Reconsidering the Welfare Costs of Business Cycles: Taking Labor Markets Seriously

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presented by
Silke Simon
173156

Examiner:
Prof. Harald Uhlig, PhD

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Abstract

The question whether or not business cycle fluctuations lead to welfare losses is of high political relevance since it implies important recommendations for the course of economic policy between interventionism and laissez-faire. In contrast to several other works, I show that in a setting with incomplete labor markets due to search externalities, the welfare costs of business cycles are substantial. Agents are willing to give up as much as 13 to 23% of consumption in order to live in an environment without fluctuations instead of living in an economy with cycles. Thus, high welfare gains can be generated by counter-cyclical economic policy measures.

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1 Introduction

When reading a newspaper article or listening to politicians who discuss the current slowdown in economic growth in Germany, one notes a strong public concern with short-term economic fluctuations. Even though the slowdown is not extraordinarily long or intensive and can well be interpreted as the recession part of a usual business cycle, measures and political intervention are called for in order to mitigate the temporarily low growth rates which are obviously perceived as detrimental to welfare.

In contrast to this public notion, Lucas (1987) has shown that the welfare gains from eliminating short-term business cycle fluctuations are very small and therefore negligible. He compares the utility of a representative, infinitely-lived agent in an environment with consumption fluctuations to his utility in an economy where the consumption path follows a constant trend. Lucas shows that the welfare gains from moving from the stochastic to the deterministic consumption path are as small as 0.01% of average consumption. An increase in the long-term growth rate, on the other hand, results in much higher welfare gains. Lucas thus comes to the conclusion that economic research and policy should focus on enhancing long-term growth instead of dealing with short-term fluctuations.
While the importance of long-term growth is widely uncontested, the idea of business cycle fluctuations being of little economic relevance has been scientifically dissented. Lucas' results have been challenged in a number of works by essentially questioning two of his assumptions: the assumption on risk-aversion and the one on complete markets.

In my analysis, I will pursue the second path and consider welfare in an economy with incomplete labor markets. In particular, in my model search costs arise on the labor market: matching between an unemployed worker and an open vacancy does not take place instantaneously, but it takes time and is costly for both sides. A worker who is searching for a job has to spend time and money on finding suitable offers and to apply; a firm that wants to fill a vacancy has to invest in selecting a candidate. These costs cannot be insured for so that they lead to welfare losses. Since a certain fraction of jobs are dissolved each period, search occurs in any economic environment, in an economy with business cycles fluctuations as well as in an economy without fluctuations so that also search costs and welfare losses arise in any setting. In an economy with uncertainty due to fluctuations, however, employment fluctuates as well, leading to higher costs and higher welfare losses than in an economy that remains on its trend growth path. It turns out that the welfare effects of search frictions are substantial: I calculate welfare gains from eliminating business cycles of 13 to 23% of average consumption; orders of magnitude larger than the results Lucas and most other researchers obtain. Partly, these astonishingly high gains might be due to the fact that I do not account for general equilibrium effects on the returns on the factors of production. This should be done in subsequent research.

My analysis is organized as follows: In section 2, I provide an overview over the literature on the question of welfare costs of business cycles, of course starting at Lucas' influential monograph of 1987 and then presenting subsequent works. Section 3 describes the model I am working with and which I analyze using the value function approach. This yields a number of solutions to the social planner's problem out of which I choose the feasible ones and carry out a welfare analysis on them. Section 5 provides the results of this analysis. In section 6, I discuss these results; section 7 concludes.

2 Literature

In his seminal work, Lucas (1987) challenges the general notion that business cycle fluctuations lead to substantial welfare losses. Comparing the utility of a representative, infinitely-lived agent in an environment with consumption fluctuations to his utility in an economy where the consumption path follows a constant trend, Lucas shows that the welfare gains from moving from the stochastic to the deterministic consumption path are small. He defines these gains as the percentage increase in average consumption in the stochastic economy that would be necessary to make the agent indifferent between the stochastic and the deterministic consumption path. This compensation turns out to be as small as 0.01% of average consumption in Lucas' calculations which implies that consumption fluctuations have a negligible impact on welfare.
This conclusion has caused contradiction and a number of works have been carried out in order to prove Lucas wrong[1]. To this end, mainly two of his assumptions have been put into question: those on individual risk aversion and on the amount of risk present in an economy.

The most obvious way to challenge Lucas' results is to modify his assumptions on individual preferences and to vary the parameter of risk aversion. In his model, Lucas assumes log utility from consumption. This corresponds to a constant relative risk aversion of unity. It has been shown, though, that risk aversion realistically takes much higher values, especially when the parameter of risk aversion is calibrated to match asset market moments. Tallarini (1996) for example sets the parameter to over 45 in order to replicate business cycle data and asset pricing data in the same model. Pursuing Lucas' calculations then leads to much higher welfare losses of more that 10% of average consumption due to business cycle fluctuations. Alvarez and Jermann (2004), however, show that Tallarini's estimate overstates the true impact of business cycles on welfare. They explain that most economic risk does not occur at business cycle frequencies. Despite high risk

[1] An overview over subsequent literature can be found in Lucas (2003).

aversion, they conclude that reducing risk by eliminating business cycle fluctuations does not increase welfare substantially as the main sources of risk are not affected.

The other debatable assumption Lucas makes concerns the market structure and the resulting amount of risk that can be removed by eliminating business cycles. In Lucas' framework with a representative agent and complete markets, all individual risk can be insured for and the only economic risk present is aggregate risk. Removing it results in small welfare gains. In an environment with imperfect markets, on the other hand, risk sharing is incomplete so that individual risk cannot be diversified away completely. In that case, the welfare gains from eliminating risk in the form of business cycles will be larger than in Lucas' setup as the amount of removable variance increases.

The idea of incomplete markets has been implemented in various ways, either focusing on imperfections on the labor market or on the asset market. The latter implies restricted possibilities to insure against risk. Imrohoroglu (1989) for example assumes that the possibility to insure against consumption fluctuations is restricted to a storage technology, i.e. there is no risk sharing. She then comes up with a loss due to business cycle fluctuations of about 0.3% of average consumption, order of magnitude lager than Lucas' estimate. In a similar model, Atkeson and Phelan (1994) include the asset market so that they can analyze the impact of reduced risk on market clearing asset prices. They show that the main welfare gain of eliminating consumption risk stems from the fact that asset prices change, although this gain is close to zero.

In addition to incomplete insurance markets, Storesletten et al (2001) consider heterogeneous agents with idiosyncratic labor market risk. This setup enables them to also evaluate distributional effects on welfare costs. Consumer heterogeneity is introduced by designing an overlapping generations model where young individuals face higher labor market risk than older workers. Krusell and Smith (2002) use a similar approach although they do not condition idiosyncratic risk on age, but assume uncertainty in individual preferences. Both papers come to the conclusion that in the absence of markets to insure against these idiosyncratic risks, welfare losses due to business cycles increase to about 0.1% of average consumption; an effect again much larger than the one Lucas calculates. Furthermore, the impact of fluctuations on welfare varies substantially between the different groups of consumers.

While all these models focus on incomplete markets for insurance, Beaudry and Pages (2001) consider frictions on the labor market and assume labor contracts to be incomplete due to contractual enforcement problems. Workers that become unemployed therefore face costs to find a new job and close a new contract. In this setup, undiversifiable risk exists in the form of the risk of becoming unemployed and then having to find a new job on the imperfect labor market, although insurance against consumption fluctuations is possible. Under these assumptions, the costs of business cycle fluctuations again increase substantially compared to Lucas' calculations and amount to up to 4.4% of average consumption.

In my analysis, I use a similar approach and consider a model by Merz (1995) that includes search externalities on the labor market so that matching a worker and a firm is costly. In contrast to Beaudry and Pages, however, in Merz' setup these costs do not stem from incomplete labor contracts, but are simply included in the form of search costs that arise for both firms and workers: a firm that is trying to fill a vacancy has to post the offer and invest time into recruiting and evaluating applicants while an unemployed worker who is looking for a job has to invest time and money into finding suitable offers and applying for them. As before, there exists an insurance against risk of consumption fluctuations[2], but the risk of becoming unemployed and facing search costs on the labor market cannot be diversified away. As this risk is affected by removing business cycle fluctuations, larger welfare gains result.

[2]Merz defines each household as "a very large extended family" with an infinite number of members where "members in each family insure each other against variations in labor income". (p. 272)

3 Model Setup

My welfare analysis is based on an RBC model defined by Merz (1995) with perfect markets for goods and capital, but an imperfect market for labor. On the labor market, search costs arise: firms that try to fill a vacancy and workers who are looking for a job do not match instantaneously, but matching takes time and is costly for both sides. For each trader on the labor market, search costs increase in the number of traders on the same side of the market and decrease in the number of traders on the opposite side; i.e. search costs of a firm increase the more firms are looking for new workers and decrease the more workers search for a new job, and vice versa. As a certain fraction of job matches is dissolved each period, search takes place all the time, but it is further intensified if the demand for labor varies. Eliminating business cycles as the driving force of variations in labor demand thus decreases search and search costs, resulting in welfare gains.

The model economy is inhabited by infinitely-lived agents that are endowed with capital K_t and time that they use for labor N_t and leisure. The labor force is constant and normalized to one. The representative agent maximizes his expected utility which is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log c_t - \frac{N_t^{1-\frac{1}{\nu}}}{1-\frac{1}{\nu}} \right] \tag{1}$$

where ν denotes the elasticity of intertemporal substitution of labor.

Output Y_t is produced using a Cobb-Douglas technology that takes capital and labor as inputs and is subject to an exogenous shock z_t

$$Y_t = \exp\left[(1-\alpha)(\mu t + z_t)\right] K_t^{\alpha} N_t^{1-\alpha} \tag{2}$$

where z_t follows an AR(1) process of the form

$$z_{t+1} = \rho z_t + \varepsilon_{t+1} \quad \varepsilon_{t+1} \sim iidN(0, \sigma_\varepsilon) \tag{3}$$

The parameter μ denotes the constant growth rate in the economy, i.e. the model exhibits balanced growth. Search intensity, vacancies, search costs, employment and unemployment of course are stationary and have to be trended in order to fit into the model.

Output can either be consumed, invested, or spent on search activity in the labor market so that the budget constraint of the economy reads

$$C_t + I_t + \exp(\mu t) A_t < Y_t \tag{4}$$

where A_t stands for search costs.

The capital stock of the economy evolves according to the usual law of motion

$$K_{t+1} = (1 - \delta) K_t + I_t \tag{5}$$

where δ is the depreciation rate of capital per period.

Search costs can be split into costs on the supply side and on the demand side of the labor market. On the supply side, a cost per unemployed worker of $c(S_t) = c_0 S_t^\eta$ arises, where S_t denotes search intensity, $c_0 > 0$, $\eta \geq 1$. On the demand side, search intensity is represented by the number of vacancies posted V_t that come at a constant advertising cost a. Search costs thus amount to

$$A_t = c(S_t)(1 - N_t) + aV_t \tag{6}$$

Search intensity and vacancies posted generate job matches M_t in the economy according to a Cobb-Douglas production function:

$$M_t = V_t^{1-\lambda} \left[S_t (1 - N_t) \right]^\lambda \tag{7}$$

Assuming an exogenous rate Ψ of dissolved job matches per period, employment then follows

$$N_{t+1} = (1 - \Psi) N_t + M_t \tag{8}$$

4 Model Analysis

The model is analyzed using a value function approach, similar to the solution strategy in Storesletten et al (2001). The analysis proceeds as follows: First the model is transformed in a suitable way and the value function is determined. The model is then calibrated for both the benchmark case with business cycles and the alternative case without fluctuations. Both models are solved numerically carrying out a value function iteration in a discretizated state space. Out of these solutions, the long-term solutions are determined and a welfare analysis is carried out on them, comparing welfare in the benchmark economy to welfare in the deterministic economy.

4.1 Value Function

Since the model exhibits balanced growth, all nonstationary variables of the model have to be detrended. These are K_t, C_t, I_t and Y_t. They are transformed into their detrended counterparts (denoted by lower case letters) by defining

$$k_t = \frac{K_t}{\exp(\mu t)} \tag{9}$$

$$c_t = \frac{C_t}{\exp(\mu t)} \tag{10}$$

$$i_t = \frac{I_t}{\exp(\mu t)} \tag{11}$$

$$y_t = \frac{Y_t}{\exp(\mu t)} \tag{12}$$

Moreover, the stochastic AR(1) process for the technology shock has to be transformed into an invariant process since the value function iteration implies calculating the utility for each possible combination of the state variables. In order to keep these values comparable to each other, they may not depend on a realization of the autoregressive process for the technology shock as it would differ from calculation to calculation. Therefore, the AR(1) process is approximated by a finite-state Markov chain. Tauchen (1986) explains such an approximation and provides specifications for the state variables and the transition matrix[3]. The initial AR(1) process can thus be approximated by a three-state Markov chain with $z_t \epsilon$ $-0.0224, 0, 0.0224\}$ and the transition matrix

$$P = \begin{pmatrix} 0.925 & 0.0075 & 0 \\ 0.0548 & 0.8904 & 0.0548 \\ 0.925 & 0.0075 & 0 \end{pmatrix} \qquad (13)$$

For each combination of states today, this leads to three optimal choices the next period: one given a low state of technology today, one given an average and one given a high state today.

In the resulting stationary economy, the choice problem of the agent can be expressed by the value function

$$V(k, N, z) = \max_{S,V,c} \left[u(c, N) + \beta E \left[V(k', N') \right] \right] \qquad (14)$$

subject to

$$\begin{aligned} k' &= (1-\delta)k + i && (15) \\ N' &= (1-\Psi)N + V^{1-\lambda} \left[S(1-N) \right]^{\lambda} && (16) \\ c &= y - i - A && (17) \end{aligned}$$

given that the aggregate resource constraint is binding.[4]

The equilibrium is then characterized by the policy functions mapping the state variables k and N into the control variables S, V and c and the laws of motion for the two state variables (16) and (17).

[3]see Appendix for details

[4]Variables with a prime denote values in the next period, variables without a prime values in the current period.

The policy functions for S and V are determined by the cost minimization problem of the firm and the worker respectively: search costs are to be minimized subject to the matching function.

$$\begin{aligned} \min_{S,V} A &= c(S)(1-N) + aV & (18) \\ \text{s.t. } V^{1-\lambda}\left[S(1-N)\right]^{\lambda} &= N' - (1-\Psi)N & (19) \end{aligned}$$

This yields the following functions:

$$\begin{aligned} S^* &= \left[\left(\frac{1-\lambda}{\lambda a}\eta c_o\right)^{\frac{1}{1-\lambda}} \frac{1}{1-N}(N' - (1-\Psi)N) \right]^{\frac{1}{\eta(1-\lambda)+\lambda}} & (20) \\ V^* &= \frac{1-\lambda}{\lambda a}\eta c_0 S^{\eta}(1-N) & (21) \end{aligned}$$

Computational details can be found in the Appendix.

The policy function for c is given by the aggregate resource constraint (18) where $A = A(S^*, V^*)$.

4.2 Calibration

For the calibration of the model parameters, I follow Merz in order to obtain results comparable to hers. To a large extend, the parameter values she proposes are rather standard. The not-so-standard parameters of the matching function and the search cost functions are determined such that the model matches the empirical findings on the first moments of certain variables.

The capital share α is set to a usual value of 0.36[5]. The depreciation rate of capital δ is assumed to be equal 0.022, the discount factor β equal 0.99[6]. For the elasticity of intertemporal substitution of labor ν, Merz chooses values between -0.5 and -1.25 which

[5]Note, though, that due to the search friction on the labor market, $(1-\alpha)$ does not denote the labor share of income, but the sum of the labor share and the return to investing in job search. (Merz p.277)

[6]This corresponds considering a period as a quarter

are all consistent with empirical findings. I use $\nu = -1.25$ because Merz reports her results for this value.

On the labor market, the transition rate from employment to unemployment Ψ is defined as the ratio of the unemployment rate to the employment rate. Merz calculates this ratio based on US data from 1959 to 1988 and obtains a value of 0.07. Job matching is determined by the matching elasticities with respect to vacancies and unemployment λ and $(1-\lambda)$. An empirical study for the US on the matching technology by Blachard and Diamond (1989) finds values of 0.4 and 0.6 respectively for these elasticities, i.e. λ is equal to 0.4.

The parameters of the search cost function are chosen such that the first moments of certain labor market variables in the model mimic their empirical counterparts. A per unit advertising cost for the firms of $a = 0.05$ leads to a rate of vacancy duration that corresponds to its empirical value as reported in Ours and Ridder (1992) for the Netherlands. The search cost function of the workers is specified as to match the average unemployment rate and unemployment duration. This implies a level of search cost for the workers of $c_0 = 0.005$ and a degree of convexity η of unity, i.e. a linear cost function.

Merz sets the persistence parameter of the technology shock ρ to 0.95 and the standard deviation of the white noise σ_ϵ to 0.007 which corresponds to the values used in Hansen (1985). For the analysis of the model without business cycle fluctuations, I set the technology shock to its unconditional mean $\bar{z} = 0$.

4.3 Numerical Solution Strategy

The equilibrium solution of the social planner's problem is computed numerically by iterating on the value function in a discrete-state space, i.e. where the possible values of state variables are restricted to a finite number within certain intervals. These intervals can for the most part be derived from the initial assumptions on the state variables. Since total time endowment is normalized to unity, N can take values in the interval [0, 1]. Due to computational restrictions (division by $1-N$), the maximum value of N is set to 0.99.

For k, the minimum value has to be zero as there is no borrowing allowed in the model economy. The maximum value of k is arbitrarily set to 100. The intervals are subdivided in 50 steps each, leading to a step length of 0.0198 for N and 2 for k and 51 possible realizations for each state variable.
As the number of realizations of the state variables is finite, the number of possible combinations of N and k (and N' and k' respectively) is finite as well. Given these possible combinations, the process of value function iteration picks the equilibrium combination $\{N', k'\}$ for each pair $\{N, k\}$ and for each level of technology (low, average and high).

The deterministic economy without business cycles is designed by setting the technology shock z to its unconditional mean $\bar{z} = 0$. In the long-term perspective of the value function approach that maximizes utility over an infinite time horizon, an economy without uncertainty will converge to its steady state. Therefore, the possible steady states of the deterministic economy are determined by selecting the combinations where the values for capital and employment stay constant over time, i.e. where $k' = k$ and $N' = N$. The utility in the deterministic economy at the steady states is then compared to the utility in the stochastic economy for the same levels of employment and capital.

The welfare effect from eliminating business cycles can be determined as the fraction by which consumption in the economy without fluctuations has to be changed in order to make an agent indifferent between living in one or the other economy. Lucas (2003) defines it as τ in

$$u[(1-\tau)c_A, N_A] = u[c_B, N_B] \tag{22}$$

where A denotes values in the deterministic economy and B values in the stochastic economy. The welfare effect can thus be considered as a kind of consumption tax that the agent in the stochastic economy would be willing to pay in order to live in the deterministic economy.

The MATLAB code that determines the equilibrium solutions for the stochastic model and for the deterministic model and calculates the welfare effects of moving from one economy to the other is provided in the Appendix[7].

[7] The code is based on a program for value function interation kindly provided by Alexander Kriwoluzky.

5 Model Results and Answer

The analysis yields a number of standard results: in the stochastic economy, agent smooth consumption so that it is rather stable in all states of the economy while the optimal level of the capital stock the next period fluctuates substantially. Other features of the results are less expected but can be explained by the specific structure of the problem.
In the following, the main results of the analyses of the stochastic economy and the deterministic economy are presented and compared. For the stochastic economy, the outcomes for an average level of technology ($z = 0$) are reported since they correspond to the values of the deterministic economy. Moreover, the results of the welfare analysis are given and interpreted.

5.1 Stochastic vs. Deterministic Economy

Figures 1 and 2 show the value of the value function for the stochastic and the deterministic economy respectively. It is invariant in employment, but depends positively on the initial level of capital. This is due to the fundamental difference between capital and labor: capital has long-term effects on the economy as it is partly transferred into the next period while labor is only effective in the one period it is supplied. In the long-run perspective of the value function which represents a maximization problem over an infinite time horizon, the short-term effect of employment becomes negligible, while the level of capital turns out to be the main choice parameter due to its long-term impact.
As expected, the value function takes much higher values in the deterministic economy than in the stochastic economy. For risk-averse agents, the absence of uncertainty increases utility substantially.

A similar structure can be observed for the optimal choice of the capital stock the next period. Like the value function, it only depends on the level of capital in the current period. Figure 3 displays it for the stochastic economy; the picture for the deterministic economy looks almost identical. A comparison between the two economies shows that in

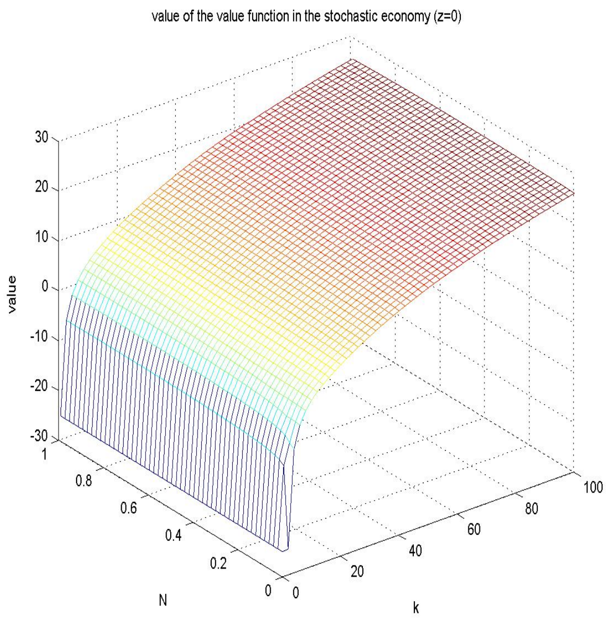

Figure 1: Value function (stochastic economy)

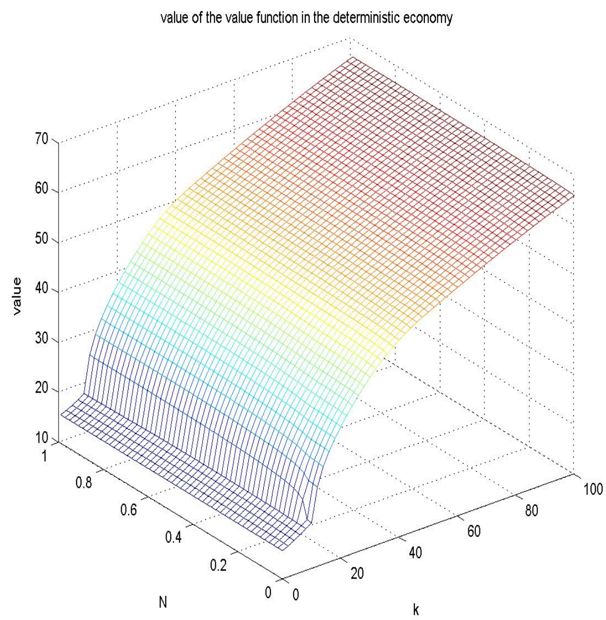

Figure 2: Value function (deterministic economy)

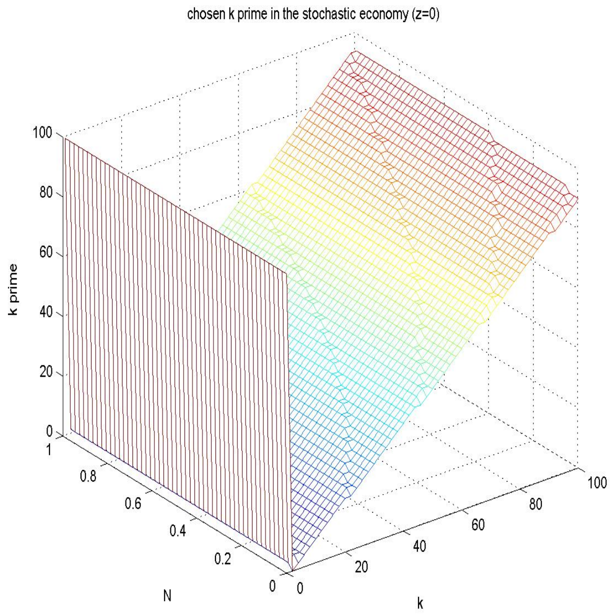

Figure 3: Optimal capital stock in the next period (stochastic economy)

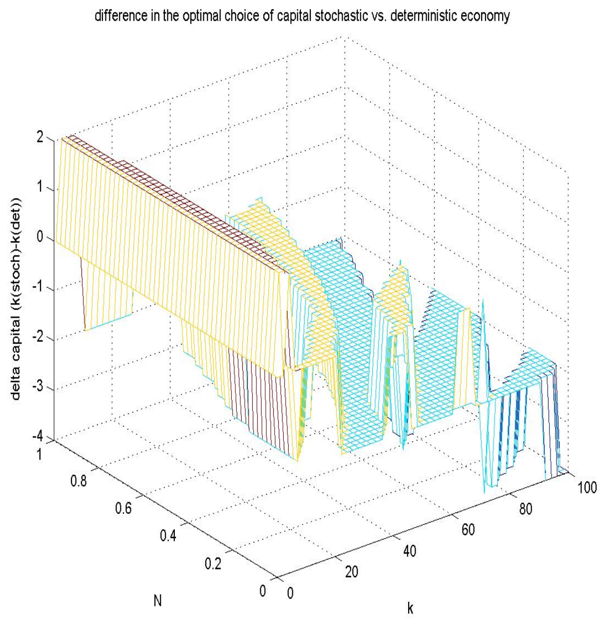

Figure 4: Difference in optimal capital stock (stochastic vs. deterministic economy)

the deterministic economy, the optimal stock of capital in the next period is higher than in the stochastic economy for many initial combinations of N and k (Fig. 4). This does not imply that capital accumulation is higher in the deterministic economy. Instead, it depicts the fact that in the deterministic setting, there exists a certain (steady state) level of capital which, once attained, is optimal for all future periods. If the current level of capital differs from this optimal capital stock, long-term utility is maximized by investing the amount of capital necessary to reach this steady state level at once and then keeping that level of capital for ever.
The very high values of chosen capital for a capital stock of about zero in the previous period occur due to the features of the value function approach: for a low capital stock, output is low as well. For most values of k', this implies negative consumption. This case is ruled out by artificially setting consumption to zero so that the costs of a high capital stock in the future are not reflected. Then, of course, the highest possible value of k' is chosen.

The optimal level of employment the next period (Fig. 5 for the stochastic economy), on the other hand, depends on both the level of the capital stock and the level of employment (at least for higher values of employment). This structure stems from the presence of search costs. Fig. 6 shows that the optimal level of employment in an economy without search cost is invariant in the current stock of employment: Since creating employment is costless, the optimal level of employment in the next period, which only depends on the stock of capital, can be reached each period at no costs no matter what employment was before. If creating employment is costly due to search costs, the level of employment has to be chosen such as to minimize long-term search costs. If employment is high, it is then optimal to maintain this high level of employment the next period, even though the optimal level that results from maximizing utility would be lower. In that way, costs of creating employment in a future period are avoided.

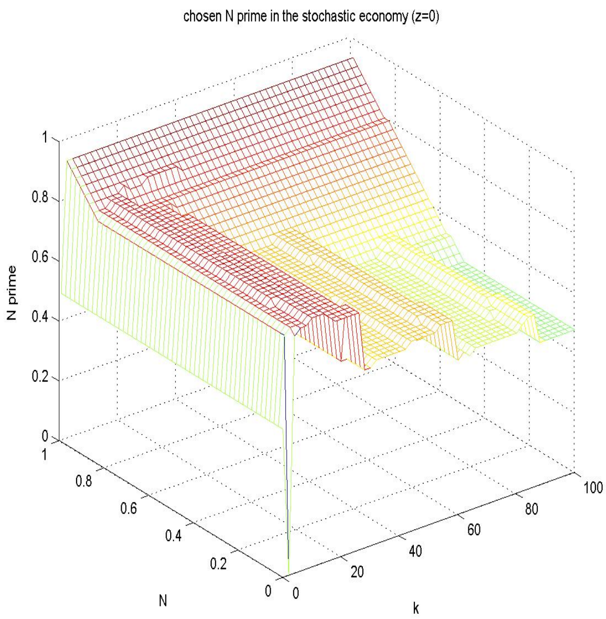

Figure 5: Optimal employment in the next period (stochastic economy)

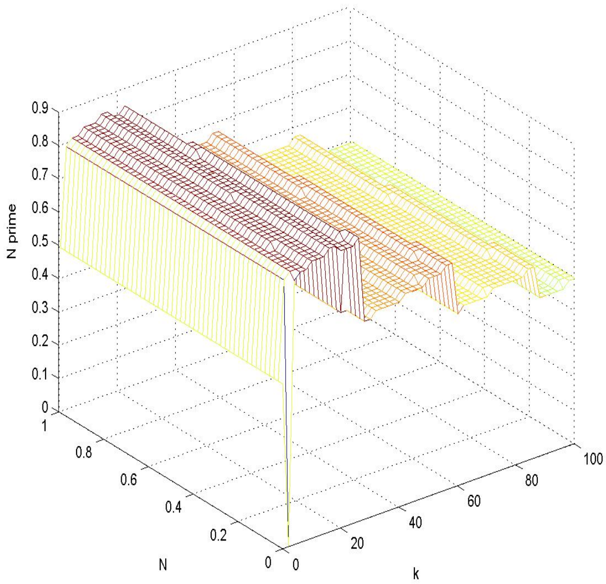

Figure 6: Optimal employment without search costs (deterministic economy)

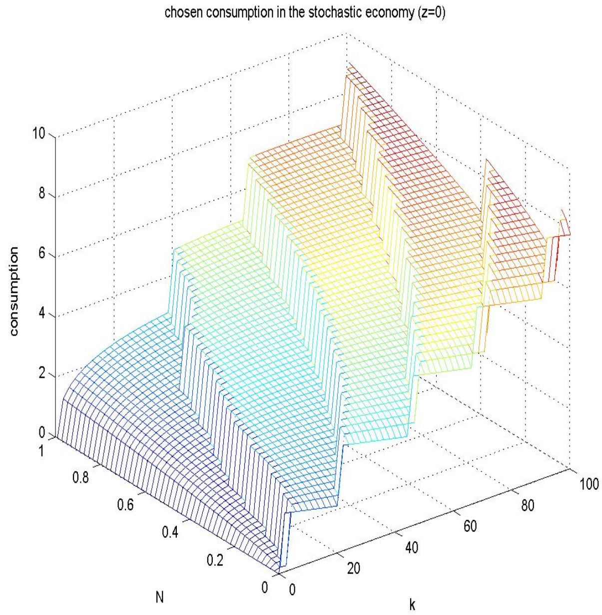

Figure 7: Consumption (stochastic economy)

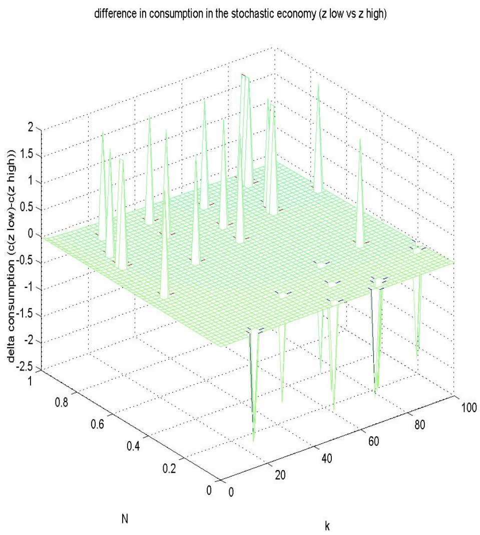

Figure 8: Difference in consumption in the stochastic economy (z low vs. z high)

Figure 7 gives the level of consumption that solves the value function for all combinations of the state variables. It is increasing in employment N, since higher employment increases output and decreases search costs. The relation between consumption and capital is more complex: On the one hand, more capital increases consumption as it leads to higher output. On the other hand, a high level of the capital stock implies a high optimal level of capital the next period (Fig. 3) which has to be financed by reducing consumption. Thus, the impact of the capital stock on the optimal level of consumption is ambiguous.

For different states of the stochastic economy, consumption is rather stable (Fig. 8). As expected from economic theory, agents smooth their consumption over the different states of the economy. The peaks, that indicate a strong difference in consumption between the two states of the economy, occur at the steps of consumption (Fig. 7) since these are slightly different from one state to the other.

5.2 Welfare Analysis

The equilibrium sequence in the deterministic economy contains 10 combinations $\{N, k, N', k'\}$ where employment and capital in the next period take the same values as in the proceeding period (Table 1). These combinations are candidates for steady states.[8] Since an economy without uncertainty will end up in its steady state sooner or later, these are the only possible solutions to the deterministic maximization problem in the long-term horizon of the value function. This is why in the following, only these combinations will be further analyzed.

[8]The existence of multiple steady states is due to the errors imposed by the discretionized state space. With the restricted information drawn from the value function iteration process, however, none of the candidate combinations can be ruled out a priori.

Table 1: Steady states in the deterministic economy

	employment N	capital k
1	0.8910	28
2	0.8910	30
3	0.8910	32
4	0.8910	34
5	0.9108	36
6	0.9108	38
7	0.9108	40
8	0.9108	42
9	0.9306	44
10	0.9306	46

For each of these pairs, one can determine the optimal choice of capital and employment the next period in the stochastic economy. Of course, in the stochastic economy, these optimal levels of capital and labor are not constant. Over a longer time horizon, though, the optimal levels of capital and employment converge to constant values within few periods in a setting corresponding to the deterministic environment (i.e. for a constant level of technology $z = 0$). In particular, for any initial combination of employment and capital given by the steady states of the deterministic economy, the values they converge to in the stochastic economy are the same: $k = 22$ and $N = 0.792$ (Fig. 9 and 10).

Since my objective is to determine welfare effects in a setting with infinite-time horizon, the situations that have to be compared are the potential long-term solutions of the deterministic economy (the possible steady states) and the corresponding long-term solutions of the stochastic economy, given the same level of technology.

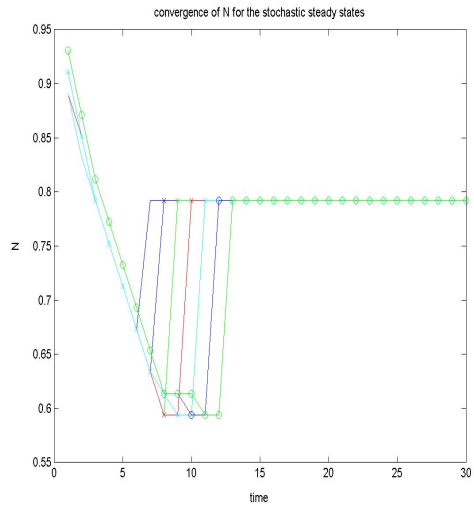

Figure 9: Convergence of employment (stochastic economy)

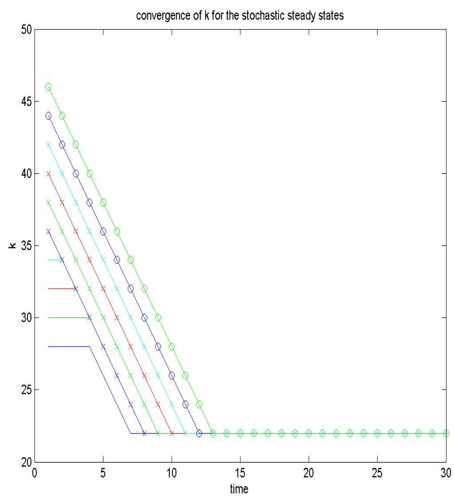

Figure 10: Convergence of capital (stochastic economy)

The welfare analysis is carried out by comparing utility in the deterministic economy to utility in the stochastic economy. In order to quantify the welfare effect of eliminating business cycles, the fraction of consumption τ is calculated that an agent in the stochastic economy would be willing to give up for living in an environment without uncertainty instead. The results for the 10 states of interest are given in Table 2.

Table 2: Welfare gains (stochastic vs. deterministic economy)

states in the deterministic economy	states in the stochastic economy	τ
0.8910; 28	0.7920; 28	0.1336
0.8910; 30	0.7920; 30	0.1452
0.8910; 32	0.7920; 32	0.1555
0.8910; 34	0.7920; 32	0.1645
0.9108; 36	0.7920; 32	0.1875
0.9108; 38	0.7920; 34	0.1946
0.9108; 40	0.7920; 36	0.2010
0.9108; 42	0.7920; 38	0.2066
0.9306; 44	0.8118; 40	0.2262
0.9306; 46	0.8118; 42	0.2307

Agents are willing to reduce consumption by as much as 13 to 23% of average consumption in order to move from a stochastic environment to an economy without uncertainty, i.e. without business cycle fluctuations. These results are orders of magnitude higher that what Lucas found and also much larger than what most other works calculated.

6 Discussion

My analysis shows that, when taking into account search frictions on the labor market, the welfare gains from eliminating business cycles increase substantially compared to a setting with complete markets. The welfare gains derived are surprisingly high compared to the results most other researchers obtain. Two main reasons account for these high values:

First, the search frictions on the labor market lead to costs and thus to welfare losses whenever employment is created. These losses occur in the an uncertain setting with business cycle fluctuations as well as in a deterministic environment as a certain number of job matches are dissolved and have to be recreated each period. In an economy with business cycles, however, fluctuations in employment are higher than in the deterministic case so that higher search costs arise and welfare losses increase. Still, the absolute amount of the increase is astonishingly high. Since search costs are calibrated to rather low values in the model, it seems implausible that they lead to welfare losses in the stochastic economy of around 20% of average consumption.

Second, my analysis suffers from a flaw that partly explains these high differences in welfare. I implicitly assume that the deterministic and the stochastic economies are identical except for their levels of capital and employment and thus their level of consumption. But this is not the case. Due to the difference in capital and employment, the returns on these factors of production differ as well. Since both the optimal stock of capital and the optimal level of employment are smaller in the stochastic economy, their marginal productivity and thus the returns are higher than in the deterministic case. The comparison between utility in the stochastic and the deterministic case is biased due to the difference in the return on capital and labor. In order to capture the pure welfare gain from eliminating business cycles, these general equilibrium effects have to be accounted for by comparing the utility in the stochastic economy to the utility in the deterministic case while holding prices for capital and labor constant. This is a topic for further analysis.

Of course it is also possible that the specifications of the model or the solution strategy are not correct. This could be checked by examining a model without search costs with the same technique. If the model specifications are correct, this should yield welfare effects close to zero.

7 Conclusion

The question whether or not business cycles fluctuations cause welfare losses is politically highly relevant since it provides a motivation for many political measures. If business cycles are detrimental for an economy, economic policy should be counter-cyclical in order to eliminate or at least reduce cycles. If economic fluctuations do not have an effect on welfare, on the other hand, there is no need to intervene and a laissez-faire type of policy is optimal.

Lucas' answer to this question clearly supports the second point of view. According to him, the welfare gains from eliminating business cycles are negligible while the gains from increasing the long-term growth trend are substantial. His recommendation for economic policy is evident: political measures should aim at enhancing long-term growth instead of reducing short-term fluctuations. Other researchers have shown that in the presence of incomplete markets and undiversifyable risk, welfare losses due to business cycles are much higher than in Lucas' calculations. In my analysis, I come to the same result. I consider a model economy with frictions on the labor market due to search costs that arise for each worker who is looking for a new job and for each firm that is trying to fill a vacancy. These costs can be zero at best, but of course never turn negative, so that welfare is reduced whenever search takes place. Since less search occurs in an environment without economic fluctuations than in a setting with business cycles, eliminating cycles results in smaller losses due to search and thus in higher welfare. I show this by using the value function approach and determining the policy functions for the deterministic and the stochastic environment respectively. I then compare the utility of an representative agent in these economies for all possible long-run solutions and find that utility is much higher in the deterministic case than in the stochastic economy: Agents would give up between 13 and 23% of average consumption in order to live in an economy without business cycles instead of living in an environment with fluctuations.

Thus, significant welfare gains can be generated by reducing uncertainty in an economy in the form of business cycles fluctuations. The political implications of this result are the reverse of what Lucas recommends: now, policy measures should be undertaken whenever possible in order to reduce fluctuations. During a recession, the economy has to be supported; in a boom, it should be cooled down.

References

[1] Alvarez, F., U. J. Jermann, (2004), "Using asset prices to measure the costs of business cycles", *Journal of Political Economy* 112, 1223-1256.

[2] Atkeson A., C. Phelan, (1994), "Reconsidering the costs of business cycles with incomplete markets", NBER Working Paper No. 4719.

[3] Beaudry, P., C. Pages, (2001), "The cost of business cycles and the stabilization value of unemployment insurance", *European Economic Review* 45, 1545-1572.

[4] Blanchard, O.J., P.Diamond (1989), The Beveridge curve, NBER Working Paper No. R1405, 1-60.

[5] Hansen, G., (1985), "Indivisible labor and the business cycle", *Journl of Monetary Economics* 16, 309-327.

[6] Krusell, P., A.A. Smith, (2002), Revisiting the welfare effects of eliminating business cycles, Manuscript, Carnegie-Mellon University.

[7] Ljungqvist, L., T. Sargent, (2004), *Recursive Macroeconomic Theory*, Second edition, MIT Press.

[8] Lucas, R.E., (1987), *Models of Business Cycles*, Basil Blackwell.

[9] Lucas, R.E., (2003), "Macroeconomic priorities", *American Economic Review* 93, 1-14.

[10] Merz, M., (1995), "Search in the labor market and the real business cycle", *Journal of Monetary Economics* 36, 269-300.

[11] van Ours, J.C., G. Ridder (1992), "Vacancies and the recruitment of new employees", *Journal of Labor Economics* 10, 138-155.

[12] Storesletten, K., C. I. Telmer, A. Yaron, (2000), "The welfare cost of business cycles revisited: finite lives and cyclical variation in idiosyncratic risk", *European Economc Review* 45, 1311-1339.

[13] Tallarini, T.D., (2000), "Risk-sensitive real business cycles", *Journal of Monetary Economics* 45, 507-532.

[14] Tauchen, G., (1985), "Finite state Markov-chain approximations to univariate and vector autoregressions", *Economics Letters* 20, 177-181.

Appendix

A Calculations

A.1 Approximation of an AR(1) Process

Tauchen (1986) shows that a continuous autoregressive process can be approximated by a finite-state Markov chain. The approximation becomes arbitrarily close to the original process the finer the grid of state variables is defined. For my purpose, a three-state Markov chain approximates the underlying AR(1) process of the form $z_{t+1} = \rho z_t + \varepsilon_{t+1}$, $\varepsilon_{t+1} \sim$ i.i.d.$N(0, \sigma_\varepsilon)$ sufficiently well. Tauchen then proposes the following specifications for the state variables and the transition matrix:

The state variables can take three values z_1, z_2 and z_3, where z_3 is set to the unconditional standard deviation (eq. 2), and $z_1 = -z_3$. z_2 is equal zero. The elements of the transition matrix are defined as

$$
\begin{aligned}
p_{i1} &= Pr\left[z_{t+1} \leq -\frac{\sigma_z}{2} | z_t = z_i\right] && (23)\\
&= Pr\left[\varepsilon_{t+1} \leq -\rho z_i - \frac{\sigma_\varepsilon}{2}\right] && (24)\\
p_{i2} &= Pr\left[z_{t+1} \leq \frac{\sigma_z}{2} | z_t = z_i\right] - Pr\left[z_{t+1} < -\frac{\sigma_z}{2} | z_t = z_i\right] && (25)\\
&= Pr\left[\varepsilon_{t+1} \leq -\rho z_i + \frac{\sigma_\varepsilon}{2}\right] - Pr\left[\varepsilon_{t+1} < -\rho z_i - \frac{\sigma_\varepsilon}{2}\right] && (26)\\
p_{i3} &= Pr\left[z_{t+1} > \frac{\sigma_z}{2} | z_t = z_i\right] && (27)\\
&= Pr\left[\rho z_i + \varepsilon_{t+1} > -\rho z_i + \frac{\sigma_z}{2}\right] && (28)\\
& && (29)
\end{aligned}
$$

These probabilities can be determined using a cumulative density function with unit variance, e.g. the standard normal distribution.

The autoregressive process $z_{t+1} = 0.95z_t + \varepsilon_{t+1}$, $\varepsilon_{t+1} \sim$ i.i.d.$N(0, 0.007)$ in Merz' model can thus be approximated by a Markov-chain with the state space $z_i \epsilon\ -0.0224, 0, 0224\}$ and the transition matrix

$$P = \begin{pmatrix} 0.925 & 0.0075 & 0 \\ 0.0548 & 0.8904 & 0.0548 \\ 0.925 & 0.0075 & 0 \end{pmatrix} \tag{30}$$

A.2 Search Cost Minimization

The optimal search intensities for workers and firms respectively are determined by the following cost minimization problem:

$$\min_{S_t, V_t} A_t = c_0(S_t)^\eta (1 - N_t) a V_t \tag{31}$$

$$\text{s.t. } V_t^{1-\lambda} \left[S_t(1 - N_t)\right]^\lambda = N_{t+1} - (1 - \Psi) N_t \tag{32}$$

The first order conditions then read:

$$\frac{\partial L}{\partial S_t} = (1 - N_t)\eta c_0 S_t^{\eta-1} - \gamma_t(\lambda \frac{V_t^{1-\lambda}\left[S_t(1 - N_t)\right]^\lambda}{S_t}) = 0 \tag{33}$$

$$\frac{\partial L}{\partial V_t} = a - \gamma_t(1 - \lambda)\frac{V_t^{1-\lambda}\left[S_t(1 - N_t)\right]^\lambda}{V_t} = 0 \tag{34}$$

$$\frac{\partial L}{\partial \gamma_t} = V_t^{1-\lambda}\left[S_t(1 - N_t)\right]^\lambda - N_{t+1} + (1 - \Psi)N_t = 0 \tag{35}$$

Combining (32) and (33) and plugging the result into (34) yields after some algebraic manipulation

$$S_t^* = \left[\left(\frac{1-\lambda}{\lambda a}\eta c_o\right)^{\frac{1}{1-\lambda}} \frac{1}{1 - N_t}(N_{t+1} - (1 - \Psi)N_t)\right]^{\frac{1}{\eta(1-\lambda)+\lambda}} \tag{36}$$

$$V_t^* = \frac{1-\lambda}{\lambda a}\eta c_0 S_t^\eta (1 - N_t) \tag{37}$$

Since the function A(t) is concave for $\eta \geq 1$, determining second derivatives is not necessary: the point given by S_t^* and V_t^* must be a minimum.

B Matlab Code

```
% OPTIMIZATION PROBLEM FOR THE STOCHASTIC ECONOMY

clear all
% define state variable N
% max for N_bar
N_bar=.99;
% define N-grid
grid_N=[0:0.0198:N_bar]; N_size=size(grid_N,2);

% define state variable k
% max for k_bar
k_bar=100;
% define k-grid
grid_k=[0:2:k_bar]; k_size=size(grid_k,2);

% no of possible combinations of N and k:
N_k=N_size*k_size;

% reduce 4-dimensional problem to a 2-dimensional one:
% define two-dimensional (N_k x 2) matrix MM in the N - k space where the
% rows contain all possible combinations of N and k (which corresponds to
% all possible combinations of N' and k').
% Then the dynamic programming problem can be considered as a problem
% with only one state variable, namely the various combinations
% of N and k today and tomorrow respectively
MM=[];
```

```
% step 1: define a vector (N_k x 1) that contains N_size times the
% possible values of k, i.e. N_size times k_grid'
for i=1:N_size MM=[MM; grid_k']; end

% step 2: add a column of zeros
MM=[zeros(N_k,1) MM];

% step 3: replace the column of zeros by a vector that contains k_size
% times each possible value for N,
% i.e. (k_size times N(1)  k_size times N(2)  ...  k_size times N_bar)'
k_ones=ones(k_size,1); j=1; for i=1:N_size
    N_value=grid_N(i);
    N_aux=N_value*k_ones;
    MM(j:k_size*i)=N_aux;
    j=j+k_size;
end

% define auxiliary matrices
% matrix for saving the sarch intensity for each combinations of state
% variables today and tomorrow
search=zeros(N_k, N_k);
% matrices for saving the value of utility for each combination of the
% state variables today and tomorrow for the three states of the shock z
% respectively
R_low=zeros(N_k, N_k); R_0=zeros(N_k, N_k); R_high=zeros(N_k,
N_k);
% matrices for saving the corresponding value of consumption
cons_low=zeros(N_k, N_k); cons_0=zeros(N_k, N_k);
cons_high=zeros(N_k, N_k);
```

```
% parameter values
alpha= .36;  % output's elasticity wrt capital stock
betta= .99;  % discount factor
delta= .022; % depreciation rate for capital
eta= 1;  % convexity of search cost function
nu= -1.25; % elasticity of intertemporal substitution of labor supply
lambda= .4;    % elasticity of job matches wrt total search effort
psi= .07;   % transistion rate employment - unemployment
mu= .004; % common growth rate
a= .05;  % per unit advertising cost
c_nought= .005; % level of search costs
rho= .95;  % autocorrelation of technology shock

% compute the utility for all possible combinations of the two states in
% two periods: N, k; N', k'
% all combinations of N and k tomorrow
for i=1:N_k
    N_prime=MM(i,1);
    k_prime=MM(i,2);
    % all combinations of N and k today
    for j=1:N_k
        N=MM(j,1);
        k=MM(j,2);
        % calculate the value of the control variables
           % for S
           S=(((((1-lambda)*eta*c_nought)/(lambda*a))^(1/(1-lambda)))*
           ((N_prime-(1-psi)*N)/(1-N)))^(1/((eta*(1-lambda))+lambda));
           % for V
```

```
V=((1-lambda)/(lambda*a))*(1-N)*eta*c_nought*(S^eta);
% check whether the search intensities are non-negative:
% if they are negative, rule out the solution by setting
% the corresponding utility to -50
if S<0
    R_low(i,j)=-50;
    R_0(i,j)=-50;
    R_high(i,j)=-50;
    cons_low(i,j)=0;
    cons_0(i,j)=0;
    cons_high(i,j)=0;
    search(i,j)=0;
% (checking non-negativity of vacanies is not necessary as V and
% S have the same sign)
else
search(i,j)=S;

% for c:
% c depends on realization of random variable z
% approximation of the AR(1) process by a three-state Markov
% chain gives the following possible realizations for z:
% -0.0224, 0, 0.0224
    % for z=-0.0224
    c_low=((exp((1-alpha)*(-0.0224))) *(k^alpha) *(N^(1-alpha)))
    -k_prime +((1-delta)*k) -(c_nought*(S^eta)*(1-N)) -(a*V);
    % check whether consumption is non-negative
    if c_low<0
        R_low(i,j)=-50;
        cons_low(i,j)=0;
```

```
else
   % if c is non-negative, the corresponding utility is computed:
   % U= log(c) - N^(1-1/nu)/(1-1/nu) and saved in R_low
    R_low(i,j)=(log(c_low)) - ((N^(1-(1/nu)))/(1-(1/nu)));
   % save the corresponding value of consumption (for plots)
    cons_low(i,j)=c_low;
end

% for z=0
c_0=((k^alpha) *(N^(1-alpha))) -k_prime +((1-delta)*k)
-(c_nought*(S^eta)*(1-N)) -(a*V);
if c_0<0
    R_0(i,j)=-50;
    cons_0(i,j)=0;
else
    R_0(i,j)=(log(c_0)) - ((N^(1-(1/nu)))/(1-(1/nu)));
    cons_0(i,j)=c_0;
end

% for z=0.0224
c_high=((exp((1-alpha)*0.0224)) *(k^alpha) *(N^(1-alpha)))
-k_prime +((1-delta)*k) -(c_nought*(S^eta)*(1-N)) -(a*V);
if c_high<0
    R_high(i,j)=-50;
    cons_high(i,j)=0;
else
    R_high(i,j)=(log(c_high)) -((N^(1-(1/nu)))/(1-(1/nu)));
    cons_high(i,j)=c_high;
end
```

```
          end

      % next combination of N and k
      j=j+1;
    end
  % next combination of N' and k'
  i=i+1;
end

% MAXIMIZATION

% define auxiliary variables
v=zeros(N_k,3); % initial value of the value function
decision_low=zeros(N_k,1); % for saving the combination of states that yields
%the maxima of the value fct for z=-0.0224
decision_0=zeros(N_k,1); % for z=0
decision_high=zeros(N_k,1); % for z=0.0224
% transition matrix for z
P=[0.925 0.0075 0;
   0.0548 0.8904 0.0548;
   0 0.0075 0.925];

% VALUE FUNCTION ITERATION
converge=10; while converge>0.01
    % the value function is defined as max[u(N,k)+betta*v(N', k')]
    % starting out from an initial value of the value function of zero,
    % compute [u(N,k)+betta*v(N', k')] for each combination of N, k; N', k'

    % for z=-0.0224
```

```
    utility_low=R_low+betta*P(1,1)*v(:,1)*ones(1,N_k)
    +betta*P(1,2)*v(:,2)*ones(1,N_k)+betta*P(1,3)*v(:,3)*ones(1,N_k);
    % determine the maximum of these values and save it together with the
    % corresponding combination of the state variables
    [tv_low decision_low]=max(utility_low);

    % for z=0
    utility_0=R_0+betta*P(2,1)*v(:,1)*ones(1,N_k)
    +betta*P(2,2)*v(:,2)*ones(1,N_k)+betta*P(2,3)*v(:,3)*ones(1,N_k);
    % determine the maximum of these values
    [tv_0 decision_0]=max(utility_0);

    % for z=0.0224
    utility_high=R_high+betta*P(3,1)*v(:,1)*ones(1,N_k)
    +betta*P(3,2)*v(:,2)*ones(1,N_k)+betta*P(3,3)*v(:,3)*ones(1,N_k);
    % determine the maximum
    [tv_high decision_high]=max(utility_high);

    TV=[tv_low' tv_0' tv_high']; % maxima for each combination of
    % N, k; N', k' for z=-0.0224, z=0 and z=0.0224 respectively
    % iterate until the iteration changes the value function
    % by less than 0.01
    converge=max(abs(v-TV));
    v=TV;
end

clear R_low R_high TV utility* save value

% OPTIMIZATION PROBLEM FOR THE DETERMINISTIC ECONOMY
```

```
% in the deterministic economy, the level of technology
% is set to zero search
% one-period utility and consumption have already been
% calculated in the optimization problem of the stochastic
% economy
search_det=search; R_det=R_0; cons_det=cons_0;

% MAXIMIZATION
% define auxiliary variables
v_det=zeros(N_k,1); decision_det=zeros(N_k,1);
% value function iteration
converge=10; while converge>0.01
    % compute [u(N,k)+betta*v(N', k')] for each
    % combination of N, k; N', k'
    utility_det=R_det+betta*v_det*ones(1,N_k);
    % determine the maximum of these values
    [tv_det decision_det]=max(utility_det);
    % iterate until the iteration changes the value function
    % by less than 0.01
    converge=max(abs(v_det-tv_det'));
    v_det=tv_det';
end

clear utility_det tv_det save value

% ORGANIZING THE RESULTS
% matrices with the variable values in optimum for
% z=-0.0224, z=0 and z=0.0224 and for the deterministic economy
respectively chosen_Ndet=zeros(N_k, 1); chosen_kdet=zeros(N_k, 1);
```

```
chosen_cdet=zeros(N_k, 1); chosen_Sdet=zeros(N_k, 1);

for i=1:N_k
   chosen_Nstoch(i,1)=MM((decision_low(i)),1);
   chosen_Nstoch(i,2)=MM((decision_0(i)),1);
   chosen_Nstoch(i,3)=MM((decision_high(i)),1);
   chosen_Ndet(i,1)=MM((decision_det(i)),1);
end for i=1:N_k
   chosen_kstoch(i,1)=MM((decision_low(i)),2);
   chosen_kstoch(i,2)=MM((decision_0(i)),2);
   chosen_kstoch(i,3)=MM((decision_high(i)),2);
   chosen_kdet(i,1)=MM((decision_det(i)),2);
end for i=1:N_k
   chosen_cstoch(i,1)=cons_low(decision_low(i),i);
   chosen_cstoch(i,2)=cons_0(decision_0(i),i);
   chosen_cstoch(i,3)=cons_high(decision_high(i),i);
   chosen_cdet(i,1)=cons_det(decision_det(i),i);
end for i=1:N_k
   chosen_Sstoch(i,1)=search(decision_low(i),i);
   chosen_Sstoch(i,2)=search(decision_0(i),i);
   chosen_Sstoch(i,3)=search(decision_high(i),i);
   chosen_Sdet(i,1)=search_det(decision_det(i),i);
end

clear cons* search*

% (N x k) matrices that contain value of the value function
% for all combinations of N and k for z=-0.0224, z=0 and z=0.0224
% and for the deterministic economy respectively
```

```
j_row=1;
value_Nklow=[];
    for i_row=1:N_size
    value_Nklow=[value_Nklow; v(j_row:i_row*k_size,1)'];
    j_row=j_row+k_size;
end
j_row=1;
value_Nk0=[];
    for i_row=1:N_size
    value_Nk0=[value_Nk0; v(j_row:i_row*k_size,2)'];
    j_row=j_row+k_size;
end
j_row=1;
value_Nkhigh=[];
    for i_row=1:N_size
    value_Nkhigh=[value_Nkhigh; v(j_row:i_row*k_size,3)'];
    j_row=j_row+k_size;
end
j_row=1;
value_Nkdet=[];
    for i_row=1:N_size
    value_Nkdet=[value_Nkdet; v_det(j_row:i_row*k_size,1)'];
    j_row=j_row+k_size;
end

clear v v_det

% (N x k) matrices that contain N' chosen for all
% combinations of N and k for z=-0.0224, z=0 and z=0.0224
```

```
% and for the deterministic economy respectively
j_row=1;
Nprime_Nklow=[];
    for i_row=1:N_size
    Nprime_Nklow=[Nprime_Nklow; chosen_Nstoch(j_row:i_row*k_size,1)'];
    j_row=j_row+k_size;
end
j_row=1;
Nprime_Nk0=[];
    for i_row=1:N_size
    Nprime_Nk0=[Nprime_Nk0; chosen_Nstoch(j_row:i_row*k_size,2)'];
    j_row=j_row+k_size;
end
j_row=1;
Nprime_Nkhigh=[];
    for i_row=1:N_size
    Nprime_Nkhigh=[Nprime_Nkhigh; chosen_Nstoch(j_row:i_row*k_size,3)'];
    j_row=j_row+k_size;
end
j_row=1;
Nprime_Nkdet=[];
    for i_row=1:N_size
    Nprime_Nkdet=[Nprime_Nkdet; chosen_Ndet(j_row:i_row*k_size,1)'];
    j_row=j_row+k_size;
end

% (N x k) matrices that contain k' chosen for all
% combinations of N and k for z=-0.0224, z=0 and z=0.0224
% and for the deterministic economy respectively
```

```
j_row=1;
kprime_Nklow=[];
    for i_row=1:N_size
    kprime_Nklow=[kprime_Nklow; chosen_kstoch(j_row:i_row*k_size,1)'];
    j_row=j_row+k_size;
end
j_row=1;
kprime_Nk0=[];
    for i_row=1:N_size
    kprime_Nk0=[kprime_Nk0; chosen_kstoch(j_row:i_row*k_size,2)'];
    j_row=j_row+k_size;
end
j_row=1;
kprime_Nkhigh=[];
    for i_row=1:N_size
    kprime_Nkhigh=[kprime_Nkhigh; chosen_kstoch(j_row:i_row*k_size,3)'];
    j_row=j_row+k_size;
end
j_row=1;
kprime_Nkdet=[];
    for i_row=1:N_size
    kprime_Nkdet=[kprime_Nkdet; chosen_kdet(j_row:i_row*k_size,1)'];
    j_row=j_row+k_size;
end

% (N x k) matrices that contain consumption chosen
% for all combinations of N and k for z=-0.0224, z=0 and z=0.0224
% and for the deterministic economy respectively
j_row=1;
```

```
c_Nklow=[];
    for i_row=1:N_size
    c_Nklow=[c_Nklow; chosen_cstoch(j_row:i_row*k_size,1)'];
    j_row=j_row+k_size;
end
j_row=1;
c_Nk0=[];
    for i_row=1:N_size
    c_Nk0=[c_Nk0; chosen_cstoch(j_row:i_row*k_size,2)'];
    j_row=j_row+k_size;
end
j_row=1;
c_Nkhigh=[];
    for i_row=1:N_size
    c_Nkhigh=[c_Nkhigh; chosen_cstoch(j_row:i_row*k_size,3)'];
    j_row=j_row+k_size;
end
j_row=1;
c_Nkdet=[];
    for i_row=1:N_size
    c_Nkdet=[c_Nkdet; chosen_cdet(j_row:i_row*k_size,1)'];
    j_row=j_row+k_size;
end

% (N x k) matrices that contain search intensity chosen
% for all combinations of N and k for z=-0.0224, z=0 and z=0.0224
% and for the deterministic economy respectively
j_row=1;
S_Nklow=[];
```

```
    for i_row=1:N_size
    S_Nklow=[S_Nklow; chosen_Sstoch(j_row:i_row*k_size,1)'];
    j_row=j_row+k_size;
end
j_row=1;
S_Nk0=[];
    for i_row=1:N_size
    S_Nk0=[S_Nk0; chosen_Sstoch(j_row:i_row*k_size,2)'];
    j_row=j_row+k_size;
end
j_row=1;
S_Nkhigh=[];
    for i_row=1:N_size
    S_Nkhigh=[S_Nkhigh; chosen_Sstoch(j_row:i_row*k_size,3)'];
    j_row=j_row+k_size;
end
j_row=1;
S_Nkdet=[];
    for i_row=1:N_size
    S_Nkdet=[S_Nkdet; chosen_Sdet(j_row:i_row*k_size,1)'];
    j_row=j_row+k_size;
end

save value

% STEADY STATE AND WELFARE COMPARISON
% determine possible combinations for the steady state
% (where N'=N and k'=k in the deterministic economy)
MM_prime=[chosen_Ndet(1:N_k,1) chosen_kdet(1:N_k,1)];
```

```
diff=MM-MM_prime;
f=diff==0;
 KK=[];
 place_ss=[];
 for i=1:N_k
     if sum(f(i,:))==2
     KK=[KK; MM(i,:)];
     place_ss=[place_ss; i]
     end
 end
 place_size=size(place_ss,1);

% TIME SEQUENCE OF N AND k IN THE STOCHASTIC ECONOMY
% starting at steady state values of the deterministic economy
periods=30; %number of periods
CC=zeros(periods,2,place_size); % for saving N and k
    for j=1:place_size
    i=place_ss(j,1); % index of steady state combination in MM
    N_today=MM(i,1);
    N_tomorrow=chosen_Nstoch(i,2);
    k_today=MM(i,2);
    k_tomorrow=chosen_kstoch(i,2);
    n=i;
      for m=1:periods
          CC(m,1,j)=N_today;
          CC(m,2,j)=k_today;
          N_today=N_tomorrow;
          k_today=k_tomorrow;
          n=decision_0(1,n);
```

```
        N_tomorrow=chosen_Nstoch(n,2);
        k_tomorrow=chosen_kstoch(n,2);
      end
    end

% calculate consumption and employment for the possible
% steady states in the deterministic and the stochastic economy
% define auxiliary variables
Nkc_det=[]; Nkc_stoch=[]; tau=[]; for j=1:place_size
    i=place_ss(j,1);
    N_conv=CC(periods,1,j); % value N converges to
    k_conv=CC(periods,2,j); % value k converges to
    % index of these combinations in MM
    for bus=1:N_k
    if MM(bus,1)==N_conv
        if MM(bus,2)==k_conv
        index=bus;
        end
    end
    end
% value consumption converges to
c_ss_det=chosen_cdet(i,1); c_ss_stoch=chosen_cstoch(index,2);
% value k converges to
k_ss_det=MM(i,2); k_ss_stoch=MM(index,2);
% value N converges to
N_ss_det=MM(i,1); N_ss_stoch=MM(index,1);
% save steady state values of N, k and c for each steady state
Nkc_det=[Nkc_det; N_ss_det, k_ss_det, c_ss_det];
Nkc_stoch=[Nkc_stoch; N_ss_stoch, k_ss_stoch, c_ss_stoch];
```

```
% tax on consumption that would make agents indifferent between
% the stochastic and the deterministic economy
tau_i=1- exp( (log(c_ss_stoch)) - (log(c_ss_det)) ); tau=[tau;
tau_i]; end

save value

% PLOTS
% results of the value function iteration
mesh (grid_k, grid_N, value_Nk0)
xlabel('k');
ylabel('N');
zlabel('value'); title('value of the value function in the
stochastic economy (z=0)'); pause

mesh (grid_k, grid_N, value_Nkdet)
xlabel('k');
ylabel('N');
zlabel('value'); title('value of the value function in the
deterministic economy'); pause


mesh (grid_k, grid_N, Nprime_Nk0);
xlabel('k');
ylabel('N');
zlabel('N prime'); title('chosen N prime in the stochastic economy
(z=0)'); pause
```

```
mesh (grid_k, grid_N, Nprime_Nkdet);
xlabel('k');
ylabel('N');
zlabel('N prime'); title('chosen N prime in the deterministic
economy'); pause

mesh (grid_k, grid_N, kprime_Nk0);
xlabel('k');
ylabel('N');
zlabel('k prime'); title('chosen k prime in the stochastic economy
(z=0)'); pause

mesh (grid_k, grid_N, kprime_Nkdet);
xlabel('k');
ylabel('N');
zlabel('k prime'); title('chosen k prime in the deterministic
economy'); pause

mesh (grid_k, grid_N, c_Nk0);
xlabel('k');
ylabel('N');
zlabel('consumption'); title('chosen consumption in the stochastic
economy (z=0)'); pause

mesh (grid_k, grid_N, c_Nkdet);
xlabel('k');
ylabel('N');
zlabel('consumption'); title('chosen consumption in the
deterministic economy'); pause
```

```
mesh (grid_k, grid_N, kprime_Nk0-kprime_Nkdet)
xlabel('k');
ylabel('N');
zlabel('delta capital (k(stoch)-k(det))');
title('difference in the optimal choice of capital stochastic vs.
deterministic economy'); pause

mesh (grid_k, grid_N, c_Nklow-c_Nkhigh)
xlabel('k');
ylabel('N');
zlabel('delta consumption (c(z low)-c(z high))');
title('difference in consumption in the stochastic economy (z low
vs z high)'); pause

% convergence of N and k
h=[1:periods];
plot(h,CC(:,1,1),'b-',h,CC(:,1,2),'g-',h,CC(:,1,3),'r-',
h,CC(:,1,4),'c-',h,CC(:,1,5),'bx-',h,CC(:,1,6),'gx-',
h,CC(:,1,7),'rx-',h,CC(:,1,8),'cx-',h,CC(:,1,9),'bo-',
h,CC(:,1,10),'go-');
xlabel('time');
ylabel('N'); title('convergence of N for the
stochastic steady states'); pause

h=[1:periods];
plot(h,CC(:,2,1),'b-',h,CC(:,2,2),'g-',h,CC(:,2,3),'r-',
h,CC(:,2,4),'c-',h,CC(:,2,5),'bx-',h,CC(:,2,6),'gx-',
h,CC(:,2,7),'rx-',h,CC(:,2,8),'cx-',h,CC(:,2,9),'bo-',
```

```
h,CC(:,2,10),'go-');
xlabel('time');
ylabel('k');
title('convergence of k for the stochastic steady states'); pause
```

I hereby confirm that I have authored this master thesis independently and without use of other than the indicated resources.

Silke Simon

Berlin, August 11, 2005