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Real Effects of Monetary Policy with Nominal Price and Wage Rigidities

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Abstract

Incorporating nominal rigidities of prices and wages is one way to analyze possible real effects of monetary policy and likewise accounting for actual economic situations, namely the observable short-term stickiness of prices and wages. Empirical studies evidence this theoretical concept resulting in the development of New Keynesian economics. In this paper I use that framework to examine the impact of a technology and a monetary policy shock on variables such as output, output gap, labor or the real wage. Impulse response functions determined from the general equilibrium of the model then show various effects. With merely sticky prices I achieve the standard results of decreasing output gap and labor in response to a technology shock. In comparison, adding sticky wages to the system causes a positive initial reaction of output gap and labor, but yields a significantly lower increase in the real wage. A monetary policy shock causes negative deviations from steady state of output, output gap and labor regardless of the assumed rigidity. However, the response of the real wage alters in the distinct specifications. I observe a negative response under a sticky prices regime, an increase assuming merely sticky wages and finally, almost no deviation from steady state if prices and wages are simultaneously subject to nominal rigidities.

Affidavit/statutory declaration

I herewith declare, in lieu of oath, that I have prepared this paper on my own, using only the materials (devices) mentioned. Ideas taken, directly or indirectly, from other sources, are identified as such.

This work has not been submitted in equal or similar form to another examination office and has not been published.

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1. Introduction

The question about real effects of monetary policy if prices and wages are subject to nominal rigidities is extensively examined in recent literature. Publications such as Gali (2001) and Goodfriend and King (1997) include examples of contributions to that subject. However so far, research does not provide a complete picture of the various influences money might have and how to model the different real complications adequately. Hence, I analyze the impact of a technology and a monetary policy shock assuming sticky wages as in Erceg, Henderson and Levin (2000). Initially, as outlined in Walsh (2003), monetary policy analysis is motivated by empirical results which prove that monetary policy indeed impacts real economic variables. Consequently, to appropriately analyze economic activities and to conduct the desired policy it is essential to understand the transmission mechanisms of how central bank behavior impacts the economy. In his book Walsh describes various ways, whereas one possibility is to include nominal rigidities. This theoretical concept provides reasonable results compared to empirical findings and resembles the motivation presented in Taylor (1998). He emphasizes that "from David Hume in the 18th century to Milton Friedman in the 20th" researchers were considering "temporary price and wage rigidities in the economy" to explain effects of money. Besides, sticky prices and wages are not only theoretically motivated but reflect reality quite well. Simple observations of prices e.g. in grocery stores or the wage contracts negotiated between unions and employers evidence a certain short-term stickiness of about one year. Apparently, prices and wages do not immediately adjust in response to economic shocks and changes in the state of the economy, respectively.

Following this line of reasoning, I use the famous framework of New Keynesian models to analyze the initial question. In particular this involves calculating the optimal decisions of firms and households, whereas producers set prices and

households set wages. I provide the log-linearized equations of the general equilibrium, including a simple Taylor rule and produce impulse responses with Matlab and the Toolkit (Uhlig (1997)). My results are conditional on these graphs but yield economically reasonable interpretations with regard to a technology shock and a monetary policy shock. I consider three different specifications namely, merely sticky prices, merely sticky wages and both assumptions simultaneously.

Hence, a one percent shock in productivity causes a negative response of the output gap and labor if prices are sticky. In contrast, nominal wage rigidity shows a positive initial response of the output gap and labor. Finally, the joint assumption of both rigidities creates a merger of the previous effects, while dampening their magnitude. Most important, however, is that the real wage does not increase as much as in either cases before. A one percent shock in monetary policy leads to decreasing output gap and labor irrespective of the assumed rigidity. However, the quality of the response of the real wage differs in all three modifications. With sticky prices the real wage declines, whereas it increases if I assume merely sticky wages. In the final case, though, both graphs seem to merge, offsetting the previous effects and resulting in almost no response of the real wage.

The presented ideas and implications are constrained to the impulse response diagrams. Therefore, a further evaluation of the effects on the various real variables with regard to their actual economic performance would be an appropriate extension. Moreover, one could examine the impact of the different parameters specified in this model or one includes other shocks. Most critically, though, is the assumption of wage setting by households since reality does not really reflect this concept. Thus, modeling the nominal wage rigidity remains an intricate task and might be subject for subsequent discussions.

In chapter 2 I briefly summarize the developments in recent research of New Keynesian models as well as on the incorporation of sticky nominal wages. The following two sections comprise the crucial assumptions of this model and part 5 contains the essential derivations to obtain the general equilibrium. Afterwards, I provide graphs showing impulse response functions of the different variables and interpret these, before the intuition behind these results is explained in chapter 7. Finally, this paper concludes by summarizing the basic results in part 8.

2. Literature

Before I start calculating and analyzing the model, a general overview of recent developments in macroeconomic literature is necessary in order to classify my solutions. Due to the limited scope of this paper and the obtained results therein, I neglect to list the voluminous literature about empirically estimating and evaluating the model implications. Some of these results are included in the cited papers and a growing acceptance of New Keynesian (NK) models has proven the consistency of theory and empirics and thus, the suitability of NK models. Instead I focus on theoretical contributions about New Keynesian models and literature which provides insights on the nominal wage rigidity assumption. In particular, this contains an introduction of the development of New Keynesian models and its specific advantages to analyze monetary policy as well as the extension of staggered wage setting. Note, however, that this list can also merely be partial and is mainly based on papers which deliver a more exhaustive list of references.

One of these is the work by Goodfriend and King (1997) who describe the evolution and the superiority of the New Neoclassical Synthesis (NNS) and provide some "guiding principles for monetary policy". The NNS is determined as a combination of the RBC model and NK economics, whereas it is important to note, that they distinguish between NK models and the NNS.¹ On the one hand RBC theory as successor of the Neoclassical Synthesis successfully includes the idea of intertemporal substitution, i.e. the intertemporal optimization approach of consumption and labor supply which in turn provides the determinants of the real interest rate. Moreover, it is capable of analyzing policy and other shocks in a dynamic stochastic context. On the other hand reality requires alternative models that incorporate not merely perfect competition and flexible prices.

¹The subsequent literature, however, uses both expressions synonymously. Consequently, I will use them as substitutes throughout my entire paper, except in this paragraph.

Hence, researchers concentrate on the development of NK ideas identifying imperfect competition as a crucial factor for money to have an impact on output if prices are sticky. Positive explanations of NK economics clarify the need to consider monetary policy in order to analyze and understand economic fluctuations whereas normative results show that "aggregate demand must be managed by monetary policy in order to deliver efficient macroeconomic outcomes." Finally, Goodfriend and King (1997) derive the NNS as a merger of NK economics and RBC components, since both rely on microeconomics. This single model reinforces the understanding of economic fluctuations by embodying intertemporal optimization and rational expectations applied on pricing and output decisions. Additionally, it incorporates insights regarding the theory and practice of monetary policy. In particular, they conclude by recommending that central banks should pursue neutral monetary policy to achieve zero inflation. With gradual price adjustments monetary policy impacts real economic activity and thus, needs to stabilize the gap between price and marginal cost referred to as the average markup. Besides these results and the broad overview, Goodfriend and King (1997) do not provide special insights or particular model assumptions especially not with regard to wage rigidities. Another paper in line with the aforementioned is that of Clarida, Gali and Gertler (1999). They, focus on the question "how to conduct monetary policy" and list sources of empirical evidences for the short-term influence of monetary policy. This non-neutrality of money and resulting monetary policy rules are mainly discussed in this paper. It provides ideas of how the interest rate should adjust to the current state of the economy if it is the instrument of the central bank. Since household's optimal decision is dependent on the future path of monetary policy as well as on current policy, a commitment analysis is conducted within Clarida et al. (1999), too. Alongside, it is worthwhile to cite the paper by Gali (2001). This most recent summary provides a list of crucial insights and findings with regard to NK models, while setting up and extending the standard model. Accordingly, these models are applied to examine the relation of money, inflation and the business cycle by deriving optimal individual decisions of firms and households with "simultaneous clearing of all markets" and incorporating nominal rigidities as "a source for monetary non-neutralities." Therein, Gali also summarizes recent findings about optimal monetary policy, the effects of technology shocks in such a model economy and gives a short introduction of the nominal wage rigidity extension.

As this is a decisive assumption of my paper, besides the New Keynesian setup, I will provide a short list of papers which concentrate on that subject. A first contribution about staggered wage setting is to be found in Taylor (1979), who states that the "accelerationist model is unspecific for the development of short-run dynamics." In order to analyze the persistence of the expected inflation rate appropriately one needs to include microeconomic foundations which determine wage and price adjustments. Accordingly, he briefly outlines a model to examine how "contract length and adjustment speeds affect aggregate demand." In Taylor (1998), however, a quite comprehensive list of papers which concentrate on various forms of price and/or wage rigidities and a summary of the development of staggered price and wage setting in macroeconomic models is provided. According to Taylor, researchers began to consider sticky prices and wages and incorporating utility functions in general equilibrium models from the 1980's onwards, stating early studies such as Deborah Lucas (1985, 1986), Levin (1989) or Yun (1994/96).² Other examples are models with monopolistic competition developed in Blanchard and Kiyotaki (1987) or in Svensson (1986). Since, these papers are not completely satisfying in explaining economic activities further studies such as King and Wolman (1996) or Chari, Kehoe and McGrattan (1998) were conducted. Their focus lies on policy analysis to examine inflation targeting procedures and studying wage and price stickiness in optimizing models, respectively. Besides his general observations and evidences which motivate staggered price and wage setting models, Taylor (1998) provides a list of papers evidencing that nominal wage rigidities are subject to extensive research.

One of numerous contributions is the seminal paper by Erceg et al. (2000). Many recent studies as well as textbooks like Walsh (2003) or Woodford (2003) cite and especially apply the calculations and resulting implications of Erceg et al. (2000). Hence, the following analysis is mainly based on their scientific work, although it is important to note, that they concentrate on optimal monetary policy and welfare analyses instead of impulse responses. In fact, my examination throughout this paper will merely apply the main ideas of their model assumptions in order to calculate impulse response functions. Nonetheless, it is noteworthy that their study compares different welfare outcomes incorporating optimal monetary policy and various alternative monetary policy rules, respectively, in a model

²For the exact reference compare Taylor (1998).

with nominal price and wage rigidities. In conclusion, it is shown that "monetary policy cannot achieve the Pareto-optimal equilibrium". Staggered price and wage contracts lead to a "tradeoff in stabilizing the output gap, price inflation and wage inflation" causing a deviation from the Pareto-optimum produced in the case with flexible prices and wages.

3. Theory

Answering the overall question about real effects of monetary policy and producing adequate results requires a New Keynesian model. Due to developments in recent literature, as abovementioned, and perpetual research in this field, the New Neoclassical Synthesis is the framework to analyze effects of monetary policy.

One of the most important facts building the basis of these models is the imperfect competition assumption. Here in particular, I assume monopolistic competition on the firm as well as on the consumer side in order to include a price and wage setting mechanism in the model. This in turn accounts for the assumption of nominal rigidities which is the crucial specification for monetary policy to impact real economic variables such as output, real wage or employment. Another key fact is the underlying microeconomic foundation namely, the individual optimization of firms and households. Together with simultaneous market clearing NK economics incorporate general equilibrium conditions quite well. Other characteristics originating from the RBC theory comprise the incorporation of stochastic shocks and the intertemporal optimization, both causing dynamics in the NK model. Finally, the possible and necessary assumption of monetary policy rules to close the system makes the analysis advantageous and feasible compared to other models. The synthesis of all aforementioned characteristics causes New Keynesian models to be best suited to examine the initial question.

4. The Model

The model which I will use to analyze how monetary policy affects real variables when nominal prices and wages are sticky is based on a standard New Keynesian approach. It is based on optimizing behavior of economic agents and thus, captures basic economic conditions quite well. In contrast to RBC theory this Dynamic Stochastic General Equilibrium (DSGE) model includes money and monetary policy, respectively. Hence, it is best suited to analyze effects of monetary policy on real variables.

The economy in this setup comprises three agents namely, households, final good firms and the monetary authority. Their optimal decisions determine the system of equations which will be examined in chapter 6. In the following I provide crucial assumptions of the model as well as the objective functions.

4.1. Households

I follow the basic micro-founded setup of a NK model which is to be found in literature. Accordingly, households preferences comprise a composite consumption good C_t , real money balances M_t/P_t and leisure $(1 - N_t(h))$, whereas $N_t(h)$ represents the hours of work of household h . The composite consumption good is defined as the sum of differentiated goods supplied by final good firms. It is assumed that there exists a continuum of these firms, where each firm i produces a single final good $C_t(i)$. Formally C_t is defined as a constant returns to scale aggregator according to Dixit and Stiglitz (1977)

$$C_t = \left[\int_0^1 C_t(i)^{\frac{1}{1+\theta_p}} di \right]^{1+\theta_p},$$

with $\theta_p > 0$ being the price elasticity of demand for the individual good i . Besides these standard assumptions, I follow the modification of Erceg et al. (2000) in order to incorporate and later on analyze nominal wage rigidities. Hence, modeling a wage setting mechanism requires further assumptions for households. They are defined on a unit interval and each of them supplies differentiated labor, that is, the different labor services $N_t(h)$, $h \in [0, 1]$ demanded by the goods producing firms are imperfect substitutes. Additionally, I assume monopolistic competition on the labor market to ensure wage setting of consumers. Consequently, the modified decision problem of the representative household is to maximize the expected sum of discounted lifetime utility with regard to consumption C_t , nominal money M_t , the asset position X_{t+1} and the nominal wage $W_t(h)$ set by household h

$$\max_{C_t, M_t, X_{t+1}, W_t(h)} E_t \left[\sum_{s=0}^{\infty} \beta^s \left(\frac{C_{t+s}^{1-\sigma} - 1}{1-\sigma} + \frac{\chi}{1-\nu} \left(\frac{M_{t+s}}{P_{t+s}} \right)^{1-\nu} - \frac{N_{t+s}(h)^{1+\phi}}{1+\phi} \right) \right],$$

with σ being the coefficient of relative risk aversion, ϕ representing the elasticity of utility from supplying labor, ν denoting the elasticity of real money balances and β referring to as the subjective discount factor with $0 < \beta < 1$. Moreover, this optimization problem is subject to the budget constraint

$$\int_0^1 P_t(i) C_t(i) di + M_t + E_t [Q_{t,t+1} X_{t+1}] = W_t(h) N_t(h) + M_{t-1} + X_t - T_t,$$

where $P_t(i)$ denotes the price for consumption good i and T_t is net tax payments of the household.¹ E_t simply represents the conditional expectation at time t . According to the theory of asset pricing, I assume complete financial markets as explained in Woodford (2003). Thus, I only distinguish nominal money M_t from the rest of the different financial instruments and their possible state contingent payoffs, denoted as X_t . This is a convenient way to incorporate the financial asset portfolio in the budget constraint which arises from the complete market assumption, without explicitly having to state the quantities agents hold of the particular assets. Additionally, it is worth emphasizing the stochastic discount factor $Q_{t,t+1}$ which will be derived later in the model analysis. However so far, it follows "the basic consumption-based pricing equation" described by Cochrane

¹ T_t represents the difference between transfers the household receives and total tax payments.

(2001). Moreover, according to his explanation the stochastic discount factor and the riskless nominal interest rate R_t are combined as follows

$$R_t = \frac{1}{E_t [Q_{t,t+1}]}.$$

Since the analysis of the model will be conducted with flexible as well as with sticky prices and wages, the consumer's objective function will be slightly modified in a second step to

$$\max_{W_t(h)} E_t \sum_{s=0}^{\infty} (\psi_w \beta)^s \left[\frac{C_{t+s}^{1-\sigma} - 1}{1-\sigma} + \frac{\chi}{1-\nu} \left(\frac{M_{t+s}}{P_{t+s}} \right)^{1-\nu} - \frac{N_{t+s}(h)^{1+\phi}}{1+\phi} \right],$$

with ψ_w being the probability with which a household cannot adjust its wage and thus, measures nominal wage rigidity according to Calvo's sticky prices model (Calvo (1983)).² Furthermore, it can be interpreted as the fraction of households that cannot change their wages i.e. the price for their labor services is fix by some time $t + s$. In this case they maximize their expected discounted future value of utility up to the period $t + s$, where the wage is increased by the "unconditional mean rate of gross inflation" (Erceg et al. (2000)) so that $W_{t+s} = \bar{\Pi}^s W_t$.

Based on these assumptions and definitions the optimal behavior of the household will be derived in the following chapter. These equations, together with the ones from the production sector will then be analyzed in a general equilibrium.

4.2. Firms

As aforementioned, the standard assumptions for the final goods producing firms comprise the idea of monopolistic competition on the goods market to allow for a price setting mechanism in the production sector. In this model the producers are defined on a unit interval, that is, there exists a continuum of such firms. Each firm produces a differentiated final good $Y_t(i), i \in [0, 1]$, what denotes them as imperfect substitutes. The goods are solely consumed by households and thus,

²The maximization with regard to the other variables remains the same.

the aggregate output Y_t is defined according to the composite consumption C_t

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{1+\theta_p}} \right]^{1+\theta_p},$$

where $Y_t = C_t$ in equilibrium. The objective function referred to as the maximization of the firm value is given by

$$\max_{P_t(i)} E_t \left[\sum_{s=0}^{\infty} Q_{t,t+s} \left(P_{t+s}(i) Y_{t+s}(i) - \int_0^1 W_{t+s}(h) N_{t+s}(h) dh \right) \right],$$

with $Q_{t,t+s}$ being the stochastic discount factor, $N_t(h)$ the labor service and $W_t(h)$ the wage of household h . Firm i charges price $P_t(i)$ for their respective good. For this particular model it is important to emphasize the labor index L_t as defined in Erceg et al. (2000), as it will be taken into account in the optimization. The firm demands this labor index which is defined according to Dixit and Stiglitz (1977) as well

$$L_t = \left[\int_0^1 N_t(h)^{\frac{1}{1+\theta_w}} dh \right]^{1+\theta_w},$$

where $\theta_w > 0$ represents the wage elasticity of labor demand. Additionally, I assume the following specific Cobb-Douglas production function as a further constraint to the maximization problem

$$Y_t(i) = X_t L_t^{1-\alpha},$$

with X_t being the total factor productivity specified by an AR(1) process and $(1 - \alpha)$ representing the labor elasticity of output, where $0 < \alpha < 1$. Here, for simplicity, I ignore any capital stock according to the argumentation in Walsh (2003). His reasoning is supported by McCallum and Nelson (1999) who argue, that for a short-run business cycle analysis the fixed capital assumption from the standard IS-LM model is justified. Although "it is possible to incorporate an endogenously determined capital stock" for a long-run business cycle analysis, such an equation obtained from optimizing behavior of economic agents is unlikely to be both "analytically tractable and empirically successful". McCallum and Nelson (1997) provide an empirical evidence for the US economy that capital and output movements are not strongly related over business cycles. This is in line

with empirical results by Cogley and Nason (1995) who showed that the capital stock and the investment "reaction on a technology shock have little effect on the short-term dynamics of output." Consequently, it is possible to neglect the capital stock.

Similar to the household's decision problem, there will be a modified version of the objective function for the producer, too. It incorporates nominal price rigidity following the sticky prices model of Calvo (1983) and has the functional form

$$\max_{P_t(i)} E_t \left[\sum_{s=0}^{\infty} \psi_p^s Q_{t,t+s} \left(P_{t+s}(i) Y_{t+s}(i) - \int_0^1 W_{t+s}(h) N_{t+s}(h) dh \right) \right],$$

where $\psi_p > 0$ is the probability with which a firm cannot reset its price and thus, represents a measure of nominal price stickiness. It is also interpreted as the fraction ψ_p of firms that cannot adjust their prices by some time $t + s$. Hence, these producers maximize their "expected discounted current and future profits" (Walsh (2003)), where the price will be updated by the "unconditional mean rate of gross inflation" (Erceg et al. (2000)) so that $P_{t+s}(i) = \bar{\Pi}^s P_t(i)$.

According to these assumptions the first order conditions will be derived in the next chapter and the model will be analyzed afterwards.

4.3. Monetary authority

In order to examine the effects of monetary policy and to close the model, a policy rule is required. Due to the limited scope of this paper I will not perform a welfare analysis and hence do not consider optimal monetary policy. Instead, I focus on using an alternative policy rule, referred to as a Taylor rule (Taylor (1993)). The original version as well as variations were empirically proven to mirror the behavior of most central banks. In my analysis I will use the simple version

$$\hat{R}_t = \phi_R \hat{R}_{t-1} + \phi_{\Pi} \hat{\Pi}_t + \phi_g g_t + z_t,$$

where ϕ_R , ϕ_{Π} and ϕ_g represent the policy parameters which are crucial for the stability of this rule. Here, the central bank adjusts the deviation of the nominal interest rate \hat{R}_t from its steady state by responding to deviations of inflation

$\hat{\Pi}_t$, the previous period nominal interest rate \hat{R}_{t-1} and output gap g_t from their steady state values. Hence, the nominal interest rate is determined endogenously. Moreover, the policy rule includes a monetary policy shock z_t which will be specified in the model analysis.

5. Model Analysis

After having defined the key assumptions, the following chapter now contains all necessary calculations determining the general equilibrium of the model. That includes the derivation of the household's optimal behavior in the first and the firm's value maximization in the second part. Thereafter, I present the flexible prices and wages solution in log-linearized version. Finally, the last part comprises the log-linearized conditions for the sticky prices case as well as the crucial equations when both, prices and wages are subject to nominal rigidities.

5.1. Household's optimization problem

Deriving the equilibrium conditions from the household's decision problem will be done in two stages. The first step involves a cost minimization approach, that is, the consumer chooses the quantity of each consumption good $C_t(i)$ in order to minimize its total spending for all differentiated goods i . Secondly, I will consider the utility maximization problem. Thereby, flexible wages are examined in section 5.1.2, but calculations with staggered nominal wage setting are provided not until 5.4.2.

5.1.1. Cost minimization

In this section I provide the results from the cost minimization problem the household solves, so as to obtain the optimal minimal cost of buying $C_t(i)$ independently of the level of the composite consumption good. The optimization problem is

$$\min_{C_t(i)} \int_0^1 P_t(i) C_t(i) di,$$

subject to

$$C_t = \left[\int_0^1 C_t(i)^{\frac{1}{1+\theta_p}} di \right]^{1+\theta_p}.$$

Using the Langrange method with λ_t being the Lagrange multiplier, the minimization yields the aggregate price P_t for the composite consumption good

$$\lambda_t = \left[\int_0^1 P_t(i)^{-\frac{1}{\theta_p}} di \right]^{-\theta_p} \equiv P_t.$$

Together with the first order condition (FOC) of this problem the aggregate demand function for firm i 's consumption good can be derived as

$$C_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\frac{1+\theta_p}{\theta_p}} C_t. \quad (5.1)$$

Finally, the optimal minimal cost for household h can be written as

$$\int_0^1 P_t(i) C_t(i) di = P_t C_t.$$

5.1.2. Utility maximization

The utility maximization problem itself can be separated into two steps as well. Here, I will provide the equations of the flexible nominal wages problem, whereas the sticky prices and wages assumptions are analyzed in a subsequent section. According to the aforementioned definitions household h solves the following decision problem

$$\max_{C_t, M_t, X_{t+1}, W_t(h)} E_t \left[\sum_{s=0}^{\infty} \beta^s \left(\frac{C_{t+s}^{1-\sigma} - 1}{1-\sigma} + \frac{\chi}{1-\nu} \left(\frac{M_{t+s}}{P_{t+s}} \right)^{1-\nu} - \frac{N_{t+s}(h)^{1+\phi}}{1+\phi} \right) \right],$$

subject to the budget constraint

$$\overbrace{\int_0^1 P_t(i) C_t(i) di}^{P_t C_t} + M_t + E_t [Q_{t,t+1} X_{t+1}] = W_t(h) N_t(h) + M_{t-1} + X_t - \tau_t$$

and subject to the aggregate demand for household h 's labor service, derived from the cost minimization problem of the final goods firm. This equation occurs due to the assumption of differentiated labor services supplied in a monopolistically competitive labor market

$$N_t(h) = \left[\frac{W_t(h)}{W_t} \right]^{-\frac{1+\theta_w}{\theta_w}} L_t.$$

Then, the first order conditions which are listed below, can be calculated using the Lagrange method.

$$\begin{aligned} \frac{\partial L}{\partial C_t} : \quad \lambda_t &= \frac{C_t^{-\sigma}}{P_t} \\ \frac{\partial L}{\partial M_t} : \quad \chi^{-1} \left(\frac{M_t}{P_t} \right)^\nu &= \frac{C_t^\sigma}{1 - R_t^{-1}} \end{aligned} \quad (5.2)$$

$$\begin{aligned} \frac{\partial L}{\partial X_{t+1}} : \quad E_t [Q_{t,t+1}] &= E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \right] \\ \Leftrightarrow \quad 1 &= \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{R_t}{\pi_{t+1}} \right] \\ \text{with} \quad R_t^{-1} &= E_t [Q_{t,t+1}] \end{aligned} \quad (5.3)$$

Equation (5.2) represents the money demand and equation (5.3) is the famous consumption Euler equation. It delivers an optimal consumption path linking consumption in the future and consumption today, combined with the definition for the nominal interest rate. Additionally, the list of FOCs comprises the derivative with respect to λ_t as well as an equation obtained by maximizing over the nominal wage. The latter yields

$$\begin{aligned} \frac{\partial L}{\partial W_t(h)} : \quad 0 &= E_t \left[\frac{1 + \theta_w}{\theta_w} N_t(h)^{1+\phi} \frac{1}{W_t(h)} + \lambda_t \left(N_t(h) - \frac{1 + \theta_w}{\theta_w} N_t(h) \right) \right] \\ &= E_t \left[\frac{W_t(h)}{P_t} \frac{C_t^{-\sigma}}{1 + \theta_w} - N_t(h)^\sigma \right] \\ \Leftrightarrow \quad N_t(h)^\phi C_t^\sigma &= \frac{1}{1 + \theta_w} \frac{W_t(h)}{P_t}. \end{aligned} \quad (5.4)$$

As a consequence of monopolistically supplied labor services one can see from equation (5.4) that the real wage ($W_t(h)/P_t$) equals a constant markup ($1 + \theta_w$) over the marginal rate of substitution ($N_t(h)^\phi C_t^\sigma$) which is different from the competitive solution.¹

5.2. Firm's optimization problem

The structure of the final goods producers decision problem resembles that of the households, as it will be solved in a two stage procedure as well. At first, I consider the cost minimization of the firm using an optimal combination of input factors. Secondly, maximizing the firm value with regard to the individual price $P_t(i)$ yields the first order conditions necessary for the general equilibrium.

5.2.1. Cost minimization

Before examining the pricing decision of the final goods producers, I will determine the optimal minimal cost for a certain level of production. The firm's objective function only considers labor as production input and hence, is defined as

$$\min_{N_t(h)} \int_0^1 W_t(h) N_t(h) dh,$$

subject to the labor index

$$L_t = \left[\int_0^1 N_t(h)^{\frac{1}{1+\theta_w}} dh \right]^{1+\theta_w}.$$

Using the Lagrange method, with λ_t being the Lagrange multiplier, the problem can be written as

$$L = \int_0^1 W_t(h) N_t(h) dh + \lambda_t \left[L_t - \left[\int_0^1 N_t(h)^{\frac{1}{1+\theta_w}} dh \right]^{1+\theta_w} \right].$$

¹The competitive solution delivers the marginal rate of substitution equal to the real wage.

Calculating and manipulating the first order conditions yield the aggregate price W_t for the labor index L_t

$$\lambda_t = \left[\int_0^1 W_t(h)^{-\frac{1}{\theta_w}} dh \right]^{-\theta_w} \equiv W_t.$$

Moreover, the labor demand equation for the individual labor service of household h is derived. Note that this relationship was already applied in the consumer's decision problem in part (5.1.2).

$$N_t(h) = \left[\frac{W_t(h)}{W_t} \right]^{-\frac{1+\theta_w}{\theta_w}} L_t \quad (5.5)$$

Finally, the optimal minimal cost can be written as

$$\int_0^1 W_t(h) N_t(h) dh = W_t L_t.$$

5.2.2. Firm value maximization

In this part I present the first order conditions of the flexible prices optimization of the producer. The sticky nominal prices and wages solution will be shown in a later part of this chapter in order to concisely structure the analysis. The firm maximizes its firm value, that is, the expected sum of discounted current and future revenues and costs with respect to its individual price $P_t(i)$. Formally this can be characterized as

$$\max_{P_t(i)} E_t \left[\sum_{s=0}^{\infty} Q_{t,t+s} \left(P_{t+s}(i) Y_{t+s}(i) - \overbrace{\int_0^1 W_{t+s}(h) N_{t+s}(h) dh}^{W_{t+s} L_{t+s}} \right) \right].$$

Solving this problem is subject to two constraints. One is the production function

$$Y_t(i) = X_t L_t^{1-\alpha},$$

where total factor productivity X_t follows an AR(1) process. Thereby, ϵ_t is identically independently distributed with $E[\epsilon_t] = 0$ and $Var[\epsilon_t] = \sigma_x^2$,

$$\log X_t = (1 - \rho_x) \log \bar{X} + \rho_x \log X_{t-1} + \epsilon_{x,t}.$$

The second constraint is the demand function for the individual good i which originates from the consumers cost minimization problem. As the output Y_t is solely consumed by households, the equilibrium condition $Y_t = C_t$ translates equation (5.1) into

$$Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\frac{1+\theta_p}{\theta_p}} Y_t.$$

Manipulating these conditions and inserting them into the objective function, yields the following modified version

$$\max_{P_t(i)} E_t \left[\sum_{s=0}^{\infty} Q_{t,t+s} \left[P_{t+s}(i) \left(\frac{P_{t+s}(i)}{P_{t+s}} \right)^{-\frac{1+\theta_p}{\theta_p}} Y_{t+s} - W_{t+s} \left(\frac{(P_{t+s}(i)/P_{t+s})^{-\frac{1+\theta_p}{\theta_p}} Y_{t+s}}{X_{t+s}} \right)^{\frac{1}{1-\alpha}} \right] \right].$$

The solution procedure includes the following steps starting by calculating the first order condition

$$\frac{\partial}{\partial P_t(i)} : 0 = E_t \left[\sum_{s=0}^{\infty} Q_{t,t+s} \left[\left(\frac{1}{1+\theta_p} P_t(i) - \frac{W_t L_t^\alpha}{(1-\alpha) X_t} \right) Y_t(i) \right] \right].$$

Before continuing it is helpful to state two standard derivations which will also be used in the later analysis. First, note the definition for the marginal product of labor, MPL_t

$$\frac{\partial Y_t(i)}{\partial L_t} = (1 - \alpha) X_t L_t^{-\alpha} = \frac{Y_t(i)}{L_t} = MPL_t.$$

The purpose of the second definition is twofold. On the one hand it delivers a general equation for the nominal and real marginal cost of firm i , denoted as $MC_t(i)$ and $mc_t(i)$, respectively. Therefore, one uses the total cost $TC = W_t L_t$ and the production function in a rearranged version, $L_t = (Y_t(i)/X_t)^{1/1-\alpha}$.

Differentiating TC with respect to output yields

$$\frac{\partial TC}{\partial Y_t(i)} = MC_t(i) = \frac{W_t L_t^\alpha}{(1 - \alpha) X_t}$$

and

$$mc_t = \frac{MC_t(i)}{P_t(i)} = \frac{W_t}{P_t(i)} \frac{L_t^\alpha}{(1 - \alpha) X_t} = \frac{W_t}{P_t(i)} MPL_t^{-1}, \quad (5.6)$$

respectively. On the other hand this definition can be inserted in the abovementioned FOC to obtain

$$0 = E_t \left[\sum_{s=0}^{\infty} Q_{t,t+s} \left[\left(\frac{1}{1 + \theta_p} P_t(i) - MC_t(i) \right) Y_t(i) \right] \right]$$

Finally this results into

$$\begin{aligned} \frac{1}{1 + \theta_p} P_t(i) - MC_t(i) &= 0 \\ \Leftrightarrow mc_t(i) &= \frac{MC_t(i)}{P_t(i)} = \frac{1}{1 + \theta_p} \end{aligned}$$

for the flexible price equilibrium which describes the real marginal cost as constant and equal for all firms. Consequently, together with equation (5.6) the real wage $W_t/P_t(i)$ is lower than the MPL_t according to

$$\frac{W_t}{P_t(i)} = \frac{1}{1 + \theta_p} MPL_t = \frac{1}{1 + \theta_p} (1 - \alpha) X_t L_t^{-\alpha}.$$

Note, that under perfect competition, $\theta_p = 0$, the real wage would equal MPL_t . Finally, the price $P_t(i)$ is calculated as a constant markup over marginal cost

$$P_t(i) = (1 + \theta_p) MC_t(i).$$

5.3. Flexible prices equilibrium

A first solution to the model will be obtained by summarizing all necessary equations from the optimization problems of the economic agents and calculating the relevant values by hand. The actual results will be delivered in a log-linearized

version, the prominent method of describing economic variables in a deviation-from-steady-state representation.

5.3.1. Collecting common equations

Here, the equilibrium characterizing equations derived in the previous section are summarized. Moreover, I state the conditions and requirements as well as possible simplifications following from the flexible prices assumption. The equilibrium equations and FOCs, respectively, are listed below.

Aggregate labor demand:	$N_t(h)$	$= \left[\frac{W_t(h)}{W_t} \right]^{-\frac{1+\theta_w}{\theta_w}} L_t$
Aggregate demand	$C_t(i)$	$= \left[\frac{P_t(i)}{P_t} \right]^{-\frac{1+\theta_p}{\theta_p}} C_t$
Production:	$Y_t(i)$	$= X_t L_t^{1-\alpha}$
Labor demand:	$mc_t(i) = \frac{MC_t(i)}{P_t(i)}$	$= \frac{W_t}{P_t(i)} \frac{L_t^\alpha}{(1-\alpha)X_t}$
with	$mc_t(i)$	$= \frac{1}{1+\theta_p}$
Wage setting:	$N_t(h)^\phi C_t^\sigma$	$= \frac{1}{1+\theta_w} \frac{W_t(h)}{P_t}$
Money demand:	$\chi^{-1} \left(\frac{M_t}{P_t} \right)^\nu$	$= \frac{C_t^\sigma}{1-R_t^{-1}}$
Euler equation:	1	$= \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{R_t}{\pi_{t+1}} \right]$
Technology:	$\log X_t$	$= (1-\rho_x) \log \bar{X} + \rho_x \log X_{t-1} + \epsilon_{x,t}$
Final goods market:	Y_t	$= C_t.$

When prices are flexible all firms charge the same price $P_t(i) = P_t$, whereas the same logic applies for the households which choose the same wage $W_t(h) = W_t$ if wages are flexible. From the equations above it follows that the real marginal cost are equal for each firm, $MC_t(i)/P_t = mc_t(i) = mc_t$. Moreover, $C_t(i) = C_t = Y_t = Y_t(i)$ and $N_t(h) = L_t$. In the following part I use these specifications to simplify and reduce the system of equations. Then, it will be log-linearized based on the modified version.

5.3.2. Log-linearization

Before continuing, I summarize the modified system which will be the basis for log-linearizing the equilibrium equations.

$$\begin{aligned}
Y_t &= X_t L_t^{1-\alpha} \\
mc_t &= \frac{W_t}{P_t} \frac{L_t^\alpha}{(1-\alpha)X_t} = \frac{1}{1+\theta_p} \\
L_t^\phi Y_t^\sigma &= \frac{1}{1+\theta_w} \frac{W_t}{P_t} \\
\chi^{-1} \left(\frac{M_t}{P_t} \right)^\nu &= \frac{Y_t^\sigma}{1-R_t^{-1}} \\
1 &= \beta E_t \left[\left(\frac{Y_{t+1}}{Y_t} \right)^{-\sigma} \frac{R_t}{\pi_{t+1}} \right] \\
\log X_t &= (1-\rho_x) \log \bar{X} + \rho_x \log X_{t-1} + \epsilon_{x,t}
\end{aligned}$$

Now I apply the well-known procedure of log-linearizing equations around the steady state. This approach is based on a first order Taylor approximation and provides a way to analyze nonlinear dynamic stochastic models. These are transformed into linear systems, i.e. all equations are linear in log-deviations of the variables. A formal description of this concept and its characteristics as well as further details are outlined in Uhlig (1997). Following this paper variables in small letters "denote [throughout my entire analysis] the logarithmic deviation of $[X_t]$ from its steady state value $[\bar{X}]$." Note, that for the nominal interest rate, the real marginal cost, real money ($M_t/P_t = m_t$) and inflation, the associated variables are \hat{R}_t , \hat{mc}_t , \hat{m}_t and $\hat{\pi}_t$, respectively. All values are interpreted as percentage deviations from its steady state value according to the subsequent reasoning, whereas x_t is a real number close to zero.

$$x_t = \log X_t - \log \bar{X} = \log \frac{X_t}{\bar{X}} = \log(1 + x_t) = \log(1 + \% \text{change}) \approx \% \text{change}$$

In the following derivation, however, I apply a simpler method and replace each variable X_t with $\bar{X}e^{x_t}$. Afterwards, I take advantage of $e^{x_t} \approx (1 + x_t)$ in conjunction with the corresponding steady state representation of each equation and restate the system in its log-linearized version.

The steady state

$$\begin{aligned}
\bar{Y} &= \bar{X} \bar{L}^{1-\alpha} \\
\bar{m}c &= \frac{\bar{W}}{\bar{P}} \frac{\bar{L}^\alpha}{(1-\alpha)\bar{X}} = \frac{1}{1+\theta_p} \\
\bar{L}^\phi \bar{Y}^\sigma &= \frac{1}{1+\theta_w} \frac{\bar{W}}{\bar{P}} \\
\chi^{-1} \left(\frac{\bar{M}}{\bar{P}} \right)^\nu &= \frac{\bar{Y}^\sigma}{1-\bar{R}^{-1}} \\
1 &= \beta \bar{R}
\end{aligned}$$

Note, that the last equation results from the definition of the inflation rate, where $\pi_{t+1} = P_{t+1}/P_t$ and hence $\bar{\pi} = \bar{P}/\bar{P} = 1$.

The log-linearized equations

$$\text{Production:} \quad y_t^f = x_t + (1-\alpha)l_t^f$$

$$\text{Labor demand:} \quad \hat{m}c_t^f = w_t^f - p_t^f - x_t + \alpha l_t^f = 0 \quad (5.7)$$

$$\text{Wage setting:} \quad \phi l_t^f + \sigma y_t^f = w_t^f - p_t^f = mrs_t \quad (5.8)$$

$$\text{Money demand:} \quad m_t^f = \frac{\sigma}{\nu} y_t^f - \frac{\hat{R}_t^f}{(\bar{R}-1)\nu} \quad (5.9)$$

$$\text{Euler equation:} \quad y_t^f = E_t[y_{t+1}^f] - \frac{1}{\sigma} \left(\hat{R}_t^f - E_t[\hat{\pi}_{t+1}^f] \right)$$

$$\text{Technology:} \quad x_t = \rho_x x_{t-1} + \epsilon_{x,t}$$

Here, the index f is used to indicate the equilibrium with flexible prices and wages. Combining these equations yields following solutions, whereas I start by using the first three equations to obtain

$$mc_t^f = \frac{\phi + \alpha + (1-\alpha)\sigma}{1-\alpha} y_t^f - \frac{\phi+1}{1-\alpha} x_t = 0. \quad (5.10)$$

Accordingly, one can calculate the flexible prices and wages solution for output y_t^f , the real wage $w_t^f - p_t^f = \zeta_t^f$, the real interest rate $r_t^f = \hat{R}_t^f - E_t[\hat{\pi}_{t+1}^f]$ and labor l_t^f as functions of the exogenous state x_t only.

$$\begin{aligned}
y_t^f &= \frac{1 + \phi}{\phi + \alpha + (1 - \alpha)\sigma} x_t \\
\zeta_t^f &= \frac{\phi + \sigma}{\phi + \alpha + (1 - \alpha)\sigma} x_t \\
r_t^f &= \frac{\sigma(\rho_x - 1)(1 + \phi)}{\phi + \alpha + (1 - \alpha)\sigma} x_t \\
l_t^f &= \frac{1 - \sigma}{\phi + \alpha + (1 - \alpha)\sigma} x_t
\end{aligned}$$

5.4. Sticky prices and wages equilibrium

Before considering joint nominal price and wage rigidities, I will present the equations describing the model merely with a price setting mechanism on the firm side. I assume flexible wages for the household and use the results of this setup as benchmark to compare my solutions from the sticky-prices-and-wages-model with.

5.4.1. The New Keynesian Model with Calvo staggered prices

This section contains the derivation of the New Keynesian Philips Curve, considering nominal price rigidities. For the households I assume flexible wages but still wage setting.² However, the consumer's log-linearized FOCs remain unchanged as well as the production function and the process assumed for technology. The only affected condition is the labor demand equation which will result in a price setting equation according to the following maximization problem incorporating

²It is noteworthy to point out, that the results with sticky prices in log-linearized form do not differ, regardless of assuming "wage setting with flexible wages" or no wage setting (i.e. households maximize utility w.r.t. labor) at all. The only difference is the markup in the case of wage setting which is important for welfare analyses. Hence, either version leads to the equation: $\phi l_t + \sigma y_t = w_t - p_t$, as the constant $1/(1 + \theta_w)$ drops out through the log-linearization.

the Calvo parameter ψ_p^s . I solve

$$\max_{P_t(i)} E_t \left[\sum_{s=0}^{\infty} \psi_p^s Q_{t,t+s} [P_{t+s}(i) Y_{t+s}(i) - W_{t+s} L_{t+s}] \right],$$

subject to the production function

$$Y_{t+s}(i) = X_{t+s} L_{t+s}^{1-\alpha} \Leftrightarrow L_{t+s} = \left(\frac{Y_{t+s}(i)}{X_{t+s}} \right)^{\frac{1}{1-\alpha}}$$

and the aggregate demand for individual good i

$$Y_{t+s}(i) = \left(\frac{P_{t+s}(i)}{P_{t+s}} \right)^{-\frac{1+\theta_p}{\theta_p}} Y_{t+s}.$$

Here, the firm which could reset its price at last in t maximizes its value over periods where it cannot adjust prices, i.e. until $t + s$, using $P_{t+s} = \bar{\pi}^s P_t$. The resulting first order condition equals

$$\frac{\partial}{\partial P_t(i)} = 0 = \sum_{s=0}^{\infty} \psi_p^s E_t [Q_{t,t+s} [-\bar{\pi}^s P_t(i) + (1 + \theta_p) MC_{t+s}] Y_{t+s}(i)]$$

with

$$MC_{t+s} = \frac{W_{t+s} L_{t+s}^{\alpha}}{(1 - \alpha) X_{t+s}}$$

Rewriting gives the price for good i

$$P_t(i) = (1 + \theta_p) \frac{\sum_{s=0}^{\infty} \psi_p^s E_t [Q_{t,t+s} MC_{t+s} Y_{t+s}(i)]}{\sum_{s=0}^{\infty} \psi_p^s E_t [Q_{t,t+s} \bar{\pi}^s Y_{t+s}(i)]}.$$

The next steps follow the derivation provided in Uhlig (2003) starting with detrending the FOC. Using $\tilde{P}_t(i) = P_t(i)/\bar{\pi}^t$, $\tilde{MC}_{t+s} = MC_{t+s}/\bar{\pi}^{t+s}$ and $\tilde{Q}_{t,t+s} = Q_{t,t+s} \bar{\pi}^s$ as well as exploiting

$$E_t[Q_{t,t+s}] = E_t \left[\beta^s \left(\frac{C_{t+s}}{C_t} \right)^{-\sigma} \frac{\tilde{P}_t}{\tilde{P}_{t+s}} \right]$$

yields

$$\begin{aligned} & \sum_{s=0}^{\infty} (\psi_p \beta)^s E_t \left[\left(\frac{C_{t+s}}{C_t} \right)^{-\sigma} \frac{\tilde{P}_t}{\tilde{P}_{t+s}} Y_{t+s}(i) \tilde{P}_t(i) \right] \\ = & (1 + \theta_p) \sum_{s=0}^{\infty} (\psi_p \beta)^s E_t \left[\left(\frac{C_{t+s}}{C_t} \right)^{-\sigma} \frac{\tilde{P}_t}{\tilde{P}_{t+s}} Y_{t+s}(i) \tilde{M} C_{t+s} \right] \end{aligned}$$

Log-linearizing this relation results in

$$\begin{aligned} & \sum_{s=0}^{\infty} (\psi_p \beta)^s E_t [-\sigma(c_{t+s} - c_t) + p_t - p_{t+s} + y_{t+s}(i) + p_t(i)] \\ = & \sum_{s=0}^{\infty} (\psi_p \beta)^s E_t [-\sigma(c_{t+s} - c_t) + p_t - p_{t+s} + y_{t+s}(i) + \hat{M} C_{t+s}]. \end{aligned}$$

This equation can be simplified further and the relation $\sum_{s=0}^{\infty} (\psi_p \beta)^s = \frac{1}{1 - \psi_p \beta}$ can be applied, since $\psi_p \beta < 1$. It follows

$$\frac{1}{1 - \psi_p \beta} p_t(i) = \sum_{s=0}^{\infty} (\psi_p \beta)^s E_t [\hat{M} C_{t+s}].$$

Since all firms which adjust their price in t set the same price, I write $p_t(i) = p_t^*$. Consequently, the FOC is rewritten as

$$p_t^* = (1 - \psi_p \beta) \sum_{s=0}^{\infty} (\psi_p \beta)^s E_t [\hat{M} C_{t+s}],$$

where the price p_t^* equals the sum of discounted current and future nominal marginal cost. By exploiting the inherent recursive structure of this relationship one can rearrange the equation to

$$p_t^* = (1 - \psi_p \beta) \hat{M} C_t + \psi_p \beta E_t[p_{t+1}^*]. \quad (5.11)$$

The next step involves deriving the aggregate price level by using the definition

$$P_t \equiv \left[\int_0^1 P_t(i)^{-\frac{1}{\theta_p}} di \right]^{-\theta_p}$$

in the following specified way

$$P_t = \left[\psi_p (\bar{\pi} P_{t-1})^{-\frac{1}{\theta_p}} + (1 - \psi_p) P_t^*{}^{-\frac{1}{\theta_p}} \right]^{-\theta_p}.$$

Detrending and rearranging yields

$$\tilde{P}_t^{-\frac{1}{\theta_p}} = \psi_p \tilde{P}_{t-1}^{-\frac{1}{\theta_p}} + (1 - \psi_p) \tilde{P}_t^{*- \frac{1}{\theta_p}}.$$

Log-linearizing around the common steady state where $\bar{\tilde{P}} = \bar{\tilde{P}}^*$ and reorganizing the resulting equation, delivers

$$(1 - \psi_p \beta) p_t^* = -\psi_p p_{t-1} + p_t,$$

which will be used to derive the price setting equation. Therefore, I insert the expression for p_t^* into (5.11) and use $\hat{M}C_t = \hat{m}c_t + p_t$ so as to obtain the relation for the aggregate inflation

$$\hat{\pi}_t = \kappa_p \hat{m}c_t + \beta E_t[\hat{\pi}_{t+1}]$$

with $\hat{\pi}_t = p_t - p_{t-1}$ and

$$\kappa_p = \frac{(1 - \beta \psi_p)(1 - \psi_p)}{\psi_p}.$$

The final manipulation is to replace the real marginal cost $\hat{m}c_t$ with an output gap representation. By subtracting $\hat{m}c_t^f$ from $\hat{m}c_t$ I get a formal version which incorporates g_t , that is the percentage deviation of actual output and output arising under the assumption of flexible prices, $g_t \equiv y_t - y_t^f$. Formally this is

$$mc_t - mc_t^f = \frac{\phi + \alpha + (1 - \alpha)\sigma}{1 - \alpha} y_t - \frac{\phi + 1}{1 - \alpha} x_t - \frac{\phi + \alpha + (1 - \alpha)\sigma}{1 - \alpha} y_t^f + \frac{\phi + 1}{1 - \alpha} x_t = 0$$

since equation (5.10) holds for both, the flexible and sticky prices equilibrium. Moreover, as $\hat{m}c_t^f = 0$ it follows, that

$$\hat{m}c_t = \frac{\phi + \alpha + (1 - \alpha)\sigma}{1 - \alpha} g_t.$$

As a consequence the New Keynesian Philips Curve in output gap formulation is

$$\hat{\pi}_t = \kappa g_t + \beta E_t[\hat{\pi}_{t+1}], \quad (5.12)$$

with

$$\kappa = \frac{\overbrace{(1 - \beta\psi_p)(1 - \psi_p)}^{\kappa_p}}{\psi_p} \frac{\phi + \alpha + (1 - \alpha)\sigma}{1 - \alpha}.$$

A second equation necessary to solve the model is the New Keynesian IS Curve and goods demand, respectively. This representation will be derived from the Euler equation as well as using the definition for the real interest rate r_t which is the percentage deviation from its steady state value

$$r_t = \hat{R}_t - E_t[\hat{\pi}_{t+1}].$$

Calculating

$$y_t - y_t^f = E_t[y_{t+1} - y_{t+1}^f] - \frac{1}{\sigma} r_t + \frac{1}{\sigma} r_t^f$$

yields

$$g_t = E_t[g_{t+1}] - \frac{1}{\sigma} \left[R_t - E_t[\hat{\pi}_{t+1}] - r_t^f \right]. \quad (5.13)$$

Equations (5.12) and (5.13) are the two crucial relationships to solve the New Keynesian model with nominal price rigidities. Nonetheless, additionally the process for technology and a monetary policy rule is required to close the model. My analysis in the subsequent chapter focuses on examining reactions of variables such as labor, output, real marginal cost or the nominal interest rate on a technology as well as on a monetary policy shock. Therefore, I use Matlab and the Toolkit program (see Uhlig (1997)) to create impulse response representations. The results to these calculations will be compared to the ones from the sticky prices and wages equilibrium and are to be found in chapter 6. However, I like to emphasize, that I will not consider nominal money demand given by equation (5.9). Using a Taylor rule as monetary policy control simply determines the money demand, i.e. money demand is endogenously determined by setting the nominal interest rate. Before I start showing and interpreting further results and diagrams, I will derive the necessary equations characterizing the sticky prices and wages setup in the following part.

5.4.2. The New Keynesian Model with Calvo staggered prices and wages

Relying on the assumptions and descriptions from previous parts and on the result of the optimization with sticky prices some general conclusions can be outlined for the calculation to follow. In this particular setup more than one equation will differ from the flexible prices and wages equilibrium solution. Following Erceg et al. (2000) I will derive the wage setting equation which is the seminal change of the model. Moreover, the New Keynesian Philips Curve is modified differently as well as the marginal product of labor mpl_t and the marginal rate of substitution mrs_t . They will be presented in an output gap representation. However, despite these alterations there is no need to completely derive the New Keynesian Philips Curve again. Rather, I can use an intermediate version and incorporate the changes through the marginal cost definition. The New Keynesian IS Curve remains the same as well as the log-linear versions of the production function, the process for technology and the Taylor rule. This part starts with the modified decision problem of the household and delivers all other necessary equations afterwards. The altered objective function for the household is as follows³

$$\max_{W_t(h)} E_t \sum_{s=0}^{\infty} (\psi_w \beta)^s \left[\frac{C_{t+s}^{1-\sigma} - 1}{1-\sigma} + \frac{\chi}{1-\nu} \left(\frac{M_{t+s}}{P_{t+s}} \right)^{1-\nu} - \frac{N_{t+s}(h)^{1+\phi}}{1+\phi} \right].$$

It will be maximized subject to the budget constraint

$$P_{t+s}C_{t+s} + M_{t+s} + E_t [Q_{t,t+s+1}X_{t+s+1}] = W_{t+s}(h)N_{t+s}(h) + M_{t+s-1} + X_{t+s} - \tau_{t+s}$$

and the aggregate labor demand equation

$$N_{t+s}(h) = \left[\frac{W_{t+s}(h)}{W_{t+s}} \right]^{-\frac{1+\theta_w}{\theta_w}} L_{t+s}.$$

Here, all households which could reset their wage in period t maximize the expected sum of their discounted current and future utility up to period $t + s$, whereas now they discount with $\psi_w \beta$. Hence, their wage will be fixed for some

³I do not maximize over C_t , X_{t+1} or M_t again since merely the wage is affected by nominal rigidities and the other FOCs remain valid from the flexible equilibrium.

time period and will only be updated by the average inflation. Using $W_{t+s}(h) = \bar{\pi}^s W_t(h)$ yields for the Lagrangian

$$L = E_t \sum_{s=0}^{\infty} (\psi_w \beta)^s \left[\frac{C_{t+s}^{1-\sigma} - 1}{1-\sigma} + \frac{\chi}{1-\nu} \left(\frac{M_{t+s}}{P_{t+s}} \right)^{1-\nu} - \frac{\left(\left[\frac{\bar{\pi}^s W_t(h)}{W_{t+s}} \right]^{-\frac{1+\theta_w}{\theta_w}} L_{t+s} \right)^{1+\phi}}{1+\phi} \right] \\ + E_t \sum_{s=0}^{\infty} (\psi_w \beta)^s \left[\lambda_{t+s} \left(\bar{\pi}^s W_t(h) \left[\frac{W_{t+s}(h)}{W_{t+s}} \right]^{-\frac{1+\theta_w}{\theta_w}} L_{t+s} + \dots \right) \right].$$

The first order condition together with $\lambda_t = \frac{C_{t+s}^{-\sigma}}{P_{t+s}}$ and $Y_t = C_t$ is then

$$\frac{\partial L}{\partial W_t(h)} = 0 = E_t \sum_{s=0}^{\infty} (\psi_w \beta)^s \left[\left(N_{t+s}(h)^\phi Y_{t+s}^\sigma - \frac{1}{1+\theta_w} \frac{\bar{\pi}^s W_t(h)}{P_{t+s}} \right) N_{t+s}(h) \right],$$

where rearranging gives the individual wage

$$W_t(h) = (1 + \theta_w) \frac{E_t \sum_{s=0}^{\infty} (\psi_w \beta)^s (N_{t+s}(h)^\phi Y_{t+s}^\sigma)}{E_t \sum_{s=0}^{\infty} (\psi_w \beta)^s \left(\frac{\bar{\pi}^s}{P_{t+s}} \right)}.$$

Using the mechanism of detrending again and inserting

$$\tilde{P}_{t+s} = \frac{P_{t+s}}{\bar{\pi}^{t+s}} \quad \text{and} \quad \tilde{W}_t(h) = \frac{W_t(h)}{\bar{\pi}_t}$$

results in the following relation

$$E_t \sum_{s=0}^{\infty} (\psi_w \beta)^s \tilde{W}_t(h) = (1 + \theta_w) E_t \sum_{s=0}^{\infty} (\psi_w \beta)^s [N_{t+s}(h)^\phi Y_{t+s}^\sigma \tilde{P}_{t+s}].$$

The log-linearization immediately yields

$$E_t \sum_{s=0}^{\infty} (\psi_w \beta)^s w_t(h) = (1 + \theta_w) E_t \sum_{s=0}^{\infty} (\psi_w \beta)^s (n_{t+s}(h)^\phi + y_{t+s}^\sigma + p_{t+s}).$$

In the following I use again the definition that for $\psi_w \beta < 1$, $\sum_{s=0}^{\infty} (\psi_w \beta)^s = \frac{1}{1-\psi_w \beta}$ and the relation $w_t(h) = w_t^*$ as all households adjusting their wage in t set the same wage. Hence, the wage can be derived as the expected sum of

discounted current and future values of the marginal rate of substitution, $mrs_{t+s} = \phi n_{t+s}(h) + \sigma y_{t+s}$, plus the price level p_t . Formally this is

$$w_t^* = (1 - \psi_w \beta) \sum_{s=0}^{\infty} (\psi_w \beta)^s E_t(mrs_{t+s} + p_{t+s}).$$

The recursive structure of this equation allows for manipulation so as to obtain

$$w_t^* = (1 - \psi_w \beta)(\phi n_t(h) + \sigma y_t + p_t) + \psi_w \beta E_t[w_{t+1}^*]. \quad (5.14)$$

As a next step I exploit the aggregate wages relation

$$W_t \equiv \left[\int_0^1 W_t(h)^{-\frac{1}{\theta_w}} dh \right]^{-\theta_w}$$

specified under Calvo wage setting to

$$W_t = \left[\psi_w (\bar{\pi} W_{t-1})^{-\frac{1}{\theta_w}} + (1 - \psi_w) W_t^*{}^{-\frac{1}{\theta_w}} \right]^{-\theta_w},$$

where W_t^* represents the wage for all households which could adjust their wage in t . Detrending and further manipulating yields

$$\tilde{W}_t^*{}^{-\frac{1}{\theta_w}} = \psi_w \tilde{W}_{t-1}^{-\frac{1}{\theta_w}} + (1 - \psi_w) \tilde{W}_t^*{}^{-\frac{1}{\theta_w}}.$$

This will be log-linearized using the common steady state relationship $\tilde{\bar{W}} = \tilde{W}^*$. Consequently, after some rearranging I obtain

$$(1 - \psi_w) w_t^* = w_t - \psi_w w_{t-1}.$$

Now, using this equation and insert for w_t^* in (5.14) as well as applying the log-linear version of the aggregate labor demand from equation (5.5),

$$n_t(h) = -\frac{1 + \theta_w}{\theta_w} w_t^* - w_t + l_t,$$

yields the wage setting equation according to the specification in Erceg et al. (2000) and Galí (2001), where $\omega_t(\text{omega}) = w_t - w_{t-1}$ denotes the percentage deviation of wage inflation from steady state and should not be confused with

w_t , the percentage deviation of nominal wage. The wage setting equation is

$$\omega_t = E_t[\omega_{t+1}] + \kappa_w(mrs_t - \zeta_t), \quad (5.15)$$

with

$$\kappa_w = \frac{(1 - \psi_w \beta)(1 - \psi_w)}{\psi_w \left(1 + \phi \frac{1 + \theta_w}{\theta_w}\right)}.$$

Moreover, it is important to emphasize, that in this specific formulation of the model the mrs_t representation is defined in an output gap version and thus, is different from the previously used way. In the following I shortly provide the steps producing the modified equation as in Erceg et al. (2000). In order to calculate I make use of

$$mrs_t^f = \phi l_t^f + \sigma y_t^f = \zeta_t^f \quad \text{and} \quad mrs_t = \phi l_t + \sigma y_t$$

as well as

$$l_t^f = \frac{y_t^f - x_t}{1 - \alpha} \quad \text{and} \quad l_t = \frac{y_t - x_t}{1 - \alpha}.$$

Finally, subtracting mrs_t^f from mrs_t yields

$$mrs_t = \zeta_t^f + \left(\frac{\phi}{1 - \alpha} \sigma \right) g_t. \quad (5.16)$$

Due to wage setting of households the New Keynesian Philips Curve has a different representation, too. I use an intermediate formulation, specified as

$$\hat{\pi}_t = \kappa_p \hat{m}c_t + \beta E_t[\hat{\pi}_{t+1}]$$

where $\hat{m}c_t = w_t - p_t - x_t + \alpha l_t = \zeta_t - mpl_t$. Note, that here the marginal product of labor is given in an output gap representation derived from the labor demand equation of the flexible equilibrium and the definition of mpl_t . Using

$$mpl_t^f = x_t - \alpha l_t^f = \zeta_t^f \quad \text{and} \quad mpl_t = x_t - \alpha l_t$$

as well as the reorganized log-linear versions of the production function, l_t^f and l_t as defined before, results in

$$\begin{aligned} mpl_t - mpl_t^f &= -\frac{\alpha}{1-\alpha}(y_t - y_t^f) \\ \Leftrightarrow \quad mpl_t &= \zeta_t^f - \frac{\alpha}{1-\alpha}g_t. \end{aligned} \tag{5.17}$$

Finally, all necessary equations to solve the model and to appropriately analyze the effects occurring from a technology or monetary policy shock are derived. I will use them in the following chapter to interpret impulse response functions of economic variables such as labor or real wage. This is done with the help of Matlab and the Toolkit program as aforementioned.

6. Model results and answer

Following the purpose of this paper and examining real effects of monetary policy when prices and wages are sticky is an intricate task. Due to the complex structure of the model and interdependent reactions of economic variables this cannot easily be solved by hand. However, there are various computer based tools available which will help finding some answers.

After having derived the equilibrium characterizing equations in the previous chapter, I present and calculate further results in this chapter. For each of the two particular model setups I provide a list of equations which will be examined with the help of the Toolkit program (Uhlig (1997)). In order to actually observe reactions of e.g. labor or output I "reverse" some of the simplifications made throughout the derivation process, that is, I extend the model by definitions of earlier stages. Then I reorganize the equations in matrix notation using three general vectors x_t , y_t and z_t and apply Matlab to generate impulse response functions in a first calibration.¹ I include and interpret the impulse response diagrams in part 6.4 by, firstly analyzing sticky prices, secondly merely sticky wages and finally a version accounting for both, nominal price and wage rigidities.

6.1. Baseline calibration

In order to use numerical methods to qualify and quantify the model results some initial parameter values need to be defined.² The baseline model which can be the benchmark to compare variations with, is specified using values defined in

¹The Matlab code showing the matrices of either model specifications can be found in appendix B.1 and B.2.

²I will only specify the parameters which are indeed used in the calibration with Matlab. Consequently, I neglect to define variables concerning money demand, ν and χ , as well as the markup θ_p .

Erceg et al. (2000) and Uhlig (2003). In fact, they provide numbers e.g. for monetary policy parameters ϕ_π , ϕ_g which are widely used in literature and which were empirically estimated in studies such as Clarida et al. (1999). Moreover, the parameters from the utility function or the Calvo variables are set according to economic intuition and to capture realistic situations, at least to some extent. Hence, I set the subjective discount factor β to 0.99, as this corresponds to a real return of about four percent a year. The preference parameters from the utility function σ and ϕ are specified to 1.5, where one would get a logarithmic representation for consumption and labor if both coefficients were 1. Labor elasticity of output, $(1 - \alpha)$ is set to 0.7, resulting in α being 0.3. Following Erceg et al. (2000) the wage markup θ_w , representing the wage elasticity of labor demand equals $1/3$.³ Additionally, the Calvo parameters for firms and households, ψ_p and ψ_w , respectively, are set to 0.75 which corresponds to an average contract duration of four quarters.⁴ Note however, that this is simply another interpretation and does not violate the previous assumptions of these parameters. In order to specify the Taylor rule I set ϕ_π to 1.5, ϕ_g to 0.5 and ϕ_R to 0.8.⁵ Finally, the autocorrelation parameter ρ_x for the productivity process is 0.95 with $\epsilon_{x,t}$ being i.i.d. with mean zero and σ_x equal to 0.712. As the monetary policy shock follows a white noise process, I only need to specify the standard deviation, σ_z which equals 0.712 as well. The following table 6.1 summarizes the abovementioned determination of parameters.

³The elasticity of substitution, θ_w , describes the substitutability of labor services and thus, influences the market power of households. For $\theta_w \rightarrow \infty$ labor services become closer substitutes and market power diminishes, whereas $\theta_w = 11$ would imply a 10 percent markup in steady state as outlined in Gali (2001).

⁴The average time period over which prices or wages are fixed is calculated as follows: $1/(1-\theta)$.

⁵Here, ϕ_R is taken from Christiano, Eichenbaum and Evans (2005).

Parameter	(economic) Explanation	Value
β	subjective discount factor related to annual real return	0.99
σ	relative risk aversion (preference parameter for consumption)	1.5
ϕ	coefficient for elasticity of utility from supplying labor	1.5
$1 - \alpha$	labor elasticity of output ($\alpha = 0.3$)	0.7
θ_w	wage elasticity of labor demand	1/3
ψ_p	probability with which a firm cannot reset price	0.75
ψ_w	probability with which a household cannot reset wage	0.75
ϕ_π	coefficient weighting inflation in Taylor rule	1.5
ϕ_g	coefficient weighting output gap in Taylor rule	0.5
ϕ_R	coeff. weighting lagged nominal interest rate in Taylor rule	0.8
ρ_x	autocorrelation for AR(1) process of technology	0.95
σ_x	standard deviation of technology shock	0.712
σ_z	standard deviation of monetary policy shock	0.712

Table 6.1.: Parameterization of the baseline model

6.2. The NK Model with Calvo staggered prices

Before presenting graphs showing impulse response functions and interpreting these, it is convenient to state the actually applied equations once more. Hence, the model with merely nominal price rigidities comprises the following equations to describe optimal behavior of the economic agents as assumed in the beginning.

$$\text{NK IS Curve:} \quad g_t = E_t[g_{t+1}] - \frac{1}{\sigma} \left(\hat{R}_t - E_t[\hat{\pi}_{t+1}] - r_t^f \right) \quad (6.1)$$

$$\text{with } r_t^f = \frac{\sigma(\rho_x - 1)(1 + \phi)}{\phi + \alpha + (1 - \alpha)\sigma} x_t$$

$$\text{NK Philips Curve:} \quad \hat{\pi}_t = \kappa_p \hat{m}c_t + \beta E_t[\hat{\pi}_{t+1}] \quad (6.2)$$

$$\text{with } \kappa_p = \frac{(1 - \beta\psi_p)(1 - \psi_p)}{\psi_p}$$

$$\text{Taylor rule:} \quad \hat{R}_t = \phi_R \hat{R}_{t-1} + \phi_\pi \hat{\pi}_t + \phi_g g_t + z_t \quad (6.3)$$

$$\text{Marginal cost:} \quad \hat{m}c_t = \frac{\phi + \alpha + (1 - \alpha)\sigma}{1 - \alpha} g_t \quad (6.4)$$

$$\textbf{Output gap: } g_t = y_t - \overbrace{\frac{1 + \phi}{\phi + \alpha + (1 - \alpha)\sigma}}^{y_t^f} x_t \quad (6.5)$$

$$\textbf{Production function: } y_t = x_t + (1 - \alpha)l_t \quad (6.6)$$

$$\textbf{MRS } \zeta_t = \phi l_t + \sigma y_t \quad (6.7)$$

$$\textbf{Technology: } x_t = \rho_x x_{t-1} + \epsilon_{x,t} \quad (6.8)$$

$$\textbf{Monetary policy: } z_t \text{ white noise } E_t[z_t] = 0, \text{ } Var[z_t] = \sigma_z^2 \quad (6.9)$$

This system contains seven unknown variables and seven equations. Following the required structure of the Toolkit program (Uhlig (1997))

$$\begin{aligned} 0 &= AAx_t + BBx_{t-1} + CCy_t + DDz_t \\ 0 &= E_t[FFx_{t+1} + GGx_t + HHx_{t-1} + JJy_{t+1} + KKy_t + LLz_{t+1} + MMz_t] \\ z_{t+1} &= NNz_t + \epsilon_{t+1}; \quad E_t[\epsilon_{t+1}] = 0, \end{aligned}$$

I define the vectors x_t , y_t and z_t . The endogenous state vector x_t contains the deviation from steady state of inflation $\hat{\pi}_t$ and the nominal interest rate \hat{R}_t , whereas the vector of other endogenous variables y_t comprises g_t , $\hat{m}c_t$, y_t , l_t and ζ_t and the exogenous state vector z_t summarizes both shocks x_t and z_t . Now, Matlab creates impulse response functions and computes the recursive equilibrium law of motion according to Uhlig (1997)

$$\begin{aligned} x_t &= PPx_{t-1} + QQz_t \\ y_t &= RRx_{t-1} + SSz_t. \end{aligned}$$

6.3. The NK Model with staggered prices and wages

As mentioned before, the altered model assumptions result in new formal relationships describing the equilibrium of this economy. Hence, firstly, I will state the characteristic equations with nominal price and wage rigidities and secondly deliver impulse response diagrams.

$$\text{NK IS Curve:} \quad g_t = E_t[g_{t+1}] - \frac{1}{\sigma} \left(\hat{R}_t - E_t[\hat{\pi}_{t+1}] - r_t^f \right) \quad (6.10)$$

$$\text{with } r_t^f = \frac{\sigma(\rho_x - 1)(1 + \phi)}{\phi + \alpha + (1 - \alpha)\sigma} x_t$$

$$\text{NK Philips Curve:} \quad \hat{\pi}_t = \kappa_p \hat{m}c_t + \beta E_t[\hat{\pi}_{t+1}] \quad (6.11)$$

$$\text{with } \kappa_p = \frac{(1 - \beta\psi_p)(1 - \psi_p)}{\psi_p}$$

$$\text{Wage setting:} \quad \omega_t = \beta E_t[\omega_{t+1}] + \kappa_w \left(\zeta_t^f + \left(\frac{\phi}{1 - \alpha} + \sigma \right) g_t - \zeta_t \right)$$

$$\text{with } \zeta_t^f = \frac{\phi + \sigma}{\phi + \alpha + (1 - \alpha)\sigma} x_t = \gamma_f x_t \quad (6.12)$$

$$\text{and } \kappa_w = \frac{(1 - \psi_w\beta)(1 - \psi_w)}{\psi_w \left(1 + \phi \frac{1 + \theta_w}{\theta_w} \right)}$$

$$\text{Taylor rule:} \quad \hat{R}_t = \phi_R \hat{R}_{t-1} + \phi_\pi \hat{\pi}_t + \phi_g g_t + z_t \quad (6.13)$$

$$\text{Marginal cost:} \quad \hat{m}c_t = \zeta_t - \gamma_f x_t + \frac{\alpha}{1 - \alpha} g_t \quad (6.14)$$

$$\text{Output gap:} \quad g_t = y_t - \overbrace{\frac{1 + \phi}{\phi + \alpha + (1 - \alpha)\sigma}}^{y_t^f} x_t \quad (6.15)$$

$$\text{Production function:} \quad y_t = x_t + (1 - \alpha)l_t \quad (6.16)$$

$$\text{Real wage change:} \quad \zeta_t = \zeta_{t-1} + \omega_t - \hat{\pi}_t \quad (6.17)$$

$$\text{Technology:} \quad x_t = \rho_x x_{t-1} + \epsilon_{x,t} \quad (6.18)$$

$$\text{Monetary policy:} \quad z_t \quad \text{white noise} \quad E_t[z_t] = 0, \quad \text{Var}[z_t] = \sigma_z^2 \quad (6.19)$$

In order to use the Toolkit once again, vectors need to be defined and matrices derived. Here, the endogenous state vector x_t contains the deviation from steady state of inflation $\hat{\pi}_t$, the nominal interest rate \hat{R}_t and the real wage ζ_t . The vector of other endogenous variables y_t comprises g_t , ω_t , $\hat{m}c_t$, y_t and l_t and the exogenous state vector z_t summarizes both shocks x_t and z_t . Now, Matlab creates impulse response functions and calculates the recursive equilibrium law of motion.

6.4. Impulse responses

Here I provide the results of my calculations and present diagrams showing impulse responses of the various economic variables. Moreover, the graphs will be interpreted with the help of the underlying equations, examining effects on real variables of a technology and a monetary policy shock if prices or wages or both are subject to nominal rigidities.

6.4.1. Technology shock

Due to its assumption, a technology shock incorporates some basic responses such as higher productivity and thus, higher output. More important for the economic analysis is however, the reaction of labor. This is extensively discussed in literature where contributions by Gali and Pau (2004), Gali (1999) or Ireland (2002) are merely three examples. In contrast to the RBC model, New Keynesian models predict more realistic ⁶ responses of labor, whereas its actual result also depends on how the monetary authority reacts on a technology shock as outlined in Gali and Pau (2004). Here, a technology shock together with a Taylor rule and nominal rigidities in various forms impact variables such as labor, inflation, real wage or output. In particular the model predicts increasing employment and output gap if wages are sticky.

6.4.1.1. Sticky prices

Figure 6.1 shows the responses of all considered variables and their propagation over a horizon of eight years.⁷ Each value is at its peak and minimum, respectively, immediately after the shock and approaches steady state subsequently except the nominal interest rate which is at minimum one quarter later. In general the results are in line with standard theoretical and empirical findings, representing a negative reaction of labor hours to a technology shock.⁸ As presented

⁶These results match empirical results much better.

⁷Figures A.1 and A.2 in appendix A.1 present the reaction functions in separated diagrams for a clear view on every curve.

⁸This is in contrast to implications from basic RBC models which predict high positive correlation between hours worked and productivity (Gali (1999)).

and empirically evidenced in Gali (1999) a "stylized model with monopolistic competition and sticky prices", i.e. New Keynesian models, causes a negative comovement of labor and productivity generated by a technology shock.

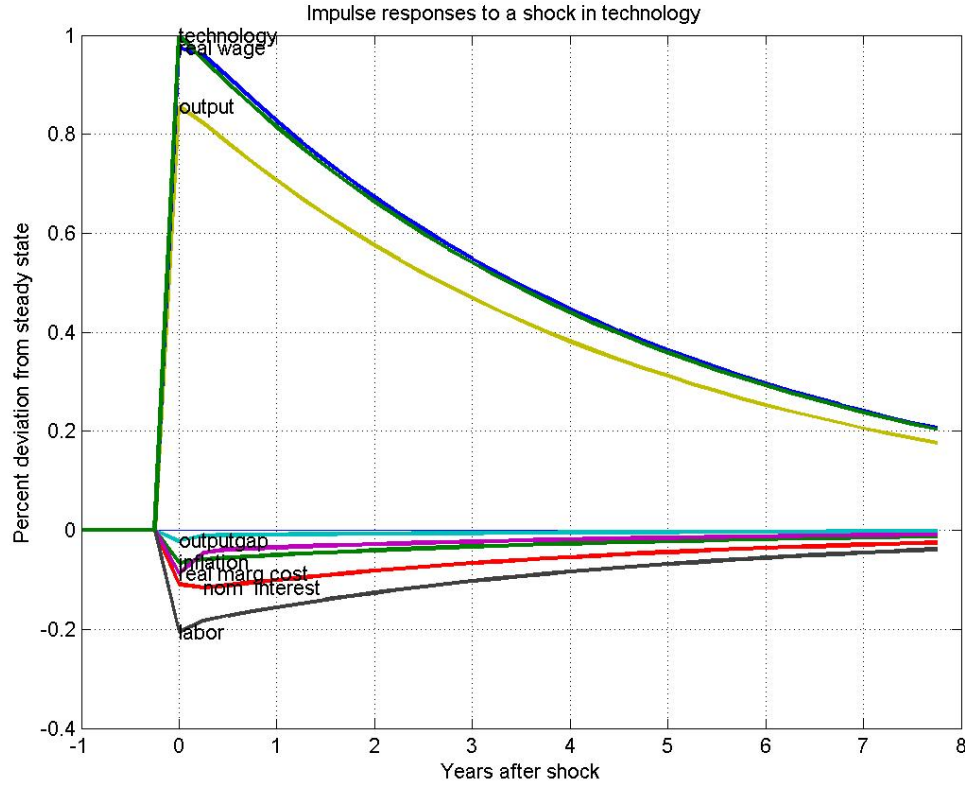


Figure 6.1.: Impulse responses for the New Keynesian model with sticky prices - Technology shock

A one percent increase in technology, i.e. a one percent positive deviation from steady state, directly affects production and causes output to rise. In case of sticky prices, however, the output does not increase proportionally to an increase in total factor productivity. In fact it rises only about 0.8 percent. Then, simple accounting as described in Gali and Pau (2004) delivers that labor hours decline as much as firms do not increase output.⁹ Applying equation (6.7) yields the result for the real wage; a more than 0.9 percent positive deviation from steady state. For the labor market to clear the demand side has to adjust as well.

⁹This can easily be seen from an input-output-diagram and a constant returns to scale production function with diminishing marginal product of labor.

Therefore, the relation of real wage $\zeta_t = w_t - p_t$ and marginal product of labor mpl_t , described by $\hat{m}c_t = w_t - p_t - mpl_t$, can be applied.¹⁰ Accordingly, a higher mpl_t results in higher real wage, since in optimum $\hat{m}c_t$ should be zero.

Regarding real marginal cost in a subsequent step one can observe a decline in figure 6.1. The 0.08 percent decrease of "real marg cost" (as denoted in the graph) can directly be seen from equation (6.4) together with the -0.02 response of output gap. Nevertheless, using $\hat{m}c_t = w_t - p_t - mpl_t$ implies that the real wage does not increase as much as productivity does. This effect occurs due to the sticky prices assumption, that is, a fraction ψ_p of firms cannot adjust prices and thus detain real wage from rising as high as it would do in the flexible prices case.¹¹

Lower marginal cost, due to the technology shock, lead to lower prices, and hence explains higher output considering the demand side, i.e. higher consumption.¹² Furthermore, the reaction of prices is reflected in the 0.06 percent decrease of inflation. Nonetheless, with nominal price rigidities not all firms can adjust prices and thus, cannot supply as much as they would like to. Consequently the sticky prices output is lower than the flexible prices output and as a result the output gap deviation is negative.¹³

Finally, the central bank reacts on a change of inflation and output gap according to the Taylor rule. The nominal interest rate is reduced in order to stabilize the output gap implied by equation (6.1), the NK IS Curve.

6.4.1.2. Sticky wages

Before considering a joint model of sticky prices and wages, it is helpful to examine the system merely with sticky wages.¹⁴ The reaction functions in figure (6.2) are

¹⁰This is simply the log-linearized version of equation 5.6.

¹¹The flexible prices solution yields $\hat{m}c_t^f = 0$ as derived in equation (5.7).

¹²Not only can firms produce more, but households are also willing to purchase more if prices decrease.

¹³With flexible prices all firms set the same price and the same output. As abovementioned this is not the case with sticky prices. Applying the Dixit Stiglitz aggregator, introduced in section 4.1, now explains the different output levels.

¹⁴For this to happen I set $\psi_p = 0.001$, i.e. only 0.001 percent of firms cannot reset its price. Hence, one could speak of flexible prices.

obviously different from the previous case. On the one hand there is a qualitative change namely, the increase of output gap and labor right after the shock. On the other hand a hump-shaped real wage curve is observable now. Additionally, there exists a curve referred to as the wage inflation which is decreasing, hump-shaped and accounts for the nominal wage rigidity.

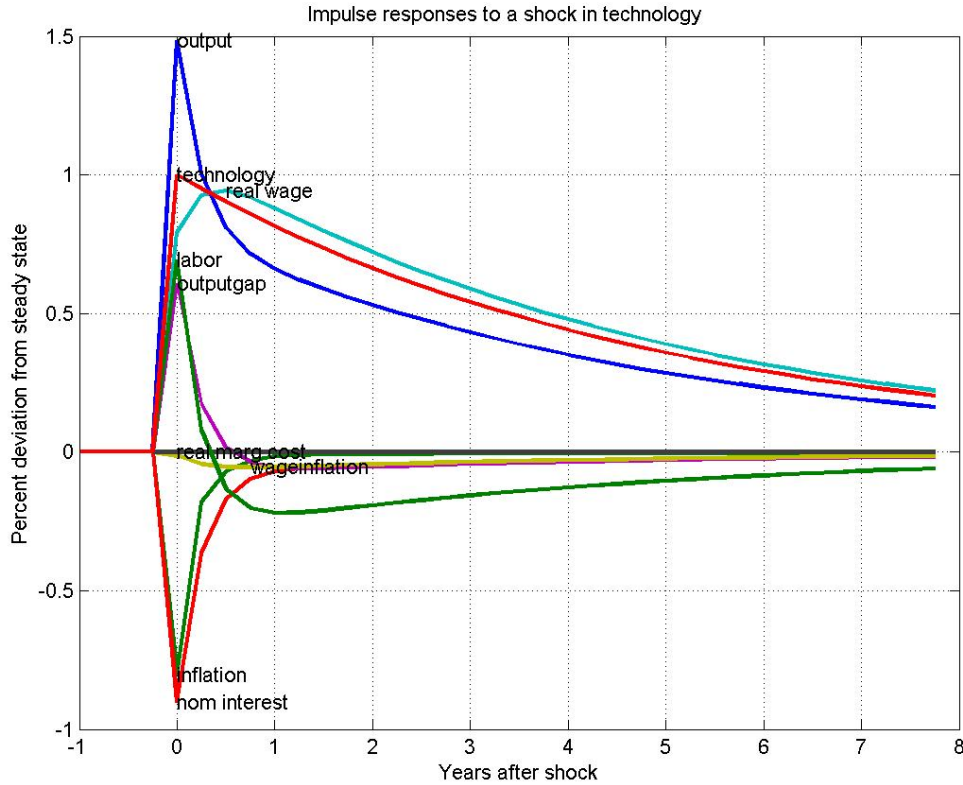


Figure 6.2.: Impulse responses for the New Keynesian model with sticky wages - Technology shock

The one percent technology shock increases productivity and output as before. However, the responses differ due to staggered wage setting of households. Starting with the labor demand side, the flexible prices solution can be applied. This implies real marginal cost of zero since firms can adjust their prices and offset any increase in the nominal wage so as to ensure $\zeta_t = mpl_t$. Thus, the final goods firm is always on its labor demand curve. Despite this theoretical result and its support by the real marginal cost curve in the diagram which shows no visible reaction, inflation decreases. Here, however the constant κ_p is so large that even a small decrease in \hat{mc}_t leads to a reaction in inflation according to equation (6.11).

On the other hand households are not on their labor supply curve anymore. In the sticky-prices-flexible-wages-case, nominal wages could adjust so as to ensure that the real wage equals the marginal rate of substitution as described in Walsh (2003). With sticky wages, though, real wage and mrs_t differ, since households can alter wages only in a staggered way. If mrs_t is greater than the real wage they would set higher nominal wages, whereas in fact only a fraction $(1 - \psi_w)$ can adjust. The gradual way of wage setting explains the hump-shaped real wage curve and its peak by more than 0.9 percent above steady state after three quarters. Directly after the shock a fraction of 0.25 households adjust causing the increase up to 0.8 percent. In the subsequent quarter again about 25 percent of households alter their nominal wages, though, real wage only increases by about 0.1 percent to 0.9. This alleviated rise occurs due to the fact that in the previous quarter 25 percent of households already adjusted, but also due to the diminishing effect of the technology shock. The gradual increase continues for one more quarter before real wages start approaching steady state again, as a consequence of the diminishing technology shock.

Moreover, the sluggish wage adjustment causes output to rise more than proportionally - about 1.4 percent above steady state - compared to the technology shock. Firms hire more workers than they would in the flexible wages case, as wages are not that high now. This reaction is quite obvious from the diagram, where labor increases by more than 0.6 percent. Due to the real wage adjustment this effect diminishes and output approaches its steady state level. After eight years it is about 0.2 percent above steady state which corresponds to the previous result in figure 6.1. Additionally, the response of labor is the lowest with -0.2 percent one year after the shock which is about the same level as with sticky prices. Note, that the intuition about decreasing labor from section 6.4.1.1 still applies.

Explaining the response of output gap is in line with the argumentation of the section above. Sticky wages output y_t is much higher than flexible-prices-and-wages-output y_t^f due to the "artificially" low wages. Hence, the positive response of output gap of about 0.6 percent is straightforward using the equation $g_t = y_t - y_t^f$. This effect diminishes with the wage adjustment, resulting in a negative deviation of g_t after one year and approaching steady state afterwards.

Although the effect on wage inflation is quite small (only -0.05 percent), the observed decline occurs since not all households can increase their nominal wage immediately after the shock. The response is the lowest three quarters after the shock and then approaches steady state.

Finally, the monetary authority increases the nominal interest rate according to equation (6.13), the Taylor rule. It reacts to the changes in $\hat{\pi}_t$ and g_t caused by the technology shock and tries to stabilize the output gap by adjusting the expected real interest rate, $\hat{R}_t - E_t[\hat{\pi}_{t+1}]$, to the real interest rate r_t^f with flexible prices and wages .

6.4.1.3. Nominal price and wage rigidities

At first glance the hybrid form of the model (see figure 6.3), that is, incorporating nominal rigidities of both prices and wages, seems to be a merger of the previous graphs, lessening or increasing effects on certain variables.¹⁵

With regard to real marginal cost a much larger negative response than before is observable. The deviation of -0.7 percent from steady state represents a large difference between the real wage and the marginal product of labor. Regarding the real wage curve which shows an increase of only 0.65 percent and taking the relation $\hat{m}c_t = w_t - p_t - \hat{m}pl_t$ into account, the reaction of "real marg cost" is quite obvious. Economically, this effect occurs due to the stickiness of w_t and p_t , i.e. both variables cannot be adjusted immediately after the shock. Hence, the real wage does not rise as much as it did in either of the previous cases. The hump-shape of the curve simply mirrors the gradual wage setting of households due to the Calvo mechanism.

Lower real marginal cost lead to a negative response of inflation. As explained before, firms decrease prices and thus, inflation deviates by -0.2 percent from steady state which is in between either of the previous results. Higher output caused by the technology shock, will be met on the demand side due to lower prices. However, it is important to note that incorporating two sources of rigidity causes output to rise by only one percent. The 1.4 percent increase of the sticky

¹⁵Figures A.3 and A.4 in appendix A.1 present the reaction functions in separated diagrams for a clear view on every curve.

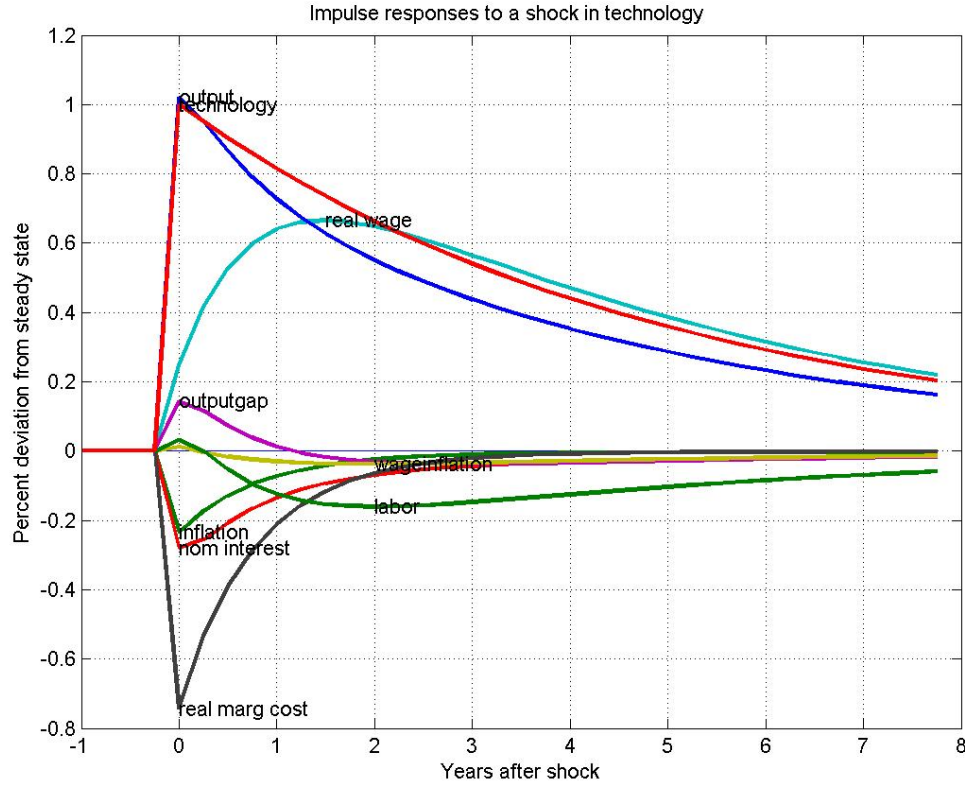


Figure 6.3.: Impulse responses for the New Keynesian model with sticky prices and wages - Technology shock

wages case is dampened by the firms' sluggish price reduction. Compared to the 0.8 percent increase in the sticky prices case it is still higher now, due to the lower aggregate wage caused by the staggered wage setting of households.

Simultaneously, the lower aggregate wage (compared to the flexible wages case) results in higher demand for labor as can be seen from the labor curve; 0.03 percent positive deviation from steady state. This reflects the response of labor from section 6.4.1.2, even though in an alleviated form.

The reaction of labor, in turn affects the output gap, though indirectly, by equation (6.15). A 0.14 percent increase of g_t in period one implies that the sticky-prices-and-wages-output is higher than the flexible outcome. This effect holds for about one year until g_t is negative again and is in line with the reasoning before. Firms demand more labor and hence produce more than with flexible wages. The downturn of labor up to -0.15 percent in the second year and the

resulting decrease of output gap by -0.03 percent in year three, show that the reasoning about labor from section 6.4.1.2 and the effect of sticky prices is still valid.

Although the response is again quite small, wage inflation actually rises in this particular calibration, i.e. the aggregate nominal wage increases more than in the steady state. The 0.015 percent increase is followed by a decrease as in the case with sticky wages. However, the lowest value is only -0.03 percent compared to -0.06 percent in figure 6.2.

The interest rate reaction is again just an endogenous response of the monetary authority according to the Taylor rule. Here however, it is not as negative as in the sticky wages case, -0.9 percent, but also not as low as in the sticky prices modification, -0.12 percent. Instead, with -0.3 percent it is in between both results, evidencing once more how both rigidities affect the response functions in the hybrid calibration.

6.4.2. Monetary policy shock

A second set of results comprise the analysis of monetary policy. How does a monetary policy shock influences the responses of the various economic variables and what are the actual real effects? According to Gali (2001) and Walsh (2003) the central bank can only influence real output as long as it affects the real interest rate via its control over the nominal interest rate. Using a nominal money growth rule, Gali (2001) then concluded that a typical monetary policy shock causes a "strong and highly persistent" effect on output. However, the application of a Taylor rule here, does not produce the same persistence, especially since there is no autocorrelation assumed for the monetary policy shock ($\rho_z = 0$). The solutions are consistent with explanations in Walsh (2003), showing that the variables return to their steady state within the first year after the shock. In this part I provide impulse response functions to a monetary policy shock assuming sticky prices, sticky nominal wages as well as both together. Moreover, it contains possible interpretations based on economic intuition.

6.4.2.1. Sticky prices

The examination of the propagation of a one percent monetary policy shock starts by considering the Taylor rule. Accordingly, the nominal interest rate rises as shown in figure 6.4, though only about 0.5 percent above steady state.¹⁶ Consequently, the short term real interest rate, $r_t = \hat{R}_t - E_t[\hat{\pi}_{t+1}]$, increases and

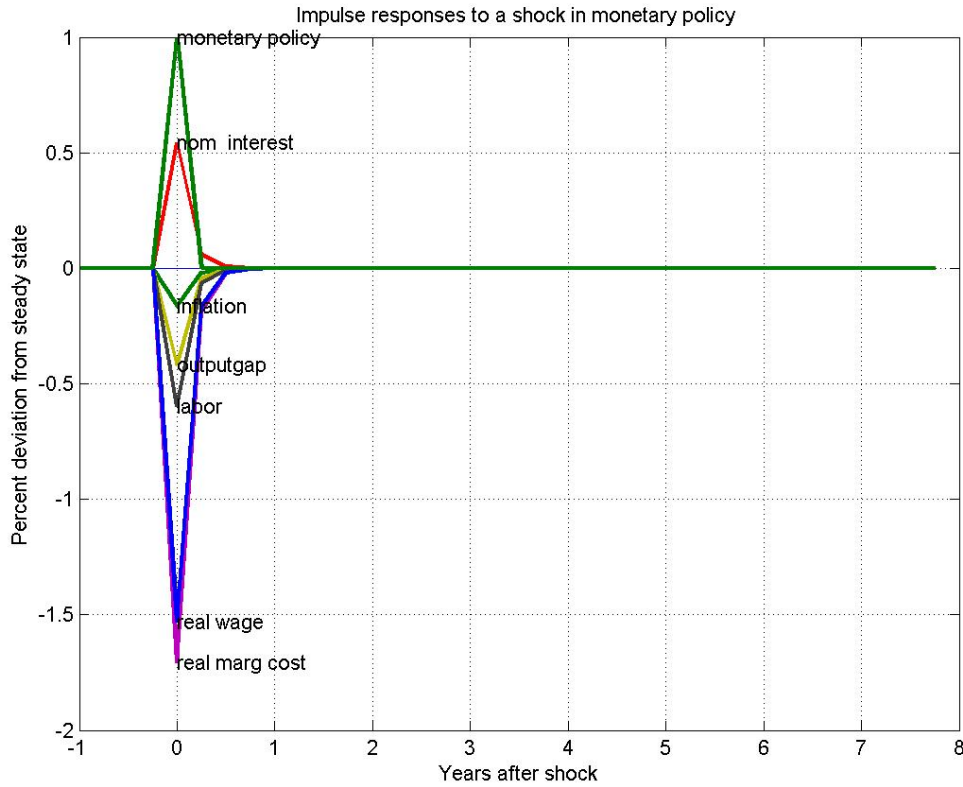


Figure 6.4.: Impulse responses for the New Keynesian model with sticky prices - Monetary policy shock

the output gap declines, applying the NK IS curve (equation(6.1)). The downturn of g_t of about 0.4 percent causes output to deviate by the same amount. This can easily be derived from equation (6.5) knowing that the deviation of flexible output y_t^f from steady state is zero.¹⁷ In conclusion, both variables move together and independent of whether one assumes sticky prices, sticky wages or both.

¹⁶Figures A.5 and A.6 in appendix A.2 present the reaction functions in separated diagrams for a clear view on every curve.

¹⁷Figure A.7 in appendix A.2 evidences this result quite clearly.

Continuing to analyze the effects on the producers side, one can observe a decline of 1.6 percent in real marginal cost (compare equation (6.4)), resulting in lower prices and finally in a negative deviation of inflation of about 0.15 percent.

Lower prices and thus, lower output and consumption, respectively, are more appropriately explained considering the consumers side of the economy. Based on the Euler equation which relates consumption today and tomorrow to the real interest rate as follows

$$E_t[c_{t+1}] - c_t = \frac{1}{\sigma}(\hat{R}_t - E_t[\hat{\pi}_{t+1}]) = \frac{1}{\sigma}r_t,$$

one can conclude that consumption (output) today falls in response to a higher real interest rate. The corresponding left-shift of the demand curve is the actual reason which causes output and prices to decline. Intuitively, this result occurs since consumers benefit from higher interest rates by postponing consumption, i.e. they spend less and invest more. In the subsequent period they get a higher payoff and can consume more.¹⁸

As a consequence of decreasing production the deviation of labor from steady state is negative, too. Following this line of reasoning and taking equation (6.7) into account, the negative reaction of real wage can be derived by simple accounting. The 1.5 percent decrease results from declines in both, the -0.4 percent response in output and the -0.6 percent deviation in labor. Reformulating the condition for the marginal rate of substitution to $w_t - p_t = \hat{m}rs_t$ (compare equation 5.8), additionally reveals one way how sticky prices impact real variables. In this case households are always on the labor supply curve and thus, adjust their nominal wages continuously to a change in mrs_t . Hence, sluggish price adjustment to a shock in monetary policy, i.e. p_t does not decrease as much as it would in the absence of nominal price rigidity and especially not as much as w_t , causes the real wage reduction here.¹⁹

¹⁸Further details on this issue and the influence of the coefficient of relative risk aversion σ , are to be found in Cochrane (2001).

¹⁹This effect is more visible by gradually lowering the parameter ψ_p . Then, more and more firms can reduce prices and $w_t - p_t$ approaches zero.

6.4.2.2. Sticky wages

The sole assumption of nominal wage rigidity and the potential real effects of a monetary policy shock have been examined in the recent literature. Contributions such as Erceg et al. (2000), Blanchard and Gali (2006) or Taylor (1998) analyze this issue with regard to welfare effects and volatility of output, real wage rigidities and empirical issues, respectively. In the following, however, I confine my analysis on presenting impulse response functions and providing some economic intuition.

Figure 6.5 shows the same reaction as in figure 6.4 for the interest rate, output gap, output and labor except for small differences in the magnitude and a slightly longer duration until the variables return to their steady state level. Inflation, real wage and marginal cost react differently than before and the wage inflation curve is introduced in addition.

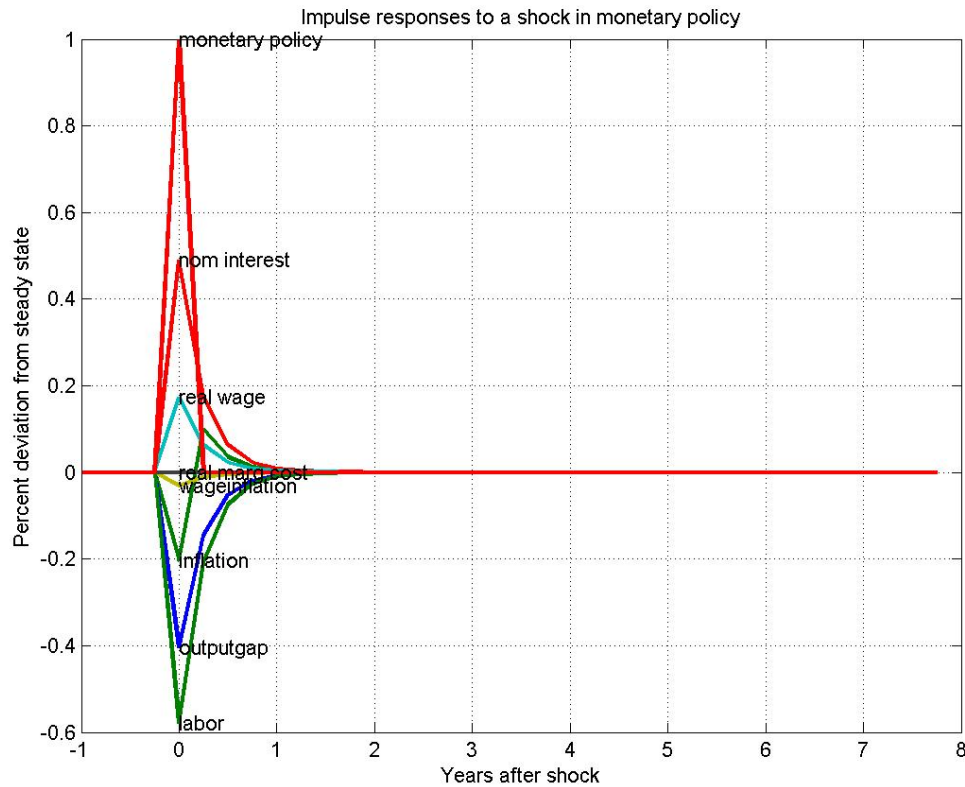


Figure 6.5.: Impulse responses for the New Keynesian model with sticky wages - Monetary policy shock

Though the nominal interest rate is less positive than with sticky prices, the reasoning remains the same. The Taylor rule determines the change of \hat{R}_t to a monetary policy shock and hence g_t and y_t decrease. This result is obtained by considering the NK IS curve and intuitively explained by the household's optimal consumption decision over time. Subsequently, lower output leads to a decline in labor. Real marginal cost show no observable reaction which is consistent with theory. Firms are on their labor demand curve due to flexible prices and thus, the condition $\hat{m}c_t = w_t - p_t - \hat{m}pl_t = 0$ holds, irrespective of the nominal wage stickiness. Inflation, however, reacts on even very little changes in real marginal cost, as can be seen from equation (6.11).²⁰

The consumers side of this system is mainly characterized by the gap between the real wage and the marginal rate of substitution. With flexible wages the condition $mrs_t = w_t - p_t$ holds and any change in mrs_t would be offset by an immediate alteration in w_t . The staggered wage decline in this case results in $mrs_t < 0 < w_t - p_t$ implying that nominal wages do not decrease as much as prices do. Technically, this difference causes wage inflation to fall, according to equation (6.12). Intuitively, this occurs simply because of decreasing nominal wages.

6.4.2.3. Nominal price and wage rigidities

The reaction functions of the hybrid form of the model are obviously influenced by both sources of rigidities. Consequently figure 6.6 presents again a mixed picture of effects.²¹ Except for the real wage, there is no change in the quality of responses. Instead, the graphs merely show alterations in the magnitude considering variables such as the nominal interest rate (0.6), the output gap and output (-0.5), labor (-0.8), inflation (-0.02) and real marginal cost (-0.2) compared to either cases before.

According to equation (6.13) the nominal interest rate rises due to a monetary policy shock, though slightly higher in this case. In turn one can observe a larger decline in output gap and output, respectively, whereas this still relies on the

²⁰A small value of ψ_p results in a very large weight parameter κ_p .

²¹Figures A.8 and A.9 in appendix A.2 present the reaction functions in separated diagrams for a clear view on every curve.

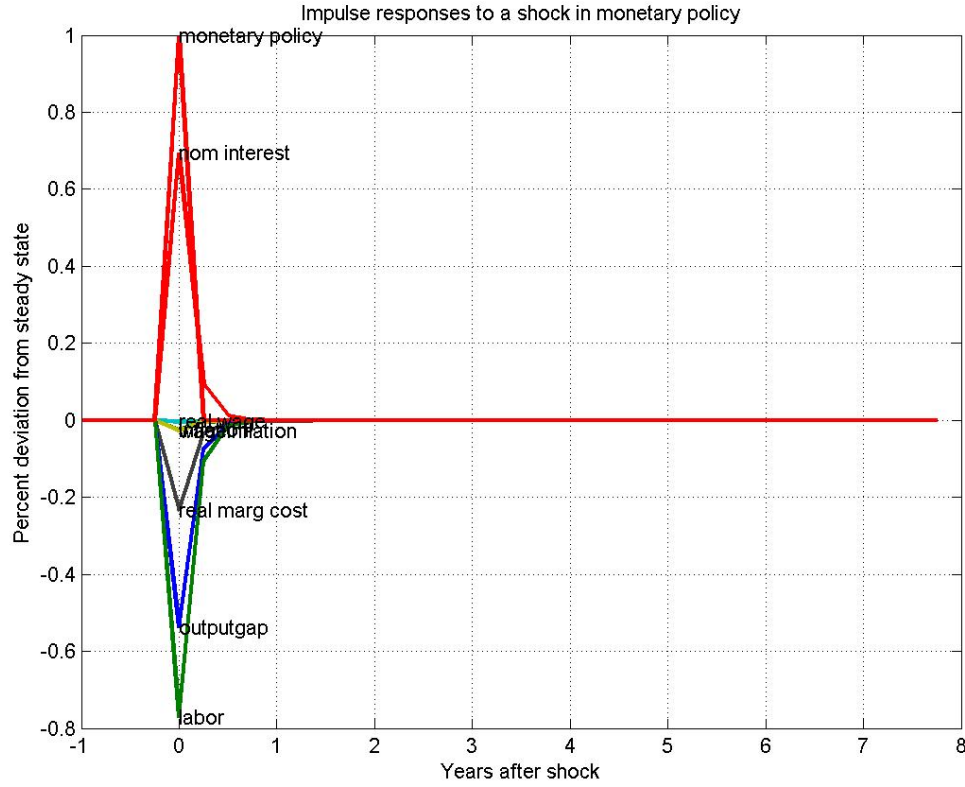


Figure 6.6.: Impulse responses for the New Keynesian model with sticky prices and wages - Monetary policy shock

intuition about the optimal consumption path of households. Lower output leads to a decline in labor which is -0.8 percent on account of a larger effect in output.

With regard to the producers side the decrease of real marginal cost, causes lower prices and thus, produces the response in inflation.²² Moreover, the negative deviation of "real marg cost" implies a gap between $w_t - p_t$ and mpl_t induced by sticky prices. The reaction is, however, not as large as with merely sticky prices, since w_t is subject to stickiness now, too. Consequently, w_t and p_t do not decline as much as they did within either of the flexible regimes separately.

The households side of the economy still incorporates the features of section 6.4.2.3 causing a downturn in wage inflation on the one hand. On the other hand the reaction of real wage is different from either cases abovementioned. A slightly

²²Lower prices also result from lower demand, i.e. a left shift of the demand curve based on household optimization.

negative deviation from steady state implies, that the positive response from the case with sticky wages is outweighed by the decline from the sticky prices setup, though obviously not in a linear way. In general the small response only evidences the joint assumption of staggered price and wage setting, i.e. w_t and p_t almost move together.²³

²³Varying the Calvo parameter ψ_w reveals this implication quite clearly. If $\psi_w \rightarrow 0$ nominal wages can adjust immediately, but as p_t is still sluggish, $w_t - p_t$ is much more negative than with $\psi_w = \psi_p$. Figure A.10 in appendix A.2 supports this result.

7. Discussion

The initial question of this paper is broadly defined and leaves room for several ways to answer and analyze the effects of monetary policy. One can discuss welfare aspects as in Erceg et al. (2000) and compare different monetary policy rules or evaluate the theoretical results from such a New Keynesian framework on the ground of estimates from econometric studies. From the numerous possibilities in economics I choose to examine the real effects of monetary policy with nominal price and wage rigidities with the help of impulse response functions and compare the various diagrams. Hence, all results will be conditional on that specific technique which is often applied to analyze DSGE models. It produces meaningful insights to intricate economic problems and possible inferences are based on these results. The following part summarizes the solutions outlined in chapter 6 using the same structure as before, that is, considering technology shocks and monetary policy shocks successively.

With respect to a **technology shock**, monetary policy plays only a minor role. The Taylor-type interest rate rule simply reacts to the state of the economy and thus, the interest rate is endogenously determined in response to changes in inflation and output gap. Nonetheless, the resulting implications regarding the sticky wages assumption are noteworthy.

A positive productivity shock in the basic model with merely **sticky prices** results in increased output, but not to the extent the technology rises. This effect is crucial for the response of labor and occurs due to the staggered price setting of firms. Not all producers can immediately reduce their prices in order for the supply to match demand. Hence, the aggregate price level is higher than in the flexible case, causing the aggregate output to be lower in turn which is reflected by the negative response of the output gap.¹ Furthermore, this reasoning

¹This effect can clearly be shown by varying the parameter ψ_p with $\psi_p \rightarrow 0$.

explains the famous result of decreasing labor in NK models. Intuitively argued, firms lower their labor input to the extent they cannot increase output. Finally, note the positive effect on real wage triggered by a higher marginal product of labor. Although real wage increases, the impact is less than in the case with flexible prices where firms follow $\hat{m}c_t = w_t - p_t - mpl_t = 0 \Leftrightarrow w_t - p_t = mpl_t$. Here, this condition is not fulfilled and creates a gap between ζ_t and mpl_t , since p_t does not adjust immediately to a change in mpl_t .

In comparison, the polar case of **sticky wages** delivers significantly different responses of variables such as output, output gap, real wage, labor or real marginal cost. According to the diagrams nominal wage rigidity causes a 1.4 percent increase in output to a one percent technology shock. Moreover, a positive output gap is observable in the first year after the shock, implying that the flexible output is lower than output with sticky wages. The underlying reasoning starts with the staggered wage setting of households. Under a flexible wages regime they would immediately adjust their nominal wage to ensure $mrs_t = w_t - p_t$. The special situation with sticky wages, though, allows only $(1 - \psi_w)$ households to increase nominal wages in response to the technology shock. Consequently, the aggregate real wage is lower than with flexible wages, resulting in higher labor demand and thus, in higher output. The hump-shaped real wage curve in figure 6.2 clearly evidences that as well as the gradual adjustment of wages in general.

The joint assumption of **sticky prices and wages** yields impulse response functions that seem to combine either of the previous graphs. Both, the output gap and labor, first reveal a positive reaction to a technology shock similar to the sticky wages case. Subsequently, both decrease and show a negative deviation from steady state, obviously caused by the impact of sticky prices. Hence, a straightforward inference is that one effect dominates the other if $\psi_p \neq \psi_w$, i.e. a higher positive response of output gap and labor is produced if $\psi_w > \psi_p$ and vice versa. Another important reaction is that of the real wage, where the hump-shape of the curve is more distinctive reaching the peak only after one and a half year. Moreover, the maximum is only 0.65 percent in this case which is lower than in either of the previous specifications. Thereby, two ways of economic reasoning are appropriate. On the hand one can argue that the sluggish nominal wage rise

compared to the sticky prices setup² causes this low value. On the other hand the staggered price increase compared to the sticky wages case³ is responsible. Either ways deliver the same response namely, a smaller difference between w_t and p_t .

A shock in **monetary policy** affects the economy through a change in the nominal interest rate. Therefore and in contrast to a technology shock, monetary policy is the driving force now. In general the calculated impulse response functions reveal less persistence, that is, the variables return to their steady state value after one year. Nonetheless, the results to a one percent shock remain economically reasonable as outlined in the previous chapter.

A monetary policy shock causes a sharp decline in real marginal cost, real wage, output and labor if I assume **sticky prices**. The transmission of a change in the interest rate to impact real variables is mainly determined by the intertemporal optimization of households. Intuitively, higher interest rates lead to postponing consumption from today to tomorrow and cause demand and thus, output to decline. Lower output decreases labor and since both are components of the marginal rate of substitution the decline in real wage which follows $\zeta_t = w_t - p_t = mrs_t$, is reasonable, too. Sticky prices impact the system exactly in that equation. If $\psi_p = 0$, w_t and p_t would adjust simultaneously by the same amount resulting in $\zeta_t = 0$. However, since $\psi_p = 0.75$ in this case, firms do not lower prices as fast as in the flexible case and therefore, real wage decreases. Varying the parameter ψ_p clearly evidences this effect and proofs the reason of these implications.

In contrast, the case of **sticky wages** shows a rise in real wage, no reaction of real marginal cost, but still the same impact on output and labor. Assuming flexible prices simply causes $\hat{m}c_t = w_t - p_t - mpl_t = 0$, i.e. the firm is on its labor demand curve. More important is the positive deviation of real wage of about 0.16 percent above steady state. This obviously results from the staggered wage setting assumption, leaving the reasoning of optimal intertemporal optimization unaffected. The sluggish adjustment of wages, however, causes a gap between mrs_t and ζ_t , since only $(1 - \psi_w)$ households can adjust. Consequently, aggregate nominal wage does not decrease as much as with flexible wages and especially not as much as prices, so that $w_t - p_t$ is actually positive.

²There, wages are assumed to be flexible.

³Here, prices are assumed to be flexible.

Again, the joint assumption of nominal **price and wage rigidities** does not produce entirely new results. Instead both effects work together showing an immediate response of real marginal cost and almost no reaction of real wage. Output and labor still decrease, although slightly more this time. Most interestingly is the impact on real wage which simply mirrors that $\psi_p = \psi_w$ and wages and prices decline simultaneously almost by the same amount to a shock in monetary policy.

In conclusion, adding sticky wages to a standard NK model with sticky prices causes a positive initial response of output and labor to a technology shock. A monetary policy shock merely alters the reaction of real wage changing from a sharp decline in the standard case to almost no response. Both impacts are economically reasonable based on the argumentation beforehand. Nonetheless, further inferences can only be drawn by comparing these results to actual economic activity, that is, using econometric studies to evaluate the results. Other possible extensions for this analysis are e.g. the application of various monetary policy rules or the incorporation of different assumptions about the wage setting mechanism. Moreover, one could examine the effects of parameters such as the relative risk aversion σ or the elasticity of utility from supplying labor ϕ . Another possibility to extend this model could be to include other shocks e.g. a cost push shock.

8. Conclusion

Examining real effects of monetary policy with nominal price and wage rigidities can be conducted in various ways. In this paper I apply the New Keynesian model which is extensively discussed in recent literature, in conjuncture with a standard Taylor rule. The analysis is confined on creating impulse response functions and interpreting the graphs of variables such as output, real wage or labor.

In particular, I define monopolistic competition on the goods and labor market according to Erceg et al. (2000) to ensure price and wage setting. After having outlined further necessary assumptions, I solve the optimization problems of the representative household and the firms in order to obtain the general equilibrium. At first, I derive the equations which characterize the flexible-prices-and-wages-equilibrium. Secondly, the sticky prices assumption is included resulting in the NK Philips curve and the NK IS curve. Finally, I extend the model and maximize the households objective function taking sticky wages into account. The systems of equations are provided in log-linearized versions in order to produce impulse responses applying Matlab and the Toolkit (Uhlig (1997)). This allows for interpreting the variables in a deviation-from-steady-state-representation.

The results are confined on the included graphs and they are analyzed separately with regard to a positive technology shock and a monetary policy shock which causes an increase in the nominal interest rate. Moreover, I compare the responses to each shock assuming merely sticky prices, only sticky wages as well as both together. A technology shock with sticky prices predicts a negative deviation of output gap and labor. In contrast, nominal wage rigidity causes a positive initial response of the output gap and labor. The common assumption of staggered price and wage setting yields a merger of both diagrams, however with a dampened magnitude of the graphs. It shows positive initial responses of the output gap and labor but decreasing reactions later on. Most important is the positive

deviation of the real wage curve which is significantly lower than in either of the previous specifications. Regarding the monetary policy shock the standard result of decreasing output and labor is valid for all three cases. The most interesting effect occurs again to the real wage. With sticky prices it decreases, whereas under sticky nominal wages real wage increases. However, in the joint case there is almost no impact on real wage observable. This implies, that both assumptions offset each other, i.e. prices and wages adjust simultaneously (only if $\psi_p = \psi_w$) and especially nearly about the same amount.

These interpretations are economically reasonable and constrained on the stated equations. Consequently, various extensions are imaginable. On the one hand, one could evaluate the implications by comparing them to actual economic activities as examined in numerous econometric studies about the US economy or Europe. On the other hand, it is possible to incorporate other shocks or different labor market assumptions to model nominal wage rigidity. Another extension of the paper might involve analyzing how the distinct parameters impact the responses of the real variables.

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A. Seperated impulse responses

A.1. Technology shock

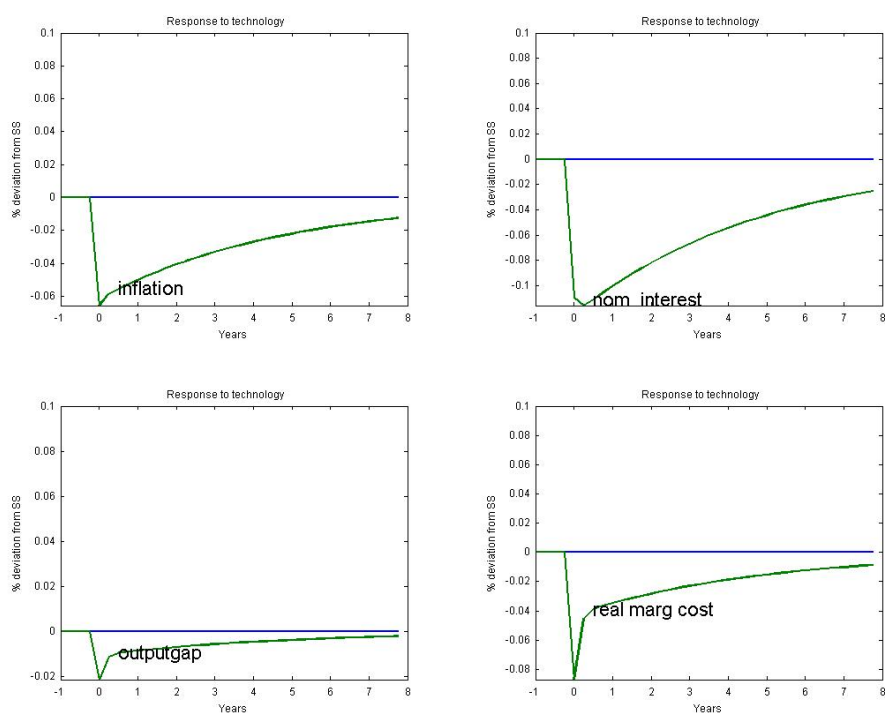


Figure A.1.: Impulse responses for the NK model with sticky prices - Technology shock - $\hat{\pi}_t, \hat{R}_t, g_t, \hat{mc}_t$

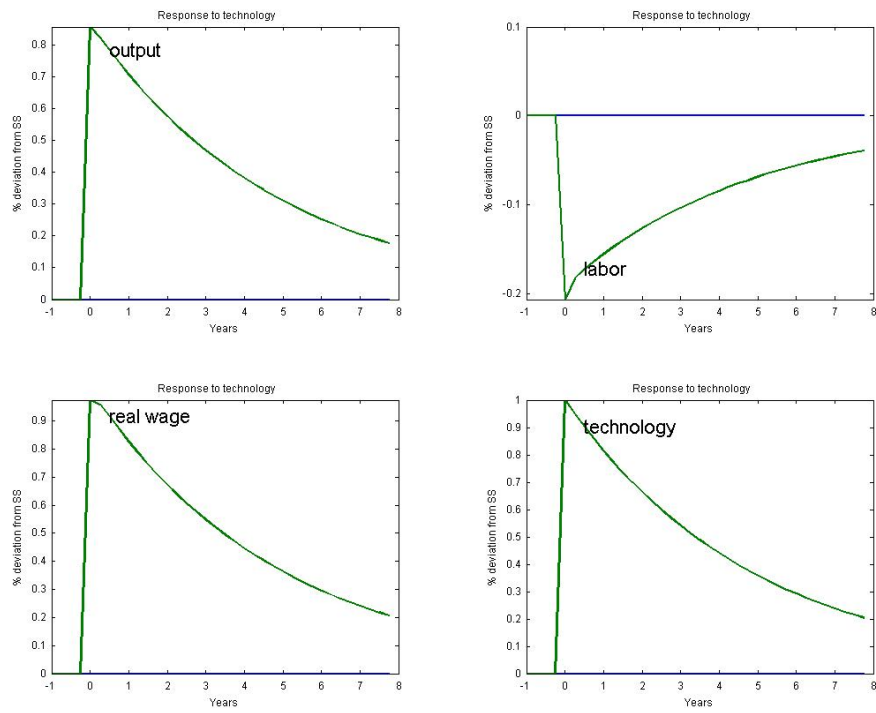


Figure A.2.: Impulse responses for the NK model with sticky prices - Technology shock - y_t, l_t, ζ_t, x_t

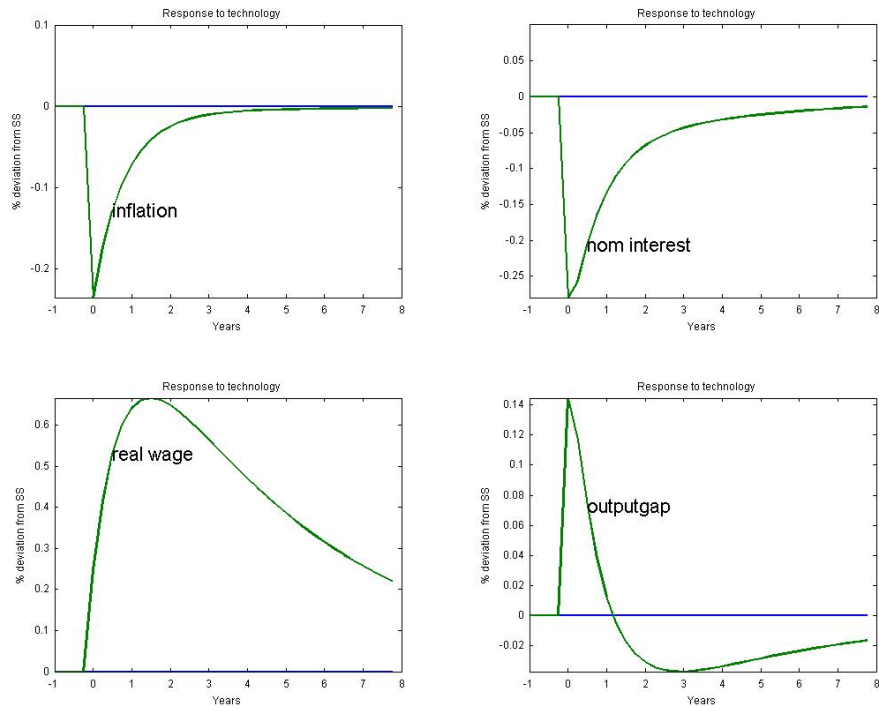


Figure A.3.: Impulse responses for the NK model with sticky prices and wages -
Technology shock - $\hat{\pi}_t, \hat{R}_t, \zeta_t, g_t$

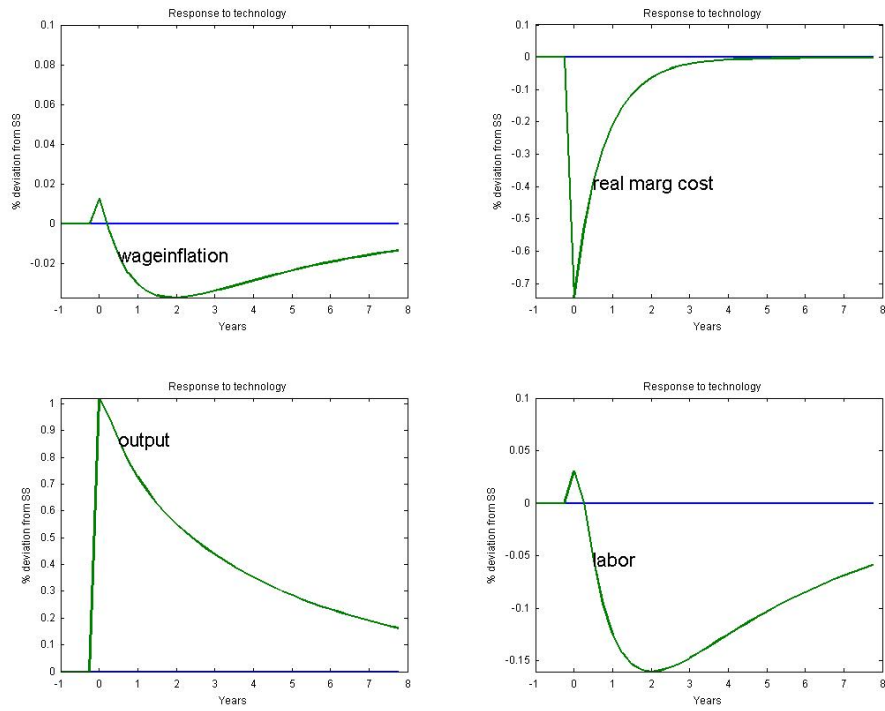


Figure A.4.: Impulse responses for the NK model with sticky prices and wages -
Technology shock - $\omega_t, \hat{m}c_t, y_t, l_t$

A.2. Monetary policy shock

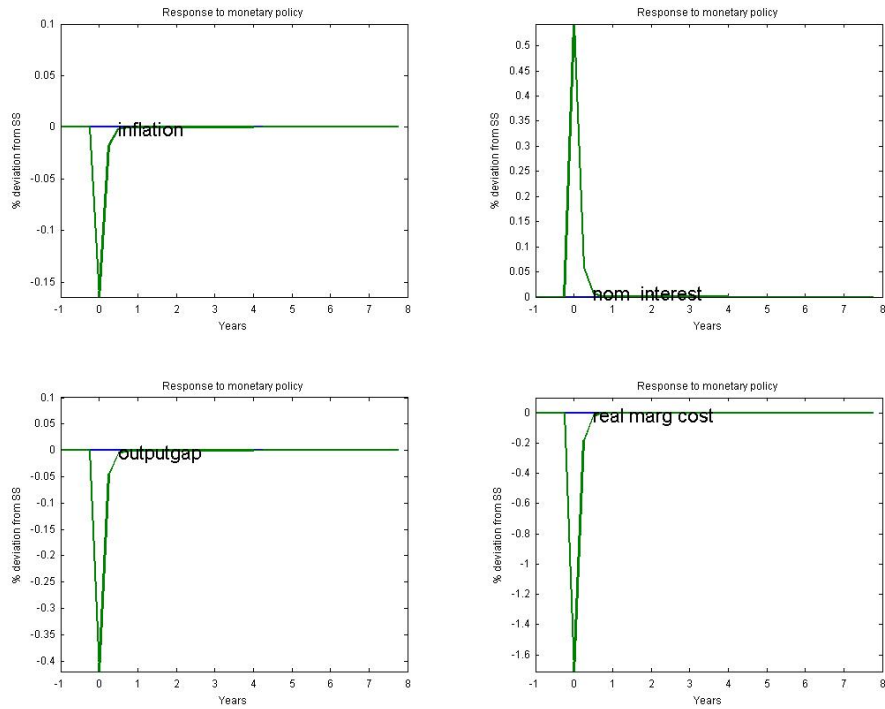


Figure A.5.: Impulse responses for the NK model with sticky prices - Monetary policy shock - $\hat{\pi}_t, \hat{R}_t, g_t, \hat{m}c_t$

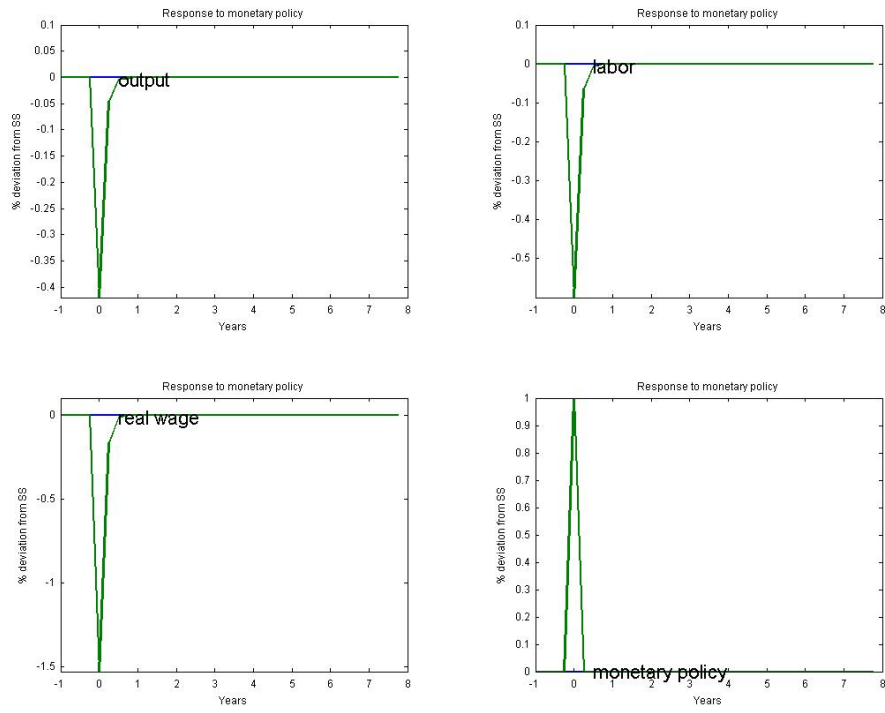


Figure A.6.: Impulse responses for the NK model with sticky prices - Monetary policy shock - y_t, l_t, ζ_t, z_t

Figure A.7 evidences that monetary policy has no real effects if prices are entirely flexible. In order to achieve the famous result of classical dichotomy, I set $\psi_p = 0.001$. Hence, there is no responses observable except for inflation.

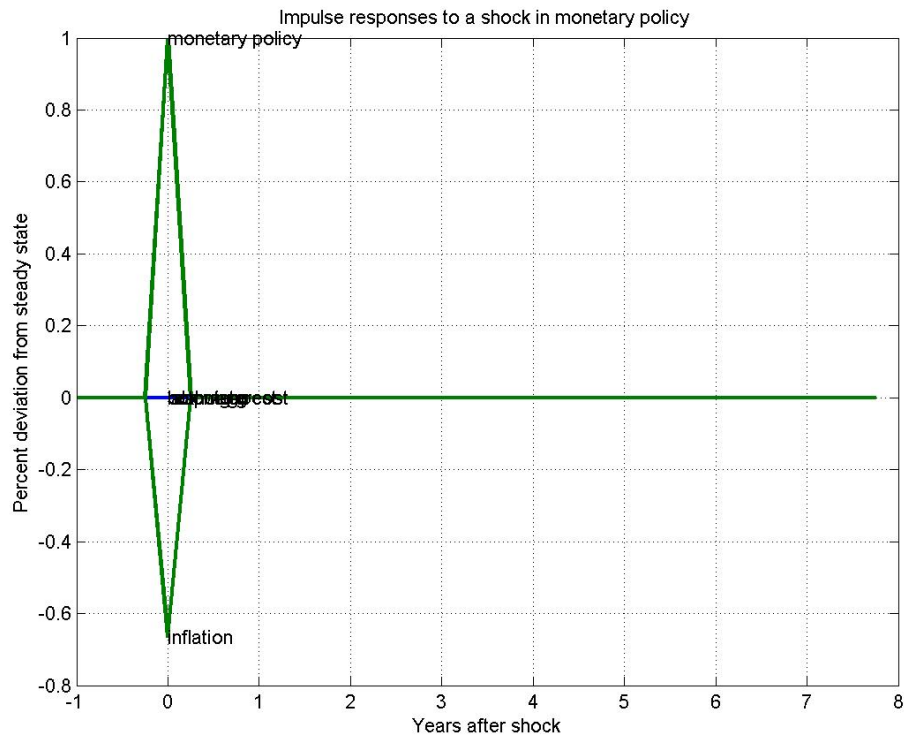


Figure A.7.: Impulse responses for the NK model with flexible prices - Monetary policy shock

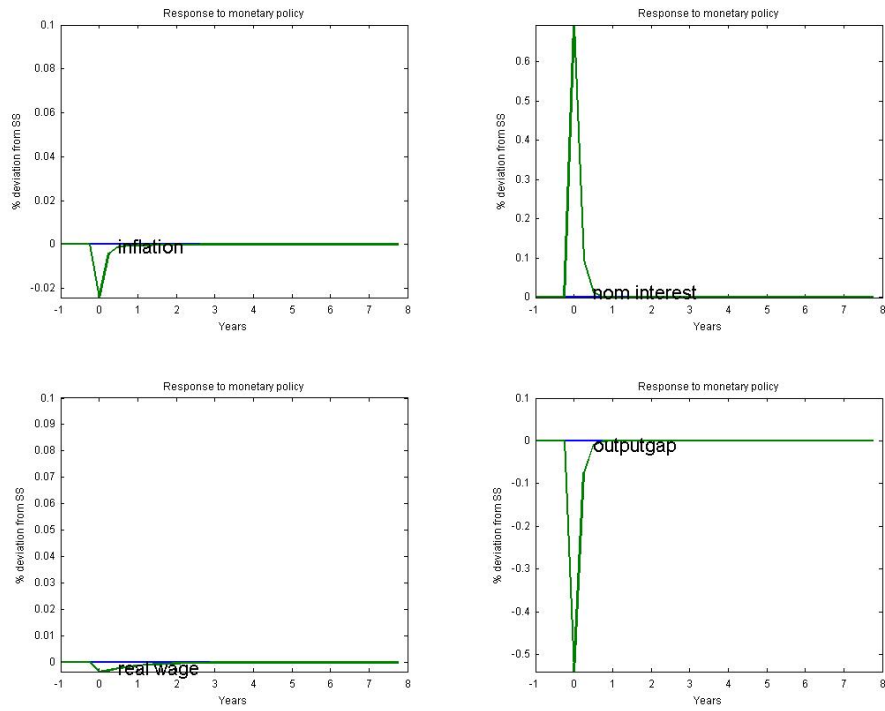


Figure A.8.: Impulse responses for the NK model with sticky prices and wages - Monetary policy shock - $\hat{\pi}_t, \hat{R}_t, \zeta_t, g_t$

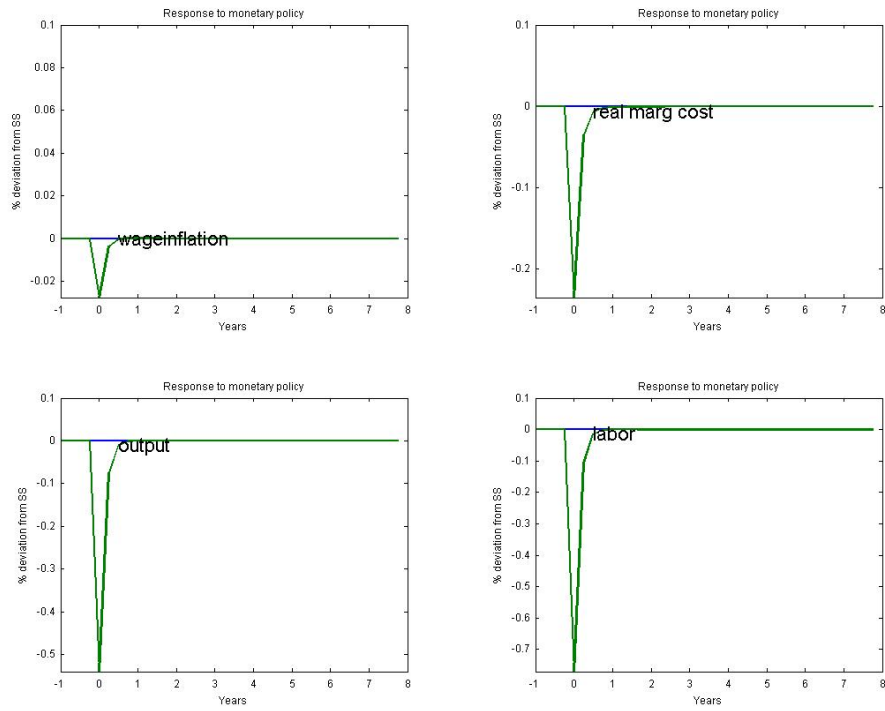


Figure A.9.: Impulse responses for the NK model with sticky prices and wages - Monetary policy shock - $\omega_t, \hat{m}c_t, y_t, l_t$

Figure A.10 is created by solely changing ψ_w to 0.5. Hence, as explained before real wage has a larger negative deviation compared to $\psi_w = 0.75$.

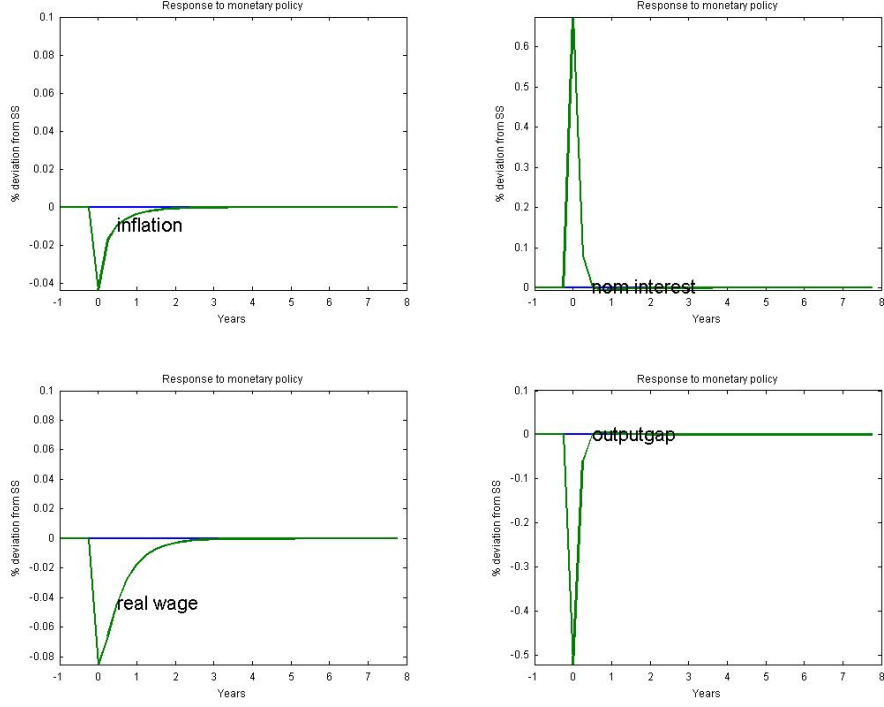


Figure A.10.: Impulse responses for the NK model with sticky prices and wages ($\psi_w = 0.5$) - Monetary policy shock - $\hat{\pi}_t, \hat{R}_t, \zeta_t, g_t$

B. Matlab code

B.1. Sticky prices

```
disp('New Keynesian Model with sticky prices');

disp('Hit any key when ready...');
pause;
% Setting parameters:
betta      = .99;      % subjective discount factor
sigma      = 1.5;      % relative risk aversion
phi        = 1.5;      % Depreciation rate for capital
alpha      = .3;      % capital share, labor elasticity of output, (1- alpha) = .7
theta_w    = 1/3;      % wage markup, wage elasticity of labor demand
psi_p      = .75;      % probability with which firm cannot reset its price
phi_pi     = 1.5;      % parameter for inflation in Taylor rule
phi_g      = .5;      % parameter for output gap in Taylor rule
phi_r      = .2;      % parameter for lagged nominal interest in Taylor rule
rho_x      = .95;      % autocorrelation for process of technology
sigma_x     = .712;    % Standard deviation of technology shock
sigma_z     = .712;    % Standard deviation of monetary policy shock
K_p        = ((1-psi_p*betta)*(1-psi_p))/psi_p

% Declaring the matrices.
VARNAMES = ['inflation      ',
            'nom  interest   ',
            'outputgap      ',
            'real marg cost  '];
```

```

        'output          ',
        'labor           ',
        'real wage       ',
        'technology       ',
        'monetary policy '];

% Endogenous state variables "x(t)": Pi(t), R(t)
% Endogenous other variables "y(t)": g(t), mc(t), y(t), l(t), zeta(t)
% Exogenous state variables "z(t)": x(t), z(t).

% DETERMINISTIC EQUATIONS:
% For [Pi(t), R(t)]:
AA = [ phi_pi, -1
       0,      0
       0,      0
       0,      0
       0,      0];
% For [Pi(t-1), R(t-1)]:
BB = [ 0, phi_r
       0, 0
       0, 0
       0, 0
       0, 0];
% For [ g(t), mc(t), y(t), l(t), zeta(t)]:
CC = [      phi_g,                0, 0,      0,      0
       ((phi+alpha+(1-alpha)*sigma)/(1-alpha)), -1, 0,      0,      0
       -1,                0, 1,      0,      0
       0,                0, 1,      -(1-alpha), 0
       0,                0, sigma, phi,      -1];
% For [x(t), z(t)]:
DD = [                0,      1
                0,      0
       -(1+phi)/(phi+alpha+(1-alpha)*sigma), 0
                -1,      0

```

```

                                0,      0];

% EXPECTATIONAL EQUATIONS:
% For [Pi(t+1), R(t+1)]:
FF = [ 1/sigma, 0
      beta, 0];
% For [Pi(t), R(t)]:
GG = [ 0, -1/sigma
      -1, 0];
% For [Pi(t-1), R(t-1)]:
HH = [ 0, 0
      0, 0];
% For [ g(t+1), mc(t+1), y(t+1), l(t+1), zeta(t+1)]:
JJ = [ 1, 0, 0, 0, 0
      0, 0, 0, 0, 0];
% For [ g(t), mc(t), y(t), l(t), zeta(t)]:
KK = [ -1, 0, 0, 0, 0
      0, K_p, 0, 0, 0];
% For [x(t+1), z(t+1)]:
LL = [ 0, 0
      0, 0];
% For [x(t), z(t)]:
MM = [ ((rho_x-1)*(1+phi))/(phi+alpha+(1-alpha)*sigma), 0
      0, 0];

% AUTOREGRESSIVE MATRIX FOR [x(t), z(t)]:
NN = [rho_x, 0
      0, 0];
Sigma = [ sigma_x^2, 0
          0, sigma_z^2];

```

B.2. Sticky prices and wages

```

disp('New Keynesian Model with sticky prices and wages');
disp('Hit any key when ready...');

```



```

pause;
% Setting parameters:
beta      = .99;      % subjective discount factor
sigma     = 1.5;      % relative risk aversion
phi       = 1.5;      % Depreciation rate for capital
alpha     = .3;       % capital share, labor elasticity of output, (1- alpha) = .7
theta_w   = 1/3;      % wage markup, wage elasticity of labor demand
psi_p     = .75;      % probability with which firm cannot reset its price
psi_w     = .75;      % probability with which hh cannot reset its wage
phi_pi    = 1.5;      % parameter for inflation in Taylor rule
phi_g     = .5;       % parameter for output gap in Taylor rule
phi_r     = .2;       % parameter for lagged nominal interest in Taylor rule
rho_x     = .95;      % autocorrelation for process of technology
sigma_x   = .712;     % Standard deviation of technology shock
sigma_z   = .712;     % Standard deviation of monetary policy shock

K_p       = ((1-psi_p*beta)*(1-psi_p))/psi_p
K_w       = ((1-psi_w*beta)*(1-psi_w))/(psi_w*(1+phi*((1+theta_w)/theta_w)))
gamma_f   = (phi+sigma)/(phi+alpha+(1-alpha)*sigma)
% Declaring the matrices.
VARNAMES = ['inflation      ',
            'nom interest   ',
            'real wage      ',
            'outputgap      ',
            'wageinflation  ',
            'real marg cost ',
            'output         ',
            'labor          ',
            'technology     ',
            'monetary policy'];
% DETERMINISTIC EQUATIONS:

% for [Pi(t), R(t), zeta(t)]:
AA = [ phi_pi, -1,          0

```

```

        0,      0,      1
        0,      0,      0
        0,      0,      0
        -1,     0,     -1];
% for [Pi(t-1), R(t-1), zeta(t-1)]:
BB = [  0, phi_r, 0
        0,      0, 0
        0,      0, 0
        0,      0, 0
        0,      0, 1];
% For [ g(t), omega(t), mc(t), y(t), l(t)]:
CC = [  phi_g,      0, 0, 0, 0
        alpha/(1-alpha), 0, -1, 0, 0
        -1,      0, 0, 1, 0
        0,      0, 0, 1, -(1-alpha)
        0,      1, 0, 0, 0];
% For [ x(t), z(t)]:
DD = [  0,      1
        -gamma_f,      0
        -(1+phi)/(phi+alpha+(1-alpha)*sigma), 0
        -1,      0
        0,      0];
% EXPECTATIONAL EQUATIONS:

% for [Pi(t+1), R(t+1), zeta(t+1)]:
FF = [  1/sigma,      0,      0
        betta,      0,      0
        0,      0,      0 ];
% for [Pi(t), R(t), zeta(t)]:
GG = [ 0,      -1/sigma,      0
        -1,      0,      0
        0,      0,      -K_w ];
% for [Pi(t-1), R(t-1), zeta(t-1)]:
HH = [ 0, 0, 0

```

```

    0, 0, 0
    0, 0, 0];
% For [ g(t+1), omega(t+1), mc(t+1), y(t+1), l(t+1)]:
JJ = [ 1, 0, 0, 0, 0
       0, 0, 0, 0, 0
       0, betta, 0, 0, 0];
% For [ g(t), omega(t), mc(t), y(t), l(t)]:
KK = [ -1, 0, 0, 0, 0
       0, 0, K_p, 0, 0
       K_w*(phi/(1-alpha)+sigma), -1, 0, 0, 0];
% For [ x(t+1), z(t+1)]:
LL = [ 0, 0
       0, 0
       0, 0];
% For [ x(t), z(t)]:

MM = [ ((rho_x-1)*(1+phi))/(phi+alpha+(1-alpha)*sigma), 0
       0, 0
       K_w*gamma_f, 0 ];
% AUTOREGRESSIVE MATRIX FOR [x(t), z(t)]:

NN = [rho_x, 0
      0, 0];

Sigma = [ sigma_x^2, 0
         0, sigma_z^2];

```