

# Master's Thesis

Comparing the Monetary Policies  
of the Fed and the ECB:  
A New Keynesian Approach

Arda Özcan

Master of Economics and Management Science  
Humboldt University of Berlin

Student Number: 500375

Examiner:  
Prof. Harald Uhlig Ph. D.

August 2, 2007

## **Abstract**

In this paper a Dynamic Stochastic General Equilibrium (DSGE) model is described for the closed and open economy case with incomplete exchange pass-through in the so-called "New Keynesian" framework following Christiano, Eichenbaum and Evans (2005) and Adolfson, Laseén, Lindé and Villani (2007), respectively. The main focus of this paper are the differences and similarities between US and EU monetary policies. A less important focus is attributed to the implications of open and closed economy models in monetary policy analysis. The models are solved and log-linearized and finally calibrated based on Smets and Wouters (2004a, 2004b and 2007). We found that monetary transmission mechanism has similar for US and EU in both models.

# Contents

<b>1</b>	<b>Introduction</b>	<b>5</b>
<b>2</b>	<b>Theory and Related Literature</b>	<b>6</b>
<b>3</b>	<b>The Model</b>	<b>8</b>
3.1	Households . . . . .	9
3.2	Intermediate Goods Firms . . . . .	14
3.3	Importing Firms . . . . .	17
3.4	Exporting Firms . . . . .	18
3.5	More Open Economy Aspects . . . . .	19
3.6	The Government . . . . .	20
3.7	The Central Bank . . . . .	21
3.8	Market Equilibrium . . . . .	22
<b>4</b>	<b>Solving the Model</b>	<b>23</b>
4.1	Steady State . . . . .	23
4.2	Log-linearized equations . . . . .	26
4.3	Calibration . . . . .	30
<b>5</b>	<b>Impulse Response Analysis</b>	<b>32</b>
<b>6</b>	<b>Concluding Remarks</b>	<b>36</b>
<b>7</b>	<b>References</b>	<b>38</b>
<b>8</b>	<b>Appendix</b>	<b>40</b>
8.1	Parameter values . . . . .	40
8.2	Figures . . . . .	42

## List of Figures

1	Interest rate shock, EU, closed economy model (1)	32
2	Interest rate shock, US, Closed economy model (1)	33
3	Interest rate shock, EU, Open economy model (1)	34
4	Interest rate shock, US, Open economy model (1)	34
5	Inflation target shock, EU, Closed economy model (1)	35
6	Inflation target shock, US, Closed economy model (1)	35
7	Interest rate shock, EU, Closed economy model (2)	42
8	Interest rate shock, US, Closed economy model (2)	43
9	Inflation target shock, EU, Closed economy model (2)	43
10	Inflation target shock, US, Closed economy model (2)	44
11	Interest rate shock, EU, Open economy model (2)	45
12	Interest rate shock, US, Open economy model (2)	46
13	Inflation target shock, EU, Open economy model (2)	47
14	Inflation target shock, US, Open economy model (2)	48
15	Preference shock, EU, Closed economy model	49
16	Preference shock, US, Closed economy model	50
17	Labor supply shock, EU, Closed economy model	51
18	Labor supply shock, US, Closed economy model	52
19	Technology shock, EU, Closed economy model	53
20	Technology shock, US, Closed economy model	54
21	Preference shock, EU, Open economy model	55
22	Preference shock, US, Open economy model	56
23	Labor supply shock, EU, Open economy model	57
24	Labor supply shock, US, Open economy model	58
25	Technology shock, EU, Open economy model	59
26	Technology shock, US, Open economy	60

# 1 Introduction

In this paper a Dynamic Stochastic General Equilibrium (DSGE) model is described for the closed and open economy case in the so-called "New Keynesian" framework. The closed economy model is based on the work of Christiano, Eichenbaum and Evans (2005) whereas the open economy model is based on the work of Adolfson, Laseén, Lindé and Villani (2007). Both papers allow for nominal and real rigidities, such as price and wage stickiness, variable capital utilization, capital adjustment costs and habit formation as common features. Following Adolfson, Laseén, Lindé and Villani (2007), incomplete exchange rate pass-through is introduced in order to capture the differences in monetary policy implementation in open and closed economy models. Furthermore, consumption, labor income and capital income taxes are incorporated for both models and accordingly, fiscal policy is rule is introduced following Trabandt and Uhlig (2006).

The main focus of this paper are the differences and similarities between US and EU monetary policies. A less important focus is attributed to the implications of open and closed economy models in monetary policy analysis. This paper can be differentiated from that of Adolfson, Laseén, Lindé and Villani (2007), as within this paper the open economy model for both the EU and US are modelled together allowing for interactions and subsequently putting forward further challenges. Both models are not estimated but instead calibrated based on Smets and Wouters (2004a, 2004b and 2007) estimations. In the absence of benchmark large-open economy models incorporating US and EU economies in the same model, calibration also puts forward further challenges. Both models have similar effects on monetary transmission mechanism. However, due to the arising miscalculations, the models presented here fails to generate standard impulse response functions for some particular shocks. Therefore, the next step, following this paper will be fixing the calculation problems and using Bayesian techniques for the estimation procedure.

The paper is organised as follows. In Section 2, a brief introduction is given about the New Keynesian principles and the evolution of such models. The open and closed economy models are described in Section 3. Further, in Section 4 the steady state definitions along with the model solution is presented. This is followed by calibration of the model. Calibration is based on Smets and Wouters (2004a, 2004b and 2007). The model is evaluated in Section 5 by analyzing impulse response functions. Finally, concluding remarks are made in Section 6.

## 2 Theory and Related Literature

For at least past two decades there is a growing interest in the so-called "New Keynesian" synthesis in analyzing monetary policies. The New Keynesian synthesis came into popular usage in the 1980's.<sup>1</sup> Earlier in the pre-Keynes world competitive markets and perfect flexibility of prices and wages were assumed. Keynes's argument was the existence of rigidity in prices and wages. One of the reasons for the prevalence of involuntary, prolonged unemployment was attributed to this stickiness in wages which causes market imperfections. Later on from 1950's until mid-1970's neoclassical synthesis, which is based on utility based maximisation of households and firms, was the dominant framework. The essence of neoclassical synthesis was "Walrasian" equilibrium (i.e. perfectly flexible prices and wages and competitive markets) in the long-run and treatment of wages and prices as given in the short-run. Along with this short-run rigidity in prices and wages, the expectations were also assumed to be fixed in the short-run or subject to ad-hoc adjustment as under the adaptive expectations hypothesis.

Further, 1970's witnessed the rise of rational expectations or New Classical approach, originally proposed by Muth (1961) and later developed by Lucas (1976). New Classical assumed perfect flexibility for prices and wages, whereas information available to economic agents was treated as imperfect. This implies that agents could use the information they had optimally but still markets could deviate from the full information equilibrium. Lucas (1976) argued that the parameters of traditional macroeconomic models depended implicitly on agents' expectations of the policy process and were unlikely to remain stable as policymakers changed their behaviour. As Rudebusch (2002) states, this critique was influential in two respects. First, it helped re-orient macroeconomic research toward models with explicit expectations and "deep" parameters of taste and technology. These models, which were to be invariant to policy shifts, included estimated first-order conditions or Euler equations, calibrated general equilibrium models with explicit optimization. Second, the Lucas critique helped change the focus of policy evaluation from consideration of alternative paths of the policy instrument to consideration of alternative policy rules, which allowed individual agents to formulate forward-looking dynamic optimization problems.

New Keynesian economics adopted the rational expectations approach as well as nominal rigidities in prices and wages. Differences between the New Classical and New Keynesians exist more in terms of modeling the supply-side and less in modeling the demand-side. (Gordon, 1990). Additionally, Romer (1993), Greenwald and Stiglitz (1993), Taylor (1997) give insights into understanding the basic principles of New Keynesian perspective.

---

<sup>1</sup>Goodfriend and Woodford (1997) give an insight into the evolution of monetary policy analysis along with the evolution of economic thoughts starting from neoclassical synthesis.

- In the long run growth path of output is determined by the supply-side.
- In the long run there is no tradeoff between inflation and unemployment, i.e. monetary neutrality in the long run.
- In the short run there is a tradeoff between inflation and unemployment as described by the Phillips curve. Given the price/wage stickiness, short run fluctuations are caused by changes in aggregate demand.
- Policy measures are affected by the expectations of private agents. Equilibrium, which does not imply cleared markets, is possible when these expectations are fulfilled.

These principles innately constitutes the Taylor (1993) rule which is the essence of the New Keynesian monetary policy analysis framework. Taylor rule specifies that the central bank sets its instrument, which is the short term interest rate, in order to react to two key variables: the deviations inflation from a target and the deviations of real output from its potential level or from its steady state. Therefore, by focusing on policy responses to these key variables, the Taylor rule implicitly captures policy responses to economic factors that affect the evolution of these key variables. Further, Dynamic Stochastic General Equilibrium (DSGE) models provided the necessary tools for the study of the optimal conduct of monetary policy, design and implementation of simple interest rate rules (Monacelli, 2003).

In this matter, sticky price closed economy model of Rotemberg and Woodford (1997) became the workhorse for monetary policy analysis. Smets and Wouters (2003), on the other hand, showing that Bayesian methods can be applied succesfully to closed economy analysis, provided a new perspective on the subject. And more recently the research was extended into open economy analysis, i.e., New Open Economy Macroeconomics (NOEM) analysis. Monetary policy analysis in the early NOEM literature was assuming complete exchange rate pass-through. However, Gali and Monacelli (2002) show that optimal monetary policy is identical in open and closed economy analysis when there is complete exchange rate pass-through to domestic prices. Accordingly, Lubik and Schorfheide (2005) develops a small open economy model with two countries, namely US and EU. They introduce endogenous deviations from purchasing power parity (PPP) via monopolistic price-setting importers that lead to imperfect pass-through. Adolfson, Laseén, Lindé and Villani (2007) provides a large-scale open economy model for EU, also assuming incomplete exchange rate pass-through both in import and export sector. The estimated model have a monetary transmission mechanism well in line with the common literature.

### 3 The Model

In this section the closed and the open economy models for the EU and the US economy are derived from the optimizing behaviour of households and firms. Closed economy model is a standard New Keynesian DSGE model with a fiscal policy incorporating sticky wages and prices, adjustment costs in investment, habit persistence in preferences and variable capital utilization. Fiscal policy is introduced following Trabandt and Uhlig (2006). Further, open economy model builds on the structure of Adolfson, Laseén, Lindé and Villani (2007) with extending the foreign economy framework. In this paper, both economies are modelled with symmetric preferences and technologies allowing for differences in price-setting, policies and exogenous shocks rather than being modelled as exogenous shocks.

Households maximize a utility function consisting of consumption and leisure in both models. The real cash balances is absent from the utility function, in line with the practice in modern New Keynesian literature. Woodford (2003) justifies the reason conceptually on the grounds of a bookkeeping cashless economy. Note also that in the open economy model, consumption and investment is composed of domestic and import goods which are supplied by the domestic and importing firms, respectively. Further, the households can save in domestic bonds and/or foreign bonds depending on the model, i.e., open versus closed. Accordingly, this decision of the households results in the uncovered interest rate parity condition in open economy analysis. In both models, the households rent capital to the intermediate goods producers. In turn, given the costs to adjusting the investment rate and the costs of varying utilization rate of the physical capital stock, they decide on how much to invest in physical capital stock. Further, each household sets his wage in Calvo(1983) setting. This is in line with the assumption that each household is a monopoly supplier of a differentiated labor service.

Firms in the both economy models, i.e, intermediate goods firms (only firm in the closed economy model), exporting firms and importing firms, produce a differentiated good and therefore, operate in a monopolistically competitive environment. They set their respective prices according to a Calvo setting, i.e., only a certain fraction of firms can reoptimize their prices in each period. Moreover, while differentiated intermediate goods are aggregated by a final good producer in the Home economy, importing firms buy the homogenous good in the Foreign economy and after differentiating it by brand-naming sells it to a final import good producer to be aggregated. On the other hand, exporting firms buy the homogenous good from the final good producer, differentiates it by brand-naming and sells it (by local currency pricing) to the importing firms operating in the Foreign economy.

Assuming price stickiness in the short run implies that exchange rate pass-



through is not complete in the short run.<sup>2</sup> Inherently, deviations from PPP are assumed to exist in the short run, but not in the long run. However, monopolistic importers charge a constant mark-up to consumers causing a l.o.p. gap even after adjusting for exchange rate movements.

Below, the closed and the open economy models will be presented along with each other. Since symmetric preferences and technologies are assumed for both economies, describing the model for the EU ( as Home economy in open economy analysis) will suffice. In this respect, variables with a "\*" denotes the US economy variables. Note also that economic agents in both economies make identical decisions, i.e. in equilibrium individual decisions coincide with the aggregate decision.

### 3.1 Households

There is a continuum of households, indexed by  $j \in [0, 1]$ , optimize

$$E_0^j \sum_{t=0}^{\infty} \beta^t \left[ d_t^c \log(C_{jt} - H_{jt}) - d_t^h A_L \frac{(h_{jt})^{1+\sigma_l}}{1+\sigma_l} \right] \quad (1)$$

where  $C_{jt}$  and  $h_{jt}$  denote the  $j^{th}$  household's consumption and labor effort levels, respectively. Consumption smoothing behaviour of the households are captured by including a habit formation,  $H_t = hC_{t-1}$  where  $h$  captures the level of habit persistence.  $\beta$  is the discount factor and two structural shocks  $d_t^c$  and  $d_t^h$  are persistent preference shocks to consumption and labor supply, respectively. Both of the shocks follow a first order autoregressive process, AR(1), given by:

$$\begin{aligned} d_t^c &= \rho_c d_{t-1}^c + \varepsilon_{dc,t} \quad \text{and} \quad \varepsilon_{dc,t} \sim N(0, 1) , \\ d_t^h &= \rho_h d_{t-1}^h + \varepsilon_{dh,t} \quad \text{and} \quad \varepsilon_{dh,t} \sim N(0, 1) \end{aligned}$$

For the open economy analysis, aggregate consumption is assumed to consist of domestically produced goods and import goods given by the following technology

$$C_t = \left[ (1 - w_c)^{\frac{1}{\eta_c}} (C_t^d)^{\frac{\eta_c - 1}{\eta_c}} + w_c (C_t^m)^{\frac{\eta_c - 1}{\eta_c}} \right]^{\frac{\eta_c}{\eta_c - 1}} \quad (2)$$

where  $C_t^d$  and  $C_t^m$  are consumption of the domestic and imported good respectively.  $w_c$  is the share of imports in consumption and  $\eta_c$  is the elasticity of

<sup>2</sup>See, for example, Devereux and Engle (2002), Corsetti and Pesenti (2002) and Lubik (2006)

substitution between consumption goods. The total expenditure on consumption is the sum of domestic and import good purchases.

$$P_t^c C_t = P_t C_t^d + P_t^m C_t^m$$

where  $P_t^c$ ,  $P_t$  and  $P_t^m$  denotes consumer price index (CPI), domestic producer prices and import prices, respectively. Maximizing the above budget constraint subject to equation (2), demand functions for domestic and import goods can be derived.

$$C_t^d = (1 - w_c) \left( \frac{P_t}{P_t^c} \right)^{-\eta_c} C_t, \quad (3)$$

$$C_t^m = w_c \left( \frac{P_t^m}{P_t^c} \right)^{-\eta_c} C_t \quad (4)$$

Plugging (3) and (4) into the budget constraint, CPI can be obtained

$$P_t^c = \left[ (1 - w_c) P_t^{1-\eta_c} + w_c (P_t^m)^{1-\eta_c} \right]^{\frac{1}{1-\eta_c}} \quad (5)$$

The  $j$ -th household's budget constraint is:

$$\begin{aligned} & (1 + \tau_t^c) C_{jt} + I_{jt} + a(u_{jt}) \bar{K}_{jt} + \frac{e_t B_{jt}^*}{P_t} + \frac{B_{jt}}{P_t} + \tau_t^k \frac{[(R_{t-1} - 1) B_{jt-1} + (R_{t-1}^* - 1) e_t B_{jt-1}^*]}{P_t} \\ & = (1 - \tau_t^y) w_{jt} h_{jt} + (1 - \tau_t^k) r_t^k u_{jt} \bar{K}_{jt} + \frac{e_t R_{t-1}^* B_{jt-1}^*}{P_t} + \frac{R_{t-1} B_{jt-1}}{P_t} + TR_t + D_{jt} \end{aligned} \quad (6)$$

where  $w_{jt}$  is the real wage,  $r_t^k$  is the real rental price of capital,  $u_{jt}$  is the varying capital utilization rate,  $a(u_{jt})$  is the physical cost of  $u_{jt}$  in resource terms,  $TR_t$  is a lump-sum transfer from the government and  $D_t$  is the profit of the firms in the economy. All interest rates are expressed in gross rates, i.e.  $R_t = 1 + r_t$  and  $R_t^* = 1 + r_t^*$ . The households invest either in government bonds  $B_t$  or foreign bonds  $B_t^*$  at time  $t$ . Note that in closed economy analysis  $B_t^* = 0$ .  $\tau_t^k$ ,  $\tau_t^c$ , and  $\tau_t^y$  denotes taxes on capital income, consumption and labour income, respectively. Moreover, households pay  $\tau_t^y w_{jt} h_{jt}$  amount of labor income tax,  $\tau_t^k r_t^k u_{jt} \bar{K}_{jt}$  amount of capital income tax on efficiently used capital,  $\tau_t^c C_{jt}$  amount of consumption tax and  $\tau_t^k [(R_t - 1) B_{jt-1} + (R_t^* - 1) e_t B_{jt-1}^*]$  amount of capital income tax on domestic and foreign bonds. Finally,  $e_t$  is the nominal exchange rate (domestic currency per unit of foreign currency).

The households can increase their capital stock by investing in additional physical capital, waiting one period to come in action, or by directly increasing the utilization rate of capital,  $u_t$ . However, the latter is delivered facing a

cost function,  $a(u_t)$  with  $a(1) = 0$ ,  $\bar{u} = 1$ . As seen in the budget constraint equation (6), adjustment costs of varying utilization rate of capital are paid by the households. Further, following Altig et al (2004), the household's stock of physical capital evolves according to

$$\bar{K}_{jt} = (1 - \delta)\bar{K}_{jt-1} + F(I_{jt}, I_{jt-1}) \quad (7)$$

where  $\delta$  denotes the physical rate of depreciation and  $I_{jt}$  denotes time  $t$  investment goods. The function  $F(\cdot)$  is the technology used to transform current and past investment into installed capital for use in the following period.  $F(\cdot)$  is given by

$$F(I_{jt}, I_{jt-1}) = \left(1 - \tilde{S} \left(\frac{I_{jt}}{I_{jt-1}}\right)\right) I_{jt}$$

Furthermore, function  $S(\cdot)$  is assumed to be increasing, convex and satisfies:  $S = S' = 0$  and  $S'' > 0$  in steady state. The steady state of the model does not depend on the value of  $S''$ , however, the dynamics do.<sup>3</sup>

In a similar way as in consumption, total investment is assumed to consist of domestically produced goods and import goods given by the following technology

$$I_t = \left[ (1 - w_i)^{\frac{1}{\eta_i}} (I_t^d)^{\frac{\eta_i - 1}{\eta_i}} + w_i (I_t^m)^{\frac{\eta_i - 1}{\eta_i}} \right]^{\frac{\eta_i}{\eta_i - 1}} \quad (8)$$

where  $w_i$  is the share of imports in investment and  $\eta_i$  is the elasticity of substitution across investment goods. Also as in the consumption goods case, the demand for domestic and import investment goods are as follows<sup>4</sup>

$$I_t^d = (1 - w_i) \left(\frac{P_t}{P_t^c}\right)^{-\eta_i} I_t, \quad (9)$$

$$I_t^m = w_i \left(\frac{P_t^m}{P_t^c}\right)^{-\eta_i} I_t \quad (10)$$

Note that for simplicity same prices are assumed for investment and consumption goods.

<sup>3</sup>For an explicit definition of investment and utilization adjustment costs, see Christiano, Motto and Rostagno (2007).

<sup>4</sup>In contrast to Adolfson, Laseén, Lindé and Villani (2007) the total expenditure on investment, assuming same price levels for investment as for consumption goods, is given by

$$P_t^c I_t = P_t I_t^d + P_t^m I_t^m$$

By using (1), (6) and (7), households solve the Lagrangian problem. The first order conditions with respect to  $C_{jt}$ ,  $B_{jt}$ ,  $B_{jt}^*$ ,  $u_{jt}$ ,  $K_{jt}$  and  $I_{jt}$  are as follows:

$$\lambda_t = \frac{d_t^c (C_t - H_t)^{-1}}{(1 + \tau_t^c)} \quad (11)$$

$$1 = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{(R_t - (R_t - 1)\tau_{t+1}^k)}{\pi_{t+1}} \right] \quad (12)$$

$$1 = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{(R_t^* - (R_t^* - 1)\tau_{t+1}^k)}{\pi_{t+1}} \frac{e_{t+1}}{e_t} \right] \quad (13)$$

$$r_t^k = \frac{a'(u_{jt})}{(1 - \tau_t^k)} \quad (14)$$

$$Q_t = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} (Q_{t+1}(1 - \delta) + (1 - \tau_{t+1}^k)r_{t+1}^k u_{t+1} - a(u_{t+1})) \right] \quad (15)$$

$$1 = Q_t \left( 1 - \tilde{S} \left( \frac{I_t}{I_{t-1}} \right) - \tilde{S}' \left( \frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right) + \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} Q_{t+1} \tilde{S}' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \right] \quad (16)$$

where,  $Q_t = \frac{\theta_t}{\lambda_t}$  and  $\pi_{t+1} = \frac{P_{t+1}}{P_t}$ . Note that the first order conditions does not depend on  $j$ , since households' aggregate decisions are assumed to coincide in equilibrium. Combining the two first order conditions with respect to  $B_{jt}$  and  $B_{jt}^*$ , the uncovered interest rate parity along with the log-linear version can be obtained

$$R_t - R_t^* E_t \frac{e_{t+1}}{e_t} = \frac{\tau_{t+1}^k}{1 - \tau_{t+1}^k} \left( E_t \frac{e_{t+1}}{e_t} - 1 \right),$$

$$\hat{R}_t - \hat{R}_t^* = \frac{1}{1 - \beta \bar{\tau}^k} E_t \Delta \hat{e}_{t+1} \quad (17)$$

This interest rate parity condition is slightly different from the standard case without taxes. Note that  $\frac{1}{1 - \beta \bar{\tau}^k} > 1$  due to the introduction of taxes. The tax on capital income causes the households to require an extra interest rate premium on their domestic bond holdings. The reason is that there is no tax paid on expected exchange rate profits. Consequently, when the exchange rate is expected to depreciate, the households anticipate larger gains from holding foreign bonds compared to holding domestic bonds since the effective tax rate differs between these positions. For the expected earnings on the domestic and foreign bond holdings to be equivalent, the domestic-foreign interest rate differential must then be larger than one when the capital tax is positive (Adolfson, Laseén, Lindé and Villani, 2007).

Wage decision of the households is more involved. Households supply different types of labour allowing them to have monopolistic power over individual wages. Labor supplied by the households are transformed into aggregate labor by the given technology:

$$L_t = \left( \int_0^1 h_{jt}^{\frac{1}{\epsilon}} dj \right)^\epsilon \quad (18)$$

where  $L_t$  denotes the aggregate labor and  $\epsilon$  controls the elasticity of substitution among different types of labor. Labor aggregator maximizes his profits subject to (18) taking as given all differentiated labor wages  $w_{jt}$  and the wage  $w_t$ . Consequently, his maximisation problem is:

$$\max_{h_{jt}} w_t L_t - \int_0^1 w_{jt} h_{jt} dj$$

From his maximisation problem, demand for j-th household's labor can be derived:

$$h_{jt} = \left( \frac{w_{jt}}{w_t} \right)^{\frac{\epsilon}{1-\epsilon}} L_t \quad (19)$$

Zero profit condition for labor aggregator's maximisation problem delivers the aggregate wage rate which depends on the individual wage rate:

$$w_t = \left( \int_0^1 w_{jt}^{\frac{1}{1-\epsilon}} dj \right)^{1-\epsilon} \quad (20)$$

Households set their wages following a Calvo's setting. In each period, a fraction  $(1 - \theta_w)$  of households reoptimize their wages. All other households can only partially set their wages by past CPI inflation. Indexation is controlled by the parameter  $\chi_w \in [0, 1]$ . When  $\chi_w = 0$  there is no indexation and when  $\chi_w = 1$  there is total indexation. This implies that if the household cannot reoptimize its wage for  $s$  periods, his normalised wage after  $s$  periods is

$$\prod_{h=1}^s (\pi_{t+h-1}^{c_w})^{\chi_w} w_{jt}$$

Therefore, the relevant part of the Lagrangian for the j-th household is:

$$\max_{w_t} E_t \sum_{s=0}^{\infty} (\beta \theta_w)^s \left[ -d_t^h A_L \frac{h_{jt+s}^{1+\sigma_l}}{1+\sigma_l} + \lambda_{t+s} (1 - \tau_{t+s}^y) \prod_{h=1}^s (\pi_{t+h-1}^{c_w})^{\chi_w} \tilde{w}_t h_{jt+s} \right]$$

subject to

$$h_{jt+s} = \left( \prod_{h=1}^s (\pi_{t+h-1}^{c_w})^{X_w} \frac{\tilde{w}_t}{w_{t+s}} \right)^{-\epsilon} L_{t+s}$$

Note that optimized wage,  $\tilde{w}_t$ , does not depend on  $j$ , since it is assumed that households who can optimize their wages, choose the same wage rate. Following is the first order condition to the above problem:

$$\tilde{w}_t = \left( \frac{\epsilon}{\epsilon - 1} \right) \frac{E_t \sum_{s=0}^{\infty} (\beta \theta_w)^s \{ d_t^h A_L h_{jt+s}^{1+\sigma_l} \}}{E_t \sum_{s=0}^{\infty} (\beta \theta_w)^s \left\{ \lambda_{t+s} (1 - \tau_{t+s}^y) \prod_{h=1}^s (\pi_{t+h-1}^{c_w})^{X_w} h_{jt+s} \right\}} \quad (21)$$

Finally, given equation (20) and indexation, the law of motion of the real wage rate can be defined as:

$$w_t^{1-\epsilon} = \theta_w (w_{t-1} \pi_{t-1}^{c_w})^{1-\epsilon} + (1 - \theta_w) (\tilde{w}_t)^{1-\epsilon} \quad (22)$$

### 3.2 Intermediate Goods Firms

There is a continuum of intermediate goods firms indexed by  $i \in [0, 1]$ ; each producing  $y_{it}$  units of goods to be used by the final good firm to produce  $Y_t$  at period  $t$ . Final good production is given by the following technology:

$$Y_t = \left( \int_0^1 y_{it}^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}} \quad (23)$$

where  $\eta$  is the elasticity of substitution. Final good firms are perfectly competitive and maximize profits subject to the production function (23), taking as given all intermediate goods prices  $p_{it}$  and the final good price  $p_t$ . Consequently, their maximization problem is:

$$\max_{y_{it}} p_t Y_t - \int_0^1 p_{it} y_{it} di$$

From the above maximization problem, demand for  $i$ -th intermediate goods firm's output can be derived:

$$y_{it} = \left( \frac{p_{it}}{p_t} \right)^{-\eta} Y_t \quad (24)$$

where  $Y_t$  is the aggregate demand and the zero profit condition delivers the aggregate price level:

$$p_t = \left( \int_0^1 p_{it}^{1-\eta} di \right)^{\frac{1}{1-\eta}} \quad (25)$$

Each intermediate good  $i$  is produced by the following Cobb-Douglas production function:

$$y_{it} = A_t K_{it}^\alpha L_{it}^{1-\alpha} - \phi \quad (26)$$

where  $\phi$  denotes the fixed cost of production and it is set to deliver zero profit in steady state.  $L_{it}$  is the labor demanded to produce the intermediate good  $i$  and  $K_t$  is the capital services stock rather than physical capital stock  $\bar{K}_{t-1}$ , since variable capital utilization is introduced in the model.  $A_t$  is the total factor productivity shock which follows a  $AR(1)$  process, given by

$$A_t = \rho_A A_{t-1} + \varepsilon_{A,t} \quad \text{and} \quad \varepsilon_{A,t} \sim N(0, 1)$$

Intermediate goods producers face a two-stage problem. In the first stage, given factor prices  $w_t$  and  $r_t^k$ , firms rent  $L_{it}$  and  $K_{it}$  in order to minimize the real cost:

$$\min_{L_{it}, K_{it}} w_t L_{it} + r_t^k K_{it}$$

subject to

$$y_{it} = \begin{cases} A_t K_{it}^\alpha L_{it}^{1-\alpha} - \phi & \text{if } A_t K_{it}^\alpha L_{it}^{1-\alpha} \geq \phi \\ 0 & \text{otherwise} \end{cases}$$

The first order conditions for this problem are:

$$w_t = (1 - \alpha) A_t K_{it}^\alpha L_{it}^{-\alpha} \quad \text{or} \quad w_t = (1 - \alpha) \frac{(y_{it} + \phi)}{L_{it}}$$

$$r_t^k = \alpha A_t K_{it}^{\alpha-1} L_{it}^{1-\alpha} \quad \text{or} \quad r_t^k = \alpha \frac{(y_{it} + \phi)}{K_{it}}$$

Furthermore, total cost is given by:

$$TC = w_t L_{it} + r_t^k K_{it} = \left( \frac{1}{1 - \alpha} \right) w_t L_{it}$$

where  $K_{it} = \frac{\alpha}{1-\alpha} \frac{w_t L_{it}}{r_t^k}$ . Given that the firms has constant returns to scale, by setting the level of labor and capital to produce one unit of good, real marginal cost,  $mc_t$  can be derived as:

$$mc_t = \left( \frac{1}{1-\alpha} \right)^{1-\alpha} \left( \frac{1}{\alpha} \right)^\alpha \frac{w_t^{1-\alpha} (r_t^k)^\alpha}{A_t}$$

Note that the real marginal cost does not depend on  $i$ : all firms receive the same technology shock and all firms rent inputs at the same price.

In the second stage, intermediate good producers choose the price that maximizes discounted real profits in a Calvo's setting. In each period, a fraction  $(1 - \theta_p)$  of firms can reoptimize their prices. All other firms can only index their prices by past inflation  $(p_{it} = \pi_{t-1}^{\chi_p} p_{it-1})$ . Indexation is controlled by the parameter  $\chi_p \in [0, 1]$  where  $\chi_p = 0$  implies no indexation and  $\chi_p = 1$  implies total indexation. It is assumed that all firms that can reoptimize their prices, choose the same price  $\tilde{p}_t$ . Maximization of the firm  $i$  is as follows:

$$\max_{\tilde{p}_t} E_t \sum_{s=0}^{\infty} (\beta \theta_p)^s \lambda_{t+s} \left[ y_{it+s} \left( \prod_{h=1}^s \pi_{t+h-1}^{\chi_p} \frac{\tilde{p}_t}{p_{t+s}} - mc_{t+s} \right) \right]$$

subject to

$$y_{it+s} = \left( \prod_{h=1}^s \pi_{t+h-1}^{\chi_p} \frac{\tilde{p}_t}{p_{t+s}} \right)^{-\eta} Y_{t+s}$$

and the first order condition to above problem is:

$$\tilde{p}_t = \left( \frac{\eta}{\eta - 1} \right) \frac{E_t \sum_{s=0}^{\infty} (\beta \theta_p)^s \lambda_{t+s} p_{t+s} mc_{t+s} y_{it+s}}{E_t \sum_{s=0}^{\infty} (\beta \theta_p)^s \lambda_{t+s} \prod_{h=1}^s \pi_{t+h-1}^{\chi_p} y_{it+s}} \quad (27)$$

Furthermore, given equation (25) and indexation, the law of motion of price level can be defined as:

$$p_t^{1-\eta} = \theta_p \left( \pi_{t-1}^{\chi_p} p_{t-1} \right)^{1-\eta} + (1 - \theta_p) (\tilde{p}_t)^{1-\eta} \quad (28)$$

Log-linearizing the equations (27) and (28), the New Keynesian Phillips curve can be derived:

$$\hat{\pi}_t = \frac{\beta}{1 + \beta \chi_p} E_t \hat{\pi}_{t+1} + \frac{\chi_p}{1 + \beta \chi_p} \hat{\pi}_{t-1} + \frac{(1 - \theta_p)(1 - \beta \theta_p)}{\theta_p (1 + \beta \chi_p)} m \hat{c}_t$$

where "hat" variables denote percentage deviations from their steady states. Note also that if  $\chi_p = 0$ , above equation boils down to a standard New Keynesian Phillips curve with no backward-looking feature.



### 3.3 Importing Firms

There is a continuum of importing firms that purchase a homogenous good at world market prices  $P_{it}^*$  which are set by their respective producers in their own currency. These foreign goods are differentiated ( i.e brand naming) by importing firms and then aggregated by a final import good producer. Finally, these import goods are sold to the households as consumption and investment goods. As mentioned above, deviations from purchasing power parity (PPP) are assumed to exist in the short run due to the existence of monopolistically competitive importers.

The final import good is a composite of  $i$  differentiated imported goods, supplied by importing firm  $i$ , and its production is given by the technology:

$$M_t = \left[ \int_0^1 M_{it}^{\frac{\eta_m-1}{\eta_m}} di \right]^{\frac{\eta_m}{\eta_m-1}}$$

where  $M_t = C_t^m + I_t^m$ . Similar to the final good producer, from the maximization problem of the final import good producer, it can be shown that each importing firm  $i$ , faces the demand for imported consumption goods and imported investment goods which are given by

$$C_{it} = \left( \frac{P_{it}^m}{P_t^m} \right)^{-\eta_m} C_t^m \quad (29)$$

$$I_{it} = \left( \frac{P_{it}^m}{P_t^m} \right)^{-\eta_m} I_t^m \quad (30)$$

The importing firms choose the price that maximizes discounted real profits in a Calvo's setting. In each period, a fraction  $(1 - \theta_m)$  of firms can re-optimize their prices. All other firms can only index their prices by past inflation ( $p_{it}^m = (\pi_{t-1}^m)^{\chi_m} p_{it-1}^m$ ). Indexation for non-optimized import prices is controlled by the parameter  $\chi_m \in [0, 1]$  where  $\chi_m = 0$  implies no indexation and  $\chi_m = 1$  implies total indexation. It is assumed that all firms that can re-optimize their prices, choose the same price  $\tilde{p}_t^m$ . Marginal cost for firm  $i$  is given by  $mc_t^m = e_t P_t^*$  and law of one price (l.o.p.) gap is defined by  $\psi_t^m = \frac{P_t^* e_t}{P_t^m}$ . If PPP holds, then  $\psi_t^m = 1$ . However, imported goods are subject to price discrimination as monopolistic importers charge a mark-up to consumers at the border. This implies that the same good can have different prices depending on where it is sold even after adjusting for exchange rate movements (Lubik, 2006).

Maximization problem of the firm  $i$  is as follows:

$$\max_{\tilde{p}_t^m} E_t \sum_{s=0}^{\infty} (\beta \theta_m)^s \lambda_{t+s} \left[ M_{it+s} \left( \prod_{h=1}^s (\pi_{t+h-1}^m)^{\chi_m} \frac{\tilde{p}_t^m}{p_{t+s}^m} - \psi_{t+s}^m \right) \right]$$

subject to

$$M_{it+s} = \left( \prod_{h=1}^s \pi_{t+h-1}^{\chi_m} \frac{\tilde{p}_t^m}{p_{t+s}^m} \right)^{-\eta_m} M_{t+s}$$

Similar to domestic intermediate good producers, first order condition to above maximization problem and the law of motion for import prices  $(p_t^m)^{1-\eta_m} = \theta_m ((\pi_{t-1}^m)^{\chi_m} p_{t-1}^m)^{1-\eta_m} + (1-\theta_m) (\tilde{p}_t^m)^{1-\eta_m}$  results in the Phillips-curve relationship between import-price inflation and the l.o.p. gap for import goods.

$$\hat{\pi}_t^m = \frac{\beta}{1+\beta\chi_m} E_t \hat{\pi}_{t+1}^m + \frac{\chi_m}{1+\beta\chi_m} \hat{\pi}_{t-1}^m + \frac{(1-\theta_m)(1-\beta\theta_m)}{\theta_m(1+\beta\chi_m)} \hat{\psi}_t^m$$

Further, in order to find the steady state to be used in the log-linearization procedure, flexible price equilibrium can be defined in the following way. In a flexible price environment the maximization problem of the importing firms is given by

$$\max_{\tilde{P}_{it}^m} \left( \tilde{P}_{it}^m - e_t P_t^* \right) (C_{it}^m + I_{it}^m)$$

subject to the demand equations (29) and (30) and  $\tilde{P}_{it}^m$  denotes the optimal flexible price of firm  $i$ . First order conditions, after rearranging and dropping the subscript  $i$  (since all firms make identical decisions), yield

$$\tilde{P}_{it}^m = \frac{\eta_m}{\eta_m - 1} e_t P_t^* \quad (31)$$

Note also that as  $\eta_m \rightarrow \infty$ , implying a horizontal demand curve for imported goods, the markup over the world market price  $P_t^*$  goes to zero. Due to nominal rigidities in the import sector incomplete exchange rate pass-through occurs.<sup>5</sup>

### 3.4 Exporting Firms

Similar to import sector, there is a continuum of exporting firms, which purchase the homogenous good at  $P_t$  from the final good producer and differentiate it by brand naming. The differentiated good is then sold at  $P_{it}^x$  (local currency pricing) in the foreign market as consumption and investment goods. So the marginal cost of exporting firms is  $mc_t^x = P_t/e_t$ . Deviations from l.o.p. is also assumed for the export goods and accordingly l.o.p. gap is defined by  $\psi_t^x = \frac{P_t}{P_t^x e_t}$ . The exporting firm  $i$  faces the following demand for its product  $X_{it}$

$$X_{it} = \left( \frac{P_{it}^x}{P_t^x} \right)^{-\eta_x} X_t$$

<sup>5</sup>Note also that in contrast to Adolfson et al (2007), the steady state markup for imported consumption and investment goods are assumed to be identical.

where  $X_t = C_t^x + I_t^x$  and  $P_t^x$  is the aggregate export price level. The exporting firms choose the price that maximizes discounted real profits in a Calvo's setting. In each period, a fraction  $(1 - \theta_x)$  of firms can reoptimize their prices. All other firms can only index their prices by past inflation ( $p_{it}^x = (\pi_{t-1}^x)^{\chi_x} p_{it-1}^x$ ). Indexation for non-optimized import prices is controlled by the parameter  $\chi_x \in [0, 1]$  where  $\chi_x = 0$  implies no indexation and  $\chi_x = 1$  implies total indexation. It is assumed that all firms that can reoptimize their prices, choose the same price  $\tilde{p}_t^x$ . The export firms maximize profits (denoted in local currency) according to the following maximization problem

$$\max_{\tilde{p}_t^x} E_t \sum_{s=0}^{\infty} (\beta \theta_x)^s \lambda_{t+s} \left[ X_{it+s} \left( \prod_{h=1}^s (\pi_{t+h-1}^x)^{\chi_x} \frac{\tilde{p}_t^x}{p_{t+s}^x} - \psi_{t+s}^x \right) \right]$$

subject to

$$X_{it+s} = \left( \prod_{h=1}^s (\pi_{t+h-1}^x)^{\chi_x} \frac{\tilde{p}_t^x}{p_{t+s}^x} \right)^{-\eta_x} X_{t+s}$$

Combining log-linearized first order condition of the above maximization problem with the log-linearized version of the law of motion for the export prices  $(p_t^x)^{1-\eta_x} = \theta_x ((\pi_{t-1}^x)^{\chi_x} p_{t-1}^x)^{1-\eta_x} + (1 - \theta_x) (\tilde{p}_t^x)^{1-\eta_x}$ , the Phillips-curve relationship between export-price inflation and the l.o.p. gap for export goods can be derived.

$$\hat{\pi}_t^x = \frac{\beta}{1 + \beta \chi_x} E_t \hat{\pi}_{t+1}^x + \frac{\chi_x}{1 + \beta \chi_x} \hat{\pi}_{t-1}^x + \frac{(1 - \theta_x)(1 - \beta \theta_x)}{\theta_x (1 + \beta \chi_x)} \hat{\psi}_t^x$$

Furthermore, given the similar setting for foreign aggregate consumption, foreign demand for the home exports is given by

$$C_t^{m*} + I_t^{m*} = C_t^x + I_t^x = \left( \frac{P_t^x}{P_t^*} \right)^{-n_c^*} (C_t^* + I_t^*) \quad (32)$$

where  $C_t^*$  and  $I_t^*$  are foreign consumption and investment, respectively and  $P_t^*$  denotes foreign price level. Note that equation (32) allows for short run deviations from the l.o.p. which occur because of the price stickiness.

### 3.5 More Open Economy Aspects

In this section various relative prices, which enter the model, will be discussed. Relative prices between domestic goods and imported goods is given by

$$\gamma_t^{m,d} = \frac{P_t^m}{P_t} \quad (33)$$

There is also the relative price between home exports and the foreign goods

$$\gamma_t^{x,*} = \frac{P_t^x}{P_t^*} \quad (34)$$

where  $P_t^{F,*}$  is the world-market price of imports. Deviations from the l.o.p. for the export goods and import goods are given by

$$\psi_t^x = \frac{P_t}{P_t^x e_t}, \quad (35)$$

$$\psi_t^m = \frac{P_t^* e_t}{P_t^m} \quad (36)$$

Combining (34) and (35) another relative price which is used by the exporting and importing firms can be obtained

$$\begin{aligned} \gamma_t^f &= \frac{P_t}{P_t^* e_t} \\ &= \psi_t^x \gamma_t^{x,*} \end{aligned} \quad (37)$$

Note that l.o.p. gap for import goods can be written as

$$\begin{aligned} \psi_t^m &= \frac{P_t^* e_t}{P_t^m} \\ &= \frac{1}{\gamma_t^f \gamma_t^{m,d}} \\ &= \frac{1}{\psi_t^x \gamma_t^{x,*} \gamma_t^{m,d}} \end{aligned}$$

Finally, the producer price and import price relative to CPI is as follows

$$\gamma_t^{c,d} = \frac{P_t^c}{P_t} \quad (38)$$

$$\gamma_t^{c,m} = \frac{P_t^c}{P_t^m} \quad (39)$$

### 3.6 The Government

The government budget constraint is given by

$$G_t + TR_t + R_{t-1}B_{t-1} = B_t + T_t \quad (40)$$

where  $T_t$  denotes tax revenues and it is as follows

$$T_t = \tau_t^c C_t + \tau_t^k [(R_{t-1} - 1) B_{t-1} + (R_{t-1}^* - 1) e_t B_{t-1}^*] + \tau_t^y w_t L_t + \tau_t^k r_t^k u_t \bar{K}_{t-1} \quad (41)$$

The three tax rates, namely labor income, capital income and consumption tax, and lump-sum transfers,  $TR_t$  are treated as shocks and follow an AR(1) process. Following Trabandt and Uhlig (2006) government debt is assumed not to deviate from its balanced growth path, i.e.  $B_{t-1} = \psi^t \bar{B}$ ,  $\forall t \geq 0$  and therefore government budget constraint can be written as

$$G_t = \psi^t \bar{B}(\psi - R_{t-1}) + T_t - TR_t$$

where  $\psi$  is set equal to 1.0075 which is consistent with the annual growth of real GDP in both economies of roughly 3 percent.

### 3.7 The Central Bank

Adjusting the Taylor (1993) rule, monetary policy authority is assumed to adjust the short run interest rate in response to deviations in CPI inflation from the inflation target ( $\pi_t^{tar}$ ), deviations in output from its steady state value and . Additionally, in open economy analysis, monetary policy rule is also assumed to be a function of real exchange rate. Accordingly, monetary policy rule is given by

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R)(\hat{\pi}_t^{tar} + \mu_r(\hat{\pi}_t^c - \hat{\pi}_t^{tar}) + \mu_y \hat{y}_t + \mu_x \hat{x}_t) + \varepsilon_{Rt} \quad (42)$$

which is a generalized backward-looking Taylor rule<sup>6</sup>.  $\hat{R}_t$  is the short run interest rate,  $\hat{\pi}_t^c$  is the CPI inflation,  $\hat{y}_t$  is the output deviation from its steady state and  $\hat{x}_t$  denotes real exchange rate, which is given by

$$\hat{x}_t = \Delta \hat{e}_t + \hat{\pi}_t^* - \hat{\pi}_t^c + \hat{x}_{t-1} \quad (43)$$

The central bank responds to the log-linearized model-consistent measure of the CPI inflation which can be derived from the equation (5).

$$\hat{\pi}_t^c = \left( (1 - w_c) (\bar{\gamma}^{d,c})^{1-\eta_c} \right) \hat{\pi}_t + \left( w_c (\bar{\gamma}^{m,c})^{1-\eta_c} \right) \hat{\pi}_t^m \quad (44)$$

where the steady state domestic goods prices and import goods prices relative to steady state CPI are given by  $\bar{\gamma}^{d,c} = \frac{\bar{P}}{\bar{P}^c}$  and  $\bar{\gamma}^{m,c} = \frac{\bar{P}^m}{\bar{P}^c}$ , respectively. Further,  $\hat{\pi}_t^{tar}$  is a time-varying inflation target of the central bank and  $\varepsilon_{Rt}$  is an interest rate shock. The motion of the inflation target is defined as an AR(1) process.

$$\hat{\pi}_t^{tar} = \rho_\pi \hat{\pi}_{t-1}^{tar} + \varepsilon_{\pi^{tar}t}$$

---

<sup>6</sup>For a more detailed analysis on exchange rate effects on monetary policy rule, see Ball(1999) and Svensson(2000).

### 3.8 Market Equilibrium

Final goods market equilibrium condition in closed economy is given by

$$C_t + G_t + I_t \leq A_t K_t^\alpha L_t^{1-\alpha} - \phi - a(u_t) \bar{K}_t \quad (45)$$

To clear the final goods market and foreign bond market in open economy analysis, the following two constraints must hold in equilibrium:

$$\tilde{C}_t + \tilde{I}_t + G_t + \tilde{X}_t - \tilde{M}_t \leq A_t K_t^\alpha L_t^{1-\alpha} - \phi - a(u_t) \bar{K}_t \quad (46)$$

$$e_t B_t^* = e_t R_t^* B_{t-1}^* + e_t P_t^x (C_t^x + I_t^x) - e_t P_t^{F,*} (C_t^m + I_t^m) \quad (47)$$

where

$$\begin{aligned} \tilde{C}_t &= C_t^d + C_t^m \\ \tilde{I}_t &= I_t^d + I_t^m \\ \tilde{M}_t &= C_t^m + I_t^m \\ \tilde{X}_t &= C_t^x + I_t^x \end{aligned}$$

Also, notice that final good market condition can be rearranged as

$$C_t^d + I_t^d + G_t + C_t^x + I_t^x \leq A_t K_{it}^\alpha L_{it}^{1-\alpha} - \phi - a(u_t) \bar{K}_t$$

By using the demand equations (3), (4), (9), (10) and (32), consumption, investment, total imports and exports can be arranged, respectively, as

$$\tilde{C}_t = C_t^d + C_t^m = \left( (1 - w_c) \left[ \frac{P_t}{P_t^c} \right]^{-\eta_c} + w_c \left[ \frac{P_t^m}{P_t^c} \right]^{-\eta_c} \right) C_t \quad (48)$$

$$\tilde{I}_t = I_t^d + I_t^m = \left( (1 - w_i) \left[ \frac{P_t}{P_t^c} \right]^{-\eta_i} + w_i \left[ \frac{P_t^m}{P_t^c} \right]^{-\eta_i} \right) I_t \quad (49)$$

$$\tilde{M}_t = C_t^m + I_t^m = w_c \left[ \frac{P_t^m}{P_t^c} \right]^{-\eta_c} C_t + w_i \left[ \frac{P_t^m}{P_t^c} \right]^{-\eta_i} I_t \quad (50)$$

$$\tilde{X}_t = C_t^x + I_t^x = \left( \frac{P_t^x}{P_t^*} \right)^{-n_c^*} (C_t^* + I_t^*) \quad (51)$$

Dividing (47) by  $P_t$  and defining  $a_t = \frac{e_t B_t^*}{P_t}$ , the foreign bond market equilibrium condition can be written as

$$a_t = a_{t-1} R_t^* \frac{e_t}{e_{t-1}} \frac{1}{\pi_t} + (\psi_t^x)^{-1} (C_t^x + I_t^x) - (\gamma_t^f)^{-1} (C_t^m + I_t^m)$$

where  $\psi_t^x = \frac{P_t}{P_t^x e_t}$  and  $\gamma_t^f = \frac{P_t}{P_t^* e_t} = \psi_t^x \gamma_t^{x,*}$

## 4 Solving the Model

In this section, first, steady state equations are given. Second, all the log-linearized equations are presented. This is followed by model calibration and calculation of the steady state values to get an insight into the equilibrium conditions for both economies and models.

### 4.1 Steady State

In steady state, the economy is assumed to reach equilibrium. Dropping the subscripts in the first order conditions for variables and in the market equilibrium equations, steady state values of interest, to be used in the analysis, can be calculated.

Using the consumption Euler equation (12), steady state gross interest rate is calculated as

$$\bar{R} = \frac{1 - \beta\bar{\tau}^k}{\beta(1 - \bar{\tau}^k)}$$

where  $\bar{\tau}^k$  is the average capital income tax rate. It is also clear to see in steady state  $\bar{R}^* = \bar{R}$  from equation (13) assuming  $\bar{e} = 1$ . Further, using capital Euler equation (15), steady state rental rate of capital can be obtained

$$\bar{r}^k = \frac{1 - \beta(1 - \delta)}{\beta(1 - \bar{\tau}^k)}$$

Cost minimization of the firms imply in the steady state

$$\bar{w}\bar{L} = (1 - \alpha)(\bar{Y} + \phi) \quad (52)$$

$$\text{and } \bar{r}^k = \alpha \frac{(\bar{Y} + \phi)}{\bar{K}} \quad (53)$$

which delivers

$$\frac{\bar{K}}{\bar{Y}} = \frac{\alpha\beta(1 - \bar{\tau}^k)}{1 - \beta(1 - \delta)} \left(1 + \frac{\phi}{\bar{Y}}\right)$$

where  $\frac{\phi}{\bar{Y}}$  is the share of fixed cost in total production. Assuming  $\bar{S} = 0$ , capital accumulation constraint (7) in steady state yields

$$\delta\bar{K} = \bar{I}$$

which in turn gives

$$\frac{\bar{I}}{\bar{Y}} = \delta \frac{\alpha\beta(1 - \bar{\tau}^k)}{1 - \beta(1 - \delta)} \left(1 + \frac{\phi}{\bar{Y}}\right)$$

Now consider the Euler equation associated with the household's capital utilization decision. In steady state full capacity utilization is assumed, i.e.  $\bar{u} = 1$  and this implies

$$\begin{aligned} a'(1) &= \bar{r}^k (1 - \bar{\tau}^k) \\ &= \frac{1 - \beta(1 - \delta)}{\beta} \end{aligned}$$

Market equilibrium conditions for the closed economy and open economy implies in steady state

$$1 = \frac{\bar{C}}{\bar{Y}} + \frac{\bar{G}}{\bar{Y}} + \frac{\bar{I}}{\bar{Y}} \quad (54)$$

$$1 = \frac{\bar{C}}{\bar{Y}} + \frac{\bar{I}}{\bar{Y}} + \frac{\bar{G}}{\bar{Y}} = \frac{\bar{C}^d}{\bar{Y}} + \frac{\bar{I}^d}{\bar{Y}} + \frac{\bar{G}}{\bar{Y}} + \frac{\bar{C}^x}{\bar{Y}} + \frac{\bar{I}^x}{\bar{Y}}$$

where steady state values  $\bar{C}$  and  $\bar{I}$  can be derived from equations (48) and (49) and are given by

$$\bar{C} = \left( (1 - w_c) (\bar{\gamma}^{c,d})^{\eta_c} + w_c (\bar{\gamma}^{c,m})^{\eta_c} \right) \bar{C}$$

$$\bar{I} = \left( (1 - w_i) (\bar{\gamma}^{c,d})^{\eta_i} + w_i (\bar{\gamma}^{c,m})^{\eta_i} \right) \bar{I}$$

where  $\bar{\gamma}^{c,d} = \frac{\bar{P}^c}{\bar{P}}$  and  $\bar{\gamma}^{c,m} = \frac{\bar{P}^c}{\bar{P}^m}$  are steady state relative prices.

Tax revenue of the government in steady state is given by

$$\frac{\bar{T}}{\bar{Y}} = \bar{\tau}^c \frac{\bar{C}}{\bar{Y}} + \bar{\tau}^k (\bar{R} - 1) \frac{\bar{B}}{\bar{Y}} + \bar{\tau}^y \frac{\bar{w}\bar{L}}{\bar{Y}} + \bar{\tau}^k \frac{\bar{r}^k \bar{K}}{\bar{Y}} \quad (55)$$

Using equations (52), (53) and (54) above equation (55) can written for the closed economy case as

$$\frac{\bar{T}}{\bar{Y}} = \bar{\tau}^c \left( 1 - \frac{\bar{G}}{\bar{Y}} - \frac{\bar{I}}{\bar{Y}} \right) + \bar{\tau}^k (\bar{R} - 1) \frac{\bar{B}}{\bar{Y}} + (\bar{\tau}^y (1 - \alpha) + \bar{\tau}^k \alpha) \left( 1 + \frac{\phi}{\bar{Y}} \right) \quad (56)$$

whereas in open economy case this differs slightly

$$\begin{aligned} \frac{\bar{T}}{\bar{Y}} &= \bar{\tau}^c \left( 1 - \frac{\bar{G}}{\bar{Y}} - \left( (1 - w_i) (\bar{\gamma}^{c,d})^{\eta_i} + w_i (\bar{\gamma}^{c,m})^{\eta_i} \right) \frac{\bar{I}}{\bar{Y}} \right) \\ &\quad + \bar{\tau}^k (\bar{R} - 1) \frac{\bar{B}}{\bar{Y}} + (\bar{\tau}^y (1 - \alpha) + \bar{\tau}^k \alpha) \left( 1 + \frac{\phi}{\bar{Y}} \right) \end{aligned} \quad (57)$$

Government budget constraint in steady state is as follows

$$\frac{\bar{G}}{\bar{Y}} = (\psi - \bar{R}) \frac{\bar{B}}{\bar{Y}} + \frac{\bar{T}}{\bar{Y}} - \frac{\bar{T}\bar{R}}{\bar{Y}}$$



Plugging equations (56) and (57) into the above equation, steady state values for the government expenditure to output ratio for closed and open economy can be calculated, respectively

$$\begin{aligned}\frac{\bar{G}}{\bar{Y}} &= \frac{1}{1 + \bar{\gamma}^c} \left[ \bar{\tau}^c \left( 1 - \frac{\bar{I}}{\bar{Y}} \right) + (\bar{\tau}^y (1 - \alpha) + \bar{\tau}^k \alpha) \left( 1 + \frac{\phi}{\bar{Y}} \right) + (\bar{\tau}^k (\bar{R} - 1) + (\psi - \bar{R})) \frac{\bar{B}}{\bar{Y}} \right] \\ \frac{\tilde{G}}{\bar{Y}} &= \frac{1}{1 + \bar{\gamma}^c} \left[ \begin{aligned} &\bar{\tau}^c \left( 1 - ((1 - w_i) (\bar{\gamma}^{c,d})^{\eta_i} + w_i (\bar{\gamma}^{c,m})^{\eta_i}) \frac{\bar{I}}{\bar{Y}} \right) \\ &+ (\bar{\tau}^y (1 - \alpha) + \bar{\tau}^k \alpha) \left( 1 + \frac{\phi}{\bar{Y}} \right) + (\bar{\tau}^k (\bar{R} - 1) + (\psi - \bar{R})) \frac{\bar{B}}{\bar{Y}} \end{aligned} \right]\end{aligned}$$

Now consider the steady state values for relative prices. Equation (5), which defines the CPI, can be written as

$$\frac{P_t^c}{P_t} = \left[ (1 - w_c) + w_c \left( \frac{P_t^m}{P_t} \right)^{1 - \eta_c} \right]^{\frac{1}{1 - \eta_c}} \quad (58)$$

$$\frac{P_t^c}{P_t^m} = \left[ (1 - w_c) \left( \frac{P_t}{P_t^m} \right)^{1 - \eta_c} + w_c \right]^{\frac{1}{1 - \eta_c}} \quad (59)$$

Combining (58), (59) and (31), evaluated in steady state gives

$$\frac{\bar{P}^c}{\bar{P}} = \left[ (1 - w_c) + w_c \left( \frac{\eta_m}{\eta_m - 1} \frac{\bar{e} \bar{P}^*}{\bar{P}} \right)^{1 - \eta_c} \right]^{\frac{1}{1 - \eta_c}} \quad (60)$$

$$\frac{\bar{P}^c}{\bar{P}^m} = \left[ (1 - w_c) \left( \frac{\eta_m - 1}{\eta_m} \frac{\bar{P}}{\bar{e} \bar{P}^*} \right)^{1 - \eta_c} + w_c \right]^{\frac{1}{1 - \eta_c}} \quad (61)$$

where  $\bar{e}$  is the steady state exchange rate and is assumed to equal 1, i.e.,  $\bar{e} = 1$ . Further, the producer price level equals the foreign price level in steady state, i.e.,  $\bar{P} = \bar{P}^{F,*}$ . Therefore, equations (60) and (61) can be described as

$$\frac{\bar{P}^c}{\bar{P}} = \left[ (1 - w_c) + w_c \left( \frac{\eta_m}{\eta_m - 1} \right)^{1 - \eta_c} \right]^{\frac{1}{1 - \eta_c}}$$

$$\frac{\bar{P}^c}{\bar{P}^m} = \left[ (1 - w_c) \left( \frac{\eta_m - 1}{\eta_m} \right)^{1 - \eta_c} + w_c \right]^{\frac{1}{1 - \eta_c}}$$

Note that if  $0 < \eta_c < \infty$  and  $1 < \eta_m < \infty$ ,  $\frac{\bar{P}^c}{\bar{P}}$  will be greater than 1 with  $w_c > 0$ . Accordingly,  $\frac{\bar{P}^c}{\bar{P}^m}$  will be smaller than 1. Combining both equations above gives

$$\frac{\bar{P}^m}{\bar{P}} = \left[ \frac{(1 - w_c) + w_c \left( \frac{\eta_m}{\eta_m - 1} \right)^{1 - \eta_c}}{(1 - w_c) \left( \frac{\eta_m - 1}{\eta_m} \right)^{1 - \eta_c} + w_c} \right]^{\frac{1}{1 - \eta_c}} = \frac{\eta_m}{\eta_m - 1}$$

In addition to the above assumption  $\bar{P} = \bar{P}^{F,*}$ , note also that the export price equals the foreign price level in steady state, i.e.,  $\bar{P}^x = \bar{P}^{F,*}$ . This implies that there are no deviations from the l.o.p. in expor sector in steady state equilibrium.

$$\bar{\psi}^x = \frac{\bar{P}}{\bar{P}^x \bar{e}} = 1$$

On the other hand, importing firms charge a markup of their cost and therefore, exchange rate pass-through in domestic import prices is imperfect in steady state.

$$\begin{aligned} \bar{\psi}^m &= \frac{\bar{P}^* \bar{e}}{\bar{P}^m} \\ &= \frac{1}{\bar{\psi}^x \bar{\gamma}^{x,*} \bar{\gamma}^{m,d}} \\ &= \frac{\bar{P}}{\bar{P}^m} \\ &= \frac{\eta_m - 1}{\eta_m} \end{aligned}$$

Finally, steady state values for the remaining relative prices are as follows

$$\begin{aligned} \bar{\gamma}^{m,d} &= \frac{\bar{P}^m}{\bar{P}} \\ \bar{\gamma}^{x,*} &= \frac{\bar{P}^x}{\bar{P}^*} = 1 \\ \bar{\gamma}^f &= \frac{\bar{P}}{\bar{e} \bar{P}^*} = 1 \end{aligned}$$

## 4.2 Log-linearized equations

For the dynamics, the equations of the model are log-linearized around their steady steady values, i.e.  $z_t = \bar{z} e^{\hat{z}_t} \approx \bar{z}(1 + \hat{z}_t)$  and the system is solved following Uhlig (1999). All the model equations are given below.

After combining equations (11) and (12) and log-linearizing, the dynamics for consumption can be obtained

$$\hat{C}_t = E_t \left[ \begin{aligned} &\frac{1}{1+h} \hat{C}_{t+1} + \frac{h}{1+h} \hat{C}_{t-1} + \frac{1-h}{1+h} \frac{\bar{\tau}^c}{1+\bar{\tau}^c} (\hat{\tau}_{t+1}^c - \hat{\tau}_t^c) + \frac{1-h}{1+h} (\hat{d}_t^c - \hat{d}_{t+1}^c) \\ &+ \frac{1-h}{1+h} \hat{\pi}_{t+1} - \frac{1-h}{1+h} (1 - \beta \bar{\tau}^k) R_t - \frac{1-h}{1+h} (1 - \beta \bar{R}) \hat{\tau}_{t+1}^k \end{aligned} \right] \quad (62)$$

In the absence of consumption and capital income taxes, above equation boils down to the standard consumption equation with habit formation used in New Keynesian models. Boldrin, Christiano and Fisher (2001) argue that the ability of standard general equilibrium models to account for the equity premium and other asset market statistics is considerably enhanced by the presence of habit formation in preferences.

The investment equation is given by

$$\hat{I}_t = E_t \left[ \frac{1}{1+\beta} \hat{I}_{t-1} + \frac{\beta}{1+\beta} \hat{I}_{t+1} + \frac{\varphi}{1+\beta} \hat{Q}_t \right] \quad (63)$$

where  $\varphi = \frac{1}{S''}$  is the elasticity of investment with respect to a one percent temporary increase in the current price of installed capital. A more persistent change in the price of capital implies a larger percentage change in investment because adjustment costs induce agents to be forward looking (Christiano et al, 2005). As mentioned before,  $S''$  does not affect the steady state analysis, but the dynamics depend on the value of  $S''$ .

Using (14) and (15), log-linearized version of Euler equation for capital can be obtained

$$\hat{Q}_t = E_t \left[ \hat{\pi}_{t+1} - (1 - \beta \bar{\tau}^k) \hat{R}_t + \beta (1 - \delta) \hat{Q}_{t+1} - \frac{\bar{\tau}^k}{1 - \bar{\tau}^k} \beta \delta \hat{\tau}_{t+1}^k + (1 - \beta (1 - \delta)) \hat{r}_{t+1}^k \right] \quad (64)$$

Introduction of capital income tax implies that interest rate changes have less effect on the current price of capital and accordingly, on the investment level compared to the case without capital income tax. Further, an expected increase in future capital income tax causes the current price of capital to decrease whereas an increase in expected inflation, rental rate of capital and price of capital implies an increase in current price of capital.

Capital accumulation is as follows

$$\hat{K}_t = (1 - \delta) \hat{K}_{t-1} + \delta \hat{I}_t \quad (65)$$

New Keynesian Phillips curves for all the firms in the economy are derived by assuming Calvo's contract model in the economy with some price-setters being backward-looking.

$$\hat{\pi}_t = \frac{\beta}{1 + \beta \chi_p} E_t \hat{\pi}_{t+1} + \frac{\chi_p}{1 + \beta \chi_p} \hat{\pi}_{t-1} + \frac{(1 - \theta_p)(1 - \beta \theta_p)}{\theta_p (1 + \beta \chi_p)} m \hat{c}_t \quad (66)$$

$$\hat{\pi}_t^m = \frac{\beta}{1 + \beta \chi_m} E_t \hat{\pi}_{t+1}^m + \frac{\chi_m}{1 + \beta \chi_m} \hat{\pi}_{t-1}^m + \frac{(1 - \theta_m)(1 - \beta \theta_m)}{\theta_m (1 + \beta \chi_m)} \hat{\psi}_t^m \quad (67)$$

$$\hat{\pi}_t^x = \frac{\beta}{1 + \beta \chi_x} E_t \hat{\pi}_{t+1}^x + \frac{\chi_x}{1 + \beta \chi_x} \hat{\pi}_{t-1}^x + \frac{(1 - \theta_x)(1 - \beta \theta_x)}{\theta_x (1 + \beta \chi_x)} \hat{\psi}_t^x \quad (68)$$

where  $\hat{\pi}_t$  is the domestic producer price inflation,  $\hat{\pi}_t^m$  is the import price inflation and  $\hat{\pi}_t^x$  is the export price inflation. Real marginal costs for domestic producers and l.o.p. gap for importing firms and exporting firms are given by

$$m\hat{c}_t = (1 - \alpha)\hat{w}_t + \alpha\hat{r}_t^k - \hat{A}_t \quad (69)$$

$$\hat{\psi}_t^m = \hat{\psi}_{t-1}^m - \Delta\hat{\psi}_t^x - \Delta\hat{\gamma}_t^{x,*} - \Delta\hat{\gamma}_t^{m,d} = \hat{\psi}_{t-1}^m + \hat{\pi}_t^* + \Delta\hat{e}_t - \hat{\pi}_t^m \quad (70)$$

$$\hat{\psi}_t^x = \hat{\psi}_{t-1}^x + \hat{\pi}_t - \hat{\pi}_t^x - \Delta\hat{e}_t \quad (71)$$

Since firms in the Calvo model would like to keep their price as a fixed markup over marginal cost, an increase in real marginal cost will spark inflationary pressures in domestic prices and accordingly, in export prices. Further, due to the partial indexation of all three firms by past inflation, New Keynesian Phillips curve implies that inflation rates depend not only on the expected inflation but also on the past inflation. Indexation is captured by the parameters  $\chi_p$ ,  $\chi_m$  and  $\chi_x$  for domestic producers, importing firms and exporting firms, respectively. Note that  $\chi_j = 0$ ,  $j \in [p, m, x]$  implies that all three New Keynesian Phillips curves are only forward-looking.

Remaning log-linearized relative prices are given by

$$\hat{\gamma}_t^{x,*} = \hat{\gamma}_{t-1}^{x,*} + \hat{\pi}_t^x - \hat{\pi}_t^* \quad (72)$$

$$\hat{\gamma}_t^{m,d} = \hat{\gamma}_{t-1}^{m,d} + \hat{\pi}_t^m - \hat{\pi}_t \quad (73)$$

$$\hat{\gamma}_t^f = \hat{\gamma}_{t-1}^f + \Delta\hat{\psi}_t^x + \Delta\hat{\gamma}_t^{x,*} \quad (74)$$

$$\hat{\gamma}_t^{c,d} = \hat{\gamma}_{t-1}^{c,d} + \hat{\pi}_t^c - \hat{\pi}_t \quad (75)$$

Similar to firms, households set wages in a Calvo setting. Log-linearizing and rearranging (21) and (22), the wage dynamics can be obtained

$$0 = E_t \left[ \begin{array}{l} \varsigma_0 \hat{w}_{t-1} + \varsigma_1 \hat{w}_t + \varsigma_2 \hat{w}_{t+1} + \varsigma_3 \hat{\pi}_t + \varsigma_4 \hat{\pi}_{t+1} + \varsigma_5 \hat{\pi}_{t-1}^c + \varsigma_6 \hat{\pi}_t^c \\ + \varsigma_7 (\hat{d}_t^c - \frac{1}{1-h} \hat{C}_t + \frac{h}{1-h} \hat{C}_{t-1} - \frac{\bar{\tau}^c}{1+\bar{\tau}^c} \hat{\tau}_t^c) + \varsigma_8 \hat{L}_t + \varsigma_9 \hat{\tau}_t^y + \varsigma_{10} \hat{d}_t^h \end{array} \right] \quad (76)$$

where  $\Omega_w = \frac{[\epsilon\sigma_l - (1-\epsilon)]}{[(1-\beta\theta_w)(1-\theta_w)]}$  and

$$\varsigma = \begin{pmatrix} \Omega_w \theta_w \\ (\sigma_l \epsilon - \Omega_w (1 + \beta \theta_w^2)) \\ \Omega_w \beta \theta_w \\ -\Omega_w \theta_w \\ \Omega_w \beta \theta_w \\ \Omega_w \theta_w \chi_w \\ -\Omega_w \beta \theta_w \chi_w \\ (1 - \epsilon) \\ -(1 - \epsilon) \sigma_l \\ -(1 - \epsilon) \frac{\bar{\tau}^y}{(1 - \bar{\tau}^y)} \\ -(1 - \epsilon) \end{pmatrix} = \begin{pmatrix} \varsigma_0 \\ \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \\ \varsigma_4 \\ \varsigma_5 \\ \varsigma_6 \\ \varsigma_7 \\ \varsigma_8 \\ \varsigma_9 \\ \varsigma_{10} \end{pmatrix}$$

Uncovered interest rate parity is given by

$$\hat{R}_t - \hat{R}_t^* = \frac{1}{1 - \beta\bar{\tau}^k} E_t \Delta \hat{e}_{t+1}$$

Capital utilization rate is as follows

$$\hat{u}_t = \hat{K}_t - \hat{K}_{t-1}$$

where  $\hat{k}_t$  is the deviation of capital services stock from its steady state value and  $\hat{K}_{t-1}$  is the deviation of physical capital stock from its steady state value. Further, after log-linearizing the Euler equation associated with the household's capital utilization decision, i.e. equation (14), following expression can be obtained

$$\hat{u}_t = \frac{1}{\xi} \hat{r}_t^k - \frac{1}{\xi} \frac{\bar{\tau}^k}{(1 - \bar{\tau}^k)} \hat{r}_t^k$$

where  $\xi$  is the inverse elasticity of capital utilization with respect to the rental rate of capital. Moreover, first order conditions to the cost minimization problem of the intermediate goods firms implies

$$\hat{L}_t = \hat{r}_t^k + \hat{K}_t - \hat{w}_t$$

Real exchange rate is given by

$$\hat{x}_t = \Delta \hat{e}_t + \hat{\pi}_t^* - \hat{\pi}_t^c + \hat{x}_{t-1} = -w_c (\gamma^{c,m})^{-(1-\eta_c)} \hat{\gamma}_t^{m,d} - \hat{\gamma}_t^{x,*} - \hat{\psi}_t^x$$

In open economy analysis supply and demand is given by

$$\begin{aligned} & \left(1 + \frac{\phi}{\bar{Y}}\right) \left(\hat{A}_t + \alpha \hat{K}_t + (1 - \alpha) \hat{L}_t\right) - (1 - \bar{\tau}^k) \bar{r}^k \frac{\bar{K}}{\bar{Y}} \left(\hat{K}_t - \hat{K}_{t-1}\right) \\ \geq & (1 - w_c) (\bar{\gamma}^{c,d})^{\eta_c} \frac{\bar{C}}{\bar{Y}} \left(\hat{C}_t + \eta_c \hat{\gamma}_t^{c,d}\right) + (1 - w_{i'}) (\bar{\gamma}^{c,d})^{\eta_{ii}} \frac{\bar{I}}{\bar{Y}} \left(\hat{I}_t + \eta_i \hat{\gamma}_t^{c,d}\right) \\ & + \frac{\bar{G}}{\bar{Y}} \hat{G}_t + \frac{\bar{X}}{\bar{Y}} \left(\hat{M}_t^* - \eta_f \hat{\gamma}_t^{x,*}\right) \end{aligned}$$

In closed economy analysis, the difference is in the demand side. It is given by

$$\hat{Y}_t = \frac{\bar{C}}{\bar{Y}} \hat{C}_t + \frac{\bar{I}}{\bar{Y}} \hat{I}_t + \frac{\bar{G}}{\bar{Y}} \hat{G}_t + (1 - \bar{\tau}^k) \bar{r}^k \frac{\bar{K}}{\bar{Y}} \left(\hat{K}_t - \hat{K}_{t-1}\right)$$

Government budget constraint

$$\frac{\bar{G}}{\bar{Y}} \hat{G}_t = \frac{\bar{T}}{\bar{Y}} \hat{T}_t - \frac{\bar{B}}{\bar{Y}} \bar{R} \hat{R}_t - \frac{\bar{T}\bar{R}}{\bar{Y}} \hat{T}\hat{R}_t$$

and accordingly, tax income is given by

$$\begin{aligned} \frac{\bar{T}}{\bar{Y}} \hat{T}_t &= \frac{\bar{C}}{\bar{Y}} \bar{\tau}^c \left( \hat{C}_t + \hat{\tau}_t^c \right) + \bar{\tau}^k \left( (\bar{R} - 1) \frac{\bar{B}}{\bar{Y}} + \alpha \left( 1 + \frac{\phi}{\bar{Y}} \right) \right) \hat{\tau}_t^k \\ &\quad + \bar{\tau}^k \frac{\bar{B}}{\bar{Y}} \bar{R} \hat{R}_{t-1} + \bar{\tau}^y \left( 1 + \frac{\phi}{\bar{Y}} \right) (1 - \alpha) \hat{\tau}_t^y + (\bar{\tau}^y (1 - \alpha) + \bar{\tau}^k \alpha) \hat{Y}_t \end{aligned}$$

Net foreign asset dynamics is as follows

$$\begin{aligned} \hat{a}_t &= \bar{R} \hat{a}_{t-1} - \bar{X} \left( \hat{\psi}_t^x + \eta_f \hat{\gamma}_t^{x,*} - \hat{M}_t^* \right) + (\bar{C}^m + \bar{I}^m) \hat{\gamma}_t^f \\ &\quad - \bar{C}^m \left( \hat{C}_t - \eta_c (1 - w_c) (\bar{\gamma}^{c,d})^{-(1-\eta_c)} \hat{\gamma}_t^{m,d} \right) \\ &\quad - \bar{I}^m \left( \hat{I}_t - \eta_i (1 - w_i) (\bar{\gamma}^{c,d})^{-(1-\eta_i)} \hat{\gamma}_t^{m,d} \right) \end{aligned}$$

CPI inflation is given by

$$\hat{\pi}_t^c = \left( (1 - w_c) (\bar{\gamma}^{c,d})^{-(1-\eta_c)} \right) \hat{\pi}_t + \left( w_c (\bar{\gamma}^{c,m})^{-(1-\eta_c)} \right) \hat{\pi}_t^m$$

Finally, monetary policy is given by

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R) (\hat{\pi}_t^{tar} + \mu_\pi (\hat{\pi}_t^c - \hat{\pi}_t^{tar}) + \mu_y \hat{y}_t + \mu_x \hat{x}_t) + \varepsilon_{Rt}$$

### 4.3 Calibration

Although the model parameters are not determined by estimation, it is found that certain parameter combinations generate standard impulse response functions. For allowing such combinations, parameter calibration, which is done at a quarterly frequency, is based mainly on Smets and Wouters (2004a, 2004b); for import and export sector and the share of capital in production on Adolfson et al. (2007); for fiscal policy parameters on Trabandt and Uhlig (2006).

The discount factor  $\beta$  is set equal to 0.99 which below implies a steady state quarterly gross real interest rate of 1.0153 and 1.016 for the EU and US, respectively. This difference is due to the capital income tax on domestic and foreign bonds. However, above we assume  $\bar{R} = \bar{R}^*$ . This problem, which could be solved by setting US discount factor infinitesimally higher or EU discount factor infinitesimally lower, is negligible. Parameter, defining the habit persistence

in consumption is set so that  $h^{EU} = 0.59$  and  $h^{US} = 0.69$ . The share of capital in production  $\alpha$  is 0.29 and 0.24 for the EU and US economy, respectively. Further, the depreciation of capital  $\delta$  is set equal to 0.015 for both economies which implies an annual depreciation rate of 6 percent.

For the monetary policy rule parameters,  $\rho_R$ ,  $\mu_\pi$  and  $\mu_y$ , in the open economy case the estimates of Smets and Wouters (2004a) and for the exchange rate parameter,  $\mu_x$  the estimates of Adolfson et al. (2007) is used. On the other hand, for closed economy case Smets and Wouters (2004b) and Smets and Wouters (2007) are used.

	Monetary Policy Rule Parameters			
	$\rho_R$	$\mu_\pi$	$\mu_y$	$\mu_x$
Open Economy				
EU	0.94	1.50	0.09	-0.02
US	0.88	1.48	0.08	-0.02
Closed Economy				
EU	0.88	1.508	0.061	
US	0.81	2.04	0.08	

Further, Trabandt and Uhlig (2006) give a detailed explanation for the consumption, labor income and capital income tax rates along with the quarterly steady state government debt and transfers to GDP ratio for both economies.

<sup>7</sup>These values are as follows

		EU	US
Labor tax rate	$\bar{\tau}^y$	0.38	0.26
Capital tax rate	$\bar{\tau}^k$	0.34	0.37
Consumption tax rate	$\bar{\tau}^c$	0.17	0.05
Gov. transfers to GDP ratio	$\frac{\bar{T}R}{\bar{Y}}$	0.19	0.11
Gov. debt to GDP ratio	$\psi \frac{\bar{B}}{\bar{Y}}$	$0.53 \times 4$	$0.61 \times 4$

<sup>7</sup>For the rest of the parameters, see Appendix.

## 5 Impulse Response Analysis

In section four, closed and open economy models for the EU and US are described in a DSGE framework. Accordingly, in this section the impulse response analysis, using Toolkit, will be based on open versus closed economy comparison. First, the effects of a shock in interest rate and inflation target will be investigated. Second, our analysis will be focused on preference, labour supply and technology shocks. Furthermore, the size of shocks are set equal to one standard deviation as in Smets and Wouters (2003).

Concerning the monetary policy rule, a few things are worth mentioning. In the common literature, interest rate smoothing parameter,  $\rho_R$  is estimated to be higher for the ECB policy rule compared to the US.<sup>8</sup> This implies that it takes longer for the ECB to converge to its interest rate target after an interest rate shock. This can be seen in Figure (1) and (2), although the model cannot capture the desired hump-shaped impulse response functions. Furthermore, a decrease in consumption, investment and government expenditures, which can be seen in Figure (7) and (8) for both economies, respectively, leads to an increase in output. These results are in line with Smets and Wouters (2004a).

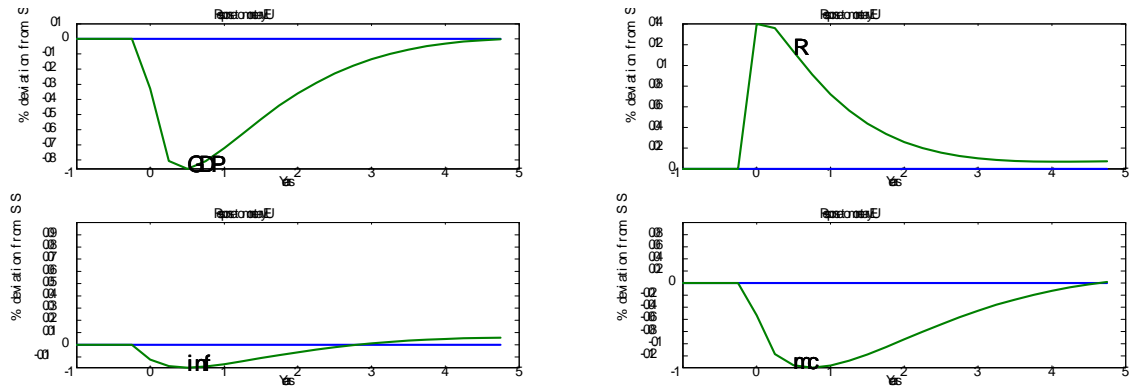


Figure 1: Interest rate shock, EU, closed economy model (1)

<sup>8</sup>See, for example, Christiano, Motto and Rostagno (2007) and Lubik and Schorfheide (2005).



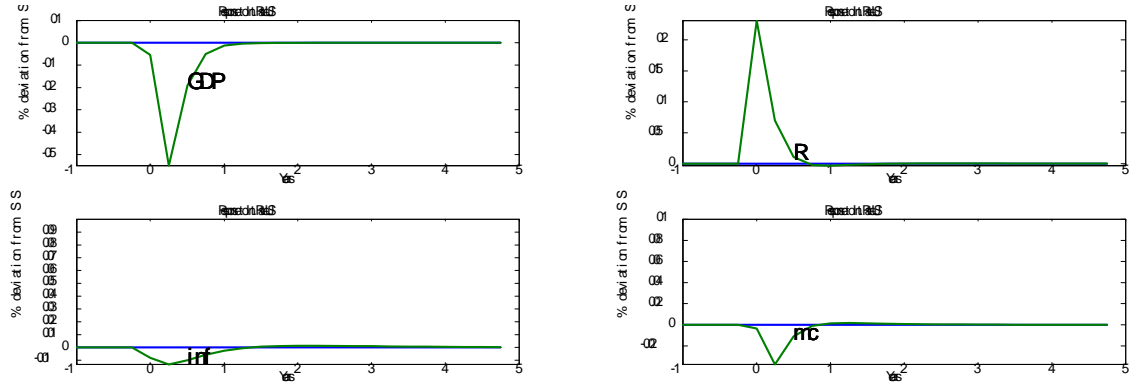


Figure 2: Interest rate shock, US, Closed economy model (1)

Figure (3) and (4) depicts the impulse response functions in open economy model following an interest rate shock. The results are similar to closed economy results, however, the deviations are bigger this time. Accordingly, similar to the results in Adolfson et al (2007), Figure (11) and (12) has implications about the open economy variables. In the EU economy, import price inflation decreases more than the domestic inflation decreases. Further, an increase in the marginal cost of the exporting firms,  $mc_t^x = P_t/e_t$ , due to a real appreciation of EU currency, puts an upward pressure on export price inflation. In this respect, given the price stickiness, l.o.p. gap for export sector,  $\psi_t^x = \frac{P_t}{P_t^* e_t}$ , increases. EU exports decrease, while imports increase causing a net foreign asset deficit. On the other hand, for the US economy, the effects are the other way around. The real appreciation is not that strong and export prices fall causing an increase in exports. The US net foreign asset acquisition increases.

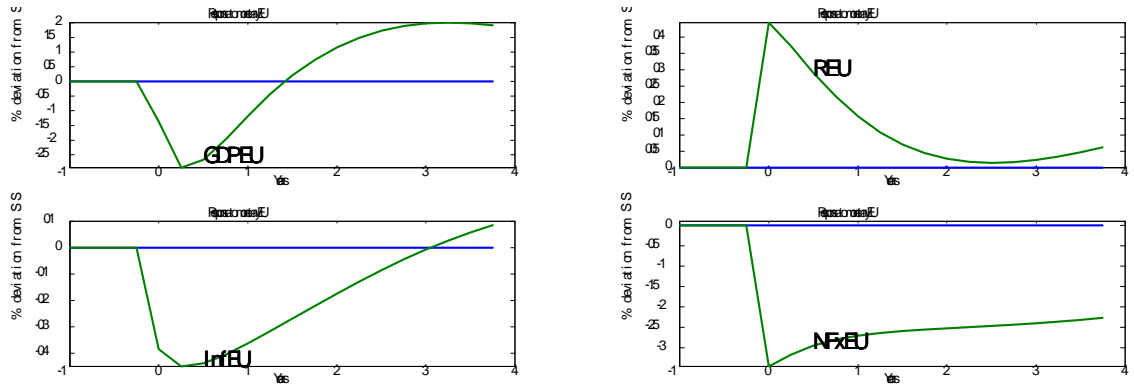


Figure 3: Interest rate shock, EU, Open economy model (1)

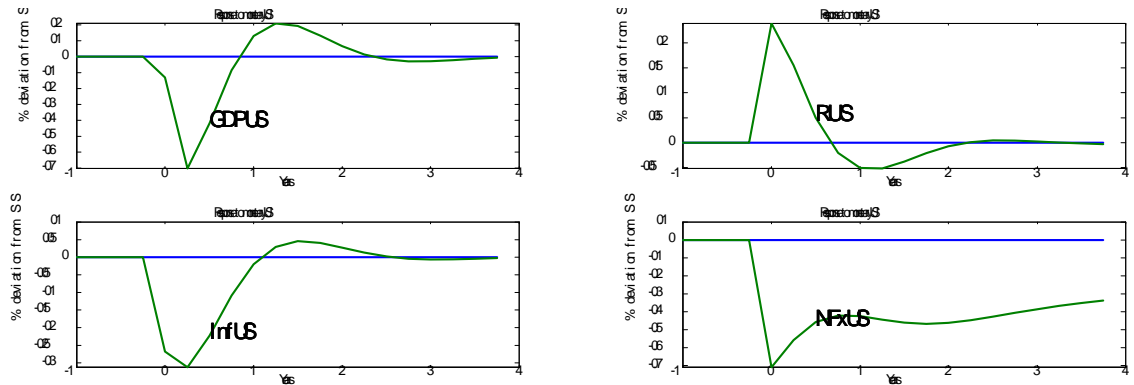


Figure 4: Interest rate shock, US, Open economy model (1)

Following figures (5) and (6) presents the deviations of output, gross real interest rate, inflation and real marginal costs from their steady state values for the EU and US economies in the closed economy model. Effects are in the same directions for both economies. Consumption, investment, government expenditures and accordingly, output increases. An upward pressure in the wages and rental rate of capital causes an increase in marginal cost in real terms and in turn inflation increases. The central banks react to this by increasing interest rates.

For further differences and similarities, we now turn to the effects of preference, labor supply and technology shocks. Figure (15) and (16) depicts impulse response functions after a shock in preferences in closed economy case for both economies. As it is shown, consumption, government expenditures and GDP

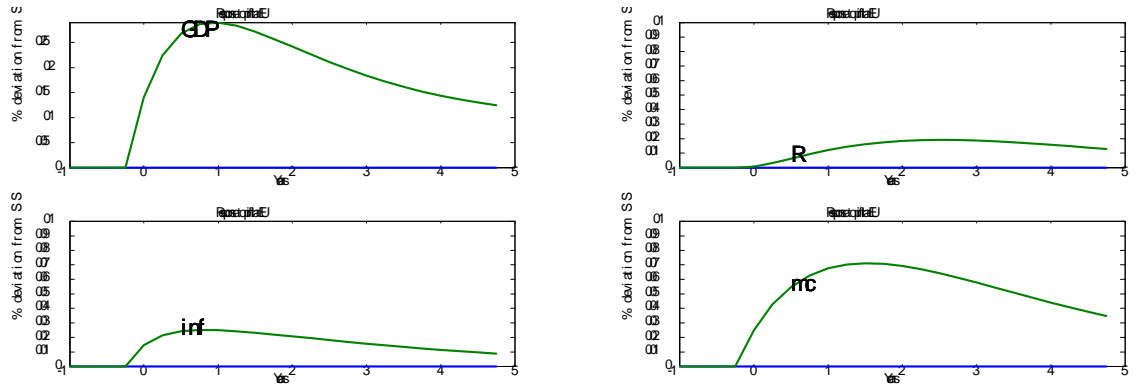


Figure 5: Inflation target shock, EU, Closed economy model (1)

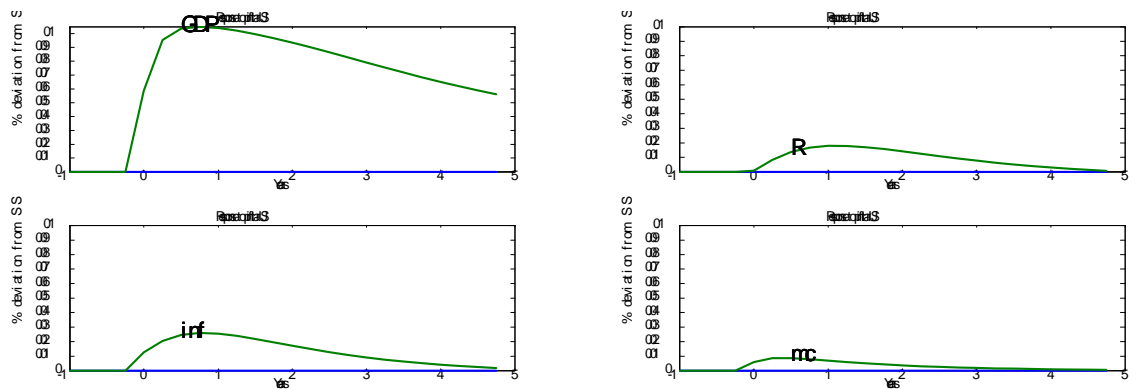


Figure 6: Inflation target shock, US, Closed economy model (1)

increases, while investment decreases. In the EU, the return on capital and real wage rates increase and accordingly, real marginal costs increase. This, in turn, creates a cost-push inflation. However, in the US, real wage rate increases only slightly while return on capital decreases. This implies that in the US real marginal costs decrease and in turn, inflation decreases. Further, the ECB reacts to this inflation increase by increasing the interest rates and the Fed reacts to this drop in inflation by decreasing the interest rates. Open economy implications of a preference shock are presented in the figures (21) and (22). This time in the EU, the decrease in domestic inflation and import price inflation pulls CPI inflation downward. However, export price inflation increases. These changes in import and export price inflation alter the demand schemes for imports and exports in the short run causing a loss in the net foreign asset position of the EU. On the contrary, in the US the effects imply an opposite responses.

Figure (17) and (18) presents the effects of a labor supply shock in the EU and US in the closed economy case. In both economies, real wage rates increase, causing marginal costs to increase and accordingly an increase in inflation. This rise in inflation engenders a rise in interest rates in both economies. In open economy case, seen in Figure (23) and (24), labor shock acts as a negative shock and generates the opposite impulse response functions for the closed economy case for both economies.

Finally, we analyse the effects of a technology shock. In Figure (19) and (20) the deviations of selected variables from their steady state values are shown. For both of the economies in closed economy case, technology shock causes the labor supply to fall and real wage rates to increase. On the other hand, real marginal costs along with the return on capital decreases. Due to the downward pressure on inflation, which is caused by the fall in marginal costs, interest rates decrease. Further, in the open economy case, which is depicted in Figure (25) and (26), consumption and output increases for both economies. Domestic and import price inflation puts an upward pressure on CPI inflation in the EU but not in the US. As l.o.p gap for the export sector decreases, accordingly, there is an increase in exports and a fall in imports in both economies. The difference between the US and EU is the response of the interest rate. In the former it decreases as a reaction to a fall in CPI inflation and in the latter it increases due to a rise in CPI inflation.

## 6 Concluding Remarks

In this paper, for US and EU we have described a closed economy and an open economy model following Christiano, Eichenbaum and Evans (2005) and Adolfson, Laseén, Lindé and Villani (2007), respectively. Furthermore, the tax structure was introduced following Tranbandt and Uhlig (2006). Both models, in line

with the New Keynesian synthesis, incorporate a number of nominal and real rigidities, such as price and wage stickiness, variable capital utilization, capital adjustment costs and habit formation. By including local currency nominal price stickiness, the open economy model also features incomplete exchange rate pass-through in both the import and export sectors.

Both models are, then calibrated according to Smets and Wouters (2004a, 2004b and 2007) estimations. Calibration imposes challenges for the open economy case, since there is a large set of parameters used. In general, the closed economy model for EU and US manages to generate impulse response function in line with Adolfson, Laseén, Lindé and Villani (2007). Monetary policy transmission mechanism for closed economy model works fine with respect to the direction of steady state deviations in the key variables, i.e., inflation, output, consumption and interest rates. However, open economy model in general, due to arising miscalculations, fails to produce such impulse responses except for interest rate, output and inflation. The next step in research, following this paper, will be fixing the miscalculations and use Bayesian estimation methods.

## 7 References

1. Adolfson, M., Laséen, S., Lindé, J., Villani, M. (2003), "Bayesian estimation of an open economy DSGE model with incomplete pass-through", *Journal of International Economics*, Volume 72, Issue 2, July 2007, Pages 481-511
2. Boldrin, M., Christiano, L., Fisher, J.,(2001) "Habit Persistence, Asset Returns, and the Business Cycle "The American Economic Review, Vol. 91, No. 1 (Mar., 2001), pp. 149-166
3. Calvo, G. (1983), "Staggered Prices In A Utility Maximizing Framework", *Journal of Monetary Economics*, September, 1983.
4. Christiano, L., Eichenbaum, M. and C. Evans (2005), "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy, *Journal of Political Economy* 113(1), 1-45"
5. Christiano, L., Motto, R. and M. Rostagno (2007), "Shocks, Structures or Monetary Policies? The Euro Area and US After 2001" *European Central Bank Working Paper No 774*, July 2007
6. Clarida Richard, Galí Jordi, and Gertler1 Mark (1999): "The Science of Monetary Policy: A New Keynesian Perspective", *Journal of Economic Literature* Vol. XXXVII (December 1999), pp.1661–1707.
7. Devereux M. and Charles Engel (2002). "Exchange Rate Pass-Through, Exchange Rate Volatility, and Exchange Rate Disconnect". *Journal of Monetary Economics*, June, 913-940
8. Goodfriend M. and B. King (1997). "The New Neoclassical Synthesis and the Role of Monetary Policy". *National Bureau of Economic Research Macroeconomics Annual*, pp.231-95.
9. Gordon, R., (1990), "What Is New-Keynesian Economics?", *Journal of Economic Literature*, Vol. 28, No. 3. (Sep., 1990), pp. 1115-1171.
10. Greenwald, B., Stiglitz, J., (1993), "New and Old Keynesians", *The Journal of Economic Perspectives*, Vol. 7, No. 1. pp. 23-44.
11. Jordi, G., and Monacelli, T., (2002) "Optimal Monetary Policy and Exchange Rate Volatility in a Small Open Economy", NBER 8905
12. Lubik, T., (2006), "A Simple, Structural, and Empirical Model of the Antipodean Transmission Mechanism", *Federal Reserve Bank of Richmond*, November 2006
13. Lubik, T., and Schorfheide, F.,(2005) "A Bayesian Look at New Open Economy Macroeconomics", *The Johns Hopkins University, Department of Economics Economics Working Paper Archive No. 521.*

14. Lucas, Robert E. (1976), "Econometric Policy Evaluation: A Critique," Carnegie-Rochester Conference Series on Public Policy 1, 19-46.
15. Monacelli, T., (2003), "Monetary Policy in a Low Pass-Through Environment", European Central Bank Working Paper No. 27
16. Romer, D., (1993), "The New Keynesian Synthesis", The Journal of Economic Perspectives, Vol. 7, No. 1. (Winter, 1993), pp. 5-22.
17. Rotemberg, J. and Woodford, M.,(1997) "An Optimization-Based Econometric Model for the Evaluation of Monetary Policy," NBER Macroeconomics Annual 12: 297-346
18. Rudebusch, G., (2002), "Assessing the Lucas Critique in Monetary Policy Models", FRBSF Working Paper, 2002-02
19. Smets F. and Wouters R.(2003): "An Estimated Dynamic Stochastic General Equilibrium model of the Euro Area", Journal of the European Economic Association, Vol 1, Issue 5, Sept 2003, 1124-1175.
20. Smets, F. and R. Wouters (2004a), "Comparing Shocks and Frictions In US and Euro Area Business Cycles: A Bayesian DSGE Approach, National Bank of Belgium Working Paper Series No. 61 - October 2004
21. Smets, F. and R. Wouters (2004b), "Forecasting with a Bayesian DSGE model -an application to the euro area.", ECB working paper series No. 722, February 2007.
22. Smets, F. and R. Wouters (2007), "Shocks and Frictions in US Business Cycles", ECB working paper series No. 389, September 2004.
23. Taylor, J. B. (1993): "Discretion versus Policy Rules in Practice", Carnegie-Rochester Series on Public Policy 23, 194-214.
24. Trabandt, M., and Harald, U., (2006), "How Far Are We From The Slippery Slope?The Laffer Curve Revisited", SFB 649 Discussion Paper 2006-023
25. Uhlig, H.(1999): "A Toolkit for analyzing Nonlinear Dynamic Stochastic Models easily" (version 2, March 1997), Oxford University Press 1999 (February), pp.30-61.
26. Woodford, M., (2003), "Interest and Prices: Foundation of a Theory of Monetary Policy", Princeton University Press

## 8 Appendix

### 8.1 Parameter values

		Open Economy		Closed Economy	
		EU	US	EU	US
Indexation wages	$\chi_w$	0.378	0.63	0.411	0.70
Indexation prices	$\chi_d$	0.199	0.24	0.469	0.66
Indexation import prices	$\chi_m$	1.00	1.00		
Indexation export prices	$\chi_x$	0.191	0.17		
Calvo prices	$\theta_d$	0.919	0.66	0.89	0.66
Calvo wages	$\theta_w$	0.655	0.80	0.798	0.70
Calvo imports	$\theta_m$	0.327	0.56		
Calvo exports	$\theta_x$	0.629	0.60		
Elast. subs. wages	$\epsilon$	1.05	1.05	1.05	1.05
Elast. subs. consumption	$\eta_c$	11.952	11		
Elast. subs. investment	$\eta_i$	2.056	2.8		
Elast. subs. imports	$\eta_m$	3.809	2.8		
Import share cons.	$w_c$	0.36	0.05		
Import share inv.	$w_i$	0.55	0.30		
Inverse elasticity of cap. util	$\xi$	0.05	0.40	0.226	0.54
Share of fixed cost in production	$\phi$	0.57	0.48	0.268	0.60
Inverse elast. of work effort	$\sigma_L$	1.94	2.45	1.570	1.83
Stand. error of preference shock	$\varepsilon_{dc}$	0.32	1.66	0.285	0.23
Stand. error of cons. tax shock	$\varepsilon_{\tau^c}$	0.01	0.01	0.01	0.01
Stand. error of capital tax shock	$\varepsilon_{\tau^h}$	0.01	0.01	0.01	0.01
Stand. error of labor supply shock	$\varepsilon_{dh}$	2.21	2.38	1.411	0.45
Stand. error of labor inc. tax shock	$\varepsilon_{\tau^y}$	0.01	0.01	0.01	0.01
Stand. error of tech. shock	$\varepsilon_A$	0.61	0.41	0.604	0.45
Stand. error of gov. transfers shock	$\varepsilon_{TR}$	0.01	0.01	0.01	0.01
Stand. error of interest rate shock	$\varepsilon_R$	0.11	0.24	0.145	0.24
Stand. error of equity prem. shock	$\varepsilon_q$	0.60	0.65	0.468	0.01
Stand. error of inf. target shock	$\varepsilon_{\pi^{tar}}$	0.05	0.05	0.05	0.05



		Open Economy		Closed economy	
		EU	US	EU	US
AR parameter of preference shock, iid	$\rho_{dc}$	0.922	0.49	0.783	0.22
AR parameter of cons. tax shock, iid	$\rho_{\tau^c}$	0.9	0.9	0.9	0.9
AR parameter of capital inc. tax shock, iid	$\rho_{\tau^k}$	0.9	0.9	0.9	0.9
AR parameter of labor inc. tax shock, iid	$\rho_{\tau^y}$	0.9	0.9	0.9	0.9
AR parameter of labor supply shock, iid	$\rho_{dh}$	0.941	0.99	0.939	0.71
AR parameter of technology shock, iid	$\rho_A$	0.951	0.99	0.953	0.95
AR parameter of gov. transfers shock, iid	$\rho_{TR}$	0.9	0.9	0.9	0.9
AR parameter of interest rate shock, iid	$\rho_{\varepsilon_r}$	0	0	0.15	0.15
AR parameter of equity premium shock, iid	$\rho_{\varepsilon_q}$	0	0	0	0
AR parameter of inf. target shock, iid	$\rho_{\pi^{tar}}$	0.975	0.975	0.975	0.975

## 8.2 Figures

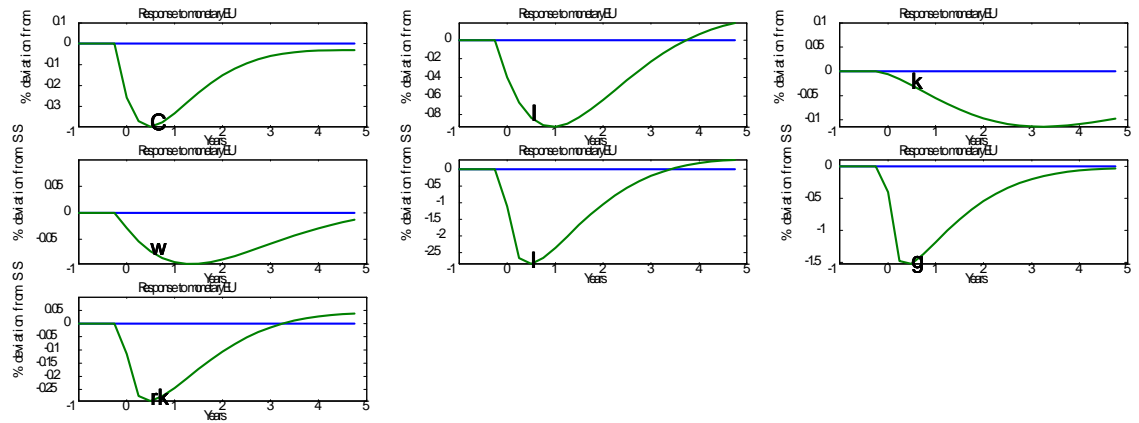


Figure 7: Interest rate shock, EU, Closed economy model (2)

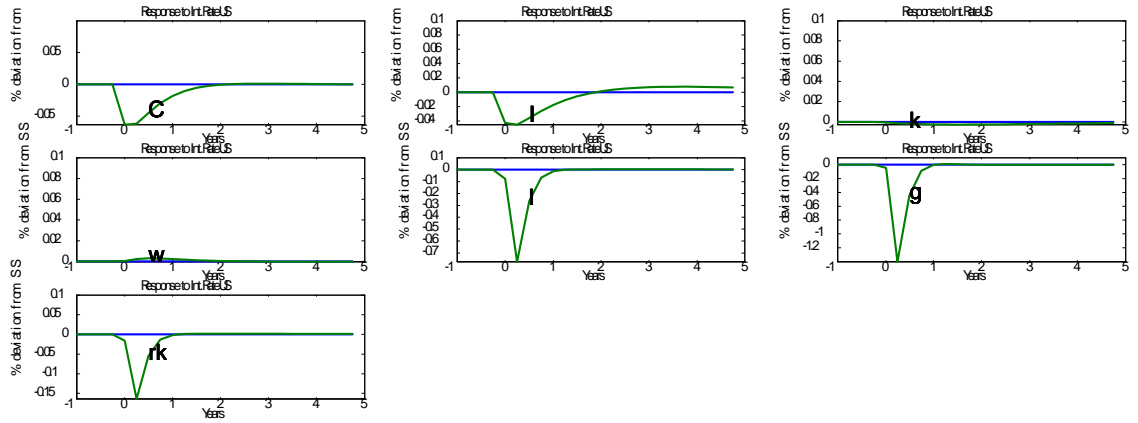


Figure 8: Interest rate shock, US, Closed economy model (2)

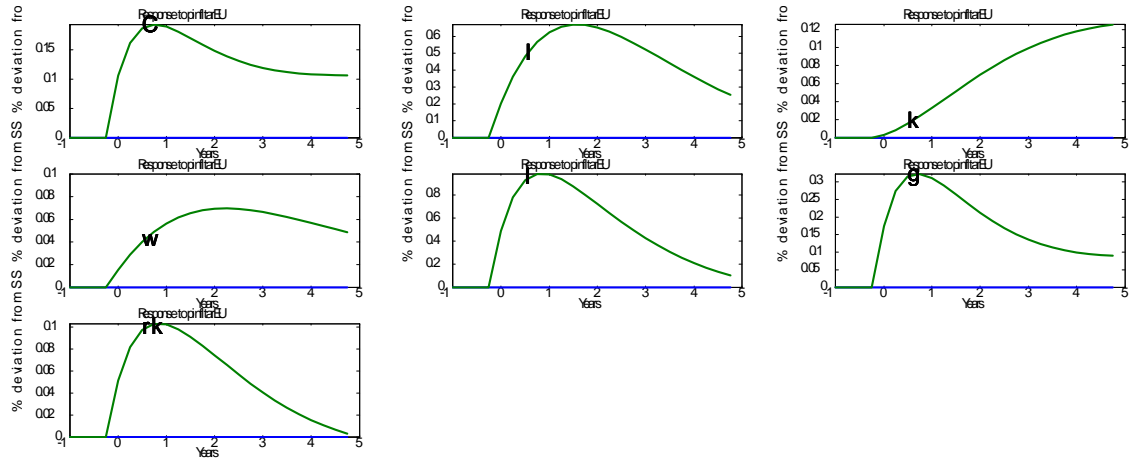


Figure 9: Inflation target shock, EU, Closed economy model (2)

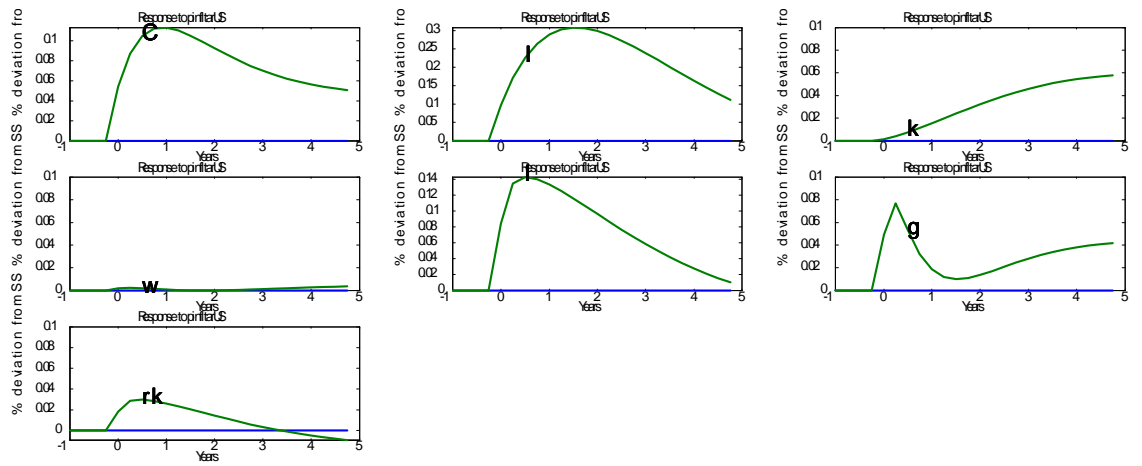


Figure 10: Inflation target shock, US, Closed economy model (2)

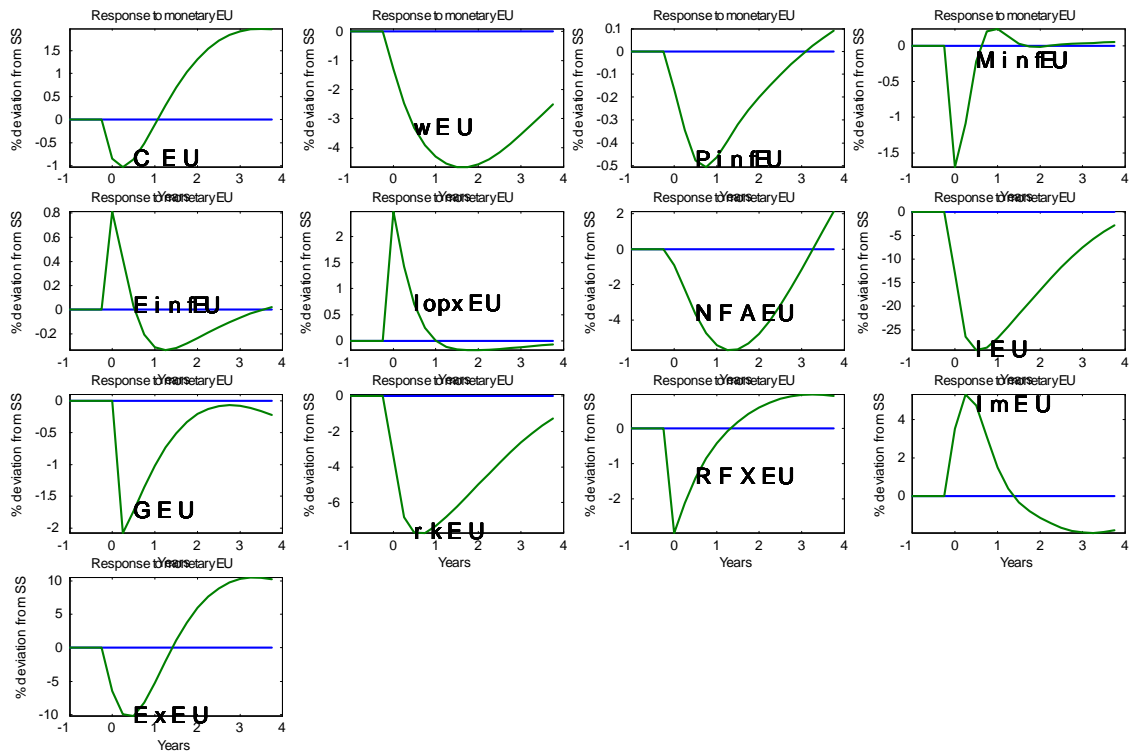


Figure 11: Interest rate shock, EU, Open economy model (2)

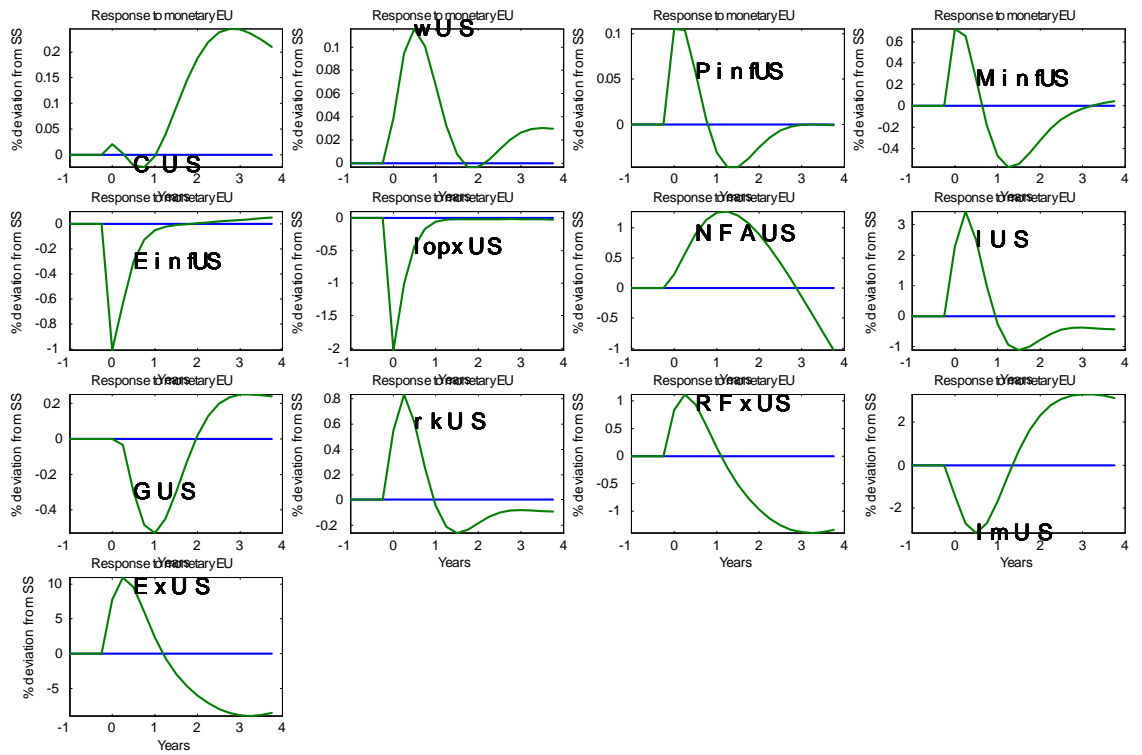


Figure 12: Interest rate shock, US, Open economy model (2)

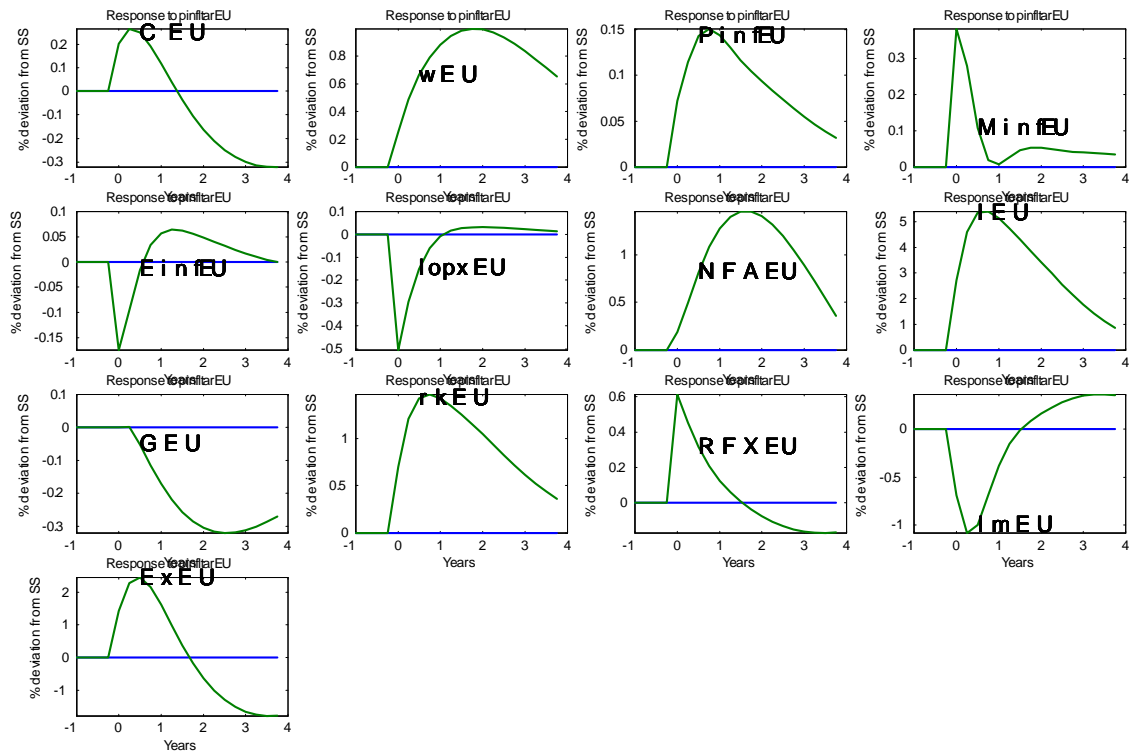


Figure 13: Inflation target shock, EU, Open economy model (2)

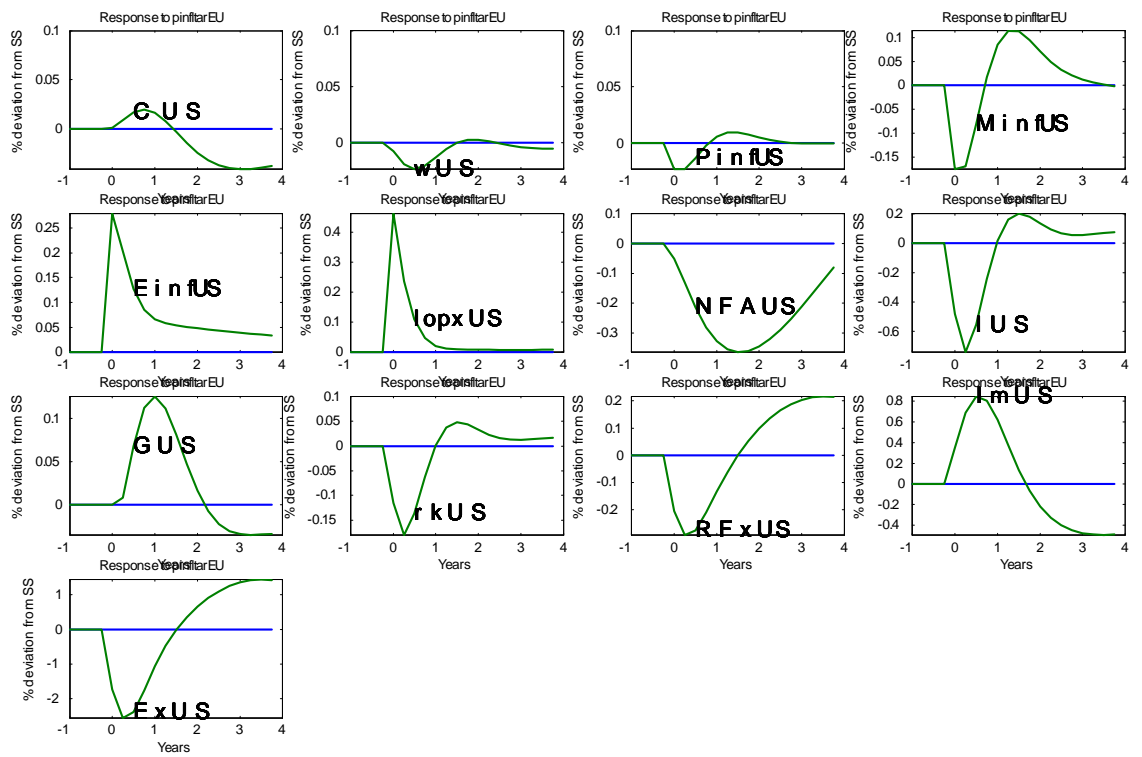


Figure 14: Inflation target shock, US, Open economy model (2)



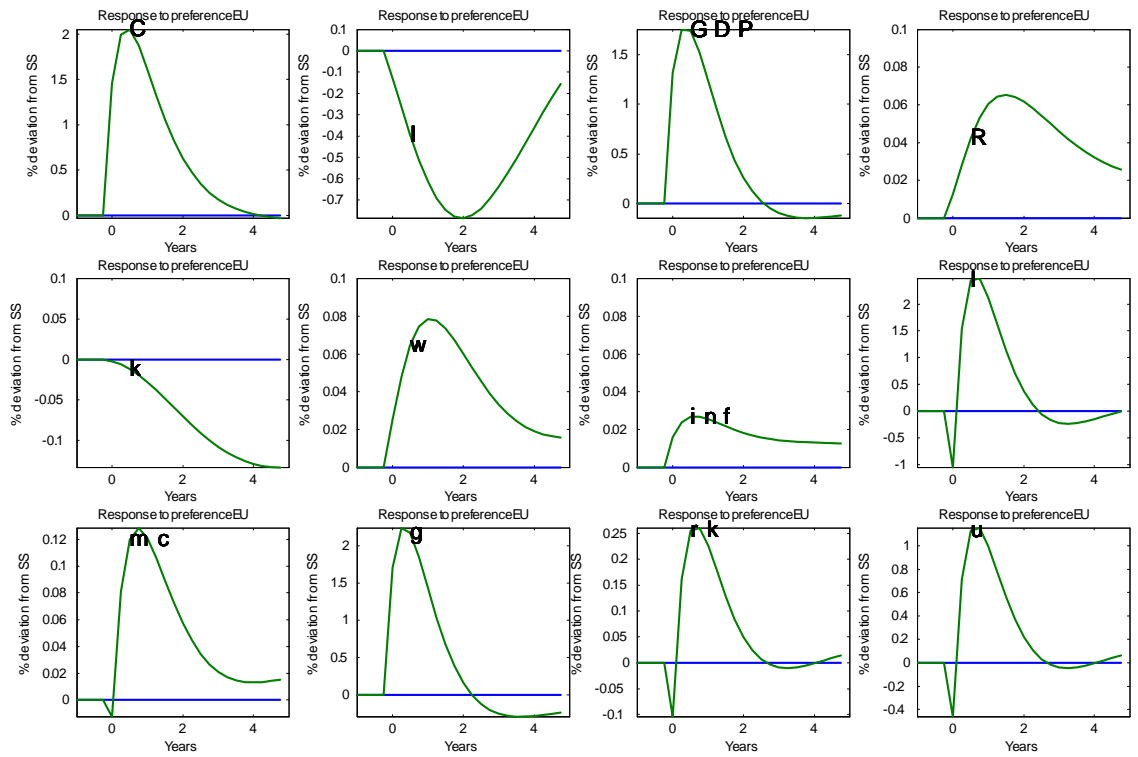


Figure 15: Preference shock, EU, Closed economy model

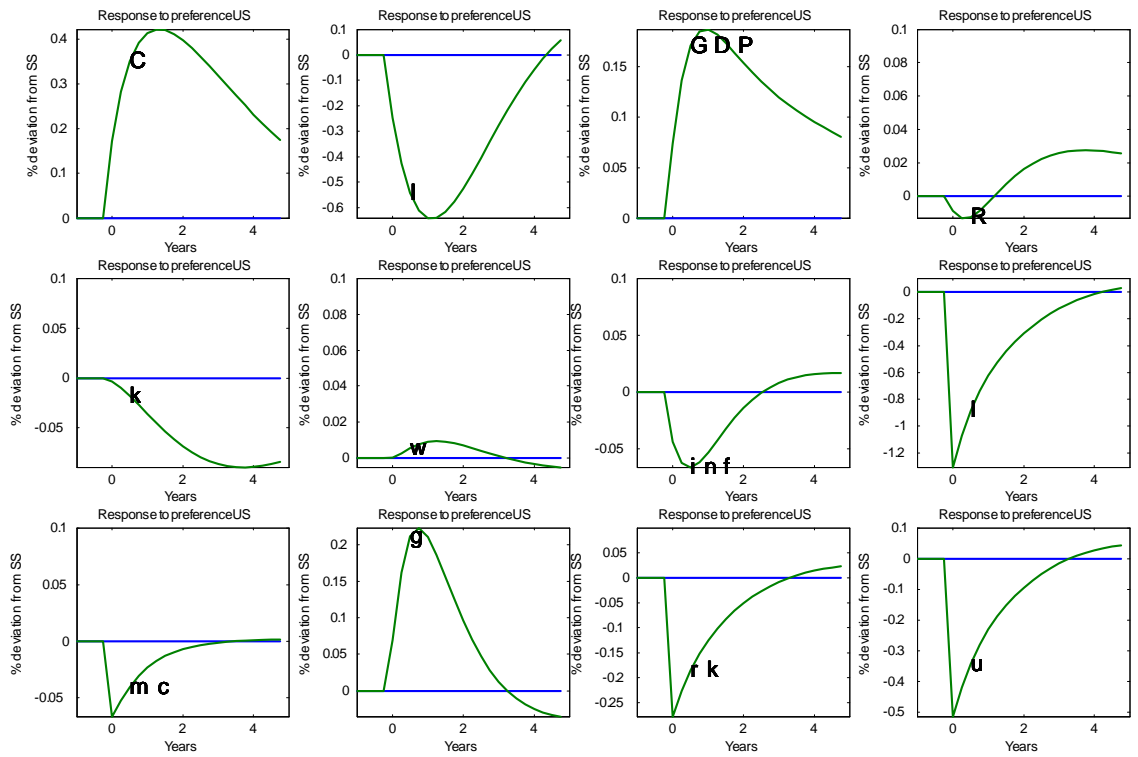


Figure 16: Preference shock, US, Closed economy model

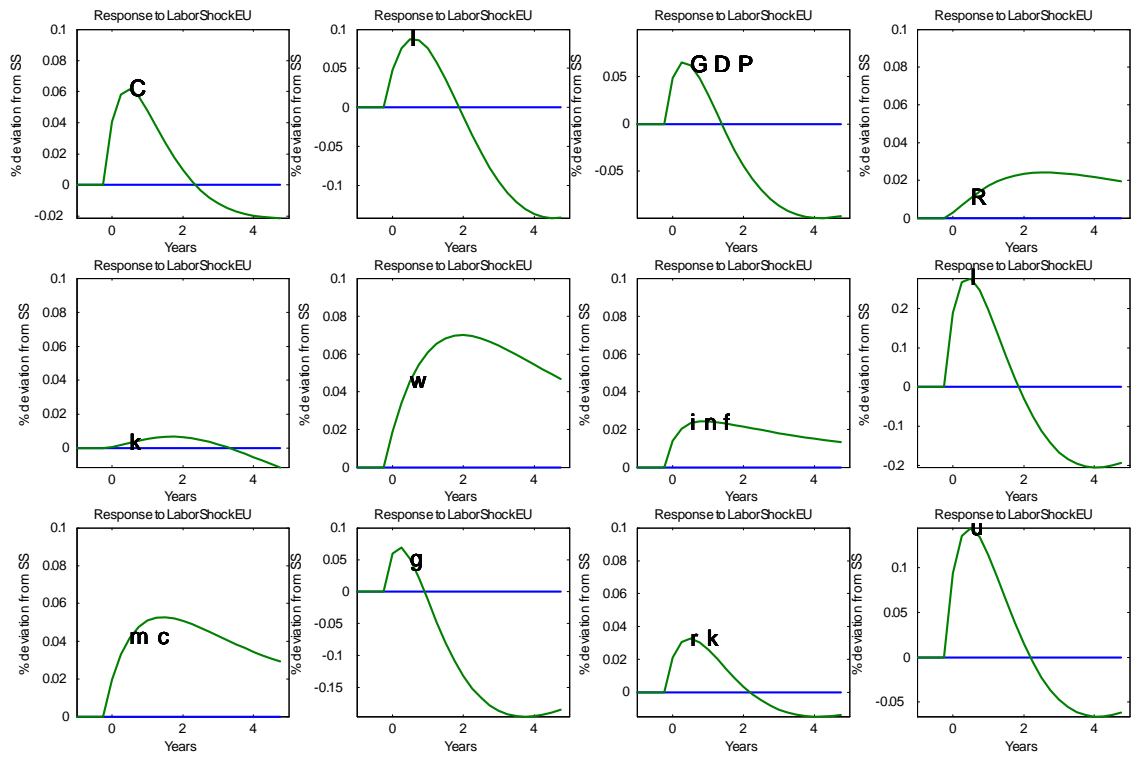


Figure 17: Labor supply shock, EU, Closed economy model

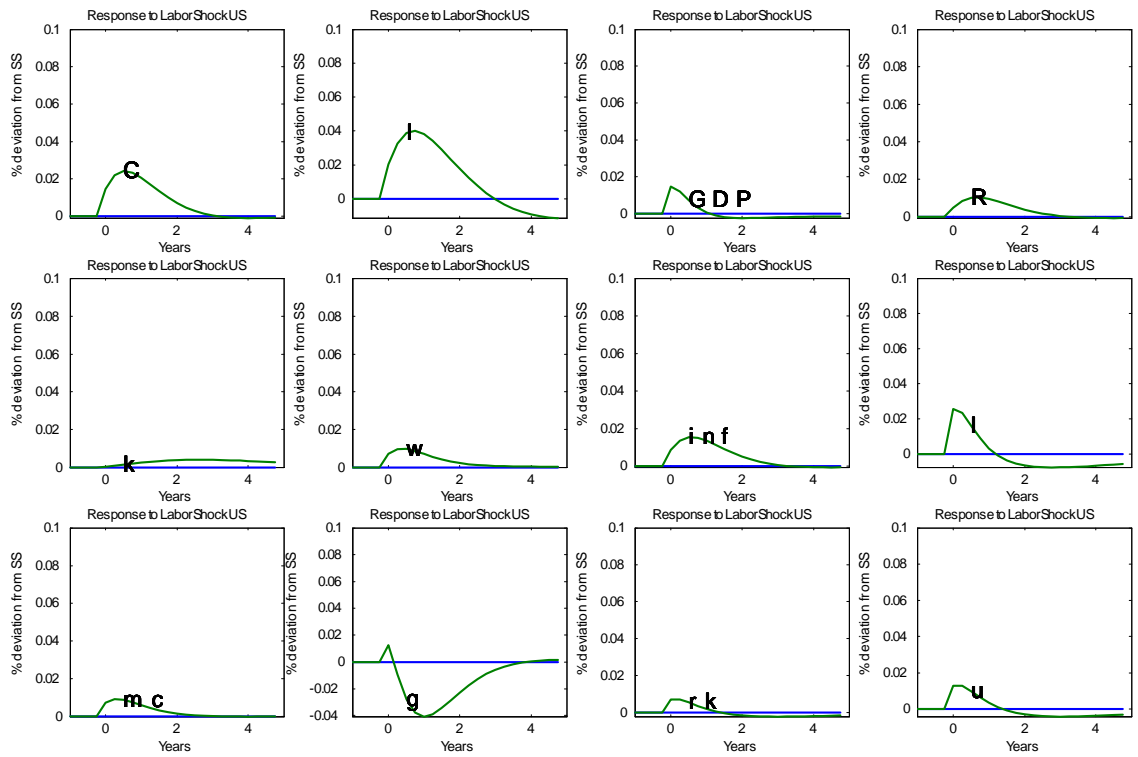


Figure 18: Labor supply shock, US, Closed economy model

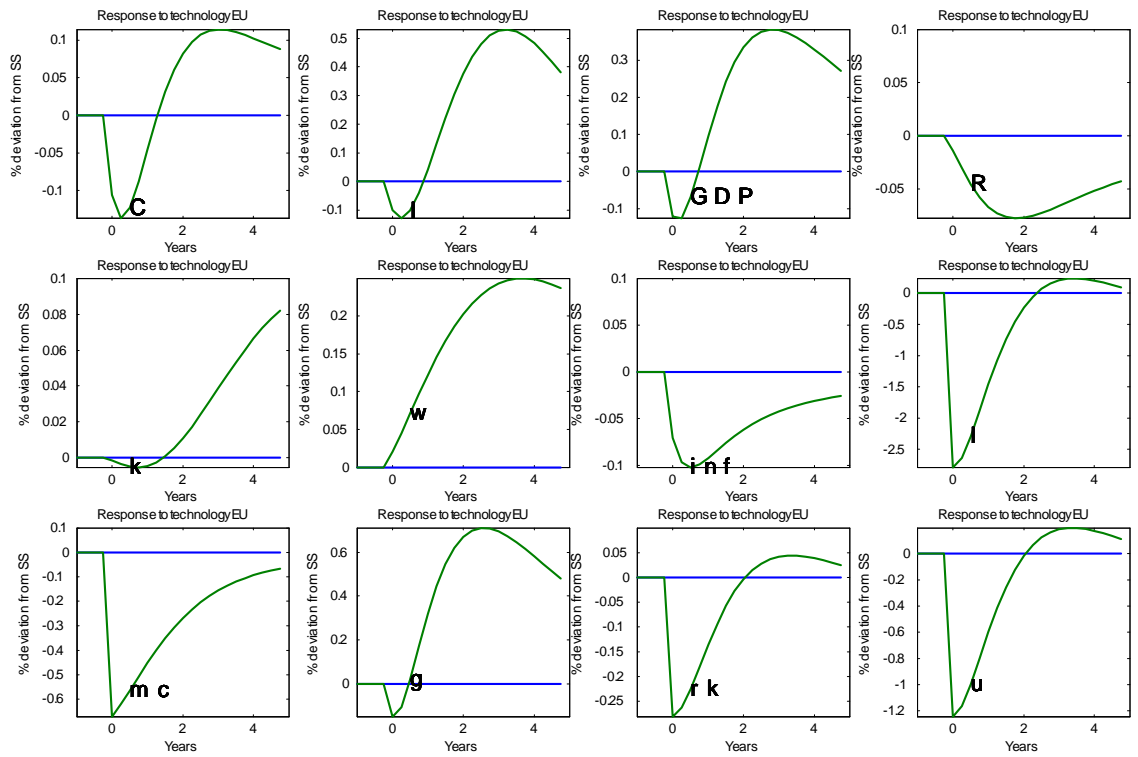


Figure 19: Technology shock, EU, Closed economy model

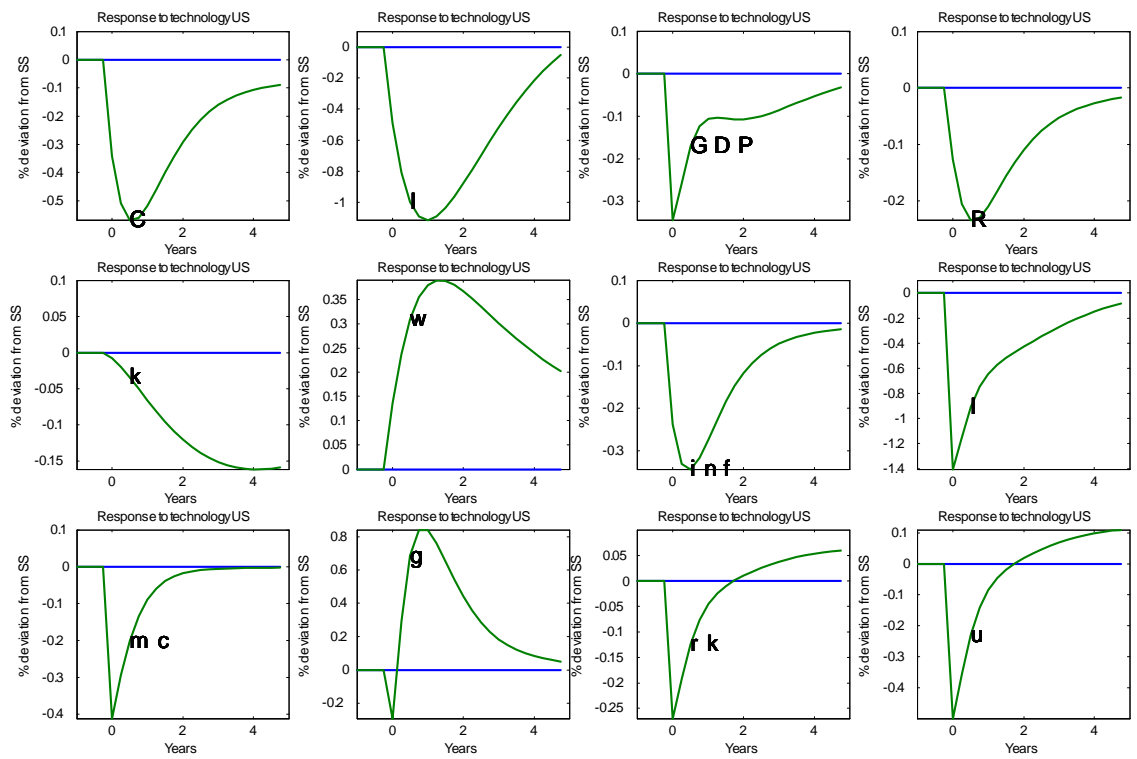


Figure 20: Technology shock, US, Closed economy model

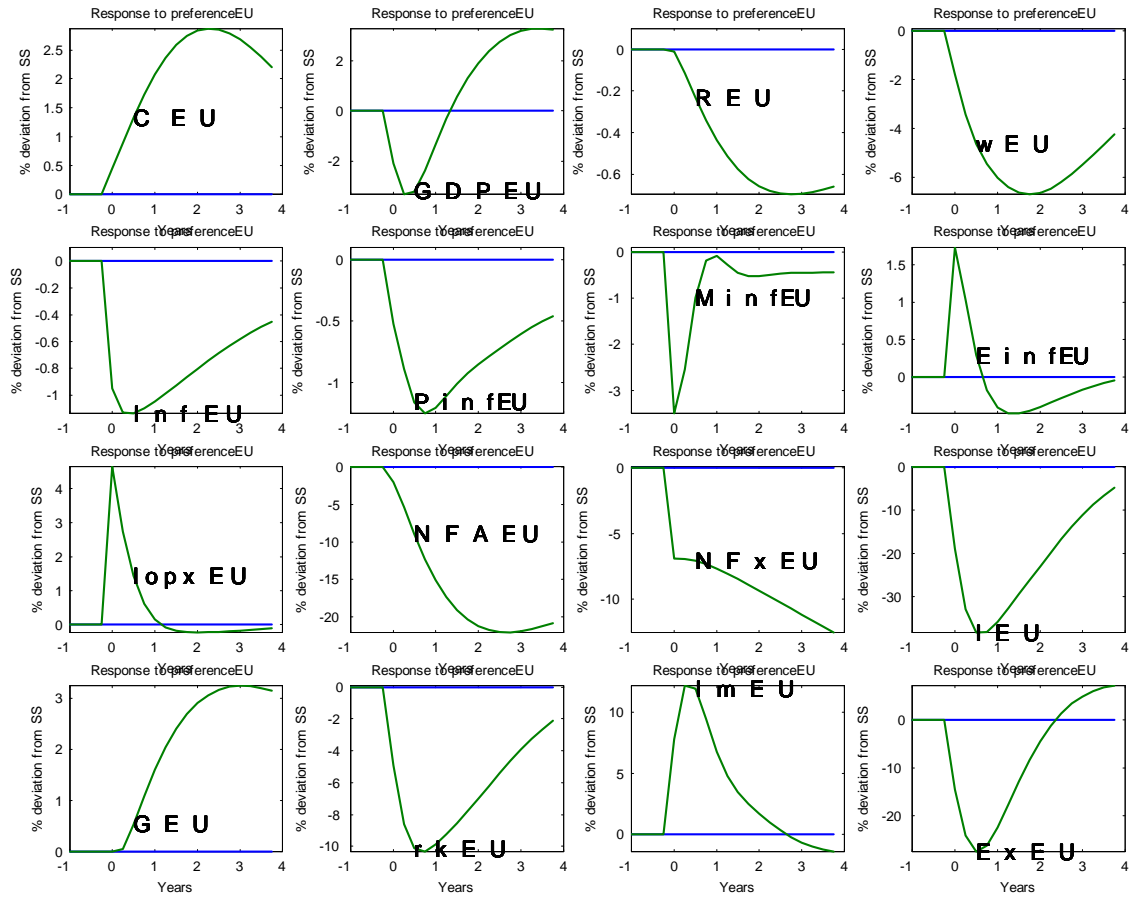


Figure 21: Preference shock, EU, Open economy model

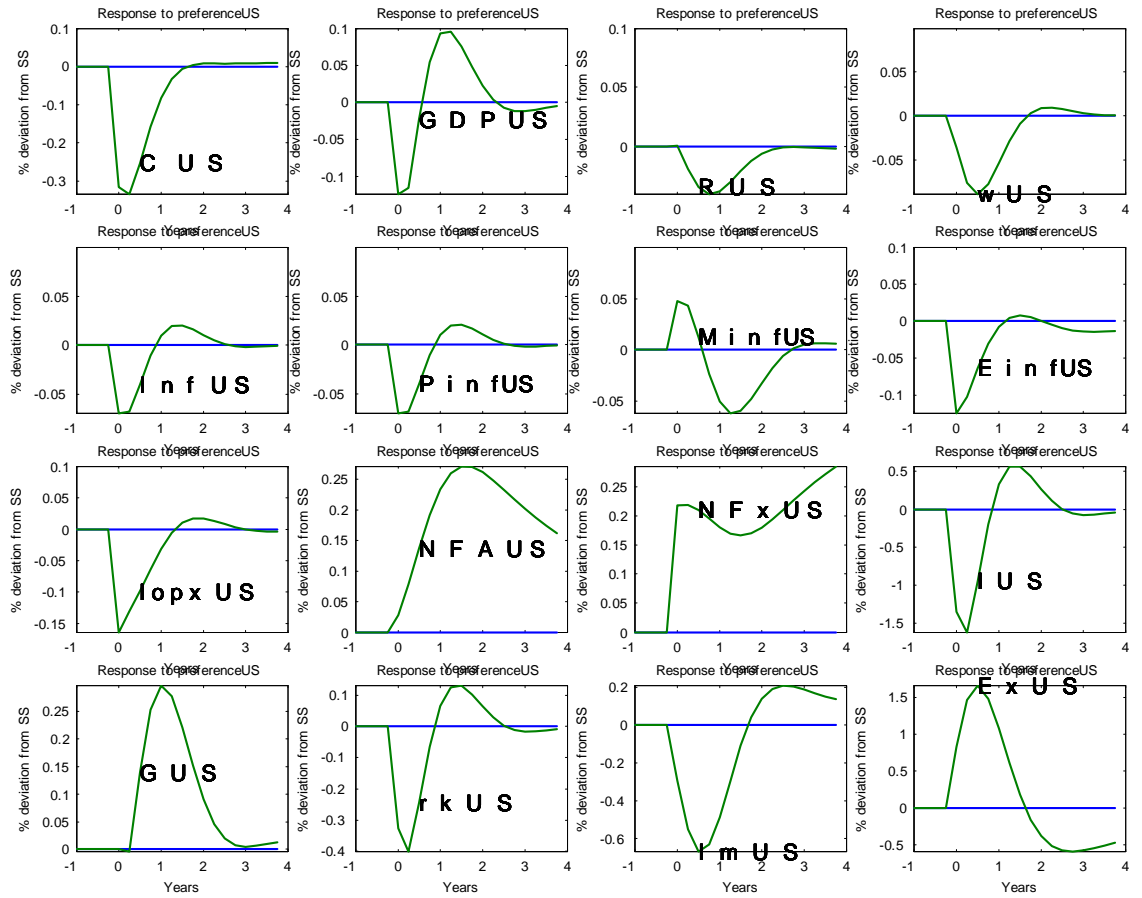


Figure 22: Preference shock, US, Open economy model



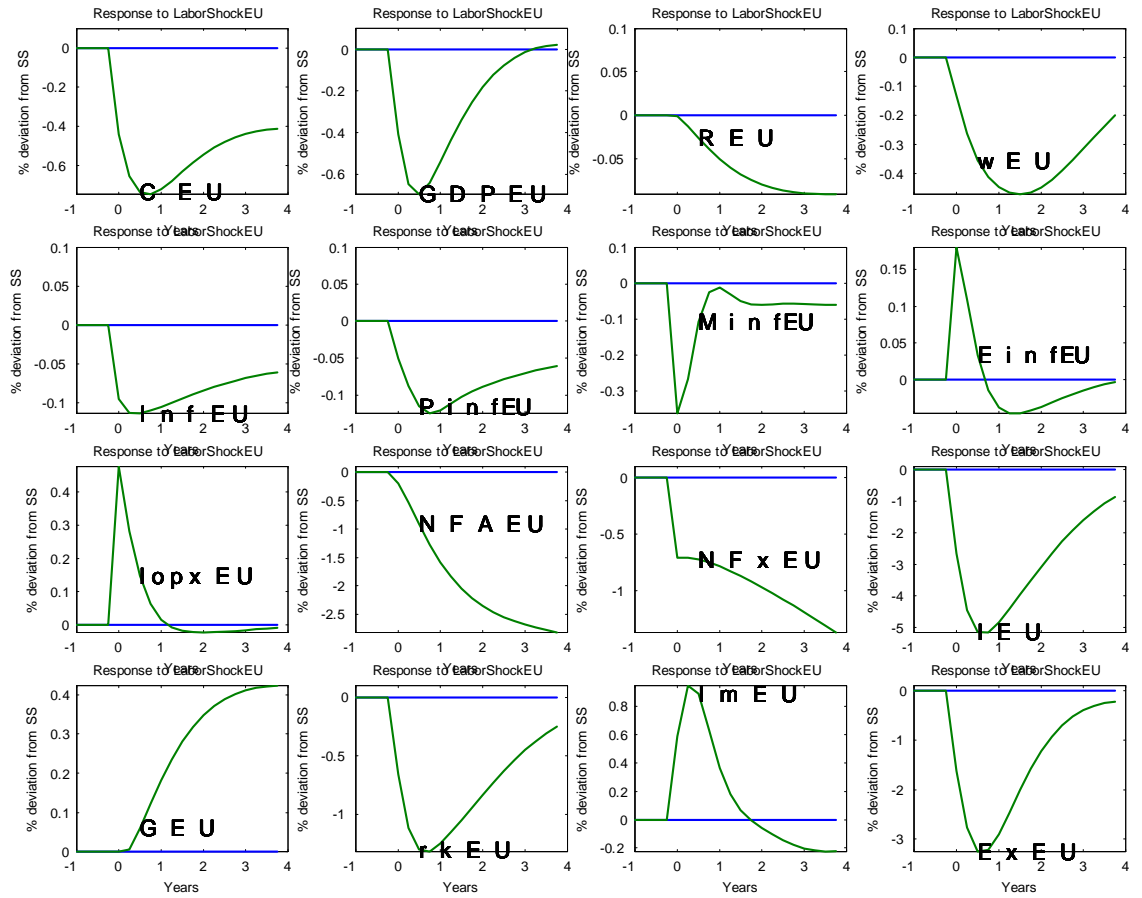


Figure 23: Labor supply shock, EU, Open economy model

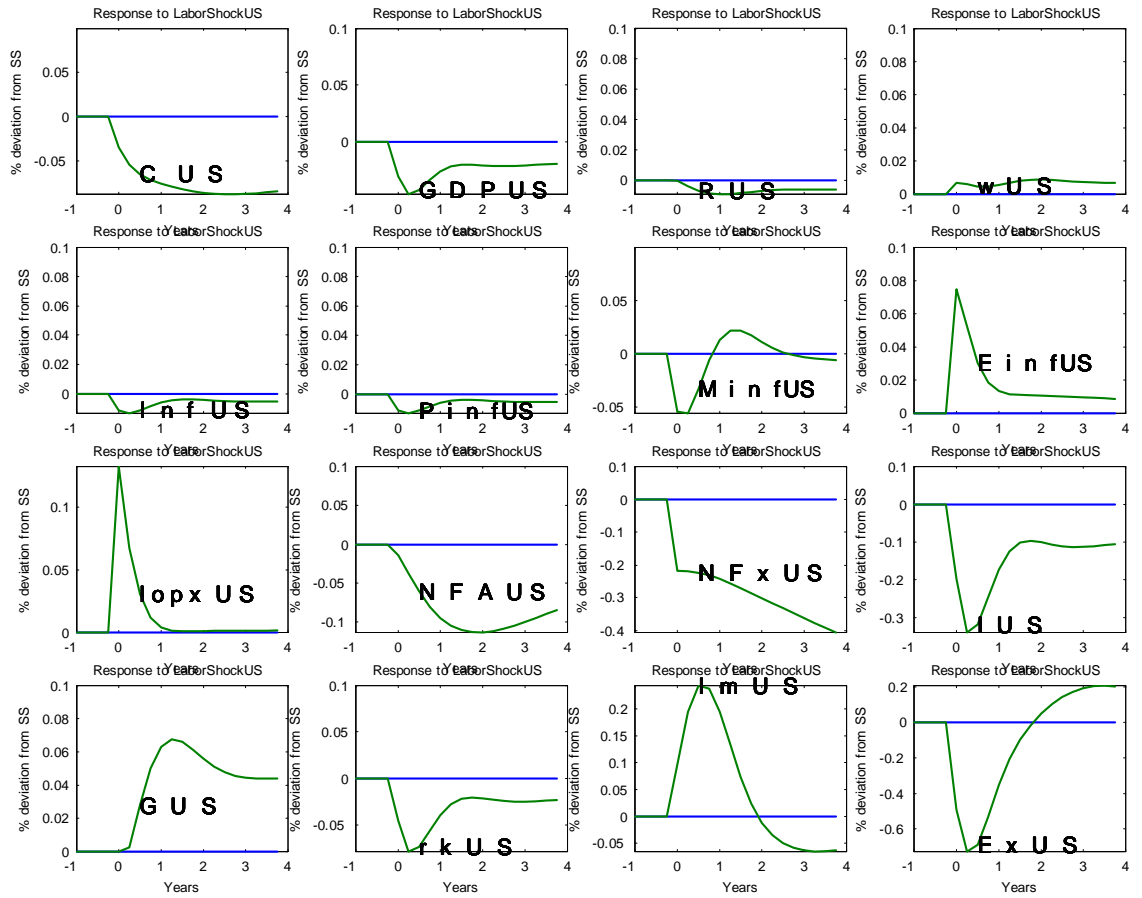


Figure 24: Labor supply shock, US, Open economy model

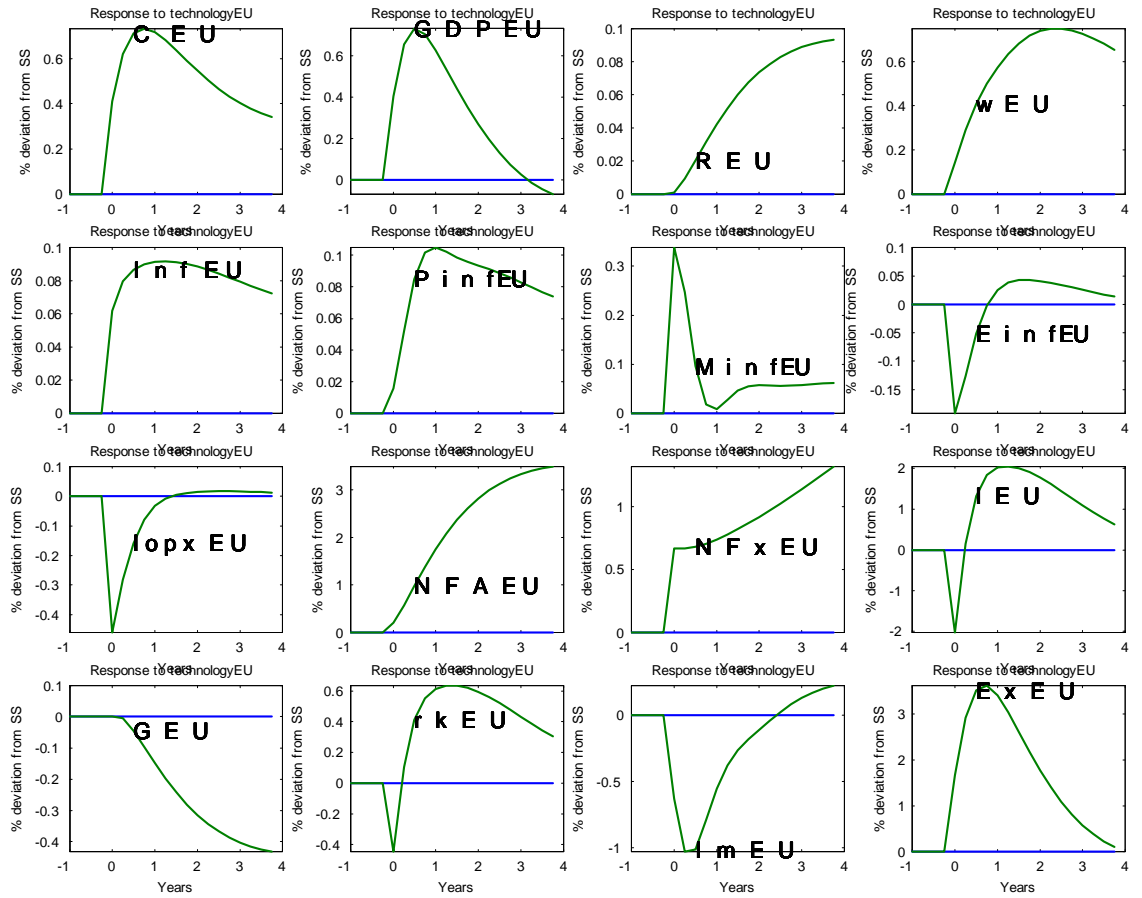


Figure 25: Technology shock, EU, Open economy model

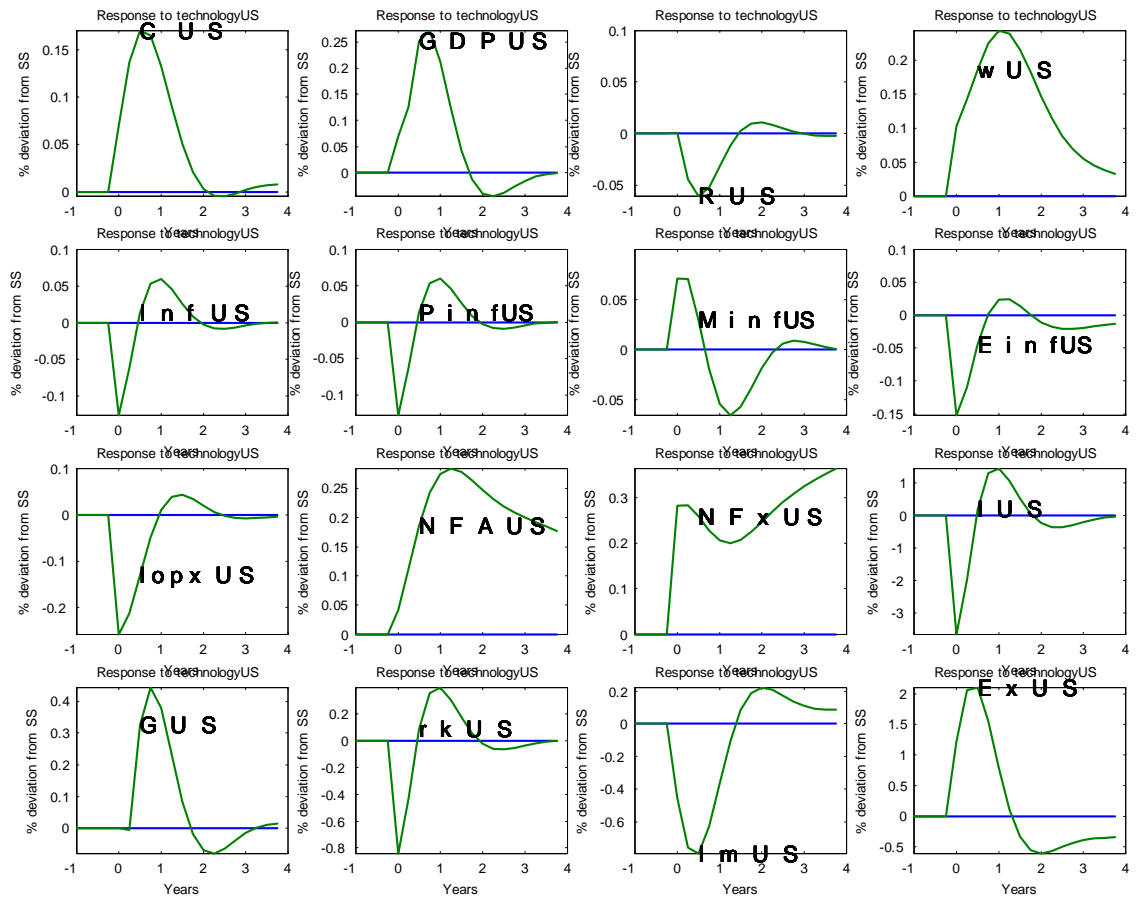
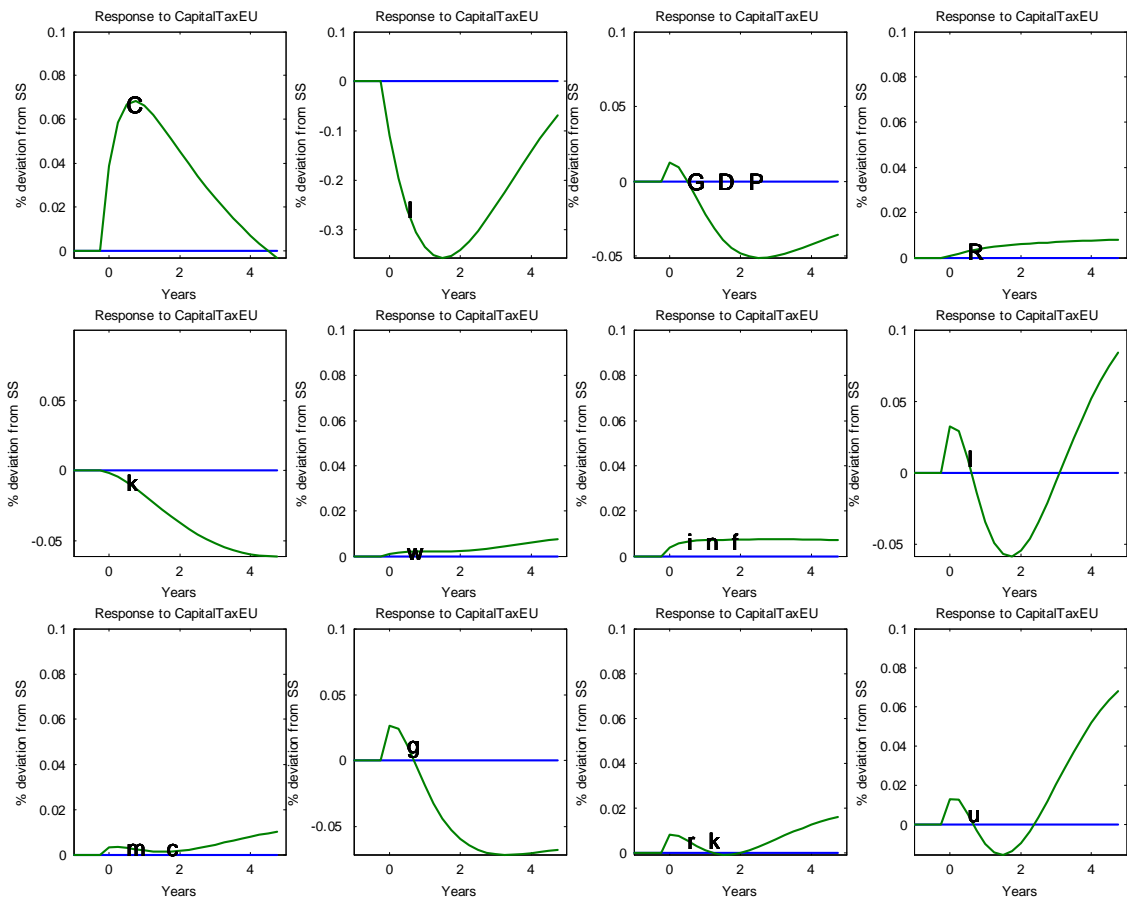
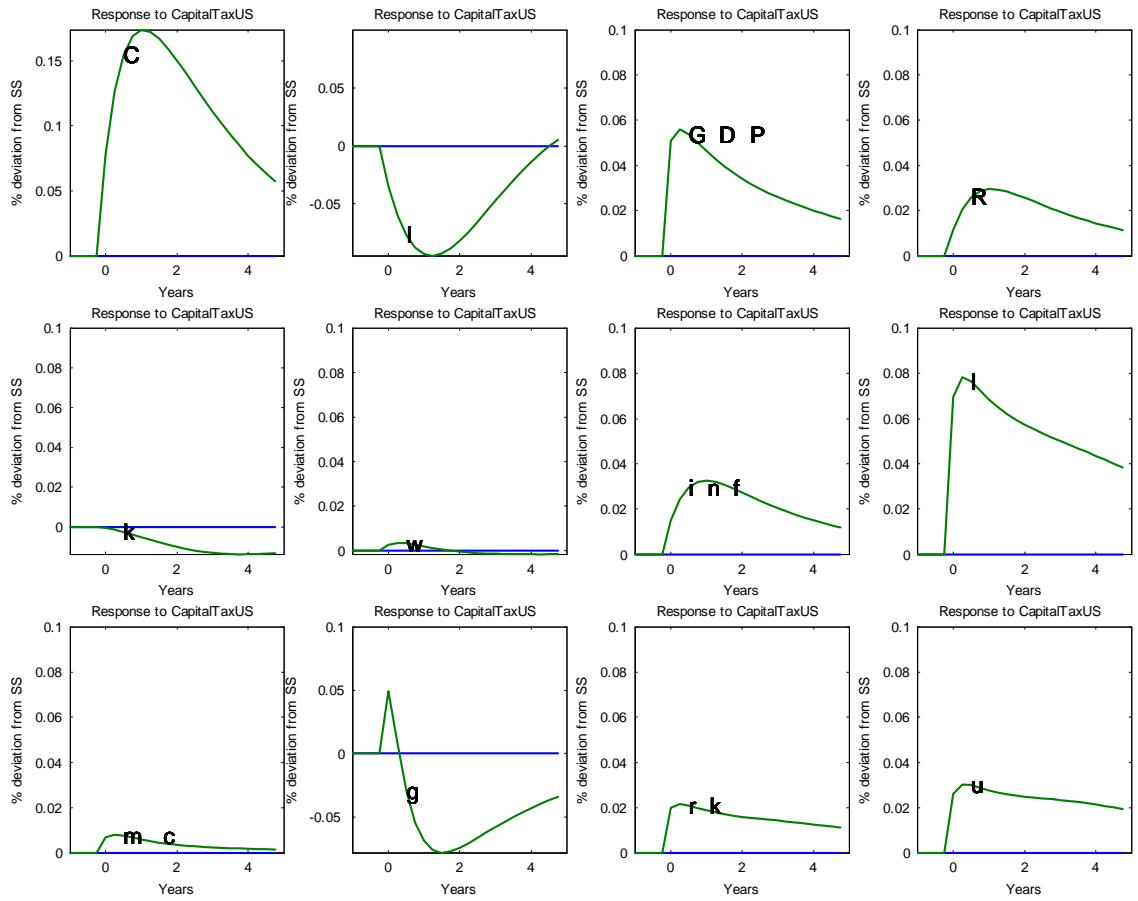


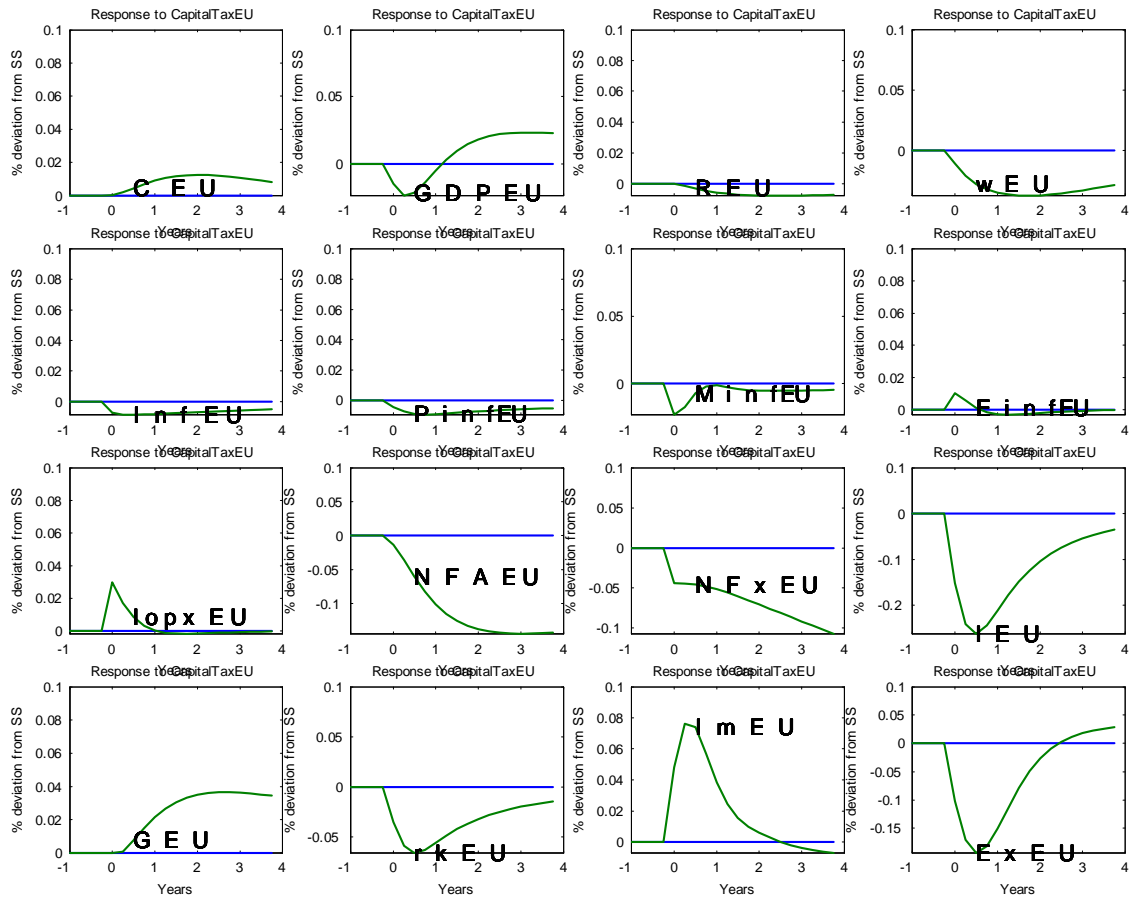
Figure 26: Technology shock, US, Open economy



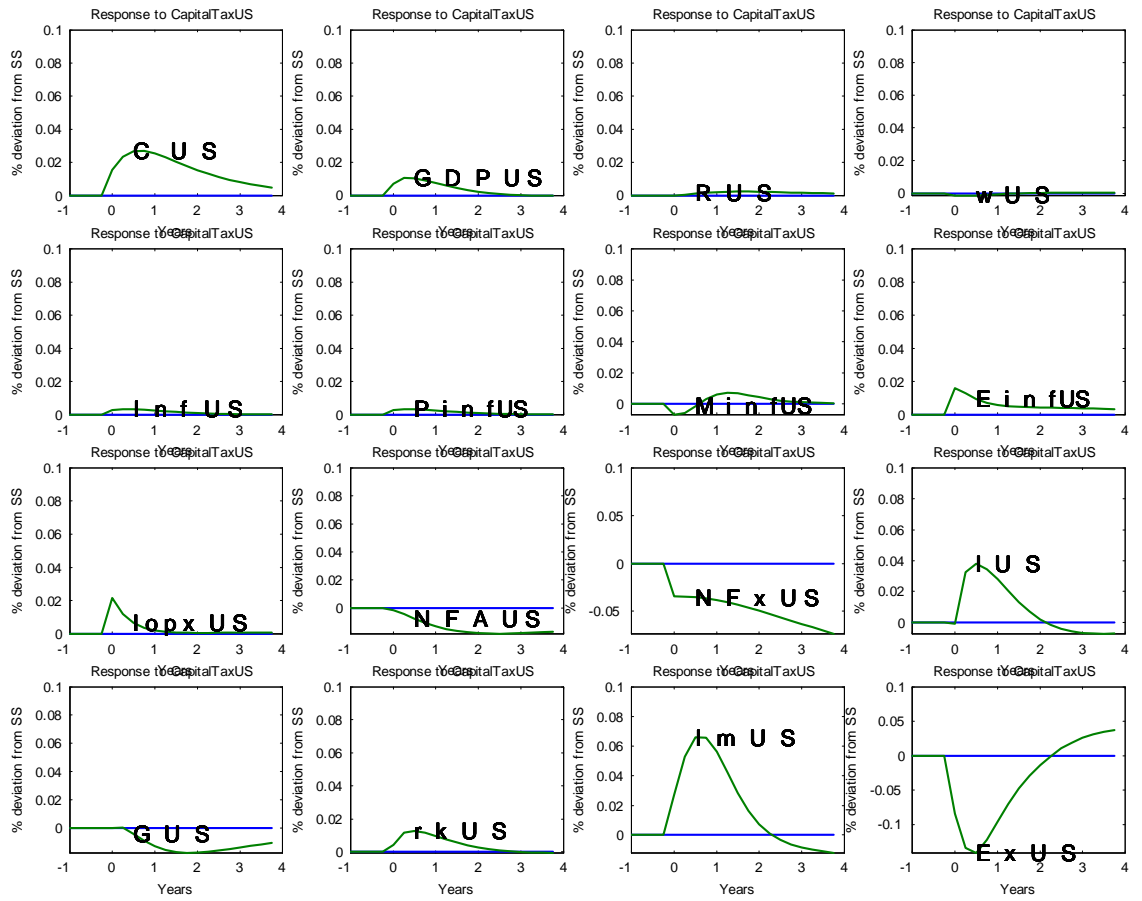
Capital income tax shock, EU, Closed economy model



Capital income tax shock, US, Closed economy model

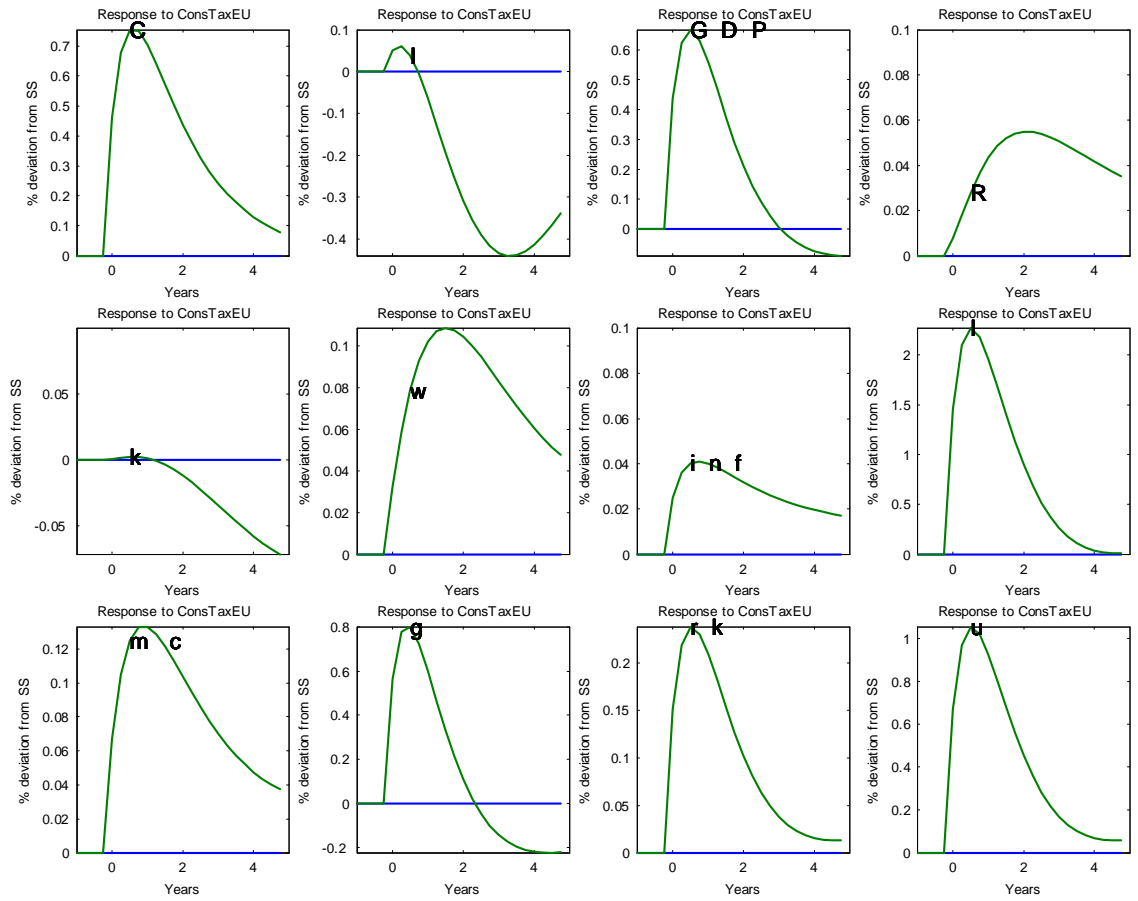


Capital income tax shock, EU, Open economy model

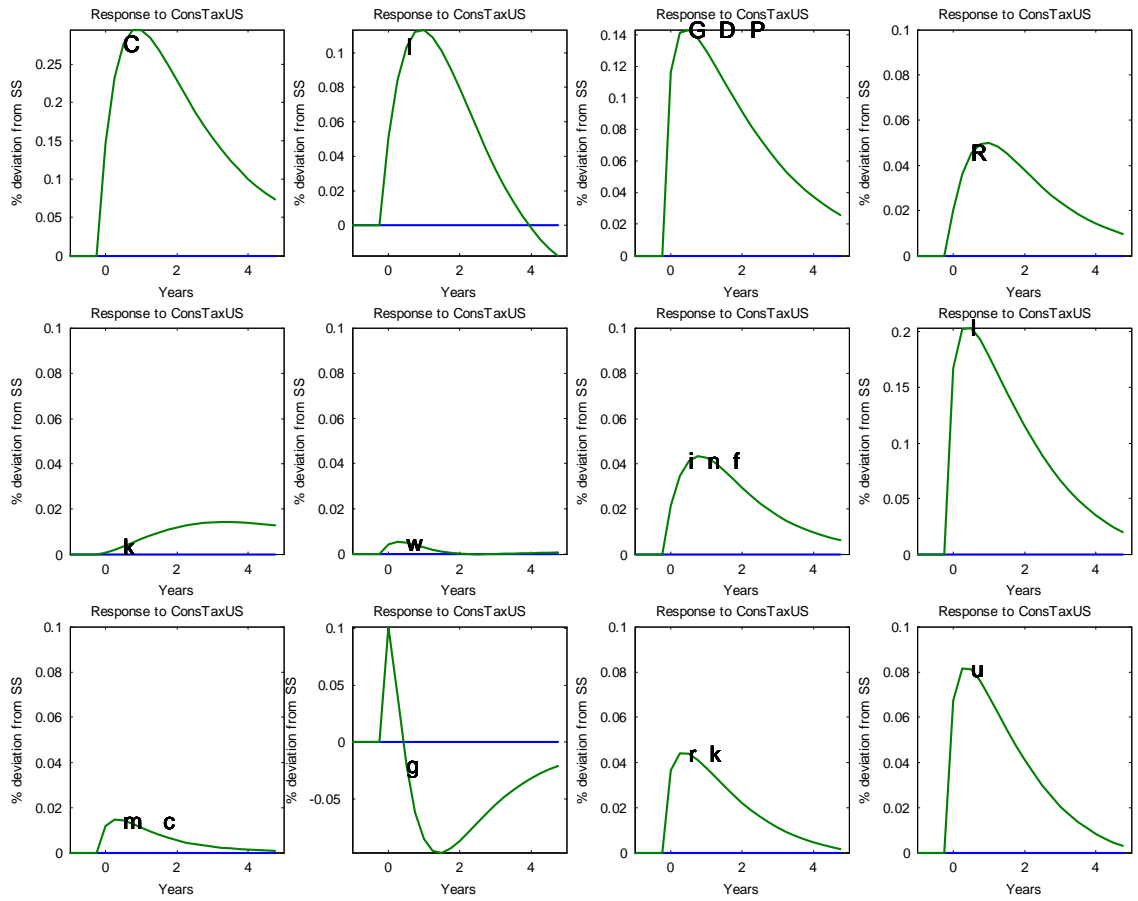


Capital income tax shock, US, Open economy model

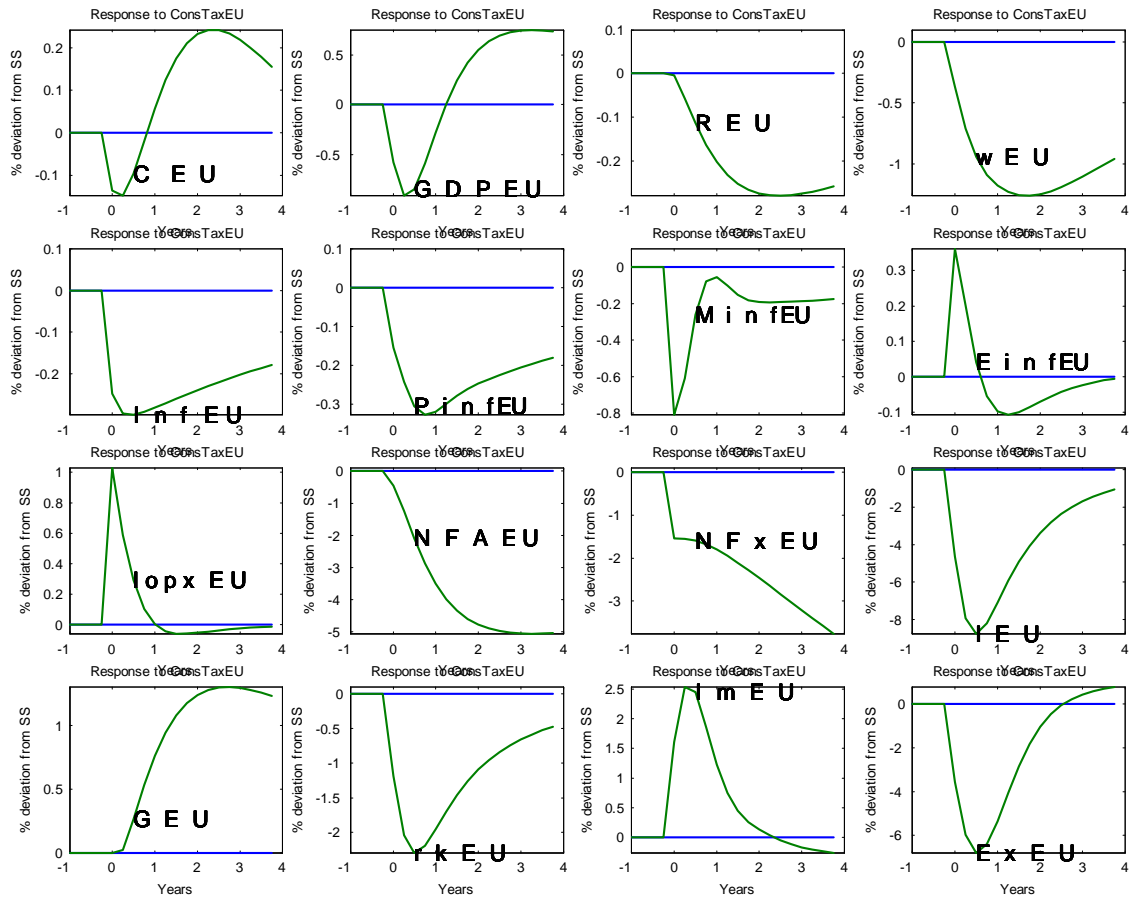




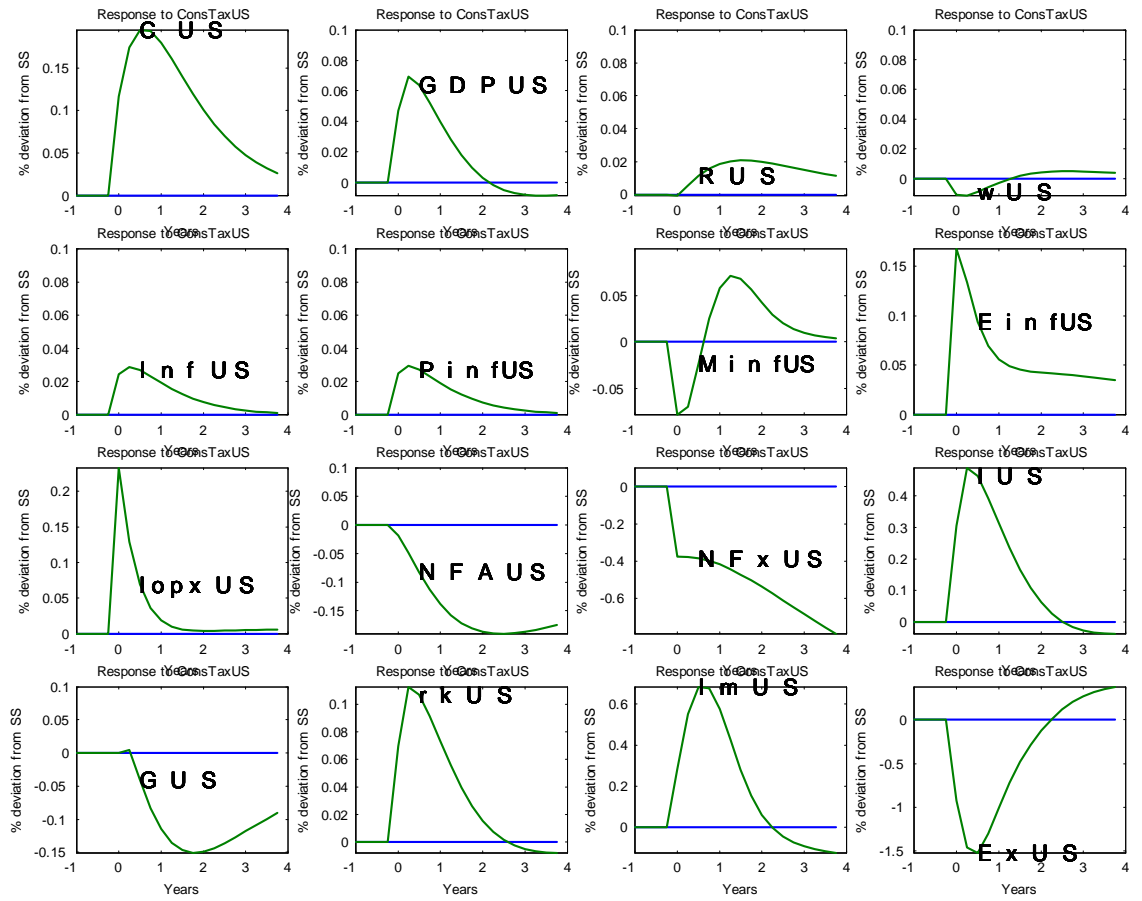
Consumption tax shock, EU, Closed economy model



Consumption tax shock, US, Closed economy model



Consumption tax shock, EU, Open economy model



Consumption tax shock, US, Open economy model