) 0 = - tau_l_bar/(1-tau_l_bar)*tau_l(t) - ch(t) + wh(t) - hh_bar/(1-hh_bar)*hh(t)
% Equ. 27) 0 = E_t [ - ch(t+1) + R(t+1) +  ch(t) ]
% Equ. 28) z(t+1) = i(t+1)
%
% Additional variable introduced for simplicity:
a =
phi*(nh_bar*qh_bar*hh_bar)^(1-rho)+(nl_bar*ql_bar*hl_bar)^(1-rho);
%
% Endogenous state variables "x(t)": k(t), L(t), Lh(t), Ll(t), Nh(t), Nl(t).
% Endogenous other variables "y(t)": y(t), x(t), tau_l(t), d(t),  R(t),
% ch(t), cl(t), wh(t), wl(t), qh(t), ql(t), nh(t), nl(t), uh(t), ul(t),
% Y(t), K(t), hh(t), hl(t), Hh(t), Hl(t).
% Exogenous state variables  "z(t)": i(t), i(t-1).
% CHECK: 28 equations, 28 variables (immigrants and immig. lagged is the same variable just in different time periods.
% Switch to that notation.  Find matrices for format:
% 0 = AA x(t) + BB x(t-1) + CC y(t) + DD z(t),
% 0 = E_t [ FF x(t+1) + GG x(t) + HH x(t-1) + JJ y(t+1) + KK y(t) + LL z(t+1) + MM z(t)],
% z(t+1) = NN z(t) + i(t+1) with E_t [ i(t+1) ] = 0.

% For k(t)          L(t)                      Lh(t)         Ll(t)            Nh(t)           Nl(t):
AA = [ 0,            0,                        0,            0,               0,              0      % Equ. 1)
       0,            0,                        0,            0,               0,              0      % Equ. 2)
       0,            0,                       -1,            0,               0,              0      % Equ. 3)
       k_bar,    (1-delta)*k_bar,              0,            0,               0,              0      % Equ. 4)
       0,          -1+alpha,                   0,            0,               0,              0      % Equ. 5)
       0,            1,                        0,            0,               0,              0      % Equ. 6)
       0,            0,                        0,            0,               0,              0      % Equ. 7)
       0,            0,                        0,            0,               0,              0      % Equ. 8)
       0,            0,                        0,            0,               0,              0      % Equ. 9)
       0,            0,                        0,            0,               0,              0      % Equ. 10)
       0,            0,                       -1,            0,               1,              0      % Equ. 11)
       0,            0,                        0,           -1,               0,              1      % Equ. 12)
       0,           -1,                        1,            0,               0,              0      % Equ. 13)
       0,           -1,                        0,            1,               0,              0      % Equ. 14)
       0,           -1,                      qh_bar,       ql_bar,            0,              0      % Equ. 15)
       0,            0,                       -1,            0,               0,              0      % Equ. 16)
       0,            0,                        0,           -1,               0,              0      % Equ. 17)
       0,            0,                        0,            0,              -1,              0      % Equ. 18)
       0,            0,                        0,            0,               0,             -1      % Equ. 19)
       0,    -(1-tau_k)*k_bar*d_bar/qh_bar,    0,            0,               0,              0      % Equ. 20)
       0,         tau_k*d_bar*k_bar,           0,            0,               0,              0      % Equ. 21)
       0,            1,                        0,            0,               0,              0      % Equ. 22)
       1,            1,                        0,            0,               0,              0      % Equ. 23)
       0,            0,                        0,            0,               1,              0      % Equ. 24)
       0,            0,                        0,            0,               0,              1      % Equ. 25)
       0,            0,                        0,            0,               0,              0   ]; % Equ. 26)


   % For k(t-1)                       L(t-1)                 Lh(t-1):
BB1 = [ 0,                             0,                     0                                           % Equ. 1)
        0,                             0,                     0                                           % Equ. 2)
        0,                             0,                     1                                           % Equ. 3)
 -(1-delta)*k_bar               -(1-delta)*k_bar,             0                                           % Equ. 4)
     1-alpha,                       1-alpha,                  0                                           % Equ. 5)
       -1,                            -1,                     0                                           % Equ. 6)
        0,                             0,                     0                                           % Equ. 7)
        0,                             0,                     0                                           % Equ. 8)
        0,                             0,                     0                                           % Equ. 9)
        0,                             0,                     0                                           % Equ. 10)
        0,                             0,                     0                                           % Equ. 11)
        0,                             0,                     0                                           % Equ. 12)
        0,                             0,                     0                                           % Equ. 13)
```

```
       0,                          0,                   0                                         % Equ. 14)
       0,                          0,                   0                                         % Equ. 15)
       0,                          0,                   1                                         % Equ. 16)
       0,                          0,                   0                                         % Equ. 17)
       0,                          0,              lambda/nh_bar*(Vh_bar/(Lh_bar-Nh_bar))^(1-lambda) % Equ. 18)
       0,                          0,                   0                                         % Equ. 19)
 (1-tau_k)*k_bar*d_bar/qh_bar,  (1-tau_k)*k_bar*d_bar/qh_bar,     0                               % Equ. 20)
 -tau_k*d_bar*k_bar,             -tau_k*d_bar*k_bar,      0                                       % Equ. 21)
       0,                          0,                   0                                         % Equ. 22)
       0,                          0,                   0                                         % Equ. 23)
       0,                          0,                   0                                         % Equ. 24)
       0,                          0,                   0                                         % Equ. 25)
       0,                          0,                   0                         ];              % Equ. 26)

   % For  Ll(t-1)                                     Nh(t-1)  :
BB2 = [ 0,                                              0                          % Equ. 1)
       0,                                              0                          % Equ. 2)
       0,                                              0                          % Equ. 3)
       0,                                              0                          % Equ. 4)
       0,                                              0                          % Equ. 5)
       0,                                              0                          % Equ. 6)
       0,                                              0                          % Equ. 7)
       0,                                              0                          % Equ. 8)
       0,                                              0                          % Equ. 9)
       0,                                              0                          % Equ. 10)
       0,                                              0                          % Equ. 11)
       0,                                              0                          % Equ. 12)
       0,                                              0                          % Equ. 13)
       0,                                              0                          % Equ. 14)
       0,                                              0                          % Equ. 15)
       0,                                              0                          % Equ. 16)
       1,                                              0                          % Equ. 17)
       0,                           1-betan-lambda*(Vh_bar/(Lh_bar-Nh_bar))^(1-lambda) % Equ. 18)
 lambda/nl_bar*(Vl_bar/(Ll_bar-Nl_bar))^(1-lambda),    0                          % Equ. 19)
       0,                                              0                          % Equ. 20)
       0,                                              0                          % Equ. 21)
       0,                                              0                          % Equ. 22)
       0,                                              0                          % Equ. 23)
       0,                                              0                          % Equ. 24)
       0,                                              0                          % Equ. 25)
       0,                                              0                  ];      % Equ. 26)


% For    Nl(t-1):
BB3 = [      0                                  % Equ. 1)
             0                                  % Equ. 2)
             0                                  % Equ. 3)
             0                                  % Equ. 4)
             0                                  % Equ. 5)
             0                                  % Equ. 6)
             0                                  % Equ. 7)
             0                                  % Equ. 8)
             0                                  % Equ. 9)
             0                                  % Equ. 10)
             0                                  % Equ. 11)
             0                                  % Equ. 12)
             0                                  % Equ. 13)
             0                                  % Equ. 14)
             0                                  % Equ. 15)
             0                                  % Equ. 16)
             0                                  % Equ. 17)
             0                                  % Equ. 18)
 1-betan-lambda*(Vl_bar/(Ll_bar-Nl_bar))^(1-lambda) % Equ. 19)
             0                                  % Equ. 20)
             0                                  % Equ. 21)
             0                                  % Equ. 22)
```

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            0                                    % Equ. 23)
            0                                    % Equ. 24)
            0                                    % Equ. 25)
            0                               ];   % Equ. 26)

  BB = [BB1, BB2, BB3];


% For y(t): output   investment            income tax
CC1 = [       0,        0,          tau_l_bar/(1-tau_l_bar)                                                     % Equ. 1)
              0,        0,        -tau_l_bar/(1-tau_l_bar)                                                      % Equ. 2)
              0,        0,                    0                                                                 % Equ. 3)
              0,    -x_bar,                   0                                                                 % Equ. 4)
             -1,        0,                    0                                                                 % Equ. 5)
              1,        0,                    0                                                                 % Equ. 6)
              0,        0,                    0                                                                 % Equ. 7)
              0,        0,                    0                                                                 % Equ. 8)
              0,        0,                    0                                                                 % Equ. 9)
              0,        0,                    0                                                                 % Equ. 10)
              0,        0,                    0                                                                 % Equ. 11)
              0,        0,                    0                                                                 % Equ. 12)
              0,        0,                    0                                                                 % Equ. 13)
              0,        0,                    0                                                                 % Equ. 14)
              0,        0,                    0                                                                 % Equ. 15)
              0,        0,                    0                                                                 % Equ. 16)
              0,        0,                    0                                                                 % Equ. 17)
              0,        0,                    0                                                                 % Equ. 18)
              0,        0,                    0                                                                 % Equ. 19)
              0,   -x_bar/qh_bar,       -tau_l_bar*wh_bar*hh_bar*nh_bar                                         % Equ. 20)
              0,        0,       -(tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar+tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar) % Equ. 21)
              1,        0,                    0                                                                 % Equ. 22)
              0,        0,                    0                                                                 % Equ. 23)
              0,        0,                    0                                                                 % Equ. 24)
              0,        0,                    0                                                                 % Equ. 25)
              0,        0,              -tau_l_bar/(1-tau_l_bar)                                   ];           % Equ. 26)


% For y(t):  dividends           return        HS consumption     LS consumption
CC2 = [         0,                 0,              0,                  1                                        % Equ. 1)
                0,                 0,              0,     -(1+tau_c)*cl_bar/((1-tau_l_bar)*hl_bar*wl_bar*nl_bar) % Equ. 2)
       (1-tau_k)*d_bar/R_bar,     -1,              0,                  0                                        % Equ. 3)
                0,                 0,              0,                  0                                        % Equ. 4)
                0,                 0,              0,                  0                                        % Equ. 5)
               -1,                 0,              0,                  0                                        % Equ. 6)
                0,                 0,              0,                  0                                        % Equ. 7)
                0,                 0,              0,                  0                                        % Equ. 8)
                0,                 0,              0,                  0                                        % Equ. 9)
                0,                 0,              0,                  0                                        % Equ. 10)
                0,                 0,              0,                  0                                        % Equ. 11)
                0,                 0,              0,                  0                                        % Equ. 12)
                0,                 0,              0,                  0                                        % Equ. 13)
                0,                 0,              0,                  0                                        % Equ. 14)
                0,                 0,              0,                  0                                        % Equ. 15)
                0,                 0,              0,                  0                                        % Equ. 16)
                0,                 0,              0,                  0                                        % Equ. 17)
                0,                 0,              0,                  0                                        % Equ. 18)
                0,                 0,              0,                  0                                        % Equ. 19)
(1-tau_k)*k_bar*d_bar/qh_bar,      0,   -(1+tau_c)*ch_bar,             0                                        % Equ. 20)
       -tau_k*d_bar*k_bar,         0,   -tau_c*ch_bar*qh_bar,   -tau_c*cl_bar*ql_bar                            % Equ. 21)
                0,                 0,              0,                  0                                        % Equ. 22)
                0,                 0,              0,                  0                                        % Equ. 23)
                0,                 0,              0,                  0                                        % Equ. 24)
                0,                 0,              0,                  0                                        % Equ. 25)
                0,                 0,             -1,                  0                        ];              % Equ. 26)
```

```
% For y(t):  HS salary                                          LS salary
CC3 = [          0,                                                -1                                  % Equ. 1)
                 0,                                                 1                                  % Equ. 2)
                 0,                                                 0                                  % Equ. 3)
                 0,                                                 0                                  % Equ. 4)
                 0,                                                 0                                  % Equ. 5)
                 0,                                                 0                                  % Equ. 6)
-alpha*phi*Y_bar/(wh_bar*Hh_bar^rho),                               0                                  % Equ. 7)
                 0,                                  -alpha*Y_bar/(wl_bar*Hl_bar^rho)                  % Equ. 8)
                 0,                                                 0                                  % Equ. 9)
                 0,                                                 0                                  % Equ. 10)
                 0,                                                 0                                  % Equ. 11)
                 0,                                                 0                                  % Equ. 12)
                 0,                                                 0                                  % Equ. 13)
                 0,                                                 0                                  % Equ. 14)
                 0,                                                 0                                  % Equ. 15)
                 0,                                                 0                                  % Equ. 16)
                 0,                                                 0                                  % Equ. 17)
                 0,                                                 0                                  % Equ. 18)
                 0,                                                 0                                  % Equ. 19)
(1-tau_l_bar)*wh_bar*hh_bar*nh_bar,                                 0                                  % Equ. 20)
-tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar,       -tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar                   % Equ. 21)
                 0,                                                 0                                  % Equ. 22)
                 0,                                                 0                                  % Equ. 23)
                 0,                                                 0                                  % Equ. 24)
                 0,                                                 0                                  % Equ. 25)
                 1,                                                 0                         ];       % Equ. 26)

% For y(t):                                            HS ratio
CC4 = [                                                  0                                                        % Equ. 1)
                                                         0                                                        % Equ. 2)
                                                         0                                                        % Equ. 3)
                                                         0                                                        % Equ. 4)
                             phi*alpha*(nh_bar*qh_bar*hh_bar)^(1-rho)/a                                           % Equ. 5)
                                                         0                                                        % Equ. 6)
                                                         0                                                        % Equ. 7)
                                                         0                                                        % Equ. 8)
                                                         0                                                        % Equ. 9)
                                                         0                                                        % Equ. 10)
                                                         0                                                        % Equ. 11)
                                                         0                                                        % Equ. 12)
                                                        -1                                                        % Equ. 13)
                                                         0                                                        % Equ. 14)
                                                         0                                                        % Equ. 15)
                                                         0                                                        % Equ. 16)
                                                         0                                                        % Equ. 17)
                                                         0                                                        % Equ. 18)
                                                         0                                                        % Equ. 19)
                                    x_bar/qh_bar-(1-tau_k)*k_bar*d_bar/qh_bar                                     % Equ. 20)
  -tau_c*ch_bar*qh_bar-tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar+r*hh_bar*wh_bar*qh_bar*uh_bar*(1-tau_l_bar)          % Equ. 21)
                                                         0                                                        % Equ. 22)
                                                         0                                                        % Equ. 23)
                                                         0                                                        % Equ. 24)
                                                         0                                                        % Equ. 25)
                                                         0                                                 ];     % Equ. 26)

% For y(t):                                            LS ratio
CC5 = [                                                  0                                                        % Equ. 1)
                                                         0                                                        % Equ. 2)
                                                         0                                                        % Equ. 3)
                                                         0                                                        % Equ. 4)
                                alpha*(nl_bar*ql_bar*hl_bar)^(1-rho)/a                                            % Equ. 5)
                                                         0                                                        % Equ. 6)
                                                         0                                                        % Equ. 7)
```

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                                 0                                                  % Equ. 8)
                                 0                                                  % Equ. 9)
                                 0                                                  % Equ. 10)
                                 0                                                  % Equ. 11)
                                 0                                                  % Equ. 12)
                                 0                                                  % Equ. 13)
                                -1                                                  % Equ. 14)
                                 0                                                  % Equ. 15)
                                 0                                                  % Equ. 16)
                                 0                                                  % Equ. 17)
                                 0                                                  % Equ. 18)
                                 0                                                  % Equ. 19)
                                 0                                                  % Equ. 20)
  -tau_c*cl_bar*ql_bar-tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar+r*hl_bar*wl_bar*ul_bar*ql_bar*(1-tau_l_bar)    % Equ. 21)
                                 0                                                  % Equ. 22)
                                 0                                                  % Equ. 23)
                                 0                                                  % Equ. 24)
                                 0                                                  % Equ. 25)
                                 0                                              ];  % Equ. 26)

% For y(t):         HS employment                           LS employment
CC6 = [                   0,                                      0                   % Equ. 1)
                          0,                                      1                   % Equ. 2)
                          0,                                      0                   % Equ. 3)
                          0,                                      0                   % Equ. 4)
    phi*alpha*(nh_bar*qh_bar*hh_bar)^(1-rho)/a,     alpha*(nl_bar*ql_bar*hl_bar)^(1-rho)/a   % Equ. 5)
                          0,                                      0                   % Equ. 6)
                          0,                                      0                   % Equ. 7)
                          0,                                      0                   % Equ. 8)
                        nh_bar,                                   0                   % Equ. 9)
                          0,                                    nl_bar                % Equ. 10)
                         -1,                                      0                   % Equ. 11)
                          0,                                     -1                   % Equ. 12)
                          0,                                      0                   % Equ. 13)
                          0,                                      0                   % Equ. 14)
                          0,                                      0                   % Equ. 15)
                          0,                                      0                   % Equ. 16)
                          0,                                      0                   % Equ. 17)
                          0,                                      0                   % Equ. 18)
                          0,                                      0                   % Equ. 19)
          (1-tau_l_bar)*wh_bar*hh_bar*nh_bar,                     0                   % Equ. 20)
        -tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar,     -tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar  % Equ. 21)
                          0,                                      0                   % Equ. 22)
                          0,                                      0                   % Equ. 23)
                          0,                                      0                   % Equ. 24)
                          0,                                      0                   % Equ. 25)
                          0,                                      0               ];  % Equ. 26)

% For y(t):     HS unemployment              LS unemployment               total output        tot. capital
CC7 = [              0,                             0,                            0,                 0   % Equ. 1)
                     0,                      r*ul_bar/nl_bar,                     0,                 0   % Equ. 2)
                     0,                             0,                            0,                 0   % Equ. 3)
                     0,                             0,                            0,                 0   % Equ. 4)
                     0,                             0,                            0,                 0   % Equ. 5)
                     0,                             0,                            0,                 0   % Equ. 6)
                     0,                             0,       alpha*phi*Y_bar/(wh_bar*Hh_bar^rho),    0   % Equ. 7)
                     0,                             0,          alpha*Y_bar/(wl_bar*Hl_bar^rho),     0   % Equ. 8)
                  uh_bar,                           0,                            0,                 0   % Equ. 9)
                     0,                          ul_bar,                          0,                 0   % Equ. 10)
                     0,                             0,                            0,                 0   % Equ. 11)
                     0,                             0,                            0,                 0   % Equ. 12)
                     0,                             0,                            0,                 0   % Equ. 13)
                     0,                             0,                            0,                 0   % Equ. 14)
                     0,                             0,                            0,                 0   % Equ. 15)
                     0,                             0,                            0,                 0   % Equ. 16)
```

```
                    0,                              0,                           0,               0   % Equ. 17)
                    0,                              0,                           0,               0   % Equ. 18)
                    0,                              0,                           0,               0   % Equ. 19)
 r*(1-tau_l_bar)*wh_bar*hh_bar*uh_bar,              0,                           0,               0   % Equ. 20)
   r*uh_bar*qh_bar*hh_bar*wh_bar,      r*hl_bar*wl_bar*ul_bar*ql_bar,            0,               0   % Equ. 21)
                    0,                              0,                          -1,               0   % Equ. 22)
                    0,                              0,                           0,              -1   % Equ. 23)
                    0,                              0,                           0,               0   % Equ. 24)
                    0,                              0,                           0,               0   % Equ. 25)
                    0,                              0,                           0,               0 ];% Equ. 26)

% For y(t):   HS hours                                  LS hours
CC8 = [        0,                                  hl_bar/(1-hl_bar)                % Equ. 1)
               0,                                          1                        % Equ. 2)
               0,                                          0                        % Equ. 3)
               0,                                          0                        % Equ. 4)
 phi*alpha*(nh_bar*qh_bar*hh_bar)^(1-rho)/a,    alpha*(nl_bar*ql_bar*hl_bar)^(1-rho)/a      % Equ. 5)
               0,                                          0                        % Equ. 6)
               0,                                          0                        % Equ. 7)
               0,                                          0                        % Equ. 8)
               0,                                          0                        % Equ. 9)
               0,                                          0                        % Equ. 10)
               0,                                          0                        % Equ. 11)
               0,                                          0                        % Equ. 12)
               0,                                          0                        % Equ. 13)
               0,                                          0                        % Equ. 14)
               0,                                          0                        % Equ. 15)
               0,                                          0                        % Equ. 16)
               0,                                          0                        % Equ. 17)
               0,                                          0                        % Equ. 18)
               0,                                          0                        % Equ. 19)
(1-tau_l_bar)*wh_bar*hh_bar*nh_bar,                        0                        % Equ. 20)
-tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar,     -tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar         % Equ. 21)
               0,                                          0                        % Equ. 22)
               0,                                          0                        % Equ. 23)
               1,                                          0                        % Equ. 24)
               0,                                          1                        % Equ. 25)
       -hh_bar/(1-hh_bar),                                 0   ];                   % Equ. 26)

% For y(t):   tot HS hours                                              tot LS hours:
CC9 = [           0,                                                           0           % Equ. 1)
                  0,                                                           0           % Equ. 2)
                  0,                                                           0           % Equ. 3)
                  0,                                                           0           % Equ. 4)
                  0,                                                           0           % Equ. 5)
                  0,                                                           0           % Equ. 6)
-((1-rho)*phi*Hh_bar^(1-rho)+rho*alpha*phi*Y_bar/(wh_bar*Hh_bar^rho)),   -(1-rho)*Hl_bar^(1-rho) % Equ. 7)
-(1-rho)*phi*Hh_bar^(1-rho),       -((1-rho)*Hl_bar^(1-rho)+rho*alpha*Y_bar/(wl_bar*Hl_bar^rho)) % Equ. 8)
                  0,                                                           0           % Equ. 9)
                  0,                                                           0           % Equ. 10)
                  0,                                                           0           % Equ. 11)
                  0,                                                           0           % Equ. 12)
                  0,                                                           0           % Equ. 13)
                  0,                                                           0           % Equ. 14)
                  0,                                                           0           % Equ. 15)
                  0,                                                           0           % Equ. 16)
                  0,                                                           0           % Equ. 17)
                  0,                                                           0           % Equ. 18)
                  0,                                                           0           % Equ. 19)
                  0,                                                           0           % Equ. 20)
                  0,                                                           0           % Equ. 21)
                  0                                                            0           % Equ. 22)
                  0,                                                           0           % Equ. 23)
                 -1,                                                           0           % Equ. 24)
                  0,                                                          -1           % Equ. 25)
                  0,                                                           0       ];   % Equ. 26)
```

```
CC = [CC1, CC2, CC3, CC4, CC5, CC6, CC7, CC8, CC9];

% For exog. var. in time period (t):
%       i(t)              i(t-1)
DD = [ 0,                   0     % Equ. 1)
       0,                   0     % Equ. 2)
       0,                   0     % Equ. 3)
       0,                   0     % Equ. 4)
       0,                   0     % Equ. 5)
       0,                   0     % Equ. 6)
       0,                   0     % Equ. 7)
       0,                   0     % Equ. 8)
       0,                   0     % Equ. 9)
       0,                   0     % Equ. 10)
       0,                   0     % Equ. 11)
       0,                   0     % Equ. 12)
       0,                   0     % Equ. 13)
       0,                   0     % Equ. 14)
       0,                   0     % Equ. 15)
       eta/qh_bar,          0     % Equ. 16)
       (1-eta)/ql_bar,      0     % Equ. 17)
       0,                   0     % Equ. 18)
       0,                   0     % Equ. 19)
       0,                   0     % Equ. 20)
       0,                   0     % Equ. 21)
       0,                   0     % Equ. 22)
       0,                   0     % Equ. 23)
       0,                   0     % Equ. 24)
       0,                   0     % Equ. 25)
       0,                   0 ];  % Equ. 26)

DD=0.4*DD; % In order to have 0.4 % shock not standard 1 %

% EXPECTATIONAL EQUATIONS:

% For  k(t+1)    L(t+1)     Lh(t+1)   Ll(t+1)    Nh(t+1)   Nl(t+1):
FF = [    0,        0,         0,        0,         0,         0];

% For     k(t)    L(t)       Lh(t)      Ll(t)    Nh(t)       Nl(t):
GG = [    0,        0,         0,        0,         0,         0];

% For   k(t-1)    L(t-1)     Lh(t-1)  Ll(t-1)    Nh(t-1)  Nl(t-1):
HH = [    0,        0,         0,        0,         0,         0];

% For y(t+1): output  investm.   income tax  divid.  return  HS consumt.  LS consump.
JJ1 = [          0,       0,         0,         0,      1,       -1,        0   ];

        %  HS sal.   LS sal.  HS ratio  LS ratio  HS employment  LS employment  HS unemployment
JJ2 = [     0,        0,         0,        0,          0,             0,              0 ] ;

       % LS umemployment total output   tot. capital   HS hours  LS hours  tot HS hours  tot LS hours
JJ3 =  [       0,           0,             0,             0,       0,          0,            0     ];

JJ = [JJ1, JJ2, JJ3];

% For y(t)  : output  investm.   income tax  divid.  return  HS consumt.  LS consump.  HS sal.   LS sal.
KK1 = [          0,       0,         0,         0,      0,        1,         0,         0,        0 ];

% For y(t)  : HS ratio  LS ratio  HS employment  LS employment  HS unemployment  LS umemployment  total output
KK2 = [          0,        0,          0,             0,             0,                0,             0   ];

% For y(t)  :  tot. capital   HS hours  LS hours  tot HS hours  tot LS hours
KK3 =  [             0,            0,       0,         0,        0      ];

KK = [KK1, KK2, KK3];
```

```
% For z(t+1):
LL =      [ 0, 0 ];

% For z(t):
MM =      [ 0, 0 ];

% AUTOREGRESSIVE MATRIX FOR z(t):
NN = [1.221257,    -0.39          % 2nd scenario
      1,            0    ];

NN = [1.221257,   -0.55             % 3rd scenario
      1,            0   ];


Sigma = [1,  0
         0,  0.0000001];

%Setting the options:

[l_equ,m_states] = size(AA);
[l_equ,n_endog ] = size(CC);
[l_equ,k_exog ]  = size(DD);


PERIOD     = 1; % The fundamental period in the model is one year.
GNP_INDEX  = 2; % Index of output among the variables selected for HP filter
IMP_SELECT = [1,7,9,22,23]; % A vector containing the indices of the variables to be plotted
HP_SELECT  = [1,7:15,20,21,24,25,28]; % Selecting the variables for the HP Filter calcs.
DO_SIMUL   = 0; % Calculates Simulations (if =1)
DO_MOMENTS = 0; % Calculates Moments (if =1)
DO_STATE_RESP = 0; % Do not calculate impulse responses to deviations of state variables
SELECT_SHOCKS = 1; % Do not show the impulse responses to the immig. lagged
HORIZON     = 50; % You need this only in case of the third (last) scenario

% Starting the calculations:

do_it;
```

E.4. UK economy

```
% Version of MATLAB code for the UK economy
% Author: Sigitas KARPAVICIUS
% Humboldt-Universität zu Berlin, SS2005
% Revised on 25th August, 2005

%disp('Hit any key when ready...');
%pause;


% Setting parameters (constant values):

alpha      = 2/3;     % Labor share
beta       = 0.96;    % Discount factor
delta      = 0.08;    % Depreciation rate
tau_c      = 0.175;   % VAT
tau_k      = 0.175;   % Tax rate on dividends
r          = 0.33;    % Unemployment benefits
A          = 2.52;    % Preference parameter
rho        = 0.8;     % Inverse of elasticity of substitution between high- and low-skilled labor
eta        = 0.07;    % Share of high-skilled workers among the immigrants
phi        = 2;       % Productivity difference between high- and low-skilled workers
```

```
lambda     = 0.5;     % Bargaining power of unemployed
betan      = 0.064;   % Job destruction rate

% Values in the steady state:

%---------------------------Given--------------------------------------

L_bar      = 31936000;% Labor in mln. in 2003
uh_bar     = 0.037;   % Unemployment among high-skilled labor
ul_bar     = 0.11098; % Unemployment among low-skilled labor
qh_bar     = 0.463;   % Ratio of high-skilled labor and all the labor
ql_bar     = 0.537;   % Ratio of low-skilled labor and all the labor
tau_l_bar  = 0.159;   % Labor income tax rate

%------------------------Calculated--------------------------------------

% R_bar - Interest factor
R_bar = 1/beta;
% d_bar - Dividends rate
d_bar = (R_bar - 1 + delta)/(1-tau_k);
% nh_bar - Employment among high-skilled labor in 2002
nh_bar = 1- uh_bar;
% nl_bar - Employment among low-skilled labor in 2002
nl_bar = 1- ul_bar;
% Total number of high-skilled economically active agents
Lh_bar = L_bar * qh_bar;
% Total number of low-skilled economically active agents
Ll_bar = L_bar * ql_bar;
% Nh_bar - Total number of high-skilled workers
Nh_bar = nh_bar * Lh_bar;
% Nl_bar - Total number of low-skilled workers
Nl_bar = nl_bar * Ll_bar;
% hh_bar - Ratio of working time and total time of high-skilled labor
% alpha*phi/((phi*(nh_bar*qh_bar*hh_bar)^(1-rho)+(nl_bar*ql_bar*hl_bar)^(1-rho))*
% *(nh_bar*qh_bar*hh_bar)^rho)*((1-tau_l_bar)*(1-hh_bar)/A-(1-tau_l_bar)*(nh_bar+r*uh_bar)*hh_bar)=
% =(1-alpha)/qh_bar*(1-tau_k-delta/d_bar);
hh_bar = 0.243;
% hl_bar - Ratio of working time and total time of low-skilled labor
hl_bar = 1/(1+A*(nl_bar+r*ul_bar));
% Hh_bar - Total amount of time worked by high-skilled agents
Hh_bar = Nh_bar*hh_bar;
% Hl_bar - Total amount of time worked by low-skilled agents
Hl_bar = Nl_bar*hl_bar;
% y_bar - Output per capita (high- and low-skilled labor)
y_bar
=(phi*(nh_bar*qh_bar*hh_bar)^(1-rho)+(nl_bar*ql_bar*hl_bar)^(1-rho))^(1/(1-rho))*((1-alpha)/d_bar)^((1-alpha)/alpha);
% k_bar - Capital stock per capita (high- and low-skilled labor)
k_bar = (1-alpha)*y_bar/d_bar;
% K_bar - Total capital stock
K_bar = L_bar*k_bar;
% Y_bar - Total output
Y_bar = L_bar*y_bar;
% x_bar - Investment per capita (low- and high-skilled)
x_bar = delta*k_bar;
% wh_bar - wage of high-skilled worker per one unit of her/his working time
wh_bar = alpha*phi*Y_bar/((phi*Hh_bar^(1-rho)+Hl_bar^(1-rho))*Hh_bar^rho);
% wl_bar - wage of low-skilled worker per one unit of her/his working time
wl_bar = alpha*Y_bar/((phi*Hh_bar^(1-rho)+Hl_bar^(1-rho))*Hl_bar^rho);
% ch_bar - consumption of each high-skilled worker
ch_bar = wh_bar*(1-tau_l_bar)*(1-hh_bar)/(A*(1+tau_c));
% cl_bar - consumption of each low-skilled worker
cl_bar = (1-tau_l_bar)*wl_bar*hl_bar*(nl_bar+ul_bar*r)/(1+tau_c);
% Vh_bar - vacancies for high-skilled jobs posted
Vh_bar = betan^(1/(1-lambda))*Nh_bar*(nh_bar/(1-nh_bar))^(lambda/(1-lambda));
% Vl_bar - vacancies for low-skilled jobs posted
Vl_bar = betan^(1/(1-lambda))*Nl_bar*(nl_bar/(1-nl_bar))^(lambda/(1-lambda));
```

```
% Declaring the matrices.

VARNAMES = ['capital      ',
            'total labor  ',
            'HS labor     ',
            'LS labor     ',
            'HS workers   ',
            'LS workers   ',
            'output       ',
            'investment   ',
            'income tax   ',
            'dividends    ',
            'return       ',
            'HS consumpt. ',
            'LS consumpt. ',
            'HS salary    ',
            'LS salary    ',
            'HS ratio     ',
            'LS ratio     ',
            'HS employment',
            'LS employment',
            'HS unempl.   ',
            'LS unempl.   ',
            'total output ',
            'total capital',
            'HS hours     ',
            'LS hours     ',
            'tot HS hours ',
            'tot LS hours ',
            'immigrants   '];


% Translating into coefficient matrices.
% The loglinearized equations are, conveniently ordered:
% Equ. 1)  0 = tau_l_bar/(1-tau_l_bar)*tau_l(t) + cl(t) - wl(t) + hl_bar/(1-hl_bar)*hl(t)
% Equ. 2)  0 = - tau_l_bar/(1-tau_l_bar)*tau_l(t) -
%              - (1+tau_c)*cl_bar/((1-tau_l_bar)*hl_bar*wl_bar*nl_bar)*cl(t) +
%              + wl(t) + nl(t) + r*ul_bar/nl_bar*ul(t) + hl(t)
% Equ. 3)  0 = (1-tau_k)*d_bar/R_bar*d(t) - R(t) - Lh(t) + Lh(t-1)
% Equ. 4)  0 = -(1-delta)*k_bar*k(t-1) - (1-delta)*k_bar*L(t-1) - x_bar*x(t) +
%              +k_bar*k(t) + (1-delta)*k_bar*L(t)
% Equ. 5)  0 = (1-alpha)*k(t-1) + (1-alpha)*L(t-1) - (1-alpha)*L(t) - y(t) +
%            + phi*alpha*(nh_bar*qh_bar*hh_bar)^(1-rho)/a*qh(t) +
%            + alpha*(nl_bar*ql_bar*hl_bar)^(1-rho)/a*ql(t) +
%            + phi*alpha*(nh_bar*qh_bar*hh_bar)^(1-rho)/a*nh(t) + alpha*(nl_bar*ql_bar*hl_bar)^(1-rho)/a*nl(t)+
%            + phi*alpha*(nh_bar*qh_bar*hh_bar)^(1-rho)/a*hh(t) + alpha*(nl_bar*ql_bar*hl_bar)^(1-rho)/a*hl(t)
% Equ. 6)  0 =  - k(t-1) - L(t-1) + L(t) + y(t) - d(t)
% Equ. 7)  0 = - alpha*phi*Y_bar/(wh_bar*Hh_bar^rho)*wh(t) + alpha*phi*Y_bar/(wh_bar*Hh_bar^rho)*Y(t) -
%            - ((1-rho)*phi*Hh_bar^(1-rho)+rho*alpha*phi*Y_bar/(wh_bar*Hh_bar^rho))*Hh(t) -
%            - (1-rho)*Hl_bar^(1-rho)*Hl(t)
% Equ. 8)  0 = - alpha*Y_bar/(wl_bar*Hl_bar^rho)*wl(t) + alpha*Y_bar/(wl_bar*Hl_bar^rho)*Y(t) -
%            - (1-rho)*phi*Hh_bar^(1-rho)*Hh(t) - ((1-rho)*Hl_bar^(1-rho)+rho*alpha*Y_bar/(wl_bar*Hl_bar^rho))*Hl(t)
% Equ. 9)  0 = nh_bar*nh(t) + uh_bar*uh(t)
% Equ. 10) 0 = nl_bar*nl(t) + ul_bar*ul(t)
% Equ. 11) 0 = - nh(t) + Nh(t) - Lh(t)
% Equ. 12) 0 = - nl(t) + Nl(t) - Ll(t)
% Equ. 13) 0 = - qh(t) + Lh(t) - L(t)
% Equ. 14) 0 = - ql(t) + Ll(t) - L(t)
% Equ. 15) 0 = qh_bar*Lh(t) + ql_bar*Ll(t) - L(t)
% Equ. 16) 0 = Lh(t-1) - Lh(t) + eta/qh_bar*i(t)
% Equ. 17) 0 = Ll(t-1) - Ll(t) + (1-eta)/ql_bar*i(t)
% Equ. 18) 0 = - Nh(t) + lambda/nh_bar*(Vh_bar/(Lh_bar-Nh_bar))^(1-lambda)*Lh(t-1) +
%            + (1-betan-lambda*(Vh_bar/(Lh_bar-Nh_bar))^(1-lambda))*Nh(t-1)
% Equ. 19) 0 = - Nl(t) + lambda/nl_bar*(Vl_bar/(Ll_bar-Nl_bar))^(1-lambda)*Ll(t-1) +
```

```
%            + (1-betan-lambda*(Vl_bar/(Ll_bar-Nl_bar))^(1-lambda))*Nl(t-1)
% Equ. 20) 0 = (1-tau_k)*k_bar*d_bar/qh_bar*k(t-1) + (1-tau_k)*k_bar*d_bar/qh_bar*L(t-1) -
%            - (1-tau_k)*k_bar*d_bar/qh_bar*L(t) - x_bar/qh_bar*x(t) - tau_l_bar*wh_bar*hh_bar*nh_bar*tau_l(t) +
%            + (1-tau_k)*k_bar*d_bar/qh_bar*d(t) - (1+tau_c)*ch_bar*ch(t) + (1-tau_l_bar)*wh_bar*hh_bar*nh_bar*wh(t)+
%            + x_bar/qh_bar-(1-tau_k)*k_bar*d_bar/qh_bar*qh(t) + (1-tau_l_bar)*wh_bar*hh_bar*nh_bar*nh(t) +
%            + r*(1-tau_l_bar)*wh_bar*hh_bar*uh_bar*uh(t) + (1-tau_l_bar)*wh_bar*hh_bar*nh_bar*hh(t)
% Equ. 21) 0 = - tau_k*d_bar*k_bar*k(t-1) - tau_k*d_bar*k_bar*L(t-1) + tau_k*d_bar*k_bar*L(t) -
%            - (tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar+tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar)*tau_l(t) -
%            - tau_k*d_bar*k_bar*d(t) - tau_c*ch_bar*qh_bar*ch(t) - tau_c*cl_bar*ql_bar*cl(t) -
%            - tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar*wh(t) - tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar*wl(t) -
%            - tau_c*ch_bar*qh_bar-tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar+r*hh_bar*wh_bar*qh_bar*uh_bar*(1-tau_l_bar)*qh(t)-
%            - tau_c*cl_bar*ql_bar-tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar+r*hl_bar*wl_bar*ul_bar*ql_bar*(1-tau_l_bar)*ql(t)-
%            - tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar*nh(t) - tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar*nl(t) +
%            + r*uh_bar*qh_bar*hh_bar*wh_bar*uh(t) + r*hl_bar*wl_bar*ul_bar*ql_bar*ul(t) -
%            - tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar*hh(t) - tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar*hl(t)
% Equ. 22) 0 = y(t) + L(t) - Y(t)
% Equ. 23) 0 = k(t) + L(t) - K(t)
% Equ. 24) 0 = hh(t) + Nh(t) - Hh(t)
% Equ. 25) 0 = hl(t) + Nl(t) - Hl(t)
% Equ. 26) 0 = - tau_l_bar/(1-tau_l_bar)*tau_l(t) - ch(t) + wh(t) - hh_bar/(1-hh_bar)*hh(t)
% Equ. 27) 0 = E_t [ - ch(t+1) + R(t+1) +  ch(t) ]
% Equ. 28) z(t+1) = i(t+1)
%
% Additional variable introduced for simplicity:
a =
phi*(nh_bar*qh_bar*hh_bar)^(1-rho)+(nl_bar*ql_bar*hl_bar)^(1-rho);
%
% Endogenous state variables "x(t)": k(t), L(t), Lh(t), Ll(t), Nh(t), Nl(t).
% Endogenous other variables "y(t)": y(t), x(t), tau_l(t), d(t),  R(t),
% ch(t), cl(t), wh(t), wl(t), qh(t), ql(t), nh(t), nl(t), uh(t), ul(t),
% Y(t), K(t), hh(t), hl(t), Hh(t), Hl(t).
% Exogenous state variables  "z(t)": i(t).
% CHECK: 28 equations, 28 variables.
% Switch to that notation.  Find matrices for format:
% 0 = AA x(t) + BB x(t-1) + CC y(t) + DD z(t),
% 0 = E_t [ FF x(t+1) + GG x(t) + HH x(t-1) + JJ y(t+1) + KK y(t) + LL z(t+1) + MM z(t)],
% z(t+1) = NN z(t) + i(t+1) with E_t [ i(t+1) ] = 0.

% For k(t)          L(t)                    Lh(t)         Ll(t)            Nh(t)          Nl(t):
AA = [ 0,            0,                      0,            0,               0,             0      % Equ. 1)
       0,            0,                      0,            0,               0,             0      % Equ. 2)
       0,            0,                     -1,            0,               0,             0      % Equ. 3)
       k_bar,    (1-delta)*k_bar,            0,            0,               0,             0      % Equ. 4)
       0,          -1+alpha,                 0,            0,               0,             0      % Equ. 5)
       0,            1,                      0,            0,               0,             0      % Equ. 6)
       0,            0,                      0,            0,               0,             0      % Equ. 7)
       0,            0,                      0,            0,               0,             0      % Equ. 8)
       0,            0,                      0,            0,               0,             0      % Equ. 9)
       0,            0,                      0,            0,               0,             0      % Equ. 10)
       0,            0,                     -1,            0,               1,             0      % Equ. 11)
       0,            0,                      0,           -1,               0,             1      % Equ. 12)
       0,           -1,                      1,            0,               0,             0      % Equ. 13)
       0,           -1,                      0,            1,               0,             0      % Equ. 14)
       0,           -1,                     qh_bar,       ql_bar,           0,             0      % Equ. 15)
       0,            0,                     -1,            0,               0,             0      % Equ. 16)
       0,            0,                      0,           -1,               0,             0      % Equ. 17)
       0,            0,                      0,            0,              -1,             0      % Equ. 18)
       0,            0,                      0,            0,               0,            -1      % Equ. 19)
       0,    -(1-tau_k)*k_bar*d_bar/qh_bar,  0,            0,               0,             0      % Equ. 20)
       0,        tau_k*d_bar*k_bar,          0,            0,               0,             0      % Equ. 21)
       0,            1,                      0,            0,               0,             0      % Equ. 22)
       1,            1,                      0,            0,               0,             0      % Equ. 23)
       0,            0,                      0,            0,               1,             0      % Equ. 24)
       0,            0,                      0,            0,               0,             1      % Equ. 25)
       0,            0,                      0,            0,               0,             0   ]; % Equ. 26)
```

```
   % For k(t-1)                          L(t-1)                   Lh(t-1):
BB1 = [ 0,                                0,                       0                                             % Equ. 1)
        0,                                0,                       0                                             % Equ. 2)
        0,                                0,                       1                                             % Equ. 3)
 -(1-delta)*k_bar                   -(1-delta)*k_bar,              0                                             % Equ. 4)
     1-alpha,                          1-alpha,                    0                                             % Equ. 5)
       -1,                               -1,                       0                                             % Equ. 6)
        0,                                0,                       0                                             % Equ. 7)
        0,                                0,                       0                                             % Equ. 8)
        0,                                0,                       0                                             % Equ. 9)
        0,                                0,                       0                                             % Equ. 10)
        0,                                0,                       0                                             % Equ. 11)
        0,                                0,                       0                                             % Equ. 12)
        0,                                0,                       0                                             % Equ. 13)
        0,                                0,                       0                                             % Equ. 14)
        0,                                0,                       0                                             % Equ. 15)
        0,                                0,                       1                                             % Equ. 16)
        0,                                0,                       0                                             % Equ. 17)
        0,                                0,                  lambda/nh_bar*(Vh_bar/(Lh_bar-Nh_bar))^(1-lambda) % Equ. 18)
        0,                                0,                       0                                             % Equ. 19)
 (1-tau_k)*k_bar*d_bar/qh_bar,  (1-tau_k)*k_bar*d_bar/qh_bar,      0                                             % Equ. 20)
 -tau_k*d_bar*k_bar,               -tau_k*d_bar*k_bar,             0                                             % Equ. 21)
        0,                                0,                       0                                             % Equ. 22)
        0,                                0,                       0                                             % Equ. 23)
        0,                                0,                       0                                             % Equ. 24)
        0,                                0,                       0                                             % Equ. 25)
        0,                                0,                       0                                 ];          % Equ. 26)

   % For  Ll(t-1)                                                  Nh(t-1)  :
BB2 = [ 0,                                                         0                          % Equ. 1)
        0,                                                         0                          % Equ. 2)
        0,                                                         0                          % Equ. 3)
        0,                                                         0                          % Equ. 4)
        0,                                                         0                          % Equ. 5)
        0,                                                         0                          % Equ. 6)
        0,                                                         0                          % Equ. 7)
        0,                                                         0                          % Equ. 8)
        0,                                                         0                          % Equ. 9)
        0,                                                         0                          % Equ. 10)
        0,                                                         0                          % Equ. 11)
        0,                                                         0                          % Equ. 12)
        0,                                                         0                          % Equ. 13)
        0,                                                         0                          % Equ. 14)
        0,                                                         0                          % Equ. 15)
        0,                                                         0                          % Equ. 16)
        1,                                                         0                          % Equ. 17)
        0,                                   1-betan-lambda*(Vh_bar/(Lh_bar-Nh_bar))^(1-lambda) % Equ. 18)
 lambda/nl_bar*(Vl_bar/(Ll_bar-Nl_bar))^(1-lambda),                0                          % Equ. 19)
        0,                                                         0                          % Equ. 20)
        0,                                                         0                          % Equ. 21)
        0,                                                         0                          % Equ. 22)
        0,                                                         0                          % Equ. 23)
        0,                                                         0                          % Equ. 24)
        0,                                                         0                          % Equ. 25)
        0,                                                         0                    ];    % Equ. 26)


% For    Nl(t-1):
BB3 = [      0                                % Equ. 1)
             0                                % Equ. 2)
             0                                % Equ. 3)
             0                                % Equ. 4)
             0                                % Equ. 5)
             0                                % Equ. 6)
```

```
            0                                    % Equ. 7)
            0                                    % Equ. 8)
            0                                    % Equ. 9)
            0                                    % Equ. 10)
            0                                    % Equ. 11)
            0                                    % Equ. 12)
            0                                    % Equ. 13)
            0                                    % Equ. 14)
            0                                    % Equ. 15)
            0                                    % Equ. 16)
            0                                    % Equ. 17)
            0                                    % Equ. 18)
 1-betan-lambda*(Vl_bar/(Ll_bar-Nl_bar))^(1-lambda) % Equ. 19)
            0                                    % Equ. 20)
            0                                    % Equ. 21)
            0                                    % Equ. 22)
            0                                    % Equ. 23)
            0                                    % Equ. 24)
            0                                    % Equ. 25)
            0                               ];   % Equ. 26)

  BB = [BB1, BB2, BB3];


% For y(t): output   investment            income tax
CC1 = [       0,        0,          tau_l_bar/(1-tau_l_bar)                                               % Equ. 1)
              0,        0,        -tau_l_bar/(1-tau_l_bar)                                                % Equ. 2)
              0,        0,                    0                                                           % Equ. 3)
              0,    -x_bar,                   0                                                           % Equ. 4)
             -1,        0,                    0                                                           % Equ. 5)
              1,        0,                    0                                                           % Equ. 6)
              0,        0,                    0                                                           % Equ. 7)
              0,        0,                    0                                                           % Equ. 8)
              0,        0,                    0                                                           % Equ. 9)
              0,        0,                    0                                                           % Equ. 10)
              0,        0,                    0                                                           % Equ. 11)
              0,        0,                    0                                                           % Equ. 12)
              0,        0,                    0                                                           % Equ. 13)
              0,        0,                    0                                                           % Equ. 14)
              0,        0,                    0                                                           % Equ. 15)
              0,        0,                    0                                                           % Equ. 16)
              0,        0,                    0                                                           % Equ. 17)
              0,        0,                    0                                                           % Equ. 18)
              0,        0,                    0                                                           % Equ. 19)
              0,   -x_bar/qh_bar,       -tau_l_bar*wh_bar*hh_bar*nh_bar                                   % Equ. 20)
              0,        0,       -(tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar+tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar) % Equ. 21)
              1,        0,                    0                                                           % Equ. 22)
              0,        0,                    0                                                           % Equ. 23)
              0,        0,                    0                                                           % Equ. 24)
              0,        0,                    0                                                           % Equ. 25)
              0,        0,              -tau_l_bar/(1-tau_l_bar)                               ];         % Equ. 26)

% For y(t):  dividends            return        HS consumption     LS consumption
CC2 = [         0,                  0,            0,                   1                                  % Equ. 1)
                0,                  0,            0,    -(1+tau_c)*cl_bar/((1-tau_l_bar)*hl_bar*wl_bar*nl_bar) % Equ. 2)
       (1-tau_k)*d_bar/R_bar,      -1,            0,                   0                                  % Equ. 3)
                0,                  0,            0,                   0                                  % Equ. 4)
                0,                  0,            0,                   0                                  % Equ. 5)
               -1,                  0,            0,                   0                                  % Equ. 6)
                0,                  0,            0,                   0                                  % Equ. 7)
                0,                  0,            0,                   0                                  % Equ. 8)
                0,                  0,            0,                   0                                  % Equ. 9)
                0,                  0,            0,                   0                                  % Equ. 10)
                0,                  0,            0,                   0                                  % Equ. 11)
                0,                  0,            0,                   0                                  % Equ. 12)
```

```
               0,                 0,           0,                 0                                 % Equ. 13)
               0,                 0,           0,                 0                                 % Equ. 14)
               0,                 0,           0,                 0                                 % Equ. 15)
               0,                 0,           0,                 0                                 % Equ. 16)
               0,                 0,           0,                 0                                 % Equ. 17)
               0,                 0,           0,                 0                                 % Equ. 18)
               0,                 0,           0,                 0                                 % Equ. 19)
(1-tau_k)*k_bar*d_bar/qh_bar,     0,  -(1+tau_c)*ch_bar,          0                                 % Equ. 20)
      -tau_k*d_bar*k_bar,         0,  -tau_c*ch_bar*qh_bar,   -tau_c*cl_bar*ql_bar                  % Equ. 21)
               0,                 0,           0,                 0                                 % Equ. 22)
               0,                 0,           0,                 0                                 % Equ. 23)
               0,                 0,           0,                 0                                 % Equ. 24)
               0,                 0,           0,                 0                                 % Equ. 25)
               0,                 0,          -1,                 0                  ];             % Equ. 26)

% For y(t):  HS salary                                 LS salary
CC3 = [        0,                                         -1                          % Equ. 1)
               0,                                          1                          % Equ. 2)
               0,                                          0                          % Equ. 3)
               0,                                          0                          % Equ. 4)
               0,                                          0                          % Equ. 5)
               0,                                          0                          % Equ. 6)
-alpha*phi*Y_bar/(wh_bar*Hh_bar^rho),                      0                          % Equ. 7)
               0,                        -alpha*Y_bar/(wl_bar*Hl_bar^rho)             % Equ. 8)
               0,                                          0                          % Equ. 9)
               0,                                          0                          % Equ. 10)
               0,                                          0                          % Equ. 11)
               0,                                          0                          % Equ. 12)
               0,                                          0                          % Equ. 13)
               0,                                          0                          % Equ. 14)
               0,                                          0                          % Equ. 15)
               0,                                          0                          % Equ. 16)
               0,                                          0                          % Equ. 17)
               0,                                          0                          % Equ. 18)
               0,                                          0                          % Equ. 19)
(1-tau_l_bar)*wh_bar*hh_bar*nh_bar,                        0                          % Equ. 20)
-tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar,     -tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar    % Equ. 21)
               0,                                          0                          % Equ. 22)
               0,                                          0                          % Equ. 23)
               0,                                          0                          % Equ. 24)
               0,                                          0                          % Equ. 25)
               1,                                          0                 ];       % Equ. 26)

% For y(t):                                      HS ratio
CC4 = [                                            0                                  % Equ. 1)
                                                   0                                  % Equ. 2)
                                                   0                                  % Equ. 3)
                                                   0                                  % Equ. 4)
                        phi*alpha*(nh_bar*qh_bar*hh_bar)^(1-rho)/a                    % Equ. 5)
                                                   0                                  % Equ. 6)
                                                   0                                  % Equ. 7)
                                                   0                                  % Equ. 8)
                                                   0                                  % Equ. 9)
                                                   0                                  % Equ. 10)
                                                   0                                  % Equ. 11)
                                                   0                                  % Equ. 12)
                                                  -1                                  % Equ. 13)
                                                   0                                  % Equ. 14)
                                                   0                                  % Equ. 15)
                                                   0                                  % Equ. 16)
                                                   0                                  % Equ. 17)
                                                   0                                  % Equ. 18)
                                                   0                                  % Equ. 19)
                             x_bar/qh_bar-(1-tau_k)*k_bar*d_bar/qh_bar                % Equ. 20)
```

```
  -tau_c*ch_bar*qh_bar-tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar+r*hh_bar*wh_bar*qh_bar*uh_bar*(1-tau_l_bar)    % Equ. 21)
                                                    0                                                     % Equ. 22)
                                                    0                                                     % Equ. 23)
                                                    0                                                     % Equ. 24)
                                                    0                                                     % Equ. 25)
                                                    0                                                ];   % Equ. 26)


% For y(t):                                       LS ratio
CC5 = [                                             0                                                     % Equ. 1)
                                                    0                                                     % Equ. 2)
                                                    0                                                     % Equ. 3)
                                                    0                                                     % Equ. 4)
                                   alpha*(nl_bar*ql_bar*hl_bar)^(1-rho)/a                                 % Equ. 5)
                                                    0                                                     % Equ. 6)
                                                    0                                                     % Equ. 7)
                                                    0                                                     % Equ. 8)
                                                    0                                                     % Equ. 9)
                                                    0                                                     % Equ. 10)
                                                    0                                                     % Equ. 11)
                                                    0                                                     % Equ. 12)
                                                    0                                                     % Equ. 13)
                                                   -1                                                     % Equ. 14)
                                                    0                                                     % Equ. 15)
                                                    0                                                     % Equ. 16)
                                                    0                                                     % Equ. 17)
                                                    0                                                     % Equ. 18)
                                                    0                                                     % Equ. 19)
                                                    0                                                     % Equ. 20)
  -tau_c*cl_bar*ql_bar-tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar+r*hl_bar*wl_bar*ul_bar*ql_bar*(1-tau_l_bar)    % Equ. 21)
                                                    0                                                     % Equ. 22)
                                                    0                                                     % Equ. 23)
                                                    0                                                     % Equ. 24)
                                                    0                                                     % Equ. 25)
                                                    0                                                ];   % Equ. 26)

% For y(t):         HS employment                           LS employment
CC6 = [                  0,                                        0                      % Equ. 1)
                         0,                                        1                      % Equ. 2)
                         0,                                        0                      % Equ. 3)
                         0,                                        0                      % Equ. 4)
    phi*alpha*(nh_bar*qh_bar*hh_bar)^(1-rho)/a,     alpha*(nl_bar*ql_bar*hl_bar)^(1-rho)/a  % Equ. 5)
                         0,                                        0                      % Equ. 6)
                         0,                                        0                      % Equ. 7)
                         0,                                        0                      % Equ. 8)
                       nh_bar,                                     0                      % Equ. 9)
                         0,                                      nl_bar                   % Equ. 10)
                        -1,                                        0                      % Equ. 11)
                         0,                                       -1                      % Equ. 12)
                         0,                                        0                      % Equ. 13)
                         0,                                        0                      % Equ. 14)
                         0,                                        0                      % Equ. 15)
                         0,                                        0                      % Equ. 16)
                         0,                                        0                      % Equ. 17)
                         0,                                        0                      % Equ. 18)
                         0,                                        0                      % Equ. 19)
         (1-tau_l_bar)*wh_bar*hh_bar*nh_bar,                       0                      % Equ. 20)
       -tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar,     -tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar  % Equ. 21)
                         0,                                        0                      % Equ. 22)
                         0,                                        0                      % Equ. 23)
                         0,                                        0                      % Equ. 24)
                         0,                                        0                      % Equ. 25)
                         0,                                        0                 ];   % Equ. 26)


% For y(t):     HS unemployment                LS unemployment                total output        tot. capital
```

```
CC7 = [               0,                                0,                         0,              0   % Equ. 1)
                      0,                         r*ul_bar/nl_bar,                  0,              0   % Equ. 2)
                      0,                                0,                         0,              0   % Equ. 3)
                      0,                                0,                         0,              0   % Equ. 4)
                      0,                                0,                         0,              0   % Equ. 5)
                      0,                                0,                         0,              0   % Equ. 6)
                      0,                                0,       alpha*phi*Y_bar/(wh_bar*Hh_bar^rho),  0   % Equ. 7)
                      0,                                0,         alpha*Y_bar/(wl_bar*Hl_bar^rho),    0   % Equ. 8)
                   uh_bar,                              0,                         0,              0   % Equ. 9)
                      0,                             ul_bar,                       0,              0   % Equ. 10)
                      0,                                0,                         0,              0   % Equ. 11)
                      0,                                0,                         0,              0   % Equ. 12)
                      0,                                0,                         0,              0   % Equ. 13)
                      0,                                0,                         0,              0   % Equ. 14)
                      0,                                0,                         0,              0   % Equ. 15)
                      0,                                0,                         0,              0   % Equ. 16)
                      0,                                0,                         0,              0   % Equ. 17)
                      0,                                0,                         0,              0   % Equ. 18)
                      0,                                0,                         0,              0   % Equ. 19)
 r*(1-tau_l_bar)*wh_bar*hh_bar*uh_bar,                  0,                         0,              0   % Equ. 20)
   r*uh_bar*qh_bar*hh_bar*wh_bar,      r*hl_bar*wl_bar*ul_bar*ql_bar,              0,              0   % Equ. 21)
                      0,                                0,                        -1,              0   % Equ. 22)
                      0,                                0,                         0,             -1   % Equ. 23)
                      0,                                0,                         0,              0   % Equ. 24)
                      0,                                0,                         0,              0   % Equ. 25)
                      0,                                0,                         0,              0 ];% Equ. 26)

% For y(t):   HS hours                                      LS hours
CC8 = [         0,                                      hl_bar/(1-hl_bar)               % Equ. 1)
                0,                                              1                       % Equ. 2)
                0,                                              0                       % Equ. 3)
                0,                                              0                       % Equ. 4)
 phi*alpha*(nh_bar*qh_bar*hh_bar)^(1-rho)/a,    alpha*(nl_bar*ql_bar*hl_bar)^(1-rho)/a   % Equ. 5)
                0,                                              0                       % Equ. 6)
                0,                                              0                       % Equ. 7)
                0,                                              0                       % Equ. 8)
                0,                                              0                       % Equ. 9)
                0,                                              0                       % Equ. 10)
                0,                                              0                       % Equ. 11)
                0,                                              0                       % Equ. 12)
                0,                                              0                       % Equ. 13)
                0,                                              0                       % Equ. 14)
                0,                                              0                       % Equ. 15)
                0,                                              0                       % Equ. 16)
                0,                                              0                       % Equ. 17)
                0,                                              0                       % Equ. 18)
                0,                                              0                       % Equ. 19)
(1-tau_l_bar)*wh_bar*hh_bar*nh_bar,                             0                       % Equ. 20)
-tau_l_bar*wh_bar*hh_bar*nh_bar*qh_bar,     -tau_l_bar*wl_bar*hl_bar*nl_bar*ql_bar      % Equ. 21)
                0,                                              0                       % Equ. 22)
                0,                                              0                       % Equ. 23)
                1,                                              0                       % Equ. 24)
                0,                                              1                       % Equ. 25)
        -hh_bar/(1-hh_bar),                                     0   ];                  % Equ. 26)

% For y(t):   tot HS hours                                                 tot LS hours:
CC9 = [           0,                                                              0            % Equ. 1)
                  0,                                                              0            % Equ. 2)
                  0,                                                              0            % Equ. 3)
                  0,                                                              0            % Equ. 4)
                  0,                                                              0            % Equ. 5)
                  0,                                                              0            % Equ. 6)
-((1-rho)*phi*Hh_bar^(1-rho)+rho*alpha*phi*Y_bar/(wh_bar*Hh_bar^rho)),   -(1-rho)*Hl_bar^(1-rho) % Equ. 7)
-(1-rho)*phi*Hh_bar^(1-rho),       -((1-rho)*Hl_bar^(1-rho)+rho*alpha*Y_bar/(wl_bar*Hl_bar^rho)) % Equ. 8)
                  0,                                                              0            % Equ. 9)
                  0,                                                              0            % Equ. 10)
```

```
                0,                                                                0           % Equ. 11)
                0,                                                                0           % Equ. 12)
                0,                                                                0           % Equ. 13)
                0,                                                                0           % Equ. 14)
                0,                                                                0           % Equ. 15)
                0,                                                                0           % Equ. 16)
                0,                                                                0           % Equ. 17)
                0,                                                                0           % Equ. 18)
                0,                                                                0           % Equ. 19)
                0,                                                                0           % Equ. 20)
                0,                                                                0           % Equ. 21)
                0                                                                 0           % Equ. 22)
                0,                                                                0           % Equ. 23)
               -1,                                                                0           % Equ. 24)
                0,                                                               -1           % Equ. 25)
                0,                                                                0       ];  % Equ. 26)

CC = [CC1, CC2, CC3, CC4, CC5, CC6, CC7, CC8, CC9];

% For exog. var. in time period (t):
%      i(t)
DD = [ 0                       % Equ. 1)
       0                       % Equ. 2)
       0                       % Equ. 3)
       0                       % Equ. 4)
       0                       % Equ. 5)
       0                       % Equ. 6)
       0                       % Equ. 7)
       0                       % Equ. 8)
       0                       % Equ. 9)
       0                       % Equ. 10)
       0                       % Equ. 11)
       0                       % Equ. 12)
       0                       % Equ. 13)
       0                       % Equ. 14)
       0                       % Equ. 15)
       eta/qh_bar              % Equ. 16)
       (1-eta)/ql_bar          % Equ. 17)
       0                       % Equ. 18)
       0                       % Equ. 19)
       0                       % Equ. 20)
       0                       % Equ. 21)
       0                       % Equ. 22)
       0                       % Equ. 23)
       0                       % Equ. 24)
       0                       % Equ. 25)
       0                  ];  % Equ. 26)

DD=0.4*DD;     % We need to multiple DD by 0.4 in order to get 0.4 % shock

% EXPECTATIONAL EQUATIONS:

% For  k(t+1)    L(t+1)     Lh(t+1)   Ll(t+1)    Nh(t+1)   Nl(t+1):
FF = [    0,        0,         0,        0,        0,         0];

% For     k(t)    L(t)       Lh(t)      Ll(t)    Nh(t)       Nl(t):
GG = [    0,        0,         0,        0,        0,         0];

% For   k(t-1)    L(t-1)     Lh(t-1)  Ll(t-1)    Nh(t-1)  Nl(t-1):
HH = [    0,        0,         0,        0,        0,         0];

% For y(t+1): output  investm.   income tax  divid.  return  HS consumt.  LS consump.
JJ1 = [          0,       0,         0,         0,      1,       -1,         0   ];

        %  HS sal.   LS sal.  HS ratio  LS ratio  HS employment  LS employment  HS unemployment
JJ2 = [     0,        0,         0,        0,          0,           0,              0 ] ;
```

```
      % LS umemployment total output   tot. capital   HS hours  LS hours  tot HS hours  tot LS hours
JJ3 =  [       0,           0,             0,            0,       0,          0,            0     ];

JJ = [JJ1, JJ2, JJ3];

% For y(t)  : output  investm.   income tax  divid.  return  HS consumt.  LS consump.  HS sal.   LS sal.
KK1 = [          0,       0,         0,         0,      0,        1,         0,         0,       0 ];

% For y(t)  : HS ratio  LS ratio  HS employment  LS employment  HS unemployment  LS umemployment  total output
KK2 = [          0,        0,          0,             0,            0,                0,                0];

% For y(t)  :  tot. capital   HS hours  LS hours  tot HS hours  tot LS hours
KK3 =  [             0,            0,       0,         0,          0     ];

KK = [KK1, KK2, KK3];

% For z(t+1):
LL =       [ 0 ];

% For z(t):
MM =       [ 0 ];

% AUTOREGRESSIVE MATRIX FOR z(t):
NN =       [ 0 ];

Sigma =    [ 1 ];


 %Setting the options:

[l_equ,m_states] = size(AA);
[l_equ,n_endog ] = size(CC);
[l_equ,k_exog ]  = size(DD);


PERIOD     = 1; % The fundamental period in the model is one year.
GNP_INDEX  = 7; % Index of output among the variables selected for HP filter
IMP_SELECT = [7,22,2,24,25]; % A vector containing the indices of the variables to be plotted
HP_SELECT  = [12,15]; % Selecting the variables for the HP Filter calcs.
DO_SIMUL   = 0; % Calculates Simulations
DO_MOMENTS = 0; % Calculates Moments
DO_STATE_RESP = 0; % Do not calculate impulse responses to deviations of state variables

% Starting the calculations:

do_it;
```

Declaration of Authorship

I hereby confirm that I have authored this master thesis independently and without use of others than the indicated resources. All passages, which are literally or in general matter taken out of the publications or other resources are marked as such.

Sigitas Karpavicius

Berlin, 26th August, 2005