Business Cycle Dynamics under Catching Up with the Joneses Preferences in Leisure

Diploma Thesis

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Abstract

In real business cycle models, the concept of habit formation, which can explain phenomena such as the *equity premium puzzle*, is mostly restricted to the consumption part of the utility function of the representative household. At least, a detailed analysis of the dynamics of a model with habit formation in both consumption and leisure does not yet exist. *A priori*, there is, however, no reason justifying such a restriction.

Thus, this diploma thesis extends standard real business cycle models by introducing the concept of external habit formation (more precisely, so-called *catching up with the Joneses* preferences) not only into the consumption part, but also into the leisure part of the utility function. Utility depends on leisure but additionally also on some benchmark level which is made up by aggregate average leisure in past periods.

We examine the general model dynamics and then concentrate on the implications for consumption- and leisure-choices after a technology shock in such an economy, scrutinizing the corresponding impulse-responses.

Our main finding is that introducing habit formation into leisure in a model which already comprises habit formation in consumption smoothes leisure-choices, but only has a very small influence on consumption-choices, let alone the other variables. This may be seen as a justification of the fact that until now, habit formation in leisure has been neglected in economic theory.

Several variations of the model (relative rather than absolute consumption and leisure, *keeping up with the Joneses* preferences rather than *catching up with the Joneses* preferences, taxes) are considered and analyzed.

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1 Introduction

In economic theory, the concept of habit formation has a long history as an alternative to the time-separable utility function employed in standard real business cycle models. It has been given consideration especially since Constantinides (1990) showed that it can solve the so-called *equity premium puzzle*, identified by Mehra and Prescott (1985).

There is already a great amount of literature on models that include habit formation into the consumption part of the utility function of the representative household. In these models, utility depends on the consumption level and additionally on some benchmark level, the habit stock, which is mostly made up by lagged values of aggregate average consumption. Most surprisingly, very few articles consider the possibility of extending external habit formation to the leisure part of the utility function, although there is *a priori* no reasonable justification to restrict it to consumption. Intuitively, if other individuals decide to spend more time on leisure today, you may well be inclined to raise your leisure level in the future, too. Thus, this diploma thesis aims at answering the economic question what business cycle dynamics look like when there is external habit formation in the form of *catching up with the Joneses* preferences in both consumption and leisure. In this context, we also ask whether the restriction of habit formation to consumption is justified or not. Consequently, we are especially interested in the impulse-responses of consumption and leisure to a technology shock in such an economy.

The methodological framework we use for the analysis of these dynamics is a real business cycle model. Habit formation for both consumption and leisure is modeled in a way that we can distinguish between a transitory and a persistent form, depending on the calibration of the parameters. Departing from a model with habit formation in consumption only, we review the general model dynamics as well as the dynamics of consumption and leisure in particular, before examining a model with habit formation only in leisure. Then we merge these two concepts and obtain a model with external habit formation in both consumption and leisure which can be regarded as the benchmark model of this thesis. We compare the impulse-responses of consumption and leisure of this benchmark model to the model with habit formation in consumption only, which has already been analyzed in literature. Thereby, we isolate the effect of habit formation in leisure. After that, we examine for what calibration of the curvature parameters the influence of habit formation in leisure on the choice of consumption is as high as possible. Finally, we also analyze the implications of habit formation in leisure for the substitution effect and the income effect in labor supply.

Our main findings are as follows: Habit formation in consumption is much more important than habit formation in leisure. While habit formation in consumption affects leisure-choices significantly, habit formation in leisure only has a weak influence on consumption-choices. Generally, introducing habit formation in leisure changes the dynamics of the model only to a very small degree. The three most influenced variables are, as expected, consumption, leisure and labor supply. The higher the curvatures of consumption and leisure is, the more important the influence of habit formation in leisure on the choice of consumption becomes. However, for microeconomically reasonable values of the curvature parameters, the influence remains very small. All these results justify that until now habit formation in leisure has been neglected in economic theory.

This thesis is organized as follows. In the next section, we present the concepts of habit formation in general and *catching up with the Joneses* preferences in particular. Here we also put this thesis into the context of the existing literature on the subject of habit formation. Section 3 points out why habit formation has been introduced into the consumption part of the utility function and provides some theoretical consideration on why there is a priori no reason against extending it to the leisure part. In section 4 we present the model and explain how the assumptions it is based on are translated into the formulas. The model is solved in section 5 and the results are examined in section 6. In section 7 we present some modifications of our basic model such as *keeping up with the Joneses* preferences instead of *catching up with the Joneses* preferences, habit formation in relative rather than absolute terms in consumption and leisure, and a model with taxes. Section 8 discusses the main results obtained and section 9 concludes.

2 Literature

In this section, we first point out the key features of habit formation before presenting its different types. We explain why habit formation has been introduced into the utility function of the representative household and review some models in literature where this has been done.

2.1 Key features of habit formation

In standard real business cycle models, the representative household maximizes a time-separable utility function subject to a budget constraint, although – according to social scientists – status seeking, habit and envy play an important role in human behavior[1]. In order to account for these features, models have been proposed where the standard utility function is replaced by a "time non-separable" version.

Relating to consumption, this means that the individual's utility depends not only on his own consumption but also on some consumption reference level, mostly lagged values of aggregate average consumption. Technically, this means that people "build up" the benchmark level (i.e. the consumption habit) over a long time. Once this has been done, the response of consumption to a technology shock is typically attenuated by this habit: People remember their past consumption-choices and only slowly deviate from them.

The consequence of introducing habit formation in consumption into a standard real business cycle model is generally a flatter consumption-deviation from its steady state or, put differently, consumption smoothing.

2.2 Different types of habit formation

2.2.1 Internal vs. external habit formation

In literature, one essentially distinguishes between "internal" and "external" habit formation. In the former case, individuals typically compare their own consumption C_t to a reference level that is made up by lagged values of their own consumption C_{t-1}, C_{t-2}, etc. Sometimes, this type of habit formation is also simply called "habit formation" and the agent is referred to as "inward-looking" (Alvarez-Cuadrado, Monteiro and Turnovsky, 2004). A typical example of a model with internal habit formation can be found in Heaton (1995).

[1] Campbell and Cochrane (1999) state that "Habit formation captures a fundamental feature of psychology: repetition of a stimulus diminishes the perception of the stimulus and responses to it." (Campbell and Cochrane, 1999, Introduction).

In the latter case, agents compare their own consumption C_t to aggregate average consumption $C_t^{(a)}$ or its lagged values. These models are alternatively known by the type of preferences employed (*catching up with the Joneses* preferences or *keeping up with the Joneses* preferences) and the agent is described as "outward-looking" (Alvarez-Cuadrado, Monteiro and Turnovsky, 2004).

2.2.2 *Catching up with the Joneses* preferences vs. *keeping up with the Joneses* preferences

In the case of external habit formation, one distinguishes preferences between *catching up with the Joneses* and *keeping up with the Joneses*. In the former case, people compare their own consumption to aggregate consumption in one or several past periods whereas in the latter case, aggregate average consumption in the same period is used as the benchmark level.

2.2.3 Absolute vs. relative terms

There are several ideas about how individual consumption should be compared to the benchmark level. Some authors simply use the difference between individual consumption and the habit stock whereas others prefer the ratio of these two values. This latter formulation has the advantage that the individual's marginal utility cannot become negative. However, Campbell and Cochrane (1999) argue that such a relative specification in habit formation eliminates changing risk aversion.

In this thesis, we concentrate on external habit formation. To be more specific, we implement a model with *catching up with the Joneses* preferences in both consumption and leisure. Habit formation in these two variables is modeled by the *differences* of individual consumption and leisure from their respective habit stocks.

2.3 Models with habit formation in literature

The first work on the microeconomic foundations of habit formation can be found in Dusenberry (1949). Overviews over habit formation in consumption are given by Deaton and Muellbauer (1980) and Deaton (1992).

2.3.1 The *equity premium puzzle* and its resolution with the help of habit formation

Over the past century, annual returns on stocks, that is on risky assets, have outperformed the riskless interest rate by about six percentage points. It is commonly accepted in economic theory that returns on stocks should be higher than returns on bonds because of the existence of a risk premium. Yet, Mehra and Prescott (1985) show that for standard general equilibrium models[2] micro- and macroeconomically reasonable values of the coefficient of relative risk aversion lead to an equity premium that is by far too small compared to the empirical value. More precisely, in their paper the authors find a value of 0.34 percent which is exceeded by the actual value of 6 percent by magnitude. This economic phenomenon is commonly known as the *equity premium puzzle*.

Constantinides (1990) shows that introducing habit formation into the consumption part of the utility function of the representative household can solve the equity premium puzzle. He argues that "the key role of habit persistence is to drive a wedge between the RRA [relative risk aversion, the author] and the inverse of the intertemporal elasticity of substitution in consumption." (Constantinides, 1990) and develops a variant of the neoclassical growth model in continuous time that accounts for habit formation in consumption. His conclusion is that "the equity premium puzzle is resolved in the sense that the model generates mean and variance of the consumption growth rate with the mean RRA coefficient as low as 2.81" (Constantinides, 1990).

Further important milestones in literature on the equity premium puzzle are the papers of Abel (1990), who introduces a utility function with habit formation in consumption in discrete time for an asset pricing model, and of Campbell and Cochrane (1999), who develop a consumption-based model with habit formation in discrete time. With their model Campbell and Cochrane (1999) are able to generate both the equity premium and a constant riskfree rate.

Abel (1999) distinguishes between two components of the equity premium: a risk premium and a term premium. He analyzes in detail these two components in a general equilibrium model with *catching up with the Joneses* preferences.

All these authors conclude that habit formation in consumption can solve the equity premium puzzle. The intuition is simple: As Fuhrer (2000) explains, consumers do not like it when consumption falls rapidly from one period to the next. Risky assets, however, make such large and rapid cuts

[2] This means models without heterogeneous agents, market imperfections, money, etc.

in consumption more probable than this is the case if the agent invests in riskless assets. Therefore, agents demand a higher equity premium to hold risky assets in a model with habit formation than this is the case in a model without.

2.3.2 Further important milestones

Ljungqvist and Uhlig (2000) analyze tax policies in an economy with *catching up with the Joneses* preferences which lead to externalities in consumption. The authors show that procyclical taxes can be used to internalize these externalities; they find that after a positive technology shock, taxes should be raised whereas in recessions they should be cut (Ljungqvist and Uhlig, 2000, Conclusion).

Lettau and Uhlig (2000) argue that introducing habit formation into consumption leads to a new problem. To be more specific the response of consumption to a shock in technology becomes far too small. The authors show that this consumption volatility puzzle can be solved by a model where technology shocks are highly persistent and labor is stochastic.

2.3.3 Habit formation in leisure

Until now, we have just considered models with habit formation in consumption but not in leisure. In this context, the closest model to ours is that analyzed by Gurdgiev (2005). The author extends the model of Ljungqvist and Uhlig (2000) by introducing *keeping up with the Joneses* preferences in leisure and then examines optimal taxation in the presence of this kind of external habit formation, as Ljungqvist and Uhlig (2000) do for their benchmark model. There is, however, no closer examination of the business cycle dynamics of this extended model and the author does not deal with the case of *catching up with the Joneses* preferences, either.

Gurdgiev (2005) finds that under *keeping up with the Joneses* preferences in both consumption and leisure, optimal tax on capital is negative, whereas optimal labor tax strongly depends on the calibration of the parameters and especially on whether habit formation in consumption is more important than habit formation in leisure or the other way around.

3 Facts

This section has two objectives: On the one hand it provides some facts which explain why habit formation has been introduced into consumption. On the other hand, we argue why there is *a priori* no reason to restrict habit formation to the consumption part of the utility function.

3.1 Business cycle facts

Already Campbell and Deaton (1989) show that in practice consumption reacts slowly to shocks. Many neo-keynesian Structural Vector Autoregressions (SVARs) investigate the reactions of consumption to a monetary shock or to a change in interest rates. For example, Christiano, Eichenbaum and Evans (2005) demonstrate that after an expansionary monetary policy shock, the response of consumption is hump-shaped and reaches its peak after about six quarters. Yet, as Fuhrer (2000) points out, in standard real business cycle models consumption reacts immediately after shocks that change lifetime resources.

To solve this puzzle of discrepancy between the empirical results of the Structural Vector Autoregressions on the one hand and the predictions of the real business cycle models on the other hand, habit formation in consumption has been introduced into the utility function of the Real Business Cycles models. For example, Bouakez, Cardia and Ruge-Murcia (2005) do so and reject the hypothesis of no habit formation. Bouakez, Cardia and Ruge-Murcia (2005) develop a model with habit formation and capital adjustment costs. They compare the impulse-responses to a monetary shock in this model with those of a Vector Autoregressions (VAR) of order 2 and find that both predict a hump-shaped consumption response, although its peak occurs one period later in the model compared to the Vector Autoregression. However, this shows that habit formation improves the prediction of the Real Business Cycles model.

3.2 Socio-economic issues

As Ljungqvist and Uhlig (2000) argue, “Envy is one important motive of human behavior”. Habit formation, more precisely *catching up with the Joneses* preferences, are well suited to formalize people’s envy in consumption. The question that has to be asked here is why envy should be restricted only to consumption. People may well be envious of others because these others have more free time.

3.3 Leisure as a special consumption good

Essentially, leisure can be seen as a special consumption good. Indeed, free time increases utility just as any usual consumption good does. The difference to other consumption goods is that leisure is directly related to labor input. However, this characteristic feature of leisure does not imply that people choose the amount of leisure they consume without comparing it to the leisure-choices of the other individuals; there can also be envy in the choice of leisure.

Thus, we basically state that when we allow habit formation in some consumption goods, we should do so in *all* of them, including the special consumption good leisure.

3.4 Too small responses to a technology shock in labor supply

Standard real business cycle models generate too small responses of labor supply to a technology shock, compared to real data. When habit formation in leisure is introduced, we expect that this smoothes leisure demand and therefore labor supply even more.

This issue might pose an inconvenience to our model. Thus, we have to examine in detail to what extent habit formation in leisure smoothes leisure demand.

4 The Model

In this section, we describe the model, which is an extension of those ones analyzed by Lettau and Uhlig (2000) and Ljungqvist and Uhlig (2000). First we explain in detail the household's utility function as it represents the crucial hypothesis of our model. Then, we state the profit maximization problem of the firm before stating the market clearing conditions.

As usual, capital Arabic letters stand for variables in levels, Greek letters stand for parameters. Superscripts $^{(d)}$ and $^{(s)}$ denote demand and supply, respectively.

4.1 Household

Consider an infinitely-lived representative household that maximizes his intertemporal utility as follows:

$$\max_{\{C_j;L_j;I_j\}_{j=t}^{\infty}} E_t \sum_{j=t}^{\infty} \beta^{j-t} \left[\frac{(C_j - V_{j-1})^{1-\gamma} - 1}{1-\gamma} + \chi \frac{(L_j - U_{j-1})^{1-\vartheta} - 1}{1-\vartheta} \right], \quad (1)$$

subject to the constraints

$$C_t + I_t = W_t N_t^{(s)} + D_t K_{t-1}^{(s)} + \Pi_t, \quad (2)$$

$$L_t + N_t^{(s)} = 1, \quad (3)$$

$$K_t^{(s)} = (1-\delta) K_{t-1}^{(s)} + I_t, \quad (4)$$

which have to be fulfilled in each period t. C_t and L_t represent the individual household's consumption and leisure in period t, respectively. Note that the utility function is additive-separable in consumption and leisure. The representative household can allocate his time endowment, normalized to 1, to supplied labor $N_t^{(s)}$ and leisure L_t.

Capital chosen in period $t-1$ and lent in period t, denoted by $K_t^{(s)}$, is the sum of capital lent in period $t-1$, depreciated by a fraction of δ, and of investment in period t, denoted by I_t.

W_t stands for real wage, D_t for dividends on capital chosen in period $t-1$ and Π_t for profit in period t, which is redistributed to the representative household.

χ measures the importance of the leisure part of the utility function relative to the consumption part and is assumed to be given and constant.

γ and ϑ stand for the curvatures of consumption and leisure, respectively.

V_t and U_t are the habit stocks of past average consumption and leisure. More specifically, they are recursively given by

$$V_{t-1} = (1 - \phi_C)\alpha_C C_{t-1}^{(a)} + \phi_C V_{t-2}, \tag{5}$$

$$U_{t-1} = (1 - \phi_L)\alpha_L L_{t-1}^{(a)} + \phi_L U_{t-2}, \tag{6}$$

where $C_t^{(a)}$ and $L_t^{(a)}$ denote aggregate average consumption and leisure in period t, respectively.

Equivalently, at any time t, V_{t-1} and U_{t-1} are completely determined by the sequences $\left\{C_j^{(a)}\right\}_{j=0}^{t-1}$ or $\left\{L_j^{(a)}\right\}_{j=0}^{t-1}$, respectively, together with V_{-1} and U_{-1}[3]:

$$V_{t-1} = (1 - \phi_C)\alpha_C \sum_{j=1}^{t} (\phi_C)^{t-j} C_{j-1}^{(a)} + (\phi_C)^t V_{-1}, \tag{7}$$

$$U_{t-1} = (1 - \phi_L)\alpha_L \sum_{j=1}^{t} (\phi_L)^{t-j} L_{j-1}^{(a)} + (\phi_L)^t U_{-1}. \tag{8}$$

We make the additional assumption that in the initial state of the economy, there is no consumption habit and no leisure habit, which means that $V_{-1} = U_{-1} = 0$. Thus, V_{t-1} and U_{t-1} can be rewritten as follows[4]:

$$V_{t-1} = (1 - \phi_C)\alpha_C \sum_{j=1}^{t} (\phi_C)^{t-j} C_{j-1}^{(a)}, \tag{9}$$

$$U_{t-1} = (1 - \phi_L)\alpha_L \sum_{j=1}^{t} (\phi_L)^{t-j} L_{j-1}^{(a)}. \tag{10}$$

The specifications of the utility function and of the process for V_t are closely related to those in the model by Ljungqvist and Uhlig (2000). The process of U_t is similar to that of V_t. Note that α_C and α_L describe the overall importance of habit formation in consumption and leisure, respectively, in the utility function. Suppose, for example, that $\alpha_C = 0$ in equation (9). This

[3]This can be derived by recursively plugging in the formulas (5) and (6).

[4]Without the additional assumption $V_{-1} = U_{-1} = 0$, equations (9) and (10) nevertheless remain good approximations for equations (7) and (8), because asymptotically, i.e. for large values of t, the second terms on the right-hand side of these two equations can be neglected, because $0 \leq \phi_C < 1$ and $0 \leq \phi_L < 1$. Intuitively, this means that, as this economy moves further away from its initial state, the initial habits V_{-1} and U_{-1} play a less and less important role.

implies $V_{t-1} = 0$, which means that habit formation in consumption does not play any role any more in the utility function.

ϕ_C and ϕ_L, by contrast, describe the persistence of the habit with respect to following periods. For instance, if ϕ_C is close to 0, consumption before period $t-1$ is very unimportant for the utility in period t, whereas consumption in period $t-1$ is very important. If, in contrast to that, ϕ_C is closer to 1, lagged consumption of all periods plays a role for consumption habit.

Using V_{t-1} and U_{t-1} rather than V_t and U_t in the utility function suggests that in period t, the values of V and U, to which the household compares consumption and leisure he is about to choose, are already given, as they have been chosen in period $t-1$. Thus, the utility function contains a *catching-up* term rather than a *keeping-up* term in consumption and leisure, as in the latter case, the utility function would contain V_t and U_t instead of V_{t-1} and U_{t-1}.

Note that, in our context of external habit formation, the individual household is assumed to be atomic, which implies that he takes the sequences of V_t and U_t as given in the maximization problem.

Following Campbell and Cochrane (1999), we define the *consumption surplus ratio* and the *leisure surplus ratio* as

$$S_t^{(C)} = \frac{C_t - V_{t-1}}{C_t}, \tag{11}$$

$$S_t^{(L)} = \frac{L_t - U_{t-1}}{L_t}, \tag{12}$$

respectively, and from that we derive the local curvatures of the utility function in consumption and leisure as follows:

$$\eta_t^{(C)} = -C_t \frac{u_{11}(C_t; L_t; V_{t-1}; U_{t-1})}{u_1(C_t; L_t; V_{t-1}; U_{t-1})} = \frac{\gamma}{S_t^{(C)}} = \gamma \frac{C_t}{C_t - V_{t-1}}, \tag{13}$$

$$\eta_t^{(L)} = -L_t \frac{u_{22}(C_t; L_t; V_{t-1}; U_{t-1})}{u_2(C_t; L_t; V_{t-1}; U_{t-1})} = \frac{\gamma}{S_t^{(L)}} = \gamma \frac{L_t}{L_t - U_{t-1}}, \tag{14}$$

where $u(C_t; L_t; V_{t-1}; U_{t-1})$ denotes the utility function of the representative household.

Several aspects are worth noting. First, as Campbell and Cochrane (1999) remark, $S_t^{(C)}$ and $S_t^{(L)}$ can be used to evaluate the state the agent is in. In extremely bad times, these values are close to zero, meaning that C_t and L_t are close to their respective habits V_{t-1} and U_{t-1}. Likewise, in extremely good times C_t and L_t are so high that habits become negligible and $S_t^{(C)}$ and $S_t^{(L)}$ converge to 1.

Second, in a usual power utility function without any habit formation (that is, with $V_t = U_t = 0$ for all periods), these curvatures could be interpreted as coefficients of constant relative risk aversions (CRRA) for consumption or for leisure, respectively, as in this case the corresponding surplus ratios would equal 1 so that they would drop out.

Third, we can identify the "wedge between the relative risk aversion of the representative household and the inverse of the intertemporal elasticity of substitution in consumption" described by Constantinides (1990) as well as an analogous wedge for leisure.

4.2 Firm

We consider a representative firm that maximizes its profit as follows:

$$\Pi_t = \max_{K_{t-1}^{(d)};N_t^{(d)}} \left[Y_t - D_t K_{t-1}^{(d)} - W_t N_t^{(d)} \right], \tag{15}$$

taking into account the standard Cobb-Douglas function of the form

$$Y_t = Z_t (K_{t-1}^{(d)})^{\theta} (N_t^{(d)})^{1-\theta}. \tag{16}$$

$K_{t-1}^{(d)}$ is demanded capital, $N_t^{(d)}$ is demanded labor and θ is the capital share, which is assumed to be constant. Z_t is an exogenous shock in technology, that is, total factor productivity (TFP). The stochastic sequence of $\log(Z_t)$ is given by an autoregressive process of order 1, AR(1):

$$\log Z_{t+1} = (1-\psi)\log \tilde{Z} + \psi \log Z_t + \epsilon_{t+1}, \ \ \epsilon_t \sim \text{i.i.d. } \mathcal{N}(0;\sigma_\epsilon^2). \tag{17}$$

where $0 < \psi \leq 1$ describes the persistence of the technology shock and $\tilde{Z}$ is some constant.

4.3 Market clearing

In equilibrium, all markets are cleared, that is:

$$Y_t = C_t + I_t, \tag{18}$$

$$N_t = N_t^{(d)} = N_t^{(s)}, \tag{19}$$

$$K_{t-1} = K_{t-1}^{(d)} = K_{t-1}^{(s)}. \tag{20}$$

5 Model Analysis

In this section we derive a solution to our model. Due to possible consumption and leisure externalities implied by the *catching-up* hypothesis, we do not reduce the model to the problem of the social planner. Instead, we start by simplifying the model and then derive the first-order conditions before calculating the steady state. Then, the next step is log-linearization, followed by solving for the recursive equilibrium law of motion.

5.1 Simplification and conventions

5.1.1 Market clearing

First of all, due to market clearing we can drop the superscripts $^{(d)}$ and $^{(s)}$ for demand and supply.

5.1.2 Household

After some algebra, the problem of the household stated in section 4 simplifies to the following problem:

$$\max_{\{C_j;N_j;K_j\}_{j=t}^{\infty}} E_t \sum_{j=t}^{\infty} \beta^{j-t} \left[\frac{(C_j - V_{j-1})^{1-\gamma} - 1}{1-\gamma} + \chi \frac{(1 - N_j - U_{j-1})^{1-\vartheta} - 1}{1-\vartheta} \right], \tag{21}$$

subject to the simplified constraint:

$$C_t + K_t = (1 - \delta + D_t) K_{t-1} + W_t N_t + \Pi_t. \tag{22}$$

From this, we obtain the following Lagrangian for the problem of the representative household:

$$\mathcal{L} = E_t \sum_{j=t}^{\infty} \beta^{j-t} \Bigg\{ \left[\frac{(C_j - V_{j-1})^{1-\gamma} - 1}{1-\gamma} + \chi \frac{(1 - N_j - U_{j-1})^{1-\vartheta} - 1}{1-\vartheta} \right] + \lambda_j \left[(1 - \delta + D_j) K_{j-1} + W_j N_j + \Pi_j - C_j - K_j \right] \Bigg\}. \tag{23}$$

As Lettau and Uhlig (2000) remark, the same symbols for individual and for aggregate consumption can be used as long as environments with a representative agent are studied. This is the case here, and additionally the same holds for individual and aggregate leisure. Thus, we may drop superscript $^{(a)}$ in the equations defining habit formation.

However, we do not plug in these simplified relationships for V_{t-1} and U_{t-1} into equation (23) on the preceding page because we consider external habit formation: The agent does not internalize the external effects.

5.1.3 Firm

The problem of the representative firm simplifies as follows:

$$\Pi_t = \max_{K_{t-1};N_t} \left[Z_t K_{t-1}^{\theta} N_t^{1-\theta} - D_t K_{t-1} - W_t N_t \right]. \tag{24}$$

5.1.4 Conventions

We define the real interest rate R_t as follows:

$$R_t = 1 - \delta + D_t. \tag{25}$$

Finally, we add a *No-Ponzi-Scheme* condition to the other equations:

$$\lim_{t\to\infty} E_0 \left[\beta^t \lambda_t K_{t+1} \right] = 0. \tag{26}$$

5.2 First-order conditions

Now we can derive the first-order conditions for the problems of the representative household and of the firm. Note again that the representative household takes V_{t-1} and U_{t-1} as given. The first-order conditions are as follows:

5.2.1 Household

$$\frac{\partial \mathcal{L}}{\partial C_t} = 0 \quad \Rightarrow \quad (C_t - V_{t-1})^{-\gamma} - \lambda_t = 0, \tag{27}$$

$$\frac{\partial \mathcal{L}}{\partial N_t} = 0 \quad \Rightarrow \quad -\chi(1 - N_t - U_{t-1})^{-\vartheta} + \lambda_t W_t = 0, \tag{28}$$

$$\frac{\partial \mathcal{L}}{\partial K_t} = 0 \quad \Rightarrow \quad -\lambda_t + \beta E_t \left[\lambda_{t+1}(1 - \delta + D_{t+1}) \right] = 0, \tag{29}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} = 0 \quad \Rightarrow \quad (1 - \delta + D_t)K_{t-1} + W_t N_t - C_t - K_t = 0. \tag{30}$$

5.2.2 Firm

Profit maximization of the representative firm yields:

$$\frac{\partial \Pi_t}{\partial K_{t-1}} = 0 \qquad \Rightarrow \qquad D_t = \theta Z_t K_{t-1}^{\theta-1} N_t^{1-\theta} = \theta \frac{Y_t}{K_{t-1}}, \tag{31}$$
$$\frac{\partial \Pi_t}{\partial N_t} = 0 \qquad \Rightarrow \qquad W_t = (1-\theta) Z_t K_{t-1}^{\theta} N_t^{-\theta} = (1-\theta) \frac{Y_t}{N_t}, \tag{32}$$

as usual for a standard Cobb-Douglas production function and perfect markets. Profit Π_t is zero:

$$\Pi_t = Y_t - D_t K_{t-1} - W_t N_t = Y_t - \theta \frac{Y_t}{K_{t-1}} K_{t-1} - (1-\theta)\frac{Y_t}{N_t} N_t = 0. \tag{33}$$

5.3 Rewriting the equations

From the first-order conditions of the problems of the household and the firm, we obtain the following equations that describe the model[5]:

$$Y_t = C_t + I_t, \tag{34}$$
$$K_t = I_t + (1-\delta) K_{t-1}, \tag{35}$$
$$Y_t = Z_t K_{t-1}^{\theta} N_t^{1-\theta}, \tag{36}$$
$$(1-\theta)\frac{Y_t}{N_t} = \chi \frac{(C_t - V_{t-1})^{\gamma}}{(L_t - U_{t-1})^{\vartheta}}, \tag{37}$$
$$R_t = 1 - \delta + \theta \frac{Y_t}{K_{t-1}}, \tag{38}$$
$$L_t = 1 - N_t, \tag{39}$$
$$V_{t-1} = (1-\phi_C)\alpha_C C_{t-1} + \phi_C V_{t-2}, \tag{40}$$
$$U_{t-1} = (1-\phi_L)\alpha_L L_{t-1} + \phi_L U_{t-2}, \tag{41}$$
$$1 = \beta E_t \left[\left(\frac{C_t - V_{t-1}}{C_{t+1} - V_t} \right)^{\gamma} R_{t+1} \right], \tag{42}$$
$$\log Z_{t+1} = (1-\psi)\log \tilde{Z} + \psi \log Z_t + \epsilon_{t+1}. \tag{43}$$

[5] We split up the budget constraint of the representative household into equations (34) and (35) with the help of equation (4) on page 14 describing capital accumulation, as we are interested in investment.

5.4 Steady State

Steady state variables are denoted by capital letters with a bar and without time subscript, because in the steady state we can drop time indices, as variables are stationary here. The following equations describe the steady state:

$$\bar{Y} = \bar{C} + \bar{I}, \tag{44}$$
$$\bar{I} = \delta\bar{K}, \tag{45}$$
$$\bar{Y} = \bar{Z}\bar{K}^{\theta}\bar{N}^{1-\theta}, \tag{46}$$
$$(1-\theta)\frac{\bar{Y}}{\bar{N}} = \chi\frac{(\bar{C}-\bar{V})^{\gamma}}{(\bar{L}-\bar{U})^{\vartheta}}, \tag{47}$$
$$\bar{R} = 1 - \delta + \theta\frac{\bar{Y}}{\bar{K}}, \tag{48}$$
$$\bar{L} = 1 - \bar{N}, \tag{49}$$
$$\bar{V} = (1-\phi_C)\alpha_C\bar{C} + \phi_C\bar{V} \Rightarrow \bar{V} = \alpha_C\bar{C}, \tag{50}$$
$$\bar{U} = (1-\phi_L)\alpha_L\bar{L} + \phi_L\bar{U} \Rightarrow \bar{U} = \alpha_L\bar{L}, \tag{51}$$
$$1 = \beta E_t\left[\left(\frac{\bar{\lambda}}{\bar{\lambda}}\right)^{\gamma}\bar{R}\right] = \beta\bar{R}, \tag{52}$$
$$\bar{Z} = \tilde{Z}. \tag{53}$$

5.5 Log-linearization

We can now proceed to log-linearize the model. Let small letters denote log-deviations of variables from their respective steady states, so that for any variable X_t it holds that $x_t = \log(X_t) - \log(\bar{X})$. For more technical information on the principles of log-linearization, see Uhlig (1997). Employing the techniques described there, we obtain the following equations[6]:

[6] We obtain equations (57) and (62) on the following page as follows: Let $F_t = C_t - V_{t-1}$ and $H_t = L_t - U_{t-1}$ which implies $\bar{F} = \bar{C} - \bar{V}$ and $\bar{H} = \bar{L} - \bar{U}$ as steady state equations as well as $\bar{F}f_t = \bar{C}c_t - \bar{V}v_{t-1}$ and $\bar{H}h_t = \bar{L}l_t - \bar{U}u_{t-1}$ as log-linearized equations. Then, equation (37) on the previous page can be written as $(1-\theta)\frac{Y_t}{N_t} = \chi\frac{F_t^{\gamma}}{H_t^{\vartheta}}$ or, in log-linearized form, as $y_t - n_t = \gamma f_t - \vartheta h_t$. Substituting f_t and h_t then yields $y_t - n_t = \gamma\left[\frac{\bar{C}c_t - \bar{V}v_{t-1}}{\bar{C}-\bar{V}}\right] - \vartheta\left[\frac{\bar{L}l_t - \bar{U}u_{t-1}}{\bar{L}-\bar{U}}\right]$. Using the steady state relationships $\bar{V} = \alpha_C\bar{C}$ and $\bar{U} = \alpha_L\bar{L}$, equation (57) on the following page is straightforward. The same technique yields equation (62) on the next page from equation (42) on the preceding page.

$$
\begin{aligned}
\bar{Y}y_t &= \bar{I}i_t + \bar{C}c_t, && (54)\\
\bar{K}k_t &= \bar{I}i_t + (1-\delta)\bar{K}k_{t-1}, && (55)\\
y_t &= z_t + \theta k_{t-1} + (1-\theta)n_t, && (56)\\
y_t - n_t &= \frac{\gamma}{1-\alpha_C}\left(c_t - \alpha_C v_{t-1}\right) - \frac{\vartheta}{1-\alpha_L}\left(l_t - \alpha_L u_{t-1}\right), && (57)\\
\bar{R}r_t &= \theta\frac{\bar{Y}}{\bar{K}}(y_t - k_{t-1}), && (58)\\
\bar{L}l_t &= -\bar{N}n_t, && (59)\\
\bar{V}v_{t-1} &= (1-\phi_c)\,\alpha_c\bar{C}c_{t-1} + \phi_C\bar{V}v_{t-2}, && (60)\\
\bar{U}u_{t-1} &= (1-\phi_l)\,\alpha_l\bar{L}l_{t-1} + \phi_L\bar{U}u_{t-2}, && (61)\\
0 &= E_t\left[-\frac{\gamma}{1-\alpha_C}\left(\Delta c_{t+1} - \alpha_C\Delta v_t\right) + r_{t+1}\right], && (62)\\
z_{t+1} &= \psi z_t + \epsilon_{t+1}. && (63)
\end{aligned}
$$

5.6 The recursive equilibrium law of motion

As all equations have been log-linearized, we can now proceed to solve for the recursive equilibrium law of motion with the method of undetermined coefficients. To do so, we implement the model into MATLAB®, using the template code provided and described by Uhlig (1997). The program code can be found in the appendix on page 69.

5.7 Calibration issues

We now calibrate the model. Table 1 on the following page gives an overview over the parameter values and steady state values assumed to calibrate the model. It also gives an indication of the sources used for the calibration. Table 2 on page 24 then displays those parameter values and steady state values that are implied by the steady state equations. Note that the curvature parameters (γ and ϑ) as well as the habit importance parameters (α_C and α_L) and the habit persistence parameters (ϕ_C and ϕ_L) are not yet fixed here, but in the different sections of the dynamics analysis. The calibration $\gamma = \vartheta = 2.372$, which we employ in most models, goes back to the fact that Lettau and Uhlig (2000) used a similar value for the curvature of consumption in their benchmark model.

Also note that some specific calibrations of our model lead to benchmark models without any habit formation or with habit formation only in

Parameter or variable	Meaning	Value	Source or reason
θ	Capital share	0.36	Kydland and Prescott (1982)
δ	Depreciation per quarter	0.025	Kydland and Prescott (1982)
ψ	Autocorrelation of technology shock	0.95	Hansen (1985)
σ_ϵ	Standard deviation of technology shock	0.00712	Hansen (1985)
$\bar{R}$	Steady state real interest rate per quarter	1.01	Uhlig (1997)
$\bar{N}$	Steady state employment level	1/3	Hansen (1985)
$\bar{Z}$	Steady state of technology shock	1	Normalization
γ	Curvature of consumption	differs	
ϑ	Curvature of leisure	differs	
α_C	Overall importance of consumption habit	differs	
α_L	Overall importance of leisure habit	differs	
ϕ_C	Persistence of consumption habit	differs	
ϕ_L	Persistence of leisure habit	differs	

Table 1: Parameter values and steady state values assumed in the benchmark model

Parameter or variable	Meaning	Value
χ	Parameter of leisure importance relative to consumption	differs
β	Discount factor	1/1.01
$\frac{\bar{Y}}{\bar{K}}$	Steady state inverse of capital ratio	0.0972
$\bar{K}$	Steady state capital level	12.7202
$\bar{I}$	Steady state investment level	0.3180
$\bar{Y}$	Steady state output level	1.2367
$\bar{C}$	Steady state consumption level	0.9187
$\bar{L}$	Steady state leisure level	2/3
$\bar{V}$	Steady state consumption habit level	differs
$\bar{U}$	Steady state leisure habit level	differs

Table 2: Parameter values and steady state values implied in the benchmark model

consumption, which are analyzed in other articles. For example, setting $\alpha_C = \alpha_L = \phi_C = \phi_L = \vartheta = 0$ and $\gamma = 1$ yields a model analyzed by Hansen (1985). The choice $\alpha_L = \phi_C = \phi_L = 0$ yields models that are analyzed by Lettau and Uhlig (2000) (with a slightly different definition of the sequence for habit formation). Three different versions are worth mentioning:

- $\vartheta = 0$ yields the benchmark model (with a utility function that is linear in leisure).
- $\vartheta \neq 0$ gives the model with power utility in leisure.
- $\vartheta \to \infty$ yields the model with fixed labor input.

6 Model Results and Answer

This section presents the most important results of our model. What implications does introducing habit formation into a standard real business cycle model have for the dynamics of the most important variables? What are the specific consequences of habit formation in leisure for these dynamics? We choose different calibrations of our parameters and examine the impulse-responses of the variables.

We first give an overview over the general dynamics of the model. After that, we scrutinize the specific implications for consumption- and leisure-choices of the representative household, examining the influences of the parameters of external habit formation: Is there a possibility to isolate the effects of the importance parameters α_C and α_L on the one hand and of the persistence parameters ϕ_C and ϕ_L on the other hand? To find the answer to this question, we fix values of the habit persistence parameters ϕ_C and ϕ_L and observe the impulse-responses for varying values of the habit importance parameters α_C and α_L. Next, we try to oppose substitution effect and income effect on leisure as a result of an increasing real wage.

6.1 General model dynamics

We begin our analysis with a brief review over the general dynamics. As habit formation may be a very long-lasting process which might take effect only very late after the initial technology shock, we distinguish between medium-run[7] (several years or even decades) and short-run (several quarters) analysis.

6.1.1 Medium run

Figure 1 on the following page displays a model without any habit formation on the one hand and a model with moderate external habit formation in both consumption and leisure on the other hand. It shows the corresponding impulse-responses of the different variable-deviations from their respective steady states to a one percent deviation in technology.

Although habit formation is supposed to have some effect on the dynamics of consumption and leisure (consumption smoothing, leisure smoothing, etc.), these variables converge back to their respective steady states, regardless

[7]We do not use the term "long run" here because this expression is generally employed for an analysis of the steady state of the model, i.e. when the economy is in equilibrium. This, however, is not of interest here. Instead, we focus on the deviations of the variables from their respective steady state values after several years have elapsed after the responsible technology shock.

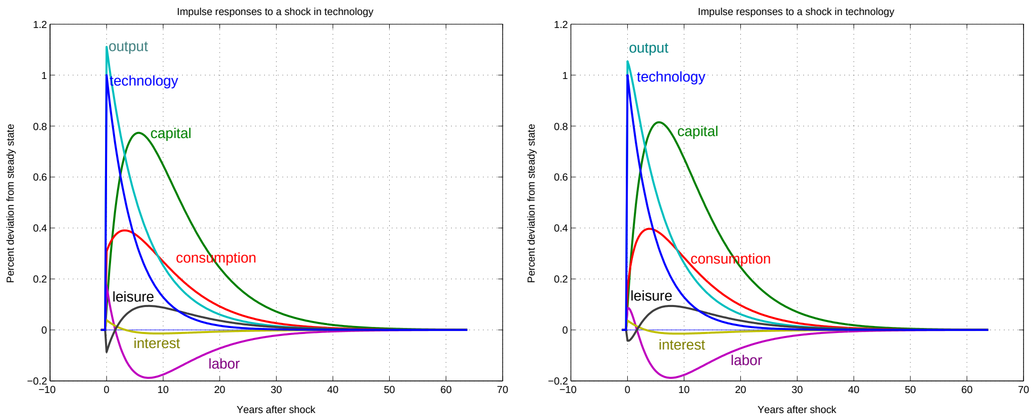

Figure 1: General medium-run dynamics of a model without (left) and with (right) external habit formation in both consumption and leisure
Calibration: $\gamma = \vartheta = 2.372$; $\alpha_C = \alpha_L = \phi_C = \phi_L = 0$ (left); $\alpha_C = \alpha_L = \phi_C = \phi_L = 0.5$ (right)

whether there is external habit formation in the model or not. Note that the respective variables reach their steady state values at approximately the same time in both models.

6.1.2 Short run

Figure 2 on the next page gives an overview over the short-run dynamics of the same models.

When these economies encounter a positive technology shock, output rises, which can be seen from the production function. More goods are available for both consumption and investment, so both of them rise. As usual, consumption volatility is constrained by the preferences of the household, to be more specific by the intertemporal elasticity of substitution, so that investment is far more volatile[8]. As investment grows, capital of course does, too. Interest first exceeds its steady state as output rises at once after the technology shock, while the capital stock increases only slowly through the channel of investment. After some periods, capital finally deviates more from its steady state than output does, so that from then on, interest deviates from its own steady state negatively. All these dynamics seem to be quite standard.

It is worth mentioning that apart from consumption-, leisure-/labor- and

[8]The deviation of investment from its steady state is not plotted in these graphs as it is too volatile and would complicate identifying the other variables.

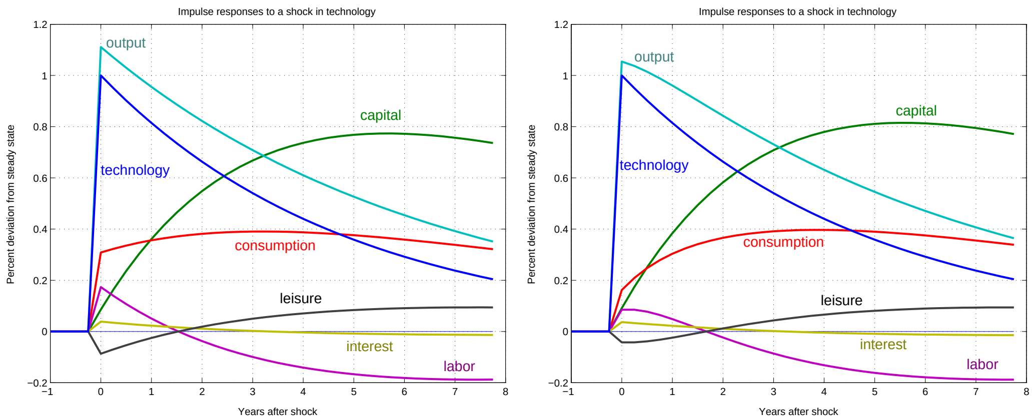

Figure 2: General short-run dynamics of a model without (left) and with (right) external habit formation in both consumption and leisure
Calibration: $\gamma = \vartheta = 2.372$; $\alpha_C = \alpha_L = \phi_C = \phi_L = 0$ (left); $\alpha_C = \alpha_L = \phi_C = \phi_L = 0.5$ (right)

capital-choices, the graphs of the other variables do not differ significantly between the models with and without habit formation.

However, in the context of external habit formation, we want to examine more in detail these consumption- and leisure-choices of the representative household. Note that, of course, the choice of leisure can be seen as a choice of labor supply, in the first place, and vice versa. This is why the graphs of these two variables look like opposite deviations from their respective steady states.

6.2 External habit formation in consumption only

We proceed our analysis of consumption- and leisure-deviations from their respective steady states with a model where there is external habit formation in consumption, but not in leisure. Setting $\alpha_L = 0$ in the benchmark setup described in section 4 yields such a model, that – with a slightly different definition of the consumption habit – has been analyzed by Lettau and Uhlig (2000) as an alternative to the benchmark model presented in their article.

6.2.1 Transitory external habit formation in consumption only

For simplification we set the habit persistence parameter ϕ_C to zero for now, so that habit formation only consists of consumption one period ago, that is $V_{t-1} = \alpha_C C_{t-1}$.

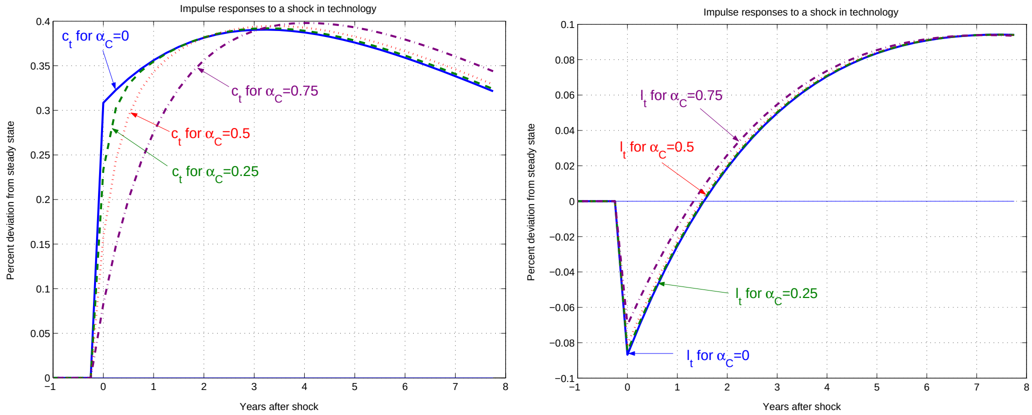

Figure 3: Importance of transitory external habit formation in consumption but not in leisure, for consumption c_t (left) and leisure l_t (right)
Calibration: $\gamma = \vartheta = 2.372$, $\phi_C = \phi_L = 0$, $\alpha_L = 0$, varying α_C

Figure 3 displays the impulse-responses of consumption- and leisure-deviations from their respective steady states for different intensities of transitory external habit formation in consumption (i.e. different α_C) when there is no habit formation in leisure ($\alpha_L = 0$).

Habit formation in consumption obviously has a *delaying effect* on consumption-deviation from its steady state. Indeed, the more important external habit formation in consumption is (that is, the bigger α_C), the smaller the instant deviation of consumption from its steady state becomes and the more time it takes for c_t to reach its peak. This becomes especially clear as one compares the graphs of c_t for $\alpha_C = 0$ (no habit formation) and $\alpha_C = 0.75$ (strong habit formation). The intuition is of course that habit formation in consumption keeps people from changing their consumption rapidly after the shock. Note that for higher values of α_C, the peak lies slightly higher.

We also observe another result described by Lettau and Uhlig (2000): When habit formation in consumption is introduced into the model, people respond to a technology shock less by adapting their consumption than in a model without habit formation, so that c_t becomes smoother. Instead they will decide to work less, for they expect that consumption in the future will not rise too much, due to habit formation in consumption. Of course, this equivalently means that people will increase their leisure in a model with habit formation in consumption compared to a model without any habit formation, as can be seen in the right graph of figure 3.

These intuitions can be demonstrated with the help of equation (37) on page 20 which can be rewritten as[9]:

$$(1-\theta)Y_t = \chi \frac{\left(C_t - (1-\phi_C)\alpha_C \sum_{j=1}^{t} (\phi_C)^{t-j} C_{j-1}\right)^{\gamma}}{\left(L_t - (1-\phi_L)\alpha_L \sum_{j=1}^{t} (\phi_L)^{t-j} L_{j-1}\right)^{\vartheta}} (1-L_t), \tag{64}$$

using equation (39) on page 20 and equations (9) and (10) on page 15. Essentially, this equation links consumption C_t to leisure L_t. An increase of the consumption habit importance parameter α_C leads to an increase of the nominator of the fraction on the right-hand side of equation (64). To compensate this effect, consumption C_t decreases and leisure L_t increases.

6.2.2 Persistent external habit formation in consumption only

Habit formation in consumption may also take a more persistent form. The corresponding impulse-responses are displayed in figure 4.

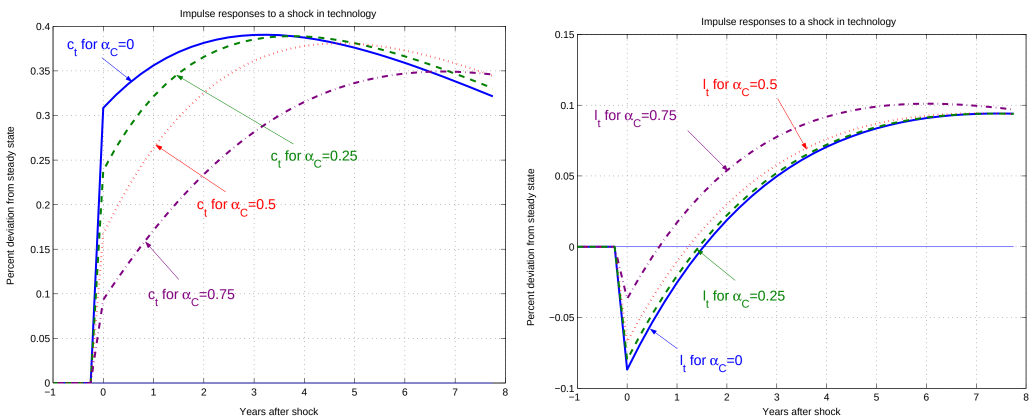

Figure 4: Importance of persistent external habit formation in consumption but not in leisure, for consumption c_t (left) and leisure l_t (right)
Calibration: $\gamma = \vartheta = 2.372$, $\phi_C = 0.75$, $\phi_L = 0$, $\alpha_L = 0$, varying α_C

Here, we observe the same *delaying effect* as in the model with transitory habit formation in consumption, but to a much higher degree. Consumption instantly after the technology shock rises much less and more slowly. The intuition behind this is that people remember past consumption for a longer

[9]Alternatively, one may also use the log-linearized version of equation (37) on page 20, i.e. equation (57) on page 22, which directly gives the implications of an increasing α_C for c_t and l_t instead of those for C_t and L_t.

time due to the higher persistence parameter ϕ_C. So, after the technology shock consumption reacts even more slowly than in the transitory model. Additionally, it can be seen that the peak of consumption-deviation from its steady state is lower for higher values of α_C. For stronger habit formation we observe a *smoothing effect*.

Interestingly, in this model with persistent habit formation in consumption, leisure rises even more than in the transitory model. Analytically, the reason can again be seen from equation (64) on the preceding page. As in the transitory case, this equation has to hold for all calibrations of α_C. As it has already been shown, when the habit importance parameter α_C rises, consumption C_t decreases more than in the transitory case. Consequently, equation (64) on the previous page implies that leisure L_t increases also more than in the transitory case. The intuition behind this is that people can generally increase their utility either by increasing their consumption (which means that they have to work more) or by increasing their leisure demand. In the persistent model, people stick to their consumption habit more than in the transitory model, let alone in the model without any habit formation. Instead, they choose to increase their utility by working less.

6.2.3 Medium-run dynamics and the effect of the habit persistence parameter

Until now, we have only evaluated short-run dynamics of consumption and leisure. However, as it has already been stated, habit formation is a long-lasting process. Figure 5 on the following page opposes consumption-choices for models with transitory (left) and persistent (right) habit formation in consumption only.

Figure 6 on the next page shows the importance of the habit persistence parameter ϕ_C for leisure in the same models.

We see that for both consumption (figure 5 on the following page) and leisure (figure 6 on the next page), the impulse-responses differ more in the case of persistent habit formation in consumption ($\phi_C = 0.75$, right graph) than in the case of transitory habit formation in consumption ($\phi_C = 0$, left graph).

We may state that the importance of habit formation in consumption is much smaller in the transitory case than in the persistent case. Furthermore, in the persistent case, the influence of habit formation on consumption- and leisure-choices lasts a bit longer.

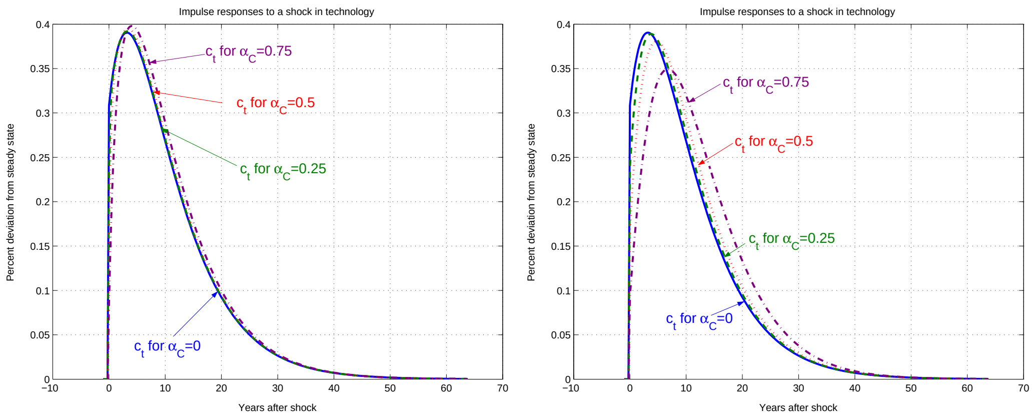

Figure 5: Medium-run dynamics of consumption and importance of the habit persistence parameter ϕ_C for consumption in a model with external habit formation in consumption only
Calibration: $\gamma = \vartheta = 2.372$, $\alpha_L = \phi_L = 0$, varying α_C, $\phi_C = 0$ (left) and $\phi_C = 0.75$ (right)

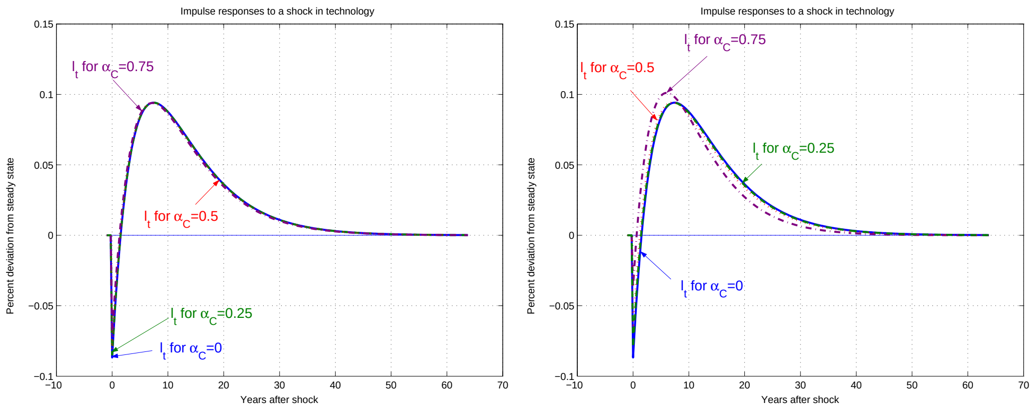

Figure 6: Medium-run dynamics of leisure and importance of the habit persistence parameter ϕ_C for leisure in a model with external habit formation in consumption only
Calibration: $\gamma = \vartheta = 2.372$, $\alpha_L = \phi_L = 0$, varying α_C, $\phi_C = 0$ (left) and $\phi_C = 0.75$ (right)

6.2.4 Interim result

From what we have analyzed in subsection 6.2, we may state that, compared to a model without any habit formation, in a model with habit formation only in consumption, consumption-choice decreases and leisure-choice increases. Thus, one observes that the impulse-responses of these variables are smoothed, especially in the case of persistent habit formation in consumption. The influence on consumption is stronger than that on leisure.

6.3 External habit formation in leisure only

Next, we are also interested in a model with external habit formation in leisure, but not in consumption. Again, we distinguish between transitory and persistent habit formation.

6.3.1 Transitory external habit formation in leisure only

We set $\phi_L = 0$ so that habit formation in leisure takes the transitory form $U_{t-1} = \alpha_L L_{t-1}$. From figure 7 we can analyze the impulse-responses in such a model, that is for $\alpha_C = 0$ and varying α_L.

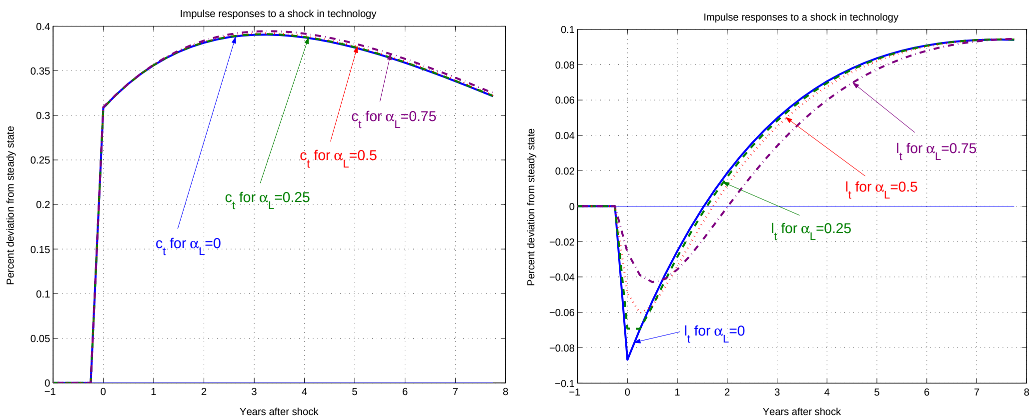

Figure 7: Importance of transitory external habit formation in leisure but not in consumption, for consumption c_t (left) and leisure l_t (right)
Calibration: $\gamma = \vartheta = 2.372$, $\phi_C = \phi_L = 0$, $\alpha_C = 0$, varying α_L

Transitory habit formation in leisure influences consumption-choices only to a very small degree[10]. For leisure, we observe that the higher the leisure

[10] A more detailed analysis of the influence of external habit formation in leisure on consumption-choices can be found in section 6.5.

habit importance parameter α_L is, the less leisure decreases after the technology shock. This means that we observe the same instant reaction as for the model with transitory external habit formation in consumption, which was analyzed in the previous section. The deviation from the steady state of leisure after some periods, however, is less for higher values of α_L.

The intuition is that due to habit formation in leisure, people want to stick to their leisure-choice of the previous period, so that for higher values of α_L, the graph of leisure-deviation from its steady state is smoothed.

6.3.2 Persistent external habit formation in leisure only

External habit formation in leisure now takes a more persistent form; people remember their leisure- and labor-choices not only from the last period, but from all past periods. The results of this modification are shown in figure 8.

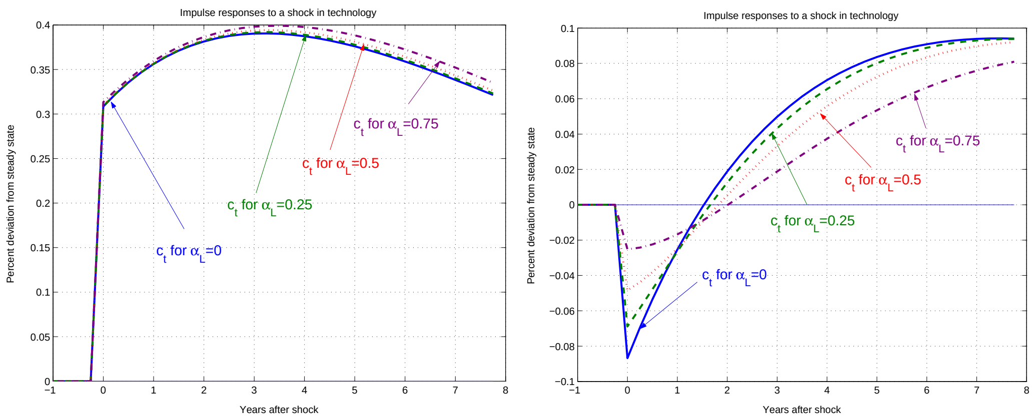

Figure 8: Importance of persistent external habit formation in leisure but not in consumption, for consumption c_t (left) and leisure l_t (right)
Calibration: $\gamma = \vartheta = 2.372$, $\phi_C = 0$, $\phi_L = 0.75$, $\alpha_C = 0$, varying α_L

We notice that with this calibration, there is a real *smoothing effect* in leisure-deviation from its steady state. For higher values of α_L (i.e. for stronger habit formation in leisure), the negative deviation of leisure in the beginning is much less distinct than in the other models considered so far. Later on, when the income effect becomes more important than the substitution effect so that leisure deviates positively from its steady state, this deviation is not as important as in a model without habit persistence, either.

Note that for the deviation of consumption from its steady state it makes almost no difference whether we consider transitory or persistent external habit formation in leisure. In both cases, external habit formation in leisure influences consumption-choices only to a very small degree.

6.3.3 Medium-run dynamics and the effect of the habit persistence parameter

Again we plot the medium-run impulse-responses in this model for the transitory and for the persistent case. Figure 9 does so for consumption and figure 10 on the next page for leisure.

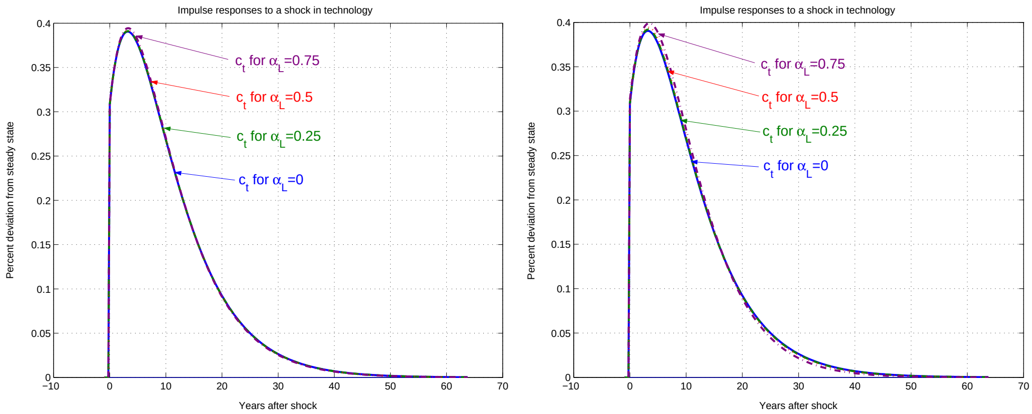

Figure 9: Medium-run dynamics of consumption and importance of the habit persistence parameter ϕ_L for consumption in a model with external habit formation in leisure only
Calibration: $\gamma = \vartheta = 2.372$, $\alpha_C = \phi_C = 0$, varying α_L, $\phi_L = 0$ (left) and $\phi_L = 0.75$ (right)

As for the model with external habit formation only in consumption, habit formation is more important if it takes a persistent form. This time, however, this difference is much smaller, for consumption it seems almost negligible.

6.3.4 Interim result

Subsection 6.3 has shown on the one hand that habit formation in leisure smoothes leisure-choices after a technology shock. On the other hand, consumption is not influenced very much by habit formation in leisure.

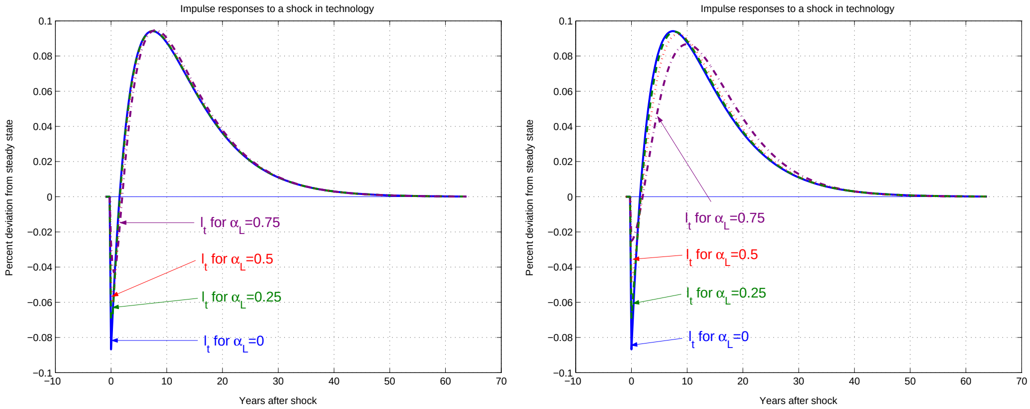

Figure 10: Medium-run dynamics of leisure and importance of the habit persistence parameter ϕ_L for leisure in a model with external habit formation in leisure only
Calibration: $\gamma = \vartheta = 2.372$, $\alpha_C = \phi_C = 0$, varying α_L, $\phi_L = 0$ (left) and $\phi_L = 0.75$ (right)

6.4 External habit formation in both consumption and leisure

Now we want to merge the two models investigated so far and analyze external habit formation in both consumption and leisure. As for the two models with habit formation in either consumption or leisure, we again distinguish between transitory and persistent habit formation.

6.4.1 Transitory external habit formation in both consumption and leisure

Figure 11 on the following page shows the corresponding impulse-responses for different values of α_C and α_L. Note that α_C and α_L are varying symmetrically here.

As expected, we once again observe the *delaying effect* for consumption. Indeed, for bigger habit importance parameters α_C and α_L it takes longer until the deviation of consumption from its steady state reaches its peak. Likewise, we observe that the graph of leisure is smoothed for stronger habit formation.

Note that the graph of consumption is almost identical to that of consumption-deviation in the model with transitory external habit formation only in consumption (left graph of figure 3 on page 28). Similarly, the graph of leisure resembles that of leisure-deviation in the model with transitory external habit formation only in leisure (right graph of figure 7 on page 32).

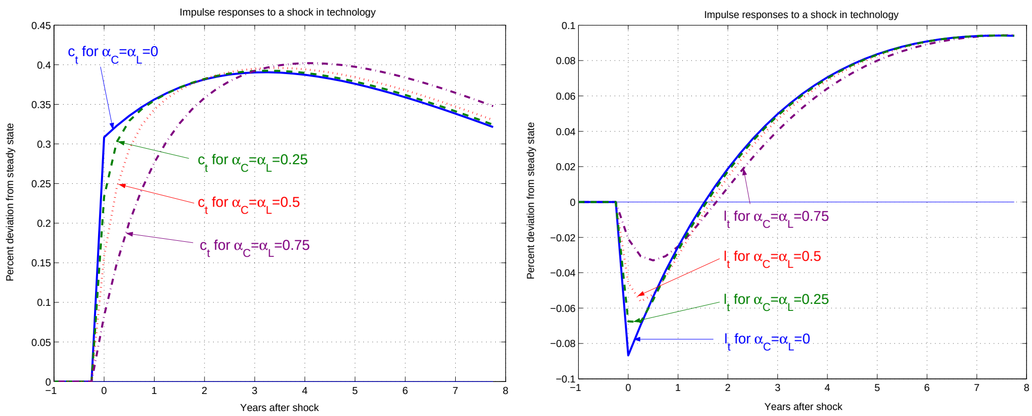

Figure 11: Importance of transitory external habit formation in both consumption and leisure, for consumption c_t (left) and leisure l_t (right)
Calibration: $\gamma = \vartheta = 2.372$, $\phi_C = \phi_L = 0$, symmetrically varying α_C and α_L

However, this latter resemblance is not as significant as the former one.

6.4.2 Persistent external habit formation in both consumption and leisure

Now we merge the models from sections 6.2.2 and 6.3.2. In figure 12 on the next page we plot impulse-responses for a model where habits consist of all lagged values of consumption and leisure rather than only values of the last period: we consider a model with high habit persistence parameters $\phi_C = \phi_L = 0.75$.

Again, the *delaying effect* in consumption is worth mentioning. Likewise, leisure is smoothed, just as in the other models.

We compare the graph of consumption to that in the model where there is only persistent external habit formation only in consumption (left graph of figure 4 on page 29) and notice that the graphs once more are almost identical. This, however, cannot be observed when comparing the graph of leisure to that of leisure in the model with persistent external habit formation only in leisure (right graph of figure 8 on page 33). Indeed, when there is persistent external habit formation in leisure only, leisure deviates less from its steady state than when there is persistent external habit formation in consumption, too.

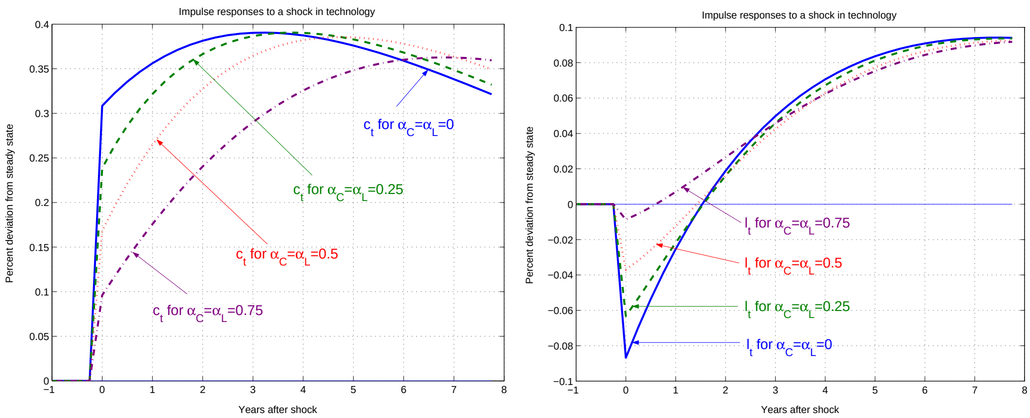

Figure 12: Importance of external habit formation in both consumption and leisure, for consumption c_t (left) and leisure l_t (right) Calibration: $\gamma = \vartheta = 2.372$, $\phi_C = \phi_L = 0.75$, symmetrically varying α_C and α_L

6.4.3 Medium-run dynamics and the effect of the habit persistence parameters

For the sake of completeness, we also plot the medium-run impulse-responses in this model for the transitory and for the persistent case. Figure 13 on the following page does so for consumption and figure 14 on the next page for leisure.

6.4.4 Interim result

Subsection has highlighted what we had already expected from the previous two subsections: the differences in the reactions of consumption are mainly due to habit formation in consumption (habit formation in leisure does not influence consumption-choices very much), whereas the differences in the reactions of leisure are due to both types of habit formation.

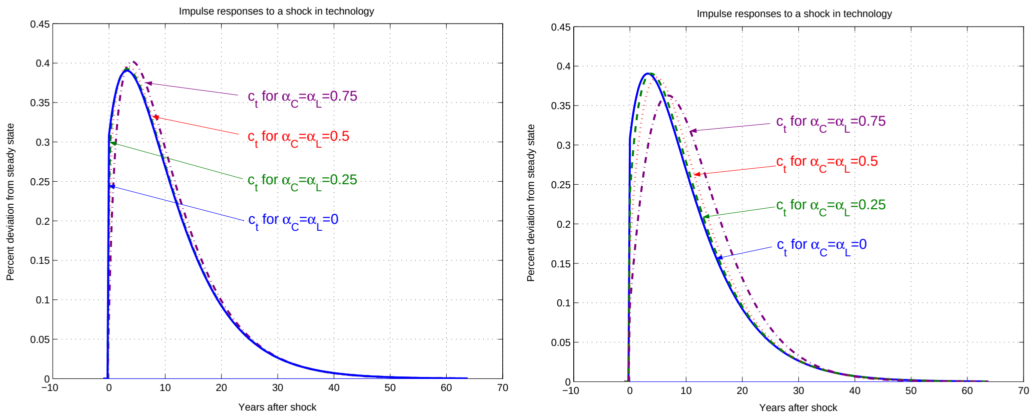

Figure 13: Medium-run dynamics of consumption and importance of the habit persistence parameters ϕ_C and ϕ_L for consumption in a model with external habit formation in both consumption and leisure
Calibration: $\gamma = \vartheta = 2.372$, symmetrically varying α_C and α_L, $\phi_C = \phi_L = 0$ (left) and $\phi_C = \phi_L = 0.75$ (right)

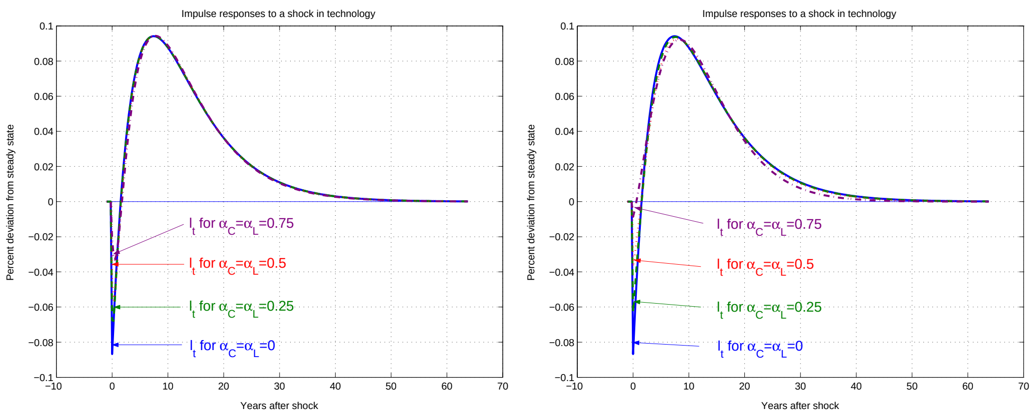

Figure 14: Medium-run dynamics of leisure and importance of the habit persistence parameters ϕ_C and ϕ_L for leisure in a model with external habit formation in both consumption and leisure
Calibration: $\gamma = \vartheta = 2.372$, symmetrically varying α_C and α_L, $\phi_C = \phi_L = 0$ (left) and $\phi_C = \phi_L = 0.75$ (right)

6.5 The influence of external habit formation in leisure on the choice of consumption

6.5.1 Introducing external habit formation in leisure into a model with external habit formation in consumption

As we can already see when comparing the consumption graphs of sections 6.2 and 6.4 (i.e. figures 3 on page 28 and 11 on page 36 as well as figures 4 on page 29 and 12 on page 37, respectively), introducing external habit formation into leisure does not influence consumption-choices very much. The reason is that with our calibration of the parameters, labor input and leisure do not deviate very much from their respective steady states already in the benchmark model. Consequently, introducing habit formation into leisure has only relatively small effects on consumption-choices. Figure 15 illustrates this for the case of transitory (left) and persistent (right) habit formation.

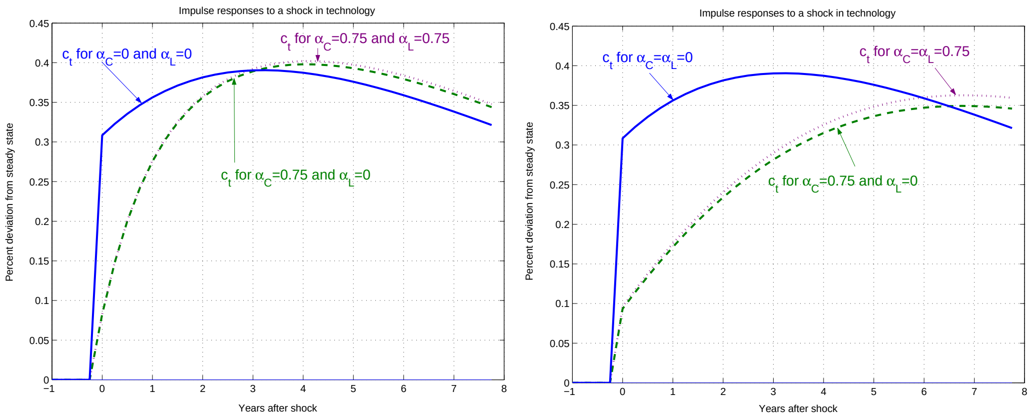

Figure 15: The effect of introducing external habit formation in leisure for consumption
Calibration: $\gamma = \vartheta = 2.372$, $\phi_C = \phi_L = 0$ (transitory, left) and $\phi_C = \phi_L = 0.75$ (persistent, right)

In both cases, the solid line shows consumption-deviation from its steady state when there is no habit formation at all. The dashed line represents a model where there is habit formation in consumption only. The dotted line finally stands for a model with habit formation in both consumption and leisure. Note that habit formation is rather strong here when it is introduced ($\alpha_C = 0.75$ and $\alpha_L = 0.75$). Yet, the dashed and the dotted lines do not differ much, especially in the transitory case.

What are the conclusions that have to be drawn here? First, external habit formation in leisure does not influence consumption-choices very much even for high values of the importance parameter α_L. Second, the influence is higher for higher values of the persistence parameter ϕ_L. And finally, with these results we might be inclined to consider extreme versions of our benchmark model in the sense that we examine a utility function that is linear in leisure ($\vartheta = 0$) or fixed in leisure ($\vartheta \to \infty$). The former case, of course, would be equivalent to introducing external habit formation in both consumption and leisure into the benchmark model of Hansen (1985).

6.5.2 Maximum influence of external habit formation in leisure on the choices of consumption

Considering these observations, it may be interesting to search for the calibration of the model where the influence of habit formation in leisure on consumption-choices is as significant as possible. We define this influence as the area between the two impulse-responses.

Consider a model with external habit formation in both consumption and leisure, that is $0 < \alpha_C < 1$ and $0 < \alpha_L < 1$. Note that, as has been argued in section 5.7, setting $\vartheta = 0$ yields a utility function which is linear in leisure, so that the effect of habit formation in leisure disappears. This can be seen from equation (57) on page 22. On the other hand, letting $\vartheta \to \infty$ gives a model with fixed labor input. In such a model, habit formation in leisure should not influence consumption-choices, either. Leisure is simply fixed, regardless whether there is habit formation in it or not. Thus, a technology shock should have the same effect as in a model with habit formation only in consumption. On the other hand, there *are* values of ϑ for which habit formation in leisure influences consumption-choices. These three facts indicate that there should be some value of ϑ for which the influence of habit formation in leisure on consumption-choices is maximum. This value may of course depend on the calibration of the other parameters.

Analytically, it is difficult to compute those parameter values yielding the highest influence of habit formation in leisure on consumption. Note, however, that higher leisure habit parameters imply a stronger influence of leisure habit on consumption. This means that, as α_L and ϕ_L converge to 1, habit formation in leisure will become more and more important for consumption. We abstract from this point by simply assuming a strong form of external habit formation, setting $\alpha_L = 0.75$. Depending on whether we consider transitory or persistent habit formation in leisure, we set $\phi_L = 0$ or $\phi_L = 0.75$.

We may now try to search for the values of γ, ϑ, α_C and ϕ_C yielding the maximum importance of habit formation in leisure for consumption-choices with the help of a four-dimensional grid[11]. Actually, this means that we search for the curvatures of consumption and leisure and for the consumption habit parameters that maximize the distance defined above.

The procedure works as follows: We define minimum and maximum values as well as step lengths for the four parameters. Note that these step lengths may not be too small, as this would soon become computationally burdensome. For example, 100 values for each parameter would lead to $100,000,000$ possible parameter combinations to be evaluated. So, we search the grid for those values of the parameters yielding the maximum distance. We choose new minimum and maximum values for the parameters that are closer to the values just obtained and make the grid smaller meshed. This procedure is iterated several times, until the parameter values converge.

The result we get from this procedure is that the more important and persistent habit formation in consumption is (the bigger α_C and ϕ_C), the bigger the influence of habit formation in leisure becomes. At the same time, a higher consumption curvature γ also leads to a higher influence of leisure habit formation on consumption. Finally, and as assumed, the influence is maximum for some leisure curvature ϑ that depends on γ.

To illustrate this graphically, we fix $\alpha_C = 0.75$ and search for the curvatures γ and ϑ of consumption and leisure that maximize the influence. Figure 16 on the next page gives us a general idea about the influence of external habit formation depending on the curvatures γ and ϑ which vary between 0 and $10,000$. The graph on the left does so for a transitory form of habit formation in consumption and leisure ($\phi_C = \phi_L = 0$) while the right one does so for persistent habit formation ($\phi_C = \phi_L = 0.75$).

Note that both graphs have a rather similar form. As already expected, for increasing values of γ and ϑ, the influence of habit formation in leisure on consumption-choices becomes more and more important. For a given value of γ, however, the maximum influence is not reached for the highest possible value of ϑ, but for some value between 0 and $10,000$ that depends on γ.

Micro- and macroeconomically, such high values of the curvature parameters are not reasonable. Thus, in figure 17 on the following page we concentrate on the intervals which correspond more to our calibration choices: $1 < \gamma < 5$ and $0 < \vartheta < 5$.

We conclude that for a higher consumption curvature parameter γ, external habit formation in leisure has a higher influence on consumption. Furthermore, the influence is higher in the persistent case of habit formation than in the transitory case.

[11]The corresponding MATLAB®-program can be found in the appendix on page 72.

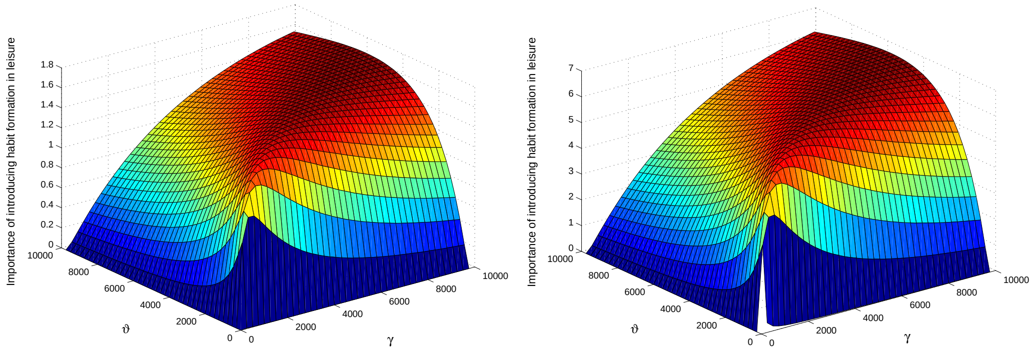

Figure 16: Maximum influence of introducing external habit formation in leisure into a model with external habit formation in consumption (transitory: left, persistent: right)
Calibration: varying γ and ϑ ($0 < \gamma < 10,000$ and $0 < \vartheta < 10,000$), $\alpha_C = \alpha_L = 0.75$, $\phi_C = \phi_L = 0$ (left) and $\phi_C = \phi_L = 0.75$ (right)

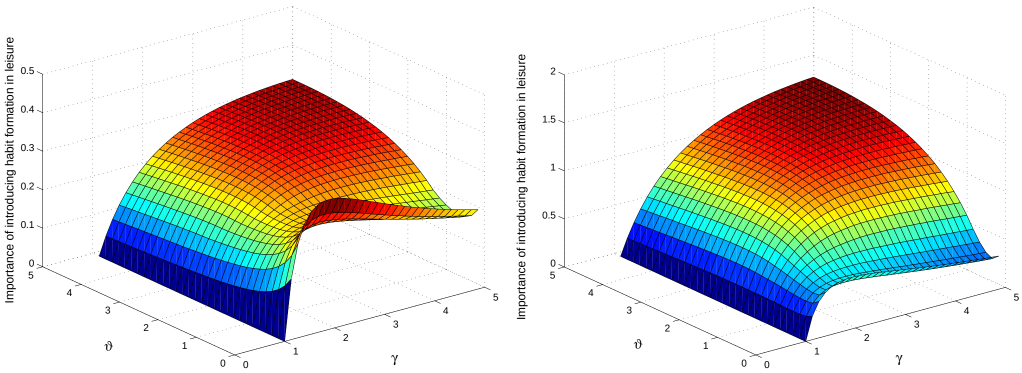

Figure 17: Maximum influence of introducing external habit formation in leisure into a model with external habit formation in consumption (transitory: left, persistent: right)
Calibration: varying γ and ϑ ($1 < \gamma < 5$ and $0 < \vartheta < 5$), $\alpha_C = \alpha_L = 0.75$, $\phi_C = \phi_L = 0$ (left) and $\phi_C = \phi_L = 0.75$ (right)

6.6 Substitution effect and income effect in labor supply

Now we want to analyze the relative importance of the substitution effect and the income effect on labor supply and leisure-choices under habit formation. To do so, we rewrite equation (37) on page 20 as follows:

$$W_t = \chi \frac{(L_t - U_{t-1})^{-\vartheta}}{(C_t - V_{t-1})^{-\gamma}}. \tag{65}$$

Log-linearization yields:

$$\begin{aligned} w_t &= \gamma \left[\frac{\bar{C} c_t - \bar{V} v_{t-1}}{\bar{C} - \bar{V}} \right] - \vartheta \left[\frac{\bar{L} l_t - \bar{U} u_{t-1}}{\bar{L} - \bar{U}} \right], & (66) \\ &= \frac{\gamma}{1 - \alpha_C} \left[c_t - \alpha_C v_{t-1} \right] - \frac{\vartheta}{1 - \alpha_L} \left[l_t - \alpha_L u_{t-1} \right]. & (67) \end{aligned}$$

We solve for l_t:

$$l_t = \frac{\gamma(1 - \alpha_L)}{\vartheta(1 - \alpha_C)} \left[c_t - \alpha_C v_{t-1} \right] - \frac{1 - \alpha_L}{\vartheta} w_t + \alpha_L u_{t-1}. \tag{68}$$

After a positive technology shock, output rises. Therefore, real wage W_t rises, too. Now, there are two different effects, both of which can be seen from equation (68). On the one hand, the relative price of leisure increases. Thus, leisure becomes less attractive and declines[12]. This is the *substitution effect*. On the other hand, income rises, and the household can afford both more consumption and more leisure, so that leisure increases[13]. This is the *income effect*.

As substitution effect and income effect operate in opposite directions, the total effect on leisure is generally unclear and depends on the calibration of the parameters in the utility function, especially on the curvatures of consumption γ and leisure ϑ, but also on the parameters that define importance and persistence of habit formation in the model:

- For high values of γ and ϑ (bigger than 5), the income effect dominates, so that leisure rises instantly after the technology shock, regardless whether there is habit formation or not in consumption, in leisure or in both of them. Intuitively, when a shock in technology occurs, both consumption and leisure rise instantly. Then, such high values of γ and ϑ indicate very low intertemporal elasticities of substitution

[12] Formally, this can be seen from equation (68): if $w_t \nearrow$ then $l_t \searrow$.

[13] This, again, can be seen from equation (68): if $w_t \nearrow$ then $c_t \nearrow$ and consequently $l_t \nearrow$.

of consumption and leisure, respectively. As a consequence, people intertemporally do not change their consumption- and leisure-choices very much, anyway. Thus, the additional feature of presence or absence of habit formation in the utility function of the model becomes totally unimportant in this case.

- The same holds for high values of γ and small values of ϑ. However, in this case the value for γ should exceed 10. The intuition here is that after a technology shock, consumption and leisure both rise instantly. The high value of γ implies a very low intertemporal elasticity of substitution for consumption, regardless whether there is habit formation in the model. Consequently, consumption keeps deviating positively from its steady state but this deviation does not increase, so that people can consume more leisure, too.

- For small and moderate values of γ and ϑ, at first the substitution effect dominates and then the income effect becomes more important. The same holds for combinations of a small value of γ and a big value of ϑ. In these cases, the actual shape of the graphs depends on the habit formation parameters more than in the previous two cases[14].

6.7 Smaller fluctuations in the series than in US data

While habit formation in the utility function can explain phenomena such as the *equity premium puzzle*, it also raises new problems. Lettau and Uhlig (2000) argue that after introducing habit formation, consumption becomes smoother and as a result its standard deviation becomes too small, compared to real time series in US data. The same holds for labor input. The authors solve this issue by introducing a model with both technology and labor shocks, with the additional assumption of a high technology persistence parameter ψ.

When habit formation in leisure is introduced into a model with habit formation in consumption, this does not make the consumption-deviation puzzle worse. From the analysis carried out so far we see that consumption-deviation from its steady state, which is already positive without habit formation in leisure, further rises when this kind of habit formation is added. An analysis of the moments of the series shows that the standard deviation of consumption indeed increases.

This does, however, not apply to leisure and labor input. The standard deviations of these two variables decrease when habit formation in leisure is introduced.

[14]The analysis of these dynamics can be found in sections 6.2, 6.3 and 6.4.

7 Variations

This section presents some extensions of our benchmark model in section 4.

7.1 *Keeping up with the Joneses* preferences rather than *catching up with the Joneses* preferences

Suppose that the individuals can observe the choice of consumption and leisure of the other individuals in period t and that they are able to adapt their own choices at once. Imagine that they do not want to fall behind the others; they want to *keep up* with them.

7.1.1 The Model

There is just one fundamental hypothesis that changes: It concerns the intertemporal utility function, which now looks as follows:

$$\max_{\{C_j;L_j;I_j\}_{j=t}^{\infty}} E_t \sum_{j=t}^{\infty} \beta^{j-t} \left[\frac{(C_j - V_j)^{1-\gamma} - 1}{1-\gamma} + \chi \frac{(L_j - U_j)^{1-\vartheta} - 1}{1-\vartheta} \right], \tag{69}$$

subject to the same constraints as in our benchmark model in section 4. Note that also the equations for V_{t-1} and U_{t-1} generally remain the same. However, we additionally suppose that $\phi_C = 0$ and $\phi_L = 0$, so that an individual compares his own consumption and leisure only to others' consumption and leisure in the same period, but not in lagged periods[15]. Moreover, we rewrite the two equations in terms of the *keeping-up* hypothesis, i.e. in terms of V_t and U_t:

$$V_t = \alpha_C C_t^{(a)}, \tag{70}$$

$$U_t = \alpha_L L_t^{(a)}. \tag{71}$$

As in the benchmark model with *catching up with the Joneses* preferences, we may drop superscript $^{(a)}$. Again, the individual household is assumed to be atomic and does not internalize these relations when maximizing his utility.

[15] Otherwise this model with $\phi_C \neq 0$ or $\phi_L \neq 0$ would contain a mixture between *keeping up with the Joneses* preferences and *catching up with the Joneses* preferences.

7.1.2 Model Analysis

The modified Lagrangian looks as follows:

$$\mathcal{L} = E_t \sum_{j=t}^{\infty} \beta^{j-t} \left\{ \left[\frac{(C_j - V_j)^{1-\gamma} - 1}{1-\gamma} + \chi \frac{(1 - N_j - U_j)^{1-\vartheta} - 1}{1-\vartheta} \right] + \lambda_j \left[(1 - \delta + D_j) K_{j-1} + W_j N_j + \Pi_j - C_j - K_j \right] \right\}. \tag{72}$$

This leads to the following new first-order conditions which replace equations (27) and (28) on page 19:

$$\frac{\partial \mathcal{L}}{\partial C_t} = 0 \Rightarrow (C_t - V_t)^{-\gamma} - \lambda_t = 0, \tag{73}$$

$$\frac{\partial \mathcal{L}}{\partial N_t} = 0 \Rightarrow -\chi (1 - N_t - U_t)^{-\vartheta} + \lambda_t W_t = 0. \tag{74}$$

Combining these first-order conditions results in two new equations:

$$(1-\theta) \frac{Y_t}{N_t} = \chi \frac{(C_t - V_t)^{\gamma}}{(L_t - U_t)^{\vartheta}}, \tag{75}$$

$$1 = \beta E_t \left[\left(\frac{C_t - V_t}{C_{t+1} - V_{t+1}} \right)^{\gamma} R_{t+1} \right], \tag{76}$$

which replace equations (37) and (42) on page 20, respectively. All steady state equations remain the same as in the benchmark model.

Log-linearization yields for habit formation:

$$\bar{V} v_t = \alpha_C \bar{C} c_t, \tag{77}$$

$$\bar{U} u_t = \alpha_L \bar{L} l_t, \tag{78}$$

which can be simplified to:

$$v_t = c_t, \tag{79}$$

$$u_t = l_t. \tag{80}$$

Equations (75) and (76) become in log-linearized form:

$$y_t - n_t = \gamma \left[\frac{\bar{C} c_t - \bar{V} v_t}{\bar{C} - \bar{V}} \right] - \vartheta \left[\frac{\bar{L} l_t - \bar{U} u_t}{\bar{L} - \bar{U}} \right], \tag{81}$$

$$0 = E_t \left[-\gamma \left(\frac{\bar{C} \Delta c_{t+1} - \bar{V} \Delta v_{t+1}}{\bar{C} - \bar{V}} \right) + r_{t+1} \right]. \tag{82}$$

Together with equations (79) and (80) on the preceding page this yields:

$$y_t - n_t = \gamma c_t - \vartheta l_t, \tag{83}$$

$$0 = E_t \left[-\gamma \Delta c_{t+1} + r_{t+1} \right]. \tag{84}$$

7.1.3 Model Results

As it can be seen from equations (83) and (84), this type of habit formation completely drops out of our log-linearized model. Figure 18 demonstrates this by displaying the general model dynamics: The two graphs in the figure for a model without and a model with *keeping up with the Joneses* preferences are completely identical and, moreover, the impulse-responses are the same ones as in our benchmark model without habit formation. This can be seen when comparing the graphs from figure 18 and the left graph from figure 2 on page 27.

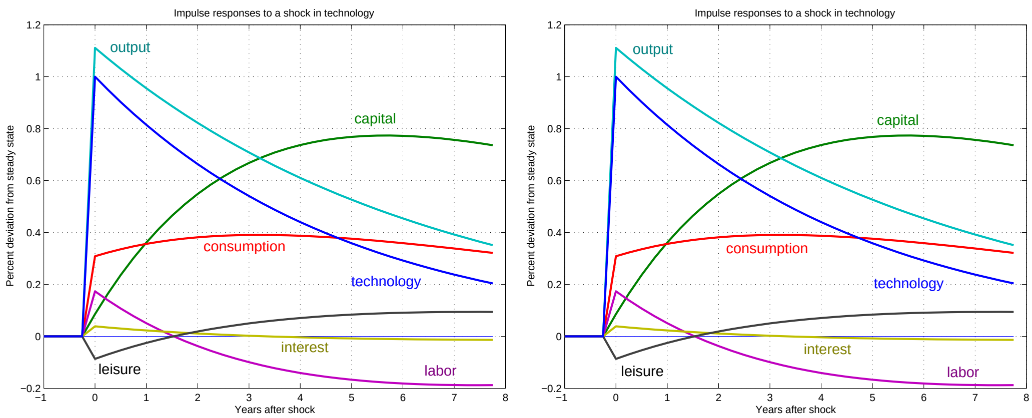

Figure 18: General dynamics of a model without (left) and with (right) external habit formation in both consumption and leisure (*keeping up with the Joneses* preferences)
Calibration: $\gamma = \vartheta = 2.372$; $\alpha_C = \alpha_L = 0$ (left); $\alpha_C = \alpha_L = 0.5$ (right)

7.2 Relative rather than absolute terms

In this subsection, we analyze a different type of *catching up with the Joneses* preferences. Instead of considering differences we rather examine the ratio of the individual's consumption relative to the corresponding habit stock, a geometric average of lagged consumption. The same holds for leisure. For consumption, such a utility function was for example constructed by Galí (1994).

7.2.1 The Model

Consider the following maximization problem of the representative household:

$$\max_{\{C_j;L_j;I_j\}_{j=t}^{\infty}} E_t \sum_{j=t}^{\infty} \beta^{j-t} \left[\frac{(C_j/V_{j-1})^{1-\gamma} - 1}{1-\gamma} + \chi \frac{(L_j/U_{j-1})^{1-\vartheta} - 1}{1-\vartheta} \right], \tag{85}$$

subject to the same constraints as in our benchmark model in section 4. The problem of the firm, the conditions of market clearing as well as the sequences of V_t and U_t remain the same.

7.2.2 Model Analysis

The Lagrangian becomes:

$$\begin{aligned} \mathcal{L} = E_t \sum_{j=t}^{\infty} \beta^{j-t} \Bigg\{ \left[\frac{(C_j/V_{j-1})^{1-\gamma} - 1}{1-\gamma} + \chi \frac{((1-N_j)/U_{j-1})^{1-\vartheta} - 1}{1-\vartheta} \right] + \\ \lambda_j \left[(1-\delta+D_j)K_{j-1} + W_j N_j + \Pi_j - C_j - K_j \right] \Bigg\}. \end{aligned} \tag{86}$$

We only give these first-order conditions that are different from those in the benchmark model:

$$\frac{\partial \mathcal{L}}{\partial C_t} = 0 \Rightarrow \frac{1}{V_{t-1}} (C_t/V_{t-1})^{-\gamma} - \lambda_t = 0, \tag{87}$$

$$\frac{\partial \mathcal{L}}{\partial N_t} = 0 \Rightarrow -\chi \frac{1}{U_{t-1}} ((1-N_t)/U_{t-1})^{-\vartheta} + \lambda_t W_t = 0. \tag{88}$$

Thus, we obtain a different Euler-equation, and also equation (37) on page 20 changes:

$$1 = \beta E_t \left[\left(\frac{V_t}{V_{t-1}} \right)^{\gamma-1} \left(\frac{C_t}{C_{t+1}} \right)^{\gamma} R_{t+1} \right], \tag{89}$$

$$(1-\theta)\frac{Y_t}{N_t} = \chi \frac{V_{t-1}}{U_{t-1}} \frac{(C_t/V_{t-1})^{\gamma}}{(L_t/U_{t-1})^{\vartheta}}. \tag{90}$$

We obtain one new steady state relation from equation (90):

$$(1-\theta)\frac{\bar{Y}}{\bar{N}} = \chi \frac{\bar{V}}{\bar{U}} \frac{(\bar{C}/\bar{V})^{\gamma}}{(\bar{L}/\bar{U})^{\vartheta}}. \tag{91}$$

This equation is used to compute the parameter χ, i.e. the importance of the leisure part of the utility function relative to the consumption part. In the log-linearized equations, this parameter is of no importance. Note that the steady state relationship implied by the new Euler-equation remains the same as before, that is: $1 = \beta \bar{R}$.

Now we can proceed to log-linearize the two new equations:

$$0 = E_t \left[(\gamma - 1)\,\Delta v_t - \gamma \Delta c_{t+1} + r_{t+1} \right], \tag{92}$$

$$y_t - n_t = \gamma c_t + (1-\gamma) v_{t-1} + \vartheta \frac{\bar{N}}{1-\bar{N}} n_t + (\vartheta - 1) u_{t-1}. \tag{93}$$

We simply have to replace the two old equations in our benchmark model in the MATLAB®-program by these two new ones and can proceed to analyze this new model with habit formation in relative rather than absolute terms.

7.2.3 Model Results

Figure 19 on the next page presents the general dynamics of such a model.

First of all, the general dynamics are the same as in our benchmark model, regardless whether there is habit formation or not: A positive technology shock leads to more output which implies both more consumption and more investment. Increasing investment makes the capital stock rise, too. The real interest rate deviates at first positively and then negatively from its steady state. Furthermore, after the technology shock, leisure at first deviates negatively and then positively from its steady state, a result that is due to the calibration of γ and ϑ.

Secondly, the quantitative effects of introducing habit formation are the same ones as in the benchmark model, too. Again, most variables are not affected by habit formation; only consumption-, labor- and leisure-deviations

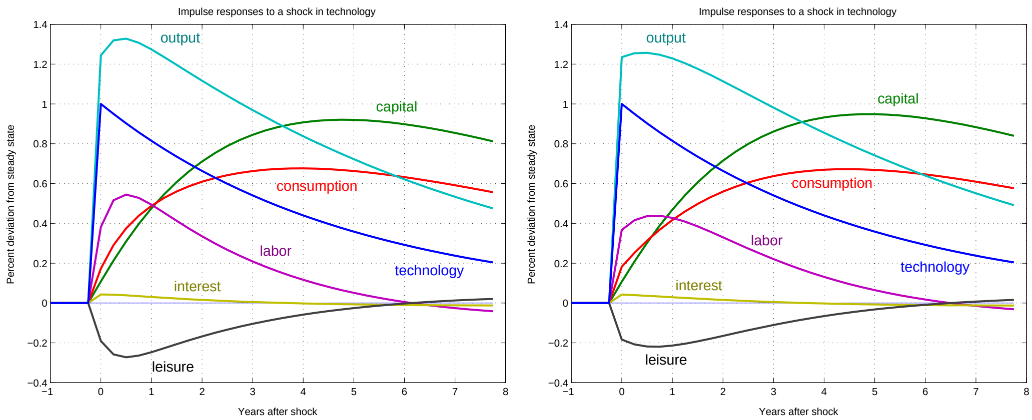

Figure 19: General dynamics of a model without (left) and with (right) external habit formation in both consumption and leisure (Relative rather than absolute terms)
Calibration: $\gamma = \vartheta = 2.372$; $\alpha_C = \alpha_L = \phi_C = \phi_L = 0$ (left); $\alpha_C = \alpha_L = \phi_C = \phi_L = 0.5$ (right)

from their respective steady states decline in absolute terms, which means that these impulse-responses become flatter.

Although the specific effects for the three variables mentioned above may slightly differ quantitatively from those obtained for the benchmark model, we may conclude from this subsection that for the general model dynamics it is not too important whether external habit formation is formally defined in absolute terms as in the benchmark model in section 4 or rather in relative terms as this has been done here.

7.3 *Keeping up with the Joneses* preferences in the model with relative terms

As *keeping up with the Joneses* preferences dropped out of our model analyzed in subsection 7.1 we want to implement them into the model of relative consumption and leisure examined in subsection 7.2.

7.3.1 The Model

The household faces the following maximization problem:

$$\max_{\{C_j;L_j;I_j\}_{j=t}^{\infty}} E_t \sum_{j=t}^{\infty} \beta^{j-t} \left[\frac{(C_j/V_j)^{1-\gamma} - 1}{1-\gamma} + \chi \frac{(L_j/U_j)^{1-\vartheta} - 1}{1-\vartheta} \right], \tag{94}$$

subject to the same constraint as in the benchmark case. Likewise, the problem of the firm and the market clearing conditions remain the same. Habit formation takes the same form as in subsection 7.1, which means that:

$$V_t = \alpha_C C_t, \tag{95}$$
$$U_t = \alpha_L L_t, \tag{96}$$

just as in the relative case under *catching up with the Joneses* preferences.

7.3.2 Model Analysis

The Lagrangian becomes:

$$\mathcal{L} = E_t \sum_{j=t}^{\infty} \beta^{j-t} \left\{ \left[\frac{(C_j/V_j)^{1-\gamma} - 1}{1-\gamma} + \chi \frac{((1-N_j)/U_j)^{1-\vartheta} - 1}{1-\vartheta} \right] + \lambda_j \left[(1-\delta+D_j)K_{j-1} + W_j N_j + \Pi_j - C_j - K_j \right] \right\}. \tag{97}$$

This leads to new first-order conditions:

$$\frac{\partial \mathcal{L}}{\partial C_t} = 0 \Rightarrow \frac{1}{V_t}(C_t/V_t)^{-\gamma} - \lambda_t = 0, \tag{98}$$
$$\frac{\partial \mathcal{L}}{\partial N_t} = 0 \Rightarrow -\chi \frac{1}{U_t}((1-N_t)/U_t)^{-\vartheta} + \lambda_t W_t = 0. \tag{99}$$

Combining the equations yields:

$$1 = \beta E_t \left[\left(\frac{V_{t+1}}{V_t} \right)^{\gamma-1} \left(\frac{C_t}{C_{t+1}} \right)^{\gamma} R_{t+1} \right], \tag{100}$$
$$(1-\theta)\frac{Y_t}{N_t} = \chi \frac{V_t}{U_t} \frac{(C_t/V_t)^{\gamma}}{(L_t/U_t)^{\vartheta}}. \tag{101}$$

Obviously, we obtain the same steady state equations as in the relative case with *catching up with the Joneses* preferences.

The two new log-linearized equations are:

$$0 = E_t \left[(\gamma - 1)\, \Delta v_{t+1} - \gamma \Delta c_{t+1} + r_{t+1} \right], \tag{102}$$
$$y_t - n_t = \gamma c_t + (1-\gamma) v_t - \vartheta l_t + (\vartheta - 1) u_t. \tag{103}$$

Furthermore, log-linearizing the habit equations (95) and (96) on the preceding page yields $c_t = v_t$ and $l_t = u_t$, so that equations (102) and (103) on the previous page simplify to:

$$0 = E_t \left[-\Delta c_{t+1} + r_{t+1} \right], \tag{104}$$

$$y_t - n_t = c_t - l_t. \tag{105}$$

7.3.3 Model Results

We can see that as in subsection 7.1, where habit formation was defined by *keeping up with the Joneses* preferences in absolute terms, habit formation drops out of our model. Figure 20 illustrates this issue: Both graphs are completely identical.

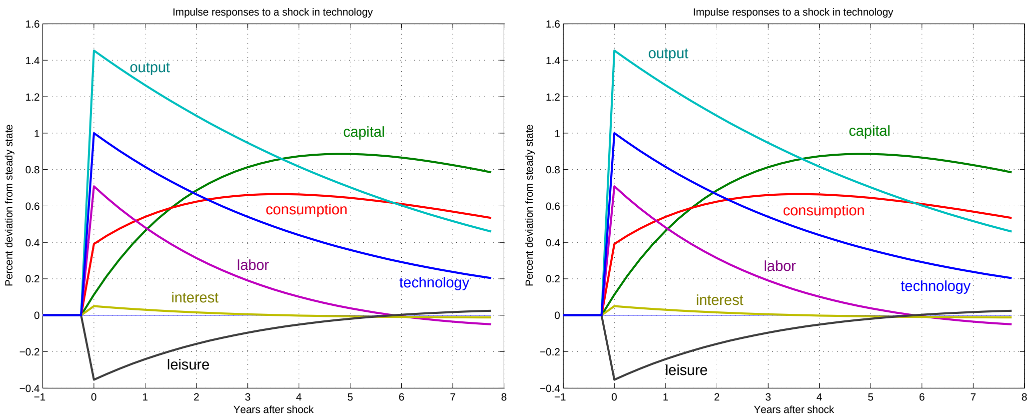

Figure 20: General dynamics of a model without (left) and with (right) external habit formation in both consumption and leisure (Relative rather than absolute terms, *keeping up with the Joneses* preferences)
Calibration: $\gamma = \vartheta = 2.372$; $\alpha_C = \alpha_L = 0$ (left); $\alpha_C = \alpha_L = 0.5$ (right)

7.4 Taxes

In this subsection, we introduce taxes into our benchmark model in section 4. Our motivation is to examine the dynamics of this new model, especially of consumption and leisure, in the presence of a tax system that is motivated by the hypothesis of fluctuating government expenditures.

7.4.1 The Model

Suppose that there is a government which acts subject to the following budget constraint:

$$G_t + S_t = \mathcal{T}_t^{(n)} W_t N_t^{(s)} + \mathcal{T}_t^{(k)} D_t K_{t-1}^{(s)}, \tag{106}$$

where G_t stands for government expenditures[16] and S_t represents lump-sum transfer from the government to the representative household. $\mathcal{T}_t^{(n)}$ and $\mathcal{T}_t^{(k)}$ denote tax rates on labor income and capital income, respectively, in period t. All taxes are payed by the representative household. We assume for simplification that equation (106) has to be fulfilled in each period t[17].

Suppose that government expenditures G_t fluctuate over time, following the AR(1) process

$$\log G_{t+1} = (1 - \psi_G) \log \tilde{G} + \psi_G \log G_t + \rho_{t+1}, \;\; \rho_t \sim \text{i.i.d. } \mathcal{N}(0; \sigma_\rho^2). \tag{107}$$

Then, equation (106) implies that also S_t, $\mathcal{T}_t^{(n)}$ and $\mathcal{T}_t^{(k)}$ fluctuate or that at least one of these three variables does so.

As in our benchmark model, we may drop superscripts $^{(d)}$ and $^{(s)}$ for demand and supply thanks to the fact that all markets are assumed to be cleared in equilibrium. The budget constraint of the representative household, derived by introducing taxes and lump-sum transfer into equation (22) on page 18, is:

$$C_t + K_t = (1 - \delta + (1 - \mathcal{T}_t^{(k)}) D_t) K_{t-1} + (1 - \mathcal{T}_t^{(n)}) W_t N_t + S_t + \Pi_t. \tag{108}$$

Although we know that fluctuating government expenditures imply fluctuating taxes, we do not know how they do so because they are not identifiable. Technically, there are two additional degrees of freedom, due to the fact that our tax system $(\mathcal{T}_t^{(n)}, \mathcal{T}_t^{(k)}, S_t)$ is made up of three variables (two different tax rates and the lump-sum transfer) that face the single variable government expenditures G_t. Thus, we must impose two additional conditions on $\mathcal{T}_t^{(n)}$, $\mathcal{T}_t^{(k)}$ and S_t in order to be able to identify them. Consequently, we assume:

$$S_t = 0, \tag{109}$$

$$\mathcal{T}_t^{(n)} = \mathcal{T}_t^{(k)} = \mathcal{T}_t, \tag{110}$$

[16] Note that these government expenditures are assumed to yield no utility at all.

[17] Put differently, the government is simply not allowed to lend or to borrow.

implying an absence of lump-sum transfers and identical tax rates. From equation (106) on the preceding page we obtain the simplified budget constraint of the government

$$G_t = \mathcal{T}_t \left(W_t N_t + D_t K_{t-1} \right) = \mathcal{T}_t Y_t. \tag{111}$$

The budget constraint of the representative household (108) on the previous page simplifies to:

$$C_t + K_t = (1 - \delta + D_t) K_{t-1} + W_t N_t + \Pi_t - \underbrace{\mathcal{T}_t \left(W_t N_t + D_t K_{t-1} \right)}_{G_t = \mathcal{T}_t Y_t}. \tag{112}$$

Intuitively, equation (112) states that the income of the household, which can be used for consumption C_t and capital formation K_t, is reduced by the amount of government expenditures G_t, compared to the benchmark model.

Finally, note that the problem of the firm remains unchanged.

7.4.2 Model Analysis

The Lagrangian for the problem of the representative household becomes:

$$\mathcal{L} = E_t \sum_{j=t}^{\infty} \beta^{j-t} \Bigg\{ \left[\frac{(C_j - V_{j-1})^{1-\gamma} - 1}{1-\gamma} + \chi \frac{(1 - N_j - U_{j-1})^{1-\vartheta} - 1}{1-\vartheta} \right] +$$
$$\lambda_j \left[(1 - \delta + (1 - \mathcal{T}_j) D_j) K_{j-1} + (1 - \mathcal{T}_j) W_j N_j + \Pi_j - C_j - K_j \right] \Bigg\}. \tag{113}$$

Maximization yields new first-order conditions:

$$\frac{\partial \mathcal{L}}{\partial N_t} = 0 \Rightarrow -\chi (1 - N_t - U_{t-1})^{-\vartheta} + \lambda_t (1 - \mathcal{T}_t) W_t = 0, \tag{114}$$
$$\frac{\partial \mathcal{L}}{\partial K_t} = 0 \Rightarrow -\lambda_t + \beta E_t \left[\lambda_{t+1} (1 - \delta + (1 - \mathcal{T}_{t+1}) D_{t+1}) \right] = 0, \tag{115}$$
$$\frac{\partial \mathcal{L}}{\partial \lambda_t} = 0 \Rightarrow (1 - \delta + (1 - \mathcal{T}_t) D_t) K_{t-1} + (1 - \mathcal{T}_t) W_t N_t - C_t - K_t = 0. \tag{116}$$

The other conditions remain unchanged. Combining the equations yields:

$$
\begin{aligned}
Y_t &= C_t + I_t + G_t, && (117)\\
K_t &= I_t + (1-\delta)K_{t-1}, && (118)\\
Y_t &= Z_t K_{t-1}^{\theta} N_t^{1-\theta}, && (119)\\
(1-\mathcal{T}_t)(1-\theta)\frac{Y_t}{N_t} &= \chi\frac{(C_t - V_{t-1})^{\gamma}}{(L_t - U_{t-1})^{\vartheta}}, && (120)\\
R_t &= 1-\delta+(1-\mathcal{T}_t)\theta\frac{Y_t}{K_{t-1}}, && (121)\\
L_t &= 1 - N_t, && (122)\\
V_{t-1} &= (1-\phi_C)\alpha_C C_{t-1} + \phi_C V_{t-2}, && (123)\\
U_{t-1} &= (1-\phi_L)\alpha_L L_{t-1} + \phi_L U_{t-2}, && (124)\\
G_t &= \mathcal{T}_t Y_t, && (125)\\
1 &= \beta E_t\left[\left(\frac{C_t - V_{t-1}}{C_{t+1} - V_t}\right)^{\gamma} R_{t+1}\right], && (126)\\
\log Z_{t+1} &= (1-\psi)\log\tilde{Z} + \psi\log Z_t + \epsilon_{t+1}, && (127)\\
\log G_{t+1} &= (1-\psi_G)\log\tilde{G} + \psi_G\log G_t + \rho_{t+1}. && (128)
\end{aligned}
$$

The new steady state equations are:

$$
\begin{aligned}
\bar{Y} &= \bar{C} + \bar{I} + \bar{G}, && (129)\\
(1-\bar{\mathcal{T}})(1-\theta)\frac{\bar{Y}}{\bar{N}} &= \chi\frac{(\bar{C}-\bar{V})^{\gamma}}{(\bar{L}-\bar{U})^{\vartheta}}, && (130)\\
\bar{R} &= 1-\delta+(1-\bar{\mathcal{T}})\theta\frac{\bar{Y}}{\bar{K}}, && (131)\\
\bar{G} &= \bar{\mathcal{T}}\bar{Y}, && (132)\\
\bar{G} &= \tilde{G}, && (133)
\end{aligned}
$$

for equations (117), (120), (121), (125) and (128), respectively. The other steady state equations remain as in the benchmark model. Note that in addition to the other parameters we need to calibrate the tax rate $\bar{\mathcal{T}} = \frac{\bar{G}}{\bar{Y}}$.

Log-linearization yields the following new equations describing the model dynamics:

$$\bar{Y}y_t = \bar{I}i_t + \bar{C}c_t + \bar{G}g_t, \tag{134}$$

$$y_t - n_t + \frac{\bar{\mathcal{T}}}{\bar{\mathcal{T}} - 1}\tau_t = \frac{\gamma}{1-\alpha_C}\left(c_t - \alpha_C v_{t-1}\right) - \frac{\vartheta}{1-\alpha_L}\left(l_t - \alpha_L u_{t-1}\right), \tag{135}$$

$$\bar{R}r_t = (1-\bar{\mathcal{T}})\theta\frac{\bar{Y}}{\bar{K}}\left[y_t - k_{t-1} - \frac{\bar{\mathcal{T}}}{1-\bar{\mathcal{T}}}\tau_t\right], \tag{136}$$

$$g_t = \tau_t + y_t, \tag{137}$$

$$g_{t+1} = \psi_G g_t + \rho_{t+1}. \tag{138}$$

These new equations can now be added to the old ones or replace the old ones in the benchmark model. The corresponding MATLAB®-program can be found in the appendix on page 75.

7.4.3 Model Results

Figure 21 presents the general model dynamics after a technology shock. Again we compare a model without any habit formation (left graph) to a model with external habit formation in both consumption and leisure (right graph).

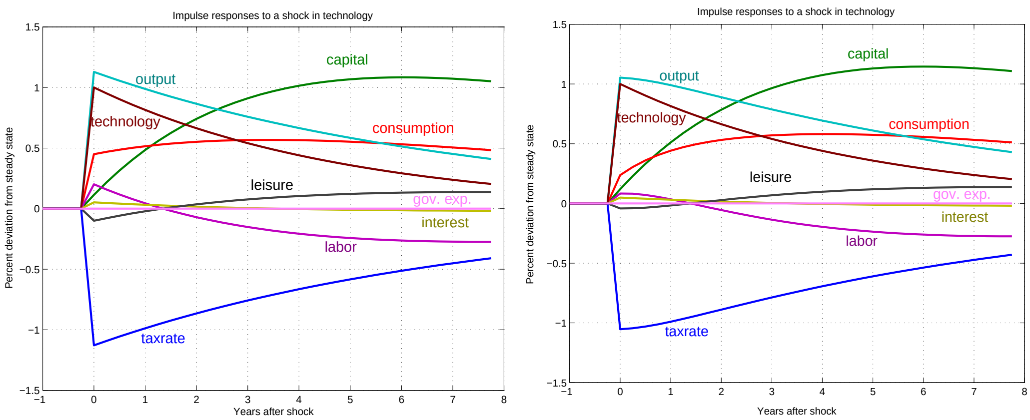

Figure 21: General dynamics of a model without (left) and with (right) external habit formation in both consumption and leisure (Taxes)
Calibration: $\bar{\mathcal{T}} = 0.25$, $\gamma = \vartheta = 2.372$; $\alpha_C = \alpha_L = \phi_C = \phi_L = 0$ (left); $\alpha_C = \alpha_L = \phi_C = \phi_L = 0.5$ (right)

For the most part, the model dynamics are the same as in the benchmark model (figure 2 on page 27). However, after the technology shock, output does not deviate as much from its steady state as it does in the benchmark model. Instead, the tax rate deviates negatively from its steady state. Nevertheless, introducing habit formation into the model has the same effects on consumption- and leisure-choices, namely a *delaying effect* on the consumption-deviation from its steady state and a *smoothing effect* on leisure and thus also on labor input.

As we consider taxes in this model, we are more interested in the effects of a shock in government expenditures than in those of a technology shock. Figure 22 shows the corresponding impulse-responses.

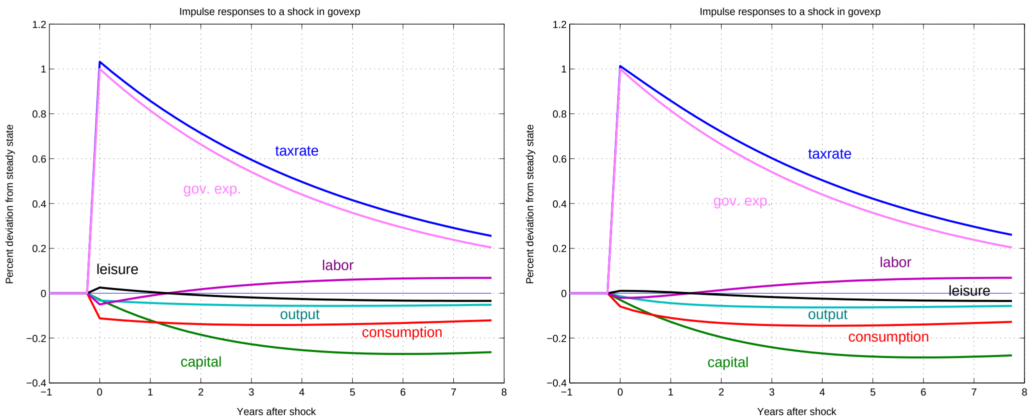

Figure 22: General dynamics of a model without (left) and with (right) external habit formation in both consumption and leisure (Taxes, responses to a shock in government expenditures) Calibration: $\bar{\mathcal{T}} = 0.25$, $\gamma = \vartheta = 2.372$; $\alpha_C = \alpha_L = \phi_C = \phi_L = 0$ (left); $\alpha_C = \alpha_L = \phi_C = \phi_L = 0.5$ (right)

A positive shock in government expenditures instantly leads to a higher tax rate because government expenditures are financed through taxes. This is also why, as government expenditures converge back to their steady state, the tax rate does so, too. As the tax rate increases after the shock in government expenditures, working becomes less attractive. Thus, leisure rises instantly after the shock in government expenditures, so that labor input falls. Additionally, due to the high government expenditures, investment (which is not plotted here) decreases significantly immediately after the shock, so that capital falls as well. As both capital input and labor input fall, output does so, too. Furthermore, because of the higher government expenditures, people consume less.

The influence of habit formation in consumption and leisure can be seen, too. As usual, on the one hand consumption deviates later from its steady state in the model with habit formation. Leisure and labor, on the other hand, become smoother when external habit formation is introduced. Generally, we may conclude that for the effect of habit formation on consumption and leisure, the type of the shock (technology shock or shock in government expenditures) only plays a minor role. Indeed, although after a technology shock consumption deviates positively from its steady state whereas after a shock in government expenditures it does so negatively, in both cases introducing habit formation into the model results in a *delaying effect*. Similarly, for leisure and labor we observe the smoothing effect, whether there is a technology shock or a shock in government expenditures.

8 Discussion

In this section we discuss the results we have obtained throughout the analysis in this diploma thesis.

First, we have seen that in the medium run, habit formation has no influence on the dynamics of the economy. All variables converge back to their steady state at approximately the same time, regardless whether there is habit formation or not. In the short run, habit formation is of more importance. Consumption deviates later and more slowly from its steady state when habit formation is introduced into consumption, a phenomenon we called the *delaying effect*. The impulse-responses of leisure and labor input become smoother when habit formation is introduced into consumption or leisure. We referred to this as the *smoothing effect*. We also observe a *smoothing effect* in consumption and leisure for more persistent forms of habit formation.

We have shown that, when habit formation is introduced into the model, the differences in the reactions of consumption are mainly due to habit formation in consumption itself, habit formation in leisure does not influence consumption-choices very much. On the other hand, the differences in the reactions of leisure are due to both types of habit formation. A more detailed analysis of the importance of external habit formation in leisure on consumption-choices yielded that the higher the values of the curvature parameters of consumption and leisure are, the more habit formation in leisure influences consumption-choices. However, for microeconomically reasonable calibrations of these parameters, this influence is very small.

Concerning the influence of habit formation in leisure on leisure-choices we found out that the income effect dominates as long as the curvature parameters are high enough. In this case leisure rises instantly after a technology shock. For smaller values of these parameters, as in our standard calibration, however, at first substitution effect and then income effect dominates so that leisure demand at first decreases and then increases.

The variations of our model also provided some interesting insight into the dynamics. When external habit formation takes the form of *keeping up with the Joneses* preferences instead of *catching up with the Joneses* preferences it has no influence in our log-linearized model. Moreover, when habit formation is modeled in relative rather than in absolute terms, the general implications for the model dynamics remain the same as in the benchmark model. Finally, introducing a government and a tax system yielded another interesting variation of the benchmark model. Here, we have seen that the effects on consumption- and leisure-choices after a shock in government expenditures are rather similar to those after a technology shock. The only difference is that the deviations of the variables after the latter shock are opposed to those after the former one[18].

All these arguments show that habit formation in consumption is more important than habit formation in leisure. Therefore, the central result of this diploma thesis and at the same time an important implication for economic theory is that when we consider models with external habit formation, we may well restrict this feature to consumption.

Generally, this result is reasonable and intuitive. As it has already been pointed out in the analysis, the deviation of leisure from its steady state after a technology shock is already very small, compared to the reactions of consumption and the other variables. Consequently, habit formation in leisure, even if it is rather strong and persistent, cannot influence the deviations of those variables very much. Equivalently, if one argues that we have examined in detail only the impulse-responses of consumption, leisure and labor input, we stress the fact that habit formation has the strongest effect on these three variables which are chosen each period by the representative agent and that already here the influence is rather small.

[18] More precisely, after a technology shock, consumption rises instantly. Leisure deviates at first negatively and then positively from its steady state. After a shock in government expenditures, consumption decreases, whereas leisure deviates at first positively and then negatively. In both cases, habit formation smoothes these impulse-responses.

9 Summary and Concluding Remarks

Motivated by the the remarkable success external habit formation in consumption has had in solving phenomena such as the *equity premium puzzle* and in explaining the slow and hump-shaped response of consumption to shocks, we have challenged the restriction of *catching up with the Joneses* preferences to consumption by extending them to the leisure part of the utility function of the representative household. In other words, this diploma thesis has investigated business cycle dynamics with external habit formation in both consumption and leisure. Furthermore, we tried to find out whether the restriction of habit formation to consumption is justified or whether it should be extended to leisure. In this context, we have extended the models analyzed by Lettau and Uhlig (2000) and Ljungqvist and Uhlig (2000).

The workhorse we used for our analysis was a real business cycle model with habit formation in both consumption and leisure. Each time, habit formation was modeled as *catching up with the Joneses* preferences and in a way that we could distinguish between a transitory and a persistent form. After developing and solving this model, in a first step we compared its general dynamics without and with habit formation and found that the impulse-responses of consumption and leisure to a technology shock were of special interest because these two variables were effected strongest by habit formation. Then, we scrutinized the impulse-responses of these variables in models with external habit formation in (i) consumption only, (ii) leisure only and (iii) both consumption and leisure. After that, we investigated in more detail the influence of external habit formation in leisure on consumption-choices, searching for what calibration of the variables this influence is maximum. In a next step, we considered the influence of habit formation in leisure on leisure-choices.

Studying the arguments we obtained throughout the analysis of our model and which are summarized in section 8, one can draw the conclusion that the overall influence habit formation in leisure is very small. Indeed, as it has been shown, the only variable that is really influenced is leisure itself. Consumption, on the other hand, is not really affected, let alone the other variables. This justifies the fact that until now, in literature and in economic theory habit formation was restricted to consumption and not introduced into leisure. Indeed, one has to consider that an extension of habit formation to leisure results in more complex models and consequently in a more intricate model analysis that is difficult to justify given the small influence of habit formation in leisure.

Although it has been shown that habit formation in leisure only plays a minor role compared to habit formation in consumption, there is still room for further research. For example, it may be of interest to investigate the model with a government and a tax system, which we introduced as a variation of our benchmark model, in more detail. Secondly, other kinds of habit formation can be considered, for example internal habit formation. Finally, one may also introduce habit formation in leisure into models with other economic features such as money, sticky prices or incomplete markets, for example.

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Appendix

Benchmark model

```
% CATCHINGABSOLUTE.M
% calculates through a real business cycle model
% with external habit formation ("Catching up with the Joneses" preferences,
% absolute terms) in both Consumption and Leisure;
% see Hildebrand, Thomas: "Business Cycle Dynamics under
% Catching up with the Joneses Preferences in Leisure",
% Diploma Thesis at the School of Business and Economics
% at Humboldt University of Berlin, July 2006
% Copyright CATCHINGABSOLUTE.M: Thomas Hildebrand

% TOOLKIT
% VERSION 2.0, MARCH 1997, COPYRIGHT H. UHLIG.
% For more information on how the Matlab Code works see H. Uhlig,
% "A toolkit for solving nonlinear dynamic stochastic models easily".
% First, parameters are set and the steady state is calculated. Next, the matrices are
% declared. In the last line, the model is solved and analyzed by calling DO_IT.M
% Copyright Toolkit-Code: H. Uhlig.
% Feel free to copy, modify and use at your own risk.
% However, you are not allowed to sell this software or otherwise impinge
% on its free distribution.

disp('Catchingabsolute: A real business cycle model with external habit formation');
disp('                  ("Catching up with the Joneses" preferences, absolute terms)');
disp('                  in both Consumption and Leisure;');
disp('                  see Hildebrand, Thomas: "Business Cycle Dynamics under');
disp('                  Catching up with the Joneses Preferences in Leisure",');
disp('                  Diploma thesis at the School of Business and Economics');
disp('                  at Humboldt University of Berlin, July 2006');

disp('Hit any key when ready...');
pause;

% Setting fixed parameters.
N_bar     = 1.0/3;  % Steady state employment is a third of total time endowment
Z_bar     = 1;      % Normalization
theta     = .36;    % Capital share
delta     = .025;   % Depreciation rate per quarter for capital
R_bar     = 1.01;   % One percent real interest per quarter
psi       = .95;    % Autocorrelation of technology shock
sigma_eps = .712;   % Standard deviation of technology shock. Units: Percent

% Setting varying parameters.
gamma     = 2.372;  % Curvature of consumption
vartheta  = 2.372;  % Curvature of leisure
alpha_C   = .5;     % Consumption habit importance parameter (set =0.00001 for no consumption habit)
phi_C     = .5;     % Consumption habit persistence parameter
alpha_L   = .5;     % Leisure habit importance parameter (set =0.00001 for no leisure habit)
phi_L     = .5;     % Leisure habit persistence parameter

% Calculating the steady state.
betta   = 1.0/R_bar;
YK_bar  = (R_bar + delta - 1) / theta; % = Y_bar / K_bar
K_bar   = (YK_bar / Z_bar)^(1.0/(theta-1)) * N_bar;
I_bar   = delta * K_bar;
Y_bar   = YK_bar * K_bar;
C_bar   = Y_bar - delta*K_bar;
L_bar   = 1.0-N_bar;
V_bar   = alpha_C * C_bar;
U_bar   = alpha_L * L_bar;
Chi     = (1 - theta) * Y_bar/N_bar * (L_bar - U_bar)^vartheta * (C_bar - V_bar)^(-gamma);

% Declaring the matrices.
VARNAMES = ['capital    ',  % Variable  1
            'conshabit  ',  % Variable  2
            'leishabit  ',  % Variable  3
            'consumption',  % Variable  4
            'output     ',  % Variable  5
            'labor      ',  % Variable  6
            'interest   ',  % Variable  7
            'investment ',  % Variable  8
            'leisure    ',  % Variable  9
            'technology ']; % Variable 10

% Translating into coefficient matrices.
% The equations are, conveniently ordered:
% 1) 0 = - I i(t) - C c(t) + Y y(t)
% 2) 0 = I i(t) - K k(t) + (1-delta) K k(t-1)
```

```
% 3) 0 = theta k(t-1) - y(t) + (1-theta) n(t) + z(t)
% 4) 0 = y(t) - n(t) - gamma * (C_bar/(C_bar-V_bar)) * c(t) + gamma * (V_bar/(C_bar-V_bar)) * v(t-1) ...
%                          - vartheta (N_bar/(1-N_bar-U_bar) * n(t) - vartheta (U_bar/(1-N_bar-U_bar) * u(t-1)
% 5) 0 = - theta Y/K k(t-1) + theta Y/K y(t) - R r(t)
% 6) 0 = L_bar * l(t) + N_bar * n(t)
% 7) 0 = -V_bar * v(t-1) + (1-phi_C) * alpha_C * C_bar * c(t-1) + phi_C * V_bar * v(t-2)
% 8) 0 = -U_bar * u(t-1) + (1-phi_L) * alpha_L * L_bar * l(t-1) + phi_L * U_bar * u(t-2)
% 9) 0 = E_t [- gamma * (C_bar/(C_bar-V_bar)) * (c(t+1)-c(t)) - gamma * (V_bar/(C_bar-V_bar)) * (v(t)-v(t-1)) + r(t+1)]
%10) z(t+1) = psi z(t) + epsilon(t+1)

% CHECK: 10 equations, 10 variables.
% Endogenous state variables "x(t)": k(t), v(t), u(t)
% Endogenous other variables "y(t)": c(t), y(t), n(t), r(t), i(t), l(t),
% Exogenous state variables  "z(t)": z(t)

% Switch to that notation.  Find matrices for format
% 0 = AA x(t) + BB x(t-1) + CC y(t) + DD z(t)
% 0 = E_t [ FF x(t+1) + GG x(t) + HH x(t-1) + JJ y(t+1) + KK y(t) + LL z(t+1) + MM z(t)]
% z(t+1) = NN z(t) + epsilon(t+1) with E_t [ epsilon(t+1) ] = 0,


% Start System 1 (8 Equations)

%      capital,                        conshabit,                      leishabit
%      k(t),                           v(t),                           u(t)
AA = [ 0,                              0,                              0                                   %Eq. 1
       - K_bar,                        0,                              0                                   %Eq. 2
       0,                              0,                              0                                   %Eq. 3
       0,                              0,                              0                                   %Eq. 4
       0,                              0,                              0                                   %Eq. 5
       0,                              0,                              0                                   %Eq. 6
       0,                              -V_bar,                         0                                   %Eq. 7
       0,                              0,                              -U_bar];                            %Eq. 8

%      capital,                        conshabit,                      leishabit
%      k(t-1),                         v(t-1),                         u(t-1)
BB = [ 0,                              0,                              0                                   %Eq. 1
       (1-delta)*K_bar,                0,                              0                                   %Eq. 2
       theta,                          0,                              0                                   %Eq. 3
       0,                              gamma * (V_bar/(C_bar-V_bar)),  - vartheta * (U_bar/(1-N_bar-U_bar))  %Eq. 4
       - theta * YK_bar,               0,                              0                                   %Eq. 5
       0,                              0,                              0                                   %Eq. 6
       0,                              phi_C*V_bar,                    0                                   %Eq. 7
       0,                              0,                              phi_L*U_bar];                       %Eq. 8

%      consumption                  output        labor                                inter.  inv.    leisure
%      c(t)                         y(t)          n(t)                                 r(t)    i(t)    l(t)
CC = [ -C_bar,                      Y_bar,        0,                                   0,      -I_bar, 0                       %Eq. 1
       0,                           0,            0,                                   0,      I_bar,  0                       %Eq. 2
       0,                           -1,           1-theta,                             0,      0,      0                       %Eq. 3
       -gamma*C_bar/(C_bar-V_bar),  1,            -1-(vartheta*N_bar/(1-N_bar-U_bar)), 0,      0,      0                       %Eq. 4
       0,                           theta*YK_bar, 0,                                   -R_bar, 0,      0                       %Eq. 5
       0,                           0,            N_bar,                               0,      0,      L_bar                   %Eq. 6
       (1-phi_C)*alpha_C*C_bar,     0,            0,                                   0,      0,      0                       %Eq. 7
       0,                           0,            0,                                   0,      0,      (1-phi_L)*alpha_L*L_bar];%Eq. 8

%      technology
%      z(t)
DD = [ 0    %Eq. 1
       0    %Eq. 2
       1    %Eq. 3
       0    %Eq. 4
       0    %Eq. 5
       0    %Eq. 6
       0    %Eq. 7
       0 ]; %Eq. 8

% End System 1


% Start System 2 (1 Equation)

%      capital,                        conshabit,                      leishabit
%      k(t+1),                         v(t+1),                         u(t+1)
FF = [ 0,                              0,                              0 ];

%      capital,                        conshabit,                      leishabit
%      k(t),                           v(t),                           u(t)
GG = [ 0,                              gamma*(V_bar/(C_bar-V_bar)),    0 ];

%      capital,                        conshabit,                      leishabit
%      k(t-1),                         v(t-1),                         u(t-1)
HH = [ 0,                              -gamma*(V_bar/(C_bar-V_bar)),   0 ];
```

```
%      consumption                     output           labor           interest  investment   leisure
%      c(t+1)                          y(t+1)           n(t+1)          r(t+1)    i(t+1)       l(t+1)
JJ = [ -gamma*(C_bar/(C_bar-V_bar)),  0,               0,              1,        0,           0 ];

%      consumption                     output           labor           interest  investment   leisure
%      c(t)                            y(t)             n(t)            r(t)      i(t)         l(t)
KK = [ gamma*(C_bar/(C_bar-V_bar)),   0,               0,              0,        0,           0 ];

%      technology
%      z(t+1)
LL = [ 0 ];

%      technology
%      z(t)
MM = [ 0 ];

% End System 2


% Start System 3 (1 Equation)

NN = [psi];

% End System 3


Sigma = [ sigma_eps^2 ];

% Setting the options.
[l_equ,m_states] = size(AA);
[l_equ,n_endog ] = size(CC);
[l_equ,k_exog  ] = size(DD);

PERIOD     = 4;                          % Number of periods per year, i.e. 12 for monthly, 4 for quarterly
GNP_INDEX  = 5;                          % Index of output among the variables selected for HP filter
IMP_SELECT = [1:10];                     % A vector containing the indices of the variables to be plotted
DO_SIMUL   = 0;                          % Calculates simulations
SIM_LENGTH = 150;
DO_MOMENTS = 0;                          % Calculates moments based on frequency-domain methods
HP_SELECT  = 1:(m_states+n_endog+k_exog);   % Selecting the variables for the HP Filter calcs.
DISPLAY_IMMEDIATELY = 1;                 % Display warnings immediately

IMP_SINGLE  = 0;                         % no single graphs for impulse responses
IMP_SELECT  = [1 4 5 6 7 9 10];          % select the variables to be plotted in impulse responses
HORIZON     = 2^5;

% Starting the calculations.
do_it;
```

Grid determining maximum influence of external habit formation in leisure on consumption

```
% This program (CATCHINGINFLUENCE.M) searches for those parameter values that yield the maximum influence
% on consumption when habit formation is introduced into leisure. Theoretically, it is a six-dimensional grid
% (gamma, vartheta, alpha_C, phi_C, alpha_L, phi_L). However, considering the four dimensions
% gamma, vartheta, alpha_C, phi_C is the really interesting part.

% Before turning this program, modify CATCHINGABSOLUTE.M so that it does not set
% the varying parameters gamma, vartheta, alpha_C, phi_C, alpha_L, phi_L
% nor HORIZON, which is done here for each iteration.
% Furthermore, turn off the information displayed and the pause in the beginning of
% the program which otherwise would appear in each iteration.

% Finally, replace "do_it;" by the following code (here as commentary;
% do so in order to turn off warnings and impulse responses for each iteration):

% DO_SIMUL    = 0;
% DO_MOMENTS  = 0;
% IMP_SELECT  = [4];
% IMP_SUBPLOT = 0;
% IMP_JOINT   = 0;
% IMP_SINGLE  = 0;
% DO_SHOCK_RESP = 1;
% DO_STATE_RESP = 0;
% DISPLAY_IMMEDIATELY = 0;
% DISPLAY_AT_THE_END = 0;
% DISPLAY_ROOTS=0;
% message = ' ';
% warnings = [];
% options;
% solve;
% if DO_IMPRESP,
% impresp;
% end;

clc;
clear;
HORIZON            = 2^10;
dummy              = 0;

disp('Parameters:');

alpha_L_min        = .9 % usually, alpha_L_min should equal alpha_L_max
alpha_L_max        = .9
alpha_L_step       = .1
alpha_L_solution   = 0;

phi_L_min          = .9 % usually, phi_L_min should equal phi_L_max
phi_L_max          = .9
phi_L_step         = .1
phi_L_solution     = 0;

alpha_C_steps      = 5
alpha_C_min        = 0.1
alpha_C_max        = 0.9
alpha_C_step       = (alpha_C_max-alpha_C_min)/alpha_C_steps
alpha_C_solution   = 0;

phi_C_steps        = 5
phi_C_min          = 0.1
phi_C_max          = 0.9
phi_C_step         = (phi_C_max-phi_C_min)/phi_C_steps
phi_C_solution     = 0;

gamma_steps        = 5
gamma_min          = 2
gamma_max          = 10
gamma_step         = (gamma_max-gamma_min)/gamma_steps
gamma_solution     = 0;

vartheta_steps     = 5
vartheta_min       = 1
vartheta_max       = 10
vartheta_step      = (vartheta_max-vartheta_min)/vartheta_steps
vartheta_solution = 0;

for alpha_L_help = alpha_L_min:alpha_L_step:alpha_L_max                  % loop alpha_L
    for phi_L = phi_L_min:phi_L_step:phi_L_max                           % loop phi_L
        for alpha_C = alpha_C_min:alpha_C_step:alpha_C_max               % loop alpha_C
            for phi_C = phi_C_min:phi_C_step:phi_C_max                   % loop phi_C
                for gamma = gamma_min:gamma_step:gamma_max               % loop for gamma
```

```
                    for vartheta = vartheta_min:vartheta_step:vartheta_max  % loop for vartheta

                        % Without habit formation in leisure
                        alpha_L = .00001;
                        catchingabsolute;
                        Resp_cons1=Resp_mat(4,:);

                        % With habit formation in leisure
                        alpha_L = alpha_L_help;
                        catchingabsolute;
                        Resp_cons2=Resp_mat(4,:);
                        Resp_cons=abs(Resp_cons2-Resp_cons1);

                        % Compute area between the two impulse responses
                        value=Resp_cons(1)/2;
                        for i = 1:(HORIZON-1)
                            value=value+(abs(Resp_cons(i)+Resp_cons(i+1)))/2;
                        end

                        if value>dummy
                            dummy=value;
                            vartheta_solution=vartheta;
                            gamma_solution=gamma;
                            alpha_C_solution=alpha_C;
                            phi_C_solution=phi_C;
                            alpha_L_solution=alpha_L_help;
                            phi_L_solution=phi_L;

                        end                                                 % end if-condition
                    end                                                     % end loop vartheta
                end                                                         % end loop gamma
            end                                                             % end loop phi_C
        end                                                                 % end loop alpha_C
    end                                                                     % end loop phi_L
end                                                                         % end loop alpha_L

% print solutions
disp('Solutions:');
alpha_L_solution
phi_L_solution
alpha_C_solution
phi_C_solution
gamma_solution
vartheta_solution
dummy
```

Graph displaying influence of external habit formation in leisure on consumption

```
% This program (CATCHINGGAMMAVARTHETA.M) creates - for given values of alpha_C, phi_C, alpha_L, phi_L -
% a graph that displays for different values of gamma and vartheta the influence on consumption when
% habit formation is introduced into leisure.

% Before turning this program, modify CATCHINGABSOLUTE.M so that it does not set
% the varying parameters gamma, vartheta, alpha_C, phi_C, alpha_L, phi_L
% nor HORIZON, which is done here.
% Furthermore, turn off the information displayed and the pause in the beginning of
% the program which otherwise would appear in each iteration.

% Finally, replace "do_it;" by the following code (here as commentary;
% do so in order to turn off warnings and impulse responses for each iteration):

% DO_SIMUL    = 0;
% DO_MOMENTS  = 0;
% IMP_SELECT  = [4];
% IMP_SUBPLOT = 0;
% IMP_JOINT   = 0;
% IMP_SINGLE  = 0;
% DO_SHOCK_RESP = 1;
% DO_STATE_RESP = 0;
% DISPLAY_IMMEDIATELY = 0;
% DISPLAY_AT_THE_END = 0;
% DISPLAY_ROOTS=0;
% message = ' ';
% warnings = [];
% options;
% solve;
% if DO_IMPRESP,
% impresp;
% end;

clear;
clc;
HORIZON       = 2^10;
phi_l         = .75;
alpha_c       = .75;
phi_c         = .75;

gamma_intervals    = 30;
gamma_min          = 1;
gamma_max          = 5;
gamma_step         = (gamma_max-gamma_min)/gamma_intervals;

vartheta_intervals = 30;
vartheta_min       = 0;
vartheta_max       = 5;
vartheta_step      = (vartheta_max-vartheta_min)/vartheta_intervals;

vartheta=vartheta_min;
for i = 1:gamma_intervals
    Gamma(i) = gamma_min+(i-1)*gamma_step;
    gamma    = Gamma(i);
    for j = 1:vartheta_intervals
        Vartheta(j) = vartheta_min+(j-1)*vartheta_step;
        vartheta    = Vartheta(j);

        % Without habit formation in leisure
        alpha_l=0.00001;
        catchingabsolute;
        Resp_cons1=Resp_mat(4,:);

        % With habit formation in leisure
        alpha_l=0.75;
        catchingabsolute;
        Resp_cons2=Resp_mat(4,:);
        Resp_cons=abs(Resp_cons2-Resp_cons1);

        % Distance
        value=Resp_cons(1)/2;
        for k = 1:(HORIZON-1)
            value=value+(abs(Resp_cons(k)+Resp_cons(k+1)))/2;
        end
        Z(i,j)=value;
    end;
end;
surf(Gamma,Vartheta,Z)
```

Model with taxes

```
% CATCHINGTAXES.M
% calculates through a real business cycle model
% with external habit formation ("Catching up with the Joneses" preferences,
% absolute terms) in both Consumption and Leisure
% and with taxes on Capital and Labor;
% see Hildebrand, Thomas: "Business Cycle Dynamics under
% Catching up with the Joneses Preferences in Leisure",
% Diploma Thesis at the School of Business and Economics
% at Humboldt University of Berlin, July 2006
% Copyright CATCHINGTAXES.M: Thomas Hildebrand

% TOOLKIT
% VERSION 2.0, MARCH 1997, COPYRIGHT H. UHLIG.
% For more information on how the Matlab Code works see H. Uhlig,
% "A toolkit for solving nonlinear dynamic stochastic models easily".
% First, parameters are set and the steady state is calculated. Next, the matrices are
% declared. In the last line, the model is solved and analyzed by calling DO_IT.M
% Copyright Toolkit-Code: H. Uhlig.
% Feel free to copy, modify and use at your own risk.
% However, you are not allowed to sell this software or otherwise impinge
% on its free distribution.

disp('Catchingtaxes: A real business cycle model with external habit formation');
disp('               ("Catching up with the Joneses" preferences, absolute terms)');
disp('               in both Consumption and Leisure');
disp('               and with taxes on Capital and Labor;');
disp('               see Hildebrand, Thomas: "Business Cycle Dynamics under');
disp('               Catching up with the Joneses Preferences in Leisure",');
disp('               Diploma thesis at the School of Business and Economics');
disp('               at Humboldt University of Berlin, July 2006');

disp('Hit any key when ready...');
pause;

% Setting fixed parameters.
N_bar     = 1.0/3;  % Steady state employment is a third of total time endowment
Z_bar     = 1;      % Normalization
tau_bar   = .25;    % Tax rate
theta     = .36;    % Capital share
delta     = .025;   % Depreciation rate per quarter for capital
R_bar     = 1.01;   % One percent real interest per quarter
psi       = .95;    % Autocorrelation of technology shock
sigma_eps = .712;   % Standard deviation of technology shock. Units: Percent
psi_g     = .95;    % Autocorrelation of government expenditures shock
sigma_rho = .712;   % Standard deviation of government expenditures shock.  Units: Percent

% Setting varying parameters.
gamma     = 2.372;  % Curvature of consumption
vartheta  = 2.372;  % Curvature of leisure
alpha_C   = .5;     % Consumption habit importance parameter (set =0.00001 for no consumption habit)
phi_C     = .5;     % Consumption habit persistence parameter
alpha_L   = .5;     % Leisure habit importance parameter (set =0.00001 for no leisure habit)
phi_L     = .5;     % Leisure habit persistence parameter

% Calculating the steady state.
betta   = 1.0/R_bar;
YK_bar  = (R_bar + delta - 1)/(theta*(1-tau_bar)); % = Y_bar / K_bar
K_bar   = (YK_bar / Z_bar)^(1.0/(theta-1)) * N_bar;
I_bar   = delta * K_bar;
Y_bar   = YK_bar * K_bar;
G_bar   = tau_bar*Y_bar;
C_bar   = Y_bar - delta*K_bar - G_bar;
L_bar   = 1.0-N_bar;
V_bar   = alpha_C * C_bar;
U_bar   = alpha_L * L_bar;
Chi     = (1 - theta)*(1-tau_bar) * Y_bar/N_bar * (L_bar - U_bar)^vartheta * (C_bar - V_bar)^(-gamma);

% Declaring the matrices.
VARNAMES = ['capital    ',  % Variable  1
            'conshabit  ',  % Variable  2
            'leishabit  ',  % Variable  3
            'consumption',  % Variable  4
            'output     ',  % Variable  5
            'labor      ',  % Variable  6
            'interest   ',  % Variable  7
            'investment ',  % Variable  8
            'leisure    ',  % Variable  9
            'taxrate    ',  % Variable 10
            'technology ',  % Variable 11
            'govexp     ']; % Variable 12

% Translating into coefficient matrices.
```

```
% The equations are, conveniently ordered:
% 1) 0 = - I i(t) - C c(t) - G g(t) + Y y(t)
% 2) 0 = I i(t) - K k(t) + (1-delta) K k(t-1)
% 3) 0 = theta k(t-1) - y(t) + (1-theta) n(t) + z(t)
% 4) 0 = y(t) - n(t) + tau_bar/(tau_bar-1) tau(t) - gamma * (C_bar/(C_bar-V_bar)) * c(t) + gamma * (V_bar/(C_bar-V_bar)) * v(t-1) ...
%                          - vartheta (N_bar/(1-N_bar-U_bar) * n(t) - vartheta (U_bar/(1-N_bar-U_bar) * u(t-1)
% 5) 0 = - (1-tau_bar) * theta Y/K k(t-1) + (1-tau_bar) * theta Y/K y(t) - tau_bar theta Y/K tau(t) - R r(t)
% 6) 0 = L_bar * l(t) + N_bar * n(t)
% 7) 0 = -V_bar *v(t-1) + (1-phi_C) * alpha_C * C_bar * c(t-1) + phi_C * V_bar * v(t-2)
% 8) 0 = -U_bar *u(t-1) + (1-phi_L) * alpha_L * L_bar * l(t-1) + phi_L * U_bar * u(t-2)
% 9) 0 = y(t) + tau(t) - g(t)
%10) 0 = E_t [- gamma * (C_bar/(C_bar-V_bar)) * (c(t+1)-c(t)) - gamma * (V_bar/(C_bar-V_bar)) * (v(t)-v(t-1))  +  r(t+1)]
%11) z(t+1) = psi z(t) + epsilon(t+1)
%12) g(t+1) = psi_g g(t) + rho(t+1)

% CHECK: 12 equations, 12 variables.
% Endogenous state variables "x(t)": k(t), v(t), u(t)
% Endogenous other variables "y(t)": c(t), y(t), n(t), r(t), i(t), l(t), tau(t)
% Exogenous state variables  "z(t)": z(t), g(t).

% Switch to that notation.  Find matrices for format
% 0 = AA x(t) + BB x(t-1) + CC y(t) + DD z(t)
% 0 = E_t [ FF x(t+1) + GG x(t) + HH x(t-1) + JJ y(t+1) + KK y(t) + LL z(t+1) + MM z(t)]
% z(t+1) = NN z(t) + epsilon(t+1) with E_t [ epsilon(t+1) ] = 0,


% Start System 1 (9 Equations)

%      capital,                         conshabit,                      leishabit
%      k(t),                            v(t),                           u(t)
AA = [ 0,                               0,                              0                                    %Eq. 1
       - K_bar,                         0,                              0                                    %Eq. 2
       0,                               0,                              0                                    %Eq. 3
       0,                               0,                              0                                    %Eq. 4
       0,                               0,                              0                                    %Eq. 5
       0,                               0,                              0                                    %Eq. 6
       0,                               -V_bar,                         0                                    %Eq. 7
       0,                               0,                              -U_bar                               %Eq. 8
       0,                               0,                              0];                                  %Eq. 9

%      capital,                         conshabit,                      leishabit
%      k(t-1),                          v(t-1),                         u(t-1)
BB = [ 0,                               0,                              0                                    %Eq. 1
       (1-delta)*K_bar,                 0,                              0                                    %Eq. 2
       theta,                           0,                              0                                    %Eq. 3
       0,                               gamma * (V_bar/(C_bar-V_bar)),  - vartheta * (U_bar/(1-N_bar-U_bar))  %Eq. 4
       -(1-tau_bar)*theta*Y_bar/K_bar,  0,                              0                                    %Eq. 5
       0,                               0,                              0                                    %Eq. 6
       0,                               phi_C*(V_bar),                  0                                    %Eq. 7
       0,                               0,                              phi_L*(U_bar)                        %Eq. 8
       0,                               0,                              0 ];                                 %Eq. 9

%      consumption                 output                      labor                                inter. inv.   leisure          taxrate
%      c(t)                        y(t)                        n(t)                                 r(t)   i(t)   l(t)             tau(t)
CC = [ -C_bar,                     Y_bar,                      0,                                   0,     -I_bar,0,               0
       0,                          0,                          0,                                   0,     I_bar, 0,               0
       0,                          -1,                         1-theta,                             0,     0,     0,               0
       -gamma*C_bar/(C_bar-V_bar),1,                           -1-(vartheta*N_bar/(L_bar-U_bar)),0,     0,     0,               tau_bar/(tau_bar-1)
       0,                          (1-tau_bar)*theta*YK_bar,0,                                   -R_bar,0,     0,               -tau_bar*theta*YK_bar
       0,                          0,                          N_bar,                               0,     0,     L_bar,           0
       (1-phi_C)*alpha_C*C_bar,    0,                          0,                                   0,     0,     0,               0
       0,                          0,                          0,                                   0,     0,     (1-phi_L)*U_bar,0
       0,                          1,                          0,                                   0,     0,     0,               1 ];

%      technology     govexp
%      z(t)           g(t)
DD = [ 0,             -G_bar % Eq. 1
       0,             0      % Eq. 2
       1,             0      % Eq. 3
       0,             0      % Eq. 4
       0,             0      % Eq. 5
       0,             0      % Eq. 6
       0,             0      % Eq. 7
       0,             0      % Eq. 8
       0,             -1];   % Eq. 9

% End System 1


% Start System 2 (1 Equation)

%      capital,                         conshabit,                      leishabit
%      k(t+1),                          v(t+1),                         u(t+1)
```

```
FF = [ 0,                               0,                              0 ];

%      capital,                         conshabit,                      leishabit
%      k(t),                            v(t),                           u(t)
GG = [ 0,                               gamma*(V_bar/(C_bar-V_bar)),    0 ];

%      capital,                         conshabit,                      leishabit
%      k(t-1),                          v(t-1),                         u(t-1)
HH = [ 0,                               -gamma*(V_bar/(C_bar-V_bar)),   0 ];

%      consumption                     output           labor           interest  investment   leisure     taxrate
%      c(t+1)                          y(t+1)           n(t+1)          r(t+1)    i(t+1)       l(t+1)      tau(t+1)
JJ = [ -gamma*(C_bar/(C_bar-V_bar)),   0,               0,              1,        0,           0,          0];

%      consumption                     output           labor           interest  investment   leisure     taxrate
%      c(t)                            y(t)             n(t)            r(t)      i(t)         l(t)        tau(t)
KK = [ gamma*(C_bar/(C_bar-V_bar)),    0,               0,              0,        0,           0           0];

%      technology      govexp
%      z(t+1)          g(t+1)
LL = [ 0               0];

%      technology      govexp
%      z(t)            g(t)
MM = [ 0               0];

% End System 2


% Start System 3 (2 Equations)
%     technology       govexp
%     z(t)             g(t)
NN = [psi,             0
      0,               psi_g];

% End System 3


Sigma = [ sigma_eps^2, 0
          0,           sigma_rho^2];

% Setting the options.
[l_equ,m_states] = size(AA);
[l_equ,n_endog ] = size(CC);
[l_equ,k_exog  ] = size(DD);

PERIOD     = 4;                             % Number of periods per year, i.e. 12 for monthly, 4 for quarterly
GNP_INDEX  = 5;                             % Index of output among the variables selected for HP filter
IMP_SELECT = [1:12];                        % A vector containing the indices of the variables to be plotted
DO_SIMUL   = 1;                             % Calculates simulations
SIM_LENGTH = 150;
DO_MOMENTS = 1;                             % Calculates moments based on frequency-domain methods
HP_SELECT  = 1:(m_states+n_endog+k_exog);   % Selecting the variables for the HP Filter calcs.
DISPLAY_IMMEDIATELY = 1;                    % Display warnings immediately

IMP_SINGLE  = 0;                            % no single graphs for impulse responses
IMP_SELECT  = [1 4 5 6 9 10 12];            % select the variables to be plotted in impulse responses
HORIZON     = 2^5;

% Starting the calculations.
do_it;
```

Declaration of Authorship

I hereby confirm that I have written this diploma thesis independently and without use of other than the indicated resources. All passages, which are literally or in general matter taken out of publications or other resources, are marked as such.

Thomas Hildebrand

Berlin, 21.07.2006