The Effects of Monetary Policy in a Multi-Sector New Keynesian Model: An Analysis Using Recent Micro Evidence on Price Changes

Master Thesis

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Abstract

Economic modelling generally assumes one-sector to represent the whole economy, whereas real economies consist of many sectors with varying price durations. We try to model this variety by setting up a multi-sector New Keynesian model based on the model outlined in Gali (2002). The Calvo parameter estimates of our model are calculated using the data set provided by Bils and Klenow (2004). We group similar price durations as sectors and form ten sectors to represent the whole economy. Our first aim is to study the responses of the multi-sector economy to policy shocks. The outcomes show that aggregate price, interest and output display interesting dynamics under such a setting. Then we try to match the responses of the multi-sector economy with one-sector models. We find that for our baseline model, the aggregate responses of the multi-sector economy can be well mimicked by a one-sector model with a price duration of around 2.9 months. This model is able to duplicate the aggregate responses, whereas the variety in the dynamics of responses can only be captured with at least two sector-models. We test our results under different parametrization. The proposed price duration for one-sector model is still able to match the aggregate impulse responses well but we see that it is not very precise in matching the volatility of the multi-sector model. Under similar settings, the two-sector performs better. Next, we change the labor assumptions of the baseline model and assume specific factor markets. The results show that prices and inflation response by little to policy shocks now. The new impulse responses are well matched with one- and two-sector models with the proposed price durations. Whereas in terms of matching the volatility, one-sector estimate performs poorly and the new price duration increases to around 4 months. However, this value lacks the precision of the previously proposed price duration in matching the aggregate response levels.

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Chapter 1

Introduction

"From Keynes on, there was wide agreement that some imperfections played an essential role in fluctuations. Nominal rigidities, played one explicit and central role in most formalizations. They were crucial to explaining why and how changes in money and other shifts in the demand for goods affected output at least in the short run." (Blanchard, 2000) Nominal rigidities always played a key role in building a bridge between the real world and macroeconomic modelling. If all prices adjusted promptly to an increase in the money supply, then money should be neutral in the short- and long-run. However empirical evidence suggests that at least in the short-run money affects output. The existence of nominal rigidities makes it possible that prices adjust only slowly and an increase in the money supply has real effects, in the short-run.

In trying to model price rigidities, the most widely used methods are those proposed by Taylor (1999) and Calvo (1983). The two models are similar, the main difference is that the Taylor model assumes fixed price duration, whereas Calvo assumes a random price duration. There are many papers that incorporate Taylor staggered price setting or Calvo staggered price setting into dynamic stochastic general equilibrium models. Some of the examples include works of Yun (1996), Woodford (1996), Smets and Wouters (2003) and Gali (2002).

One of the main criticism of these models is that they cannot generate realistic inflation and output persistence. As a result, the research trying to match the real data with these models gained momentum. For example, Fuhrer and Moore (1995) suggested a new contracting specification with relative real wages so that the model exhibited inflation persistence. Gali and Gertler (1999) derived a relationship between marginal cost and inflation (rather than output and inflation) and showed that such a model could better match the real behavior of inflation.

Another criticism of these models is that the exogenous price duration assumed in these models is inconsistent with microevidence on prices. Early empirical research on price duration suffered from limited availability of data. The early research found that there was great heterogeneity in price duration across different products, but the general conclusion was that prices do not change frequently. Recently, Bils and Klenow (2004) have studied Bureau of Labor Statistics (BLS) data on the frequency of price changes. The data covers about 70% of consumer spending for the years 1995 to 1997. Bils and Klenow find that prices change much more frequently than suggested by the earlier literature. In their data the median price duration is 4.3 months. Furthermore, Bils and Klenow find that the price duration differs a lot across categories of goods. They mention that a multi-sector model calibrated to match the empirical distribution of price durations behaves like a one-sector model with the median price duration.

This thesis sets up and solves a multi-sector New Keynesian model. We base our multi-sector model on the one sector model outlined in Gali (2002), since this is one of the most widely used papers for pedagogical and applied purposes. Our main question is then whether the responses of the economy to monetary policy shocks differ between a one-sector and a multi-sector model and if any one-sector model can match the aggregate responses of the multi-sector model in terms of volatility and magnitude. Most available models in the literature are one sector models, however in reality there are many sectors with different Calvo parameters. As mentioned in Taylor (1999): "*There is great deal of heterogeneity in wage and*

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price setting ... frozen orange juice prices change every two weeks, while magazine prices change every three years!".

In trying to answer the above mentioned questions, we calibrate our multi-sector model using the empirical distribution of the price adjustment parameters listed in Bils and Klenow (2004). We group the similar frequency of price adjustment values as sectors, and form 10 sectors with the help of this data set. The method we have used in forming these sectors are presented in the calibration part. Then we show the responses of the multi-sector economy to a shock in the policy rule and interpret the outcomes. Next, we compare the results to one sector economies with different Calvo parameters. We find that the calibration of the one sector economy with a monthly Calvo parameter of 0.71 is able to match the aggregate responses of the multi-sector economy. This result is even more flexible than Bils and Klenow. We also point out that the richness of the impulse response patterns of the multi-sector economy, however, can only be captured by modelling the economy with at least two sectors (one at the sticky and the other at the flexible end). Then we do sensitivity analysis on the robustness of our results. The results show that under different parametrization the proposed price durations for one sector and two sector models continue to mimic the impulse response levels of the multi-sector model. In the analysis of the standard deviations, with the proposed price duration, we find slight differences between the volatility of multi-sector and one-sector models. Two-sector model performs well in this part. Later we modify the model by introducing specific factor markets. Our guess is that this assumption could lead to changes in the behavior of aggregate responses that could increase the price durations. To be able to match the volatility of the new model, price duration of one-sector model increases and moves closer to the value proposed by Bils and Klenow. However, the previously proposed price duration is able to mimic the multi-sector impulse responses slightly better, there is a trade off between two alternatives. We conclude that for our model and using the data set provided by Bils and Klenow, the price duration that can match the level of impulse responses of a multi-sector economy turns out to be around 2.9 months. Furthermore, a two-

sector model with a flexible and a sticky sector is able to summarize the dynamics in the multi-sector economy better than one-sector models.

There are some papers that study two sector New Keynesian models. The first example is Ohanian et al.(1995). They form a hybrid model with one flexible and one sticky sector where the nominal prices in the sticky sector are set one period in advance. They study the responses of the economy to monetary shocks and find that the real effects of the disturbances differ across the two sectors. Aoki (2001) works with a similar model with one sticky and one flexible sector and studies the implications for optimal monetary policy. Additionally, Barsky, House and Kimball (2003) model an economy where prices of durables are flexible. Finally, Bils and Klenow (2002), Smets and Wouters (2004) mention about two sector models in their papers.

The paper will be organized as follows: Next, we will provide a brief literature review about research on price stickiness, and then review the recent work, mainly that of Bils and Klenow since our paper is based on this research results. In chapter 4 and 5, we introduce our model and analyze it. Chapter 6 reports the results of the implementation of the model in matlab. We also do model comparisons in this chapter. Chapter 7 proposes an extension to the baseline model. Finally we provide a discussion section and then conclusion at the end.

Chapter 2

Literature

2.1 Research on Price Stickiness

2.1.1 Micro Evidence on Consumer Prices

Research on price stickiness dates back to 1927 when Frederick Mills (1927) published first work in this area. He studied Bureau of Labor Statistics (BLS) data which covered about 200 goods at the period 1890 to 1926. The main findings of his research was that the frequency distribution of price changes showed a u-shaped pattern with the prices of many goods displaying strongly sticky behavior and the prices of as many goods displaying strongly flexible behavior. Gardiner Means (1935) followed the work of Mills and his research conclusions became known as Means' theory. He mainly differentiated between two markets, the traditional market with flexible prices and the administered market with inflexible administered prices.

Stigler and Kindahl (1970) worked with data from similar industries to Means and tried to improve his results. Later, Carlton (1986) studied the producers prices between 1957-1966 using the data set by Stigler and Kindahl and found that the time between adjustments of transactions prices was between 5.9 and 19.2 months.

Mussa (1981) collected data on the newspaper prices during German hyperinflation and reported that prices changed more often with the increase in inflation, but during the days of highest inflation, the prices did not change everyday. Cechetti (1986) studied the prices of magazines measured annually. The average number of years between price changes varied from 7 years in 1950s to 3 years in 1970s.

Lach and Tsiddon (1992, 1996) worked with meat products and wine at retail level in Israel. From their research, they concluded that stores usually change the prices of their products at the same time (within-store synchronization). Another important research was conducted by Kashyap (1995). He studied the monthly mail order catalogues between 1953 to 1987. Since his work covered a long period, he worked with standardized products which might have more sticky prices. Kashyap found an average price duration of 14.7 months.

Blinder et al. (1998) conducted surveys with firms on their price setting behavior. The surveys included a set of questions on the frequency of price changes and factors affecting the behavior of price changes. The research covered about 200 companies. Blinder et al. reported a median frequency of price change around one year. Buckle and Carlson (2000) also did survey research on producer price duration in New Zealand. The average duration was found to be 6.7 months.

These studies, indeed, were based on a limited number of product groups. In contrast, Bils and Klenow (2004) study the frequency of price changes for 350 categories of 70.000 to 80.000 goods and services in U.S which comprises %70 consumption expenditures. They work with the data used to calculate the consumer price index and find a median price duration of 4.3 months. The study by Bils and Klenow has triggered similar studies for Europe. Recent works include those of Aucremanne and Dhyne (2004) on Belgian data, Dias, Dias and Neves (2004) on Portugese data and Baudry, Bihan, Sevestre and Tarieu (2004) on French Data.

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2.1.2   Macro Evidence on Consumer Prices

The literature on macroeconomic evidence include the works of Gali, Gertler and Lopez-Salido (2001, 2003) and Smets and Wouters (2003). Gali, Gertler and Lopez-Salido (2001, 2003) estimate a Phillips curve for the Euro area that is derived from a model of staggered price setting. They employ "newly constructed aggregate historical data set" to estimate the possible value of the Calvo parameter, the baseline estimate turns out be somewhere around four to six quarters for the Euro area, and two or three quarters for U.S, suggesting that prices are less rigid in the U.S. Also, Smets and Wouters (2003) use data on 7 key macroeconomic variables for the Euro area (real GDP, real consumption, real investment, the GDP deflator, real wages, employment and nominal interest rate) to estimate the coefficients of their DSGE model with rigidities. They estimate a price duration of two and a half years. Until recently, these values were widely used to calibrate New Keynesian Models. However, they are inconsistent with the micro evidence on consumer prices.

Chapter 3

Facts and Theory

3.1 Recent Microeconomic Evidence on Price Changes

The work by Bils and Klenow (2002, 2004) has attracted much attention and following their work, micro data on consumer prices has been studied in many countries. Since our thesis is based on research results of this paper, we will first give a summary of their work and then briefly introduce some of the similar recent research in this area.

Bils and Klenow work with BLS data (Bureau of Labor Statistics) which covers 350 categories of consumer products and around 70% of all consumer expenditures. The price quotes are collected from 22.000 outlets in 88 geographical areas. The consumer expenditures not covered include mainly household insurance, residential rent and used cars. The data for the period 1995-1997 is used for calculation of reported frequencies.

The BLS data report the monthly frequency of price change on a mixture of monthly and bimonthly frequencies. The average frequencies in the paper are calculated from this data assuming that the probability of price change one month

and then changing back to the previous level next month is zero.[1] They note that this might understate the true frequencies as it can neglect temporary sales. The probability of price change is then calculated assuming that prices can change at any moment as $-ln(1 - \lambda)$ where λ is the monthly frequency of price change. The resulting weighted mean monthly frequency of price change is 26.1% and the weighted median is 20.9%. Bils and Klenow report their finding as "*For the median category, the time between price changes averages 4.3 months*" (2004, p.951). Additionally, they argue that excluding sales the median time would be 5.5 months and controlling for item substitutions does not affect the conclusions. Examining the consumer good categories they find that services are stickier than goods, raw products are more flexible than processed products and services.

Bils and Klenow also examine the responses of a multi-sector model (with different Calvo parameters matching the distribution of price change frequencies found in the data) to nominal and real shocks and compare these responses to the response of a one sector model. Their conclusion is as follows: "*One sector models with prices fixed for 4 months, roughly the median duration in the empirical distribution, most closely matches the aggregate response in the multi-sector models*" (2004, p.952).

Finally, they test whether the time series pattern in the data match the time series pattern of inflation predicted by a Taylor or a Calvo model. They find that "*...for nearly all consumer goods, these models predict inflation rates that are much more persistent and much less volatile than we observe. The models particularly over-predict persistence and under predict volatility for goods with less frequent price changes*"[2] (2002, p.32).

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[1] For the details of the calculation, see Bils and Klenow(2004), p.951

[2] For a comparison with the work of Fuhrer and Moore(1995), see Bils and Klenow(2004), p.972.

Recently studies have been conducted using micro data for European countries. The most comprehensive studies include those of Aucremanne and Dhyne (2004) on Belgium data , Dias, Dias and Neves (2004) on Portugese data and Baudry, Bihan, Sevestre and Tarieu (2004) on French Data. Some of the common results of these studies are as follows: Price setting is very heterogeneous across product categories; prices of energy and (unprocessed) food products change more often than other product categories, and services usually show the most rigid behavior; the frequency of price change is higher for inflationary periods and periods with important shocks; and there is time dependency in the pricing behavior. Thus, they all imply the need of modelling with more than one sector to capture the variability in the real data. We will now present the frequency of price adjustment periods that they have found but we would also like to provide some insights to these studies since they are conducted using different methods.

Luc Aucremanne and Emmanuel Dhyne (2004) work with a Federal Public Service data set which is also the data set used to calculate Belgian National and Harmonized Index of Consumer Prices. The observations include the periods from January 1989 to January 2001. (Thus, the data survey does not include effects of switching to euro.) The data set covers 68% of Belgian CPI. Product categories not covered are mainly housing, rents, electricity, gas, telecommunications, newspaper and insurance services.

The observations used for this research are collected on a monthly basis except for seasonal product categories. Furthermore all types of rebates and promotions are included in the analysis but not those that relate to summer and winter sales periods. Also, products observed in clothing and wear class include very standardized products like white t-shirts and black socks which are likely to show more price rigidities. Their results show that an important proportion of CPI is characterized by flexible prices with 12% of CPI having less than two months of average price duration. On the other hand, 13% of CPI exhibits an average price duration of at least two years. They find a median price duration of 13.25 months.

Dias, Dias and Neves (2004) carry out their research using national statistics institutes' data on consumer and producer prices[3]. For consumer prices, there are two data sets covering two different periods, from January 1992 to December 1997 (CPI1) and from January 1997 to December 2002 (CPI2). The research is restricted to the period ending in January 2001 due to the possible effects of currency change and conducted using CPI2 due to consistency problems. Also they exclude the annually reported items from the data set.

In this research items are sampled, where they note that "*an item designates a product at its most elementary level. That is, it stands for a homogeneous product although different brands and packages of the same item may be classified under the same code.*"(2004, p.6). Moreover, the study is conducted using the actual price the consumer faces but information on occurrence of sales or promotions is also available. Inflation in the period between 1992-1997 fell from 7.4 to 1.2 percent for goods. For the period 1998-2002 the inflation rate was between 2 and 4 percent. They conclude that "*on average, almost 1 in every 4 prices is changed in a month*" (this may be a result of large weight of food in the CPI) and they find a median price duration of 8.5 months.

Finally, Baudry, Bihan, Sevestre and Tarrieu (2004) work with monthly price quotes collected by INSEE. These quotes are also used for the calculation of the French CPI. Sample period is from 1994:7 to 2003:2 and covers %65 of the CPI (so the observations include the period of euro-cash change over). The categories of goods and services not included in the sample are "*centrally collected prices, and products like fresh foods and rents*". All price changes are assumed regular thus includes any sales or temporary rebates. As a result they report a median price duration of 6.2 months with a very asymmetric distribution.

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[3]Dias and Dias, Neves also attempt to describe the producer price behavior, they report that the results for the producers products are very similar to those of consumer products but their median price duration is almost one year. One thing to note is that most flexility comes from the Petroleum refinement industry, by excluding this industry from the data set the frequency of price changes reduces to %14 with a median average duration of 14 months.

These empirical studies imply no common value for the Calvo parameter. This may reflect the differences in the degree of price stickiness or the differences in the methodology across countries. However, almost all studies suggest that prices change more often than the 11 months calibration used in models. Most importantly, these studies confirm that the price duration differ substantially across sectors.

Chapter 4

Model

4.1 New Keynesian Model with n Sectors

Our framework is based on the simple Gali (2002) New Keynesian DSGE model with Calvo sticky prices, which we extend to n sectors with different price stickiness parameters. The economy consists of a continuum of households, n perfectly competitive firms whose output is used in the production of the final good, and a continuum of intermediate firms producing differentiated goods for these n firms. The n perfectly competitive firms define the n main sectors in the economy. We believe that Gali (2002) model is suitable for answering our questions at hand since it is considered a benchmark in the New Keynesian Literature and its assumptions closely match most of the recent work. Furhermore, Bils and Klenow use a similar model in their work (except for Taylor contracts and the form of the utility function), thus our results can be comparable.

4.1.1 Households

We assume a continuum of infinitely lived agents on the interval [0,1]. All households are identical, consume the final good and supply one unit of labor. A typical

household seeks to maximize a discounted sum of the utilities of the form

$$E_0 \sum_{t=0}^{\infty} \quad \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{l_t^{1+\phi}}{1+\phi} \right], \tag{4.1}$$

where β is the intertemporal discount factor with $0 < \beta < 1$, subject to the period budget constraint:

$$\mathrm{P}_t C_t + \frac{B_{t+1}}{R_t} \leq B_t + W_t l_t + \tau_t. \tag{4.2}$$

Here, P_t denotes the aggregate price index (price of the final good) and C_t is households consumption of the final good[1]. B_{t+1} is the nominal value of the bond portfolio in period $t+1$ that the household owns at the end of period t, R_t ($R_t = Q_{t,t+1}^{-1}$, stochastic discount factor) denotes the nominal riskless interest rate on bonds purchased at date t, τ_t is the lump-sum transfers (taxes) paid by consumers and W_t is the nominal wage rate which is common in all industries and clearing of the labor market requires:

$$\int_0^1 l_{1,t}(i)di + \int_0^1 l_{2,t}(i)di + ... + \int_0^1 l_{n,t}(i)di = l_t. \tag{4.3}$$

We assume complete financial markets, in that the households portfolio of bonds contains claims in a wide array of instruments, therefore the bond portfolio held by the agent contains securities from all sectors.

Money does not appear in the system directly, as in most of the works of Gali, instead we will introduce later a policy rule that will specify the nominal interest rate.

4.1.2 Firms

We consider a dynamic economy with final good production composed of n firms characterizing the n sectors. The composite goods of these sectors are produced

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[1]We build the model from output side but the same model could be constructed from consumption side, in this case P would again denote the aggregate price index and C would be consumption bundle composed of goods from n sectors and there would be no difference in the analysis in terms of demand conditions. See Aoki(2001), p.58 and Woodford(2003), p.147 for details.

by a continuum of differentiated intermediate good firms in monopolistic competition[2].

Final Goods Firms

The production function of the final goods firms is defined by CES technology :

$$Y_t = \left[(\epsilon_1)^{\frac{1}{\eta}} (Y_{1,t})^{\frac{\eta-1}{\eta}} + (\epsilon_2)^{\frac{1}{\eta}} (Y_{2,t})^{\frac{\eta-1}{\eta}} + ... + (\epsilon_n)^{\frac{1}{\eta}} (Y_{n,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}. \qquad (4.4)$$

Where $Y_1...Y_n$ are end products of sector $1...n$ respectively, ϵ_j defines the share of good j in the total basket with $\sum_{j=1}^{n} \epsilon_j = 1$ and η is the elasticity of substitution between the sectors[3]. The profit maximization problem of a final goods firm takes the form:

$$max \quad P_t Y_t - P_{1,t} Y_{1,t} - P_{2,t} Y_{2,t} - ... - P_{n,t} Y_{n,t}.$$

where Y_t is defined as above (4.4). As previously mentioned, final goods firms could be interpreted as firms forming the consumption bundles.

N Sectors

Final goods of each sector are produced with a constant returns to scale production function:

$$Y_{j,t} = \left[\int_0^1 Y_{j,t}(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \qquad ; \quad \forall j = 1, ...n, \qquad (4.5)$$

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[2]This assumption is widely used in the literature. See Blanchard(2000), p.1391 or Goodfriend and King(1997), p.252 for an overview.

[3]In this model elasticity of substitution between the sectors does not have much impact on the aggregate variables. This is due to our assumptions on labor and wages. In the extensions part we will set up a version close to Woodford(2003) assumptions with specific factor markets. In that case, the elasticities will have a more sizeable effect. Additionally, in most of the previous work with two sector or two country models this elasticity is assumed to be unitary which leads to the following aggregate output and price definitions (we will solve for price index in the next section):

$$Y_t = \frac{(Y_{1,t})^{\eta_1} (Y_{2,t})^{\eta_2} ... (Y_{n,t})^{\eta_n}}{\eta_1^{\eta_1} \eta_2^{\eta_2} ... \eta_n^{\eta_n}} \qquad P_t = (P_{1,t})^{\eta_1} (P_{2,t})^{\eta_1} ... (P_{n,t})^{\eta_n}$$

where $Y_{j,t}(i)$ is the good produced by intermediate firm i in sector j and elasticity of substitution within each sector (θ) is greater than one. The firms producing final goods in each sector are faced with a similar profit maximization problem final goods firms but are subject to the production function (4.5).

Intermediate Goods Firms

The behavior of intermediate firms govern the functioning of our economy. Here, we introduce their basic assumptions which we will be examining in the model analysis part in detail.

The output of an intermediate goods firm (i) in industry (j) equals labor services hired by firm (i) times a technology parameter that is common to all firms and all industries:

$$Y_{j,t}(i) = A_t l_{j,t}(i), \tag{4.6}$$

where log-technology follows the AR(1) process

$$log(A_t) = \rho_a log(A_{t-1}) + \varepsilon_{a,t}. \tag{4.7}$$

where $\varepsilon_{a,t}$ is white noise and $0 \leq \rho_a < 1$.
The profit function for the intermediate goods firm i in sector j is

$$(Y^d(P_{t,j}(i); P_{t,j}; P_t, Y_t) P_{t,j}(i) - (1 - \tau_t) TC(Y^d(P_{t,j}(i); P_{t,j}; P_t, Y_t); W_t, A_t)),$$

where τ represents employment subsidies and $Y^d(P_{t,j}(i); P_{t,j}; P_t, Y_t)$ is the demand curve that firm i faces.

Price setting Final goods prices are perfectly flexible whereas intermediate goods firms set their prices according to the sticky price scenario proposed by Calvo (1983)[4]. A fraction $1 - \alpha_j$ of firms in sector j reset their prices each period, independently of the time elapsed since the last adjustment. Remaining α_j of

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[4]Price rigidities can also be modelled using Taylor contracts or convex costs of price adjustment, see Gali(2002), p.4 or Woodford(2003), p.177 for a reference.

firms continue charging their old prices. What is special in this case is that the Calvo stickiness parameter differs across the sectors.

4.1.3 Monetary Policy Rule

We will assume that the central bank sets the nominal interest rate following a simple Taylor rule, thus responds to the deviations of inflation and output gap from their steady state values. The original rule was proposed by Taylor (1993) and since then have been widely used due to its simplicity and ability to match the behavior of the federal funds rate during the postwar period[5].

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[5]See Taylor(1993).

Chapter 5

Model Analysis

We will now begin solving the model, first we find the demand equations, calculate first order conditions then log-linearize these results around their non-stochastic steady-state. Throughout this section the log-linearization will be defined as $X_t = \bar{X}.e^{\hat{x}_t}$ where $\hat{x}_t = log(\frac{X_t}{X})$. Then we will arrange our equations to be consistent with the notation of the matrices defined in Uhlig (1995). This will allow us to implement the toolkit[1] package in Matlab.

5.1 Households

We begin the analysis with the first order conditions for households. As a result of our assumption on labor, the conditions expressed in aggregate variables turn out to be the same as one sector case:

$$C_t^{-\sigma} = \lambda_t P_t, \tag{5.1}$$

$$l_t^{\phi} = W_t \lambda_t, \tag{5.2}$$

$$1 = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{R_t P_t}{P_{t+1}} \right]. \tag{5.3}$$

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[1]For information on Toolkit see web page http://www.wiwi.hu-berlin.de/wpol/html/toolkit.htm.

Equations (5.1), (5.2) and (5.3) are the well-known results of maximization with respect to consumption, labor and bond portfolio at date t+1, where λ_t represents the lagrangian multiplier for the budget constraint. Log-linearizing above equations we have:

$$\hat{w}_t - \hat{p}_t = \sigma \hat{c}_t + \phi \hat{l}_t, \tag{5.4}$$

$$\hat{c}_t = -\frac{1}{\sigma}(\hat{r}_t - E_t \hat{\pi}_{t+1}) + E_t \hat{c}_{t+1}. \tag{5.5}$$

where the first equation combines first order condition for consumption and labor, the second equation is the Euler equation for the economy. In the Euler equation $\hat{r}_t$ and $\hat{\pi}_{t+1}$ are the percentage deviation of nominal riskless interest rate at date (t) and inflation at date $(t+1)$ from their steady state [2].

5.2 Firms

5.2.1 Final Goods Firms

The final goods firms maximize their profits according to the previously defined profit functions subject to their technology constraints (4.4) taking as given the final goods price P_t and the prices of sector final goods $P_{j,t}$, where $j = 1 \ldots n$. The profit maximization then leads to the following demand schedules for n sectors:

$$Y_{1,t} = (\frac{P_{1,t}}{P_t})^{-\eta} Y_t \epsilon_1 \qquad \ldots \qquad Y_{n,t} = (\frac{P_{n,t}}{P_t})^{-\eta} Y_t \epsilon_n. \tag{5.6}$$

Also as a result of zero profit condition, we have:

$$P_t = \left[\epsilon_1 (P_{1,t})^{1-\eta} + \epsilon_2 (P_{2,t})^{1-\eta} + \ldots + \epsilon_n (P_{n,t})^{1-\eta} \right]^{\frac{1}{1-\eta}}. \tag{5.7}$$

Which defines the overall price index in the economy.

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[2]Indeed, in the actual Gali(2002) model, there is an additional term -the log of time discount rate β- since Gali(2002) paper applies logs instead of log-linearizing around the steady state. Nevertheless, both cases do not differ since constants will eventually drop out in the matlab application.

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5.2.2 N Sectors

The profit maximization of the sector final goods firms given prices $P_{j,t}$ and $P_{j,t}(i)$ gives the demand curves for intermediate goods firms within each sector:

$$Y_{j,t}(i) = (\frac{P_{j,t}(i)}{P_{j,t}})^{-\theta} Y_{j,t} \qquad ; \quad j = 1, 2 \dots n. \tag{5.8}$$

Once more, the zero profit condition requires that the price levels are defined as following in each sector:

$$P_{j,t} = \left[\int_0^1 P_{j,t}(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}. \tag{5.9}$$

5.2.3 Some Relations

Before we continue with intermediate firms, it is convenient to define some identities combining prices and output among sectors and relating them to the overall output of the economy[3].

First, define the relative price of good i at time t in sector j to the overall price in sector j as $X_{j,t}(i) = P_{j,t}(i)/P_{j,t}$, and relative price levels of each sector with respect to the general price level as $X_{j,t} = P_{j,t}/P_t$, for $j = 1 \dots n$ from now on. Log-linearizing general price index (5.7) around a symmetric steady state leads to the following equation for prices:

$$\hat{p}_t = \epsilon_1 \hat{p}_{1,t} + \epsilon_2 \hat{p}_{2,t} \dots + \epsilon_n \hat{p}_{n,t}, \tag{5.10}$$

By subtracting $\hat{p}_{t-1}$ from both sides this can be converted to a definition combining the inflation rate of the sectors to give the aggregate inflation rate of the economy[4]:

$$\hat{\pi}_t = \epsilon_1 \hat{\pi}_{1,t} + \epsilon_2 \hat{\pi}_{2,t} \dots + \epsilon_n \hat{\pi}_{n,t}. \tag{5.11}$$

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[3]In deriving these relationships, we refer to the notation in Aoki(2001).

[4]To follow the literature we call $P_{j,t}/P_{j,t-1}$ as inflation rate in sector j, note that in fact they are relative price increases. From now on when we mention a sectoral inflation rate, we do it to be consistent with the literature.

Also, using the definition for $\hat{x}_{j,t}$, equation (5.10) can be further simplified to relate the relative prices in sectors as follows:

$$0 = \epsilon_1 \hat{x}_{1,t} + \epsilon_2 \hat{x}_{2,t} \ldots + \epsilon_n \hat{x}_{n,t}. \tag{5.12}$$

Finally, we need to relate sector outputs to the aggregate output. This is already embedded in the demand equations for the sectors. Log-linearizing equations (5.6) and inserting in the definition of $\hat{x}_{1,t} \ldots \hat{x}_{n,t}$, we have for each sector:

$$\hat{y}_{1,t} = \hat{y}_t - \eta \hat{x}_{1,t},$$

$$\hat{y}_{2,t} = \hat{y}_t - \eta \hat{x}_{2,t},$$

$$\vdots$$

$$\hat{y}_{n,t} = \hat{y}_t - \eta \hat{x}_{n,t}. \tag{5.13}$$

5.2.4 Intermediate Goods Firms

The total nominal cost of intermediate goods firms in each industry is defined by:

$$TC(Y_{j,t}(i), W_t, A_t) = (1 - \tau) W_t l_{j,t}(i). \tag{5.14}$$

For each sector, taking derivative of (5.14) with respect to output, dividing by the overall price level and log linearizing leads to the real marginal cost definition[5]:

$$\hat{mc}_t = \hat{w}_t - \hat{p}_t - \hat{a}_t. \tag{5.15}$$

which turns out to be the same for all sectors due to our assumptions on labor markets and technology ($\hat{mc}_t = \hat{mc}_{j,t} = \hat{mc}_{j,t}(i)$).

It is convenient to have an aggregate production function for each sector. Log-linearizing (4.6) we have up to a first order approximation the following relationship for production of each sector[6]:

$$\hat{y}_{j,t} = \hat{a}_t + \hat{l}_{j,t}, \tag{5.16}$$

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[5]The subsidy term does not drop out in Gali(2002) for the same reason as in the case of Euler equation .

[6]We have $Y_{j,t} = (\int_0^1 Y_{j,t}(i)^{\frac{\theta-1}{\theta}} di)^{\frac{\theta}{\theta-1}}$ and if we insert the definition of $Y_{j,t}(i) = A_t l_{j,t}(i)$

For future reference we need to define l_t. Since the steady-state value of technology parameter is constant and same for all sectors, relative steady state values of output and labor among the sectors turn out to be proportional and we get a similar expression for the aggregate variables as[7] :

$$\hat{y}_t = \hat{a}_t + \hat{l}_t. \tag{5.17}$$

Now we can insert equation (5.17) and the optimality condition (5.4) of the household into the definition of real marginal cost (using the market clearing condition $\hat{c}_t = \hat{y}_t$), to have:

$$\hat{mc}_t = (\sigma + \phi)\hat{y}_t - (1 + \phi)\hat{a}_t. \tag{5.18}$$

Once again, this equation holds for all sectors.

Flexible Price Equilibrium

If we assume that all the firms in all sectors can adjust prices optimally each period, then profit maximization problem will imply that producers will set their prices according to the markup equation:

$$P_{j,t}(i) = \frac{\theta}{\theta - 1} MC_{j,t}(i).$$

which is the same as one-sector model. Hence, the real marginal costs will equal $\frac{\theta-1}{\theta}$. Moreover, since the prices are identical, the output of each sector will be a fraction of the total output, and this fraction will be the share of each sector in the total basket. Then we will have the following equations (here super-index f indicates the flexible-price equilibrium) :

$$\hat{y}_t^f = \frac{1+\phi}{\sigma+\phi}\hat{a}_t^f, \tag{5.19}$$

---

we get $Y_{j,t} = (\int_0^1 A_t l_{j,t}(i)^{\frac{\theta-1}{\theta}} di)^{\frac{\theta}{\theta-1}}$. However, one would like to relate sectoral aggregate $l_{j,t}$ to sectoral aggregate $Y_{j,t}$. This is achieved using the "*alternative price index*" definition by Yun(1996) which is $P_t^a = (\int_0^1 P_{j,t}(i)^{-\theta} di)^{\frac{-1}{\theta}}$ and the demand conditions (5.8). Hence, in fact we have that aggregate labor for each sector is equal to $l_{j,t} = \frac{Y_{j,t}}{A_t}(\int_0^1 \left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{-\theta} di)$. See Gali and Monacelli(2005) p.732 for details.

[7]See appendix A for derivation.

$$\hat{c}_t^f = \hat{y}_t^f, \tag{5.20}$$

$$\hat{l}_t^f = \frac{1-\sigma}{\sigma+\phi}\hat{a}_t^f, \tag{5.21}$$

$$\hat{rrt}_t^f = \hat{r}_t^f - E_t[\hat{\pi}_{t+1}^f] = \frac{\sigma(1+\phi)(\rho_a-1)}{\sigma+\phi}\hat{a}_t^f, \tag{5.22}$$

representing the equilibrium processes for aggregate output, consumption, labor and real rate in the flexible price equilibrium. These levels will be regarded as natural levels in what follows.

Equilibrium with Staggered Price Setting

As we have previously mentioned, intermediate goods firms set their prices according to Calvo (1983) sticky price scenario. When intermediate goods firms in sector j get a chance to reset their price at time t, they maximize the following objective function:

$$max \sum_{k=0}^{\infty} \alpha_j^k E_t \Big[Q_{t,t+k}(P_{j,t}(i)Y_{j,t+k}^d(i) - TC_{j,t+k}(Y_{j,t+k}^d(i); P_{t+k}; Y_{t+k})) \Big], \tag{5.23}$$

Subject to the demand constraints of the firm in sector j:

$$Y_{j,t+k}^d = \left(\frac{P_{j,t}(i)}{P_{j,t+k}}\right)^{-\theta} \left(\frac{P_{j,t+k}}{P_{t+k}}\right)^{-\eta} Y_{t+k}\epsilon_j. \tag{5.24}$$

The maximization problem leads to the following first order conditions for all sectors[8]:

$$\sum_{k=0}^{\infty} \alpha_j^k E_t \Big[Q_{t,t+k}Y_{j,t+k}^d(i)(P_{j,t}(i) - \frac{\theta}{\theta-1}MC_{t+k}) \Big] = 0. \tag{5.25}$$

In the above equation, the term inside the brackets looks very similar to the result of an optimization in the flexible prices. We can interpret this equation as sticky price firms setting their price as an average of future expected marginal costs times the markup (as opposed to the flexible price case where they can constantly apply optimal pricing, thus care for the current marginal cost only).

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[8]Derivations are available in Appendix B.

Log-linearizing, rearranging terms and iterating forward gives the log-linear rule:

$$\hat{p}^*_{j,t} = (1 - \beta\alpha_j) \sum_{k=0}^{\infty} (\beta\alpha_j)^k E_t(\hat{mc}^n_{t+k}). \tag{5.26}$$

Where $\hat{p}^*_{j,t}$ is the (log-deviation of the) price chosen by firms charging new prices at time t in sector j and $\hat{mc}^n_{t+k}$ is the (log-deviation of the) nominal marginal cost. However, unlike Gali (2002), we prefer to continue from now on using relative prices previously defined for simplification of sectoral and aggregate price-index interdependencies. Through some calculation we have the following equations for each sector relating marginal costs to current and future inflation:

$$\hat{\pi}_{j,t} = \kappa \hat{mc}_t - \kappa \hat{x}_{j,t} + \beta E_t(\hat{\pi}_{j,t+1}), \tag{5.27}$$

where,

$$\kappa = \frac{(1 - \alpha_j\beta)(1 - \alpha_j)}{\alpha_j}.$$

As a result of the difference between the sticky price outcomes and flexible price outcome, the economy will face an output gap defined as :

$$\widehat{\widetilde{y}} = \hat{y}_t - \hat{y}^f.$$

Under some circumstances, the variation in output gap can be matched with the variation in marginal costs[9]. Following Gali (2002) paper, using marginal cost definition (5.18) and the definition of natural rate of output, output gap and marginal cost will be related as:

$$\widehat{\widetilde{y}}(\sigma + \phi) = \hat{mc}_t.$$

Finally combining this definition with equation (5.27) will yield an expression for New Keynesian Phillips curve in terms of output gap for each sector :

$$\hat{\pi}_{j,t} = \kappa_0 \widehat{\widetilde{y}} - \kappa \hat{x}_{j,t} + \beta E_t(\hat{\pi}_{j,t+1}), \tag{5.28}$$

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[9] *"In the standard sticky price framework without variable capital, there is an approximate proportionate relation between marginal cost and output. With variable capital, the relation is no longer proportionate"* Gali and Gertler(1999), p.201.

where,

$$\kappa_0 = \frac{(1 - \alpha_j \beta)(1 - \alpha_j)(\sigma + \phi)}{\alpha_j},$$

$$\kappa = \frac{(1 - \alpha_j \beta)(1 - \alpha_j)}{\alpha_j},$$

for $j = 1 \ldots n$.

The New Keynesian Phillips curves for the n sectors economy differs from one sector model with the inclusion of aggregate price level and aggregate output gap in its definition. The sectors are no more independent and the behavior of the overall economy affects the inflation dynamics of the sectors.

In addition, we write the Euler equation (5.5) in terms of output gap and natural rate of interest to have the New Keynesian IS curve for our Economy:

$$\widehat{\tilde{y}}_t = -\frac{1}{\sigma}(\hat{r}_t - E_t\{\hat{\pi}_{t+1}\} - \widehat{rr}_t^f) + E_t\{\widehat{\tilde{y}}_{t+1}\}. \tag{5.29}$$

Thus we see that the IS curve for the economy resembles the one sector case.

To close our system, we add the simple Taylor policy rule to specify the level of interest rate :

$$\hat{r}_t = \phi_{pi}\hat{\pi}_t + \phi_{\tilde{y}}\widehat{\tilde{y}}_t + \hat{\xi}_t, \tag{5.30}$$

with,

$$\hat{\xi}_t = \rho_\xi \hat{\xi}_{t-1} + \epsilon_{\xi,t}.$$

The above equations (5.28) and (5.29) with the policy rule and exogenous processes for $\hat{a}_t$ and $\hat{\xi}_t$ describe the dynamics of our economy.

Now we have to define the equations needed to implement the model in matlab. We will use the definitions of inflation (aggregate and for each sector), the demand relations (5.13) as well as a link between all relative prices (5.12), the definition of natural rate of output (5.19) also (5.28), (5.29) plus the policy rule and the two

exogeneous processes, then apply these equations to matrices of the form described in Uhlig (1995) to be able to solve for the recursive equilibrium law of motion for the economy of the form :

$$x_t = \mathbf{P}x_{t-1} + \mathbf{Q}z_t$$
$$y_t = \mathbf{R}x_{t-1} + \mathbf{S}z_t$$

where x_t is a vector of endogenous state variables, y_t is a vector of other endogenous variables, z_t is a list of exogenous stochastic processes and P, Q, R, S are the solution matrices.

Chapter 6

Model Results and Answer

We begin this section with calibrating the multi-sector model, then we present the impulse responses of the economy to policy and technology shocks and interpret the results. To provide an answer to our questions, we continue on with the responses to policy shock. We compare the impulse responses and standard deviations of multi-sector model to one-sector models with differing Calvo parameters. We try to find the best match, finally we check the robustness of our conclusion.

6.1   Calibration

To be able to implement our model using the data set provided by Bils and Klenow (2004) we calibrate the model at a monthly frequency[1].

The monthly calibration value of β is standard in the literature. We have set $\beta = 0.997$. Next, the following parameters are selected to match Bils and Klenow (2002). We assume that the elasticity of substitution between goods within each sector is $\theta = 10$ which implies a steady state markup of around %10. The elasticity of substitution between each sector is set to 1, this leads to Cobb-Douglas

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[1]Most of the recent work on New Keynesian models are implemented at a quarterly frequency.

production function specification for the final goods [2]. We run the model for 10 sectors and each sector has the same weight in the production function of the final good. Thus, we assume $\epsilon_j = 10\%$ for each sector. Bils and Klenow (2004) give 50% weight for each sector in the two-sector model.

Furthermore, we need to define the value for ϕ, inverse of labor supply elasticity. It is not fixed in the literature, Gali and Monecelli (2005) give a value of 3, Yun (1996) proposes a value of 4 whereas micro-evidence shows a value around 0.6 (Greenwood et al.,1988). We calibrate our model with $\phi = 1$ in the baseline model, this is also the value assumed by Christiano and Eichenbaum (1997) and Barsky et al.(2003). Furthermore, in the sensitivity analysis part we will test our model with different values for this parameter.

We define the Calvo parameter values for the ten sectors using the empirical distribution of price adjustment parameters provided by Bils and Klenow (2004). Using this information, we group the categories with similar frequency of price changes as a sector, with each sector having 10% share in the total cdf. Next, within the sectors we find the weight of each category of goods in the cdf and multiply this value with their assumed frequency parameter to find the category's relative share. Then we sum over these values, correct for the small deviations in the total cdf for the group and find the overall frequency of price change for the related sector. Subtracting these values from 1 and rounding leads to the values given in table 6.1[3].

For the values of simple Taylor rule, we follow the following intuition. The original Taylor rule assumes the period to be one year. In the original rule the coefficient of output gap is 0.5 and the coefficient of inflation rate is 1.5. The output gap is a level so it should appear in the same units in monthly, quarterly and annual models. Since we work with monthly values, we convert the coefficient

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[2]Smets and Wouters(2004) mention the same value for their model.

[3]The calculations are available in the excel table provided with the thesis.

Variable	Value	Description
β	0.997	Subjective discount factor
ϕ	1	Inverse of labor supply elasticity
θ	10	Elasticity of substitution within each sector
σ	1	Coefficient of relative risk aversion
η	1	Elasticity of substitution between sectors
$\phi_{\widetilde{y}}$	0.0416	Coefficient of output gap in the Taylor rule
ϕ_π	1.5	Coefficient of inflation rate in the Taylor rule
ρ_a	0.95	Autocorrelation of technology shock
ρ_ξ	0.00	Autocorrelation of policy shock
σ_a	0.712	Standard deviation of technology shock
σ_ξ	1	Standard deviation of policy shock
ϵ_j	0.10	Weight of each sector in the economy
α_1	0.95	Percentage of firms which cannot reset their prices in sector 1
α_2	0.92	Percentage of firms which cannot reset their prices in sector 2
α_3	0.90	Percentage of firms which cannot reset their prices in sector 3
α_4	0.86	Percentage of firms which cannot reset their prices in sector 4
α_5	0.82	Percentage of firms which cannot reset their prices in sector 5
α_6	0.76	Percentage of firms which cannot reset their prices in sector 6
α_7	0.69	Percentage of firms which cannot reset their prices in sector 7
α_8	0.62	Percentage of firms which cannot reset their prices in sector 8
α_9	0.56	Percentage of firms which cannot reset their prices in sector 9
α_{10}	0.32	Percentage of firms which cannot reset their prices in sector 10

Table 6.1: Calibration values for the baseline model

of output gap to monthly by dividing by 12 which gives a value of 0.0416[4]. The interest rate is already monthly in the model by monthly parameter value for β, we do not need to adjust this term and take it as 1.5 like the original Taylor rule.

Finally, we set the values of the shock parameters. The policy shock is assumed to be a white noise process with $\rho_\xi = 0$. Autocorrelation of technology shock is set to 0.95 with a standard deviation of 0.712. Lastly, the standard deviation of policy shock is set to 1.

6.2 Impulse Responses

We present the impulse responses of a unit shock to policy rule in Figure 6.1. Keeping in mind that sector 1 is the stickiest sector and sector 10 is the most flexible, the immediate response of the economy to an unexpected increase in the interest rate is a drop in output. At time zero all sectoral outputs drop followed by falls in price levels. In sector 1 the price level falls gradually whereas in sector 10 the price level falls suddenly and sharply. We see more persistence in the output responses of sticky price sectors, as expected. However, for the flexible price sectors, after the sudden drop, the output levels rise and become positive. For sector 10 this pattern is very obvious. The flexible firms enjoy the increase in the output till the economy returns back to the steady state level. The intuition behind this is as follows: When the economy faces the shock, sticky price firms cannot react to the shock by immediately decreasing their prices. Every period only a small number of firms adjust to the new price level, keeping the prices of sticky price sectors relatively higher than flexible price sectors. This leads to a substitution effect, whose size is determined by the value of the coefficient η, the elasticity of substitution between the sectors. With our calibration, output of the flexible price firms increases together with their price level and this continues on till the economy reaches to the steady state level.

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[4]See Woodford (2003), p.245 and Woodford and Giannoni(2002), p.4 divides the coefficient of output gap in the Taylor rule by 4 to have a quarterly value.

General price level first drops down by about -0.2% and then moves back to the level of sticky prices, reflecting a long term effect. Thus the price level overshoots and sticky price sectors define the new price level in the economy. Taking into consideration that we have only three very sticky sectors, this dominance over the economy is interesting. Inflation responses are inline with prices. There is persistent and negative inflation in the sticky price sector whereas for the flexible sectors, after sometime inflation becomes positive matching the responses of prices in this sector. All return to the steady state value of zero as the economy adjusts. Finally for the output gap , following gross output, we have deficit. The increases in the output of flexible price sectors are not as strong to have impact on these two aggregate variables.

Next, we turn to the effects of a shock to the level of technology in a multi-sector economy (Figure 6.2). As the shock increases the productivity, output rises and prices decline correspondingly, leading to a decrease in inflation. This time the impulse responses are similar between the various sectors. One interesting point to note is that the resulting increase in the output levels of flexible sectors are higher than the increase in the sticky sectors. We, in fact, see some stealing effect between sector 1 and sector 10. This is due to the immediate response of flexible price firms to the new conditions in the economy which leads to lower price levels in this sector, however this is not very strong. Finally, the decrease in inflation is matched with the decrease in interest rate.

6.3 Model Comparison

Now we try to match the impulse responses of multi-sector model with a one-sector model. For comparisons, we attach more importance to the level of output
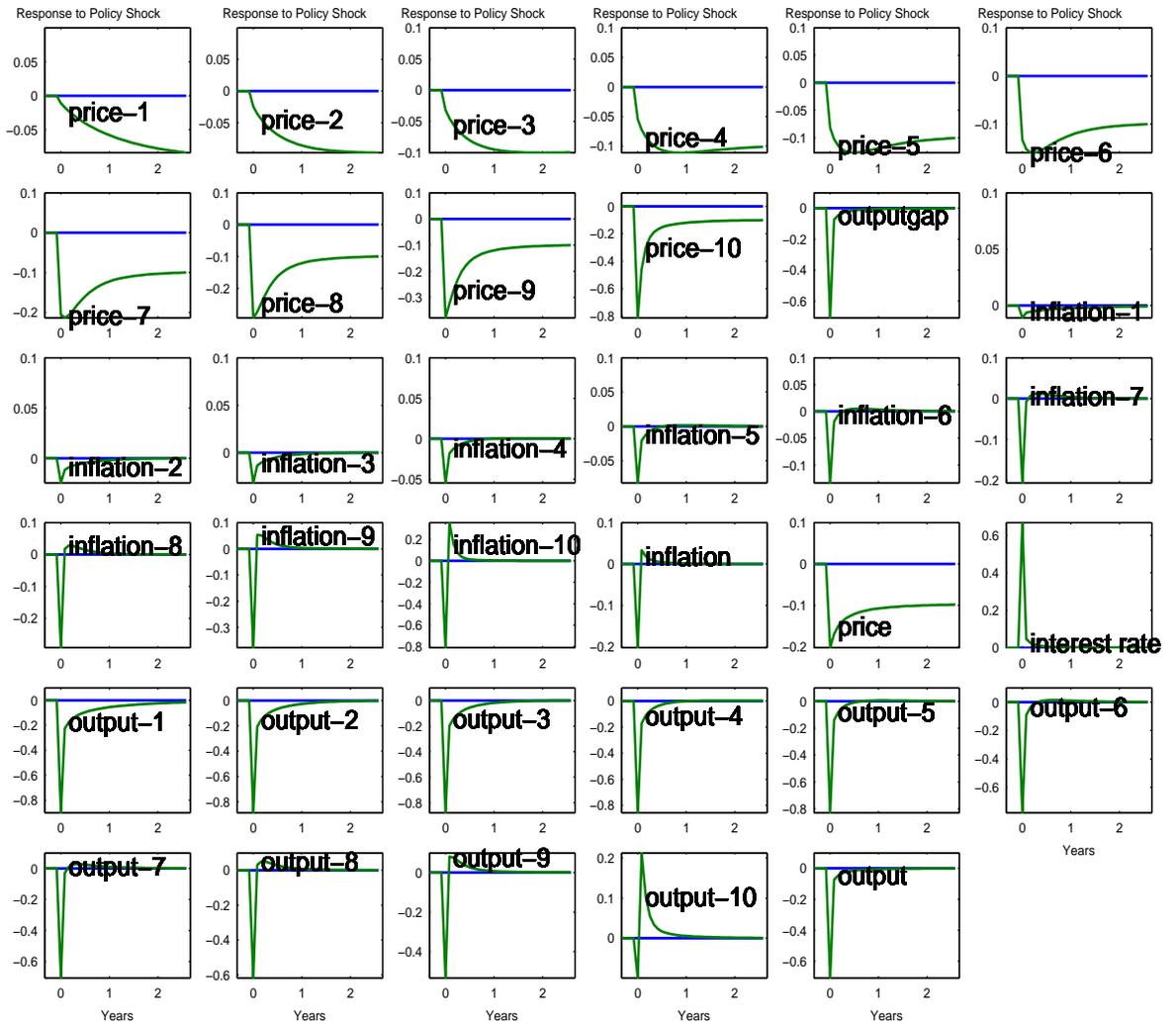
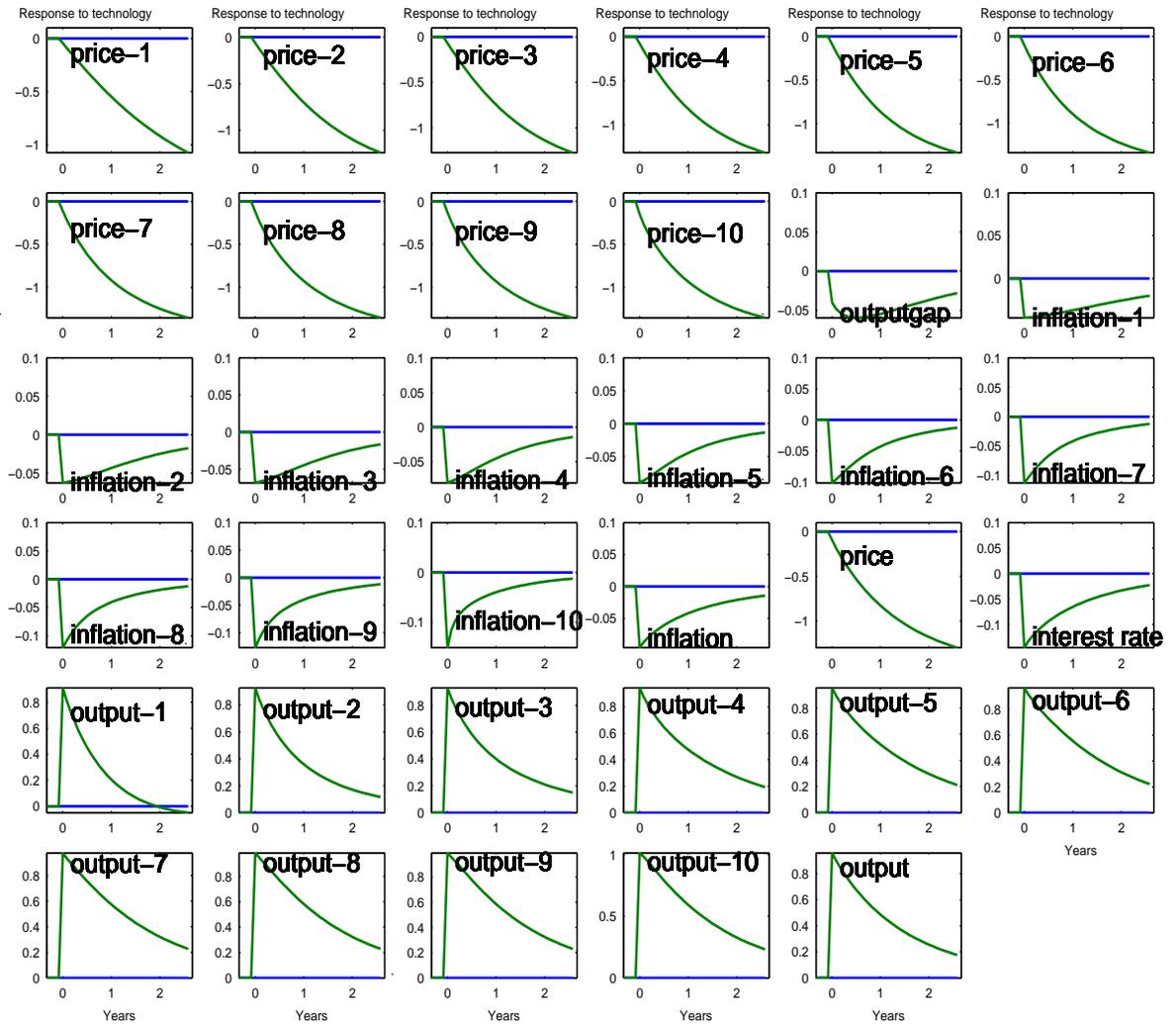Figure 6.1: Impulse Responses to a Shock to Policy Rule, baseline model.

Figure 6.2: Impulse Responses to a Shock to Technology, baseline model.

gap, but we also care for the other aggregate variables. Table 6.2 shows time zero impulse response levels of the aggregate variables to a policy shock in one- and multi-sector models :

	Output gap	Interest rate	Inflation
10 sector	−0.71	0.67	−0.2
1 sector with $\alpha = 0.71$	−0.71	0.71	−0.17
1 sector with $\alpha = 0.70$	−0.70	0.70	−0.18

Table 6.2: The values in the table correspond to the value of the impulse response function to a policy shock at t=0

The two one-sector impulse response values are very similar, we select $\alpha = 0.71$. This value corresponds to a price duration of 2.9 months. This is a low value, compared to the calibration used in the sticky price models and it is also lower than the duration found by Bils and Klenow (4 months). Figure 6.3 displays the impulse responses of the 0.71 calibration and figure 6.4 shows the impulse responses had we used the prevailing duration of around 11 months in the literature. The value corresponds to about 0.92% Calvo parameter to be used in the monthly calibration. There are differences in the magnitudes of responses. The decrease in the output gap and the increase in the interest rate are 0.2% more, also the responses of the prices and inflation are very small for $\alpha = 0.92\%$.

The one-sector model with a Calvo parameter $\alpha = 0.71$ displays similar aggregate responses to the multi-sector model, however it cannot capture the dynamics in the response patterns of multi-sector model. Therefore, it is not very descriptive in conveying the actual behavior of the economy. This variability can only be captured by a model with at least two sectors. We tried to find the two-sector combination that could mimic price, inflation, output gap behavior of the multi-sector model. We simulated different combinations and found that a two sector model with a sticky sector with $\alpha_1 = 0.91$ and a flexible sector with $\alpha_2 = 0.56$ matched
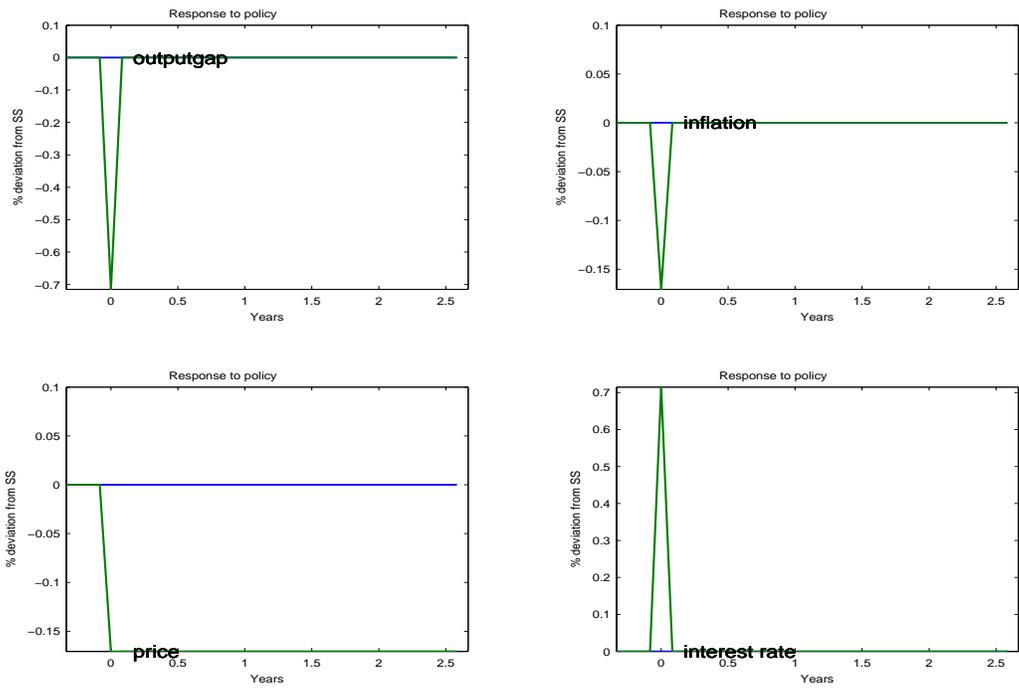
39

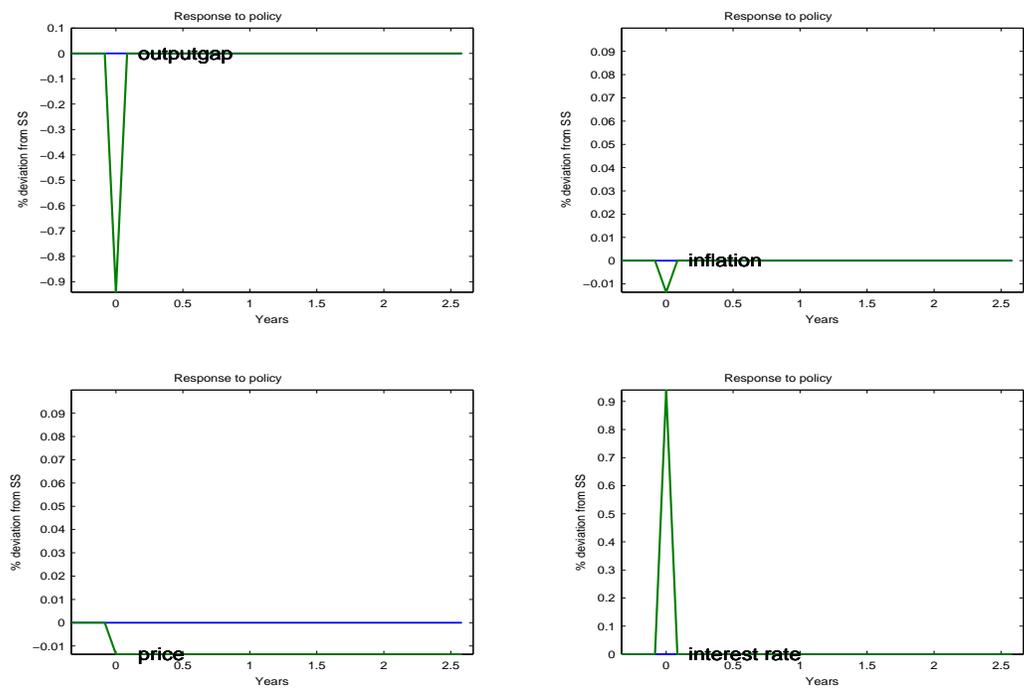Figure 6.3: Impulse Responses to a Shock to Policy Rule, $\alpha = 0.71$.

Figure 6.4: Impulse Responses to a Shock to Policy Rule, $\alpha = 0.92$.

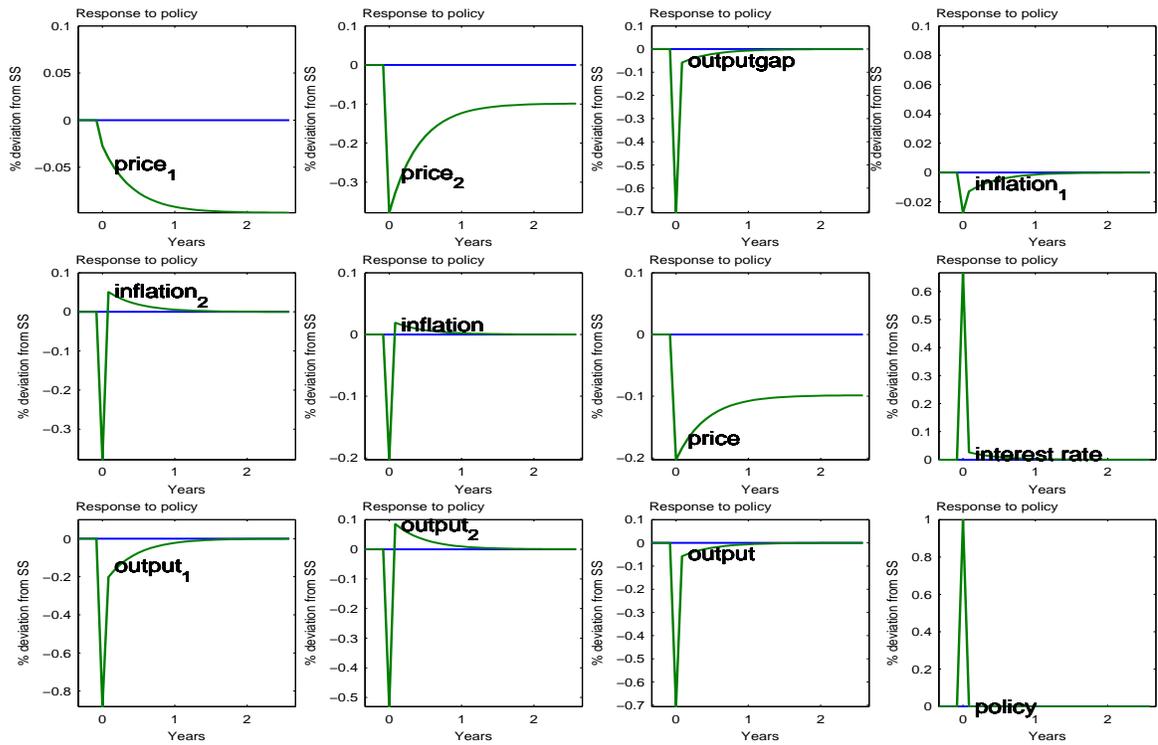Figure 6.5: Impulse Responses to a Shock to Policy Rule, $\alpha_1 = 0.91$ and $\alpha_2 = 0.56$.

almost all the impulse response patterns of the multi-sector model (Figure 6.5).

Next, to have more precise conclusions, we check the standard deviations of the models. The standard deviations of multi-sector, one-sector (for $\alpha = 0.71$ and $\alpha = 0.70$) and two-sector models ($\alpha_1 = 0.91$ and $\alpha_2 = 0.56$) are presented in table 6.3. One-sector model with $\alpha = 0.71$ can match the volatility of multi-sector model with some slight difference and the two-sector model works quite well.

	Output gap	Interest rate	Inflation	Price	Output
10 sector	0.7086	0.69	0.2445	1.0356	1.5651
1 sector ($\alpha = 0.70$)	0.6928	0.7201	0.2222	1.0946	1.6220
1 sector ($\alpha = 0.71$)	0.7080	0.7345	0.2137	1.0837	1.6265
2 sectors ($\alpha_1 = 0.91$, $\alpha_2 = 0.56$)	0.7078	0.6885	0.2437	1.0282	1.5549

Table 6.3: Comparison of standard deviations of multi-sector and one- and two-sector models based on HP-filtered, frequency domain calculations of moments.

6.4 Sensitivity Analysis

As discussed in the calibration part, some of the parameter values are not standard in the literature. These are mainly the inverse of labor supply elasticity (ϕ) and the coefficient of relative risk aversion (σ). In this section we apply a sensitivity analysis on the robustness of our conclusions to changes in these parameters. We change the coefficient of relative risk aversion to 2, 3, 4 and 5; the inverse labor supply elasticity to 0.6 and 3 respectively.

The results are presented in table 6.3. The volatility of one-sector and two-sector models are matched well with the multi-sector model for small values of σ. For one-sector model, as the parameter values begin to disperse away from the benchmark calibration, the accuracy of the proposed Calvo parameter slightly decreases. Yet, the differences with the multi-sector is never too striking. The two-

sector model performs quite well. The highest difference (in terms of matching the volatility of output gap) occurs when we calibrate the models with $\sigma = 5$ and $\phi = 3$. With this calibration, for one-sector model, the volatility of the output gap of the multi-sector model can be matched with an alpha value of 0.76, but this value is not able to mimic the volatility of the other aggregate variables precisely. Moreover, in terms of matching the level of impulse responses, $\alpha = 0.71$ performs better than $\alpha = 0.76$[5]. This result is generally valid for the other parameter value changes in table 6.4 as well. In terms of matching the impulse response levels of the multi-sector economy, the baseline values seems robust.

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[5]See Appendix C for a comparison of the impulse responses for this calibration.

$\sigma = 5$	$\phi = 0.6$			$\phi = 3$		
	multi-sector	one-sector	two-sectors	multi-sector	one-sector	two-sectors
output gap	0.1752	0.1644	0.1777	0.1678	0.1540	0.1713
interest rate	0.8430	0.8653	0.8411	0.8906	0.9109	0.8905
inflation	0.2359	0.2160	0.2355	0.3897	0.3606	0.3915
price	1.4458	1.4654	1.4050	2.5990	2.5802	2.5671
output	0.4085	0.4434	0.4021	0.7019	0.7444	0.6941
$\sigma = 4$	$\phi = 0.6$			$\phi = 3$		
	multi-sector	one-sector	two-sectors	multi-sector	one-sector	two-sectors
output gap	0.2164	0.2042	0.2191	0.1997	0.1879	0.2025
interest rate	0.8347	0.8582	0.8328	0.8479	0.8747	0.8476
inflation	0.2326	0.2129	0.2322	0.3694	0.3400	0.3710
price	1.4038	1.4287	1.3656	2.3826	2.3695	2.3595
output	0.5009	0.5417	0.4934	0.8117	0.8551	0.8039
$\sigma = 3$	$\phi = 0.6$			$\phi = 3$		
	multi-sector	one-sector	two-sectors	multi-sector	one-sector	two-sectors
output gap	0.2832	0.2694	0.2861	0.2492	0.2413	0.2508
interest rate	0.8214	0.8469	0.8195	0.7884	0.8240	0.7879
inflation	0.2280	0.2084	0.2277	0.3471	0.3158	0.3480
price	1.3394	1.3718	1.3048	2.0956	2.0905	2.0821
output	0.6472	0.6963	0.6384	0.9623	1.0054	0.9550
$\sigma = 2$	$\phi = 0.6$			$\phi = 3$		
	multi-sector	one-sector	two-sectors	multi-sector	one-sector	two-sectors
output gap	0.4108	0.3958	0.4136	0.3371	0.3376	0.3369
interest rate	0.7969	0.8257	0.7950	0.6973	0.7460	0.6971
inflation	0.2216	0.2016	0.2211	0.3294	0.2925	0.3290
price	1.2281	1.2723	1.1996	1.7025	1.7084	1.6998
output	0.9146	0.9754	0.9039	1.1831	1.2234	1.1767

Table 6.4: Comparison of standard deviations of multi-sector, one- and two-sector models based on HP-filtered, frequency-domain calculations of moments under different parameterizations.

Chapter 7

Model Extensions

7.1 Extended Model

The assumption of common factor markets is often criticized in the literature as it assumes that "*all factor prices are instantaneously equalized across suppliers of different goods*"[1]. In reality, it takes time for factors to adjust. The specific factor markets assumption makes it possible that the factors adjust slowly with the high degree of strategic complementarity it assumes. The baseline model outcomes show very flexible prices even compared to Bils and Klenow (2004). We guess that introducing specific factor markets assumption can increase the price duration needed to match the multi-sector model in terms of volatility and magnitude. Also, we want to know how a multi-sector economy behaves under this assumption to a shock to policy rule.

Hence, we adjust the labor assumptions of the baseline multi-sector model. Now different types of labor are needed to produce each good in each sector. All households are identical, and each household supplies all the types labor used to produce all goods in all sectors. We will present the model roughly, however it is almost the same model as Woodford (2003), except that we assume same

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[1]See Woodford (2003) p.166.

production technology for all firms and all sectors ,and we have a continuum of goods between 0 and 1 produced within each sector. The first difference compared to the baseline model is in the definition of the maximization problem of households which now becomes:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \int_0^1 \frac{l_{1,t}(i)^{1+\phi}}{1+\phi} di - \int_0^1 \frac{l_{2,t}(i)^{1+\phi}}{1+\phi} di - ... - \int_0^1 \frac{l_{n,t}(i)^{1+\phi}}{1+\phi} di \right],$$
(7.1)

where β is the intertemporal discount factor with $0 < \beta < 1$, subject to the period budget constraint:

$$P_t C_t + \frac{B_{t+1}}{R_t} \leq B_t + \int_0^1 l_{1,t}(i) W_{1,t}(i) di + \int_0^1 l_{2,t}(i) W_{2,t}(i) di + ... + \int_0^1 l_{n,t}(i) W_{n,t}(i) di + \tau_t.$$
(7.2)

Coming to the first order conditions, Euler equation continues to hold but now we have labor supply equations of the form:

$$\hat{w}_{j,t}(i) - \hat{p}_t = \sigma \hat{c}_t + \phi \hat{n}_{j,t}(i).$$
(7.3)

All the relations between the sectors remain valid but we have a difference in the marginal cost definition of intermediate firms. This time average marginal cost in sector (j) will be defined as:

$$\hat{mc}_{j,t+k} = (\sigma + \phi)\hat{y}_t - (1+\phi)\hat{a}_t + \phi[-\theta(\hat{x}_{j,t}(i) - \sum_{i=1}^{k} \hat{\pi}_{j,t+i}) - \eta \hat{x}_{j,t}].$$
(7.4)

The natural rate of output for the economy will be defined in the same way as before and subtracting it from both sides of above equation, we have:

$$\hat{mc}_{j,t+k} = (\sigma + \phi)\hat{\tilde{y}}_t + \phi[-\theta(\hat{x}_{j,t}(i) - \sum_{i=1}^{k} \hat{\pi}_{j,t+i}) - \eta \hat{x}_{j,t}].$$
(7.5)

The pricing decision of sticky price firms are as previously defined, thus we can insert the above equation in :

$$\sum_{k=0}^{\infty} (\beta \alpha_j)^k E_t \{\hat{x}_{j,t}(i) + \hat{x}_{j,t} - \sum_{i=1}^{k} \hat{\pi}_{t+i}\} = \sum_{k=0}^{\infty} (\beta \alpha_j)^k E_t \{\hat{mc}_{j,t+k}\},$$
(7.6)

to have the New Keynesian Phillips curves of each sector as [2]:

$$\hat{\pi}_{j,t} = \kappa_1 \hat{\tilde{y}}_t - \kappa_2 \hat{x}_{j,t} + \beta E_t(\hat{\pi}_{j,t+1}),$$
(7.7)

---

[2]The derivation of the NKPC is very similar to Aoki(2001).

where,

$$\kappa_1 = \frac{(1 - \alpha_j\beta)(1 - \alpha_j)}{\alpha_j} \frac{(\sigma + \phi)}{(1 + \phi\theta)},$$

$$\kappa_1 = \frac{(1 - \alpha_j\beta)(1 - \alpha_j)}{\alpha_j} \frac{(1 + \phi\eta)}{(1 + \phi\theta)},$$

for $j = 1 \ldots n$.

We can then combine the above equations with the New-Keynesian IS curve, the identities of the baseline model and the policy rule and apply to matlab.

7.2 Extended Model Results

7.2.1 Impulse Responses

We run the model with our benchmark calibration $\theta = 10$ and $\eta = 1$ and report the impulse responses to a policy shock in figure 7.1. The results are as expected given that the extended multi-sector model has strong strategic complementarity. The intuition is as follows: The two prices are said to be strategic complements when the increase in one price level leads to a corresponding increase in the other. Thus, the existence of strategic complementarity among different sector prices restrains the flexible price firms from adjusting their price level as they would under different settings. Hence, aggregate inflation and aggregate price level react by less to the policy shock now, whereas there is a strong decline in the level of output [3].

7.2.2 Model Comparisons and Sensitivity Analysis

The $t = 0$ impulse response-levels of the extended model can be well replicated by the one-sector calibration previously assumed ($\alpha = 0.71$). Whereas when we compare the standard deviations, we find that this calibration value is not able

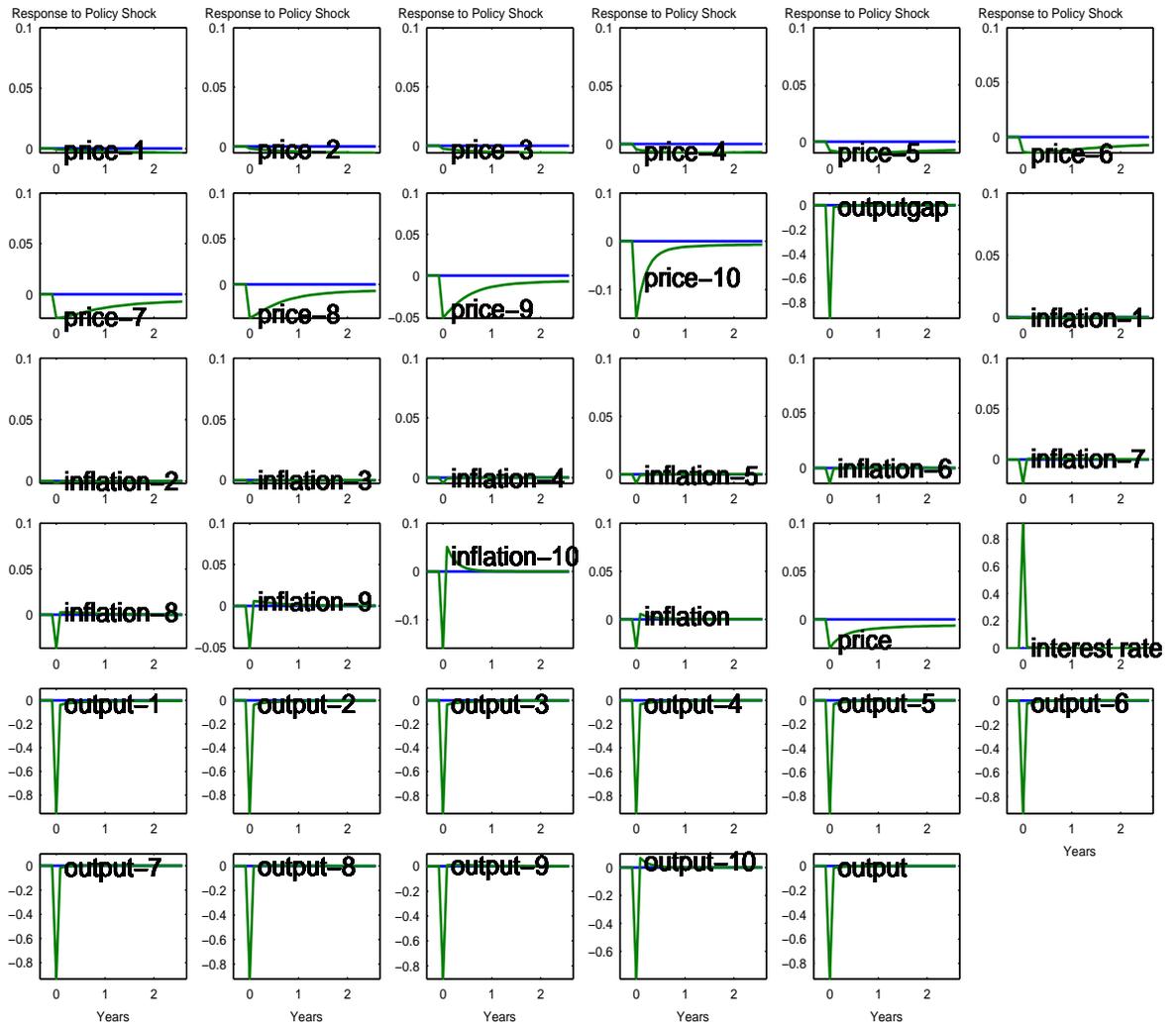[3]On strategic complementarity see Woodford(2003) pp.161-163.

Figure 7.1: Impulse Responses to a Shock to Policy Rule, extended model.

to capture the volatility of the multi-sector model. The best value for one-sector models turns out to be $\alpha = 0.82$ which is very similar to the value reported by Bils and Klenow (2004). However, this value cannot match the aggregate response levels as good as the previously reported value with $\alpha = 0.71$. (We have also noted a similar result in the sensitivity analysis for the benchmark model.)

	output gap	interest rate	inflation
10 sector	0.925	0.92	0.03
$\alpha_1 = 0.71$	0.93	0.94	0.02
$\alpha_1 = 0.82$	0.95	0.95	0.007
$\alpha_1 = 0.91$ and $\alpha_2 = 0.56$	0.93	0.92	0.027

Table 7.1: The values in the table correspond to the value of the impulse response function to a policy shock at t=0, extended model.

For the two sector models, $\alpha = 0.91$ and $\alpha = 0.56$ calibration is able to match the level of impulse responses of the multi-sector economy. In terms of second moments, its performance only slightly decreases. We present the results below:

	Output gap	Interest rate	Inflation	Price	Output
10 sector	1.0394	0.9174	0.0864	0.5631	1.3643
1 sector ($\alpha = 0.71$)	0.9515	0.9347	0.0988	0.7525	1.5595
1 sector ($\alpha = 0.82$)	1.0419	0.9473	0.0621	0.4791	1.4067
2 sector ($\alpha = 0.56$ and $\alpha = 0.91$)	1.0553	0.9230	0.0876	0.5674	1.3509

Table 7.2: Comparison of standard deviations of multi-sector and one- and two-sector models based on HP-filtered frequency domain calculations of moments.

We also performed a sensitivity analysis of the extended model for the parameter value combinations of $\eta = 1, 2$; $\phi = 0.6, 3$ and $\sigma = 2, 5$ [4]. The results follow the analysis in the baseline model. We can summarize them as follows: In terms

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[4]In Appendix C, we present the most representative tables for this analysis.

of matching the volatility of the output gap, the one-sector Calvo parameter value of the baseline model does not perform very well, the two sector values are consistent and the new proposed Calvo parameter value of one-sector model performs better than its baseline counterpart. However, generally this new value is not able to match the impulse response levels of the multi-sector model better than the baseline value for this parameter.

Chapter 8

Discussion

The first aim of this thesis was to build a multi-sector economy and study the impulse responses to a shock to policy rule. The attained impulse responses look quite descriptive, the interactions between flexible and sticky sectors introduce different dynamics to the economy. Even with just analyzing the behavior of prices, output, interest rate and inflation, we can have a good summary of what is going on in the economy. Introducing the sectoral variety is like introducing an additional dimension to the system that is usually ignored. We see that especially price and inflation aggregates show complicated impulse response patterns.

We have then tried to find the one-sector model that can capture the dynamics of the multi-sector economy most precisely. Our results for the baseline model show that one-sector model can produce similar impulse response levels to the multi-sector economy by calibrating the model with $\alpha = 0.71$. This conclusion seems robust to the changes in the parameter values and works well when we introduce a modification to the structure of the model. The volatility of the aggregate variables, however, change when we vary the parameter values or the assumptions of the economy. In terms of matching the output gap, the Calvo parameter value needed to be imposed on one-sector model usually rises and comes closer to the parametrization in the literature. Yet the values never imply very

sticky prices. While the multi-sector economy aggregate variables incorporate a variety of dynamics that one-sector models cannot capture, we sometimes faced a trade off between matching the volatility and matching the impulse response levels. The most difficult was to produce the behavior of inflation and prices, thus there was always a difference in one-sector and multi-sector outcomes of these variables. We should keep in mind these problems when we mention that one sector with $\alpha = 0.71$ is able to match the impulse responses levels of the multi-sector economy well.

The two-sector model is proposed to overcome these difficulties embedded in one-sector models. By introducing one more sector, we are able to explore the additional dimension. The two sector calvo parameters of the baseline model represent a good summary of the multi-sector environment and display consistency among different parameterizations and assumptions. Even if we try to improve the results, this is attained with altering the proposed percentage values of Calvo parameter by only one or two percent less/more.

Finally, one shortcomings of this analysis that restrains it from stronger generalizations is that these conclusions are attained with the data set provided by Bils and Klenow (2004) and their methodology of studying the data influences the definition of the sectors' pricing behavior in the multi-sector model. This affects in turn the price durations found in one- and two-sector models. Since there is no consistency in the micro-evidence and the methodologies applied by different studies, the results can differ when we work with different data sets.

Chapter 9

Summary and Conclusion

We have modelled a multi-sector economy using the frequency of price adjustment distribution provided by Bils and Klenow (2004) to analyze the impulse responses of the economy to a shock to policy rule. We have shown that interactions between the sticky and flexible price firms produced complicated dynamics compared to one sector models. Especially the impulse response patterns of price and inflation were unexpected.

Since most models in the literature assume single sectors, our next question was whether the aggregate responses of the multi-sector economy could be matched by one-sector models, and with which Calvo parameter value could we achieve such results. To answer these question, we compared multi-sector model with one-sector models in terms of level of impulse responses to policy shock and volatility. The results showed that, for the baseline model the multi-sector economy could be well replicated by a one-sector economy with a price duration of 2.9 months. We then checked the sensitivity of the results to the choice of parameters. The proposed duration showed slight differences in terms of matching the volatilities, whereas the impulse responses continued to perform well. Later, we introduced a modification to the model, with the guess that it could lead to differences in the behavior of the responses and thus increases in price durations. The results were

interesting, the proposed price duration could mimic the impulse response levels but was insufficient in terms of matching the volatility. On the other hand, the price duration that could match the volatility could not perform better than the proposed value in matching the response levels. This new duration was close to the results of Bils and Klenow (2004). By examining other examples we concluded that as we depart from the benchmark calibration, there is generally a trade of between matching the volatility and matching the impulse response levels.

The one-sector models could match aggregate response levels well but they were not descriptive in terms of conveying the complicated dynamics of multi-sector economy. These patterns could only be captured by models employing at least two-sectors. We found a two-sector model calibration that could match the volatility and impulse response levels of the multi-sector economy. The proposed price durations for the two-sector model led to descriptive impulse response patterns, moreover, they could also perform well with different parameterizations and settings.

To sum up, we conclude that using Bils and Klenow data set for defining the frequency of price adjustment parameters of multi-sector economy, one-sector models with price duration of around 3 months can match the aggregate impulse response levels of multi-sector models but they are not very precise in matching the second moments (standard deviations) under different settings. The dynamics of the impulse response patterns, however, can only be captured with economies with two or more sectors. Finally, we note that the impulse responses to monetary policy show informative patterns that should be modelled to be able to understand the actual behaviors of aggregate variables.

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Appendix A

Aggregating Labor

We had the following demand equations relating sectoral output to the aggregate output:

$$Y_{1,t} = (\frac{P_{1,t}}{P_t})^{-\eta} Y \epsilon_1 \qquad \ldots \qquad Y_{n,t} = (\frac{P_{n,t}}{P_t})^{-\eta} Y \epsilon_n.$$

And the following production functions for each sector:

$$Y_{1,t} = Al_{1,t},$$
$$Y_{2,t} = Al_{2,t},$$
$$\vdots$$
$$Y_{n,t} = Al_{n,t}. \tag{A.1}$$

It is clear from the demand relations that in the steady state (since prices will be the same for all sectors), output of each sector will be proportional to their share ϵ_j in the basket.

Moreover, from (A.1), we have that the technology parameter is the same among sectors and will be equal to 1 in the steady state. Hence, since the steady state productions will be proportional to the relative shares; the steady state labor will also be proportional to the relative shares.

Labor will be log-linearized as follows:

$$\underbrace{\int_0^1 l_{1,t}(i)di}_{l_{1,t}+} + \underbrace{\int_0^1 l_{2,t}(i)di}_{l_{2,t}\ldots} \ldots + \underbrace{\int_0^1 l_{n,t}(i)di}_{+ l_{n,t} = l_t,} = l_t,$$

in the steady-state :

$$\bar{l}_1 + \quad \bar{l}_2 \ldots \bar{l}_n = \bar{l},$$

Thus, log-linearization gives :

$$\bar{l}_1 \hat{l}_{1,t} + \bar{l}_2 \hat{l}_{2,t} \ldots + \bar{l}_n \hat{l}_{n,t} = \bar{l} \hat{l}_t. \tag{A.2}$$

which can be rewritten in terms of output and technology:

$$\frac{\bar{l}_1}{\bar{l}}(\hat{y}_{1,t} - \hat{a}_t) + \frac{\bar{l}_2}{\bar{l}}(\hat{y}_{2,t} - \hat{a}_t) + \ldots + \frac{\bar{l}_n}{\bar{l}}(\hat{y}_{n,t} - \hat{a}_t) = \hat{l}_t, \tag{A.3}$$

Inserting the general level of output :

$$\frac{\bar{l}_1}{\bar{l}}(\hat{y}_t - \eta\hat{x}_{1,t} - \hat{a}_t) + \frac{\bar{l}_2}{\bar{l}}(\hat{y}_t - \eta\hat{x}_{2,t} - \hat{a}_t) + \ldots + \frac{\bar{l}_n}{\bar{l}}(\hat{y}_t - \eta\hat{x}_{n,t} - \hat{a}_t) = \hat{l}_t, \tag{A.4}$$

using the steady state definition of labor will give :

$$\hat{l}_t = \hat{y}_t - \hat{a}_t - \eta(\frac{\bar{l}_1}{\bar{l}}\hat{x}_{1,t} + \frac{\bar{l}_2}{\bar{l}}\hat{x}_{2,t} + \ldots + \frac{\bar{l}_n}{\bar{l}}\hat{x}_{n,t}). \tag{A.5}$$

Where the term inside parenthesis is equation (5.12) in the model analysis, therefore is equal to zero.

Appendix B

Sticky Price Solution

Firm in sector j=1,...n solves:

$$max \sum_{k=0}^{\infty} \alpha_j^k E_t \left[Q_{t,t+k}(P_{j,t}(i)Y_{j,t+k}^d(i) - TC_{j,t+k}(Y_{j,t+k}^d(i); P_{t+k}; Y_{t+k})) \right],$$

(B.1)

subject to:

$$Y_{j,t+k}^d = \left(\frac{P_{j,t}(i)}{P_{j,t+k}} \right)^{-\theta} \left(\frac{P_{j,t+k}}{P_{t+k}} \right)^{-\eta} Y_{t+k} \epsilon_j.$$

(B.2)

with $0 < \alpha_j < 1$.

This maximization problem leads to the following FONC :

$$\sum_{k=0}^{\infty} \alpha_j^k E_t \left[Q_{t,t+k}(1-\theta) \left(\frac{P_{j,t}(i)}{P_{j,t+k}} \right)^{-\theta} \left(\frac{P_{j,t+k}}{P_{t+k}} \right)^{-\eta} Y_{t+k} \epsilon_j \right.$$

$$\left. + \theta MC_{t+k}(i) \left(\frac{P_{j,t}(i)}{P_{j,t+k}} \right)^{-\theta} \left(\frac{P_{j,t+k}}{P_{t+k}} \right)^{-\eta} \frac{Y_{t+k}}{P_{j,t}(i)} \epsilon_j \right] = 0.$$

(B.3)

Inserting the definition of demand :

$$Y_{j,t+k}^d = \left(\frac{P_{j,t}(i)}{P_{j,t+k}} \right)^{-\theta} \left(\frac{P_{j,t+k}}{P_{t+k}} \right)^{-\eta} Y_{t+k} \epsilon_j,$$

and since $MC_t = MC_{j,t} = MC_{j,t}(i)$, the FONC can be rewritten as :

$$\sum_{k=0}^{\infty} \alpha_j^k E_t \left[Q_{t,t+k} Y_{j,t+k}^d(i)(P_{j,t}(i) - \frac{\theta}{\theta-1}MC_{t+k}) \right] = 0.$$

(B.4)

Using the Euler equation definition :

$$Q_{t,t+k} = \beta^k E_t \left((\frac{C_{t+k}}{C_t})^{-\sigma} \frac{P_t}{P_{t+k}} \right),$$

the first order condition becomes :

$$\sum_{k=0}^{\infty} (\beta\alpha_j)^k E_t \left[\left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+k}} Y_{j,t+k}^d(i)(P_{j,t}(i) - \frac{\theta}{\theta-1} MC_{t+k}) \right] = 0. \qquad \text{(B.5)}$$

Now, define $MC_{t+k}/P_{t+k} \equiv mc_{t+k}$ and $P_{j,t}(i)/P_{t+k} \equiv X_{j,t}(i)X_{j,t} \prod_{i=1}^{k} \Pi_{t+i}^{-1}$ and we have

$$\sum_{k=0}^{\infty} (\beta\alpha_j)^k E_t \left\{ \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} P_t Y_{j,t+k}^d(i) X_{j,t}(i) X_{j,t} \prod_{i=1}^{k} \Pi_{t+i}^{-1} \right\}$$
$$= \frac{\theta}{\theta-1} \sum_{k=0}^{\infty} (\beta\alpha_j)^k E_t \left\{ \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} P_t Y_{j,t+k}^d(i) mc_{t+k} \right\}. \qquad \text{(B.6)}$$

calculate the steady state values for : $X_{j,t}(i) X_{j,t} \left(\prod_{i=1}^{k} \Pi_{t+i} \right)^{-1}$ and $\frac{\theta}{\theta-1} mc_{t+k}$. Since $P_{j,t} = P_{j,t}(i) = P_t$ in the steady state first expression will equal 1. From flexible price equilibrium, $mc_{t+k}^f = \frac{\theta-1}{\theta}$, which will also equate the second steady state ratio to 1.

So log-linearizing the above equation we have :

$$\sum_{k=0}^{\infty} (\beta\alpha_j)^k E_t \{\hat{x}_{j,t}(i) + \hat{x}_{j,t} - \sum_{i=1}^{k} \hat{\pi}_{t+i}\} = \sum_{k=0}^{\infty} (\beta\alpha_j)^k E_t \{\hat{mc}_{t+k}\}, \qquad \text{(B.7)}$$

since $\hat{x}_{j,t}(i)$ and $\hat{x}_{j,t}$ do not depend on index k, using $\sum_{k=0}^{\infty} (\beta\alpha_j)^k = \frac{1}{1-\beta\alpha_j}$ rewrite equation (B.7) as :

$$\hat{x}_{j,t}(i) + \hat{x}_{j,t} = (1 - \beta\alpha_j) \sum_{k=0}^{\infty} (\beta\alpha_j)^k E_t \{\hat{mc}_{t+k} + \sum_{i=1}^{k} \hat{\pi}_{t+i}\}, \qquad \text{(B.8)}$$

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Using $(1 - \beta\alpha_j) \sum_{k=0}^{\infty} (\beta\alpha_j)^k \sum_{i=1}^{k} \hat{\pi}_{t+i} = \sum_{k=1}^{\infty} (\beta\alpha_j)^k \hat{\pi}_{t+k}$ we have[1],

$$\hat{x}_{j,t}(i) + \hat{x}_{j,t} = (1 - \beta\alpha_j) \sum_{k=0}^{\infty} (\beta\alpha_j)^k E_t\{\widehat{mc}_{t+k}\} + \sum_{k=1}^{\infty} (\beta\alpha_j)^k \hat{\pi}_{t+k}, \qquad \text{(B.9)}$$

and we have :

$$\hat{x}_{j,t}(i) + \hat{x}_{j,t} = (1 - \beta\alpha_j)\widehat{mc}_t + \beta\alpha_j E_t\{\hat{\pi}_{t+1}\} + \beta\alpha_j\{\hat{x}_{j,t+1}(i) + \hat{x}_{j,t+1}\}, \quad \text{(B.10)}$$

From this equation we derive the log-linear rule (5.26) by rewriting in terms of prices and nominal variables and iterating forwards. Using $\hat{p}_{j,t}^*$,

Let $\hat{x}_{j,t}^* = \frac{\alpha_j}{1-\alpha_j}\hat{\pi}_{j,t}$ and $\hat{x}_{j,t+1} = \hat{\pi}_{j,t+1} + \hat{x}_{j,t} - \hat{\pi}_{t+1}$.

The second identity is directly provable by multiplication. For the first one, since $(1 - \alpha_j)$ of firms adjust their prices and the remaining (α_j) continue to charge their old prices, the price index (5.9) becomes:

$$P_{j,t} = \{(1 - \alpha_j)(P_{j,t}^*)^{1-\theta} + \alpha_j(P_{j,t-1})^{1-\theta}\}^{\frac{1}{1-\theta}}, \qquad \text{(B.11)}$$

Log-linearizing around common-steady state we have :

$$\hat{p}_{j,t} = (1 - \alpha_j)\hat{p}_{j,t}^* + \alpha_j\hat{p}_{j,t-1}, \qquad \text{(B.12)}$$

Subtracting $\hat{p}_{j,t}$ from both sides and rearranging using the definitions in "some relations" part gives the above identity for $\hat{x}_{j,t}^*$.

Hence our equation becomes,

$$\frac{\alpha_j}{1 - \alpha_j}\hat{\pi}_{j,t} + \hat{x}_{j,t} = (1-\beta\alpha_j)\widehat{mc}_t + \beta\alpha_j E_t\{\hat{\pi}_{t+1}\} + \beta\alpha_j\{\frac{\alpha_j}{1 - \alpha_j}\hat{\pi}_{j,t+1} + \hat{\pi}_{j,t+1} + \hat{x}_{j,t} - \hat{\pi}_{t+1}\}.$$

Finally, cancelling terms and multiplying both sides by $\frac{1-\alpha_j}{\alpha_j}$ leads to the equation (5.27) in the text.

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[1] See Aoki(2001).

Appendix C

Model Comparisons

C.1 Sensitivity Analysis-Baseline Model

In this appendix, we present the impulse response levels at $t = 0$ together with the impulse response graphs for the parametrization $\sigma = 5$ and $\phi = 3$ since this calibration leads to the highest difference in the volatility of output gap among the models. Time zero impulse response levels to a shock to monetary policy are as follows:

	output gap	interest rate	inflation
multi-sector	-0.155	0.73	-0.175
$\alpha_1 = 0.71$	-0.155	0.77	-0.145
$\alpha_1 = 0.76$	-0.168	0.84	-0.1
$\alpha_1 = 0.91$ and $\alpha_2 = 0.56$	-0.155	0.73	-0.177

Table C.1: The values in the table correspond to the value of the impulse response function to a policy shock at t=0, with $\sigma = 5$ and $\phi = 3$.

For $\alpha = 0.76$, the standard deviations of the aggregate variables are 0.1684, 0.9632, 0.3402, 2.5297, 0.7377 for output gap, interest rate, inflation, price and output respectively. The impulse response functions for the multi-sector economy,

the two one-sector economies and the two-sector economy are as presented next. Note that $\alpha = 0.71$ performs better than $\alpha = 0.76$ in matching the impulse response levels of interest rate and output gap.
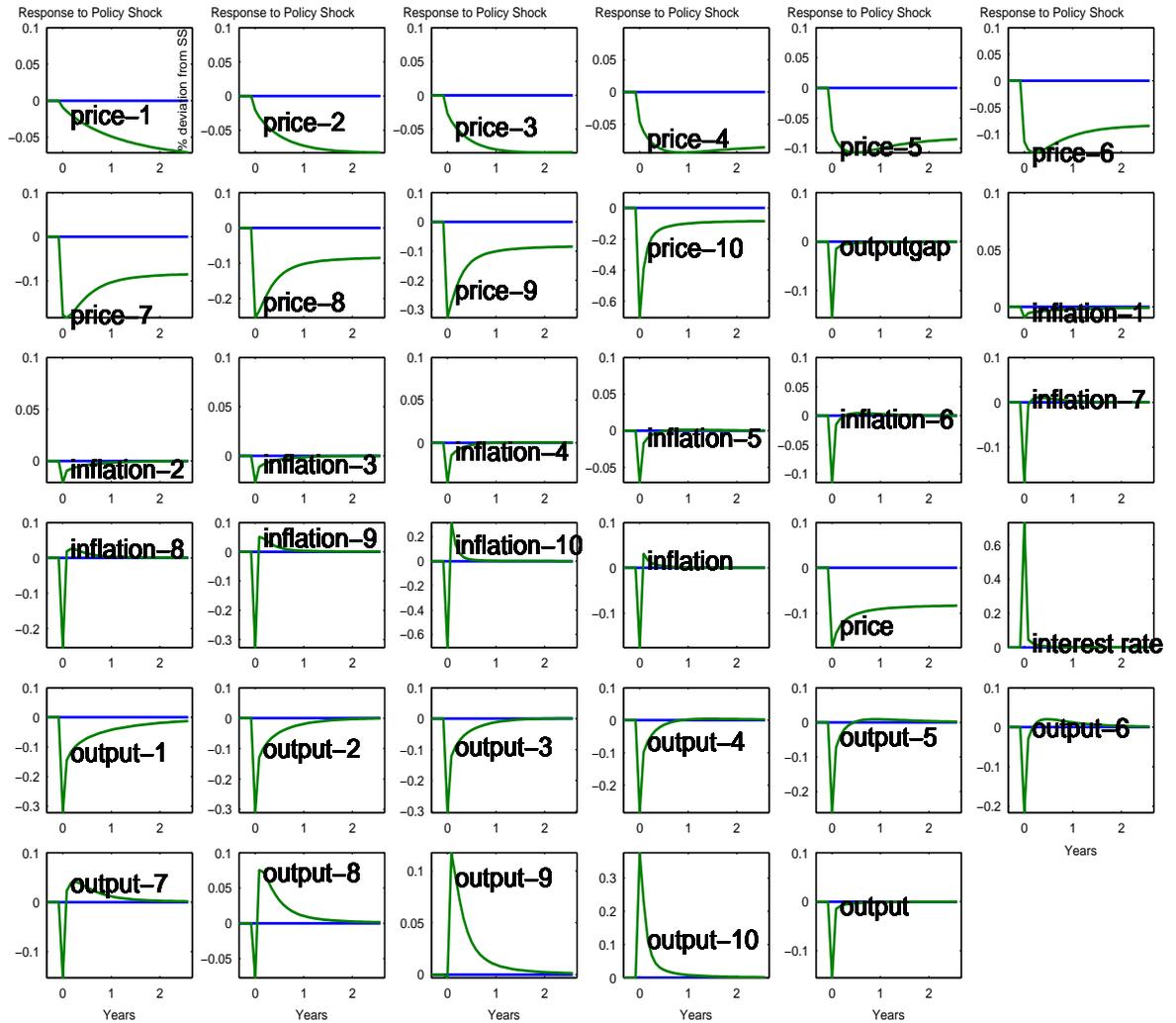

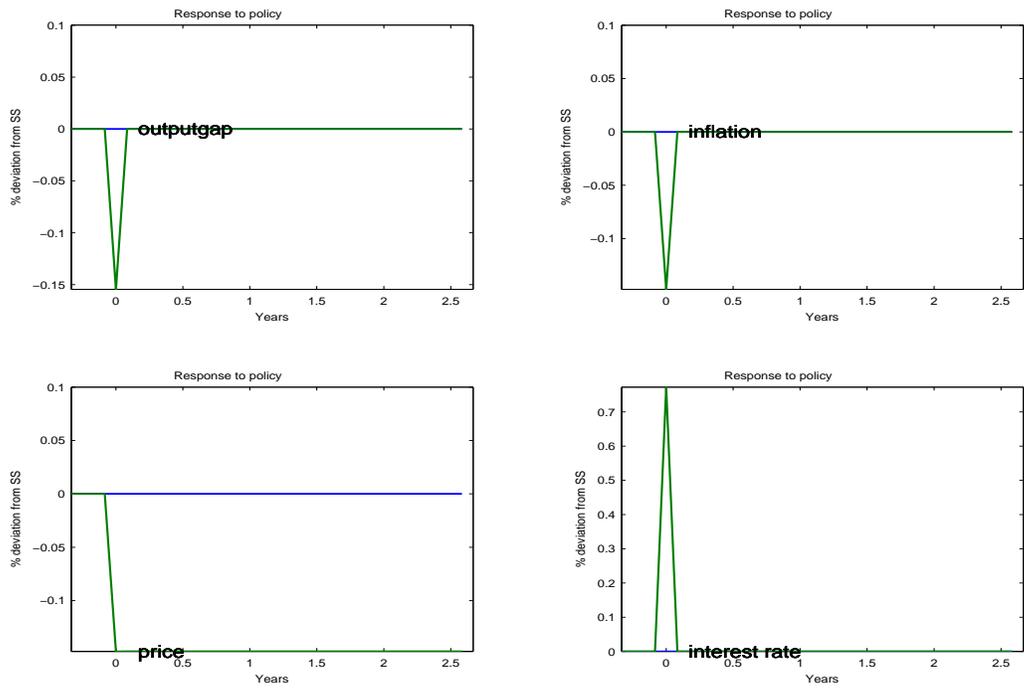Figure C.1: Impulse Responses to a Shock to Policy Rule, $\sigma = 5$ and $\phi = 3$.

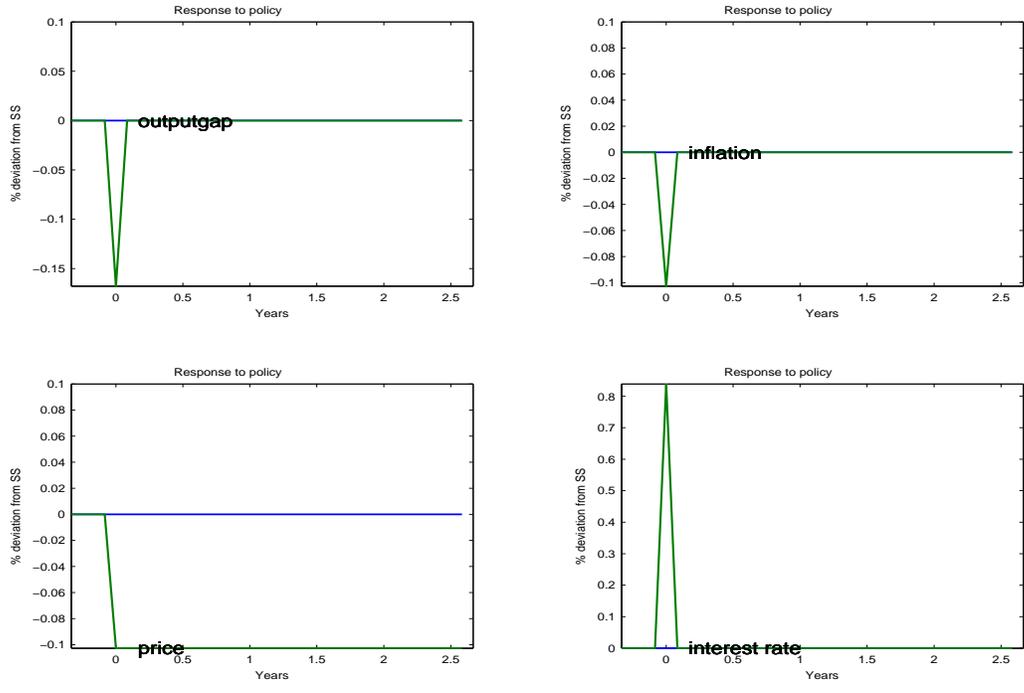Figure C.2: Impulse Responses to a Shock to Policy Rule, $\alpha = 0.71$; $\sigma = 5$, $\phi = 3$.

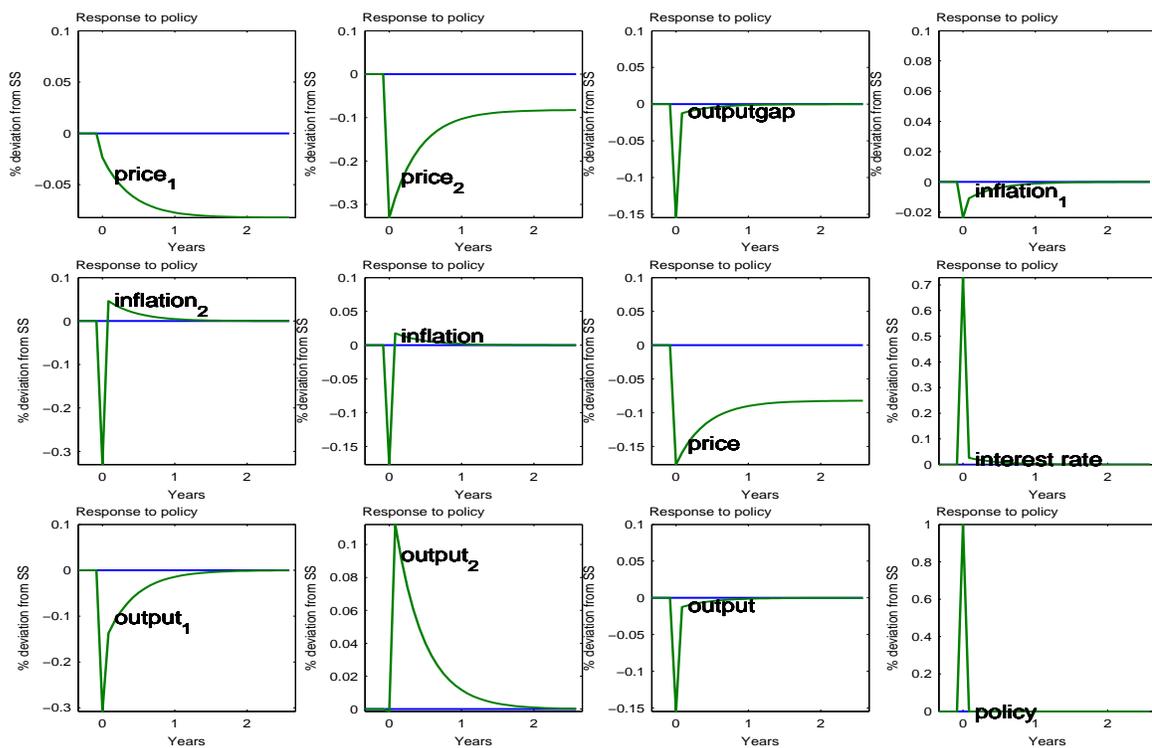Figure C.3: Impulse Responses to a Shock to Policy Rule, $\alpha = 0.76$; $\sigma = 5$, $\phi = 3$.

Figure C.4: Impulse Responses to a Shock to Policy Rule, $\alpha_1 = 0.91$ and $\alpha_2 = 0.56$; $\sigma = 5$ and $\phi = 3$

C.2   Sensitivity Analysis-Extended Model

Here, we present the most representative results (in terms of volatility) of the sensitivity analysis for the extended model.

$\sigma = 2$ $\eta = 1$ $\phi = 0.6$				
	multi-sector	$\alpha = 0.82$	$\alpha = 0.71$	$\alpha_1 = 0.56\ \alpha_2 = 0.91$
output gap	0.5941	0.5715	0.4938	0.6126
interest rate	0.9430	0.9700	0.9598	0.9484
inflation	0.1189	0.0878	0.1297	0.1206
price	0.7997	0.6790	0.9945	0.7885
output	0.7312	0.7747	0.8989	0.7143
$\sigma = 5$ $\eta = 1$ $\phi = 0.6$				
	multi-sector	$\alpha = 0.82$	$\alpha = 0.71$	$\alpha_1 = 0.56\ \alpha_2 = 0.91$
output gap	0.2828	0.2588	0.2082	0.2958
interest rate	0.9668	0.9858	0.9814	0.9718
inflation	0.1433	0.1038	0.1511	0.1456
price	0.9791	0.8041	1.1652	0.9590
output	0.2939	0.3214	0.3931	0.2832
$\sigma = 5$ $\eta = 2$ $\phi = 3$				
	multi-sector	$\alpha = 0.82$	$\alpha = 0.71$	$\alpha = 0.56\ \alpha = 0.91$
output gap	0.5797	0.5077	0.3620	0.5842
interest rate	0.9851	0.9910	1.0120	0.9860
inflation	0.1059	0.0916	0.1780	0.1045
price	0.7042	0.7111	1.3809	0.6615
output	0.2824	0.3403	0.4824	0.2789

Table C.2: Comparison of standard deviations of multi-sector and one and two sector models based on HP-filtered frequency domain calculations of moments.

Declaration of Authorship

I hereby state that I have written this paper on my own and with full acknowledgement of the used resources.

Aysegül Sanli,

Berlin, 9th August 2005.